Orbit Determination and Prediction, and Computer Programs

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Orbit determination and prediction programs are needed to generate ephemerides for the satellite. Orbit determination is from tracking data consisting of angles only, and is based on a modified version of a method by R. E. Briggs and J. W. Slowey of the Smithsonian Institution. Trends in the data due to perturbations from a Keplerian orbit are removed before this process, and estimates of the orbital elements from individual passes are combined statistically to produce refined estimates. Ephemeris calculation is by a semi-analytic method in which deviations from a Keplerian orbit are obtained by integrating the perturbing forces. The programs to implement these procedures have been written for both the IBM 7090 and the IBM 1620 computers.

AUTHOR

I. INTRODUCTION

The following paper describes the methods and programs used in the Telstar project for the purposes of orbit determination and ephemeris calculation. The orbit determination process involves the computation of orbital elements from tracking data obtained during each pass, and subsequent refinement by combining such single-pass estimates. The tracking data are in terms of angular observations only. The ephemeris calculations involve standard procedures for computation of Keplerian orbits and perturbations due to the earth's oblateness.

It is well known that in the problem of orbit determination from angular data only, three observations (each observation consisting of two angles and a time) are not sufficient to determine an orbit if the three sightlines are coplanar. If the three sightlines are nearly coplanar, the computed orbital elements may reflect large uncertainties which are not necessarily due to observational errors. Hence, the method used is based on the determination of a set of orbital elements from four observations.

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This method is a modified form of the method described in Ref. 1. In the modified form, initial estimates of the orbital elements are computed from the first and last of the four observations, supplemented by estimates of the ranges corresponding to them (Section 2.1). By a method of successive approximations the two ranges are adjusted until an agreement with the second and third observations is secured in a least-squares sense (Section 2.2).

With typically many more than four observations in one pass through the visibility zone of an angular tracker, the observations are divided into four nonoverlapping blocks, each block containing the same number of observations, N. Taking one observation at a time from each block, in serial order, N sets of orbital elements are computed. These N sets are combined into a single set of intrapass average orbital elements and an associated covariance matrix (Section III).

Trends in the data due to perturbations induced by the earth's oblateness are removed by a method which is essentially the same as that described in Ref. 2 (Section IV).

Sets of intrapass average orbital elements, and their associated covariance matrices, from two or more passes, are combined into a set of interpass average orbital elements and an associated covariance matrix (Section V). The method used is similar to the method described in Refs. 3 and 4, inasmuch as it was motivated by the desire to avoid the necessity of pooling all of the observational data from two or more passes in order to derive refined estimates of the orbital elements, as would have to be done in the classical "differential corrections" method commonly ascribed to K. F. Gauss (1777–1855). However, the method used differs from the referenced method in two respects, viz., (a) the covariance matrix associated with each set of intrapass average orbital elements is related to the actual observational data for the pass, and (b) the necessity of computing the partial derivatives of all of the observed angles (numbering $8N$ in each pass) with respect to each of the orbital elements is avoided. On the other hand, this method gives single-pass estimates of the orbital elements which are biased even when the observational errors are not biased. These biases may be appreciable for short passes associated with low altitudes of the satellite near perigee. Methods for removing or reducing these biases have been under study but were not ready for use before the launching of the Telstar satellite on July 10, 1962.

This orbit determination method was designed to permit effective antenna pointing operations with the use of a modest computing facility. The program implementation (Section VI) consists of two major program subsystems. The first of these is the orbit determination program (Section VIII), which determines the characteristics of the satellite orbit from tracking information. The second program (Section IX) computes orbit predictions from a knowledge of these orbit characteristics. The operational results obtained in using these methods and programs are discussed (Section X).

II. ORBIT DETERMINATION FROM ANGLE-ONLY DATA

2.1 *Orbit Determination from Two Observations and Estimated Ranges*

Two observations (each observation consisting of two angles and a time) and estimates of the ranges (i.e., topocentric distances) along the two sightlines are sufficient to establish two points P_1 and P_2 through which an orbit can be passed at the times of observation by only one set of orbital elements. Denoting the geocentric distances by r_1 and r_2, and the geocentric angular difference by θ_{12}, we have

$$r_1 = \frac{a(1 - l^2 - m^2)}{1 + l}, \tag{1}$$

$$r_2 = \frac{a(1 - l^2 - m^2)}{1 + l \cos \theta_{12} + m \sin \theta_{12}}, \tag{2}$$

where

$$\begin{aligned} l &= e \cos \omega, \\ m &= e \sin \omega, \end{aligned} \tag{3}$$

a is the semi-major axis, e is the eccentricity, and ω is the argument of perigee referred to the first sightline. From (1) and (2), we have

$$\alpha_1 l + \alpha_2 m = \alpha_3 \tag{4}$$

where

$$\alpha_1 = \cos \theta_{12} - \frac{r_1}{r_2},$$

$$\alpha_2 = \sin \theta_{12},$$

$$\alpha_3 = \frac{r_1 - r_2}{r_2}.$$

It is convenient to regard either l or m as an independent variable. Actually, in order to avoid an indeterminacy and to improve accuracy, preference is given to l if $|\alpha_1| \leqq |\alpha_2|$, and to m if $|\alpha_1| > |\alpha_2|$. In either case, the other two of the three quantities l, m, and a are determined by (1) and (4). These, in turn, through Kepler's equation, determine a travel time between P_1 and P_2. A search is then made for the value of l which gives the observed travel time. For bounded orbits (the only ones of interest for ground-to-ground communications) the search is confined to the interval (l_-, l_+), where

$$l_\pm = \frac{1}{\alpha_1^2 + \alpha_2^2} (\alpha_1\alpha_3 \pm |\alpha_2| \sqrt{\alpha_1^2 + \alpha_2^2 - \alpha_3^2}).$$

With a, l, m (hence also e and ω) determined, the time of perigee passage, τ, is determined through Kepler's equation.

2.2 *Orbit Determination from Four Observations*

The two observations involved in the procedure described in the preceding section are the first and last of a set of four observations. Subsequent to that procedure, the four angles corresponding to the times of the second and third observations are computed and compared with the observed angles. The sum, Φ, of the squares of the differences between the computed and observed angles is regarded as a function of the estimated ranges D_1 and D_4 associated with the first and last of the four observations. The quantity Φ is next minimized with respect to D_1 and D_4 by a method which is analogous to the classical "differential corrections" method. (With only three observations corresponding to coplanar sightlines there would be only one angular difference, and therefore D_1 and D_3 would be indeterminate.) This method involves the solution of two simultaneous equations which are linear in the corrections to D_1 and D_4, with coefficients which are quadratic in the first-order partial derivatives of the computed angles with respect to D_1 and D_4. The terms which do not involve the corrections to D_1 and D_4 are products of the first-order partial derivatives and the angular differences. Since the partial derivatives are functions of D_1 and D_4, the minimization of Φ is an iterative procedure which is terminated when the values of a, e, ω, and τ are sufficiently stabilized. Detailed formulas are given in Ref. 5.

With a, e, ω, and τ determined, the orientation of the plane through P_1, P_2, and the center of the earth gives the values of Ω and i, where Ω is the longitude of the ascending node and i is the inclination of the orbital plane.

III. ORBIT DETERMINATION FROM $4N$ OBSERVATIONS IN ONE PASS

The combined procedures described in Sections 2.1 and 2.2 are applied to as many sets of four observations as may be drawn from all of the reliable observations for each pass in accordance with the method of selection indicated in the third paragraph of Section I. The N sets of orbital elements are then combined into a single set of intrapass average orbital elements. In addition, an associated covariance matrix (an estimate of the variability of the mean in a sample of size N drawn from a correlated multivariate population) is computed in accordance with the standard formulas

$$\tilde{C}_{\overline{\alpha\alpha}} = \frac{\sum_{i=1}^{N} (\alpha_i - \bar{\alpha})^2}{N(N-1)}, \qquad \tilde{C}_{\overline{\alpha\beta}} = \frac{\sum_{i=1}^{N} (\alpha_i - \bar{\alpha})(\beta_i - \bar{\beta})}{N(N-1)},$$

where α stands for each of the six orbital elements (with average $\bar{\alpha}$), and β for each of the other five (with average $\bar{\beta}$).

A typical result of the single-pass routine, as described up to this point, is shown in Tables I and II. The orbital elements listed as "exact value" were used to generate tracking angles. These angles, combined with random errors from a normal population with a standard deviation of 0.2 milliradian, were processed. It may be noted that were it not for the strong correlation between some of the orbital elements, errors in

TABLE I

	Ω, degrees	i, degrees	a, feet	e	ω, degrees	τ, seconds
Exact value	144.4462	46.9190	31,567,194	0.240764	171.6756	47,953.227
Sample mean	144.4455	46.9184	31,573,342	0.240688	171.6349	47,950.120
Standard deviation of sample mean	0.0018	0.0018	6,212	0.000120	0.0374	2.852

TABLE II — CORRELATION COEFFICIENTS

	Ω	i	a	e	ω	τ
Ω	1	0.13189	0.64082	−0.57898	−0.67162	−0.64998
i	0.13189	1	−0.49292	0.52030	0.44004	0.47260
a	0.64082	−0.49292	1	−0.96559	−0.99058	−0.99680
e	−0.57898	0.52030	−0.96559	1	0.92798	0.94376
ω	−0.67162	0.44004	−0.99058	0.92798	1	0.99797
τ	−0.64998	0.47260	−0.99680	0.94376	0.99797	1

the elements of the order of the standard deviations shown could result in pointing angles for the same pass with errors as large as 10 times the standard deviation of the original tracking errors.

A caveat should also be noted with respect to the precision of computation of the covariance matrix. Any matrix which purports to be a covariance matrix must have a nonnegative determinant. Due to the high correlation among the elements a, e, ω, and τ, however, values of 10^{-8} for the determinant of the correlation matrix are common. Errors of the order of 0.1 per cent in some covariances could result in a matrix with a negative determinant. Such a matrix can still serve as a guide in judging the reliability of the orbital elements obtained, but the use of this matrix for interpass orbit refinement would very likely lead to absurd results, such as negative variances.

IV. TREND REMOVAL

Since the procedures described in Sections 2.1 and 2.2 are based on the assumption that the orbit is Keplerian, it is important to determine the extent to which it is necessary and sufficient to correct for deviations from that assumption. Such deviations, usually called perturbations, are induced by the asphericity of the earth, drag, radiation pressure, etc. Preliminary computations, confirmed by tests with artificial data, indicated that for the orbit and satellite under consideration here it would be necessary and sufficient to correct only for the earth's oblateness. The corrections are made to the observational data. Detailed formulas for the corrections are given in Ref. 5. These formulas involve the orbital elements which, however, do not need to be known to high accuracy for the purposes of trend removal. If sufficiently accurate values of the orbital elements are not available for trend removal, they may be obtained by including trend removal in the iterative routine of Sections 2.1 and 2.2 after the first values of the orbital elements have been obtained without trend removal.

Table III shows the importance of trend removal for the effects of oblateness. The same input data, which included the effects of oblateness, were used in both runs. The errors in the second run (without trend removal) are not acceptable. In particular, the error in the semi-major axis could lead to an error in predicted pointing angles of as much as 1.5° after only one period.

Table IV shows the speed of convergence, with trend removal, in the absence of initial estimates of the orbital elements. After only one iteration (one-half minute additional computing time for 200 observa-

TABLE III

	Ω, degrees	i, degrees	a, feet	e	ω, degrees	τ, seconds
Exact osculating elements at the center of the pass	144.4439	46.9170	31,566,742	0.240879	171.6124	47,950.421
Results of run no. 1 (with trend removal)	144.4439	46.9169	31,566,884	0.240875	171.6118	47,950.365
Results of run no. 2 (without trend removal)	144.4372	46.9152	31,542,821	0.241313	171.7500	47,961.173

TABLE IV

	Ω, degrees	i, degrees	a, feet	e	ω, degrees	τ, seconds
Exact osculating elements at the center of the pass	144.4439	46.9170	31,566,742	0.240879	171.6124	47,950.421
Results of run no. 1	144.4377	46.9149	31,543,423	0.241311	171.7487	47,960.978
Results of run no. 2	144.4439	46.9169	31,566,909	0.240874	171.6117	47,950.352
Results of run no. 3	144.4439	46.9169	31,566,886	0.240875	171.6118	47,950.362

tions, on an IBM-7090 computer), acceptable orbital elements were obtained.

V. COMBINATION OF SINGLE-PASS ORBITAL ELEMENTS

The method of combining single-pass estimates of the orbital elements is based on a matrix formula derived briefly as follows. Let $\bar{x}$ be a vector (i.e. a one-column matrix) estimate of the vector z, with ave $\{\bar{x} - z\} = 0$ and cov $\{\bar{x} - z\} = A$, where A is a covariance matrix. Similarly, let $\bar{y}$ be another estimate of z, with ave $\{\bar{y} - z\} = 0$ and cov $\{\bar{y} - z\} = B$. If $\bar{x}$ and $\bar{y}$ obey independent multivariate normal probability distributions, the "maximum likelihood" estimate of z is the $\bar{z}$ which minimizes the quadratic form

$$\bar{Q} = (\bar{z} - \bar{x})' \cdot A^{-1} \cdot (\bar{z} - \bar{x}) + (\bar{z} - \bar{y})' \cdot B^{-1} \cdot (\bar{z} - \bar{y})$$

where the primes denote transposition. Thus,

$$A^{-1} \cdot (\bar{z} - \bar{x}) + B^{-1} \cdot (\bar{z} - \bar{y}) = 0,$$

whence,

$$\bar{z} = (A^{-1} + B^{-1})^{-1} \cdot (A^{-1}\bar{x} + B^{-1}\bar{y}) \tag{5}$$

with covariance matrix

$$C = (A^{-1} + B^{-1})^{-1}. \tag{6}$$

In fact, it may be easily verified that

$$\bar{Q} = [\bar{z} - C\cdot(A^{-1}\bar{x} + B^{-1}\bar{y})]'\cdot C^{-1}\cdot[\bar{z} - C\cdot(A^{-1}\bar{x} + B^{-1}\bar{y})]$$
$$+ \text{ terms independent of } \bar{z}.$$

A somewhat longer derivation without the normality assumption, in which the main diagonal (variance) elements of C are minimized, leads to the same results.

Formulas (5) and (6) require three matrix inversions which result in an intolerable loss of accuracy in cases of highly correlated estimates of the orbital elements. This difficulty is relieved to a very large extent by using the equivalent formulas

$$\bar{z} = w_1\bar{x} + w_2\bar{y} - (w_1P - w_2Q)(P + Q)^{-1}(\bar{x} - \bar{y}), \tag{7}$$

$$C = \tfrac{1}{2}[w_1A + w_2B - (w_1P - w_2Q)(P + Q)^{-1}(A - B)] \tag{8}$$

where $P = AG$, $Q = BG$, G is an arbitrary six-by-six matrix, and w_1, w_2 are any two six by six matrices whose sum is a unity matrix (see Appendix A). Formulas (7) and (8) require only one matrix inversion. The matrix G can be constructed so that the matrix $(P + Q)$ is well suited for inversion.

As a matter of additional necessity, formulas (7) and (8) were further transformed by the introduction of matrices U, V, defined by $U = SAS$, $V = SBS$, where S is a diagonal matrix whose elements are

$$S_{ii} = (A_{ii} + B_{ii})^{-\frac{1}{2}},$$

so that the diagonal elements of the matrix $(U + V)$ are unity. Restricting w_1, w_2 to diagonal matrices, then,

$$\bar{z} = w_1\bar{x} + w_2\bar{y} - R(\bar{x} - \bar{y}), \tag{9}$$

$$C = \tfrac{1}{2}[w_1A + w_2B - R(A - B)], \tag{10}$$

where

$$R = S^{-1}(w_1\hat{P} - w_2\hat{Q})(\hat{P} + \hat{Q})^{-1}S, \tag{11}$$

$\hat{P} = UH$, $\hat{Q} = VH$, and $H = S^{-1}G$. The formal construction of the arbitrary matrix H is not necessary. The matrices $\hat{P}$, $\hat{Q}$, and $\hat{P} + \hat{Q}$ are obtained by linear combinations of rows and/or of columns of the matrices U, V, and $U + V$ according to rules which are easily programmed for a digital computer.

Two details must be noted in the use of these formulas for combining sets of orbital elements. The first detail is that the orbital elements are actually "osculatory" orbital elements which vary with time; therefore, each set is necessarily referred to a specific "epoch." Hence, before combining two sets, the set referred to the earlier epoch must be "updated" to the later epoch. In updating a set of orbital elements, it must also be noted that the "time of perigee passage" is actually the "time of mth perigee passage," where m is a specific number, usually different from the one for the set referred to the later epoch. The second detail to be noted is that the covariance matrix for the set referred to the earlier epoch must also be updated.

If $\tilde{C}_1$ is the covariance matrix to be updated, the updated covariance matrix is given by the formula

$$C_1 = J\tilde{C}_1J',$$

where J is the Jacobian of the updated orbital elements with respect to the orbital elements from which they were predicted. Even in the hypothetical case of Keplerian orbits, in which all of the orbital elements, with the possible exception of τ, are constants, the Jacobian may differ from a unity matrix. For example, if the updating is through m times the period $2\pi\sqrt{\tilde{a}_1^3/k}$, so that

$$\tau_1 = \tilde{\tau}_1 + 2\pi m\sqrt{\tilde{a}_1^3/k},$$

then,

$$\partial\tau_1/\partial\tilde{a}_1 = 3\pi m\sqrt{\tilde{a}_1/k}.$$

The results of a test problem of this hypothetical sort are shown in Table V, in which the updating was through one period. The standard deviation of the improved estimate of the semi-major axis is approximately $\frac{1}{150}$ times the average of the corresponding standard deviations for the two runs. The improved estimate is in error by only 52 feet.

Table VI shows the results of a more realistic test problem in which the input data included perturbations due to the earth's oblateness. With "no updating" of the orbital elements and the covariance matrix from the earlier pass, except only to the extent required in the hypothetical case of Keplerian orbits, the "improved" semi-major axis is in error by 5094 feet, which is inconsistent with the standard deviation of only 73 feet. However, with updating of the orbital elements, taking account of the effects of the earth's oblateness, the error is only 72 feet.

VI. PROGRAM DESCRIPTION

The computer program system required to track a satellite and generate steering information for the communications antenna is divided into

Table V

		Ω, degrees	i, degrees	a, feet	e	ω, degrees	τ, seconds
Exact Values		144.5662	47.5190	32,107,194	0.253364	175.2756	49,218.920 or 58,853.603
Pass No. 1	Single-pass estimates	144.5638	47.5181	32,102,870	0.253387	175.3081	49,221.250
	Standard deviations	0.0017	0.0014	5,884	0.000124	0.0318	2.609
Pass No. 2	Single-pass estimates	144.5688	47.5196	32,116,970	0.253228	175.2114	58,848.774
	Standard deviations	0.0017	0.0006	8,683	0.000100	0.0543	4.325
Combination of passes No. 1 and No. 2	Combined estimates	144.5667	47.5193	32,107,246	0.253340	175.2741	58,853.608
	Standard deviations	0.0010	0.0005	46	0.000023	0.0026	0.128

Table VI

		Ω, degrees	i, degrees	a, feet	e	ω, degrees	τ, seconds
Exact values at t = 54,046 sec.		144.4439	46.9170	31,566,742	0.240879	171.6124	47,950.422
Exact values at t = 64,081 sec.		144.2383	46.9111	31,562,512	0.241047	171.8311	57,341.533
Pass No. 1 (center of the pass ≈ 54,046 sec.)	Single-pass estimates (t = 54,046 sec.)	144.4461	46.9183	31,562,047	0.241027	171.6318	47,952.202
	Standard deviations	0.0038	0.0024	11,077	0.000227	0.0638	4.936
Pass No. 2 (center of the pass ≈ 64,081 sec.)	Single-pass estimates (t = 64,081 sec.)	144.2410	46.9111	31,557,704	0.241211	171.8561	57,343.464
	Standard deviations	0.0025	0.0014	13,931	0.000176	0.0932	6.946
Combination of the passes No. 1 and No. 2; no updating	Combined estimates	144.3523	46.9400	31,557,418	0.240653	171.7592	57,344.864
	Standard deviations	0.0018	0.0010	73	0.000030	0.0044	0.186
Combination of the passes No. 1 and No. 2; updating included	Combined estimates (t = 64,081 sec.)	144.2412	46.9110	31,562,440	0.241105	171.8262	57,341.304
	Standard deviations	0.0018	0.0010	73	0.000030	0.0044	0.186

two major subsystems. The first of these is the orbit determination program, TELETRACK, which determines the characteristics of the satellite orbit from tracking information. The second major program, TELEPATH, computes orbit predictions from a knowledge of these orbital characteristics.

The division of the program system into these two parts is not only natural, but is also dictated by systems considerations. One of the requirements on the system was to minimize the amount of data transmissions. Ephemeris data to steer the communications antenna can be generated from the six orbital elements, and a division of the program system into two components linked together only by these six numbers achieves this requirement if each ground station is provided with suitable computational facilities. Stations having communications antennas require the program TELEPATH and updated sets of the orbital parameters. Stations having tracking antennas process the tracking data with TELETRACK and broadcast the updated elements to other stations as they become available.

The IBM 1620 computer was chosen to provide on-site computations. The IBM 7090 computer was used, however, for the initial development of the program systems. This was done for two reasons. First of all it was desirable to take advantage of the more powerful facilities and speed of the larger computer to facilitate the development and testing of the methods employed in the program system. Secondly, it was desirable to have the complete program system available at the Whippany, N. J., location of Bell Telephone Laboratories as a back-up to the on-site computer centers. Experience has shown that it is absolutely essential to have these duplicate programs available for testing and checking of the on-site operations.

By the nature of the 7090 and 1620 computers, different operating philosophies are required for each. The speed of the 7090 and turnaround times inherent in a large computation center are such that the programs must be as automatic as possible. However, they must also be flexible enough to allow selected programs from the system to be performed when necessary. Towards this end the following system evolved. The entire set of 7090 programs can be run consecutively as a single automatic chain job. Each program communicates to the following program through a magnetic tape, but as far as the computation center is concerned each program is a separate job. As a consequence, each program can also be run independently (with input provided by cards) since it is an entity in itself. The hidden gain in this system is the fact that there is only the one flexible version of each program, thus eliminating confusion and mistakes.

For the 1620, which is devoted entirely to this problem, and which is a slower machine, such completely automatic operation is not necessary. The system can be run automatically, but is usually run with more direct operator intervention. This allows greater flexibility and the ability to monitor intermediate results. On the 1620 the two major program systems are broken down conveniently into several program components. Each of these programs runs independently of the others, receiving input data generated by one of them and preparing output data for another. Operation of the program systems is achieved by loading and running one of the program components at a time. The various program components are stored on magnetic tape, and each program in the system loads the next program into the computer from this tape. Transfer of data between the programs is accomplished by punched cards, magnetic tapes and common memory storage. The method of data transfer in a particular instance depends upon the nature and quantity of the data.

Numerous error conditions were anticipated while the programs were being written. Many of these are handled automatically by the programs themselves. Some must be taken care of by manual intervention.

VII. INERTIAL COORDINATES AND ORBITAL ELEMENTS

All orbital calculations must, of course, be referenced to an inertial (or near inertial) coordinate system. The basic system used in these programs is the usual earth-centered, right-handed rectangular system. The X-Y plane coincides with the earth's equatorial plane, the X-axis is parallel with the line of equinoxes, and the Z-axis passes through the North pole. The orientation of the earth in this system at the time of an observation is obtained from UT_2 at time of observation and the Greenwich Mean Sidereal Time at 0 hours UT of date. Conversion from Mean Sidereal Time to Apparent Sidereal Time is made using the Equation of Equinoxes at 0 hours UT of date; interpolation of this number to the time of observation was deemed unnecessary.

The satellite orbit is described by means of the osculating orbital elements, consisting of

(a) semi-major axis
(b) eccentricity
(c) right ascension of ascending mode
(d) inclination angle
(e) argument of perigee, and
(f) time of perigee passage.

These elements specify the ellipse osculatory to the satellite orbit at some instant in time. These six numbers are therefore accompanied by an epoch specifying the time of osculation. The time of perigee passage specifies the perigee passage immediately preceding the epoch and is stated in seconds relative to the epoch.

The following paragraphs describe in some detail the two program systems, TELETRACK and TELEPATH.

VIII. TELETRACK PROGRAM SYSTEM

The TELETRACK program system processes tracking data in terms of azimuth and elevation to produce estimates of the six orbital elements describing the satellite orbit. It processes tracking data from one pass over the tracking station at a time to produce a "single-pass estimate." Single-pass estimates are combined to provide "combined estimates." The combining of several single-pass estimates provides a statistical averaging of the several independent estimates and a refinement based on the separation in time of the various independent estimates.

A flow chart of TELETRACK is shown in Fig. 1. Each of the major program components and the modes of data transfer between them are shown. A few of the program switches which control the mode of operation of the system are also shown.

8.1 *TELED*

TELED is the input/edit section of TELETRACK. Inputs to this program are

(a) tracking data consisting of time, azimuth and elevation for one pass, and

(b) data cards containing date and number of pass, identification of the tracking station and satellite, meteorological conditions during the pass, GMST at 0^h of date, estimates of the orbital elements, number of data sets to be selected (N), and values of the mode control switches for TELETRACK.

TELED reads the tracking data from tape and performs format and units conversion. Data points for which the precision tracker was not in autotrack or for which the signal-to-noise ratio level was not above a predetermined level (usually 4 or 5 db) are rejected. Furthermore, data points for which the elevation is below 7.5° or above 82.5° are rejected. The specified number ($4N$) of data points is selected from the group satisfying these criteria. The set of data so selected is distributed as uniformly as possible over the available set.

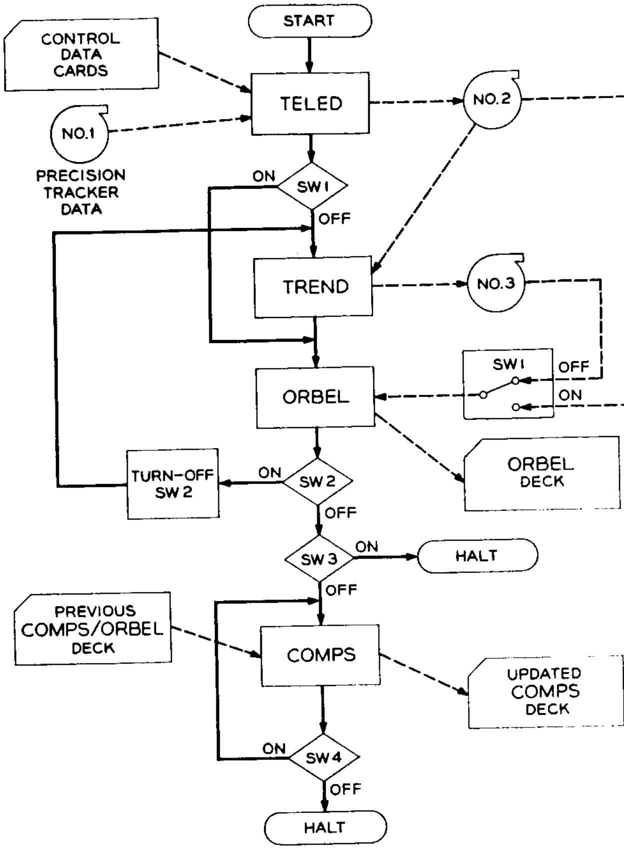

Fig. 1 — TELETRACK flow chart.

Boresight and refraction corrections are then applied to the selected data points. Following this, coordinate conversions are performed to transform the data from the topocentric azimuth-elevation system to the inertial coordinate system. Deviations of the vertical from the normal to the geodetic spheroid are accounted for in this process. Since range data are not available, the results of the coordinate conversion are in terms of the direction cosines relative to the inertial system of the observed sight lines. Also computed are the coordinates in inertial space of the tracker at the time of each observation.

Outputs from TELED are stored on magnetic tape for subsequent programs. The primary output consists of $4N$ data points. Each data point contains time of observation, the three direction cosines of the observed sight line, sine and cosine of the right ascension of the tracker at the time of the observation, and the inertial coordinates of the tracker at the time of the observation. Various other data are also stored on this tape for subsequent programs.

8.2 *TREND*

The direction cosines produced by TELED are adjusted by TREND to produce the set which would have been obtained had the satellite been moving in an unperturbed, elliptical orbit throughout the pass. These adjustments are described in detail in Section IV above. The time of osculation (t_c) between the perturbed and unperturbed orbits was selected by TELED to correspond to the center of the pass and is passed on to TREND via tape.

As noted above, an estimate of the orbital elements at time t_c is needed. These are obtained by updating to time t_c the elements supplied on the input cards to TELED. Program ORBFIX is used for this purpose, details of which are given below.

The output from TREND consists primarily of the adjusted direction cosines for each observation. These are stored on a tape which is identical in format with the TELED output tape. By making these formats identical it is possible under one of the modes of operation to bypass TREND if estimates of the orbital parameters are not available.

8.3 *ORBFIX*

As mentioned above, ORBFIX updates a set of osculating orbital elements valid at one epoch to another epoch. The program essentially makes use of the subroutine OBLATE with only minor additional bookkeeping operations. The subroutine OBLATE is a numerical integration routine in true anomaly which integrates in steps of 0.08 radian the first-order oblateness perturbation equations to provide the desired corrections. The equations also include sufficient second-order terms to allow taking steps of 2π, so that in actual use steps of 2π are taken until a value π or closer to the desired point is reached. The program then integrates either forward or backward in small steps to reach the desired point exactly. It is also possible to go only in 2π steps in cases where only limited accuracy is required. This results in a large time saving.

8.4 *ORBEL*

Calculation of the orbital parameters is performed by the program ORBEL. The input data normally consist of the adjusted direction cosines from TREND. In the absence of initial estimates of the orbital elements, however, ORBEL can process the unadjusted data from TELED. The $4N$ observations are divided into four nonoverlapping groups. A set of four observations is obtained by selecting one observation from each group. N independent estimates of the orbital elements are calculated from the resultant N sets of observations. Averages, variances and covariances of the six elements for one pass are calculated from these. Details on the methods are given in Sections II and III.

The estimates of the ranges required in producing the first set of elements are normally produced by TREND during the trend removal procedure. In the absence of trend removal these estimates must be supplied on the input data cards to TELED. Subsequent estimates of range are derived by ORBEL itself from its previous estimates of the elements.

The output from ORBEL consists of a set of cards (an "ORBEL deck") containing the single-pass estimates of the orbital elements, the standard deviations of those elements, and the correlation coefficient matrix. Pass number and the corresponding epoch are also stored on these cards. These cards are filed away for possible future use.

The information on these cards is also retained in memory for use by the combination of passes program, COMPS.

8.5 *COMPS*

Combination of the estimates from the various passes is accomplished by the program COMPS. The method employed is described in Section V above. The inputs consist of two sets of orbital elements, standard deviations and correlation coefficient matrices. The first set, obtained from input cards, is either from a single ORBEL run or from an earlier COMPS run. The second set is from the current ORBEL run and is usually supplied directly by ORBEL through common memory storage. Under some modes of operation, however, the second set is supplied by cards.

The output from COMPS is a set of cards (the "COMPS deck") identical in content and format with the ORBEL deck. These cards are filed to maintain a permanent record of the combined orbital elements. The output data also replace the data from the first input set in memory in case certain operating modes are selected.

8.6 *Modes of Operation*

Several mode-control switches are provided to permit selection of one of a number of possible operating modes. The more significant of these switches are shown in Fig. 1. Each switch is identified by a number and consists of a one-bit variable which is read from an input data card and stored in memory.

In the normal mode of operation it is assumed that some estimate of the orbital elements is available for trend removal and that a combination of the ORBEL output with an earlier COMPS output is wanted. All switches are set in the "off" condition and the sequence of operations is TELED, TREND, ORBEL and COMPS in that order. The first set of inputs to COMPS is determined by the operator, who selects the proper COMPS deck, and the second set is supplied directly by ORBEL.

An alternative mode of operation is to stop the program after the single-pass estimate is produced by ORBEL and then combine a number of such estimates in a "batch combination" at a later time. This is accomplished by turning switches 3 and 4 on and accumulating a number of ORBEL decks. These decks are fed to COMPS in order by time, with the earliest deck first. COMPS reads the first two decks and combines them, producing a combined estimate valid at the time of the second set. This in turn is combined with the third set to produce a combination of the first three decks valid at the time of the third. This process continues until all decks have been combined into a single estimate valid at the time of the last set.

Another mode of operation is available in case estimates of the orbital elements are poor or unavailable. By turning switch 1 on, trend removal is skipped initially, and ORBEL is given unadjusted data with which to estimate the elements. If switch 2 is also on, ORBEL will call on TREND after computing this initial estimate of the orbital elements. This estimate is passed on to TREND for use in adjusting the data. Switch 2 is turned off, the data are adjusted, and then ORBEL is called upon a second time, this time to process data with trend due to perturbations removed.

IX. TELEPATH PROGRAM SYSTEM

The ephemeris generation for the Telstar satellite is carried out by a trio of programs collectively known as the TELEPATH program. The three individual programs are called MUVIS, COKE, and ACEXP, and are complete entities in themselves, solving distinctly separate portions of the problem. The MUVIS program is solely concerned with finding times of future visibility or mutual visibility and updating the orbital elements to these time periods. Its output is a listing of future passes which is in itself useful, and a set of cards which serves as input to the COKE program. The COKE program generates a theoretical ephemeris for each pass as determined by the input cards, and outputs it on tape. The COKE program can also be used by itself to re-create any pass for which orbital elements are available. Both programs exist in almost identical form both for the IBM 7090 and the IBM 1620. The only differences in the programs are due to storage limitations in the 1620. This results in some extra tape manipulations in the 1620 programs which are unnecessary on the 7090. The final program, ACEXP, exists only on the 1620 and is used for adding predistortion and refraction corrections to the theoretical ephemeris.

Fig. 2 shows the flow chart of the 1620 program with its various operating options. A more detailed description of the program follows, without reference to machine.

9.1 *MUVIS*

This program takes a set of osculating orbital elements at an epoch and using them predicts when the satellite will be visible at a designated site, and when it will be mutually visible with a second designated site. The emphasis in this program is speed with only a limited amount of accuracy. It is envisaged that this program will be used for planning and general information, and thus the methods used were chosen with this in mind.

Basically, the program steps time by some increment, predicts the satellite's position in inertial coordinates for the new time, checks for visibility and mutual visibility, and continues. There is naturally a fair amount of bookkeeping associated with executing these steps, but they are essentially the heart of the program.

Since the program consists of many iterations through the basic loop outlined above, it was felt worthwhile to streamline it as much as was possible. Towards this end the following steps were taken.

(*i*) The program takes variable time steps. A coarse step is used until visibility is determined, and at this point a finer step is used for a more refined estimate. This feature is carried one step further by permitting a time step of close to a full period after visibility ends, or when the satellite appears to be moving away from visibility.

(*ii*) When the satellite's position is calculated at some time, osculat-

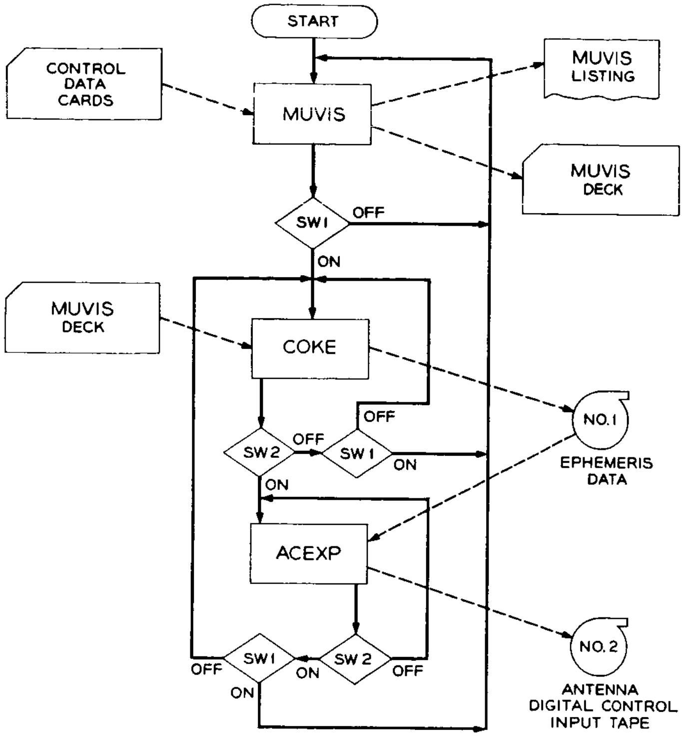

Fig. 2 — TELEPATH flow chart.

ing orbital elements are used which are valid at most one-half a period away. This enables the program to update the elements in steps of 2π, which results in a large time saving. The errors introduced by not completely updating the orbital elements are far less than the accuracy desired.

(*iii*) The determination of visibility at a site is not done by the obvious method of computing elevation and checking for a positive angle. The reason for this is that once the satellite's coordinates have been obtained, this method requires at least a square root, an arc tangent, and approximately thirteen multiplications. The method used instead requires only seven multiplications with the resultant saving in time. Instead of computing elevation, the program passes a plane through the site tangent to the earth. This plane, which can be considered a ground plane at the site, has the equation

$$\alpha X_s + \beta Y_s + \gamma Z_s - R_e = 0$$

where X_s, Y_s, Z_s are the inertial coordinates of the site, and R_e is the distance from the center of the earth to the site. If, however, the coordinates of the satellite are substituted in the equation, then a value other than zero is usually obtained. If this value is minus, then the satellite lies on the same side of plane as the center of the earth and is therefore below the horizon, but if the value is positive, then the satellite is on the opposite side of the plane which is above the horizon and is therefore visible. The determination of visibility is thus reduced to evaluating α and β, which are time varying due to the rotation of the earth, evaluating the four-term expression, and testing the sign of the result. The evaluation of α and β can be done using the previous values of α and β with only four multiplications. It should again be noted that the price for this increase of speed is the loss of some accuracy. This method does not fully take into account the earth's oblateness. The result is that the plane is not exactly tangent to the earth, and a small amount of inaccuracy is to be expected. (About 140 feet of error in placing the site.) This method is also limited in that it cannot predict rise and set at any angle other than 0°, but this information can always be obtained later from the COKE program.

When the program has determined a period of visibility, it updates the orbital elements to either the center of the mutual visibility if any exists or otherwise simply the center of visibility, and punches out the elements plus other pertinent data on the pass. The program also prints out the pass number, rise and set times at the ground station, start and end of mutual visibility, and the maximum elevation seen at the site.

This procedure is continued until a final time is reached. At this point the program finishes any pass it may be working on and then stops.

9.2 *COKE*

The COKE program uses the MUVIS results to generate an ephemeris that is exact but omits physical effects such as refraction and antenna distortion. Thus the tape can be generated ahead of time and just before the pass corrected for both meteorological conditions and the boresight corrections of the antenna to be used (there is only one antenna at each site at present).

The program uses an Encke type method (see Ref. 6, p. 176) to solve for the satellite's position at four-second intervals. These are computed both forward and backward from the center of the pass or the center of mutual visibility, whichever the orbital elements have been updated to. Thus some rearrangement of data must be done to output the ephemeris in time order. The heart of the program is the same integration program used in the trend-removal portion of TELETRACK. Therefore only the peripheral programming needed to convert the results to pointing angles, to rearrange and output the results, and to control the direction and length of integration had to be written from scratch.

9.3 *ACEXP3*

The ephemeris tape required as input to the antenna digital control during a pass is generated by the expander program, ACEXP. In addition to generating this tape, this program also produces the "mission printout," a listing of pertinent data regarding a pass over the ground station. The main input to this program is the data tape from COKE, containing time, azimuth, elevation and range at four-second intervals.

One data point on the ephemeris tape contains the following information:

(a) time
(b) azimuth and elevation positions
(c) azimuth and elevation first differences
(d) azimuth and elevation second differences
(e) azimuth and elevation predistortions, and
(f) gain factor.

Azimuth and elevation first differences for the ith data point are computed according to

$$D^1 = \tfrac{1}{2}(P_{i+1} - P_{i-1})$$

where P_i represents azimuth or elevation position. The second differences are computed according to

$$D^2 = P_{i+1} - 2P_i + P_{i-1} .$$

Azimuth and elevation predistortions, which are discussed in Ref. 7, are estimated to be functions of elevation only and to be of the form

$$PD_A = (a + bE)/\cos E$$

$$PD_E = c + dE.$$

Current values for the parameters are

$a = 0.015$ degree
$b = -0.000786$
$c = 0.057$ degree
$d = -0.000712.$

The ground station transmitter gain factor, discussed more fully in Ref. 8, is computed as a function of range as follows

$$\gamma = \frac{127}{128}\alpha[1 + 0.0791(\alpha - 1)]$$

$$\begin{aligned} \text{where } \alpha &= \log_{10} S/S_{\min} && \text{for} \quad S_{\min} < S < 10\, S_{\min} \\ &= 0 && \text{for} \quad S \leqq S_{\min} \\ &= 1 && \text{for} \quad 10\, S_{\min} < S. \end{aligned}$$

$S_{\min}$ is chosen according to the characteristics of the pass over the site.

Elevations are corrected for refraction as follows. Index of refraction is computed according to

$$n - 1 = (0.776 \times 10^{-4}\, p + 0.372\, e/T)/T$$

where T is temperature in degrees K, p is air pressure in millibars, and e is water vapor pressure in millibars (Ref. 9, pp. 13–15). The correction

$$\Delta E = (n - 1) \cot E$$

is added to the elevation, E, before putting it on the ephemeris tape.

The mission printout is generated to aid the operating personnel during a satellite pass. Tabular data at one-minute intervals specify time, azimuth, elevation, range, one-second increments in azimuth, elevation and range, and Doppler shift. From a knowledge of the azimuth rates the program predicts when (if at all) the horn antenna will lose autotrack due to excessive azimuth rates. The angular distances between the satellite and the sun are also computed, and if they come within 2° of each other an appropriate warning is included in the mission printout.

X. OPERATIONAL RESULTS

These programs have been a part of the Bell System satellite communications ground station operational system since the July 10, 1962, launch. Initial predictions were based on the launch and injection data, corrected by the few observations possible in the first six orbits. From the sixth orbit on, predictions were based entirely on track data acquired at the Andover site. By the seventeenth orbit (the second day), the orbital elements had been refined sufficiently so that the horn-reflector antenna autotrack could acquire the satellite using the predicted angles. From that point on the normal mode of acquisition was from the predicted angles, and the use of the auxiliary antennas as acquisition aids was generally not required. This means that predictions have generally been within $\pm 0.2°$, at least at the horizon where acquisition is usually achieved.

The launch of the Telstar satellite was carefully planned to put the apogee in the northern hemisphere to maximize the periods of mutual visibility in the early phases of the experiment. During the first few weeks following the launch, the prediction accuracy was very good. Samples of the results of orbit determination and prediction during this period are shown in Table VII. The predicted angles, extending five days ahead, were generated from orbital elements computed from precision angular tracking data obtained during the preceding five days. The observed angles were obtained from the precision tracker. It should be noted that errors in azimuth should be multiplied by the cosine of the elevation in order to convert them to errors in sightline angle on a par with the errors in elevation.

With apogee in the northern hemisphere, the tracking periods were long (over 30 minutes). As perigee precessed toward the northern hemisphere and the tracking periods became shorter, a gradual degradation in prediction accuracy was noted. While the prediction accuracies were sufficient for the daily antenna pointing operations at Andover, they proved inadequate for providing pointing information for the optical experiment at Holmdel[10] and for determining satellite positions for the radiation effects study.[11] These uses of the predictions require accuracies of $0.1°$ and both require that the satellite positions be related to geographical sites other than that at which the track data are acquired.

This prompted a renewed study of the orbit determination method and the program implementation. This investigation revealed that this method is quite sensitive to observational bias, particularly when the track data are obtained from short passes rising to high elevations. This sensitivity can be reduced by using only tracking passes of 30 minutes or more in which the maximum elevations do not exceed 50°. However, that is a severe restriction to place on a single tracking site with a highly eccentric orbit such as that under consideration here. In addition, it was found that the approximate methods used to account for the perturbations due to the earth's oblateness were inadequate except when the line of apsides is nearly parallel (as in July, 1962) or nearly perpendicular to the line of the nodes. Programs providing more complete perturbation calculations have been written and are presently undergoing tests.

From this study it was concluded that to achieve prediction accura-

TABLE VII

Date and Pass Number	Time (hrs-min, UT)		Azimuth (degrees)		Elevation (degrees)	
			Observed	Predicted	Observed	Predicted
7/23 #124	21	35	195.16	195.20	20.50	20.48
	21	41	188.83	188.86	29.37	29.38
	21	47	179.58	179.60	38.28	38.29
	21	53	165.12	165.14	46.35	46.37
	21	59	142.81	142.80	51.28	51.29
	22	05	115.18	115.21	49.24	49.22
	22	11	92.13	92.15	39.03	39.04
	22	17	77.19	77.19	24.08	24.08
7/24 #133	21	17	187.80	187.81	25.73	25.72
	21	23	179.40	179.40	34.40	34.40
	21	29	166.95	166.97	42.47	42.44
	21	35	148.30	148.32	48.26	48.26
	21	41	123.75	123.75	48.91	48.88
	21	47	99.92	99.93	42.22	42.20
	21	53	82.60	82.60	29.77	29.77
7/25 #143	23	43	238.23	238.24	22.01	22.03
	23	49	237.62	237.61	32.26	32.29
	23	55	235.99	236.00	44.29	44.29
7/26 #143	00	01	231.28	231.31	58.89	58.94
	00	07	207.91	207.83	76.43	76.44
	00	13	99.76	99.82	72.91	72.91
	00	19	80.65	80.65	46.23	46.22
7/26 #151	20	31	183.69	183.68	21.11	21.12
	20	37	175.55	175.59	29.39	29.42
	20	43	164.51	164.54	37.05	37.04
	20	49	149.13	149.17	42.90	42.91
	20	55	129.16	129.21	45.17	45.18
	21	01	107.72	107.77	41.98	42.02
	21	07	89.57	89.59	33.43	33.44
	21	13	76.29	76.29	21.36	21.35
7/27 #160	20	08	181.81	181.84	18.66	18.67
	20	14	173.80	173.85	26.84	26.84
	20	20	163.25	163.30	34.25	34.31
	20	26	149.03	149.11	40.15	40.19
	20	32	130.89	130.95	42.99	42.98
	20	38	110.85	110.93	41.09	41.09
	20	44	92.90	92.93	34.18	34.20
	20	50	78.98	79.01	23.62	23.63

cies of 0.1° or better, the angular observations must be taken from more than one geographical point or, if from a single tracking site, the angular observations must be supplemented by an additional independent track measurement, such as slant range to the satellite. A program system avoiding the shortcomings of the present method is now under active development. This system uses a modified method of combining passes and improved perturbation calculations, and has the ability of including slant range measurements and data from several tracker sites.

The orbit determination method described meets the objective of minimizing the computer requirements by eliminating the mass storage requirements and time-consuming iterative procedures inherent in the classical differential corrections technique. As described, the method and programs are adequate for providing acquisition information for autotracking communications antennas if the tracking restrictions can be met. For a single tracking site, these restrictions imply a perigee of 1000 nautical miles or more. If lower orbits must be handled or greater accuracies are required, the improvements mentioned above should be considered.

APPENDIX A

Derivation of Equations (7) and (8)

Since $A^{-1}\bar{x} = (A^{-1} + B^{-1})\bar{x} - B^{-1}\bar{x}$, (5) may be written in the form

$$\bar{z} = \bar{x} - (A^{-1} + B^{-1})^{-1} B^{-1}(\bar{x} - \bar{y}).$$

Now,

$$\begin{aligned}(A^{-1} + B^{-1})^{-1} B^{-1} &= [B(A^{-1} + B^{-1})]^{-1} \\ &= [1 + BA^{-1}]^{-1} \\ &= [(A + B)A^{-1}]^{-1} \\ &= A(A + B)^{-1}.\end{aligned}$$

Hence,

$$\bar{z} = \bar{x} - A(A + B)^{-1} (\bar{x} - \bar{y}).$$

Since, by (5), we may interchange $\bar{x}$ and $\bar{y}$ provided that A and B are also interchanged, we have

$$\bar{z} = \bar{y} + B(A + B)^{-1} (\bar{x} - \bar{y}).$$

Thus, if w_1 and w_2 are any two six-by-six matrices whose sum is a unity matrix,

$$\bar{z} = w_1\bar{x} + w_2\bar{y} - (w_1A - w_2B)(A + B)^{-1} (\bar{x} - \bar{y}).$$

Substituting $A = PG^{-1}$ and $B = QG^{-1}$, and noting that

$$(A + B)^{-1} = G(P + Q)^{-1}$$

$$w_1A - w_2B = (w_1P - w_2Q)\,G^{-1}$$

we get (7).

Noting that the right-hand member of (6) is a half of that of (5) if we replace $\bar{x}$ by A and $\bar{y}$ by B, (8) follows from (7).

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