N64-17227* OP

1759.

CODE-1

CR-53197

MISSING

P 117-125

OF APPENDIX IV

OTS PRICE

XEROX 10.10 ph

MICROFILM 3.95 mf

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

S64-14075

2

(NASA Contract NAS 7-100)
(NASA CR-53197; JPL-TM-33-139) OTS: $10.10 ph, $3.95 mf

Technical Memorandum No. 33-139

WEAR ANALYSIS OF NONLUBRICATED SPUR GEARS

James Clyde Randall Jun, 1963 125 p refs

W. J. Schimandle, Chief
Spacecraft Development Section

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA
June, 1963

1304823

Prepared Under Contract No. NAS 7-100
National Aeronautics & Space Administration

CONTENTS

I. Introduction 1

II. Friction, Surface Damage, and Wear 4

III. Short Discussion on Gear-Load Calculations 19

IV. Fundamentals of Involute Spur Gears 23

V. Gear Wear Tests and Results 46

IV. Summary 76

References 80

Appendix A: Sliding Velocity of Gears 82

Appendix B: Calculation of Test-Gear Tooth Loads and Stresses Using Buckingham's Formulae and American Standards Association Specification B6.11-1951 84

Appendix C: Calculation of Test-Gear Tooth Loads and Stresses Using Buckingham's Formulae and Tuplin's Method for Effective Error 96

Appendix D: Wear Rate Data 107

Appendix E: Calculation of Wear Rates 110

Appendix F: Calculation of Fatigue Lives 116

TABLES

1. Wear rates and calculated stresses for material combinations and loads 64

2. Wear rates and stresses for test materials 66

3. Stresses and calculated fatigue lives for test gears 75

FIGURES

1. Comparison of friction and adhesion for steel on indium 7

2. Comparison of calculated particle size and measured average particle size 10

3. Comparison of dynamic loads 22

4. Two mating involute profiles 25

5. Dimensions for gear teeth and rim 36

6. Factors pertaining to stretch of rim 38

7. Relationship of measured tooth-to-tooth error by center distance deviation to actual pitch error of gear teeth. . . 38

8. Comparison of ratio of insertion time to natural period (t_1/T_1) and ratio of effective error in action to actual error in action 44

9. Surface fatigue curves for 303 stainless steel and 2024-T4 aluminum 45

10. Gear test fixture 48

11. Wear rate for 303 stainless steel on 303 stainless steel (no load, 3800 rpm) 54

12. Wear rate for 303 stainless steel on 2024-T4 aluminum (no load, 3800 rpm) 54

13. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum (no load, 3800 rpm) 55

14. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum treated with molybdenum disulphide (no load, 3800 rpm) 55

15. Wear rate for anodized 2024-T4 aluminum on anodized 2024-T4 aluminum (no load, 3800 rpm) 56

16. Wear rate for 303 stainless steel on delrin (no load, 3800 rpm) 56

17. Wear rate for 303 stainless steel on 303 stainless steel (4 in. -oz, 380 rpm) 57

18. Wear rate for 303 stainless steel on 2024-T4 aluminum (4 in. -oz, 380 rpm) 57

FIGURES (Cont' d)

19. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum (4 in. -oz, 380 rpm) 58

20. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum treated with molybdenum disulphide (4 in. -oz, 380 rpm) 58

21. Wear rate for anodized 2024-T4 aluminum on anodized 2024-T4 aluminum (4 in. -oz, 380 rpm) 59

22. Wear rate for 303 stainless steel on delrin (4 in. -oz, 380 rpm) 59

23. Wear rate for 303 stainless steel on 303 stainless steel (6 in. -oz, 76 rpm) 60

24. Wear rate for 303 stainless steel on 2024-T4 aluminum (3 in. -oz, 76 rpm) 60

25. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum (3 in. -oz, 76 rpm) 61

26. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum treated with molybdenum disulphide (3 in. -oz, 76 rpm) 61

27. Wear rate for anodized 2024-T4 aluminum on anodized 2024-T4 aluminum (3 in. -oz, 76 rpm) 62

28. Wear rate for 303 stainless steel on delrin (3 in. -oz, 76 rpm) 62

29. Design curve for 303 stainless steel indicating calculated stress and corresponding depth-of-wear rate. 67

30. Design curve for 2024-T4 aluminum indicating calculated stress and corresponding depth-of-wear rate. 68

31. Design curve for anodized 2024-T4 aluminum indicating calculated stress and corresponding depth-of-wear rate 69

32. Design curve for anodized 2024-T4 aluminum treated with molybdenum disulphide indicating calculated stress and corresponding depth-of-wear rate 70

33. Design curve for delrin indicating calculated stress and corresponding depth-of-wear rate 71

ABSTRACT

17227 nn

This paper establishes a method of determining wear rates for non-lubricated, fine-pitch, precision instrument spur gears. The concepts of wear and the problems associated with applying these concepts to the unique action of spur-gear surfaces are discussed. The properties of the involute curve are included only to the extent that is deemed necessary to analyze thoroughly the gear-wear problem. Wear data for test gears run at various loads and speeds are collected to determine the wear rates for the most popular materials in use today. Design curves are made for five materials (or surfaces) relating wear rates to calculated Hertz' stresses:

1. 303 stainless steel
2. 2024-T4 aluminum
3. Anodized 2024-T4 aluminum
4. Anodized 2024-T4 aluminum treated with molybdenum disulphide
5. Delrin

Design curves consisting of any one or combination of the above materials can be used to analyze wear rates of a gear train. In addition, the wear rates are established in a depth-of-wear per revolution so that the expected life of a system can be determined, realizing that some systems can tolerate more wear than other systems before they can be said to have failed.

In addition to the data presented in the wear charts, this paper proposes a method for using the wear data to select between two popular methods of computing dynamic load; namely, the American

I. INTRODUCTION

Wear is the loss of material from surfaces that slide and bear on each other. Almost all mechanisms must have very precise sliding surfaces in order to function properly. The deterioration of these surfaces because of wear usually has a detrimental effect on the performance of such mechanisms. Although wear occurs from the time that these surfaces come into contact, the wear rate determines the life of the mechanism until it is deemed useless.

Studies on wear have been somewhat neglected historically because of the inability to accurately measure wear on surfaces. This was not a problem in the past, however, because of the inability to hold close tolerances. For example, tolerances as large as a couple of thousandths of an inch were as close as could be held, and a wear of a few tenths of thousandths of an inch was not important. The "state of the art" has progressed, however, and tolerances of a few tenths of thousandths of an inch are held quite easily. A wear of a few tenths of thousandths of an inch becomes relatively important in this case. Recent introductions of radioactive-isotope measuring techniques make accurate and reproducible wear results possible, which will undoubtedly stimulate interest in the subject of wear in the future.

In the discussion of wear it should be pointed out that there are specific methods to combat wear. One of the most significant ways to reduce the wear rate is with a lubricant. A lubricant is a low shear-strength material placed between the sliding surfaces. A lubricant does not prevent wear, but, reduces it by reducing the number of contact points between the sliding surfaces. Although a lubricant is generally thought of as an oil, grease, or some sort of dry-film, some precious metal platings have been used as lubricants because of their low shear-strengths.

There are reasons, however, why a lubricant should not be used at all times. At high altitudes, for example, low vapor-pressure oils or greases tend to evaporate, leaving harmful residues on the surfaces. Excessive amounts of lubricating fluids could cause extreme power losses and ultimate failures because of their heating and braking effects. Another serious problem associated with a lubricant is that it often carries contaminants to the sliding parts. As the surfaces bear on each other, particles are abraded away and tend to be carried by the lubricant, which adds to the wear problem. Filtering of the lubricant is possible, of course, but a more complex system results.

It is apparent that a lubricating system is quite necessary when near infinite life is required. Quite frequently, however, systems are designed in which the expected life need be only a few hours. Many instrumentation devices, for example, are designed with an operating life of only 1000 hr. With this in mind, many designers have abandoned the lubrication system to lower costs and complexity. Unfortunately, very little information exists for predicting wear life for sliding and rolling surfaces in the absence of a lubricant.

This paper has been prepared to establish a means of predicting wear life for a special type of sliding and rolling surface; namely, those surfaces of nonlubricated, fine-pitch, precision, instrument spur gears. Many nonlubricated gear trains have been built on an intuitive basis, as to the size of gears and materials to use to minimize wear. That is, in the absence of specific gear formulae to accurately predict life, most design work has been done by trial and error. Often a change in materials would improve the life of a gear train by many orders of magnitude.

Many material combinations are conceivable in a gear train, but only the most popular combinations of steel, aluminum, and delrin will be considered in this paper. Delrin is a stabilized form of nylon which has good dimensional stability. Gears made of various other plastics have been used in nonlubricated gear trains and appear to wear quite well. The chief disadvantage to plastics, however, is their poor dimensional stability, low strength, and low elastic modulus in comparison with metals.

II. FRICTION, SURFACE DAMAGE, AND WEAR

In the sliding of two surfaces over each other, there appears to be two major experimental observations. One observation is that the area of contact between two surfaces is very small. Although the techniques of grinding and polishing have advanced to the point where surface finishes within 100- to 1000-Å units are not unusual, intimate contact is still anticipated since the range of molecular attraction is only a few Angstroms. The area over which the surfaces are within molecular range will, even for carefully prepared surfaces, be quite small. The contact pressures at these small contact areas are high enough to cause plastic deformation of the surfaces.

The second observation is that the sliding speeds at which typical sliding members are used cause the surface temperatures to rise to very high values. When one body slides over another, some of the work done against the frictional force is liberated as heat between the surfaces. This heat is then carried away from the surfaces by conduction, convection, and radiation. Quite primitive calculations, however, indicate that very high surface temperatures are attained even with moderate loads and speeds. This high temperature tends to promote plastic flow and wear, caused by the softened surface.

A third item which needs investigation is the type of interaction between the moving surfaces and the physical changes which occur in them during sliding. One of the most striking conclusions drawn from extensive studies of the friction of metals is that the magnitude of the frictional force and the extent and type of surface damage caused by sliding are determined primarily by the relative physical properties of the two sliding surfaces. Specifically, the behavior is quite dependent upon the relative hardness of the two surfaces and, if the sliding speeds are high, upon their relative softening or melting points.

Sliding friction can be broken down into two main types:

1. Hard surface sliding on a soft one
2. Surfaces of similar hardness sliding on each other

Many tests have been conducted, but just the results of one test will be indicated here. Bowden and Tabor[1] performed experiments with the following results: In Case 1, the hard surface sliding on a softer one resulted in high wear in the soft surface, as expected, and a coefficient of friction of about 0.9. A groove was dug out of the softer material and very little wear was perceptible on the harder material. In Case 2, with similar surfaces sliding together, the damage was profound for both parts, and the coefficient of friction was considerably higher at 1.2. Although no actual degree of wear was given for these tests, the results are conclusive in that the coefficient of friction was considerably higher for similar surfaces than for dissimilar surfaces.

The friction force, however, is unique among the factors involved in rubbing. It used to be generally acknowledged that wear and friction were directly related. However, there appears to be a mounting number of careful experiments on the subject that demonstrate that this cannot be generally true. For example, there can be a high friction force with a very low amount of wear, and vice versa. Whittaker[2]

---

[1] F. P. Bowden, and D. Tabor, The Friction and Lubrication of Solids. (Oxford, Clarendon Press, 1954) pp. 78-79.

[2] E. J. W. Whittaker, "Friction and Wear," Nature, 159 (1947) p. 54.

and Savage[3] have shown, generally from typical wear data, that not more than 1% of the frictional work could have been absorbed by removing the worn-off material directly, and, in general, the actual proportion was very much less than 1%. Thus, a correlation between friction and wear is not necessarily to be expected.

Friction does affect wear indirectly through the intermediate factor of temperature. If high friction is present, the materials may tend to adhere to one another more easily because of elevated temperature, thus adding to the wear problem. In addition, the elevated temperatures may change the hardness of the materials, which would have a marked effect on the wear characteristics exhibited by two materials in operation.

A factor which should be called to mind when attempting to reach some conclusions about wear from friction is the relationship between the coefficient of friction and the coefficient of adhesion. It has been shown that a near linear relationship holds between friction and adhesion. This point is mentioned since the amount of cold welding, and thereby wear, is directly related to the adhesion coefficient of the materials. These observations support von Mises' relationship that $u^2 = 0.3v^2 - 0.3$ for plastic flow under combined normal and tangential stresses. A plot of the coefficient of friction and the coefficient of adhesion for relative motion between steel and indium[4] is shown in Fig. 1. The curve is essentially the same for most metals.

---

[3] R. H. Savage, "Graphite Lubrication," Journal of Applied Physics, 19 (1948) p. 1

[4] F. P. Bowden and D. Tabor, The Friction and Lubrication of Solids. (Oxford, Clarendon, Press, 1954) p. 313.

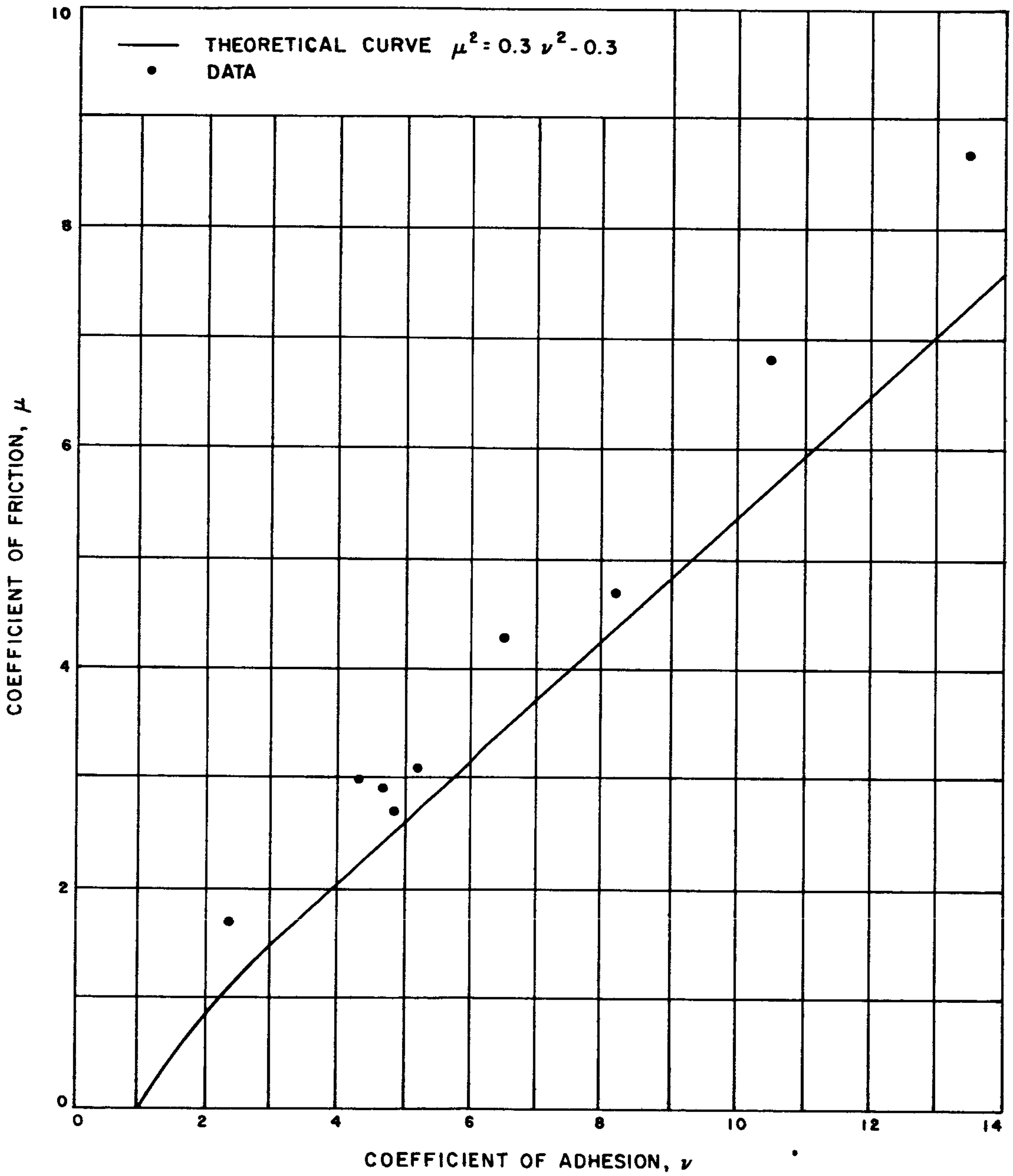

Fig. 1. Comparison of friction and adhesion for steel on indium

A wear equation has been proposed by Archard[5] and a few others. The equation is as follows:

$$v = KW/P \tag{1}$$

where:

v = volume loss per unit distance of sliding

K = wear constant

W = normal load

P = flow pressure of the materials

The W/P term is generally considered to determine the real area of contact between the sliding surfaces. The constant K is related to the nature of the wear process itself. Experiments by Spurr[6] in 1955 have shown that this equation gives the rate of wear of small flat samples of wax loaded against a rotating disk, provided the variation of P with surface temperature is considered. Additional experiments indicated, as one would expect, that the surface finish of the disk had a very marked effect on the value of K. A few papers have been published on the effect of surface roughness on wear in the absence of lubrication. Brownsdon[7] showed in 1936 that wear did increase with increasing

---

[5] J. F. Archard, "Contact and Rubbing of Flat Surfaces," Journal of Applied Physics, 24 (1953) p. 981.

[6] R. T. Spurr, "Creep and Static Friction," British Journal of Applied Physics, 6 (1955) p. 402.

[7] H. W. Brownsdon, "Metallic Wear," Journal of the Institute of Metals, 18 (1936) p. 15.

surface roughness. Later Taylor and Holt[8] found that wear was approximately proportional to the surface roughness as determined by a profilometer. Thus, it appears that surface finish may be an important variable in wear.

Mr. Rabinowicz[9], of Massachusetts Institute of Technology (MIT), has done a considerable amount of work in the specific area of wear, and has expounded several theories that laboratory investigations tend to confirm. One theory is that the same materials always wear in fragments of the same characteristic size. The particle size is related to the amount of elastic energy the material can absorb before it yields. Calculations show that the fragment size has a lower limit. This is because the shattering of the material transforms the elastic energy into surface energy. Since there is a limit to the amount of energy a given material can absorb, there is also a limit to the fragment size. The energy required to form a wear particle at the surface must be provided by the elastic energy of the material in the immediate vicinity. The minimum fragment size calculated on the basis of elastic energy should be, for a specific material, closely related to the characteristic size of its wear particles.

Figure 2 illustrates the relationship between calculated minimum particle size and actual average particle size.[10] As expected, the actual

---

[8] R. H. Taylor and W. Holt, "Effect of Roughness of Cast Iron Brake Drums in Wear Tests of Brake Linings," Journal of Research, National Bureau of Standards, 27 (1941) p. 395.

[9] Ernest Rabinowicz, "Wear," Scientific American, 206 (January, 1962) pp. 127-136.

[10] Ibid., p. 132.

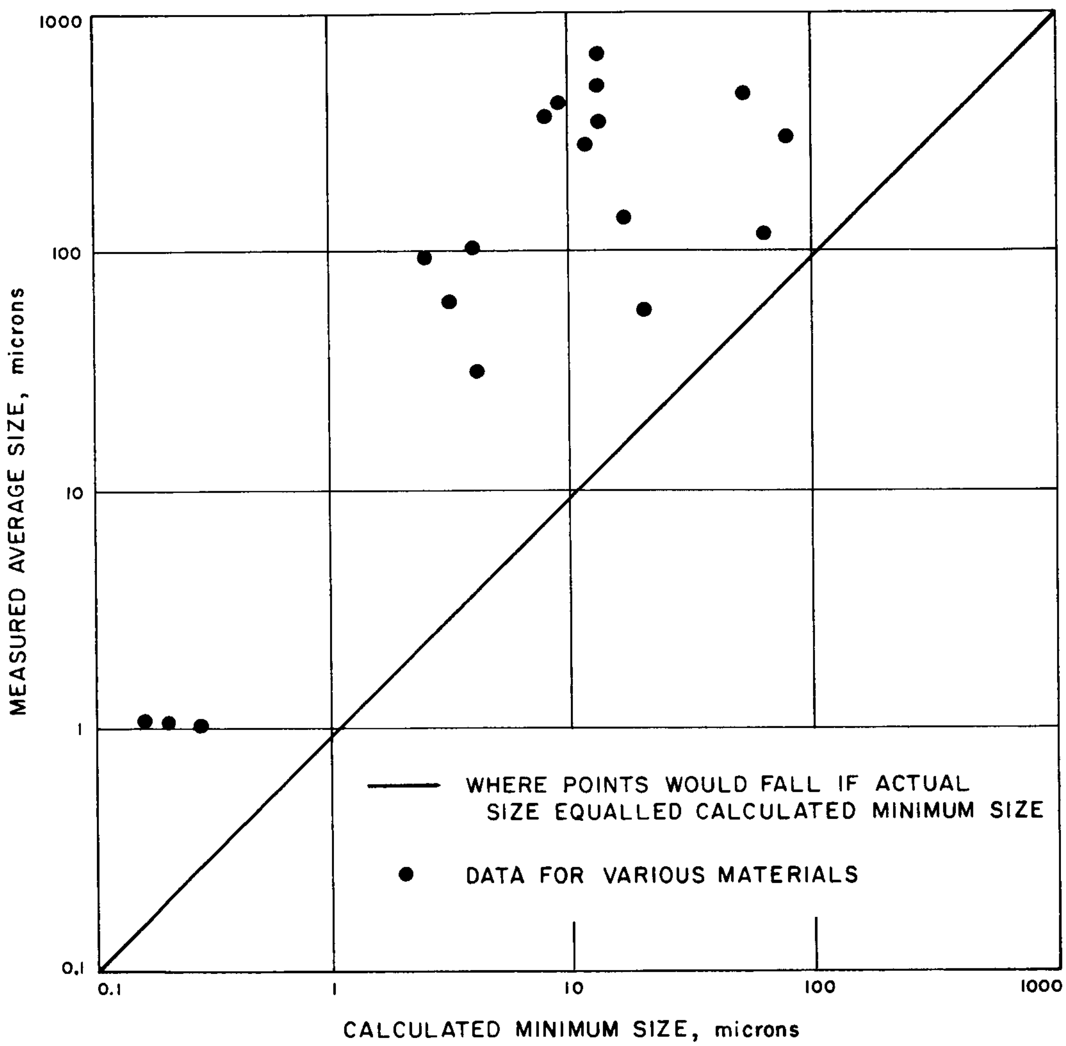

Fig. 2. Comparison of calculated particle size and measured average particle size

average particle size is slightly larger than the calculated minimum particle size. In a system in which wear particles of a certain size are being generated, the surfaces take on a corresponding roughness. The height of the hills and valleys on the surface will be roughly the same as the diameter of the average wear particle. As particles are worn away, the surface finish tends to remain the same.

If the theory proposed by Rabinowicz is correct, which preliminary investigations seem to bear out, it can account for the wear phenomenon as we observe it. At first, there seems to be a period of very rapid wear followed by a longer period of time at a much reduced wear rate. The period of rapid wear could be caused by the rough surfaces working together during the first hours of operation. As particles are generated, however, the surfaces become polished to a finish roughly equivalent to the particle size, and the wear rate declines to a fairly constant rate. Wear would then continue at this rate until the part is "worn-out" because of excessive clearance or play. This wear would be the abrasive type in that particles would be worn off by the sliding action.

In addition, Mr. Rabinowicz[11] has done some research on a lesser known type of wear called adhesive wear. When two smooth surfaces slide over each other, patches of one surface adhere to the other and are pulled away. Adhesive wear results from the strong forces established between atoms that come into intimate contact with one another. When a bond is made between two atoms, there is a certain likelihood

---

[11] Ibid., p. 129.

that, when the contact is broken, the break will not occur at the original boundary. Instead the break will occur within the surface layers of one of the materials and an adhesive wear fragment will be produced. Moreover, this adhesive wear occurs in two materials merely contacting as well as sliding.

In an experiment performed by Rabinowicz at the University of Cambridge, a copper rod with a hemispherical end was pressed against a steel surface. The rod was pressed with a 2-lb force perpendicular to the steel surface with no tangential motion allowed. By radiation-tracer techniques, it was found that 10^{-9} g of copper had transferred to the steel and that 10^{-10} g of steel had transferred to the copper. It was also found that as much as 10^{-6} g could be transferred if the rod were pressed against the plate at an angle, but not allowed to slide. Although all the laboratory conditions are not available so that one could determine these unit stresses, the above information gives a useful description of adhesive wear in a qualitative manner.

Adhesive wear can be reduced, but it cannot be eliminated. As surfaces operate, for example, material is worn away, leaving clean surfaces to contact the mating parts. If these surfaces are running in air, they usually have a chance to oxidize slightly before coming into contact again. These impurities impede the adhesion of the surfaces and the amount of adhesive wear is reduced. In a vacuum environment, however, these same surfaces are more susceptible to adhesive wear in that they do not oxidize as they do in air. A clean bare surface is exposed during operation because of abrasion and these surfaces show excellent tendencies to wear adhesively.

A simple formula has been developed by Archard[12] for adhesive wear. This formula assumes a certain probability k that intimate contact will result between two contacting surfaces:

$$v = kWl/3p \tag{2}$$

where:

v = volume of wear, mm^3

W = load on sliding surfaces, kg

l = sliding distance, mm

p = penetration hardness of the softer contacting surface, kg/mm^2

k = probability of intimate contact

Thus, the wear is directly proportional to the load and sliding distance, and inversely proportional to the penetration hardness, according to Archard.

The above equation, however, makes no reference to surface finish or sliding velocity. As Mr. Rabinowicz[13] has pointed out, the surface finish seems to affect only the early wear, but, after this period, wear rates tend to stabilize because of the generation of constant-size wear particles. Rabinowicz has assumed cubic particles and equated the volume energy of the particle, $\sigma_r^2 d^3/2E$, with the surface energy of the particle, $6\gamma d^2$. This resulted in a diameter for the particle, $d = 12E\gamma/\sigma_r^2$. Experimental results (see Fig. 2) confirm

---

[12] J. F. Archard, "Contact and Rubbing of Flat Surfaces," Journal of Applied Physics, 24 (1953) p. 983.

[13] Ernest Rabinowicz, "Wear," Scientific American, 206 (January, 1962) p. 135.

this assumption, and it seems logical, therefore, to assume that surface finish has very little effect on the wear rate after the initial run-in period. Research by Archard also tends to indicate that sliding velocity does have an effect on wear, but this is ordinarily slight, and thus not accounted for in the expression he proposed. He maintains that the effects of sliding velocity on wear are negligible if this sliding velocity is less than 500 ft/min.

In a wear problem, it is generally the depth-of-wear rather than the volume that is important. In addition, the penetration hardness of a given material is not usually as well-known as the yield strength, although a correlation does exist between the two. Rewriting Eq. (2) in English units, considering penetration depth in terms of yield strength, and finding the depth of wear rather than volume of wear, Archard obtained the following equation:

$$h = kWl/9As \tag{3}$$

where:

h = depth of wear, in.

W = load, lb

l = sliding distance, in.

A = surface area, in.^{2}

s = yield strength of softer material, lb/in.^{2}

k = probability of intimate contact

The wear rate of the harder material is less than the softer material by the following expression:

$$h_2 = (s_1/s_2)^2 h_1 \tag{4}$$

where:

h_1 = depth of wear of the softer material, in.

h_2 = depth of wear of the harder material, in.

s_1 = yield strength of the softer material, lb/in.^{2}

s_2 = yield strength of the harder material, lb/in.^{2}

The equations show the effect that changing the yield strength, load, area, or sliding distance has on the depth of wear.

In determining the amount of abrasive wear of a sliding system, Eq. (2-4) are applicable provided the probability coefficient k is replaced by an abrasive constant K. The abrasive constant is on the order of 10^{-2} to 10^{-3}, while the probability coefficient for adhesive wear is on the order of 10^{-4} to 10^{-6} in a normal Earth atmosphere. Thus, in the atmosphere, one would expect the wear of a nonlubricated system to be largely of the abrasive type, with a very minute portion being worn away by adhesion. In the hard vacuum of space, however, where pressures are on the order of 10^{-12} to 10^{-16} mm of mercury, the likelihood of adhesion will increase, and one would expect most of the wear to occur because of adhesion and a small amount because of abrasion. It seems unlikely that the magnitude of abrasive wear would change significantly from the atmosphere to very low pressures since the same general wear-particle size would be generated. It is only the relative amount of abrasive wear with respect to adhesive wear that would change from the atmosphere to a hard vacuum.

Although the rolling and sliding action of surfaces gives both abrasive and adhesive wear, the quantities would seem to be additive if considered separately. In predicting the life of a nonlubricated system, the designer is not usually interested in the mode of failure, but simply the length of time to failure. Since the wear quantities are additive, the adhesive constant k and the abrasive constant K could be added together to form one constant which would yield the wear rate caused by combined abrasive and adhesive action. Equation (3) could be rewritten as:

$$h = K_1 Wl/9As \qquad (5)$$

where:

K_1 = constant for adhesive and abrasive wear, K + k

Since W is the applied load and A is the surface area, the quantity W/A is a stress value, and the quantity S_c can be substituted for W/A to find the depth of wear in terms of stress:

$$h = K_1 S_c l/9s \qquad (6)$$

where:

S_c = compressive stress, lb/in.^{2}

Deviating to some extent from Rabinowicz' theories, Mr. Maschmeyer[14] claims that wear failures are caused by exceeding the endurance limits of the material at which time particles begin to flake from the surface. Maschmeyer expounded this theory based upon wear patterns he had observed in the operation of gears. First, there seemed to be a run-in phase in which slight wear was observed. Sharp edges and surface imperfections were burnished smooth and residual compressive stresses were induced in the microscopic surface of the contact area.

[14] A. H. Maschmeyer, "Wear Life of Aluminum Gears," Product Engineering, 27 (September, 1956) p. 162.

This run-in period was followed, Maschmeyer continued, by a phase in which the contact area was mechanically stabilized and no appreciable wear took place. Finally, the endurance limit was reached, and the contact surface fatigued, increasing the wear rate sharply.

Several authors, notably Buckingham,[15] agree that wear is a fatigue phenomenon. While this theory may be true for a well-lubricated gear system in which the effects of abrasion and adhesion are greatly reduced, this writer believes that surface fatigue is a definite life problem, but that abrasion and adhesion will play an increasingly important role since precision gear trains are continually being designed without lubrication. Factors such as adhesion will become critical with no lubrication and at reduced pressures. It does not seem reasonable to attribute all wear to surface fatigue.

Experience tells us that lubrication definitely reduces wear, but most people will be willing to admit that surface fatigue is not dependent upon intimate contact between sliding or rolling surfaces; yet the main function of a lubricant is to prevent intimate contact. Thinking into the problem a bit deeper, it may be logical to assume that rolling surfaces would be more likely to fail because of surface fatigue than because of sliding surfaces. This conclusion can be reached fairly easily when one thinks of an abrasive particle being released in both systems. In the sliding system, the particle would be carried along and gouge material out as it proceeded. In the rolling surfaces, however, the particle would merely be embedded in the surface with very

---

[15] Earle Buckingham, Analytical Mechanics of Gears. (New York, McGraw-Hill, 1949) pp. 527-529.

little wear occurring. The sliding surfaces would seem most likely to wear out, whereas the rolling surfaces would tend to fatigue.

In this Chapter, some general theories on wear have been outlined. Wear has been with us since the beginning of time, but the technology of wear is still in its infancy. Many theories are offered with little or no proof or experimentation to substantiate them. At the onset, it would appear that the work being done by Rabinowicz at MIT is probably the most advanced in the country. A considerable amount of work in the field of wear is going on at MIT and most of the theories, although quite recent, seem to support physical observations far better than most theories presented heretofore. If these theories are correct, wear will occur during the entire operation because material is being abraded and pulled away from the surface. As Maschmeyer and others point out, however, the number of stress cycles reaches the endurance limits of the materials and the wear rate rises sharply because of gross removals of the tooth profiles. Thus, if the criterion that determines depth-of-wear is very low, the surfaces will fail by pure wear, according to Rabinowicz. If the criterion is selected higher, the surfaces will most likely fail because they exceed the surface fatigue limits rather than because of adhesive and abrasive wear.

III. SHORT DISCUSSION ON GEAR-LOAD CALCULATIONS[16]

In the analysis of wear on specific surfaces, namely nonlubricated spur gears, it would seem imperative that an accurate estimate of gear loads and stresses be determined. This chapter gives a brief historical background of gear calculations and points out the lack of precision that exists in the field.

In 1879, John Cooper made an investigation of the strength of gear teeth and found that there were then in use about 48 well-established rules for working strengths of gear teeth. These rules differed, in the extreme cases, by about 500%. In a later study by William Harkness in 1886, an examination of the literature dating back to 1796 indicated that, according to the constants and formulae used by various authors, there were differences of 1500% in the calculated power capacity of a given gear set. In 1892, Lewis presented a paper to the Engineer's Club of Philadelphia entitled "Investigation of the Strength of Gear Teeth", which introduced a formula for the load capacity of gears:

$$W_T = S p_n F y \tag{7}$$

where:

W_T = total load, lb

S = safe working stress, lb/in.^{2}

F = face width, in.

p_n = circular pitch, in.

y = tooth form factor

---

[16] Earle Buckingham, Analytical Mechanics of Gears. (New York, McGraw-Hill, 1949) pp. 385-389.

Values of y and S were tabulated by Lewis for various materials and speeds. Although the Lewis formula is generally accepted in industry today, some arguments have arisen as to the meaning of the total load. Errors on gear tooth profiles, caused by elastic deformation under load or by inaccuracies of production, act to change the relative velocities of the rotating members. This varying velocity of the rotating members results in a varying load cycle on the gear teeth. The amount of load variation depends largely upon the effective masses of the rotating gears, amount of effective errors-in-action, and the speed of the gears. It is not unusual for the dynamic load on a gear tooth to be several times the static or transmitted load. The problem, however, is in accurately determining the amount of this dynamic load.

In the early 1900's, Buckingham generated a set of formulae which tended to be a very popular method in calculating dynamic loads and which, in turn, could be used in conjunction with the Lewis formula. Buckingham did a considerable amount of gear analysis and testing at MIT, and his dynamic load formulae are the most widely used in industry today. His formulae are generally acknowledged to give loads considerably higher than the actual loads, thus incorporating a safety factor in a rather inexact science.

In the past 20 years, however, numerous formulae based upon empirical and analytical considerations have been generated in an attempt to predict more closely the actual dynamic loads. These methods vary in magnitude by approximately 1000% for the amount of power that can be transmitted by a given gear set. Figure 3 illustrates this variance for some of the more popular methods.[17]

Although gears have been used for many centuries, this chapter shows that the state of the art in gearing has not progressed to the point at which accurate gear calculations can be made. It appears, however, that a valid wear analysis can be performed if a good correlation exists between the actual loads and the calculated values of load.

---

[17] Darle W. Dudley, Gear Handbook. (New York, McGraw-Hill, 1962), pp. 14-31.

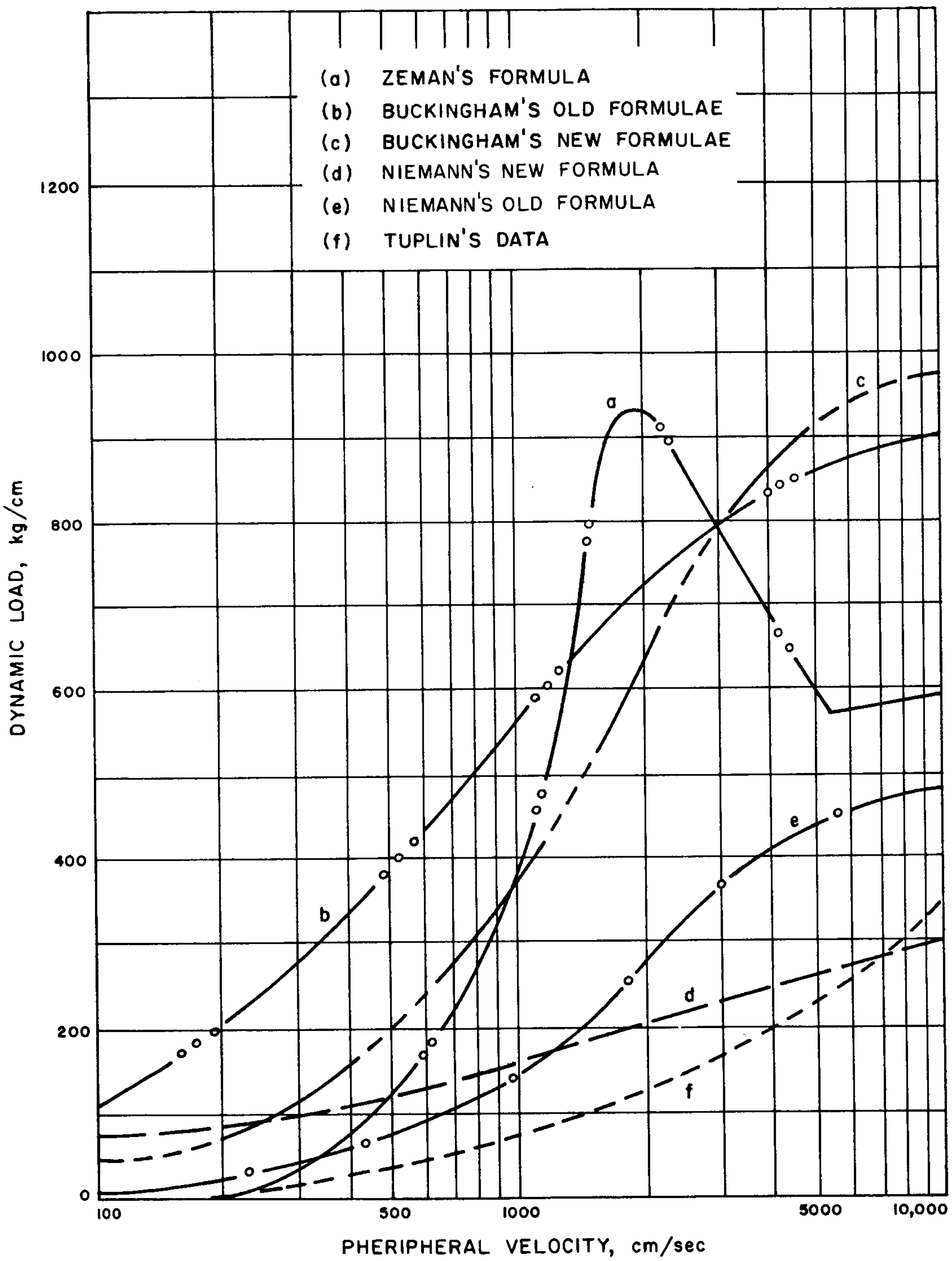

Fig. 3. Comparison of dynamic loads

IV. FUNDAMENTALS OF INVOLUTE SPUR GEARS

The wear analysis of gears first necessitates a clear understanding of the involute spur gear. A brief description of the involute curve and gear formulae are presented in order to make a thorough analysis of the wear of spur gears.

At the present time, the involute curve is used almost exclusively for spur gear tooth profiles. The involute curve is the curve that is described by the end of a line that is rolled without slip from the circumference of a circle called the base circle. The length of the generating line that is rolled from the base circle is the radius of curvature of the involute curve at any particular point on the curve. The involute curve meets all the requirements for a gear tooth profile, the most important being the transmission of uniform rotary motion. In order to transmit uniform rotary motion, the values of momentary pitch radii as defined by the instant center must remain in the same proportion to each other for all operating positions of the contacting profiles.

An advantage of the involute curve is its ability to transmit uniform motion even though the center distance be varied. If one involute, for example, rotating at a uniform rate, acts against another involute profile of the same pitch and pressure angle, it will transmit a uniform angular motion to the second profile regardless of the distance between the centers of the two base circles.

Figure 4 shows two involute curves with the generating lines at equal angular intervals.[18] The part bc on one involute curve comes into contact with hi on the second involute curve. Since bc is much nearer to its base circle than is hi to its base circle, the arc bc is much shorter than the arc hi. The two profiles must slide against each other a distance equal to their difference in length in order to have uniform rotary motion. The length cd is still much shorter than its mating section ij, but the amount of sliding will not be as large as with the previous sections of the mating profiles because the difference in lengths is not as large. The sections de and jk are almost equal in length, so very little sliding occurs in this portion of the gear profiles. It should be noted that the profile ef on the first involute curve is slightly longer than its mating section kl on the second involute, and the small amount of sliding will now act in the opposite direction. The rate of sliding between two involute curves acting against each other is proportional to the distance from the point of contact to the instant center or pitch point. The sliding velocity starts quite high, reduces to zero at the pitch point, then increases again to a maximum value in the opposite direction. The only point at which there is pure rolling action is the pitch point; at all other portions of the profile some sliding action occurs.

The sliding velocity at any point on a pair of involute gear teeth can be derived. The magnitude of the sliding velocity will be the difference in velocities of the ends of the generating lines of the

---

[18] Earle Buckingham, Analytical Mechanics of Gears. (New York, McGraw-Hill, 1949) p. 67.

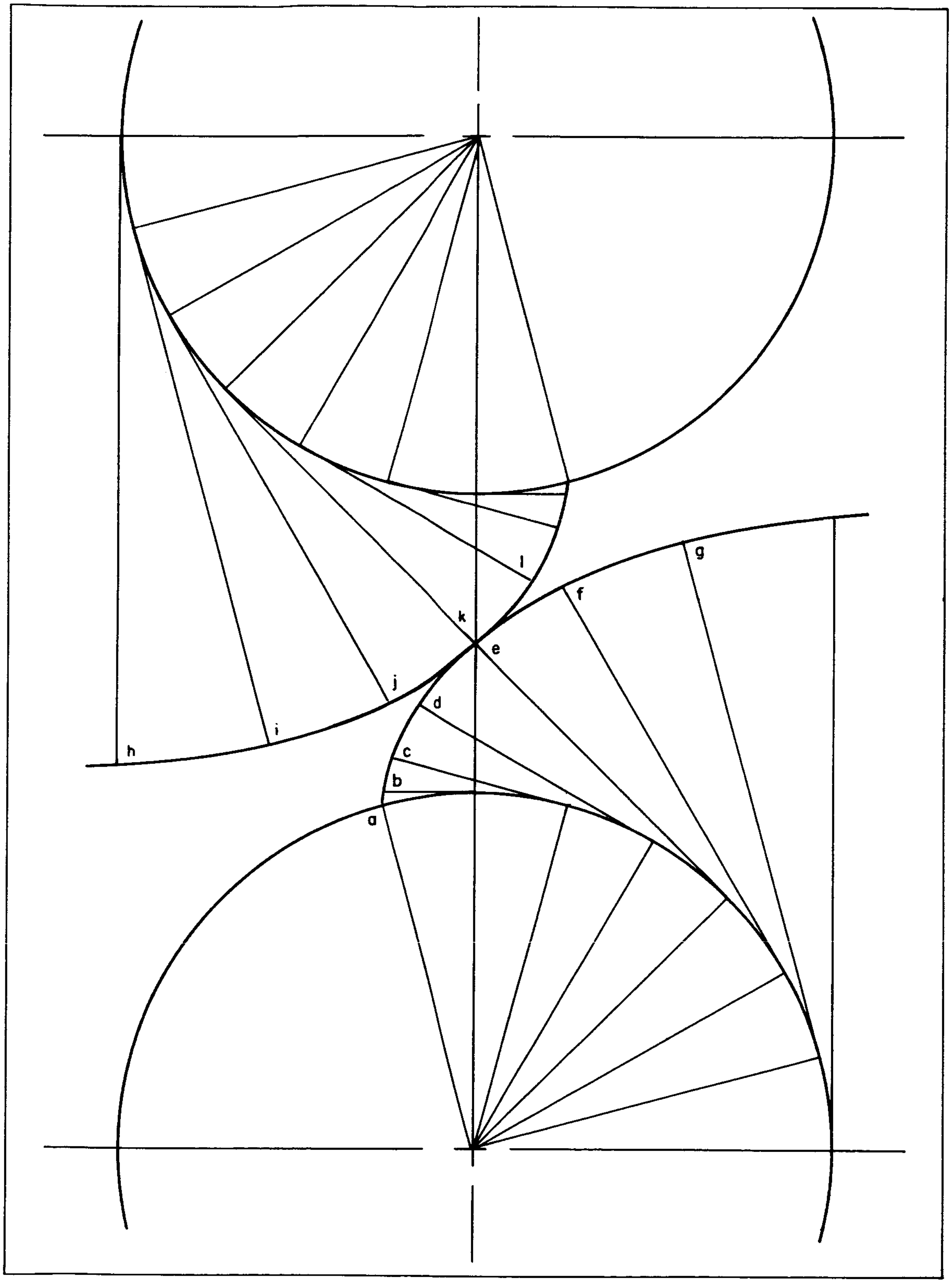

Fig. 4. Two mating involute profiles

involute curves as they pass through the line of action. The angular velocity of these generating lines will be the same as the angular velocity of the gears themselves since the profiles and the gear blank are in one rigid piece. The actual sliding velocities will be the products of these relative angular velocities and the lengths of the generating lines or radii of curvatures of the involute curves. If we let:

ω_1 = angular velocity of driving gear, rad/min

ω_2 = angular velocity of driven gear, rad/min

n = speed of driving gear, rev/min

V = pitch line velocity of gears, ft/min

V_s = sliding velocity, ft/min

R_1 = pitch radius of driving gear, in.

R_2 = pitch radius of driven gear, in.

C = center distance, in.

R_{b1} = base circle radius of driving gear, in.

R_{b2} = base circle radius of driven gear, in.

ϕ = pressure angle, deg

R_{c1} = radius of curvature of driving gear at r_1, in.

R_{c2} = radius of curvature of driven gear at r_2, in.

r_1 = radius of driving gear tooth at point in question, in.

r_2 = radius of driven gear tooth at point in question, in.

The pitch line velocity can be written

$$V = 2\pi R_1 n/12 = R_1 \omega_1/12$$

from which the angular velocity for the driving gear is found

$$\omega_1 = 12V/R_1$$

and similarly for the driven gear

$$\omega_2 = 12V/R_2$$

The angular velocity of the driven gear is related to the driving gear by

$$\omega_2 = R_1\omega_1/R_2$$

The sliding velocity is

$$V_s = (R_{c1}\,\omega_1 - R_{c2}\,\omega_2)/12$$

By geometry

$$R_{c1} + R_{c2} = C \sin\phi$$

$$R_{c1} = \sqrt{{r_1}^2 - {R_{b1}}^2}$$

$$R_{c2} = \sqrt{{r_2}^2 - {R_{b2}}^2} = C \sin\phi - \sqrt{{r_1}^2 - {R_{b1}}^2}$$

These expressions are combined and simplified to yield the sliding velocity at a given radius

$$V_s = V(1/R_1 + 1/R_2)\left(\sqrt{{r_1}^2 - {R_{b1}}^2} - R_1 \sin\phi\right) \tag{8}$$

As stated previously, the sliding velocity is continually changing as the gear teeth operate. For the wear analysis, some sort of an average velocity would be appropriate. Since the sliding velocity is nonlinear with respect to the arc length traveled, an average sliding velocity can be obtained by integrating under the sliding velocity versus angular position curve and dividing by the angular arc through which the sliding velocity takes place. Before this can be done, however, the length of the contact arc must be established.

The arc of action is the arc through which one tooth travels from the time it first makes contact with its mating tooth until it ceases to be in contact. The arc of action is often separated into the arc of approach and the arc of recess. The arc of approach is the arc through which the tooth moves from the point at which it first comes into contact with its mating tooth until contact is made at the pitch point. The arc of recess is the arc through which the tooth moves from contact at the pitch point until it ceases to be in contact with its mating tooth. The following equations can be written:

$$\beta_a = (\sqrt{R_{o2}^2 - R_{b2}^2} - R_2 \sin \phi)/R_{b1} \tag{9}$$

$$\beta_r = (\sqrt{R_{o1}^2 - R_{b1}^2} - R_1 \sin \phi)/R_{b1} \tag{10}$$

where:

β_a = arc of approach

β_r = arc of recess

R_{o1} = outside radius of driving gear, in

R_{o2} = outside radius of driven gear, in

The average sliding velocity can be found by dividing the area under the velocity curve by the arc length. In Eq. (8) for sliding velocity, it can be seen that the conditions of sliding velocity are most severe when $r_1 = R_{b1}$ and $r_2 = R_{o2}$ on the approach side and when $r_1 = R_{o1}$ and $r_2 = R_{b2}$ on the recess side. The average sliding velocity then becomes:

$$\overline{V}_s = R_{b1} \Big/ \left(\sqrt{R_{o2}^2 - R_{b2}^2} - R_2 \sin\phi\right) \int_{r_1=R_{b1}}^{r_1=R_1} V(1/R_1 + 1/R_2)$$

$$\left(\sqrt{r_1^2 - R_{b1}^2} - R_1 \sin\phi\right) dr_1$$

$$+ R_{b1} \Big/ \left(\sqrt{R_{o1}^2 - R_{b1}^2} - R_1 \sin\phi\right) \tag{10}$$

$$\int_{r_1=R_1}^{r_1=R_{o1}} V\left(1/R_1 + 1/R_2\right)\left(\sqrt{r_1^2 - R_{b1}^2} - R_1 \sin\phi)\right) dr_1$$

which reduces to:

$$\overline{V}_s = R_{b1} V(1/R_1 + 1/R_2)/2\left(\sqrt{R_{o2}^2 - R_{b2}^2} - R_2 \sin\phi\right)$$

$$\left[R_1\sqrt{R_1^2 - R_{b1}^2} + R_{b1}^2 \log R_{b1} - R_{b1}^2 \log\left(R_1 + \sqrt{R_1^2 - R_{b1}^2}\right)\right.$$

$$\left.- 2R_1^2 \sin\phi + 2R_1R_{b1} \sin\phi\right]$$

$$+ R_{b1} V(1/R_1 + 1/R_2)/2\left(\sqrt{R_{o1}^2 - R_{b1}^2} - R_1 \sin\phi\right) \tag{11}$$

$$\left[R_{o1}\sqrt{R_{o1}^2 - R_{b1}^2} + R_{b1}^2 \log\left(R_1 + \sqrt{R_1^2 - R_{b1}^2}\right)\right.$$

$$- R_1\sqrt{R_1^2 - R_{b1}^2} - R_{b1}^2 \log\left(R_{o1} + \sqrt{R_{o1}^2 - R_{b1}^2}\right)$$

$$\left.- 2R_1R_{o1} \sin\phi + 2R_1^2 \sin\phi\right]$$

With Eq. (11) the average sliding velocity can be calculated for a given gear set.

In addition to the sliding velocity, the load on a gear tooth is an important wear consideration. In high-speed applications, high pitch line velocities develop very high dynamic loads while the gears are transmitting extremely low values of torque. The load on the gear teeth consists of a transmitted load plus a dynamic load and can be expressed as follows:

$$W_T = (W_t + W_d)/F \tag{12}$$

where:

W_T = total load, lb/in.

W_t = transmitted load, lb

W_d = dynamic load, lb

F = face width, in.

Since the transmitted load is equal to the torque divided by the pitch radius, Eq. (12) can be rewritten:

$$W_T = T/RF + W_d/F \tag{13}$$

where:

T = transmitted torque, in-lb

R = pitch radius, in.

The magnitude of the dynamic load W_d is a function of the pitch line velocity, effective errors-in-action, mass effects, and pressure angle. Errors-in-action are any errors in the tooth profile that cause the contact action of the gears to differ slightly from the pure conjugate action. This error-in-action causes changes in the relative angular velocity of the mating gears and excites a varying load cycle on the gear teeth. The amount of error-in-action has been defined by the American Standards Association:[19]

[19] American Standards Association, "Inspection of Fine Pitch Gears," American Standards Association, Specification B6.11-1951, (1951) pp. 5-9.

$$E_1 = \text{total composite error} + 1/2 \text{ tooth-to-tooth error} \tag{14}$$

According to Buckingham,[20] there are two distinct load surges during each tooth engagement. The first surge is caused by small changes in the relative velocity of either gear and usually occurs during the first portion of engagement. This surge is an acceleration load, since any deviation from pure involute action results in a camming action of the gear teeth. As the succeeding pair of teeth comes into contact, the acceleration load becomes zero on the first set of teeth. However, as the gear teeth come together for a second time during one mesh, a second surge occurs called the impact load. This impact load and the acceleration load constitute the dynamic load. If the inertia load is small, as is usually the case for instrument gearing, the effective mass of a gear set may be expressed as:

$$m_e = m_p m_g/(m_p + m_g) \tag{15}$$

where:

m_p = effective mass of pinion

m_g = effective mass of gear

The effective mass of an individual spur gear is the polar moment of inertia divided by the radius squared. Since the polar moment of inertia equals $wR^2/2g$, Eq. (15) can be written as follows:

$$m_e = w_p w_g/2g(w_p + w_g) \tag{16}$$

where:

w_p = weight of pinion, lb

w_g = weight of gear, lb

g = acceleration due to gravity, in/sec^2

[20] Earle Buckingham, Analytical Mechanics of Gears. (New York, McGraw-Hill, 1949) pp. 427-452.

The mean acceleration force f_m resulting from the acceleration and impact load is expressed by Buckingham as:

$$f_m = f_a f_d/(f_a + f_d) \tag{17}$$

and for a 20-deg, pressure angle gear system:

$$f_a = 0.0012(1/R_p + 1/R_g)m_e V^2 \tag{18}$$

$$f_d = W_t + F\left[0.111E_e/(1/2E_p + 1/E_g)\right] \tag{19}$$

and the magnitude of the dynamic load is:

$$W_d = \sqrt{f_m(2f_d - f_m)} \tag{20}$$

Now Eq. (20) could be substituted into Eq. (13) to yield an expression for the total load per inch of face width.

The compressive stress developed on the gear teeth can be derived using the Hertz equation for stresses on friction disks.[21] The maximum compressive stress for two disks in contact is:

$$S_c^2 = 0.35W_T(1/R_1 + 1/R_2)/(1/E_1 + 1/E_2) \tag{21}$$

where:

S_c = maximum compressive stress, lb/in.^{2}

R_1 = radius of one disk, in.

R_2 = radius of other disk, in.

E_1 = modulus of elasticity of one disk, lb/in.^{2}

E_2 = modulus of elasticity of other disk, lb/in.^{2}

W_T = total load, lb/in.

---

[21] Ibid., p. 528.

For spur gears:

$$R_1 = D_p \sin \phi / 2$$

$$R_2 = D_g \sin \phi / 2$$

$$1/R_1 + 1/R_2 = 2(1/D_p + 1/D_g)/\sin \phi$$

Substituting these equations into Eq. (21), one obtains the maximum compressive stress for a set of spur gear teeth

$$S_c = 0.84 \sqrt{\left[(D_p + D_g)/D_p D_g\right]\left[E_p E_g/(E_p + E_g)\right]\left[W_T/\sin \phi\right]} \quad (22)$$

where:

D_p = pitch diameter of pinion, in.

D_g = pitch diameter of gear, in.

Thus, Eq. (22) gives the maximum compressive stress for a given set of gears operating under a certain total load per unit face width.

As mentioned previously, the error-in-action is taken as the maximum total composite error plus one-half the maximum tooth-to-tooth error, according to the American Standards Association Specification B6.11-1951. There seems to be some doubt, however, that this is the appropriate value to use. Experience indicates that the Buckingham formulae yield much larger values for dynamic loads than actually occur when this value for error-in-action is used. Tuplin,[22] of the University of Sheffield in England, has conducted numerous tests on actual gears in operation, and has derived his own values for errors-in-action, based largely on empirical considerations.

[22] W. Tuplin, "Dynamic Loads on Gear Teeth," Machine Design, 25 (October, 1953) pp. 203-211.

Tuplin indicates that an effective error-in-action should be used, which is in some proportion to the circular pitch error. Also, the effective error-in-action is based on the speed of operation of the gears. The speed of operation of the gears is not determined by the pitch line velocity alone, but by the natural period of the dynamic system as well. The dynamic system consists of two gears, each rotatable about its axis and each coupled with the engaged teeth. Thus, the system is an angularly vibrating one with two masses elastically connected. If the gear blanks are very stiff, the elasticity is that of the gear teeth alone. The remaining parts suffer no comparable distortion and may be regarded as rigid masses. If the gear blank is highly elastic, however, the elasticity of the blank must also be taken into account.

The compliance, the inverse of stiffness, is used because the compliance of an assembly of loaded members is simply the sum of the compliances of the components. Tuplin has found the compliance of two mating gear teeth rigidly "built in" to the mass of the gear blank. Elastic flattening of the tooth surfaces is neglected since it is very small in comparison with other compliances. The expression for the compliance of two mating gear teeth at the pitch circle is:

$$1/k_t = 3(1/E_p + 1/E_g) \tag{23}$$

This expression is nearly independent of tooth thickness because an increase in thickness is compensated for by an increase in length of the tooth. This expression was derived by empirical investigations over a wide range of diametral pitches.

If a tooth is cut in a very thin rim, however, the tooth is not "built in," and extra compliances need consideration. The bending moment at the root of the tooth causes the part of the rim below the tooth to undergo angular displacement and the tooth has more compliance because of this displacement. The compliance of this displacement has been estimated by arbitrarily assuming that the tilt of the rim under the tooth is caused by radial shear deformation in the rim sections,[23] ABCD and EFGH in Fig. 5. If a spur gear is taken with a load f per unit face width applied at the midpoint of the working depth, a moment is produced about the axis 0 of $f(0.4p_n + 0.5H)$. This moment is balanced by an equal shear force in the planes BC and EH. The magnitude of each shear force is:

$$f(0.4p_n + 0.5H)/0.8p_n$$

which simplifies to:

$$f(0.5 + 0.625H/p_n)$$

The shear stress in the vertical sections between AD and BC, and between EH and FG, is the shear force divided by the depth of the rim (unit width was assumed when f was selected):

$$f(0.5/H + 0.625/p_n)$$

Thus, the upward motion of BC relative to AD and the downward motion of EH relative to FG is:

$$(f/G)(0.5/H + 0.625/p_n)\,0.2p_n$$

where:

G = shear modulus of gear material, $\mathrm{lb/in.}^2$

The tilt of the prism BEHC is this movement divided by one-half the base CH. The horizontal movement of the tooth at the point of

---

[23] _Ibid._, p. 208.

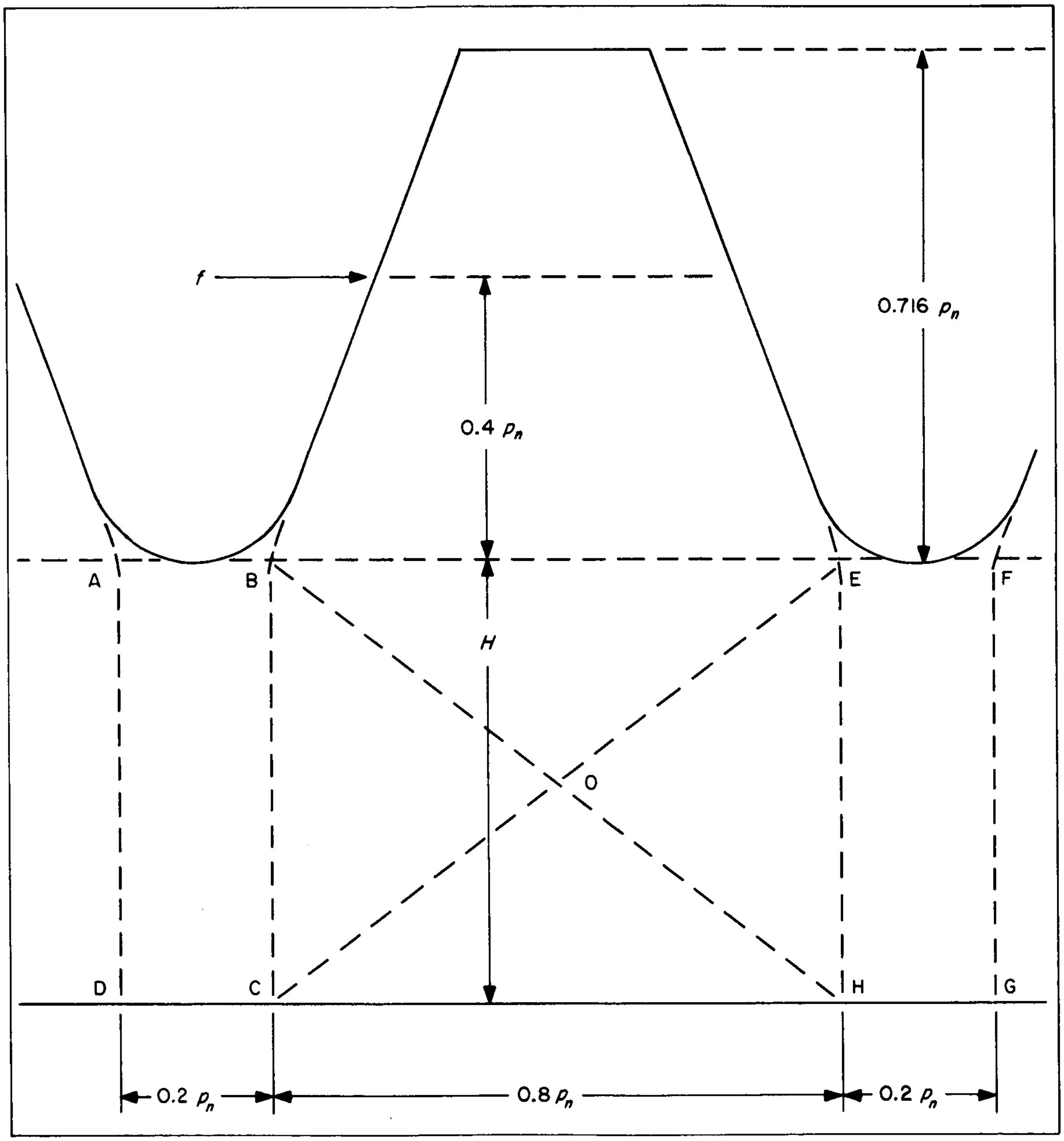

Fig. 5. Dimensions for gear teeth and rim

application of the load is the tilt multiplied by $0.4p_n$, since B and E have no horizontal movement due to this compliance. Thus, the compliance for the tooth in the rim is:

$$1/k_r = (1/G)(0.1p_n/H + 0.125)$$

Taking a as the ratio of rim depth to circular pitch, H/p_n, the expression for compliance becomes:

$$1/k_r = (1/G)(0.1/a + 0.125) \tag{24}$$

In addition to this tilting of the prism under the tooth, the rim undergoes some circumferential strain because of the transmission of the tooth load to the mass of the rim. Figure 6 represents the circumference of a toothed rim subject to a tangential force f per unit width at A.[24] Rotational acceleration is prevented by the tangential force f per unit width at B. (The effective depth of the rim may be somewhat larger than H because of the stiffening effect of the teeth. The effective depth is taken as $H + 0.2p_n$ or $(a + 0.2)p_n$.) If x is the circumferential movement of A relative to B because of the lengthening of arc AB and the shortening of arc ACB, the tension force in AB per unit width is the product of strain, effective depth, and the modulus of elasticity:

$$(x/AB)(a + 0.2)p_nE = (x/\theta)(a + 0.2)p_nE/r$$

The compressive force in arc ACB is given by a similar expression, and the sum of the two forces is equal to f:

$$f = (x/r)(a + 0.2)p_nE(1/\theta + 1/2\pi - \theta)$$

---

[24] *Ibid.*, p. 210.

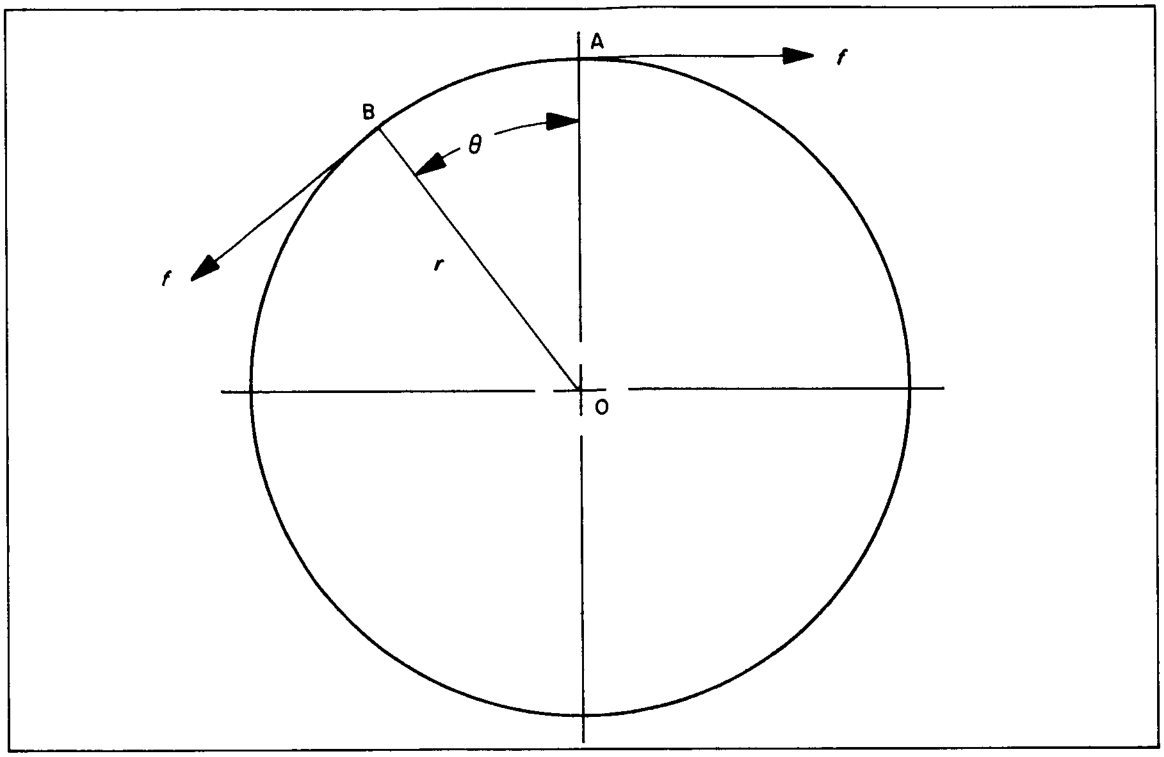

Fig. 6. Factors pertaining to stretch of rim

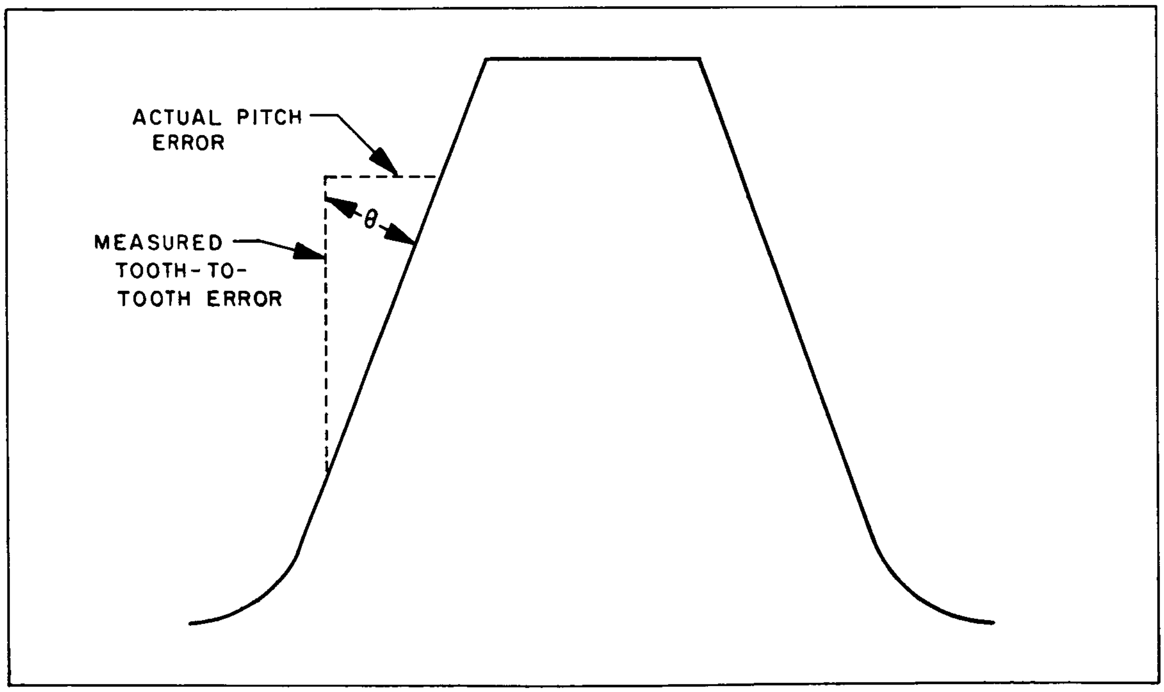

Fig. 7. Relationship of measured tooth-to-tooth error by center distance deviation to actual pitch error of gear teeth

The compliance of the rim for unit loading at points separated by an angle θ is:

$$r\theta(2\pi - \theta)/(a + 0.2)p_n E 2\pi$$

The average value of compliance for all values of θ from 0 to is:

$$1/k_c = \left[r/(2\pi^2)(a + 0.2)p_n E\right] \int_0^{\pi} \theta(2\pi - \theta)\,d\theta$$

$$= \pi r/(a + 0.2)p_n 3E$$

Since $2\pi r/p_n$ equals the number of teeth, N, in a spur gear, the circumferential compliance for the rim can be written as:

$$1/k_c = N/6(a + 0.2)E \tag{25}$$

The total compliance of the elastic connection between the two gear masses is simply the total of all the individual compliances:

$$\begin{aligned} 1/k_T = {} & 3(1/E_p + 1/E_g) + (1/G_p)(0.125 + 0.1/a_p) \\ & + (1/G_g)(0.125 + 0.1/a_g) \\ & + (1/6)\left[N_p/(a_p + 0.2)E_p + N_g/(a_g + 0.2)E_g\right] \end{aligned} \tag{26}$$

Tuplin continues by finding moments of inertia for the two mating gears. Since, in instrument gearing, the connected masses of the shafts and other parts are generally quite small in comparison with the gears themselves, only the gear inertia need be considered. Since all compliances discussed heretofore are for unit face widths, the inertias for the gears must also be per unit face width. The equivalent mass of each gear is the moment of inertia for a unit face width of the gear divided by the square of the radius of the pitch circle. This is in agreement with Buckingham except that Buckingham carries his work through on an actual gear thickness rather than working with unit face-width values. Although the effects are not exactly linear because of end effects, etc., no problem is anticipated using the two criterias since Tuplin's work is used only to find the effective error-in-action. The effective mass for Tuplin's formulation can be found by using values as calculated from Buckingham's approach and dividing by the face width. The result is an effective mass per unit face-width.

If the effective mass and the compliance of the dynamic system are known, the natural period of vibration can be calculated. The natural period for a system with two masses is:

$$T_1 = 2\pi \sqrt{\frac{1/k_T}{1/M_p + 1/M_g}} \tag{27}$$

The time of insertion of a pitch error is simply the time for the gear tooth to advance one position. This can be found by dividing the circular pitch by the pitch line velocity:

$$t_1 = 5p_n/V \tag{28}$$

where:

p_n = circular pitch, in.

V = pitch line velocity, ft/min

t_1 = insertion time, sec

If the time of insertion of a pitch error is large in comparison with the natural period T_1, the effective error-in-action is low and the dynamic loads are low. If, on the other hand, the time of insertion is very small in comparison with the natural period, the effective error-in-action is large, and a high dynamic load results.

Tuplin's research indicates that the adjacent pitch errors are much more important than the composite pitch errors in the determination of dynamic loads,[25] whereas the American Standards Association came to the opposite conclusion. Tuplin's line of thinking would seem to be correct in that the pitch error of adjacent teeth would cause most of the accelerations of the gear masses. Since the tooth-to-tooth error is usually measured by mating the gear with a master gear and observing the change in center distance as the gears rotate, the pitch error can be found by multiplying this tooth-to-tooth error by the tangent of the pressure angle. This fact is illustrated in Fig. 7. Since there are two gears in contact, this error is doubled to find the actual pitch errors for two mating gears:

$$E_1 = 2(\text{tooth-to-tooth error}) \tan \phi \tag{29}$$

The significant factor in this analysis is the summing of maximum pitch errors of any two adjacent teeth in one gear with the maximum pitch errors of any two adjacent teeth in the mating gear. The use of this sum is pessimistic to a certain degree since the maximum errors do not usually come together in every revolution of the pinion. If the number of teeth in the gear is exactly divisible by the number of teeth in the pinion, the maximum errors may never come together. Since there is some small probability that maximum errors will come together each revolution, it seems wise to use this value for calculations and realize that actual dynamic loads will never exceed the calculated values, thus incorporating conservatism in the analysis.

With the actual pitch error now defined in Eq. (29), an effective pitch error can be ascertained from the ratio of insertion time t_1 to

[25] Ibid., p. 210.

the natural period of vibration T. Tuplin has assumed that the pitch error can be equivalent to one of three types of wedges acting on the gear teeth.[26] The three types are:

1. Uniform taper
2. Concave circular arc
3. Convex circular arc

Rigorous calculations for each of the above types of wedges have been made for estimating effective pitch errors. One thing that became apparent, however, was that the type of wedge assumed seemed to have very little effect on the effective errors-in-action. The ratio of effective errors-in-action to the actual errors-in-action can be related to the ratio of insertion time to natural period of the dynamic system. Such a relationship is indicated in Fig. 8.[27] Thus an effective error-in-action can be calculated by knowing the actual pitch error from Eq. (29) and the ratio t_1/T_1. This effective error-in-action is then used with Buckingham's formulae to establish dynamic loads, which, in turn, are used with Hertz' equation to find surface compressive stresses. In addition, an important conclusion can be drawn from Fig. 8. If the ratio of t_1/T_1 is greater than 3.0, the effective error-in-action becomes essentially zero, and the dynamic load can be neglected. Thus, a criterion is established to determine when incorporation of dynamic loads in the gear analysis is necessary.

---

[26] Ibid., p. 205.

[27] Ibid., p. 207.

In the general discussion on wear in Chapter II, it was mentioned that several authorities were of the opinion that wear is a surface fatigue phenomenon. The surface fatigue life for a particular gear set can be found by first calculating the compressive stress from Hertz' equations. The average number of load cycles to failure for this stress can be determined from a fatigue life chart. Such a chart is illustrated in Fig. 9 for 303 stainless steel and 2024-T4 aluminum alloy.[28] The expected hours of operation can be derived as follows:

Let

E = expected life, hr

c = number of cycles to failure

n = angular velocity of gear being investigated, rpm

For every revolution of the investigated gear, any one surface undergoes one stress cycle. The expected life in minutes for the gear would simply be c/n. The expected life in hours would then become:

$$E = c/60n \tag{30}$$

[28] A. H. Maschmeyer, "Wear Life of Aluminum Gears," Product Engineering, 27 (September, 1956) p. 163.

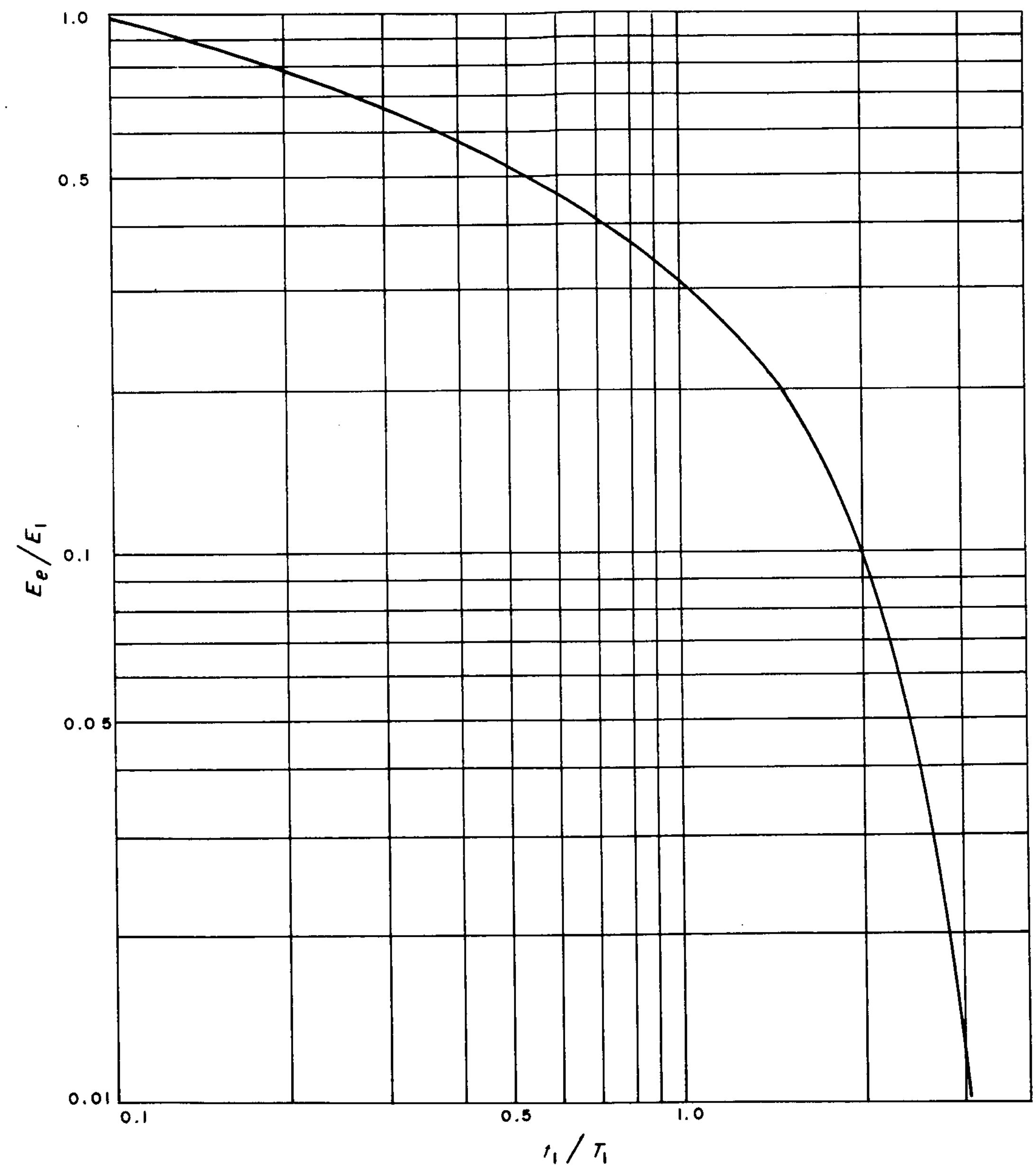

Fig. 8. Comparison of ratio of insertion time to natural period (t_1/T_1 and ratio of effective error in action to actual error in action

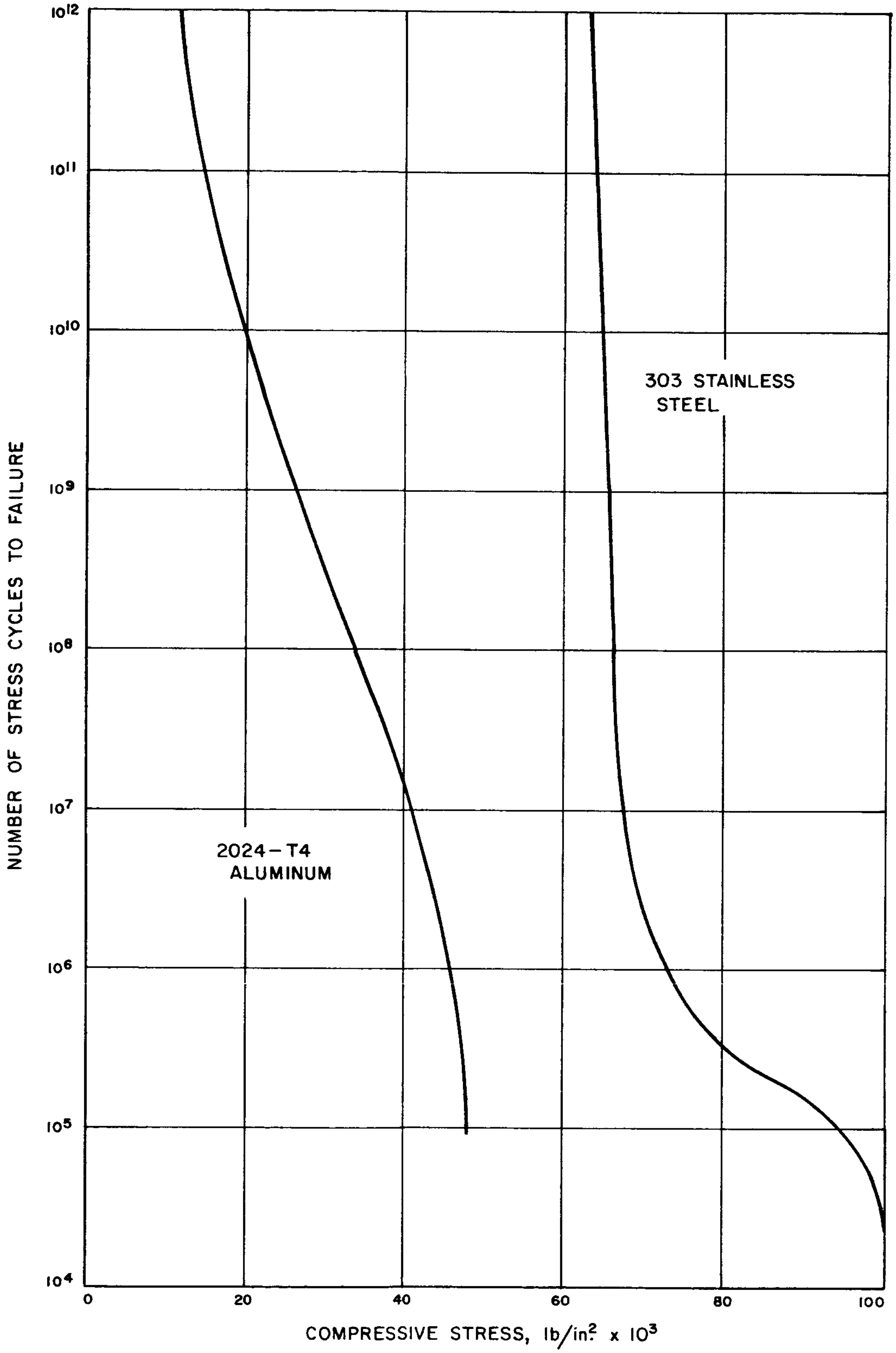

Fig. 9. Surface fatigue curves for 303 stainless steel and 2024-T4 aluminum

V. GEAR WEAR TESTS AND RESULTS

The analysis of wear on spur gears requires actual wear testing. Several wear test machines have been developed and used in the past in which two cylinders run against each other. With gearing between the two cylinders, the proportion of sliding to rolling action is variable, and the contact pressure can be varied to give the surface stress required. Since gears behave by sliding and rolling in varying proportions, it was decided that the best wear test for gears would be to measure wear of actual gears in operation rather than cylinders, or some other form of friction surfaces. Another factor which would be difficult to produce on rubbing surfaces other than gears is the surface finish left by a gear hob or shaper. Although it has been fairly well established that surface finish has little or no effect on wear, it is believed that wear data obtained directly from the gears would be more representative than would that obtained by duplicating all the variables on other rolling and sliding parts.

A gear testing fixture was built by simply using two 1/4-in. plates to hold the bearings, and by spacing these about an inch apart. These plates were precision jig-bored to reduce the possibility of misalignment of the gear sets and to insure proper operating center distances. A direct current gear motor was placed at the center and gear trains were originated at this point, extending radially in six directions. Precision 1 gears were used, and all had a pitch diameter of 1 in. and 96 diametral pitch. Precision 1 is an American Gear Manufacturers Association designation and indicates a total composite error of

0.0010 in. or less and a tooth-to-tooth error of 0.0004 in. or less. A stainless steel gear was placed on the motor to drive six aluminum gears. This procedure tended to equalize wear among the driving pinion and the driven gears. From the aluminum gears, the test gear trains were run. Figure 10 shows the test fixture with the top plate removed and one test gear train. Three meshes were used for each material combination in order to obtain statistical information about the wear patterns.

The material combinations used for the tests were:

1. 303 stainless steel on 303 stainless steel
2. 303 stainless steel on 2024-T4 aluminum
3. 303 stainless steel on 2024-T4 anodized aluminum
4. 303 stainless steel on 2024-T4 anodized aluminum treated with molybdenum disulphide
5. 303 stainless steel on delrin
6. 2024-T4 anodized aluminum on 2024-T4 anodized aluminum

Although many other material selections and combinations are available and in use, the above combinations are the most widely used in precision, nonlubricated gear trains. A set of useful tables or formula developed for these materials would be quite useful. The molybdenum disulphide was tested merely to ascertain the effects of a dry-film lubricant. Other dry-film lubricants are available, but the molybdenum disulphide seems to be the most popular because of its versatility. Graphite, for example, is a good dry-film lubricant under certain conditions. In a high vacuum, however, graphite tends to lose its absorbed moisture and become a severe abrasive. Molybdenum

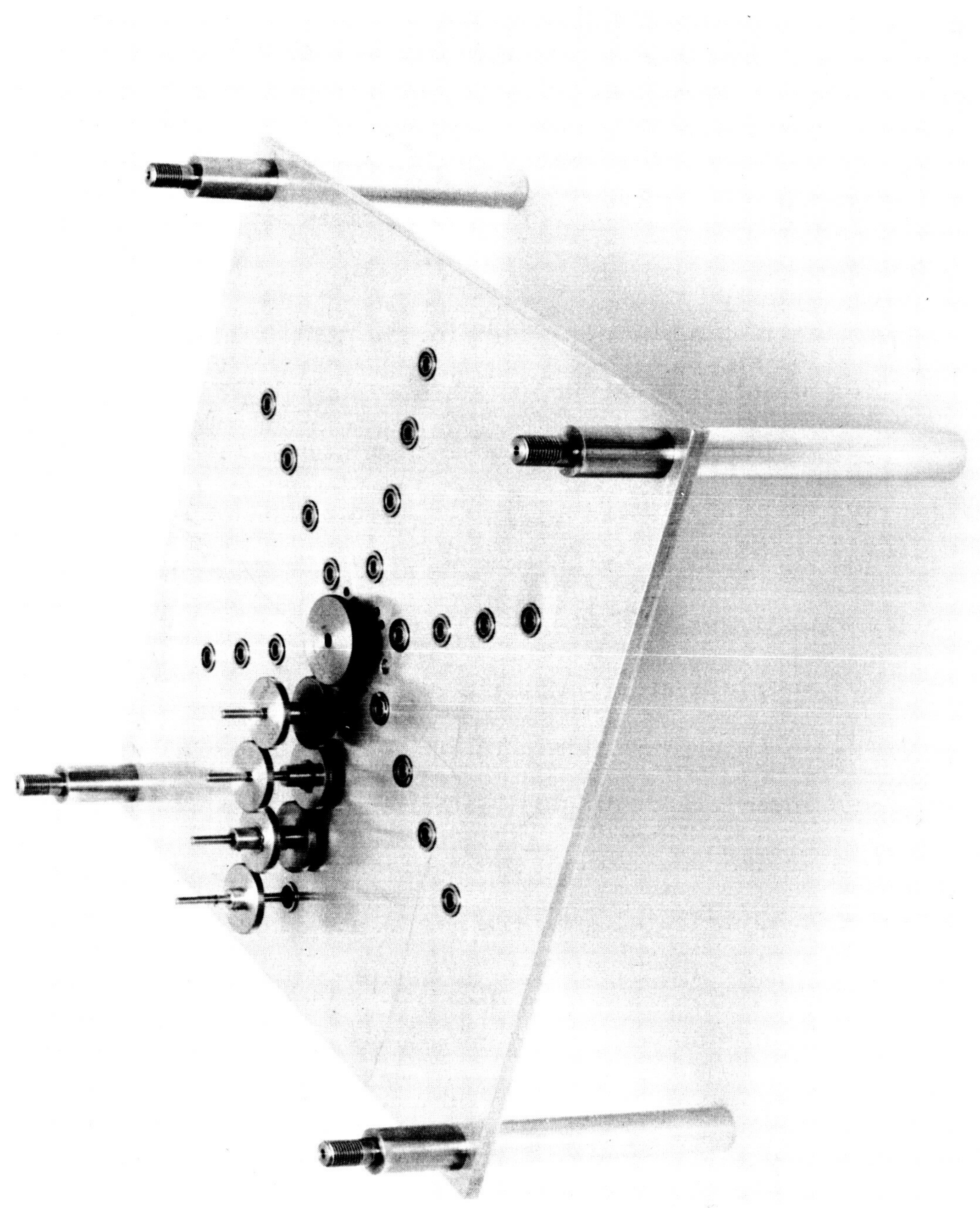

Fig. 10. Gear test fixture

disulphide, on the other hand, retains its lubricity quite well in a vacuum and is a good dry-film lubricant for space applications.

First, it was decided to use a friction clutch at the end of each train to provide the load on the gear teeth. However, with the possibility of placing the package on board a space probe for gear wear checks at extremely low pressures, a method was looked for which would consume a low amount of power. It was finally decided to load the gears in a manner so that the motor need only be large enough to supply the losses in the gear train.

Western Gear Company tests two sets of gears at once.[29] The gears are clamped on two parallel shafts with a small motor connected to one of the shafts. One gear is then rotated on its shaft until the torsional deflection in the opposite shaft gives rise to a tooth load equivalent to the actual working conditions. The small motor then brings the gears up to the desired speed. The gears are transmitting the design torques, but the motor is simply supplying the losses in order to maintain the speed.

The loading for the gear testing in this paper was accomplished in a similar manner. Antibacklash gears were used to establish the tooth loads. The tooth loads were established by using a torque wrench set at the desired load. One face of the antibacklash gear was held while the other face was turned through an angle with the torque wrench. When the load was reached, the mating pinion was brought into contact so that the antibacklash faces could not rotate, thus giving the desired preload on the gear teeth.

---

[29] Joseph Beggs, Mechanism, (New York, McGraw-Hill, 1955) pp. 94-95.

The motor used was a direct-current type. The tests were run at various speeds in order to obtain various data points that incorporated both dynamic and transmitted loads. The speed of the gear trains was varied by adjusting the motor voltage. Rotational speeds of 76, 380, and 3800 rpm were selected that resulted in pitch line velocities of 20, 100, and 1000, ft/min, respectively.

Class 7 bearings were used on the gear shafts to reduce the amount of play that might feed back to the wear measurements. All gear shafts were shimmed to reduce the radial play of the bearings to a minimum. The gears were checked for surface finish and found to have surfaces on the order of 16 μin. The gears were assembled on the gear shafts and the entire assemblies were ultrasonically cleaned to remove traces of lubricants or foreign matter. This cleaning was deemed necessary since the gear manufacturing process usually involves cutting or machining oils and an oil coating to reduce the possibility of rust or corrosion. A small amount of this oil would most likely have large effects on the observed life of a gear set. Although gears generate particles as they operate, it was also felt necessary to perform these tests in a dust free environment. This procedure would tend to reduce the hazards of foreign particles starting the chain reaction of the wearing process.

The wear was ascertained by measuring the rotation of one gear while the mating gear was held. A dial indicator and lever arms were used to record this angular rotation in inches. Although the gear meshes were antibacklashed, the amount a gear space exceeded the gear tooth thickness could be measured the same manner as regular backlash. Instead of measuring backlash as the free play from one direction to the other, the space is measured by pulling against the antibacklash spring from one side to the other side. More care must be used, however, since the limits are not as easily discernible. In order to eliminate eccentricities, total composite errors, and tooth-to-tooth errors, scribe lines were put on the gear faces so that "backlash" readings would always be taken at the same relative position. The increase in "backlash" is a direct measure of the depth of wear on the tooth surface. The amount of wear per revolution or cycle can be determined from backlash as follows:

$$h = \Delta B/60nt \tag{31}$$

where:

h = depth of wear rate, in./cycle

ΔB = change in backlash (finial - initial backlash), in.

n = speed of gear, rpm

t = time over which ΔB is taken, hr

The "backlash" was closely monitored during the early periods of operation when the wear rates were most erratic. As the tests progressed, wear data was taken at increased intervals. The tests were terminated when the "backlash" had increased by a value of 0.004 in. Although somewhat arbitrary, this value was selected since it represented roughly 25% of the tooth thickness on the pitch circle of a 96 pitch gear. The tooth thickness for a 96 pitch gear is approximately 0.016 in. at the pitch line. With a removal of 0.004 in. from the tooth thickness, the gear could no longer be classified as a precision gear. Since different systems can tolerate a different amount of profile removal before accuracy is impaired, the wear was determined in a rate of depth of wear per revolution so that the data could readily be used by many people designing systems.

In the discussion of gear testing, it should be pointed out that this testing was performed on gears of equal numbers of teeth on both the pinion and gear. Most authorities agree that gear trains should be designed with a "hunting tooth" arrangement to reduce wear. The "hunting tooth" arrangement is simply the selection of the gears of the proper number of teeth so that several revolutions are made before any two mating teeth come into contact again. The testing was purposely not conducted in this manner for several reasons. First, it is obviously not possible to design all gear trains with the "hunting tooth" scheme. Further, it was important, if possible, to achieve a reasonably accurate set of wear rates that could be used for all cases of nonlubricated gear designs, whether for the "hunting tooth" or the ordinary multiple arrangement. In short, a conservative but plausible method for predicting gear life was sought.

As mentioned previously, each material combination was tested at three values of load and speed. One series of tests run at 20 ft/min and a load of 3 in. -oz (6 in. -oz for 303 stainless steel on 303 stainless steel) was intended to be primarily a transmitted-load test, with little or no dynamic load. At the other extreme, a second series of tests, run at 1000 ft/min and with no load, was intended to a pure dynamic-loading test. The last series of run tests at 100 ft/min and 4 in. -oz was intended to be a combination of both dynamic loading and transmitted load. The three tests were performed to establish wear rates at various stress levels and also to use the wear rates to check on dynamic load calculations. Although loads cannot be found directly by knowing the wear rates, one can determine rather easily whether the calculated stresses are of the right order of magnitude. The higher the stresses for a given gear set, for example, the higher the wear rate will be. With this information, one can ascertain, to a limited degree, which methods are most nearly correct in calculating dynamic loads.

Stresses have been calculated for the above loads and speeds using Buckingham's formulae as outlined in Chapter IV. Two calculations of stress were made for each value of load and speed, using the two methods of determining effective errors-in-action. One stress calculation was based upon Eq. (14), while the other calculation was based on Tuplin's work. The calculations are performed in Appendices B and C.

The data collected for the gear test is given in Appendix D. Figures 11 through 28 illustrate this data pictorially. From the data, wear rates can be established. With the wear rate for the total gear set known, the amount of wear for the pinion or the gear can be determined from Eq. (4). For test gears of the same material, the wear rate on each gear is one half the total wear of the gear set. For gears of different materials, however, the softer gear is worn a greater percentage of the total than the harder gear. In several of the gear tests conducted, 303 stainless steel was run on itself, or on aluminum or delrin. Since the ratio of the yield strength of either aluminum or

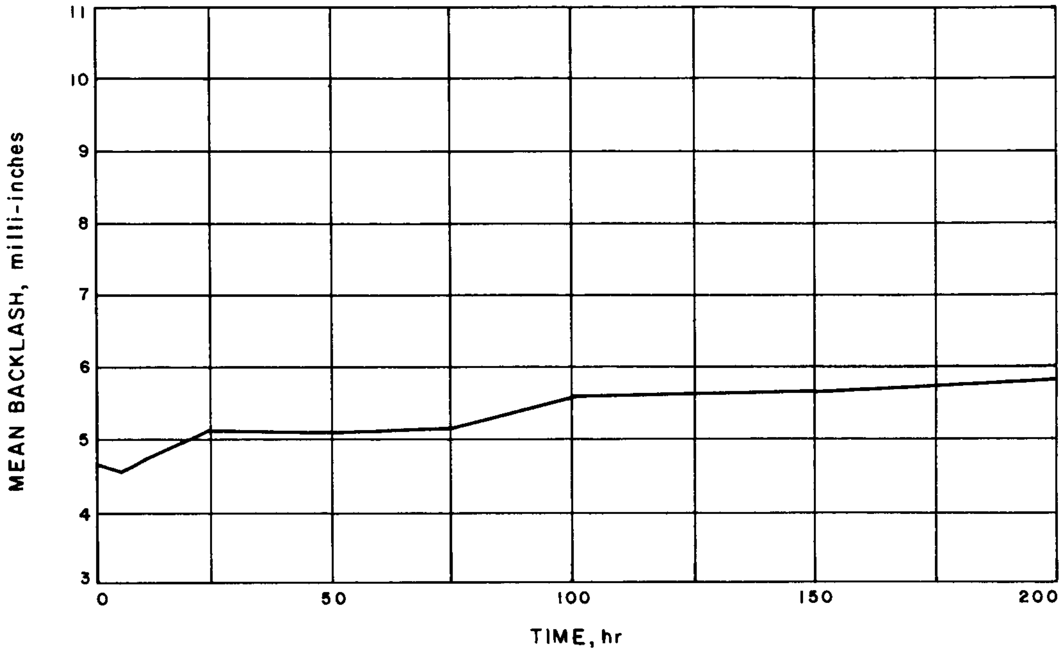

Fig. 11. Wear rate for 303 stainless steel on 303 stainless steel (no load, 3800 rpm)

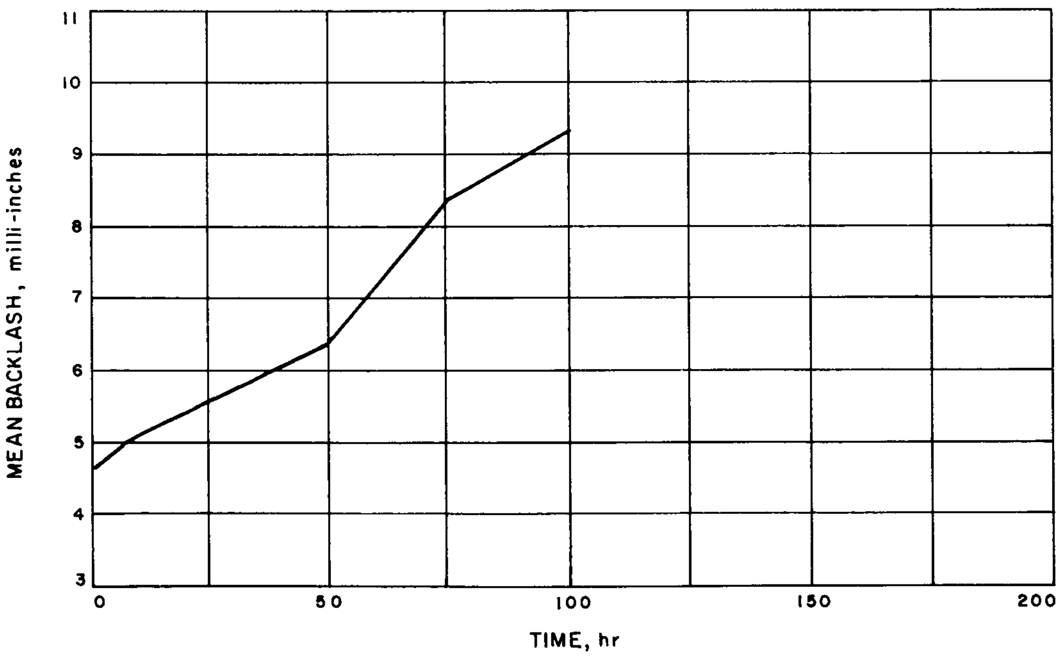

Fig. 12. Wear rate for 303 stainless steel on 2024-T4 aluminum (no load, 3800 rpm)

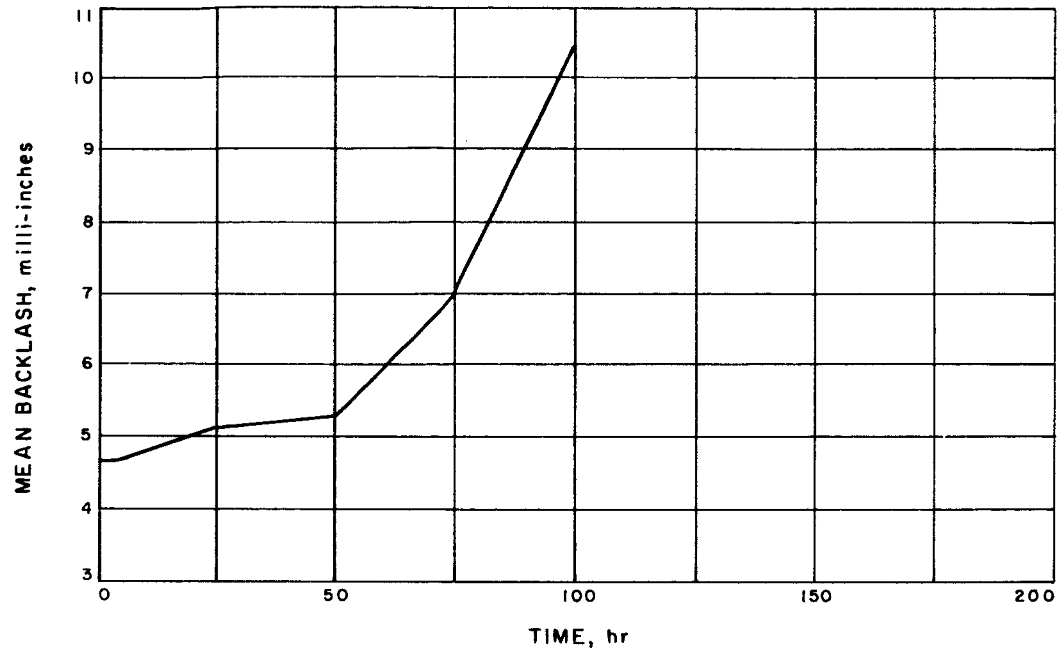

Fig. 13. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum (no load, 3800 rpm)

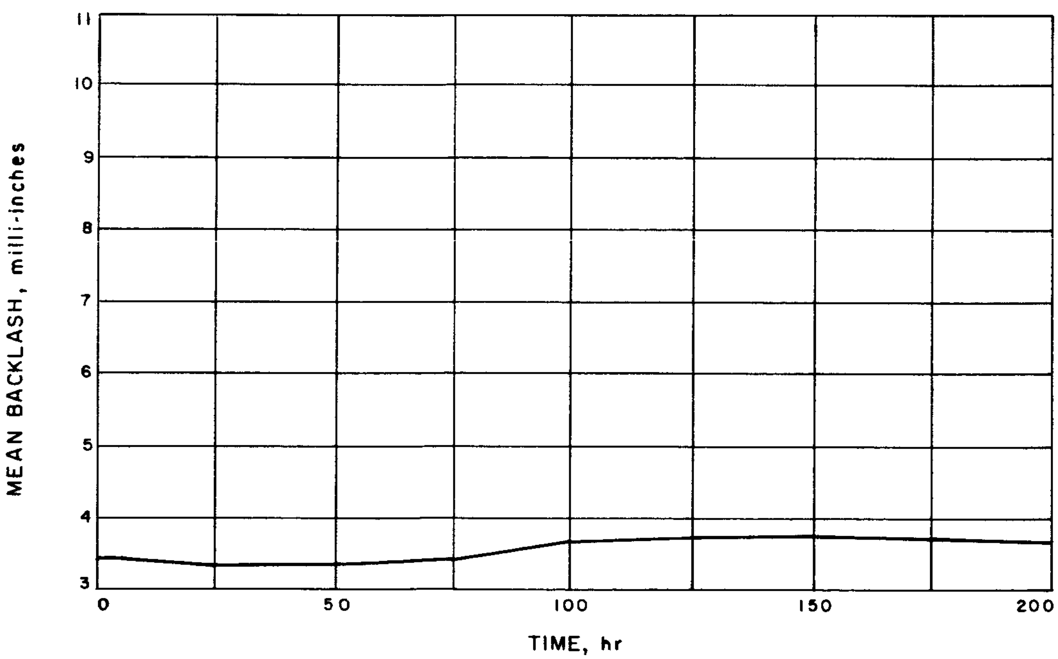

Fig. 14. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum treated with molybdenum disulphide (no load, 3800 rpm)

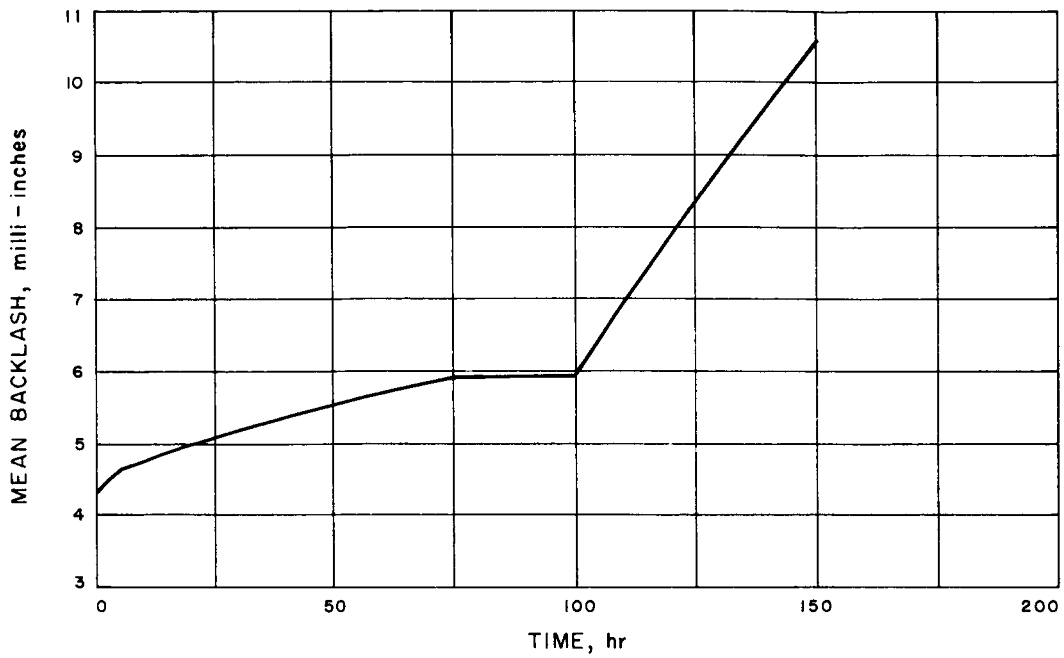

Fig. 15. Wear rate for anodized 2024-T4 aluminum on anodized 2024-T4 aluminum (no load, 3800 rpm)

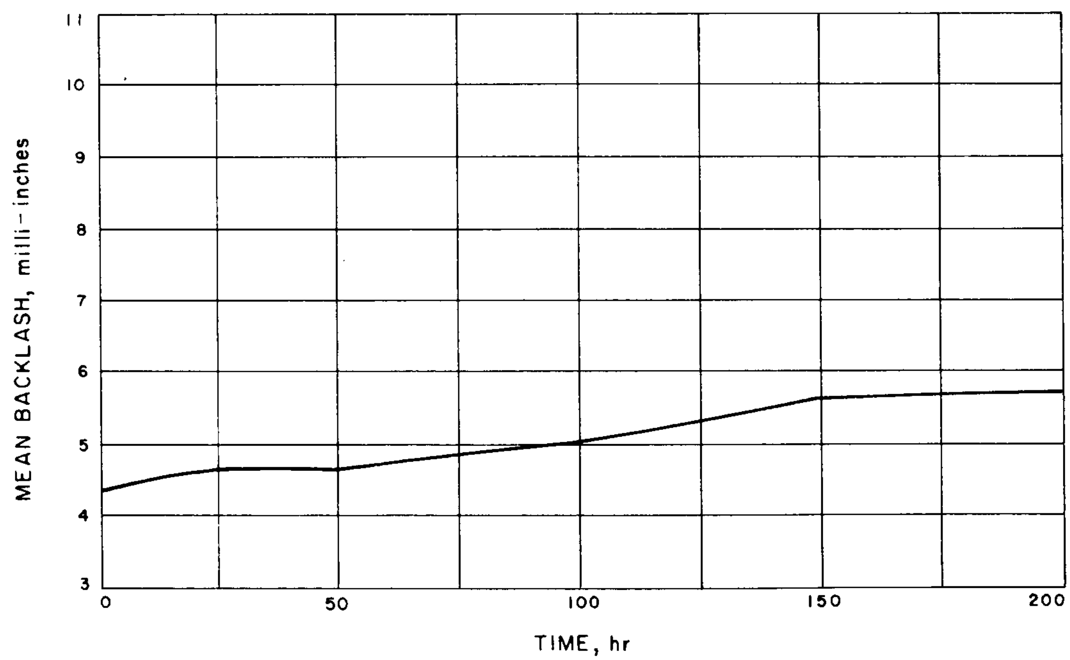

Fig. 16. Wear rate for 303 stainless steel on delrin (no load, 3800 rpm)

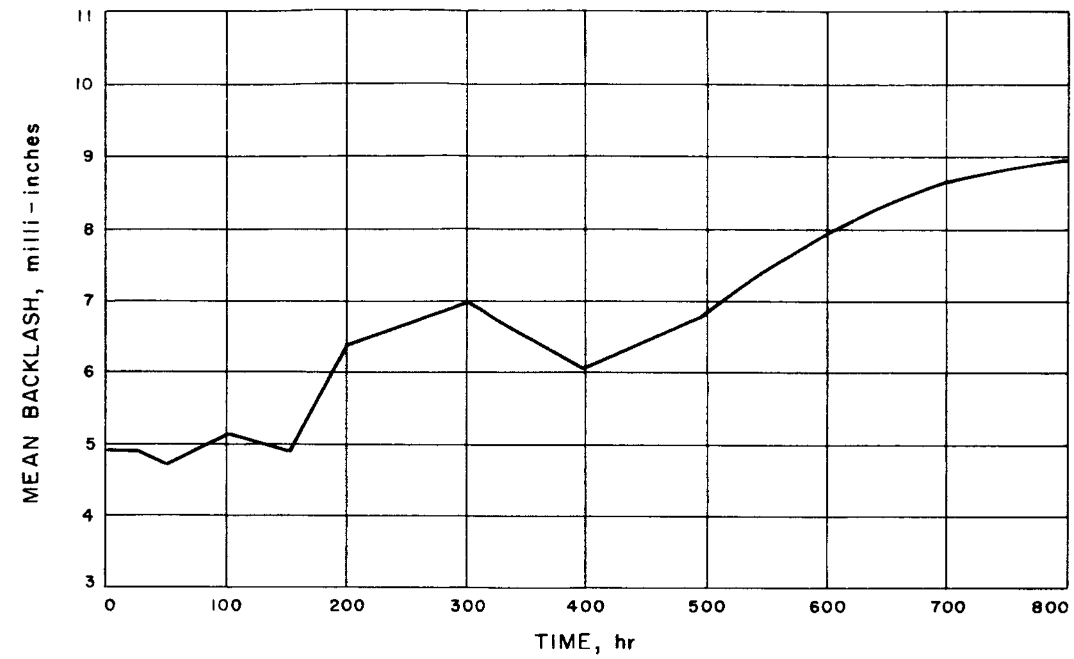

Fig. 17. Wear rate for 303 stainless steel on 303 stainless steel (4 in. -oz, 380 rpm)

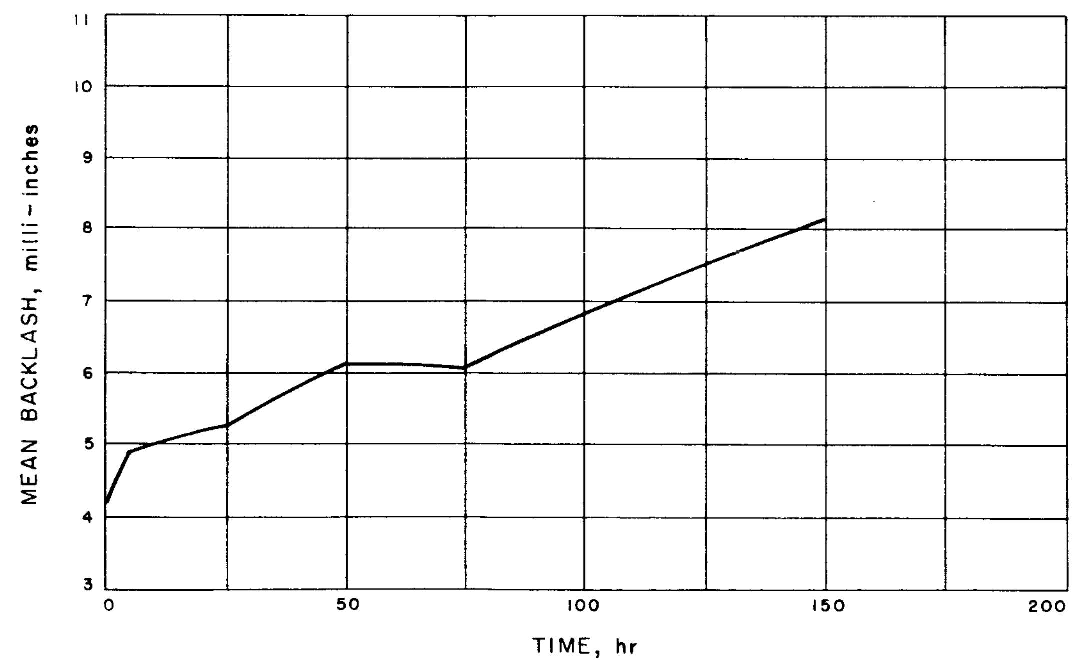

Fig. 18. Wear rate for 303 stainless steel on 2024-T4 aluminum (4 in. -oz, 380 rpm)

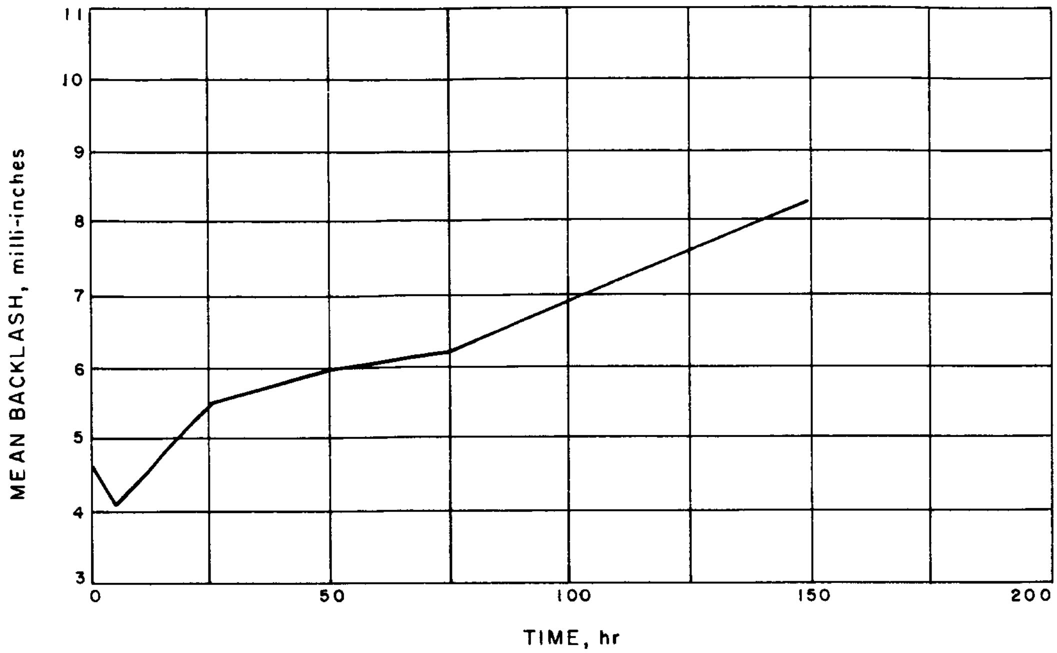

Fig. 19. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum (4 in. -oz, 380 rpm)

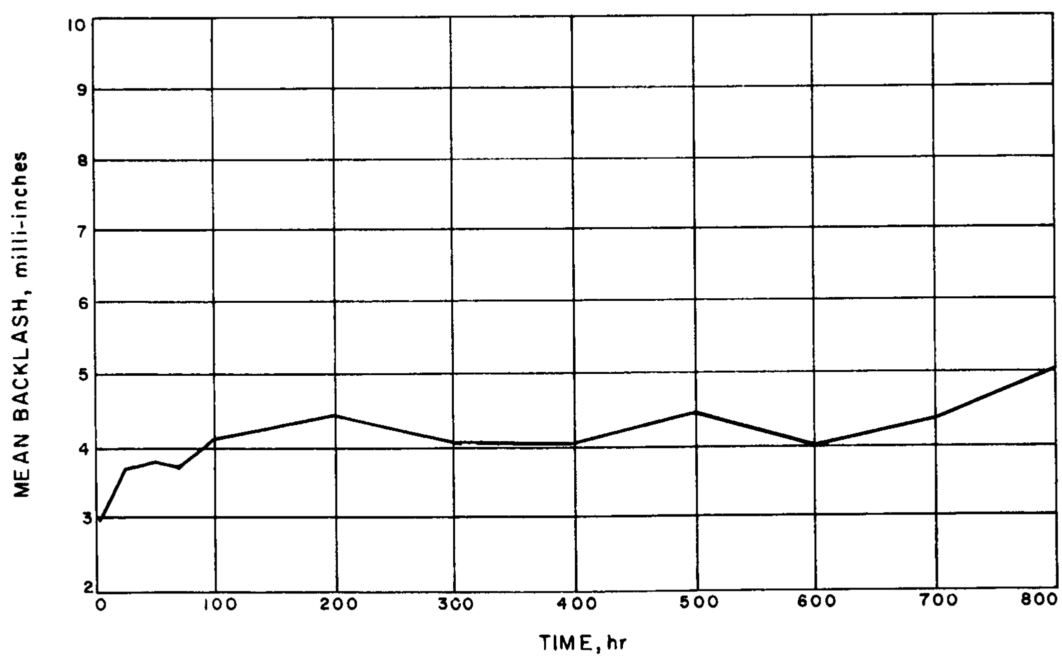

Fig. 20. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum treated with molybdenum disulphide (4 in. -oz, 380 rpm)

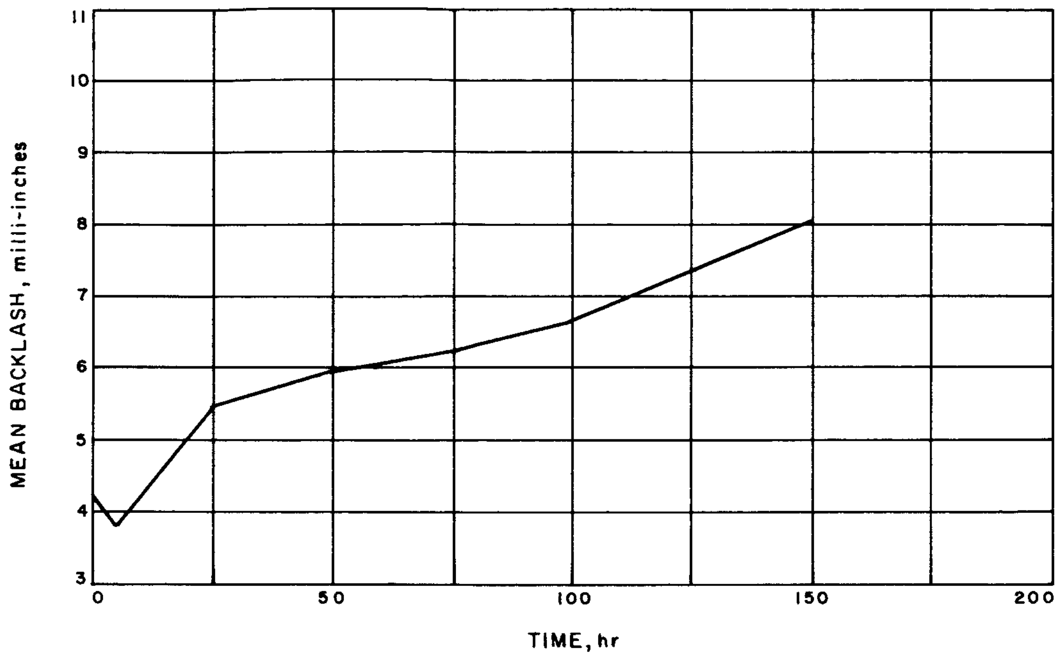

Fig. 21. Wear rate for anodized 2024-T4 aluminum on anodized 2024-T4 aluminum (4 in. -oz, 380 rpm)

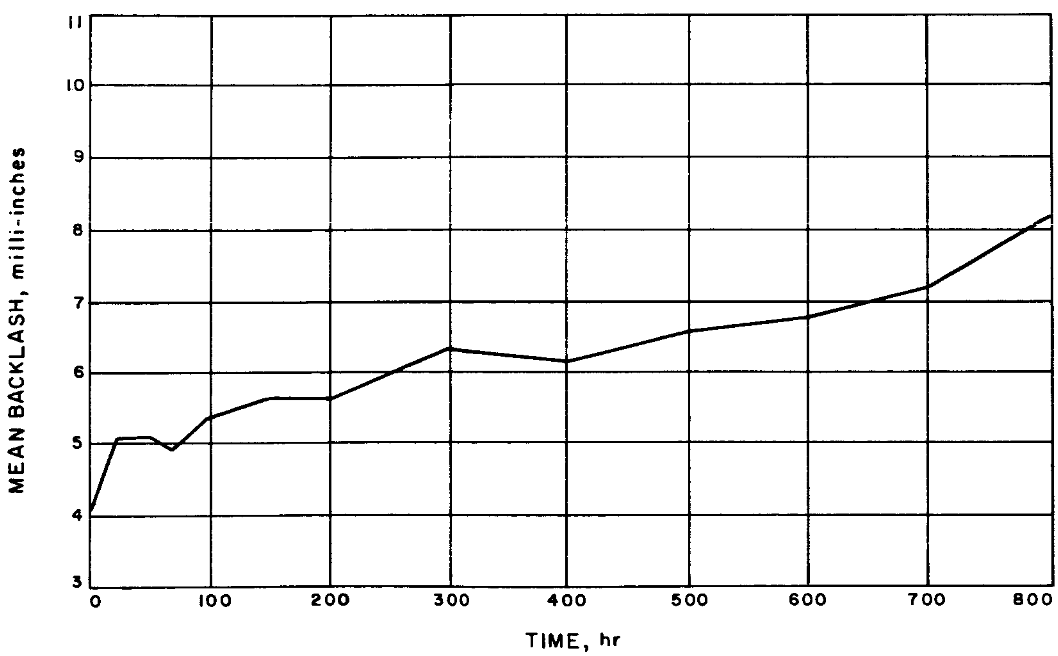

Fig. 22. Wear rate for 303 stainless steel on delrin (4 in. -oz, 380 rpm)

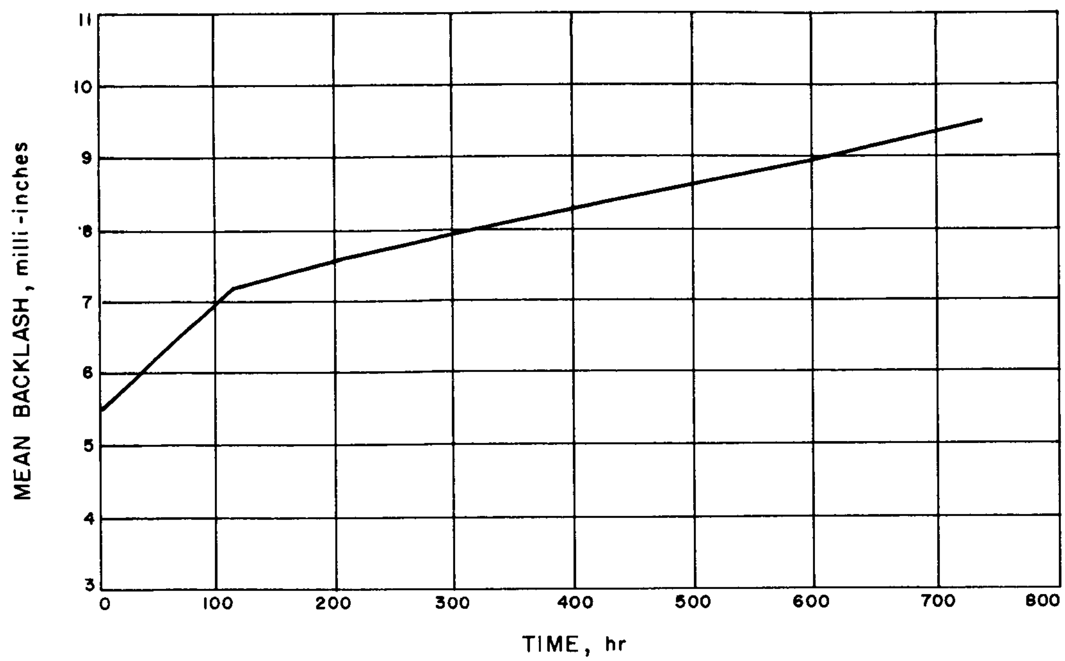

Fig. 23. Wear rate for 303 stainless steel on 303 stainless steel (6 in. -oz, 76 rpm)

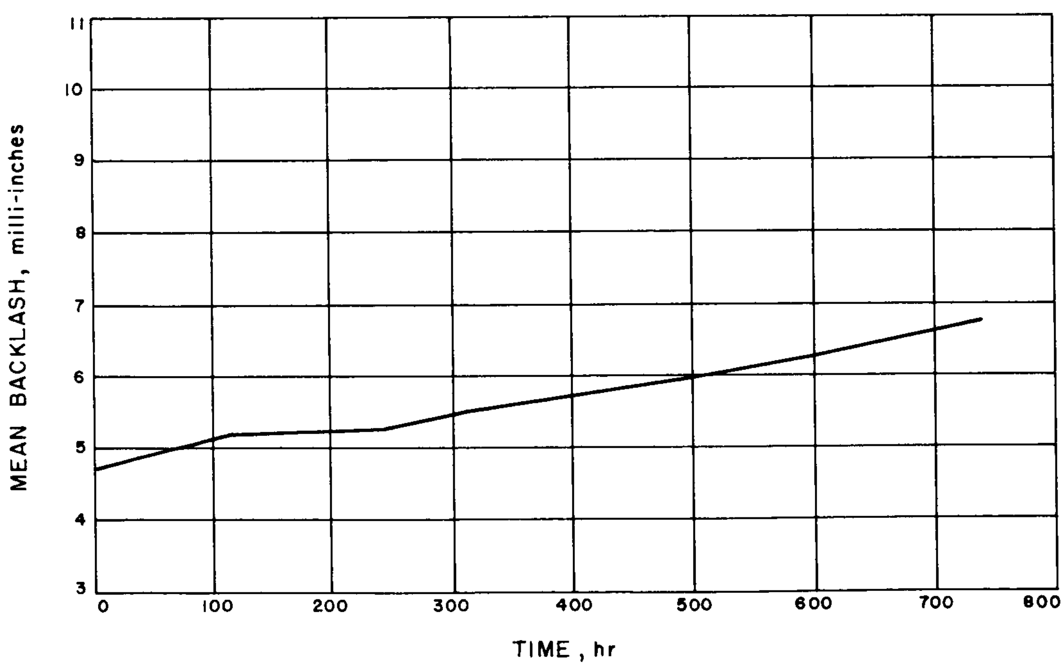

Fig. 24. Wear rate for 303 stainless steel on 2024-T4 aluminum (3 in. -oz, 76 rpm)

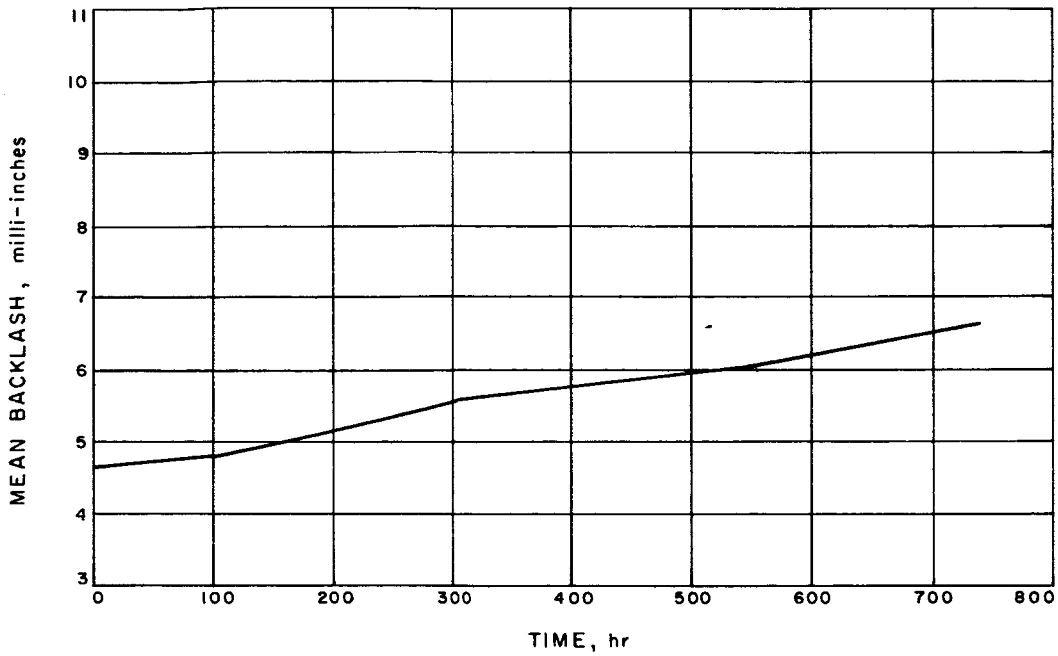

Fig. 25. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum (3 in. -oz, 76 rpm)

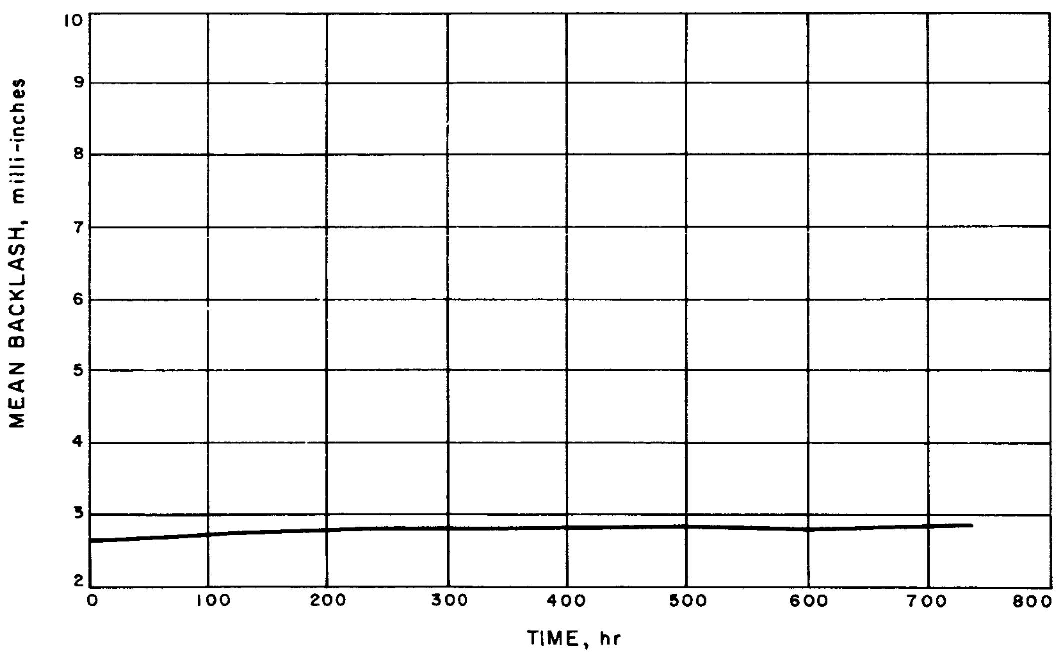

Fig. 26. Wear rate for 303 stainless steel on anodized 2024-T4 aluminum treated with molybdenum disulphide (3 in. -oz, 76 rpm)

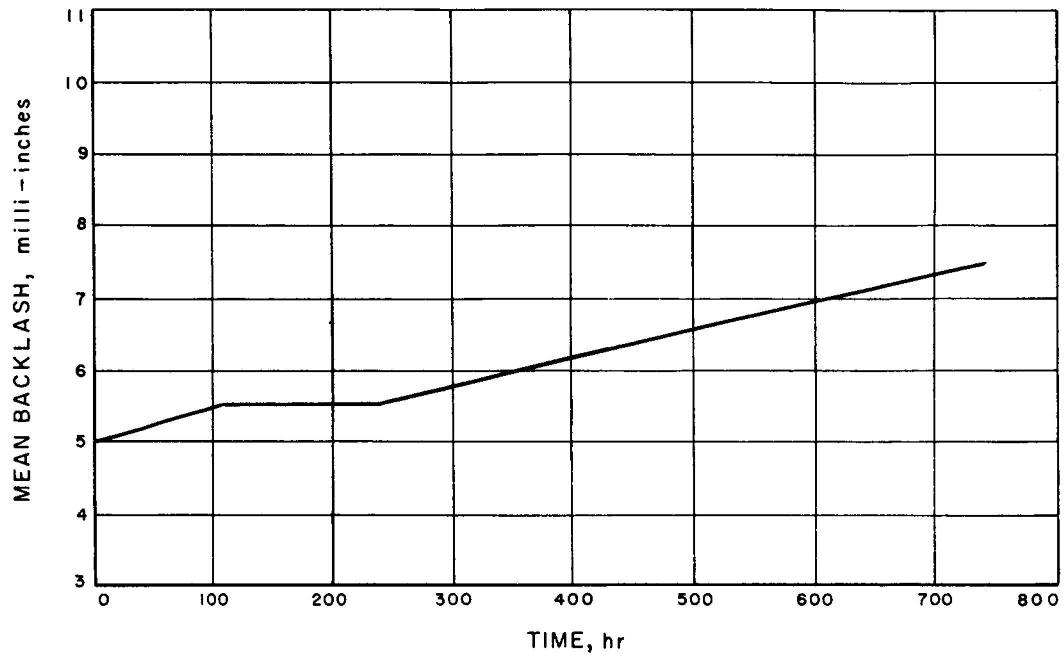

Fig. 27. Wear rate for anodized 2024-T4 aluminum on anodized 2024-T4 aluminum (3 in. -oz, 76 rpm)

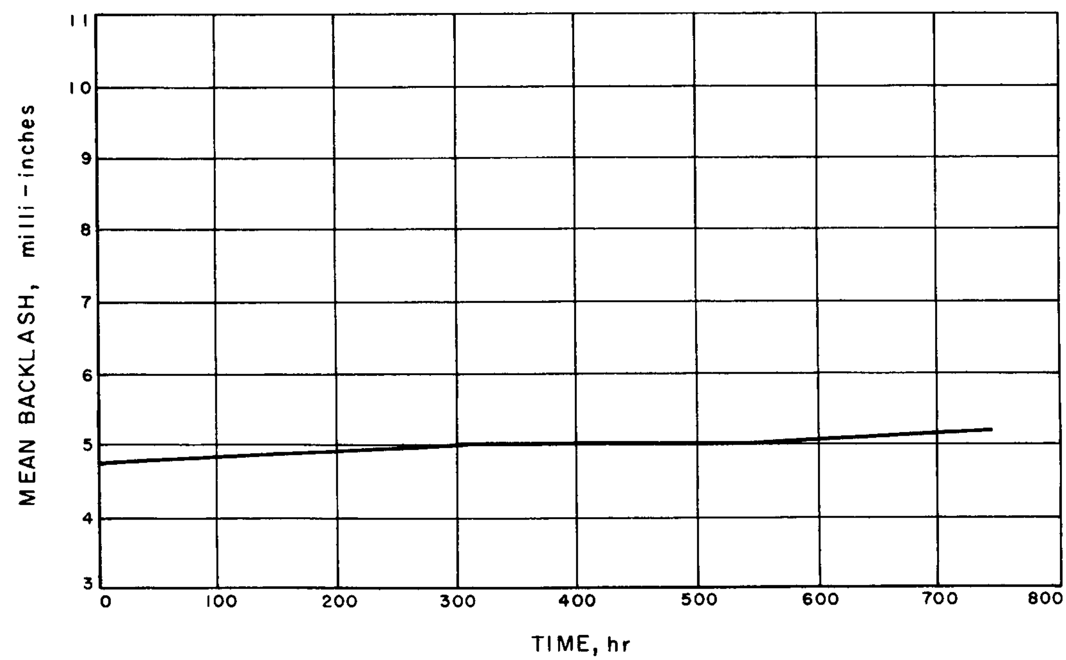

Fig. 28. Wear rate for 303 stainless steel on delrin (3 in. -oz, 76 rpm)

delrin to 303 stainless steel is small, the square of this value is even smaller, and most of the wear occurs in the softer aluminum or delrin. For the purposes of this paper, the total amount of wear will be assumed to be totally on either the delrin or aluminum when these materials are mated with stainless steel. For stainless steel or aluminum mated against itself, one half the wear is assumed to occur on each surface. Observations of the test gears under a microscope tend to support these assumptions.

Wear rates are calculated in Appendix E and the results are shown in Table 1, with calculated stresses. Investigation of this Table gives some indication of which stress calculations are more valid. The wear rates correlate quite well with the stresses calculated from Tuplin's method. On the other hand, the stresses as calculated using the American Standards Association Specification B6.11-1951 seem to have a negative correlation with the wear rate. With 303 stainless steel mating against 303 stainless steel, for example, the higher stress values seem to result in lower wear rates. Since this is contrary to the general theories on wear, all stresses used for purposes of establishing wear rates will be calculated using Tuplin's analysis for effective error-in-action, Buckingham's formulae for dynamic loads, and Hertz' equations for compressive stresses.

A second table can be derived from Table 1 giving the wear rate on a specific surface for a certain value of stress. These results are indicated in Table 2. From the information in Table 2, design curves can be drawn which relate calculated stresses to wear depth rates for the five materials and surfaces. These curves are illustrated in Fig. 29 through 33. Although Eq. (6) indicates that the relationship

Table 1. Wear rates and calculated stresses for material combinations and loads

Material and Load	Stress,* lb/in.²	Stress,** lb/in.²	Wear rates pico-inches/cycle
303 Stainless Steel on 303 Stainless Steel:			
a. 1000 ft/min, no load	66,000	19,500	11.6
b. 100 ft/min, 4 in.-oz	32,300	22,400	111
c. 20 ft/min, 6 in.-oz	29,500	27,400	605
303 Stainless Steel on 2024-T4 Aluminum:			
a. 1000 ft/min, no load	38,200	9,750	202
b. 100 ft/min, 4 in.-oz	21,300	16,900	1170
c. 20 ft/min, 3 in.-oz	16,100	15,100	575
303 Stainless Steel on Anodized 2024-T4 Aluminum:			
a. 1000 ft/min, no load	38,200	9,750	237
b. 100 ft/min, 4 in.-oz	21,300	16,900	1070
c. 20 ft/min, 3 in.-oz	16,100	15,100	585
303 Stainless Steel on Anodized 2024-T4 Aluminum Treated With MoS_2:			
a. 1000 ft/min, no load	38,200	9,750	14.6
b. 100 ft/min, 4 in.-oz	21,300	16,900	110
c. 20 ft/min, 3 in.-oz	16,100	15,100	59.4

*From ASA Spec. B6.11-1951

**From Tuplin's method

Table 1 (Cont'd)

Material and Load	Stress,* lb/in.^{2}	Stress,** lb/in.^{2}	Wear rates pico-inches/cycle
303 Stainless Steel on Delrin:			
a. 1000 ft/min, no load	3,720	1,960	29.4
b. 100 ft/min, 4 in.-oz	3,940	3,720	232
c. 20 ft/min, 3 in.-oz	3,270	3,220	118
Anodized 2024-T4 Aluminum on Anodized 2024-T4 Aluminum:			
a. 1000 ft/min, no load	27,200	5,740	82
b. 100 ft/min, 4 in.-oz	17,400	14,700	560
c. 20 ft/min, 3 in.-oz	13,500	12,800	363

*From ASA Spec. B6.11-1951

**From Tuplin's method

Table 2. Wear rates and stresses for test materials

Material	Stress, lb/in^2	Wear rate, pico-inches/cycle
303 Stainless Steel	19,500	11.6
	22,400	111
	27,400	605
2024-T4 Aluminum	9,750	202
	15,100	575
	16,900	1170
Anodized 2024-T4 Aluminum	5,740	82.0
	9,750	237
	12,800	363
	14,700	560
	15,100	585
	16,900	1070
Delrin	1,960	29.4
	3,220	118
	3,720	232
Anodized 2024-T4 Aluminum With MoS_2	9,750	14.6
	15,100	59.4
	16,900	110

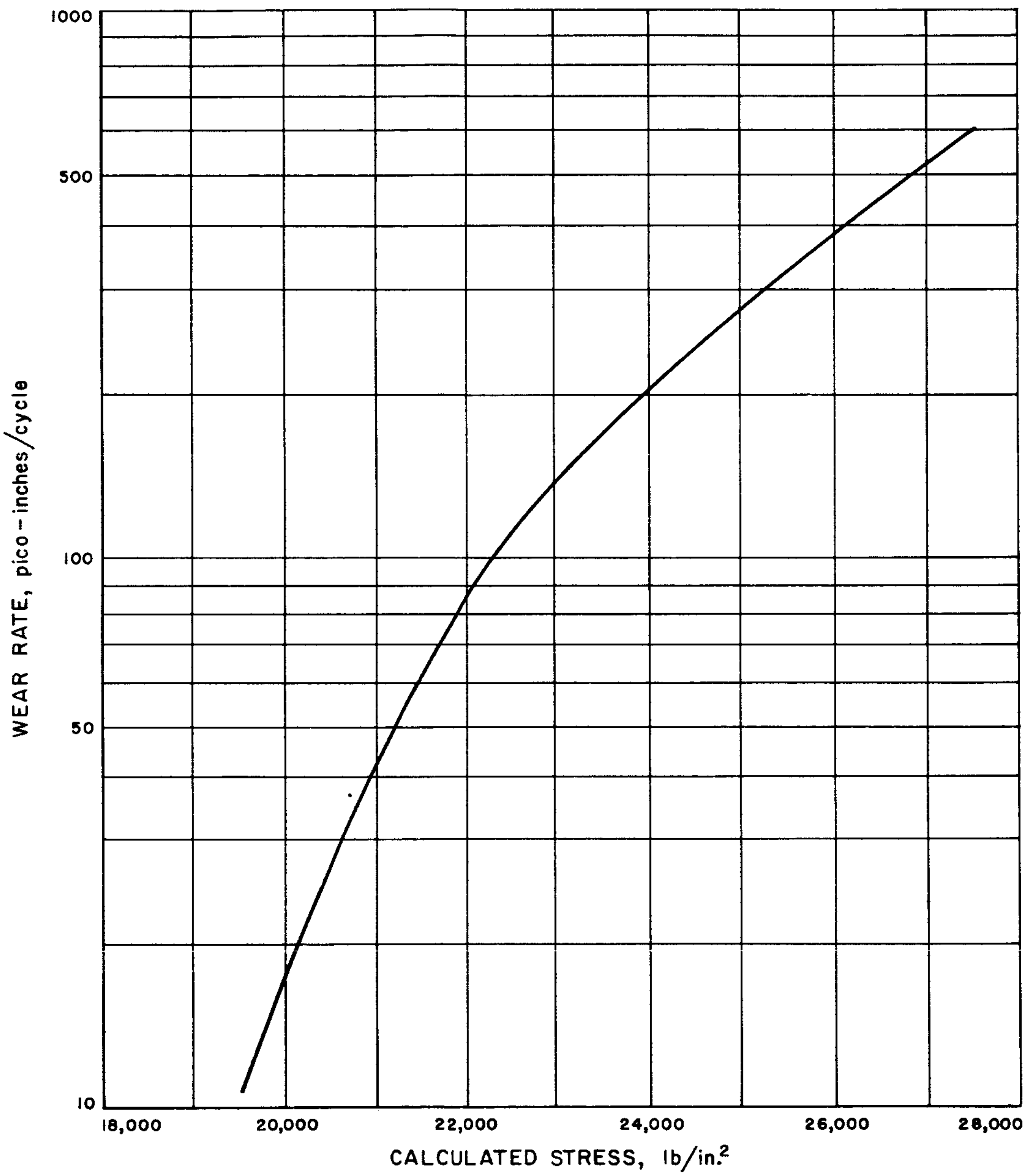

Fig. 29. Design curve for 303 stainless steel indicating calculated stress and corresponding depth-of-wear rate

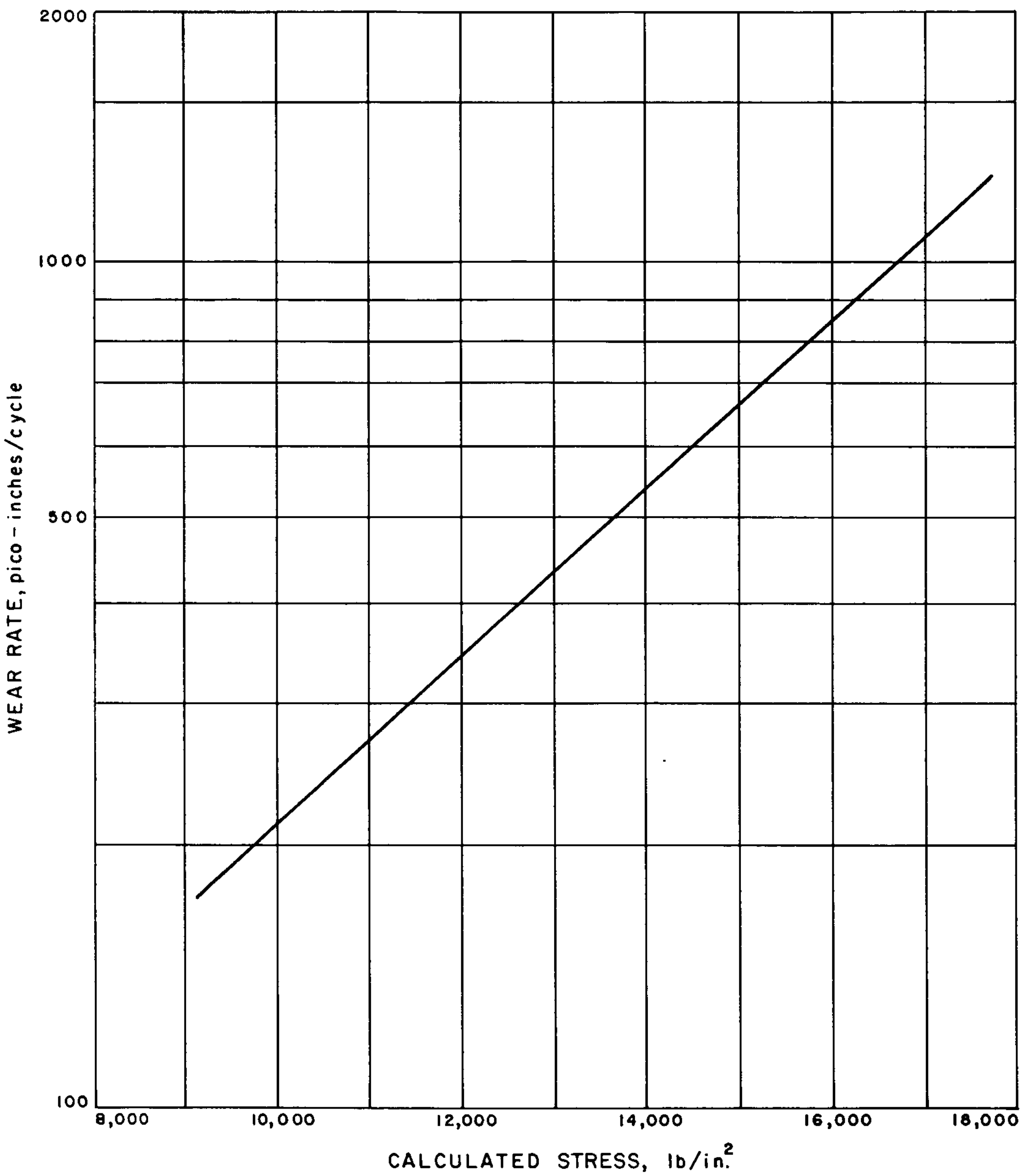

Fig. 30. Design curve for 2024-T4 aluminum indicating calculated stress and corresponding depth-of-wear rate

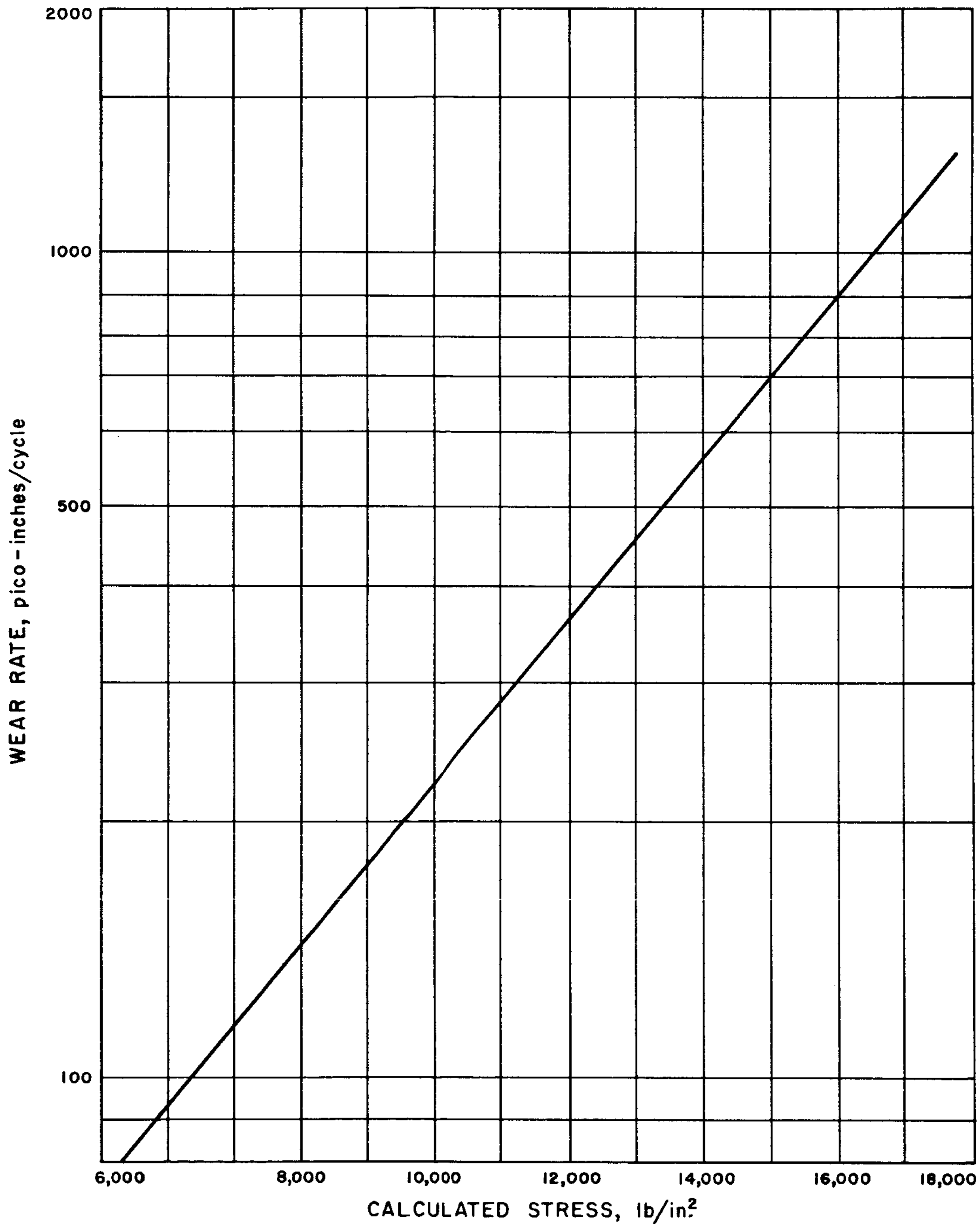

Fig. 31. Design curve for anodized 2024-T4 aluminum indicating calculated stress and corresponding depth-of-wear rate

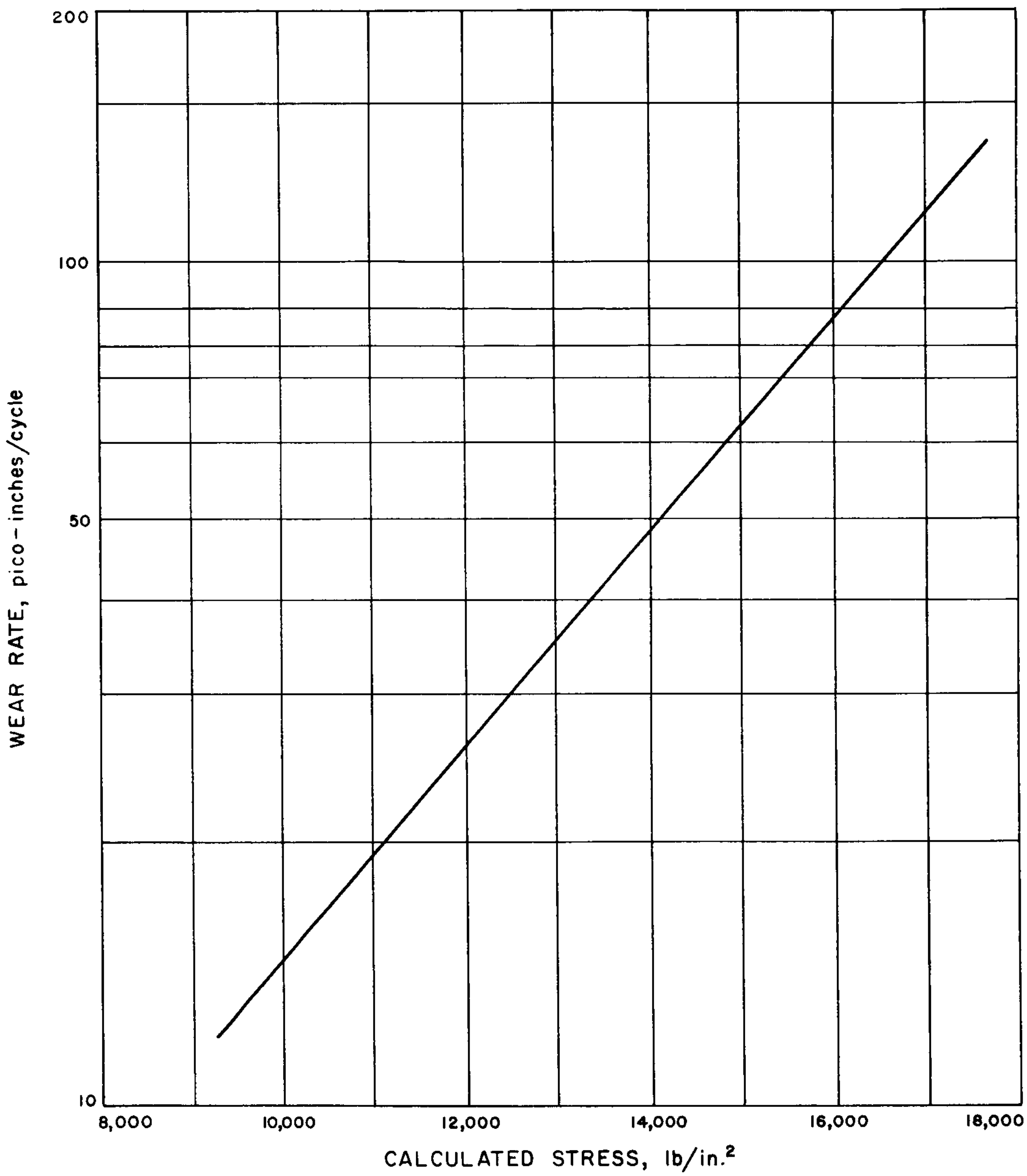

Fig. 32. Design curve for anodized 2024-T4 aluminum treated with molybdenum disulphide indicating calculated stress and corresponding depth-of-wear rate

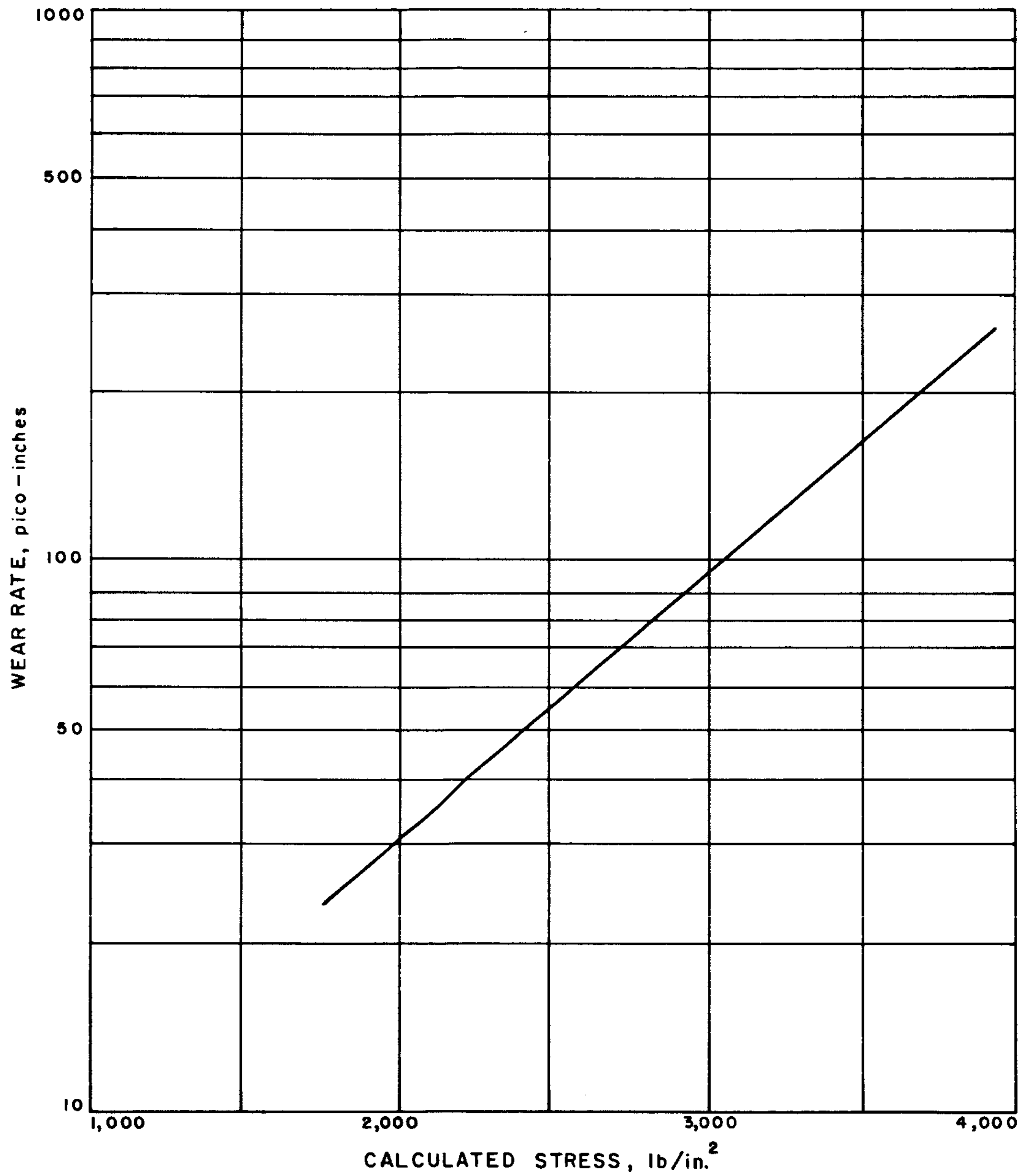

Fig. 33. Design curve for delrin indicating calculated stress and corresponding depth-of-wear rate

between stress and wear should be linear, the data points fall more nearly in a straight line if the stress and wear depth rates are plotted on logrithmic scales. In any case, the rate of wear for each gear of a set can be determined by knowing the stress levels and using the appropriate wear curves. For a typical gear set in which the pinion is usually much smaller than the gear, the depth-of-wear per revolution can be found for both the pinion and the gear. Then, realizing that the pinion travels through more revolutions than the gear in proportion to the number of teeth on each gear, the total wear rate for the gear set can be determined.

As an example, assume that it is desirable to determine the wear rate of a pinion and gear of not necessarily the same material, and that the gear is five times the diameter of the pinion. The stress can be calculated as previously outlined, and, with the design curves, wear-depth rates can be established for this stress for both the pinion and the gear. The wear-depth rates will be in terms of depth-of-wear per revolution of the gear in question. For every revolution of the gear, the pinion will have traveled through five revolutions. Thus, the total wear per revolution of the gear, or five revolutions of the pinion, would simply be the sum of the wear-depth rates per revolution of the gear and five times the wear per revolution of the pinion. This can be generalized into an equation for establishing total wear rates:

$$W_{Tp} = W_p + (N_p/N_g)W_g \tag{32}$$

or

$$W_{Tg} = (N_g/N_p)W_p + W_g \tag{33}$$

where:

W_{Tp} = total wear of gear set per revolution of the pinion, in./cycle

W_{Tg} = total wear of gear set per revolution of the gear, in./cycle

W_p = wear of pinion per revolution, in./cycle

W_g = wear of gear per revolution, in./cycle

N_p = number of teeth in pinion

N_g = number of teeth in gear

If the pinion is of one material and the gear of another, the appropriate chart is consulted and W_p is found for the pinion. In a similar manner, W_g is found for the gear. These terms are then combined by either Eq. (32) or (33) to arrive at the total wear rate for the gear set.

It is interesting to note the effects of the molybdenum disulphide on wear rates. The wear rate decreased by an order of magnitude from plain anodized aluminum to anodized aluminum treated with molybdenum disulphide. It cannot be ascertained whether the same effect could be observed on other test surfaces because of the limited testing that was performed. It can definitely be concluded that the molybdenum disulphide dry-film lubricant does help, but its use in precision instrument gearing is not recommended where high accuracy is required. The molybdenum disulphide coating ranges from 0.0002 to 0.0005 in. thick and this amount would radically change the characteristics of the involute gear surface.

In the discussion of wear, it was pointed out that several authors believed that wear is a surface-fatigue phenomenon. Fatigue lives have been calculated in Appendix F for the test gears using the fatigue curves in Fig. 9. Table 3 summarizes stresses and fatigue lives for the test gears. Since 10^6 hr is roughly equivalent to 100 yr, it can be rather emphatically said that the wear observed on the test gears was of some other nature than fatigue. This statement does not imply that fatigue is not an important problem. It does imply, however, that wear and surface fatigue are vastly different when applied to nonlubricated surfaces. Wear, in addition to fatigue, must be considered whenever nonlubricated gear systems are proposed. Fig. 29 through 33 provide a basis for predicting lives of nonlubricated gear trains caused by wear alone.

Table 3. Stresses and calculated fatigue lives for the test gears

Material and Load*	Stress, lb/in.^{2}	Fatigue life, hr
303 Stainless Steel on 303 Stainless Steel:		
a. 1000 ft/min, no load	19,500	∞
b. 100 ft/min, 4 in.-oz	22,400	∞
c. 20 ft/min, 6 in.-oz	27,400	∞
303 Stainless Steel on 2024-T4 Aluminum:		
a. 1000 ft/min, no load	9,750	∞
b. 100 ft/min, 4 in.-oz	16,900	2.6×10^6
c. 20 ft/min, 3 in.-oz	15,100	22×10^6
2024-T4 Aluminum on 2024-T4 Aluminum:		
a. 1000 ft/min, no load	5,740	∞
b. 100 ft/min, 4 in.-oz	14,700	4.3×10^6
c. 20 ft/min, 3 in.-oz	12,800	66×10^6

*Fatigue data not available for delrin

VI. SUMMARY

The application of the wear data to qualitatively analyze dynamic loads and stresses is one of the most significant discoveries to result from this study. Since the object of this paper was to find wear rates for calculated stresses, this might sound as though the "cart is being placed before the horse." But the fact remains that the observed wear gave definite clues as to which stress calculation seemed correct. Using this as a guide, it was then decided to use Tuplin's method to establish an effective error-in-action rather than using the American Standards Association Specification B6. 11-1951. This effective error-in-action was then used with Buckingham's equations to establish dynamic loads, and Hertz's equation was used to find the corresponding stress. It should be noted that the use of B6. 11-1951 resulted in dynamic loads much higher than actual loads, and, as such, gave results which tended to be safe.

Wear rates were established for five materials or surface conditions with the parts running in a normal atmosphere and nonlubricated. They were:

1. 303 stainless steel
2. 2024-T4 aluminum
3. Anodized 2024-T4 aluminum
4. Anodized 2024-T4 aluminum treated with molybdenum disulphide
5. Delrin

The wear rate for gear sets can be established for any combination of the above materials. Although all the data were taken on test gears of

96 diametral pitch, the wear-rate curves are applicable to all fine-pitch gears. A change in pitch effectively changes the sliding distance by changing the length of the tooth. This is compensated for, however, by the volume of wear, and, thus, the depth of wear would remain the same.

To summarize the method used to establish wear rates for nonlubricated spur gears:

1. Find transmitted load from: $W_t = T/R$, lb

2. Find the pitch line velocity from: $V = (\text{rpm})\pi R/6$, ft/min

3. Find the time of insertion from: $5p_n/V$, sec

4. Find the effective mass of each gear from:

   $m = \text{Weight}/(386 \times F)$, lb-sec^2/in.^{2}

5. Find the rim depth from:

   H = outside radius - tooth thickness - bore, in.

6. Find a from: $a = H/p_n$

7. Find the compliance of the gear set from:

$$1/k_T = 3(1/E_p + 1/E_g) + (1/G_p)(0.125 + 0.1/a) + (1/G_g)(0.125 + 0.1/a) + (1/6)\left[N_p/(a + 0.2)E_p + N_g/(a + 0.2)E_g\right]$$

8. Find the natural period of the gear set from:

$$T_1 = 2\sqrt{\frac{1/k_T}{1/M_p + 1/M_g}}, \text{ sec}$$

9. Find the insertion time to natural period ratio from:

$$t_1/T_1$$

10. Find the actual error-in-action from:

$$E_1 = 2(\text{tooth-to-tooth error}) \tan \phi$$

11. Find the ratio E_e/E_1 from Fig. 8 and the ratio t_1/T_1

12. Find the effective error-in-action E_e from 10 and 11 above:

13. Find f_a from:

$$f_a = 0.0012(1/R_p + 1/R_g) \left[M_p M_g/(M_p + M_g)\right] V^2$$

14. Find f_d from:

$$f_d = W_t + F \left[(0.111)E_e/(1/2E_p + 1/E_g)\right]$$

15. Find f_m from:

$$f_m = f_a f_d/(f_a + f_d)$$

16. Find the dynamic load from:

$$W_d = \sqrt{f_m(2f_d - f_m)} \text{ , lb}$$

17. Find the total load from:

$$W_T = (W_t + W_d)/F, \text{ lb/in.}$$

18. Find the stress from:

$$S_c = 0.84 \sqrt{(E_p E_g)/(E_p + E_g) \quad (D_p + D_g)/(D_p D_g) \quad W_T/\sin \phi}$$

19. Find the wear depth per revolution for the pinion and the gear from Fig. 29 through 33

20. Find the total wear rate for the gear set from:

$$W_{Tp} = W_p + (N_p/N_g)W_g \quad \text{or}$$

$$W_{Tg} = W_g + (N_g/N_p)W_p \text{ , in./revolution}$$

Although only five materials or surfaces are covered in this paper, they are the most widely used in industry today. The techniques have been established here, and an extension of this work could be performed. There are undoubtedly other materials and surfaces of interest, as well as different conditions. Testing in a vacuum, for example, would most likely lead to a different set of wear curves. At any rate, the work in this paper does provide a basis for establishing wear rates for precision, instrument, nonlubricated, fine pitch, spur gears for a very limited number of materials and/or surfaces. The reader is cautioned that the design curves relate actual wear to calculated stresses. If a safety factor is desired, the wear rates should be increased proportionately.

REFERENCES

1. American Gear Manufacturers Association. "Standard Nomenclature of Gear Tooth Wear and Failure." American Gear Manufacturers Association, Specification 110.02, 1954.

2. American Standards Association. "Inspection of Fine Pitch Gears," American Standards Association, Specification B6.11-1951, 1951.

3. Archard, J. F. "Contact and Rubbing of Flat Surfaces." Journal of Applied Physics, 24:981-988, 1953.

4. Beggs, Joseph S. Mechanism. New York, McGraw-Hill, 1955.

5. Bowden, F. P. and D. Tabor. The Friction and Lubrication of Solids. Oxford, Clarendon Press, 1950.

6. Brownsdon, H. "Metallic Wear." Journal of the Institute of Metals, 18:15-27, 1936.

7. Buckingham, Earle. Analytical Mechanics of Gears. New York, McGraw-Hill, 1949.

8. Buckingham, Earle. Spur Gears. New York, McGraw-Hill, 1928.

9. Burwell, John T. Mechanical Wear. New York, American Society for Metals, 1950.

10. Burwell, J. and C. Strang. "On the Empirical Law of Adhesive Wear." Journal of Applied Physics, 23:18-28, 1952.

11. Carmichael, Colin. Kent's Mechanical Engineers' Handbook. New York, John Wiley, 1955.

12. Dudley, Darle W. Gear Handbook. New York, McGraw-Hill, 1962.

13. Dudley, Darle W. Practical Gear Design. New York, McGraw-Hill, 1954.

14. Maschmeyer, A. H. "Wear Life of Aluminum Gears." Product Engineering, 27:160-166, September, 1956.

15. Merritt, H. E. Gears. New York, Pitman Publishing Company, 1955.

16. Price, R. "Gears Have Special Lube Problems." Plant Engineering, 16:119-123, May, 1962.

17. Rabinowicz, Ernest. "New Coefficients Predict Wear of Metal Parts." Product Engineering, 29:71-73, June, 1958.

18. Rabinowicz, Ernest. "Stick and Slip." Scientific American, 194:109-118, May, 1956.

19. Rabinowicz, Ernest. "Surface Energy and Friction and Wear Phenomena." Journal of Applied Physics, 32:1440-1444, 1961.

20. Rabinowicz, Ernest. "Wear." Scientific American, 206:127-136, January, 1962.

21. Read, Robert H. "Metal Wear." Frontier, 25:8-13, August, 1962.

22. Roark, R. J. Formulas for Stress and Strain. New York, McGraw-Hill, 1943.

23. Ryder, E. A. "A Gear and Lubricant Tester." American Society for Testing Materials Bulletin, 148:69-73, October, 1947.

24. Savage, R. "Graphite Lubrication." Journal of Applied Physics, 19:1-10, 1948.

25. Spurr, R. T. "Creep and Static Friction." British Journal of Applied Physics, 6:402-403, 1955.

26. Spurr, R. and T. Newcomb. "The Friction and Wear of Various Materials Sliding Against Unlubricated Surfaces of Different Types and Degrees of Roughness." Proceedings of the Conference on Lubrication and Wear. October 1-3, 1957, London. The Institution of Mechanical Engineers, 1957. pp. 269-275.

27. Taylor, R. and W. Holt. "Effect of Roughness of Cast Iron Brake Drums in Wear Tests of Brake Linings." Journal of Research, National Bureau of Standards, 27:395, 1941.

28. Tuplin, W. Gear Load Capacity. New York, John Wiley, 1962.

29. Tuplin, W. "Dynamic Loads on Gear Teeth." Machine Design, 25:203-211, October, 1953.

30. Underwood, O. A. "Operating Life of Gears." Product Engineering, 32:65-74, May, 1962.

31. Way, S. "Pitting Due to Rolling Contact." Journal of Applied Mechanics, 2:49-58, 1935.

32. Whittaker, E. J. W. "Friction and Wear." Nature, 159:541, 1947.

APPENDIX A

Sliding Velocity of Gears

The maximum sliding velocity of a gear set is produced at the point where the gears first come into contact and at the point where the gears last contact during a given mesh. The maximum sliding velocity for a given radius from Eq. 8 is:

$$V_s = V\left(1/R_1 + 1/R_2\right)\left(\sqrt{r_1^{\,2} - R_{b1}^{\,2}} - R_1 \sin\phi)\right)$$

The maximum sliding velocity achievable with a set of precision fine-pitch gears can be found as follows:

1. Maximum practical pitch line velocity is 1000 ft/min
2. Coarsest pitch is about 48 diametral pitch
3. Pressure angle ϕ of 20° is almost universally used
4. The smallest radii (R_1 and R_2) for 48 pitch gears is about 0.300 in.

From the above assumptions and the standard configurations, the following information can be listed:

1. $R_1 = R_2 = 0.300$ in.
2. $R_{b1} = R_{b2} = (0.300) \cos 20° = 0.282$ in.
3. $R_{o1} = R_{o2} = 0.321$ in.
4. $V = 1000$ ft/min

Thus, the maximum sliding velocity to be expected with a set of precision instrument spur gears is:

$$V_s = 1000\left(1/0.3 + 1/0.3\right)\left(\sqrt{(.321)^2 - (.282)^2} - 0.3 \sin 20°\right)$$

$$V_s = 6667\,(\sqrt{0.103 - 0.079} - 0.102)$$

$$V_s = 6667\,(0.155 - 0.102) = 353 \text{ ft/min}$$

The maximum sliding velocity for the test gears can be calculated in a similar manner. For test gears:

1. $R_1 = R_2 = 0.500$ in.
2. $R_{b1} = R_{b2} = (0.500) \cos 20° = 0.470$ in.
3. $R_{o1} = R_{o2} = 0.5104$
4. $V = 20$, 100, and 1000 ft/min

<u>For 20 ft/min</u>

$$V_s = 20 \left(1/0.5 + 1/0.5\right)\left(\sqrt{(0.500)^2 - (0.470)^2} - 0.5 \sin 20°\right)$$

$$= 20 (4) (\sqrt{.0293} - 0.171)$$

$$V_s = 80 (0.1711 - 0.171) = 0.008 \text{ ft/min}$$

<u>For 100 ft/min</u>

$$V_s = 100 \left(1/0.5 + 1/0.5\right)\left(\sqrt{(0.500)^2 - (0.470)^2} - 0.5 \sin 20°\right)$$

$$= 100 (4) (\sqrt{.0293} - 0.171)$$

$$V_s = 400 (0.1711 - 0.171) = 0.04 \text{ ft/min}$$

<u>For 1000 ft/min</u>

$$V_s = 1000 \left(1/0.5 + 1/0.5\right)\left(\sqrt{(0.500)^2 - (0.470)^2} - 0.5 \sin 20°\right)$$

$$= 1000 (4) (\sqrt{.0293} - 0.171)$$

$$V_s = 4000 (0.1711 - 0.171) = 0.4 \text{ ft/min}$$

APPENDIX B

Calculation of Test Gear-Tooth Loads and Stresses Using Buckingham's Formulae and American Standards Association Specification B6.11-1951

For purposes of the calculations, the following numbers will be used:

1. 303 Stainless Steel:

   (a). $\rho = 0.283$ psi

   (b). $E = 30 \times 10^6$ psi

   (c). $G = 12 \times 10^6$ psi

2. 2024-T4 Aluminum:

   (a). $\rho = 0.095$ psi

   (b). $E = 12 \times 10^6$ psi

   (c). $G = 4 \times 10^6$ psi

3. Delrin:

   (a). $\rho = 0.052$ psi

   (b). $E = 0.41 \times 10^6$ psi

   (c). $G = 0.16 \times 10^6$ psi

4. Error-in-action:

$$E_1 = \text{total composite error} + 1/2\ (\text{tooth-to-tooth error})$$

$$= 0.0010 + 1/2\ (0.0004) = 0.0012 \text{ in.}$$

The calculations are divided into three sections, one for each value of load and speed.

1. $V = 1000$ ft/min, no load

   $\therefore W_t = 0$ lb

(a). 303 Stainless Steel on 303 Stainless Steel

$$m_e = w_p w_g / 2_g (w_p + w_g)$$

$$w_p = \pi R^2 F \rho$$

$$w_p = (3.14)(0.5)^2 (0.0625)(.283)$$

$$= 0.014 \text{ lb}$$

$$w_g = (3.14)(0.5)^2 (0.125)(.283)$$

$$= 0.028 \text{ lb}$$

$$m_e = (0.014)(0.028)/\left[2(386)(0.014 + 0.028)\right]$$

$$= 1.21 \times 10^{-5} \text{ lb-sec}^2/\text{in.}$$

$$f_a = 0.012 (1/R_1 + 1/R_2) m_e V^2$$

$$= 0.0012 (1/0.5 + 1/0.5)(1.21 \times 10^{-5})(1000)^2$$

$$= 0.058$$

$$f_d = W_t + F\left[(0.111) E_1/(1/2E_p + 1/E_g)\right]$$

$$= 0 + .062\left[(0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/30)\right]$$

$$= 165$$

$$f_m = f_a f_d/(f_a + f_d)$$

$$= (165)(0.058)/(0.058 + 165)$$

$$= 0.058$$

$$W_d = \sqrt{f_m (2f_d - f_m)}$$

$$= \sqrt{(0.058)(330 - 0.058)}$$

$$= 4.37 \text{ lb}$$

Total Load

$$W_T = (W_d + W_T)/F$$

$$W_T = (4.37 + 0.0)/0.062$$

$$= 70 \text{ lb/in.}$$

Stress

$$S_c = 0.84 \quad E_p E_g/(E_p + E_g) \quad (D_p + D_g)/D_p D_g \quad W_T/\sin 20°$$

$$= 0.84 \quad (15 \times 10^6)\,(1)\,(70/0.34)$$

$$= 66,000 \text{ psi}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

$$w_p = 0.014 \text{ lb from (a)}$$

$$w_g = (3.14)\,(0.5)^2\,(0.125)\,(0.095)$$

$$= 0.0093 \text{ lb}$$

$$m_e = (0.0093)\,(0.014)/\left[2(386)\,(0.0093 + 0.014)\right]$$

$$= 7.25 \times 10^{-6} \text{ lb-sec}^2\text{/in.}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(7.25 \times 10^{-6})\,(1000)^2$$

$$= 0.0348$$

$$f_d = 0.062\,\left[(0.111)\,(0.0012)/(10^{-6}/60 + 10^{-6}/13)\right]$$

$$= 83$$

$$f_m = (83)\,(0.0348)/(83 + 0.0348)$$

$$= 0.0348$$

$$W_d = \sqrt{(0.0348)\,(166 - 0.0348)}$$

$$= 2.40 \text{ lb}$$

Total Load

$$W_T = (0.0 + 2.40)/0.062$$

$$= 38.8 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)\,(12)\,(10^6)/42\right]\,(1)\,(38.8/0.34)}$$

$$= 38{,}200 \text{ psi}$$

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

$$w_g = 0.093 \text{ lb from (b)}$$

$$w_p = (3.14)\,(0.5)^2\,(0.062)\,(0.095)$$

$$= 0.0047 \text{ lb}$$

$$m_e = (0.0047)\,(0.0093)/\left[2\,(386)\,(0.0047 + 0.0093)\right]$$

$$= 4.1 \times 10^{-6} \text{ lb-sec}^2/\text{in.}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(4.1 \times 10^{-6})\,(1000)^2$$

$$= 0.0197$$

$$f_d = 0.062\left[(0.111)\,(0.0012)/(10^{-6}/24 + 10^{-6}/13)\right]$$

$$= 72 \text{ lb}$$

$$f_m = (72)\,(0.0197)/(72 + 0.0197)$$

$$= 0.0197$$

$$W_d = \sqrt{(0.0197)\,(144 - 0.0197)}$$

$$= 1.69 \text{ lb}$$

Total Load

$$W_T = (0.0 + 1.69)/0.062$$

$$= 27.2 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(12)\,(12)\,(10^6)/24\right]\,(1)\,(27.2/0.34)}$$

$$= 27{,}200 \text{ psi}$$

(d). 303 Stainless Steel on Delrin

$$w_p = 0.014 \text{ lb from (a)}$$

$$w_g = (3.14)(0.5)^2 (0.125)(0.052)$$

$$= 0.0051 \text{ lb}$$

$$m_e = (0.0051)(0.014)/\left[2(386)(0.0051 + 0.014)\right]$$

$$= 4.83 \times 10^{-6} \text{ lb-sec}^2/\text{in.}$$

$$f_a = 0.0012 (1/0.5 + 1/0.5)(4.83 \times 10^{-6})(1000)^2$$

$$= 0.0232$$

$$f_d = 0.062 \left[(0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/0.41)\right]$$

$$= 5.5$$

$$f_m = (5.5)(0.0232)/(5.5 + 0.0232)$$

$$= 0.0232$$

$$W_d = \sqrt{(0.0232)(11 - 0.0232)}$$

$$= 0.505 \text{ lb}$$

Total Load

$$W_T = (0.0 + 0.505)/0.062$$

$$= 8.15 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)(0.41)(10^6)/30\right](1)(8.15/0.34)}$$

$$= 3,720 \text{ psi}$$

2. $V = 100$ ft/min, 4 in. -oz load

$$W_t = (4 \text{ in.-oz})/0.5 \text{ in.} \quad (1 \text{ lb}/16 \text{ oz})$$

$$= 0.500 \text{ lb}$$

(a). 303 Stainless Steel on 303 Stainless Steel

$$w_g = 0.028 \text{ lb from (1a)}$$

$$w_p = 0.028 \text{ lb also (face width increased)}$$

$$m_e = (0.028)^2/\left[2(386)(0.028 + 0.028)\right]$$

$$= 1.81 \times 10^{-5} \text{ lb-sec}^2/\text{in.}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(1.81 \times 10^{-5})\,(100)^2$$

$$= 0.00087$$

$$f_d = 0.500 + 0.062\left[(0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/30)\right]$$

$$= 166$$

$$f_m = (166)(0.00087)/(166 + 0.00087)$$

$$= 0.00087$$

$$W_d = \sqrt{(0.00087)(332 - 0.00087)}$$

$$= 0.539 \text{ lb}$$

Total Load

$$W_T = (0.500 + 0.539)/.062$$

$$= 16.8 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)(30)(10^6)/30\right](1)(16.8/0.34)}$$

$$= 32{,}300 \text{ psi}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

w_p = 0.028 lb from (a)

w_g = 0.0093 lb from (1b)

$$m_e = (0.028)(0.0093)/\left[2(386)(0.028 + 0.0093)\right]$$

$$= 9.05 \times 10^{-6} \text{ lb-sec}^2/\text{in.}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(9.05 \times 10^{-6})\,(100)^2$$

$$= 0.00043$$

$$f_d = 0.500 + 0.062\left[(0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/12)\right]$$

$$= 83$$

$$f_m = (83)(0.00043)/(83 + 0.00043)$$

$$= 0.00043$$

$$W_d = \sqrt{(0.00043)(166 - 0.00043)}$$

$$= 0.267 \text{ lb/in.}$$

<u>Total Load</u>

$$W_T = (0.500 + 0.267)/0.062$$

$$= 12.4 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(30)(12)(10^6)/42\right](1)(12.4/0.34)}$$

$$= 21{,}000 \text{ psi}$$

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

w_g = 0.0093 lb from (1b)

w_p = 0.0093 lb also (face width increased)

$$m_e = (0.0093)^2 / \left[2(386)\,(0.0093 + 0.0093)\right]$$

$$= 6.03 \times 10^{-6} \text{ lb-sec}^2/\text{in.}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(6.03 \times 10^{-6})\,(100)^2$$

$$= 0.00029$$

$$f_d = 0.500 + 0.062\left[(0.111)\,(0.0012)/(10^{-6}/24 + 10^{-6}/12)\right]$$

$$= 72$$

$$f_m = (72)\,(0.00029)/(72 + 0.00029)$$

$$= 0.00029$$

$$W_d = \sqrt{(0.00029)\,(144 - 0.00029)}$$

$$= 0.204 \text{ lb}$$

<u>Total Load</u>

$$W_T = (0.500 + 0.204)/0.062$$

$$= 11.3 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(12)\,(12)\,(10^6)/24\right](1)\,(11.3/0.34)}$$

$$= 17{,}400 \text{ psi}$$

(d). 303 Stainless Steel on Delrin

$$w_p = 0.028 \text{ lb from (2a)}$$

$$w_g = 0.0051 \text{ lb from (1d)}$$

$$m_e = (0.028)\,(0.0051)/\left[2(386)\,(0.028 + 0.0051)\right]$$

$$= 5.60 \times 10^{-6} \text{ lb-sec}^2/\text{in.}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(5.60 \times 10^{-6})\,(100)^2$$

$$= 0.000269$$

$$f_d = 0.500 + 0.062 \ (0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/0.41)$$

$$= 6.0$$

$$f_m = (6.0)\ (0.000269)/(6.0 + 0.000269)$$

$$= 0.000269$$

$$W_d = \sqrt{(0.000269)\ (12 - 0.000269)}$$

$$= 0.057 \text{ lb}$$

<u>Total Load</u>

$$W_T = (0.500 + 0.057)/0.062$$

$$= 9.15 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84 \sqrt{\left[(30)\ (0.41)\ (10^6)/30\right]\ (1)\ (9.15/0.34)}$$

$$= 3,940 \text{ psi}$$

3. $V = 20$ ft/min, 3 in. -oz load

(6 in. -oz for steel on steel)

$$W_t = (3 \text{ in. -oz})/0.5 \text{ in.} \quad (1 \text{ lb}/16 \text{ oz}) = 0.375 \text{ lb}$$

(0.750 lb for steel on steel)

(a). 303 Stainless Steel on 303 Stainless Steel

$$m_e = 1.81 \times 10^{-5} \text{ lb-sec}^2/\text{in. from (2a)}$$

$$f_a = 0.0012\ (1/0.5 + 1/0.5)\ (1.81 \times 10^{-5})\ (20)^2$$

$$= 0.000035$$

$$f_d = 0.750 + 0.062 \left[(0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/30)\right]$$

$$= 166$$

$$f_m = (166)\ (0.000035)/(166 + 0.000035)$$

$$= 0.000035$$

$$W_d = \sqrt{(0.000035)(332 - 0.000035)}$$

$$= 0.108 \text{ lb}$$

<u>Total Load</u>

$$W_T = (0.750 + 0.108)/0.062$$

$$= 13.8 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(30)(30)(10^6)/60\right](1)(13.8/0.34)}$$

$$= 29{,}500 \text{ psi}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

$$m_e = 9.05 \times 10^{-6} \text{ lb-sec}^2\text{/in. from (2b)}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(9.05 \times 10^{-6})\,(20)^2$$

$$= 0.0000174$$

$$f_d = 0.375 + 0.062\left[(0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/12)\right]$$

$$= 83$$

$$f_m = (83)(0.0000174)/(83 + 0.0000174)$$

$$= 0.0000174$$

$$W_d = \sqrt{(0.0000174)(166 - 0.0000174)}$$

$$= 0.054 \text{ lb}$$

<u>Total Load</u>

$$W_T = (0.375 + 0.054)/0.062$$

$$= 6.9 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)\,(12)\,(10^6)/42\right]\,(1)\,(6.9/0.34)}$$

$$= 16{,}100 \text{ psi}$$

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

$$m_e = 6.03 \times 10^{-6} \text{ lb-sec}^2/\text{in. from (2c)}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(6.03 \times 10^{-6})\,(20)^2$$

$$= 0.0000116$$

$$f_d = 0.375 + 0.062\left[(0.111)\,(0.0012)/(10^{-6}/24 + 10^{-6}/12)\right]$$

$$= 72$$

$$f_m = (0.0000116)\,(72)/(72 + 0.0000116)$$

$$= 0.0000116$$

$$W_d = \sqrt{(0.0000116)\,(144 - 0.0000116)}$$

$$= 0.041 \text{ lb}$$

Total Load

$$W_T = (0.375 + 0.041)/0.062$$

$$= 6.7 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(12)\,(12)\,(10^6)/24\right]\,(1)\,(6.7/0.34)}$$

$$= 13{,}500 \text{ psi}$$

(d). 303 Stainless Steel on Delrin

$$m_e = 5.60 \times 10^{-6} \text{ lb-sec}^2/\text{in. from (2d)}$$

$$f_a = 0.0012\,(1/0.5 + 1/0.5)\,(5.60 \times 10^{-6})\,(20)^2$$

$$= 0.0000108$$

$$f_d = 0.375 + 0.062 \left[(0.111)(0.0012)/(10^{-6}/60 + 10^{-6}/0.41)\right]$$

$$= 6.0$$

$$f_m = (6.0)(0.0000108)/(6.0 + 0.0000108)$$

$$= 0.0000108$$

$$W_d = \sqrt{(0.0000108)(12 - 0.0000108)}$$

$$= 0.0114$$

Total Load

$$W_T = (0.375 + 0.0114)/0.062$$

$$= 6.25 \text{ lb/in.}$$

Stress

$$S_c = 0.84 \sqrt{\left[(30)(0.41)(10^6)/30\right](1)(6.25/0.34)}$$

$$= 3,270 \text{ psi}$$

APPENDIX C

Calculation of Test Gear Tooth Loads and Stresses Using Buckingham's Formulae and Tuplin's Method for Effective Errors

For purposes of the calculations, the same numbers used in the previous calculations will now be used, with the following exceptions:

1. Actual Error:

$$E_1 = 2\,(\text{tooth-to-tooth error})\tan\phi$$

$$= 2\,(0.0004)\,(0.364) = 0.00029 \text{ in.}$$

2. Rim depth:

$$H = \text{radius of gear} - \text{depth of tooth}$$

$$= 0.500 - 0.021 = 0.479 \text{ in.}$$

3. Ratio of rim depth to circular pitch:

$$a = H/p_n = 0.479/0.032 = 15$$

The calculations are divided into three sections, one for each value of load and speed:

1. $V = 1000$ ft/min, no load

$$\therefore W_t = 0 \text{ lb}$$

(a). 303 Stainless Steel on 303 Stainless Steel

$$1/k_T = 3\,(1/E_p + 1/E_g) + 1/G_p\,(0.125 + 0.1/a)$$

$$+ 1/G_g\,(0.125 + 0.1/a)$$

$$+ 1/6\left[N_p/(a + 0.2)\,E_p + N_g/(a + 0.2)\,E_g\right]$$

$$= 3\,(1/15 \times 10^6) + (2/12 \times 10^6)\,(0.125 + 0.1/15)$$

$$+ 1/3\left[(96)/(30 \times 10^6)\,(15 + 0.2)\right]$$

$$1/k_T = 12 \times 10^{-6}/60 + 1.3 \times 10^{-6}/60 + 4.2 \times 10^{-6}/60$$

$$= 2.92 \times 10^{-7}$$

$$m = w/gF$$

$$m_p = m_g = (0.014)/(386)(0.062)$$

$$= 5.8 \times 10^{-4}$$

$$T_1 = 2\pi \sqrt{(1/k_T)/(1/m_p + 1/m_g)}$$

$$= 2\pi \sqrt{(2.92 \times 10^{-7})/(2/5.8 \times 10^{-4})}$$

$$= 5.8 \times 10^{-5} \text{ sec}$$

$$t_1 = p_n/V$$

$$= (0.032)(60)/(1000)(12) = 1.6 \times 10^{-4} \text{ sec}$$

$$t_1/T_1 = (1.6 \times 10^{-4})/(5.8 \times 10^{-5})$$

$$= 2.76$$

$$E_e/E_1 = 0.035 \text{ at } t_1/T_1 = 2.76 \text{ from Fig. 8}$$

$$E_e = 0.035\, E_1$$

$$= (0.035)(0.00029) = 1.01 \times 10^{-5} \text{ in.}$$

$$f_a = 0.058 \text{ from page 87}$$

$$f_d = 0.062 \left[(0.111)(1.01 \times 10^{-5})/(10^{-6}/60 + 10^{-6}/30)\right]$$

$$= 1.39$$

$$f_m = (0.058)(1.39)/(1.39 + 0.058)$$

$$= 0.056$$

$$W_d = \sqrt{(0.056)(2.78 - .056)}$$

$$= 0.390 \text{ lb}$$

Total Load

$$W_T = (0.0 + 0.390)/0.062$$

$$= 6.3 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)\,(30)\,(10^6)/60\right]\,(1)\,(6.3/0.34)}$$

$$= 19,500 \text{ psi}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

$$m_p = 5.8 \times 10^{-5} \text{ from (1a)}$$

$$m_g = (0.0093)/(0.125)\,(386)$$

$$= 1.94 \times 10^{-5}$$

$$1/k_T = 3\,(10^{-6}/30 + 10^{-6}/12)$$

$$+ (10^{-6}/12)\,(0.125 + 0.1/15)$$

$$+ (10^{-6}/4)\,(0.125 + 0.1/15)$$

$$+ 1/6\left[96/(15 + 0.2)\,(30 \times 10^6)\right.$$

$$\left.+ 96/(15 + 0.2)\,(12 \times 10^6)\right]$$

$$= 21 \times 10^{-6}/60 + 0.66 \times 10^{-6}/60$$

$$+ 1.98 \times 10^{-6}/60 + 2.10 \times 10^{-6}/60$$

$$+ 5.25 \times 10^{-6}/60$$

$$= 5.17 \times 10^{-7}$$

$$T_1 = 2\pi\sqrt{(5.17 \times 10^{-7}/(10^4/5.8 + 10^4/1.94)}$$

$$= 5.45 \times 10^{-5} \text{ sec}$$

$$t_1 = 1.6 \times 10^{-4} \text{ sec from (1a)}$$

$$t_1/T_1 = (1.6 \times 10^{-4})/(5.45 \times 10^{-5})$$

$$= 2.94$$

$$E_e/E_1 = 0.02 \text{ at } t_1/T_1 = 2.94 \text{ from Fig. 8}$$

$$E_e = (0.02)(0.00029) = 0.58 \times 10^{-5}$$

$$f_a = 0.0348 \text{ from page 88}$$

$$f_d = 0.062\left[(0.111)(0.58 \times 10^{-5})/(10^{-6}/60 + 10^{-6}/12)\right]$$

$$= 0.40$$

$$f_m = (0.40)(0.0348)/(0.40 + 0.0348)$$

$$= 0.032$$

$$W_d = \sqrt{(0.032)(0.80 - 0.032)}$$

$$= 0.157 \text{ lb}$$

Total Load

$$W_T = (0.0 + 0.157)/0.062$$

$$= 2.54 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)(12)(10^6)/42\right](1)(2.54/0.34)}$$

$$= 9,750 \text{ psi}$$

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

$$1/k_T = 6(10^{-6}/12) + (10^{-6}/2)(0.125 + 0.1/15)$$

$$+ (1/3)\left[96/(15 + 0.2)(12 \times 10^6)\right]$$

$$= 6 \times 10^{-6}/12 + 0.79 \times 10^{-6}/12 + 2.1 \times 10^{-6}/12$$

$$= 7.4 \times 10^{-7}$$

$$T_1 = 2\pi \sqrt{(7.4 \times 10^{-7})/(2/1.94 \times 10^{-4})}$$

$$= 5.32 \times 10^{-5} \text{ sec}$$

$$t_1 = 1.6 \times 10^{-4} \text{ sec from (1a)}$$

$$t_1/T_1 = (1.6 \times 10^{-4})/(7.4 \times 10^{-7})$$

$$= 3.00$$

$$E_e/E_1 = 0.015 \text{ at } t_1/T_1 = 3.00 \text{ from Fig. 8}$$

$$E_e = (0.015)(0.00029) = 0.44 \times 10^{-5}$$

$$f_a = 0.0197 \text{ from page 89}$$

$$f_d = 0.062 \left[(0.111)(0.44 \times 10^{-5})/(10^{-6}/24 + 10^{-6}/12)\right]$$

$$= 0.264$$

$$f_m = (0.264)(0.0197)/(0.264 + 0.0197)$$

$$= 0.0183$$

$$W_d = \sqrt{(0.0183)(0.328 - 0.0183)}$$

$$= 0.075 \text{ lb}$$

Total Load

$$W_T = (0.0 + 0.075)/0.062$$

$$= 1.21 \text{ lb/in.}$$

Stress

$$S_c = 0.84 \sqrt{\left[(12)(12)(10^6)/24\right](1)(1.21/0.34)}$$

$$= 5{,}740 \text{ psi}$$

(d). 303 Stainless Steel on Delrin

$$m_p = 5.8 \times 10^{-4} \text{ from (1a)}$$

$$m_g = (0.0051)/(386)\,(0.125)$$

$$= 1.06 \times 10^{-4}$$

$$1/k_T = 3\,(10^{-6}/30 + 10^{-6}/0.41)$$

$$+ (10^{-6}/12)\,(0.125 + 0.1/15)$$

$$+ (10^{-6}/0.16)\,(0.125 + 0.1/15)$$

$$+ 1/6\,\left[96/(15 + 0.2)\,(30 \times 10^{6})\right.$$

$$\left. + 96/(15 + 0.2)\,(0.41 \times 10^{6})\right]$$

$$= 6 \times 10^{-6}/60 + 439 \times 10^{-6}/60 + 0.66 \times 10^{-6}/60$$

$$+ 49.5 \times 10^{-6}/60 + 2.1 \times 10^{-6}/60$$

$$+ 154 \times 10^{-6}/60$$

$$= 10.9 \times 10^{-6}$$

$$T_1 = 2\pi\sqrt{(10.9 \times 10^{-4})/(10^4/5.8 + 10^4/1.06)}$$

$$= 19.6 \times 10^{-5} \text{ sec}$$

$$t_1 = (1.6 \times 10^{-4}) \text{ sec from (1a)}$$

$$t_1/T_1 = (1.6 \times 10^{-4})/(19.6 \times 10^{-5})$$

$$= 0.818$$

$$E_e/E_1 = 0.36 \text{ at } t_1/T_1 = 0.818 \text{ from Fig. 8}$$

$$E_e = (0.36)\,(0.00029)$$

$$= 1.04 \times 10^{-4} \text{ in.}$$

$$f_a = 0.0232 \text{ from page 90}$$

$$f_d = 0.062\left[(0.111)(1.04\times10^{-4})/(10^{-6}/60+10^{-6}/0.41)\right]$$

$$= 0.458$$

$$f_m = (0.458)(0.0232)/(0.458 + 0.0232)$$

$$= 0.0221$$

$$W_d = \sqrt{(0.0221)(0.916 - 0.0221)}$$

$$= 0.140 \text{ lb}$$

Total Load

$$W_T = (0.140 + 0.0)/0.062$$

$$= 2.27 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)(0.41)(10^6)/30\right](1)(2.27/0.34)}$$

$$= 1,960 \text{ psi}$$

2. $V = 100$ ft/min, 4 in.-oz load

$$W_t = (4 \text{ in.-oz}/0.5 \text{ in.})(1 \text{ lb}/16 \text{ oz})$$

$$= 0.500 \text{ lb}$$

(a). 303 Stainless Steel on 303 Stainless Steel

$$t_1/T_1 = (1000/100)(2.76) = 27.6 \text{ from (1a)}$$

$$\therefore E_e = 0, \quad W_d = 0$$

Total Load

$$W_T = (0.500 + 0.0)/0.062$$

$$= 8.07 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(30)\,(30)\,(10^6)/60\right]\,(1)\,(8.07/0.34)}$$

$$= 22{,}400 \text{ psi}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

$$t_1/T_1 = (1000/100)\,(2.94) = 29.4 \text{ from (1b)}$$

$$\therefore E_e = 0, \quad W_d = 0$$

<u>Total Load</u>

$$W_T = (0.0 + 0.500)/0.062$$

$$= 8.07 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(30)\,(12)\,(10^6)/42\right]\,(1)\,(8.07/0.34)}$$

$$= 16{,}900 \text{ psi}$$

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

$$t_1/T_1 = (1000/100)\,(3.00) = 30.0 \text{ from (1c)}$$

$$\therefore E_e = 0, \quad W_d = 0$$

<u>Total Load</u>

$$W_T = (0.0 + 0.500)/0.062$$

$$= 8.07 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(12)\,(12)\,(10^6)/24\right]\,(1)\,(8.07/0.34)}$$

$$= 14{,}700 \text{ psi}$$

(d). 303 Stainless Steel on Delrin

$t_1/T_1 = (1000/100)(0.818) = 8.18$

$\therefore E_e = 0, \quad W_d = 0$

Total Load

$$W_T = (0.500 + 0.0)/0.062$$

$$= 8.07 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)(0.41)(10^6)/30\right](1)(8.07/0.34)}$$

$$= 3,720 \text{ psi}$$

3. $V = 20$ ft/min, 3 in.-oz load

(6 in.-oz for steel on steel)

$W_t = (3 \text{ in.-oz}/0.5 \text{ in.})(1 \text{ lb}/16 \text{ oz})$

$= 0.375$ lb

(0.750 lb for steel on steel)

(a). 303 Stainless Steel on 303 Stainless Steel

$t_1/T_1 = (1000/20)(2.76) = 138$ from (1a)

$\therefore E_e = 0, \quad W_d = 0$

Total Load

$$W_T = (0.0 + 0.750)/0.062$$

$$= 12.1 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)(30)(10^6)/60\right](1)(12.1/0.34)}$$

$$= 27,400 \text{ psi}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

$$t_1/T_1 = (1000/20)(2.94) = 147$$

$$\therefore E_e = 0, \quad W_d = 0$$

<u>Total Load</u>

$$W_T = (0.0 + 0.375)/0.062$$

$$= 6.05 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(30)(12)(10^6)/42\right](1)(6.05/0.34)}$$

$$= 15{,}100 \text{ psi}$$

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

$$t_1/T_1 = (1000/20)(3.00) = 150$$

$$\therefore E_e = 0, \quad W_d = 0$$

<u>Total Load</u>

$$W_T = (0.0 + 0.375)/0.062$$

$$= 6.05 \text{ lb/in.}$$

<u>Stress</u>

$$S_c = 0.84\sqrt{\left[(12)(12)(10^6)/24\right](1)(6.05/0.34)}$$

$$= 12{,}800 \text{ psi}$$

(d). 303 Stainless Steel on Delrin

$$t_1/T_1 = (1000/20)(0.818) = 41$$

$$\therefore E_e = 0, \quad W_d = 0$$

Total Load

$$W_T = (0.0 + 0.375)/0.062$$

$$= 6.05 \text{ lb/in.}$$

Stress

$$S_c = 0.84\sqrt{\left[(30)(0.41)(10^6)/30\right](1)(6.05/0.34)}$$

$$= 3,220 \text{ psi}$$

APPENDIX D

Wear Rate Data

1. 1000 ft/min, no load

Dec. 19, 1962 - Jan. 4, 1963:

Time, hr	Average backlash, milli-in.					
	1**	2	3	4	5	6
0	4.71	4.33	4.38	4.71	3.43	4.60
5.0	4.88	4.43	4.60	4.60	3.50	4.77
25	5.71	4.71	5.10	5.17	3.33	5.10
50	6.38	4.67	5.50	5.10	3.33	5.27
75	8.43	4.83	5.94	5.21	3.43	7.05
100	9.33*	5.05	5.94	5.60	3.76	10.0*
150	——	5.60	10.1*	5.67	3.83	——
200	——	5.67	——	5.77	——	——

*Test terminated - wear reached 0.004 in.

**Material Combination Code

1. 303 Stainless Steel on 2024-T4 Aluminum
2. 303 Stainless Steel on Delrin
3. Anodized 2024-T4 Aluminum on Anodized 2024-T4 Aluminum
4. 303 Stainless Steel on 303 Stainless Steel
5. 303 Stainless Steel on Anodized 2024-T4 Aluminum Treated with Molybdenum Disulphide
6. 303 Stainless Steel on Anodized 2024-T4 Aluminum

2. 100 ft/min, 4 in. -oz load

Nov. 2, 1962 - Dec. 19, 1962:

Time, hr	Average backlash, milli-in.					
	1	2	3	4	5	6
0	4.12	3.95	4.22	4.95	3.05	4.67
5.0	4.95	4.62	3.83	3.83	3.66	4.05
25	5.38	5.05	5.45	4.73	3.67	5.50
50	6.11	5.05	6.00	4.72	3.78	6.00
75	6.05	4.88	6.22	4.88	3.72	6.22
100	6.84	5.33	6.78	5.16	4.12	7.00
150	8.11*	5.67	8.05*	4.95	4.22	8.34*
200	—	5.62	—	6.34	4.50	—
300	—	6.34	—	7.00	4.05	—
400	—	6.16	—	6.05	4.16	—
500	—	6.55	—	6.83	4.50	—
600	—	6.77	—	7.88	4.00	—
700	—	7.16	—	8.60	4.43	—
800	—	8.17*	—	9.00*	5.05	—

*Test terminated - wear reached 0.004 in.

3. 20 ft/min, 3 in. -oz load

(6 in. -oz for steel on steel)

Jan. 10, 1963 - Feb. 22, 1963:

Time, hr	Average backlash, milli-in.					
	1	2	3	4	5	6
0	4. 83	4. 77	5. 05	5. 44	2. 67	4. 67
115	5. 17	4. 80	5. 55	7. 17	2. 70	4. 82
237	5. 26	4. 93	5. 55	7. 60	2. 76	5. 36
311	5. 50	4. 96	5. 84	7. 93	2. 75	5. 63
571	6. 05	5. 04	6. 84	8. 77	2. 81	6. 08
740	6. 75	5. 17	7. 50	9. 50	2. 87	6. 64

APPENDIX E

Calculations of Wear Rates

1. 1000 ft/min, no-load, 3800 rpm

(a). 303 Stainless Steel on 303 Stainless Steel

$$\text{Wear Depth Rate} = \frac{(\text{final - initial backlash})}{(\text{rpm})\ (60\ \text{min/hr})\ (\text{time})}$$

$$= (5.77 - 4.71)\ (10^{-3})/(3800)\ (60)\ (200)$$

$$= 23.2 \times 10^{-12} \text{ in./cycle}$$

Wear rate equally divided between pinion and gear

$$\therefore w = 1/2\ (23.2 \times 10^{-12})$$

$$= 11.6 \times 10^{-12} \text{ in./cycle}$$

(b). 303 Stainless on 2024-T4 Aluminum

Wear Depth Rate

$$= (9.33 - 4.71)\ (10^{-3})/(3800)\ (60)\ (100)$$

$$= 202 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 202 \times 10^{-12} \text{ in./cycle}$$

(c). 303 Stainless on Anodized 2024-T4 Aluminum

Wear Depth Rate

$$= (10.0 - 4.60)\ (10^{-3})/(3800)\ (60)\ (100)$$

$$= 237 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 237 \times 10^{-12} \text{ in./cycle}$$

(d). 303 Stainless Steel on Anodized 2024-T4 Alumnium Treated with Molybdenum Disulphide

Wear Depth Rate

$$= (3.83 - 3.43)(10^{-3})/(3800)(60)(150)$$

$$= 14.6 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 14.6 \times 10^{-12} \text{ in./cycle}$$

(e). 303 Stainless Steel on Delrin

Wear Depth Rate

$$= (5.67 - 4.33)(10^{-3})/(3800)(60)(200)$$

$$= 29.4 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on delrin

$$\therefore w = 29.4 \times 10^{-12} \text{ in./cycle}$$

(f). Anodized 2024-T4 Aluminum on Anodized 2024-T4 Aluminum

Wear Depth Rate

$$= (10.1 - 4.38)(10^{-3})/(3800)(60)(150)$$

$$= 164 \times 10^{-12} \text{ in./cycle}$$

Wear rate equally divided between pinion and gear

$$\therefore w = 1/2\,(164 \times 10^{-12})$$

$$= 82 \times 10^{-12} \text{ in./cycle}$$

2. 100 ft/min, 4 in.-oz load, 380 rpm

(a). 303 Stainless Steel on 303 Stainless Steel

Wear Depth Rate

$$= (9.00 - 4.95)(10^{-3})/(380)(60)(800)$$

$$= 223 \times 10^{-12} \text{ in./cycle}$$

Wear rate equally divided between pinion and gear

$$\therefore w = 1/2\,(223 \times 10^{-12})$$

$$= 111 \times 10^{-12} \text{ in./cycle}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

Wear Depth Rate

$$= (8.11 - 4.12)(10^{-3})/(380)(60)(150)$$

$$= 1170 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 1170 \times 10^{-12} \text{ in./cycle}$$

(c). 303 Stainless Steel on Anodized 2024-T4 Aluminum

Wear Depth Rate

$$= (8.34 - 4.67)(10^{-3})/(380)(60)(150)$$

$$= 1070 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 1070 \times 10^{-12} \text{ in./cycle}$$

(d). 303 Stainless Steel on Anodized 2024-T4 Aluminum Treated with Molybdenum Disulphide

Wear Depth Rate

$$= (5.05 - 3.05)(10^{-3})/(380)(60)(800)$$

$$= 110 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 110 \times 10^{-12} \text{ in./cycle}$$

(e). 303 Stainless Steel on Delrin

Wear Depth Rate

$$= (8.17 - 3.95)(10^{-3})/(380)(60)(800)$$

$$= 232 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on delrin

$$\therefore w = 232 \times 10^{-12} \text{ in./cycle}$$

(f). Anodized 2024-T4 Aluminum on Anodized 2024-T4 Aluminum

Wear Depth Rate

$$= (8.05 - 4.22)(10^{-3})/(380)(60)(150)$$

$$= 1120 \times 10^{-12} \text{ in./cycle}$$

Wear rate divided equally between pinion and gear

$$\therefore w = 1/2\,(1120 \times 10^{-12})$$

$$= 560 \times 10^{-12} \text{ in./cycle}$$

3. 20 ft/min, 3 in. -oz load (6 in. -oz load for steel on steel), 76 rpm

(a). 303 Stainless Steel on 303 Stainless Steel

Wear Depth Rate

$$= (9.50 - 5.44)\,(10^{-3})/(76)\,(60)\,(740)$$

$$= 1210 \times 10^{-12} \text{ in./cycle}$$

Wear rate equally divided between pinion and gear

$$\therefore w = 1/2\,(1210 \times 10^{-12})$$

$$= 605 \times 10^{-12} \text{ in./cycle}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

Wear Depth Rate

$$= (6.75 - 4.83)\,(10^{-3})/(76)\,(60)\,(740)$$

$$= 575 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 575 \times 10^{-12} \text{ in./cycle}$$

(c). 303 Stainless on Anodized 2024-T4 Aluminum

Wear Depth Rate

$$= (6.64 - 4.67)\,(10^{-3})/(76)\,(60)\,(740)$$

$$= 585 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 585 \times 10^{-12} \text{ in./cycle}$$

(d). 303 Stainless Steel on Anodized 2024-T4 Aluminum Treated with Molybdenum Disulphide

Wear Depth Rate

$$= (2.87 - 2.67)(10^{-3})/(76)(60)(740)$$

$$= 59.4 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on aluminum

$$\therefore w = 59.4 \times 10^{-12} \text{ in./cycle}$$

(e). 303 Stainless Steel on Delrin

Wear Depth Rate

$$= (5.17 - 4.77)(10^{-3})/(76)(60)(740)$$

$$= 118 \times 10^{-12} \text{ in./cycle}$$

Wear rate totally on delrin

$$\therefore w = 118 \times 10^{-12} \text{ in./cycle}$$

(f). Anodized 2024-T4 Aluminum on Anodized 2024-T4 Aluminum

Wear Depth Rate

$$= (7.50 - 5.05)(10^{-3})/(76)(60)(740)$$

$$= 726 \times 10^{-12} \text{ in./cycle}$$

Wear rate equally divided between pinion and gear

$$\therefore w = 1/2\,(726 \times 10^{-12})$$

$$= 363 \times 10^{-12} \text{ in./cycle}$$

APPENDIX F

Calculation of Fatigue Lives

1. V = 1000 ft/min, no-load, 3800 rpm

   (a). 303 Stainless Steel on 303 Stainless Steel

   From Fig. 9

   $$c = \infty \text{ at stress } = 19{,}500 \text{ psi}$$

   $$E = c/60 \times n$$

   $$= \infty/(60)\,(3800)$$

   $$= \infty \text{ hr}$$

   (b). 303 Stainless Steel on 2024-T4 Aluminum

   From Fig. 9

   $$c = \infty \text{ at stress } = 9{,}750 \text{ psi}$$

   $$E = \infty/(60)\,(3800)$$

   $$= \infty \text{ hr}$$

   (c). 2024-T4 Aluminum on 2024-T4 Aluminum

   From Fig. 9

   $$c = \infty \text{ at stress } = 5{,}740 \text{ psi}$$

   $$E = \infty/(60)\,(3800)$$

   $$= \infty \text{ hr}$$

2. V = 100 ft/min, 4 in. -oz load, 380 rpm

   (a). 303 Stainless Steel on 303 Stainless Steel

   From Fig. 9

   $$c = \infty \text{ at stress } = 22{,}400 \text{ psi}$$

   $$E = \infty/(60)\,(380)$$

   $$= \infty \text{ hr}$$

(b). 303 Stainless Steel on 2024-T4 Aluminum

From Fig. 9

$c = 6 \times 10^{10}$ at stress $= 16,900$ psi

$E = (6 \times 10^{10})/(60)\ (380)$

$= 2.6 \times 10^6$ hr

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

From Fig. 9

$c = 1 \times 10^{11}$ at stress $= 14,700$ psi

$E = (1 \times 10^{11})/(60)\ (380)$

$= 4.3 \times 10^6$ hr

3. $V = 20$ ft/min, 3 in. -oz load (6 in. -oz for steel on steel), 76 rpm

(a). 303 Stainless Steel on 303 Stainless Steel

From Fig. 9

$c = \infty$ at stress $= 27,400$ psi

$E = \infty/(60)\ (76)$

$= \infty$ hr

(b). 303 Stainless Steel on 2024-T4 Aluminum

From Fig. 9

$c = 1 \times 10^{11}$ at stress $= 15,100$ psi

$E = (1 \times 10^{11})/(60)\ (76)$

$= 22 \times 10^6$ hr

(c). 2024-T4 Aluminum on 2024-T4 Aluminum

From Fig. 9

$$c = 3 \times 10^{11} \text{ at stress } = 12{,}800 \text{ psi}$$

$$E = (3 \times 10^{11})/(60)\,(76)$$

$$= 66 \times 10^{6} \text{ hr}$$