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GEORGE C. MARSHALL SPACE FLIGHT CENTER

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HEAT AND MASS TRANSFER IN BINARY INERT GAS FLOW FOR DISTRIBUTIONS OF TEMPERATURE AND CONCENTRATION RENDERING THE PROPERTIES NEARLY CONSTANT

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Ernst W. Adams, John D. Warmbrod, C. Lee Fox, and Robert M. Huffaker

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ABSTRACT

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Injection of a foreign gas into the laminar air boundary layer is considered. The mixture properties are arbitrary functions of temperature T and foreign gas concentration w. Unless the properties are constant, similarity transformations are valid only at the stagnation point or for a wall at constant pressure. Solutions of the three similarity equations are quite cumbersome because triple iterations are involved at the wall to satisfy three conditions at the outer edge of the boundary layer. A correlation formula of rigorous numerical solutions for q_w/q_{wo} has been derived in Reference 14 for the constant pressure case. Here, q_w is the heat transfer in the presence of mass transfer, whose absence is denoted by subscript o. This correlation formula is linear in the mass transfer rate $\rho_w v_w$, and, therefore, fails for large values of $\rho_w v_w$.

A simple engineering solution method of the similarity equations is proposed here, which gives those points of the q_w/q_{wo} versus $\rho_w v_w$ relationship for which five property parameters are nearly constant across the boundary layer. If these five conditions are satisfied, the differential equations in similarity variables can be uncoupled. By use of an auxiliary graph, the momentum equation can be integrated directly as an initial value problem. This solution is used to determine q_w by quadratures. Correlation formulas for q_w are presented for both the constant pressure and the stagnation point cases.

The five conditions on the mixture properties can be satisfied in an approximate way for the injection of H_2O, H_e, or H_2 into air, provided dissipation effects are sufficiently small. Comparison to the correlation formula of Reference 14 shows very good agreement if the Mach number $M_\infty = 0$, and some difference for $M_\infty = 3$. The results for H_2O-air mixtures

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cover the range of q_w/q_{wo} values from unity to values as low as 0.5. Only very small injection rates are compatible with the five conditions if H_e or H_2 is injected into air. The theory is worked out in this paper for the case of constant pressure and air as the primary flowing medium.

Author

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

Symbol	Definition	
a	m/sec	speed of sound
B	$= - [f_w(m+1)Pr/2](Nu/\sqrt{Re})_{f_{w=o}}$	
C	$= \mu\rho/\mu_\infty\rho_\infty$	Chapman-Rubesin parameter
c	$= c_{p_f}/c_{p_a}$	auxiliary parameter
c_p	kcal/kg°K	specific heat
D	m^2/sec	oridnary diffusion coefficient
f		non-dimensional stream function
h	kg/sec m^2	heat transfer coefficient
i	kcal/kg	enthalpy
k	kcal/m °K sec	thermal conductivity
m		Euler number in u_∞ = const. x^m
M_∞		Mach number
M		molecular weight
N_u	$= - x \left(\frac{\partial T}{\partial y}\right) w/(T_w - T_\infty)$	Nusselt number
Pr	$= \mu c_p/k$	Prandtl number
q_w	kcal/m^2 sec	heat transfer at the wall
R	kcal/kg °K	universal gas constant
Re	$= \rho x u_\infty/\mu$	Reynolds number
Sc	$= \nu/D$	Schmidt number

LIST OF SYMBOLS (Cont'd)

Symbol		Definition
T	°K	absolute temperature
t	$= T/T_\infty$	temperature ratio
u	m/sec	velocity component parallel to the wall
v	m/sec	velocity component normal to the wall
w		concentration of foreign gas
x	m	coordinate parallel to the wall
y	m	coordinate normal to the wall
α_1		see equation (29)
α_2		see equation (29)
φ_ρ	$= \rho/\rho_\infty$	
φ_μ	$= \mu/\mu_\infty$	
φ_k	$= k/k_\infty$	
φ_c	$= c_p/c_{p_\infty}$	
φ_{af}	$= [c_{p_a}(T) - c_{p_f}(T)/c_p(T_\infty)$	
ρ	kg/m^3	density
μ	kg/m sec	viscosity
ν	$= \mu/\rho$ m^2/sec	kinematic viscosity
ψ	kg/m sec	stream function
η		similarity variable
ξ		dummy variable for η

LIST OF SYMBOLS (Cont'd)

Symbol		Definition
ζ		dummy variable for η
κ	$= c_p/c_v$	ratio of specific heats

SUBSCRIPTS

a = air

f = foreign gas

w = wall

o = no mass transfer

i = incompressible

∞ = outer edge of boundary layer

SUPERSCRIPT

$*$ = value taken at reference temperature, see equation (40).

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I. INTRODUCTION

The problem of simultaneous heat and mass transfer began to attract considerable attention about ten years ago. A two-way interaction between these transfer rates occurs in case of ablating surfaces, and a one-directional interaction in case of transpiration cooling. Ablation is experienced by objects traversing an atmosphere at sufficiently high speed; ablation also takes place under suitable conditions if a medium flowing through a chemical reactor undergoes a reaction with a solid or a liquid deposited in the reactor. Transpiration cooling of the surfaces of wings, gas turbine blades, etc., has been studied extensively in the past.

Even though the assumptions upon which boundary layer theory is based are violated in case of normal mass transfer across a wall at rest, experiments confirm pertinent boundary layer solutions within measuring accuracy. Laminar boundary layer solutions are physically valid only for sufficiently small injection rates normal to the surface because of stability considerations. Solutions of the stationary boundary layer equations may be used to approximate nonstationary flight situations. An exceptional case occurs when the ablation rate depends not only on the instantaneous heat transfer rate but also on the transient conduction of heat in the interior of the wall whose surface is ablating (see Reference 1). If time-dependent effects may be ignored, the double interaction between heat and mass transfer can be analyzed for a given flight state by interpolating between a number of simple-interaction solutions with prescribed mass injection.

Theoretical contributions to the heat and mass transfer problem under discussion deal predominantly with stationary, binary, and laminar boundary layer flow consisting of a main ("primary") stream and an injected ("foreign") constituent. This problem is governed by a system of four simultaneous differential equations, (1) through (4) in Section 2, representing conservation of matter, momentum, foreign species, and energy. Up to now, these equations have been solved rigorously only under simplifying assumptions on the mixture properties and/or the flow parameters. Inspection of equations (1) through (4) yields the conditions under which either the mass concentration w(x, y) of the foreign gas is determined as a function of the temperature distribution T(x, y) or the distributions of both w(x, y) (see Reference 3, p. 608 and Reference 9, p. 458). Similarity solutions of equations (1) through (4) have been published for both constant and variable mixture properties. If the Mach number, the temperature difference across the boundary layer, and the foreign gas concentration are sufficiently small, constant mixture properties may be assumed. This case has been treated in References 15 and 25. Similarity solutions then hold true for any Euler number m. If the mixture properties are allowed to depend on temperature and foreign gas concentration, similarity solutions exist only for the stagnation point ($m = 1$) and for zero pressure gradient ($m = 0$). The case $m = 1$ is dealt with in References 16 and 17; the case $M = 0$ is treated in References 11 and 14. In some references, homogeneous or heterogeneous chemical reactions of assumed order between the constituents are included in the discussion, e.g., Chapter 19 of Reference 3. Solutions for more than two chemical constituents have been derived in connection with dissociation or ionization of the constituents in the mixture, e.g., References 21 and 23.

Even if a similarity transformation has been introduced, the system of simultaneous nonlinear differential equations offers formidable resistance to numerical solutions, provided the mixture properties are allowed to depend on both foreign gas concentration w and temperature T as is predicted by the kinetic theory of gases, e.g., References 6 and 19. This situation becomes evident by considering that three boundary conditions at the outer edge of the boundary layer have to be satisfied by simultaneous iterations for the gradients of velocity, temperature, and concentration at the wall. The momentum equation in similarity variables becomes independent of the differential equations for T and w if the produce of viscosity μ and density ρ is assumed to be constant. However, for the compressible boundary layer flow of a single gas, similarity solutions show a significant difference if the case of constant $\rho\mu$ is dependent on T according to the equation of state and Sutherland's viscosity law for air, e.g., Chapter 4 of Reference 27.

It is shown in this report for mixtures consisting of air and H_2O vapor, He, or H_2 that it is possible for any given set of boundary values T_w and T_∞ to determine the wall concentration w_w so that the relevant mixture properties take approximately the same values at the wall and at the outer edge of the boundary layer. Under suitable conditions specified in this report, these mixture properties then must be nearly constant across the entire boundary layer. If it may be assumed that $\rho\mu$ = const. in the boundary layer, the momentum equation in, similarity variables can be solved independently of the other differential equations. The equation for the foreign gas concentration and, subsequently, the one for the temperature distribution, can then be solved by quadratures involving the solution of the momentum equation. Evidently, the numerical work of this solution method is very much smaller than the work needed to integrate the simultaneous system of three similarity differential equations with split boundary conditions. This is particularly true since experience shows that numerical instabilities in these integration procedures limit their range of applicability. Also, it is impossible to check the accuracy of numerical integrations of the differential equations by means as simple as the ones needed to check the quadratures involved in the method proposed in this paper.

II. THE DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

According to page 457 of Reference 9, the stationary, two-dimensional, and laminar boundary layer flow of a binary, compressible mixture is described by the following system of differential equations if both pressure gradient and thermal diffusion are negligible and if the mixture is chemically inert:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} - \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 0, \tag{2}$$

$$\rho^{u} \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} - \frac{\partial}{\partial y}\left(D \frac{\partial w}{\partial y}\right) = 0, \text{ and} \tag{3}$$

$$\rho^{c_p u} \frac{\partial T}{\partial x} + \rho^{c_p v} \frac{\partial T}{\partial y} - \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) - \mu \left(\frac{\partial u}{\partial y}\right)^2 + \rho D \,(c_{p_a} - c_{p_f}) \frac{\partial T}{\partial y} \frac{\partial w}{\partial y} = 0. \tag{4}$$

In the equations (2), (3), and (4), the third term represents, respectively, the diffusion of momentum, matter, and heat. The last two terms in equation (4) stand for dissipated mechanical energy and for internal energy exchanged due to the diffusion flows of primary and injected medium, respectively.

The following boundary conditions are specified:

$$u(x, 0) = 0, \quad v(x, 0) = v_w(x), \quad w(x, 0) = w_w(x), \quad \text{and } T(x, 0) = T_w(x) \tag{5}$$

at the wall y = 0; and

$$\lim_{y \to \infty} u = u_\infty(x), \quad \lim_{y \to \infty} w = w_\infty(x), \quad \text{and} \quad \lim_{y \to \infty} T = T_\infty(x) \tag{6}$$

at the outer edge of the boundary layer. The quantities v_w, w_w, u_∞, w_∞, and T_∞ have to be given. Since the wall is supposedly impermeable to the primary gas, convection of this gas toward the wall is balanced by diffusion of this gas in the opposite direction, i.e.,

$$\rho_w v_w (1 - w_w) = - \rho_w D_w \left(\frac{\partial w}{\partial y}\right)_w \ . \tag{7}$$

Provided suitable initial conditions have been prescribed at some upstream station x = const., e.g., at the stagnation point x = 0, equations (1) through (6) completely determine the functions u(x, y), v(x, y), w(x, y), and T(x, y). In the following, this system is solved for the temperature gradient $\partial T(x, 0)/\partial y$ at the wall. Under the stationary conditions stipulated, the heat transfer rate $q_w = - k_w \, \partial T(x, 0)/\partial y$ is balanced by radiation from the surface and/or by the heat absorbed by the coolant as it is transported across the wall.

As in Reference 11, the following nondimensional similarity variables are introduced:

$$\eta = \frac{1}{2}\sqrt{\frac{u_\infty}{v_\infty x}} \int_0^y \frac{dy}{\varphi_\mu(y)} \quad \text{and} \tag{8}$$

$$f(\eta) = \frac{\psi(x,y)}{\sqrt{u_\infty \nu_\infty x}} \ . \tag{9}$$

The continuity equation (1) is identically satisfied by the stream function ψ for the two-dimensional case at hand. Equations (8) and (9) then determine the velocity components as follows:

$$u \equiv \frac{\rho_\infty}{\rho} \frac{\partial \psi}{\partial y} = \frac{u_\infty}{2} \frac{f'(\eta)}{c(\eta)} \tag{10}$$

$$v \equiv - \frac{\rho_\infty}{\rho} \frac{\partial \psi}{\partial x} = - \frac{\rho_\infty}{\rho} \frac{1}{2} \sqrt{\frac{u_\infty \nu_\infty}{x}} \; [f(\eta) - \eta f'(\eta)], \tag{11}$$

where $C(\eta) = \psi_\mu \, \psi_\rho$. Equation (11) implies

$$f(0) = f_w = - 2 \, \frac{\rho_w \, \nu_w}{\rho_\infty \, u_\infty} \sqrt{\frac{u_\infty x}{\nu_\infty}} \; . \tag{11a}$$

The similarity transformation given by equations (8) and (9) implies $v_w(x) = x^{-1/2}$. The momentum equation (2) becomes

$$f(\eta) \frac{d}{d\eta} [C^{-1}(\eta) \; f'(\eta)] + \frac{d^2}{d\eta^2} \; [C^{-1}(\eta) \; f'(\eta)] = 0. \tag{12}$$

Additional similarity conditions are introduced by

$$w = w(\eta) \quad \text{and} \quad T = T(\eta) \tag{13}$$

so that w_w = const. and T_w = const. Equations (3) and (4) then take the form

$$f(\eta) \frac{dw(\eta)}{d\eta} + \frac{d}{d\eta} \left(\frac{1}{S_c(\eta)} \; \frac{dw(\eta)}{d\eta} \right) = 0 \text{ and} \tag{14}$$

$$\varphi_c(\eta) \; f(\eta) \frac{d}{d\eta} \left(\frac{T(\eta)}{T_\infty} \right) + \frac{1}{P_{r_\infty}} \frac{d}{d\eta} \left[\frac{\varphi_k(\eta)}{\varphi_\mu(\eta)} \frac{d}{d\eta} \left(\frac{T(\eta)}{T_\infty} \right) \right] + \frac{k_\infty - 1}{4} M_\infty^2 \left[\frac{d}{d\eta} \left(C^{-1}(\eta) \; \frac{df(\eta)}{d\eta} \right) \right]^2$$

$$- \frac{\varphi_{a_f}(\eta)}{S_c(\eta)} \frac{dw(\eta)}{d\eta} \frac{d}{d\eta} \left(\frac{T(\eta)}{T_\infty} \right) = 0, \tag{15}$$

where the relation $a_\infty^2 = c_{p_\infty} (\kappa_\infty - 1) T_\infty$ has been used. The coefficients φ_μ, φ_ρ, φ_c, and φ_k in the system of differential equations (12), (14), and (15) depend on $w(\eta)$ and $T(\eta)$. The coefficient φ_{af} is a function of $T(\eta)$ only. The boundary conditions (5) and (6) take the following form:

$$f'(0) = 0, \quad f(0) = f_w, \quad w(0) = w_w, \quad T(0) = T_w, \tag{16}$$

$$\lim_{\eta \to \infty} f'(\eta) = 2, \quad \lim_{\eta \to \infty} w(\eta) = w_\infty, \quad \lim_{\eta \to \infty} T(\eta) = T_\infty, \text{ and} \tag{17}$$

$$f(0) = \frac{w'(0)}{S_c(0)\ [1 - w(0)]}. \tag{18}$$

Equations (12) and (14) through (17) define a split boundary-value problem for a system of three nonlinear ordinary differential equations of the seventh order. Solutions with regard to Pr = Sc = 1 and $M_\infty = \varphi_{af} = 0$ are particularly simple, but constitute rather poor approximations to solutions with variable properties. Under the assumptions of $M_\infty = 0$ and constant properties, the authors have solved equations (12) and (14) through (18) in a wide range of Pr, Sc, c, and f_w. A correlation formula for the results is presented in Appendix B. A numerical solution of this problem for any given set of coefficients $\varphi_\mu(T, w)$, $\varphi_\rho(T, w)$, $\varphi_k(T, w)$, $\varphi_c(T, w)$, and $\varphi_{af}(T, w)$ requires an iterative technique which employs the four given initial conditions (16) and three iteratively corrected initial assumptions of $f''(0)$, $w'(0)$, and $T'(0)$ to satisfy the conditions (17). The boundary values at the wall have to be consistent with equation (18). It is desirable to uncouple these differential equations in order to reduce the amount of numerical work involved in the solution procedure.

III. SOLUTIONS OF THE DIFFERENTIAL EQUATIONS

The differential equations (12), (14), and (15) can be uncoupled if the following relations hold true:

$$C(\eta) = \frac{\rho(\eta)}{\rho_\infty} \frac{\mu(\eta)}{\mu_\infty} \equiv 1, \tag{19}$$

$$\frac{\varphi_k(\eta)}{\varphi_\mu(\eta)} = \frac{k(\eta)\ \mu_\infty}{k_\infty\ \mu(\eta)} \equiv 1, \tag{20}$$

$$\varphi_c(\eta) = \frac{c_p(\eta)}{c_{p_\infty}} \equiv 1, \tag{21}$$

$$\frac{Sc(\eta)}{Sc_\infty} = \frac{\mu(\eta)\ \rho_\infty\ D_\infty}{\rho(\eta)\ D(\eta)\ \mu_\infty} \equiv 1, \text{ and} \tag{22}$$

$$\frac{\varphi_{a_f}(\eta)}{\varphi_{a_{f_\infty}}} = \frac{c_{p_a}(\eta) - c_{p_f}(\eta)}{c_{p_{a_\infty}} - c_{p_{f_\infty}}} \equiv 1. \tag{23}$$

Conditions (20) and (21) imply $Pr(\eta) = Pr_\infty$.

Figure 10 presents the relation between f"(0) and f(0) which follows from numerical solutions of equation (12) under the assumption of $C(\eta) \equiv 1$. If f_w is prescribed, the three initial conditions are known and thus equation (12) can be integrated as an initial value problem. Equations (12) and (19) yield the relation

$$\int_0^\eta f(\eta)\, d\eta = -\ln \frac{f''(\eta)}{f''(0)} . \tag{24}$$

Equations (14), (22), and (24) give for Sc = const.

$$w(\eta) - w(0) = w'(0) \int_0^\eta \left[\frac{f''(\eta)}{f''(0)}\right]^{Sc} d\eta \tag{25}$$

and

$$w'(0) = \frac{w_\infty - w_w}{\displaystyle\int_0^\infty \left[\frac{f''(\eta)}{f''(0)}\right]^{Sc} d\eta} . \tag{26}$$

The parameter w'(0) can be eliminated between equations (18) and (26) so that

$$\frac{w_w - w_\infty}{1 - w_\infty} = \frac{f_w \; Sc \int_0^\infty \left[\frac{f''(\eta)}{f''(0)}\right]^{Sc} d\eta}{f_w \; Sc \int_0^\infty \left[\frac{f''(\eta)}{f''(0)}\right]^{Sc} d\eta - 1} , \tag{27}$$

where the right-hand side is a function of $f(0) = f_w$ only. Figure 11 presents the relation between w_w and f_w which follows for $w_\infty = 0$ from equation (27).

Equation (15) can be written to give

$$t''(\eta) + t'(\eta) \left[Pr \; f(\eta) - \frac{Pr}{Sc} \varphi_{a_f} \; w'(\eta)\right] + Pr \frac{k_\infty - 1}{4} M_\infty^2 \left[\frac{d^2 f(\eta)}{d\eta^2}\right]^2 = 0, \tag{28}$$

where $t(\eta) = T(\eta)/T_\infty$ and Pr, Sc, and φ_{af} are constants. The short notations

$$\alpha_1 = \frac{Pr}{Sc} \varphi_{a_f} \frac{w_\infty - w_w}{\int_0^\infty \left[\frac{f''(\eta)}{f''(0)}\right]^{Sc} d\eta} \quad \text{and} \quad \alpha_2 = Pr \frac{k_\infty - 1}{4} M_\infty^2 \; [f''(0)]^{Pr} \tag{29}$$

are employed, which are defined since $f''(0) \geqq 0$. Integration of equation (28) gives

$$t'(\eta) = \left[\frac{f''(\eta)}{f''(0)}\right]^{Pr} \left\{\exp\left[\alpha_1 \int_0^\eta \left(\frac{f''(\xi)}{f''(0)}\right)^{Sc} d\xi\right]\right\} \cdot$$

$$\left\{t'(0) - \alpha_2 \int_0^\eta \left[f''(\xi)\right]^{2-Pr} \left[\exp\left(- \alpha_1 \int_0^\xi \left(\frac{f''(\zeta)}{f''(0)}\right)^{Sc} d\zeta\right)\right] d\xi\right\} . \tag{30}$$

In view of the boundary condition $\lim_{\eta \to \infty} t(\eta) = 1$, a quadrature of equation (30) yields

$$t'(0) \equiv \frac{T'(0)}{T_\infty} = \left\{ \frac{T_\infty - T(0)}{T_\infty} + \alpha_2 \int_0^\infty \left\{ \left(\frac{f''(\eta)}{f''(0)} \right)^{Pr} \left[\exp \left(\alpha_1 \int_0^\eta \left(\frac{f''(\xi)}{f''(0)} \right)^{Sc} d\xi \right) \right] \right\} \cdot \right.$$

$$\left. \cdot \frac{\displaystyle \int_0^\eta \left[\left(f''(\xi) \right)^{2-Pr} \left[\exp \left(- \alpha_1 \int_0^\xi \left(\frac{f''(\zeta)}{f''(0)} \right)^{Sc} d\zeta \right) \right] d\xi \right] d\eta \right\}}{\displaystyle \left\{ \int_0^\infty \left(\frac{f''(\eta)}{f''(0)} \right)^{Pr} \exp \left[\alpha_1 \int_0^\eta \left(\frac{f''(\xi)}{f''(0)} \right)^{Sc} d\xi \right] d\eta \right\}} . \tag{31}$$

Equation (31) shows that the contribution of dissipation to heat transfer is governed by the parameter $(k_\infty - 1)\, M_\infty^2\, T_\infty / (T_\infty - T(0))$. The function $f''(\eta)/f''(0)$ in the right-hand side of equation (31) is obtained by integrating equation (12). Equation (31) includes the case of zero mass transfer ($f_w = w_w = w_\infty = 0$), $\alpha_1 = 0$. Because of the boundary condition (19), equation (12) gives in this case for constant properties

$$f''(0) = 1.328. \tag{32}$$

For $f_w = M_\infty = 0$, both α_1 and α_2 equal zero. Equation (31) then gives the well known expression

$$\left\{ \frac{d}{d \frac{y}{L}} \left[\frac{T(\eta) - T_\infty}{T_w - T_\infty} \right] \right\}_{\eta=0} \sqrt{\frac{x}{L}} \sqrt{\frac{\nu_\infty}{u_\infty L}} = - \frac{Nu}{\sqrt{Re}} = - \frac{1.328^{Pr}}{2} \frac{1}{\displaystyle \int_0^\infty [f''(\eta)]^{Pr} \, d\eta} . \tag{33}$$

See Chapter XIV of Reference 24.

In two special cases, the repeated quadratures in equation (31) can be reduced to one single quadrature in the expression for α_1. If Pr = Sc = 1,

$$\left[\frac{d\left(\dfrac{T(\eta) - T_\infty}{T_w - T_\infty}\right)}{d\,\dfrac{y}{L}}\right]_{\eta=0} 2\,\frac{\mu(0)}{\mu_\infty}\sqrt{\frac{x}{L}}\sqrt{\frac{\nu_\infty}{u_\infty L}} = \frac{-\alpha_1 + 2\,\dfrac{\alpha_2 T_\infty}{T_\infty - T_w}}{\exp\left[\dfrac{2\alpha_1}{f''(0)}\right] - 1} - \frac{f''(0)}{\alpha_1}\,\frac{\alpha_2 T_\infty}{T_\infty - T_w}\,. \tag{34}$$

If Pr = Sc and $M_\infty = 0$,

$$\left[\frac{d\,\dfrac{T(\eta) - T_\infty}{T_w - T_\infty}}{d\,\dfrac{y}{L}}\right]_{\eta=0} 2\,\frac{\mu(0)}{\mu_\infty}\sqrt{\frac{x}{L}}\sqrt{\frac{\nu_\infty}{u_\infty L}} = \frac{-\alpha_1}{\exp\left[\varphi_{a_f}(w_\infty - w_w)\right] - 1}\,. \tag{35}$$

The heat transfer rate for any values of Pr, Sc, and M_∞ can be expressed as follows by use of equation (8):

$$q_w = k(0)\,\frac{dT(0)}{d\eta}\,\frac{d\eta(0)}{dy} = \frac{1}{2x}\,\frac{k(0)}{\mu(0)}\,\mu_\infty\sqrt{\frac{xu_\infty}{\nu_\infty}}\,\frac{dT(0)}{d\eta}\,. \tag{36}$$

The numerical values of $\mu(0)$ and $k(0)$ may be taken from Figures 2 and 3, respectively, if the injection of H_2O vapor, He, or H_2 into air is considered.

IV. DISCUSSION OF THE VALIDITY OF THE ASSUMPTIONS (19) - (23) ON THE MIXTURE PROPERTIES

Mixture Properties

The simultaneous system of differential equations (12), (14), and (15) can be uncoupled if the binary mixture flow represented by these equations satisfies conditions (19) through (23) within a reasonable margin of error for a triple of boundary values T_∞, T_w, and w_w. Clearly, these conditions can be satisfied rigorously by assuming constant mixture properties. Condition (23) can always be met in an approximate way because the specific heat c_p of any gas increases together with temperature, e.g., Figure 4. If $T_\infty > T_w$ and $w_\infty < w_w$, the conditions (19) through (22) can be satisfied only if each one of the mixture parameters $\mu\rho$, μ/k, c_p, and Sc is influenced in the same direction by changes of either temperature T or concentration w. Results of pertinent methods listed in Appendix A show that this situation prevails if H_2O vapor, He, or H_2 is injected into air (see Figures 5 through 9). This condition of the same influences exerted by T and w fails to be satisfied for several investigated foreign gases with molecular weights exceeding the one of air.

As a first approximation, conditions (19) through (23) are relaxed and satisfied by equating values at the wall and at the outer edge of the boundary layer. By employing the T- and w-dependencies of the mixture properties presented in Figures 1 through 5, these relaxed conditions have been satisifed approximately for given temperatures T_∞ and T_w listed in Table I by calculating suitable values of w_w. Equations (25), (26), (30), and (31) have been evaluated for $T(\eta)$ and $w(\eta)$ with regard to case 3 listed in Table I (see Figures 12 and 13). The functions $T(\eta)$ for $M_\infty = 0$ and $w(\eta)$ presented in Figures 12 and 13, respectively, have been used to evaluate conditions (19) through (23) in the range $0 \doteq \eta < \infty$. The results of this calculation appear in Figure 14. At least in this one special case, the margin of error in satisfying conditions (19) through (22) is smaller inside the boundary layer than at the wall. It may be conjectured for $M_\infty = 0$ that it is sufficient to satisfy the relaxed conditions (19) through (23) by employing only given boundary values at the wall and at the outer edge of the boundary layer, instead of the unknown functions $T(\eta)$ and $w(\eta)$. A glance at Figure 13 shows that the relaxed form of conditions (19) through (23) is not applicable unless the difference between the temperature profiles for a given Mach number M_∞ and for $M_\infty = 0$ is sufficiently small. This consideration is verified by the final results presented in Figure 15.

V. RELATIONS FROM THE REFERENCE ENTHALPY METHOD USED FOR COMPARISON

The reference enthalpy method derived by Eckert in Reference 7 gives the expression

$$q_{wo} = h_i \ (i_r - i_w) \tag{37}$$

for the heat transfer at an impermeable solid wall for compressible air boundary layer flow with constant pressure. The recovery enthalpy can be expressed as follows

$$i_r = i_\infty + \frac{r_i}{2} M_\infty^2 \ c_{p_\infty} \ (k_\infty - 1) \ T_\infty. \tag{38}$$

For laminar air flow, the recovery factor can be approximated by

$$r_i = \sqrt{Pr} \, . \tag{39}$$

The temperature-dependent gas properties are introduced at a reference enthalpy defines as

$$i^* = i_\infty + 0.5\,(i_w - i_\infty) + 0.22\,(i_r - i_\infty); \tag{40}$$

i^* gives the reference temperature T^* by use of enthalpy tables for air. For two-dimensional laminar flow along a surface with constant properties, pressure, and temperature, the local heat transfer coefficient is obtained from the relation

$$h_i = \rho u_\infty \frac{0.332}{Re^{1/2}\, Pr^{2/3}} . \tag{41}$$

It has been shown in Reference 7 that equation (41) is a correlation formula for heat transfer in high-speed flow with variable properties if ρ, Re, and Pr are evaluated at the reference temperature T^*, i.e., $Re(x) = \rho^* x u_\infty / \mu^*$ and $Pr = \mu^* c_p^* / k^*$. Equations (36) through (41) give for $w_\infty = 0$

$$\frac{q_w}{q_{wo}} = \frac{\dfrac{k(T_w,\ w_w)}{(T_w,\ w_w)} \sqrt{\dfrac{\rho(T_\infty)\ \mu(T_\infty)}{\rho(T^*)\ \mu(T^*)}} \left[\dfrac{\mu(T^*)\ c_p(T^*)}{k(T^*)}\right]^{2/3} \dfrac{dT(0)}{d\eta}}{0.664 \left[i_\infty + \sqrt{\dfrac{\mu(T^*)\ c_p(T^*)}{k^*}}\ \dfrac{M_\infty^2}{2}\, c_{p_\infty}\,(k_\infty - 1)\, T_\infty - i_w\right]} . \tag{42}$$

For constant pressure, it has been shown in Reference 14 that exact results for heat transfer in the presence of mass transfer cooling with a foreign gas (e.g., References 2, 8, and 26) can be approximated by the following expression:

$$\frac{q_w}{q_{wo}} = 1 - 1.82 \sqrt[3]{\frac{M_a}{M_f}}\ \frac{\rho_w\ \nu_w}{\rho_\infty\ u_\infty} \sqrt{\frac{\rho_\infty\ x u_\infty}{\mu_\infty}\ \frac{1}{C^*}} \quad ; \tag{43}$$

C^* is given by

$$C^* = \frac{\rho(T^*)\ \mu(T^*)}{\rho_\infty\ \mu_\infty} . \tag{44}$$

Equation (43) is valid over a wide range of values M_∞, T_∞, T_w, and $\rho_w v_w$. Figures 20 and 21 in Reference 14 indicate for the injection of H_2 and He, respectively, that evaluations of equation (43) deviate by less than approximately 0.05 from rigorous results derived by other investigators. These rigorous results and pertinent experimental data show, however, that q_w/q_{wo} ceases to be a linear function of the mass transfer rate $\rho_w v_w$ as $q_w/q_{wo} \rightarrow 0$. For this reason, it is questionable whether or not equation (43) is valid for large mass transfer rates $\rho_w v_w$.

VI. RESULTS OF THE DERIVED METHOD

Table 1 presents triples of numbers T_∞, w_w, and T_w for which the relaxed form of conditions (19) through (23) has been satisfied within the margin of error also listed in this table. The slopes of both $\rho\mu$ and μ/k as functions of w in Figures 6 and 7, respectively, show why the cases appearing in Table 1 cover a wide w_w-range for H_2O-air mixtures and only small values of w_w for He-air and H_2-air mixtures. The width of the bands presenting Sc and c_p as functions of w for T = const. in Figures 9 and 5, respectively, confirms this situation.

When the five conditions (19) through (23) have been satisfied in the outlined approximate way, suitable average values of Pr, Sc, and φ_{af} can be estimated. For each one of the cases listed in Table I, Figure 11 has been used to determine f_w as a function of Sc and w_w. Figure 10 gives f''_w in terms of f_w. The momentum equation (12) can then be integrated as an initial value problem to give $f(\eta)$. The resulting function $f''(\eta)/f''(0)$ is substituted into equation (31) to obtain $T'(0)$ by quadratures. Equation (42) finally yields q_w/q_{wo}. For $M_\infty = 0$, the numerical evaluations involved in this method can be reduced to a few hand calculations by employing the correlation formula (B-2) in Appendix B.

The marked points in Figure 15 present results of equation (42), which is a consequence of the derived method, for the H_2O-air mixtures listed in Table I. The curves in Figure 15 have been obtained from the correlation method presented in Reference 14. Equation (43) gives the upper line when evaluated for the molecular weight ratio of H_2O and air. The lower curve represents a special correlation formula shown in Figure 22 of Reference 14 for H_2O and air. The agreement between points referring to $M_\infty = 0$ and the lines is remarkably close, particularly in view of the accuracy of the correlation method of Reference 14, where rigorous solutions for H_2O and air cover a band of certain lateral width around the lower straight line, which is also approximated by the upper straight line representing equation (43) within a certain margin of error. While the agreement between the derived method and the correlation method of Reference 14 is good for H_2O-air mixtures in a low-speed boundary layer, the evaluations of equation (42) differ significantly from the lower straight line for $M_\infty = 3$. This is due to the fact that the influences of $T(\eta)$ and $w(\eta)$ on the pertinent property parameters given by equations (19) through (23) do not compensate each other in an approximate way as they do for $M_\infty = 0$, since the temperature profiles reach a maximum inside the boundary layer for $M_\infty = 3$ (see Figure 13). This trend is particularly strong when T_W and T_∞ differ only by a relatively small quantity, see case no. 2 in Figures 13 and 15. Points for considerably higher mass transfer rates f_W could have been calculated by specifying larger values of $(T_\infty - T_W)$. This is due to the fact that the values of w_W which satisfy conditions (19) through (23) increase together with $(T_\infty - T_W)$.

VII. REVIEW AND APPLICATIONS OF THE DERIVED METHOD

A laminar air boundary layer with injection of foreign gas is considered here. The mixture properties may depend in any prescribed way on temperature T and foreign gas concentration w. The partial differential equations representing conservation of matter, injected species, momentum, and energy can be reduced to three simultaneous ordinary differential equations by a similarity transformation which is valid both at the stagnation point and for a constant pressure wall. Several investigators have solved this simultaneous system of similarity equations by rigorous numerical methods for the case of constant pressure. The solution procedures employed are quite cumbersome because a triple iteration is necessary to satisfy three boundary conditions at the outer edge of the boundary layer. Available rigorous numerical solutions have been correlated in Reference 14 and represented by a linear dependency between q_W/q_{WO} and the injection rate $\rho_W v_W/\rho_\infty u_\infty$, whose slope coefficient is a function of M_a/M_f, Re(x), and $\rho^*\mu^*/\rho_\infty\mu_\infty$. While this correlation formula treats the case of constant pressure in a satisfactory manner, only individual numerical solutions for special given values of T_∞, M_∞, and T_W have been published so far for the stagnation point.

Because of this situation, a relatively simple calculation method is proposed in this paper which gives q_W/q_{WO} for both the stagnation point and the case of constant pressure. To show its performance and limitations, this method is worked out here only for the latter case. The method rests on the fact that the three ordinary differential equations in similarity variables can be uncoupled if the mixture parameters $\mu\rho$, μ/k, Sc, and c_p, and $(c_{p_a} - c_{p_f})$ are constant. To render these parameters independent of the solutions $T(\eta)$ and $w(\eta)$ to be determined, these five conditions are relaxed so that only the values at the wall are equated to the values at the outer edge of the boundary layer. Comparison to the correlation solution compiled in Reference 14 shows that an approximate matching of the relaxed form of the five conditions is sufficient unless dissipation has a significant influence on the temperature profile. These relaxed conditions can be satisfied in an approximate way for the injection of H_2O, He, or H_2 into air because changes of either T or w have the same influences on each one of the mixture parameters $\mu\rho$, μ/k, Sc, and c_p in these three cases. This situation does not exist, however, for several investigated foreign gases whose molecular weight exceeds the one of air. The relaxed form of the conditions can be satisfied in the approximate manner for He or H_2 if the injection rates are very small. In case of H_2O injection into air, however, values of q_w/q_{wo} as low as 0.5 can be obtained for moderate values of $(T_\infty - T_w)$. If this temperature difference is sufficiently large, the mass transfer rate $\rho_w v_w/\rho_\infty u_\infty$ of H_2O vapor may reach values outside the range of applicability of the correlation formula (43) which is linear in $\rho_w v_w/\rho_\infty u_\infty$.

If the five conditions on the mixture properties are satisfied in the outlined approximate way, the heat transfer parameter q_w/q_{wo} is obtained by integration of equation (12) and quadratures in the right-hand side of equation (31). For $M_\infty = 0$, the numerical evaluations can be reduced to a few hand calculations by employing the correlation functions presented in Appendix B for the Euler numbers m = 0 (constant pressure), m = 0.5, and m = 1 (stagnation point).

In conclusion it may be pointed out that the method derived in this paper can give simple engineering solutions of the heat and mass transfer problem which in some applications exceed the range of validity of the correlation formula (43) of Reference 14. Evidently, the derived method is valid for primary gases other than air and for foreign gas concentrations $w_\infty \neq 0$. The method can be generalized to include homogeneous or heterogeneous chemical reactions of given order. Also, the method can be rewritten to account for a mass transfer vector component tangential to the wall.

APPENDIX A

THE MIXTURE PROPERTIES

1. Viscosity, Diffusion, and Thermal Conductivity Coefficient

Viscosity, ordinary diffusion, and thermal conductivity coefficients were determined by use of a rigorous kinetic theory of gases of low density initially developed for pure gases by Chapman (Reference 6) and independently by Enskog at the same time. Hirschfelder and associates (Reference 19, pp. 514-540) extended this theory to include binary mixtures. The Chapman-Enskog theory expresses the transport coefficients in terms of the potential energy of attraction between a pair of gas molecules. Since exact potential functions are not known, empirical relations are commonly used to describe interactions between molecules. According to Reference 19, transport properties of pure gases can be expressed in terms of molecular weight, pressure, temperature, specific heat, a characteristic diameter σ of the molecule, and collision integrals which represent the deviation of the molecules from the rigid sphere model. The collision integral is a function of a dimensionless temperature ($\kappa T/\epsilon$) and depends upon the potential model under consideration. The parameters ϵ/κ (°K) and σ(Å) are often referred to as "force constants"; ϵ is a characteristic energy of attraction between two molecules, and κ is the Boltzmann constant.

In calculations of the viscosity and thermal conductivity of H_2 and He, tabulated values of the collision integrals based on the Lennard-Jones potential model were taken from Reference 19, pp. 1126-1127. Force constants were also obtained from Reference 19, p. 110. The expression used for calculating thermal conductivity was developed on a semi-empirical basis by Eucken (Reference 12) and was also derived theoretically by Hirschfelder, Reference 19, p. 534.

Hirschfelder's general mixture equations of viscosity, Reference 19, p. 531, mass diffusivity, Reference 19, p. 539, and thermal conductivity, Reference 19, p. 535, were used in computations for H_2-air and He-air mixtures. No additional input data are needed for the use of these equations except the mole fraction of foreign gas and the force constants of the mixture. Empirical combining laws were used to obtain mixture force constants which account for interaction between unlike molecules,

$$\sigma_{12} = 0.5(\sigma_1 + \sigma_2) \quad \text{and} \quad \epsilon_{12} = \sqrt{\epsilon_1 \epsilon_2}, \tag{A.1}$$

(see p. 222 of Reference 19).

Approximate, semi-empirical relations were used as a comparison method for determining viscosity and thermal conductivity of H_2-air and He-air mixtures. The expression used for viscosity calculations (Reference 5, p. 5) was first developed by Buddenberg and Wilke (Reference 4, p. 1345) and simplified according to kinetic theory by Wilke (Reference 28, p. 57). In order to evaluate Wilke's equations, the temperature, concentration, molecular weights, and viscosities of the pure gases are needed. The pure component viscosities were obtained by use of Hirschfelder's modification of the Chapman-Enskog theory outlined previously. A semi-empirical method for thermal conductivity derived by Enskog and presented by Carlson and Schneider (Reference 5, p. 13) was used in the same manner as the Wilke method for viscosity. Pertinent tables in Reference 10 have been used for comparison.

Computations for the viscosity and thermal conductivity of water vapor were based on the Stockmayer potential function which is a more realistic model for polar molecules than is the Lennard-Jones model. The collision integrals for this case were taken from tables in Reference 22, pp. 23-33. Force constants for water molecules were obtained from Reference 19, p. 599. Force constants for H_2O-air mixtures were determined from equation (A-1) and were then multiplied by a correction factor (Reference 19, p. 600) to account for the effective potential energy of attraction between a polar and nonpolar molecule. The viscosity and thermal conductivity for H_2O-air mixtures were then determined by the theoretical method used for H_2-air and He-air mixtures. Diffusion coefficients for H_2O-air mixtures were calculated according to an empirical equation given by Eckert (Reference 9, p. 512). Figures 2, 3, and 8 present viscosity, thermal conductivity, and ordinary diffusion coefficient, respectively, as functions of foreign gas concentration w for selected temperatures T.

2. Specific Heat and Density

Experimental values of the specific heats of pure gases (References 13, 18, and 20) were used in all cases except in calculations of the thermal conductivity of H_2-air and He-air mixtures (see Figure 4). For this case, the specific heats of the pure components were determined by use of the energy principle of equipartition. The actual contributions of the various energies for each degree of freedom of the molecule to the total energy of the molecule were estimated according to expressions given by Hirschfelder in Reference 19, pp. 116-119. The specific heats of mixtures were determined by the following ideal gas relationship:

$$c_p(T, w) = c_{p_f}(T)\, w + c_{p_a}(T)\, (1 - w), \qquad \text{(A-2)}$$

(Reference 5, p. 3). Densities of pure gases and mixtures were determined from the following ideal gas relationship:

$$\rho_i = \frac{p_i M_i}{RT} \quad \text{and} \quad \rho = \frac{PM_2}{RT[1 + W(\frac{M_2}{M_1} - 1)]}, \qquad \text{(A-3)}$$

(Reference 5, p. 3). The average molecular weight of air was taken as 28.97 (79% N_2 and 21% O_2). Figures 1 and 5 present the density and the specific heat as functions of foreign gas concentration w for selected values of temperature T.

APPENDIX B

CORRELATION FORMULAS FOR $Nu/\sqrt{Re}$ IN CASE OF $M_\infty = 0$

The similarity equations (12) and (14) through (18) have been solved numerically on a digital computer under conditions of $M_\infty = 0$, constant properties, and Euler numbers $m = 0$, $m = 0.5$, and $m = 1$. The Euler number is defined by $u_\infty(x) = c\ x^m$; $m = 0$ and 1, respectively, represent the cases under discussion in this paper, i.e., constant pressure and stagnation point. The numerical results for the heat transfer parameter

$$Nu/\sqrt{Re} = -\ x\ \frac{\partial T(x,\ 0)}{\partial y} / (T_w - T_\infty)\sqrt{Re}$$

have been correlated as functions of the pertinent constant parameters, Pr, Sc, $c = c_{pf}/c_{pa}$, and B where $B = -\ f_w Pr (m + 1)/2\ [Nu/\sqrt{Re}]_{f_w=0}$ is the blowing factor. The correlation functions are valid in the following bounds:

$$\left\{\begin{array}{ccccc} 0.0 & \leqq & B & \leqq & 0.1 \\ 0.3 & \leqq & Pr & \leqq & 5.0 \\ 0.25 & \leqq & c & \leqq & 5.0 \\ 0.2 & \leqq & Sc & \leqq & 5.0 \end{array}\right. \qquad \text{(B-1)}$$

For $0.25 \leqq c \leqq 1.0$, the blowing factor may vary in the bounds $0 \leqq B \leqq 0.3$. For $m = 0$,

$$Nu/\sqrt{Re} = 0.33203 Pr^{0.3489} - B g_1(Sc,\ c) F_1(Pr) \left\{1 + S_1(Pr,\ c) \left[H_1(c) + \right.\right.$$

$$\left.\left. + A_1(Sc)\ A_2(c) - \frac{10\ E_1(Pr,\ c)}{g(Sc,\ c)\ F_1(Pr)}\right]\right\}, \qquad \text{(B-2)}$$

where

$$F_1(Pr) = 0.835 + 0.18\ Pr,$$

$$g_1(Sc,\ c) = 0.2394 + (c - 1) \left[(-0.00312\ c + 0.18213 + Sc(0.00059\ c - \right.$$

$$\left. - 0.03298) + Sc^2(-0.00005\ c + 0.00364)\right].$$

$$S_1(Pr,\ c) = -\frac{(Pr - 1)\ (c - 1)}{|Pr - 1|\ |c - 1|} \quad \text{and 0 if } Pr = 1 \text{ or } c = 1,$$

$$H_1(c) = |c - 1| \left[\frac{0.115}{0.63 + c}\right],$$

$$A_1(Sc) = Sc^{1/4}$$

$$A_2(c) = 0.21066 - 0.05479\ c + 0.01058\ c^2, \text{ and}$$

$$E_1(Pr,\ c) = c^{0.6407} \left[-0.33 + 2.19\ c + 0.63\ Pr + 0.25/Pr\right] 10^{-3}.$$

For $m = 0.5$,

$$Nu/\sqrt{Re} = 0.4775Pr^{0.3777} - Bg_2(Sc,\ c)\ F_2(Pr) \left\{1 + H_2(c) + G(c)\ E_2(Pr)\right\} \tag{B-3}$$

where

$$F_2(Pr) = 0.76 + 0.265\ Pr,$$

$$g_2(Sc,\ c) = 0.3036 + (c - 1) \left[0.25633 - 0.05076\ Sc + 0.00583\ Sc^2\right]$$

$$H_2(c) = (1 - c) \left[0.27902 - 0.44556\ c + 0.18848\ c^2 - 0.02194\ c^3\right],$$

$$G(c) = (c - 1)/|c - 1| \quad \text{and 0 if } c = 1, \text{ and}$$

$$E_2(Pr) = 0.40581 - 0.29358/Pr - 0.05363\ Pr + 0.02649/Pr^2.$$

For $m = 1.0$,

$$Nu/\sqrt{Re} = 0.57047Pr^{0.3858} - Bg_3(Sc,\ c)\ F_3(Pr) \left\{1 + S_1(Pr,\ c) \left[H_3(c) + {} + E_3(Pr)\right]\right\}, \tag{B-4}$$

where

$$F_3(Pr) = 0.762 + 0.275\ Pr,$$

$$g_3(Sc,\ c) = 0.3577 + (c - 1)\left[0.30035 - 0.05438\ Sc + 0.00589\ Sc^2\right],$$

$$H_3(c) = (1 - c)\left[0.26302 - 0.30873\ c + 0.12054\ c^2 - 0.01369\ c^3\right], \text{ and}$$

$$E_3(Pr) = 0.45561 - 0.32596/Pr - 0.06026\ Pr + 0.02916\ Pr^2.$$

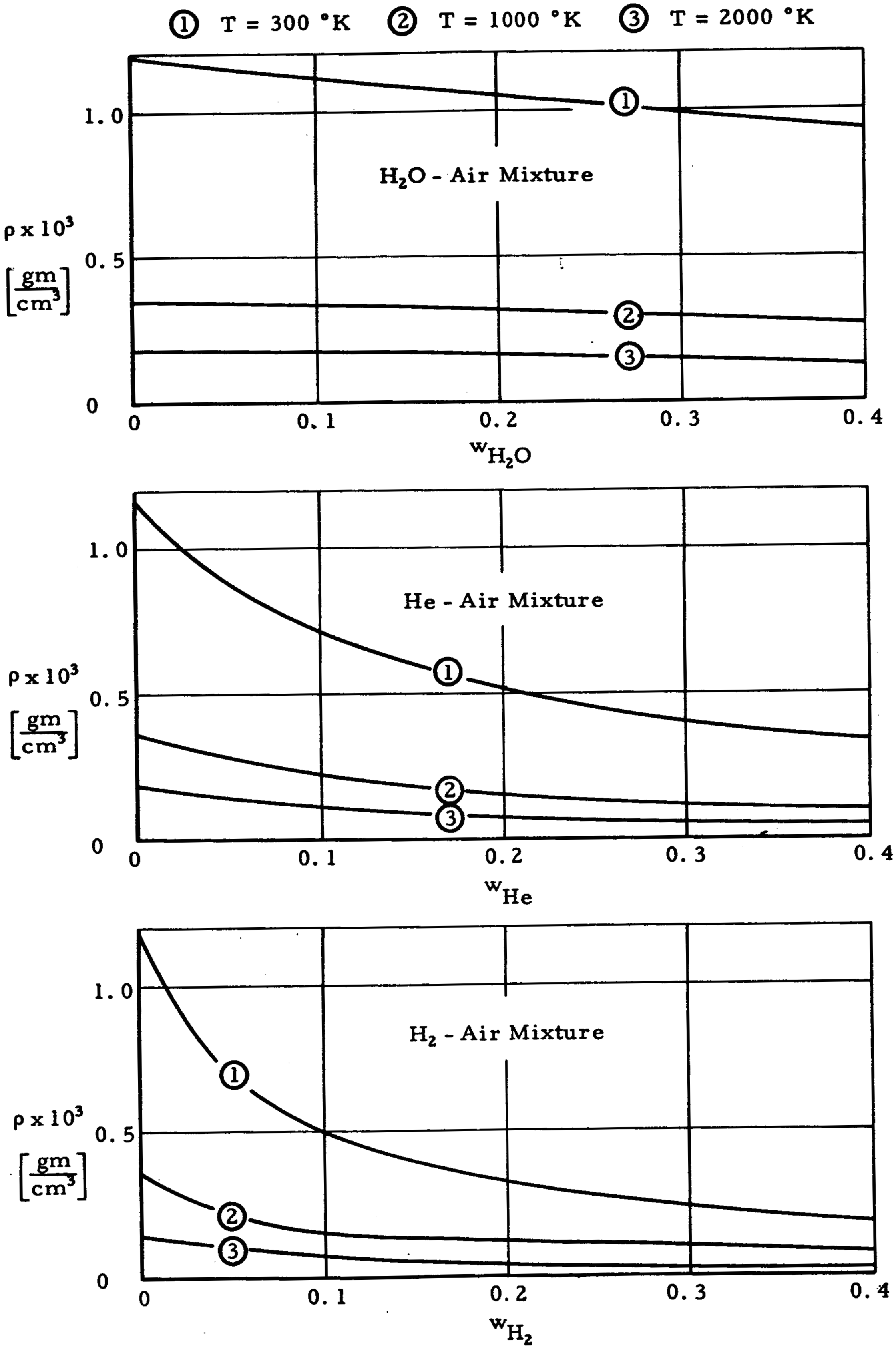
Comparison of the computer solutions of equations (12) and (14) through (18) to the correlation formulas (B-2) through (B-4) shows a maximum error of 0.032 of $Nu/\sqrt{Re}$ in the bounds defined by the relations (B-1). $Nu/\sqrt{Re}$ varies in the bounds $0.158 \leq Nu/\sqrt{Re} \leq 1.025$ in this region. The average error of $Nu/\sqrt{Re}$ in this region is approximately equal to 0.01.

NOTE: The correlation formulas presented in Appendix B have been derived by Mr. F. E. McKinney of the George C. Marshall Space Flight Center.

TABLE 1

ERRORS IN SATISFYING THE RELAXED FORM OF CONDITIONS (19) - (23)

CASE NO.	GAS - AIR	T_w(°K)	T_∞(°K)	W_w	$(\varphi_\rho\ \varphi_\mu)_w$	$(\varphi_\mu/\varphi_k)_w$	Sc_w/Sc_∞	$(\varphi_{cp})_w$	$\frac{c_{p_{air}}(T_w)-c_{pf}(T_w)}{c_{p_{air}}(T_\infty)-c_{pf}(T_\infty)}$
1	H_2O-Air	800	1000	.051	1.055	.995	1.078	1.007	.889
2	H_2O-Air	1500	2000	.201	1.009	.893	1.153	1.125	.889
3	H_2O-Air	300	1000	.302	1.087	.901	1.170	1.193	.735
4	H_2O-Air	500	1500	.349	1.102	.895	1.357	1.108	.719
5	He-Air	1500	2000	.0056	1.072	.848	1.045	.922	1.022


① T = 300 °K
② T = 1000 °K
③ T = 2000 °K
H_2O - Air Mixture
$\rho \times 10^3$
$\left[\frac{gm}{cm^3}\right]$
1.0
0.5
0
0.1
0.2
0.3
0.4
w_{H_2O}
①
②
③
He - Air Mixture
$\rho \times 10^3$
$\left[\frac{gm}{cm^3}\right]$
1.0
0.5
0
0.1
0.2
0.3
0.4
w_{He}
①
②
③
H_2 - Air Mixture
$\rho \times 10^3$
$\left[\frac{gm}{cm^3}\right]$
1.0
0.5
0
0.1
0.2
0.3
0.4
w_{H_2}
①
②
③

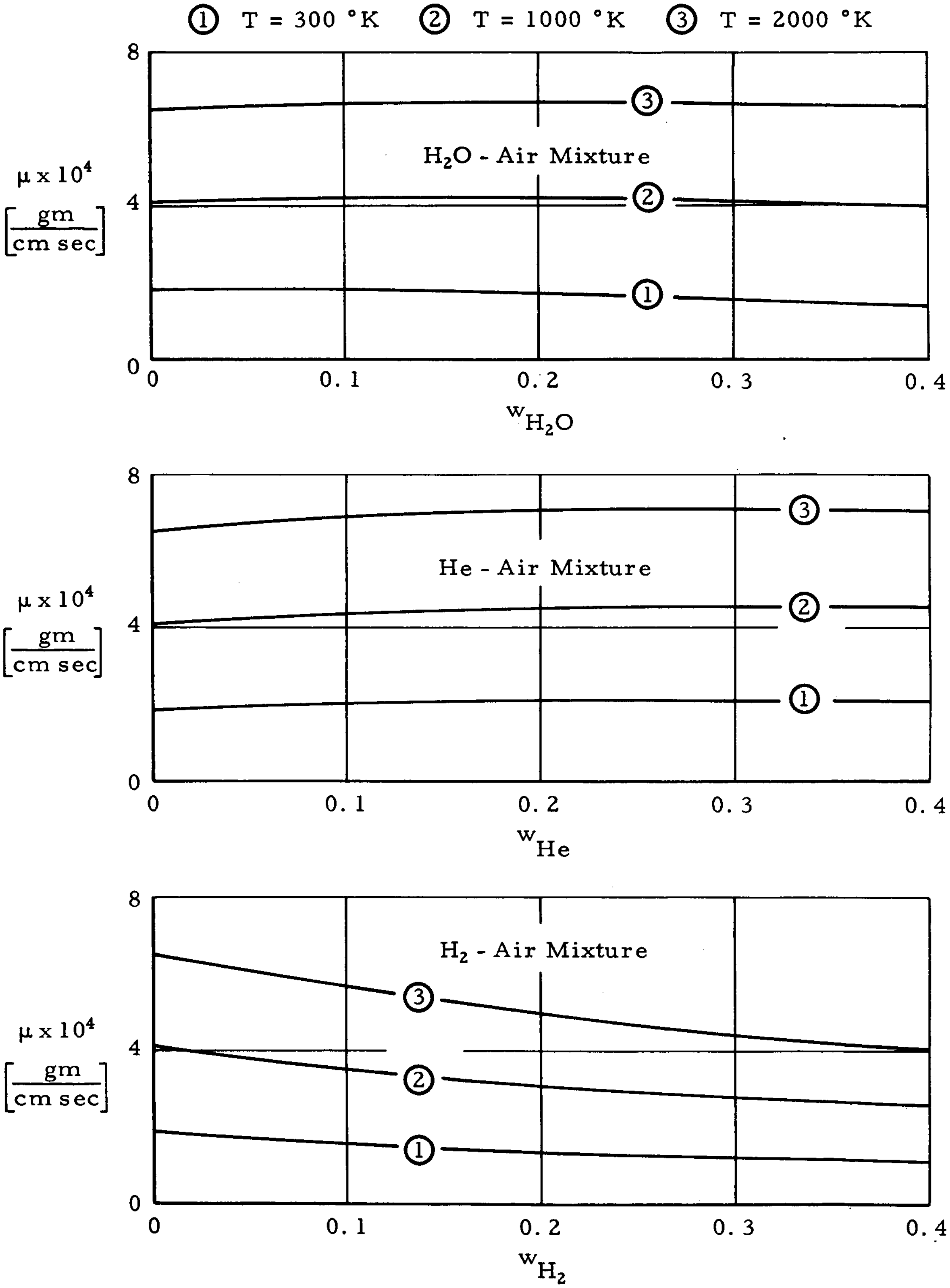


FIG. 2. VISCOSITY OF MIXTURES

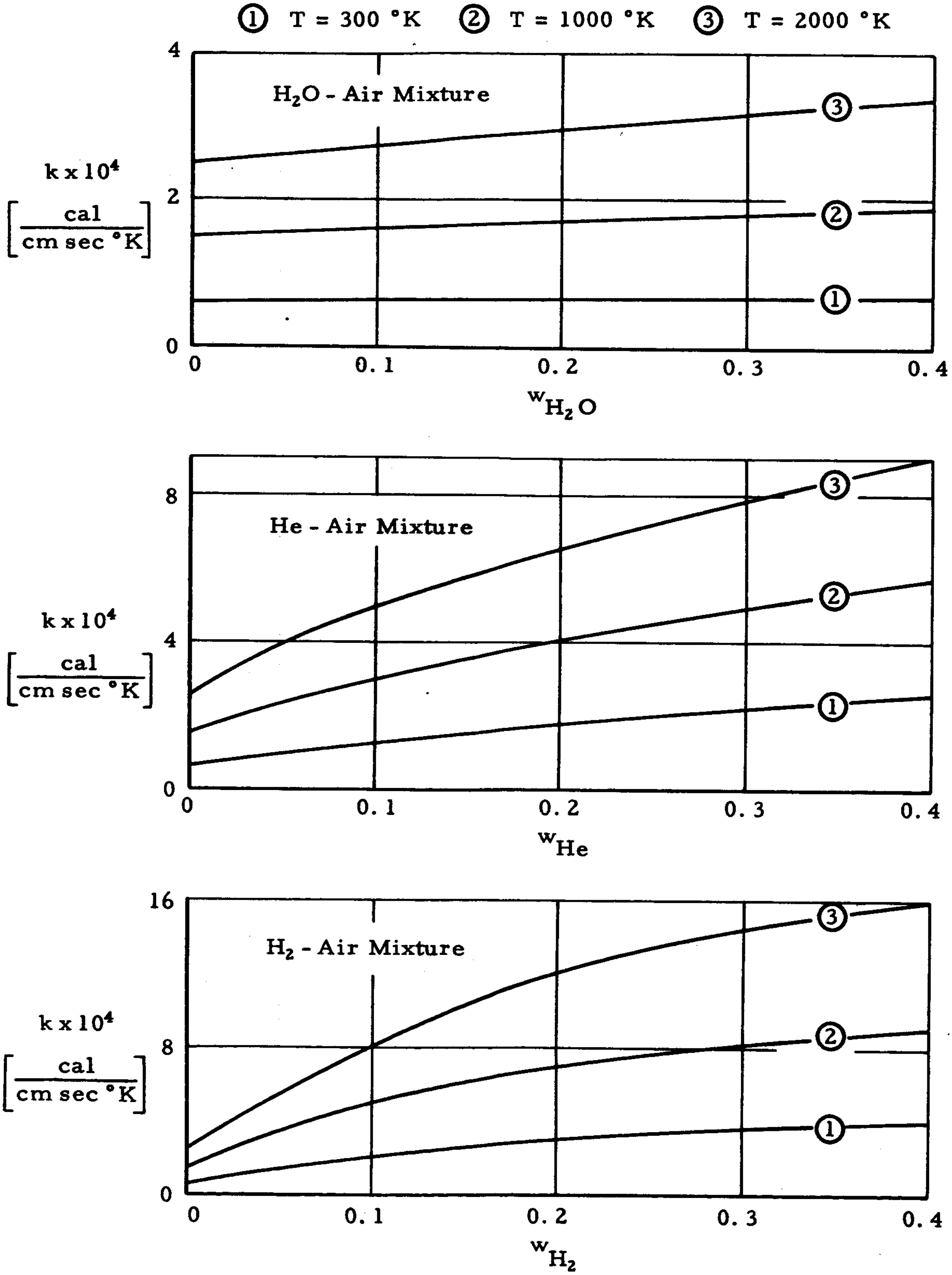


FIG. 3. THERMAL CONDUCTIVITY OF MIXTURES

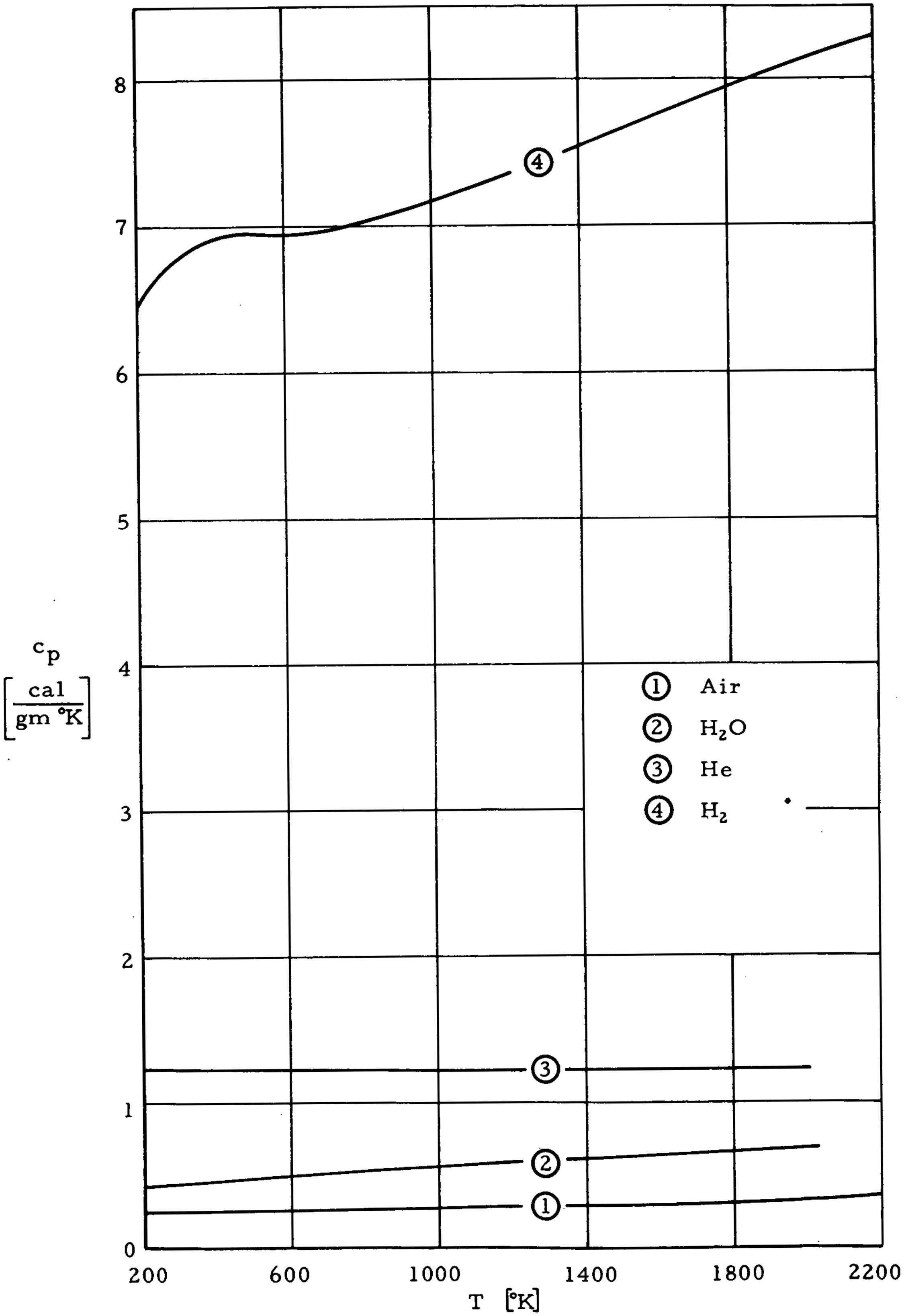


FIG. 4. SPECIFIC HEAT OF PURE GASES

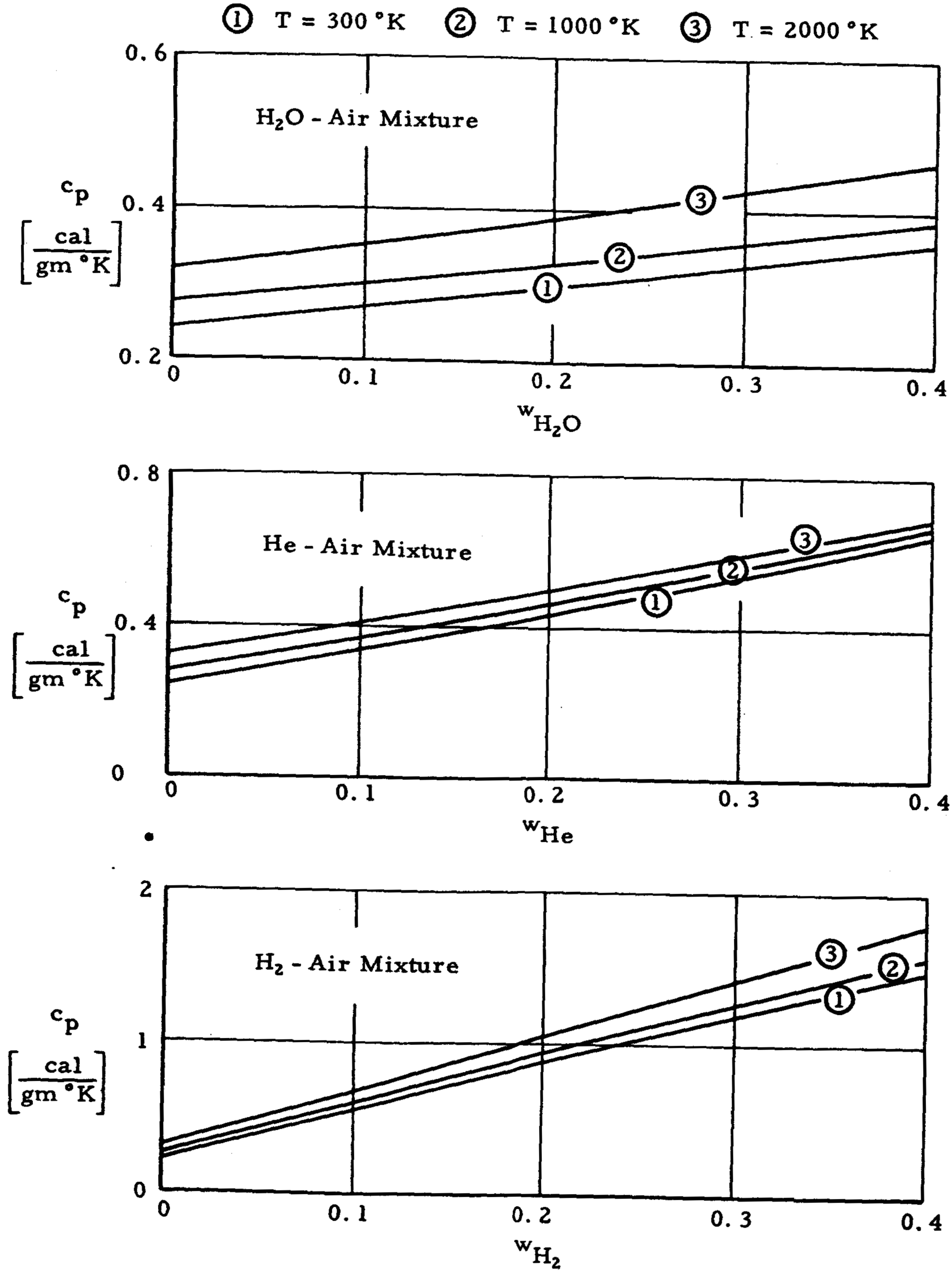


FIG. 5. SPECIFIC HEAT OF MIXTURES

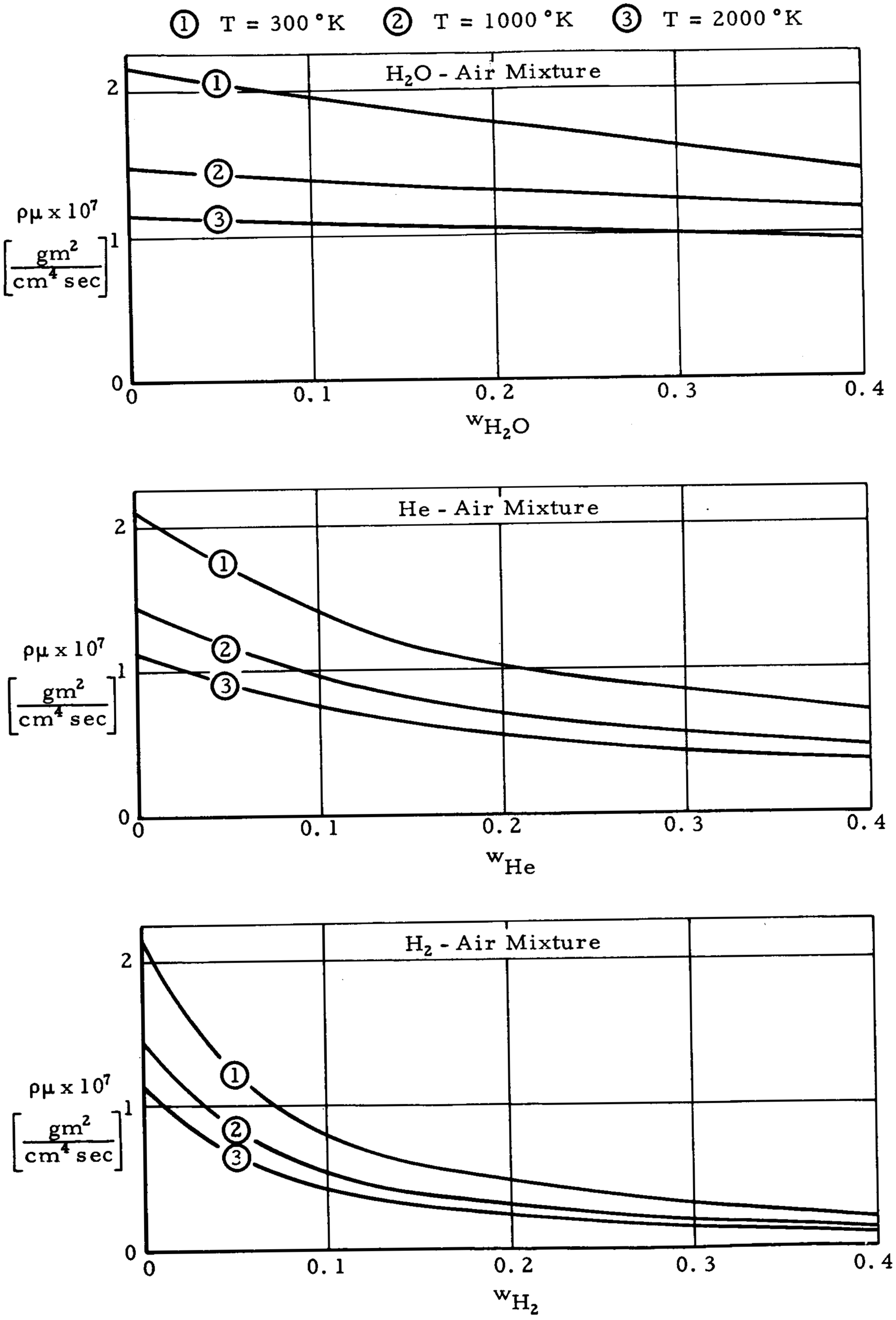


FIG. 6. DENSITY-VISCOSITY FACTOR FOR MIXTURES

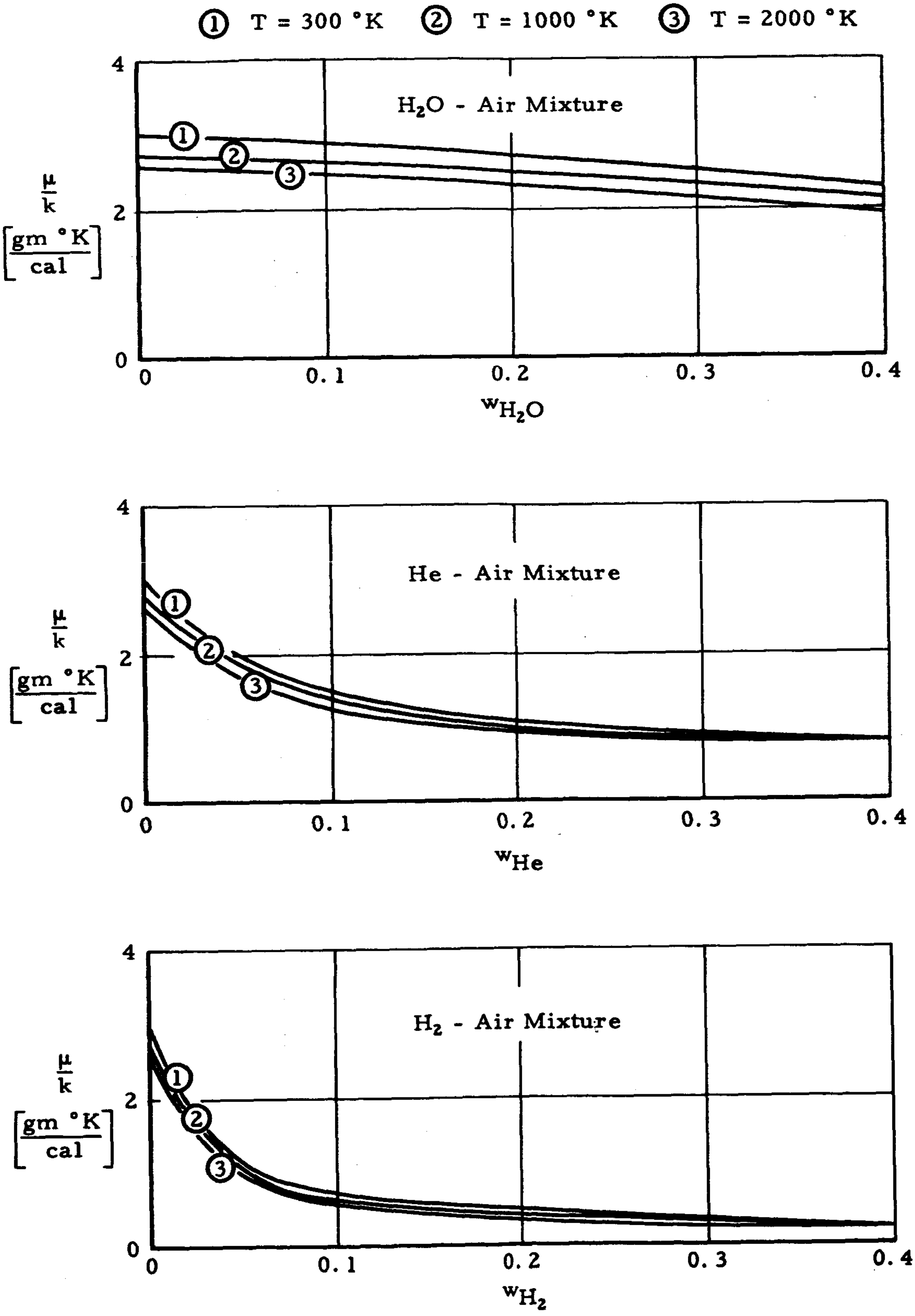


FIG. 7. VISCOSITY-THERMAL CONDUCTIVITY FACTOR FOR MIXTURES

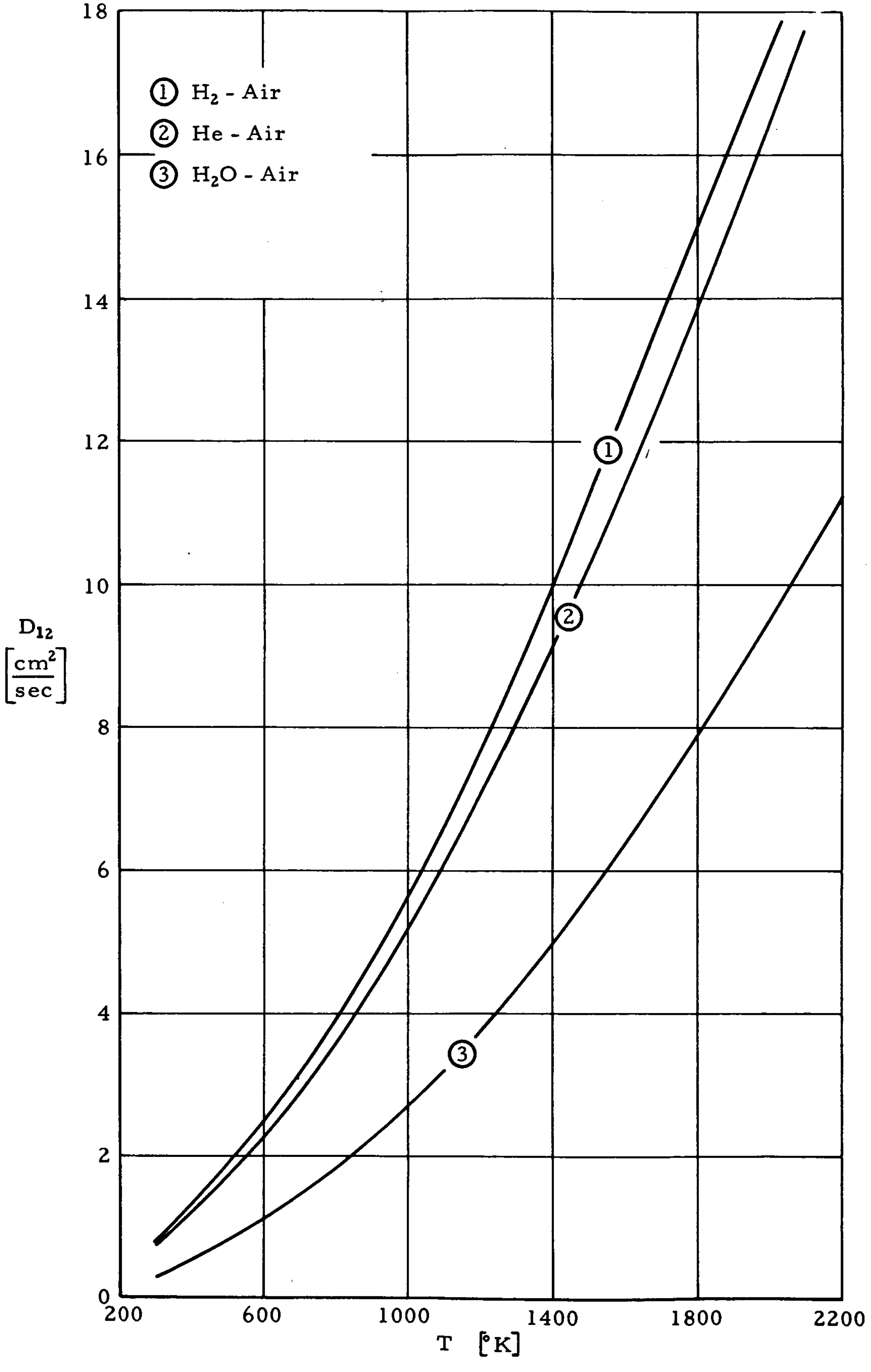


FIG. 8. ORDINARY DIFFUSION COEFFICIENT FOR MIXTURES

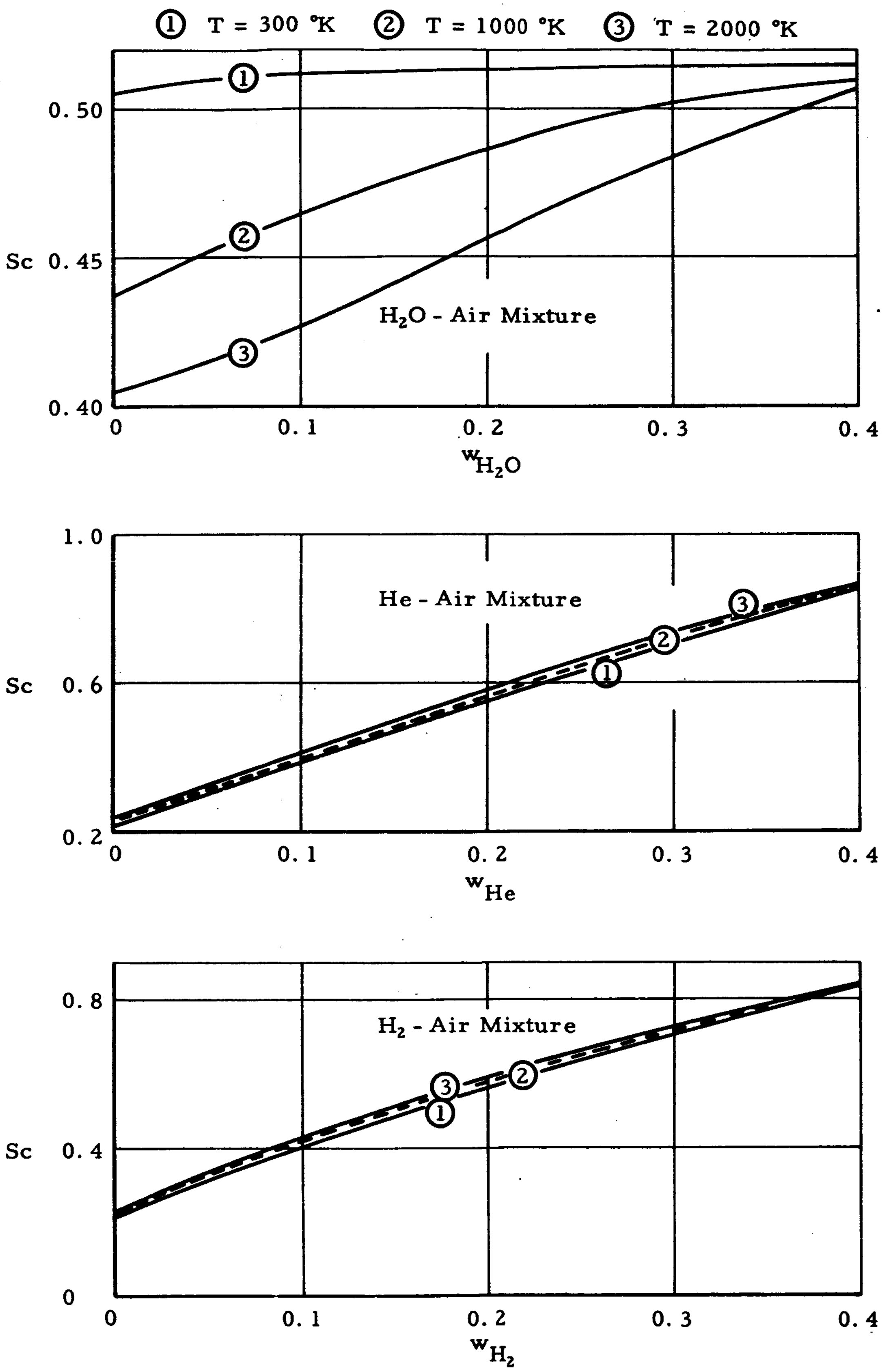


FIG. 9. SCHMIDT NUMBER FOR MIXTURES

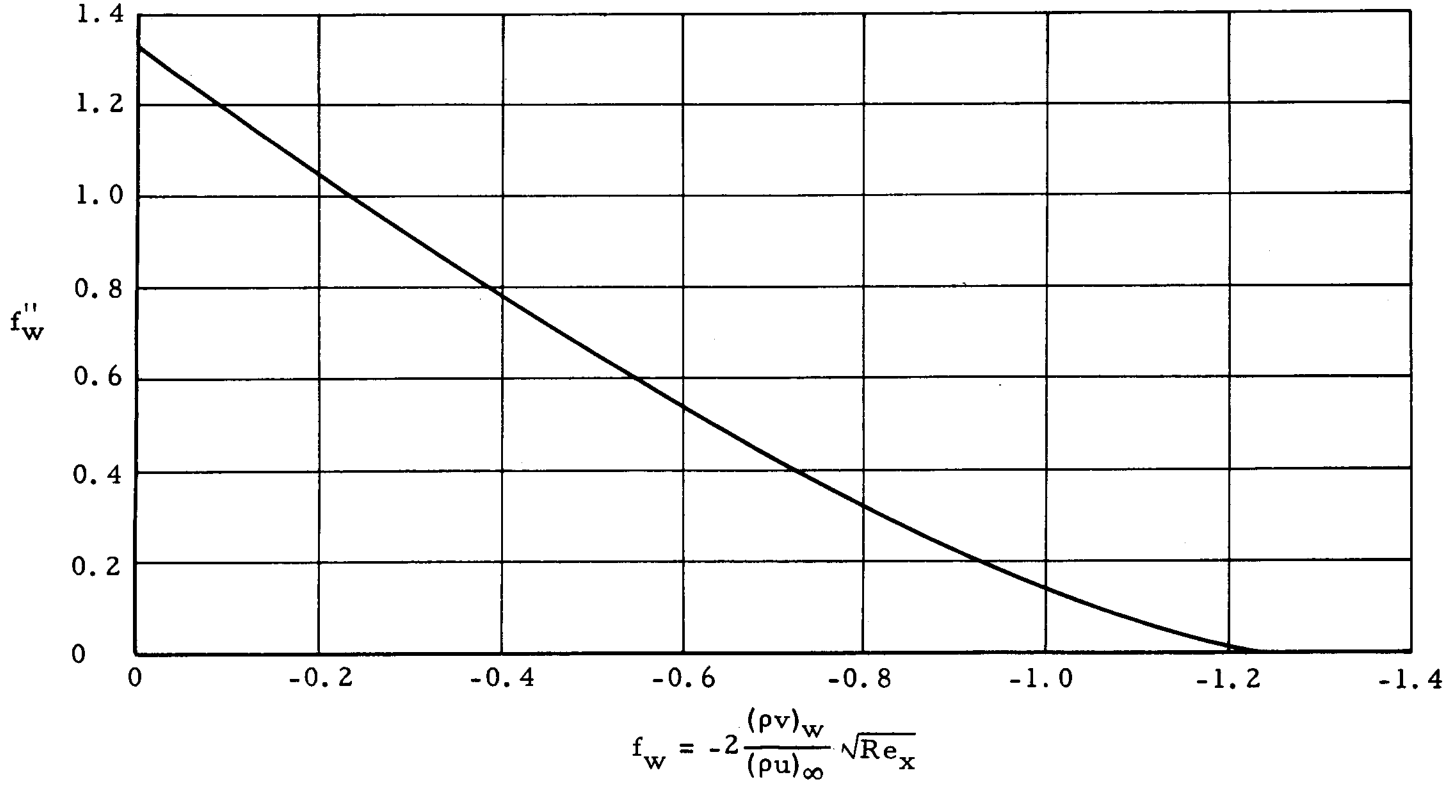


FIG. 10. RELATIONSHIP BETWEEN f''_W AND f_W, FOLLOWING FROM SOLUTIONS OF EQ. (12)

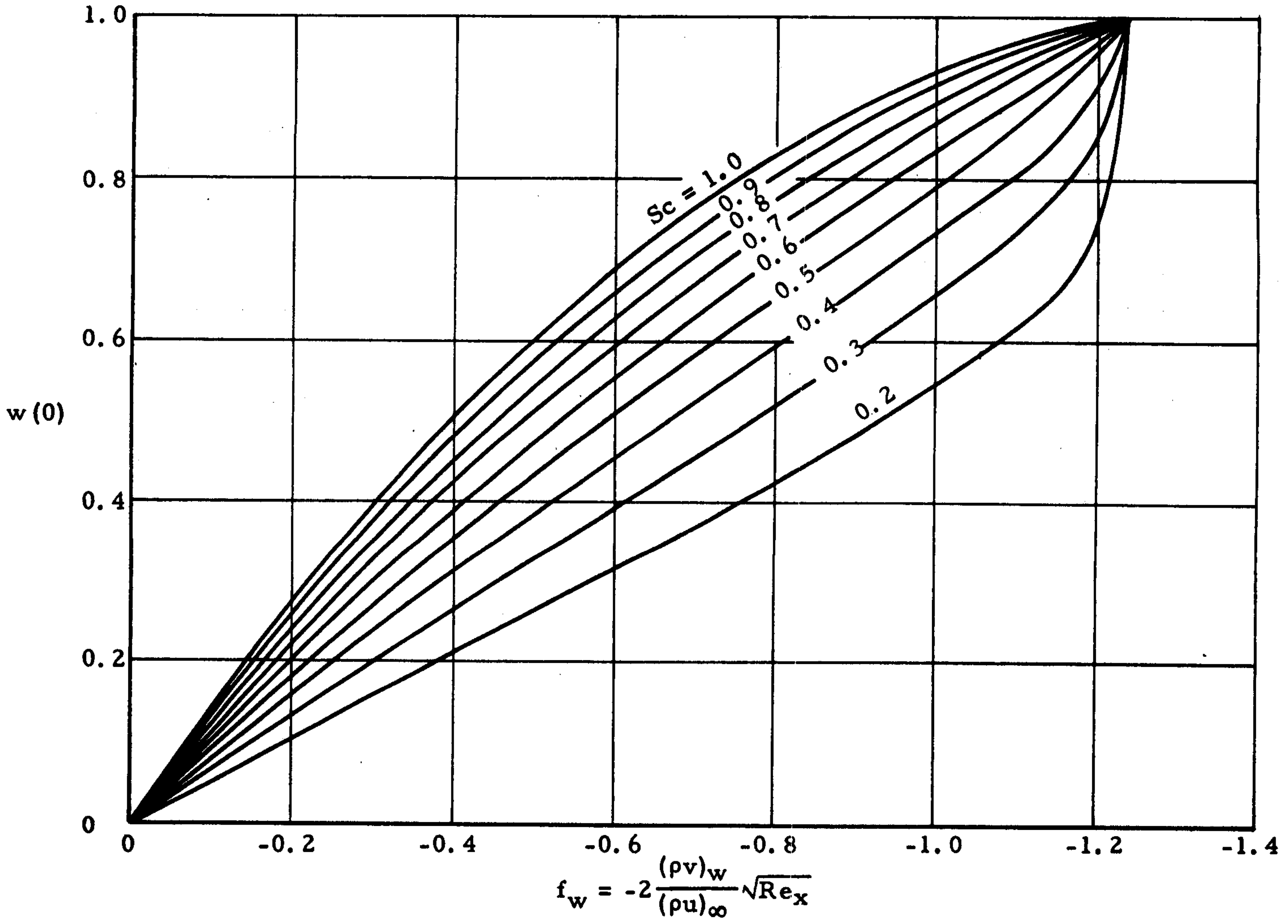
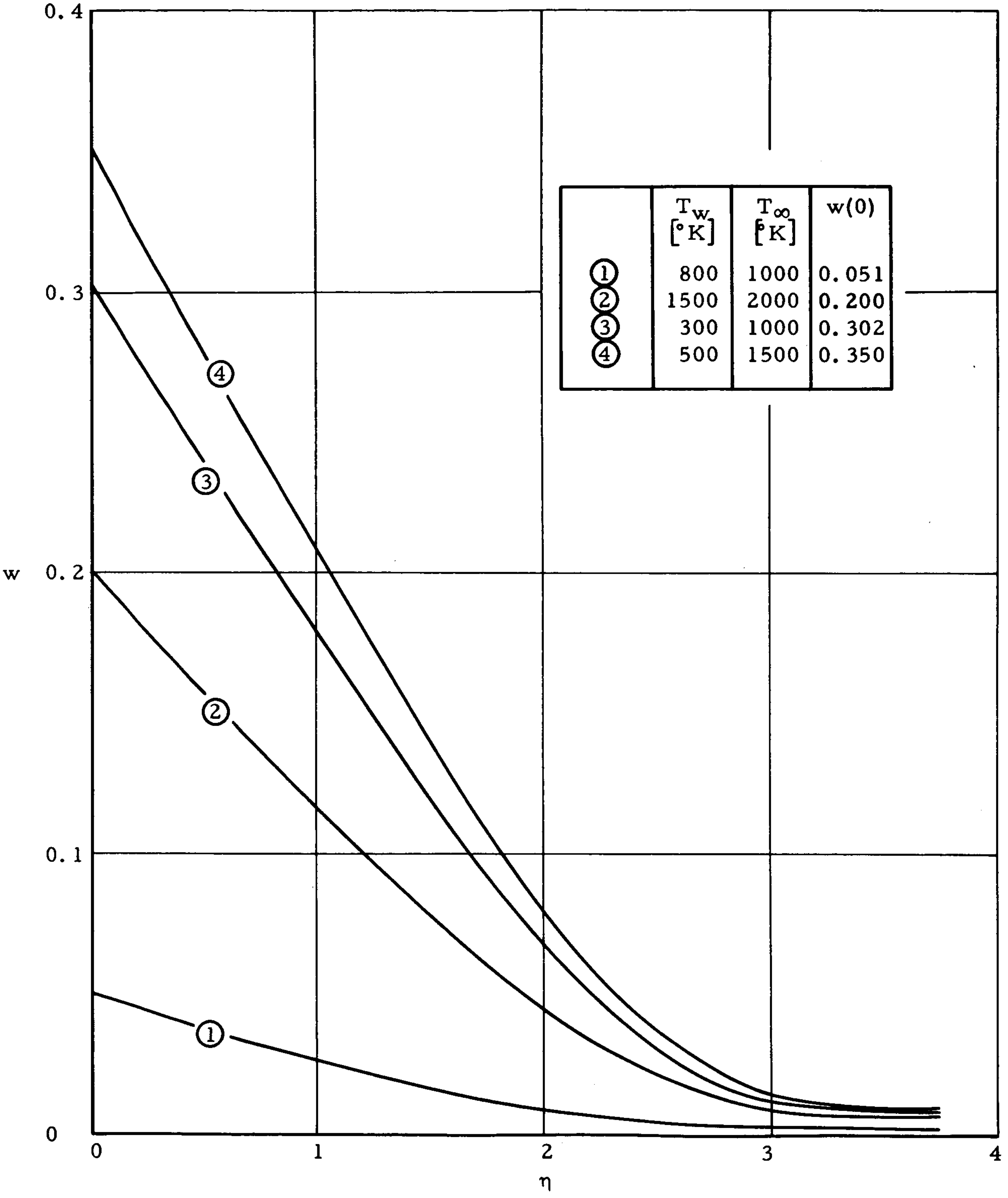


FIG. 11. RELATIONSHIP BETWEEN w_W AND f_W, FOLLOWING FROM EQ. (25)



	T_w [°K]	T_∞ [°K]	w(0)
①	800	1000	0.051
②	1500	2000	0.200
③	300	1000	0.302
④	500	1500	0.350

FIG. 12. CONCENTRATION PROFILES

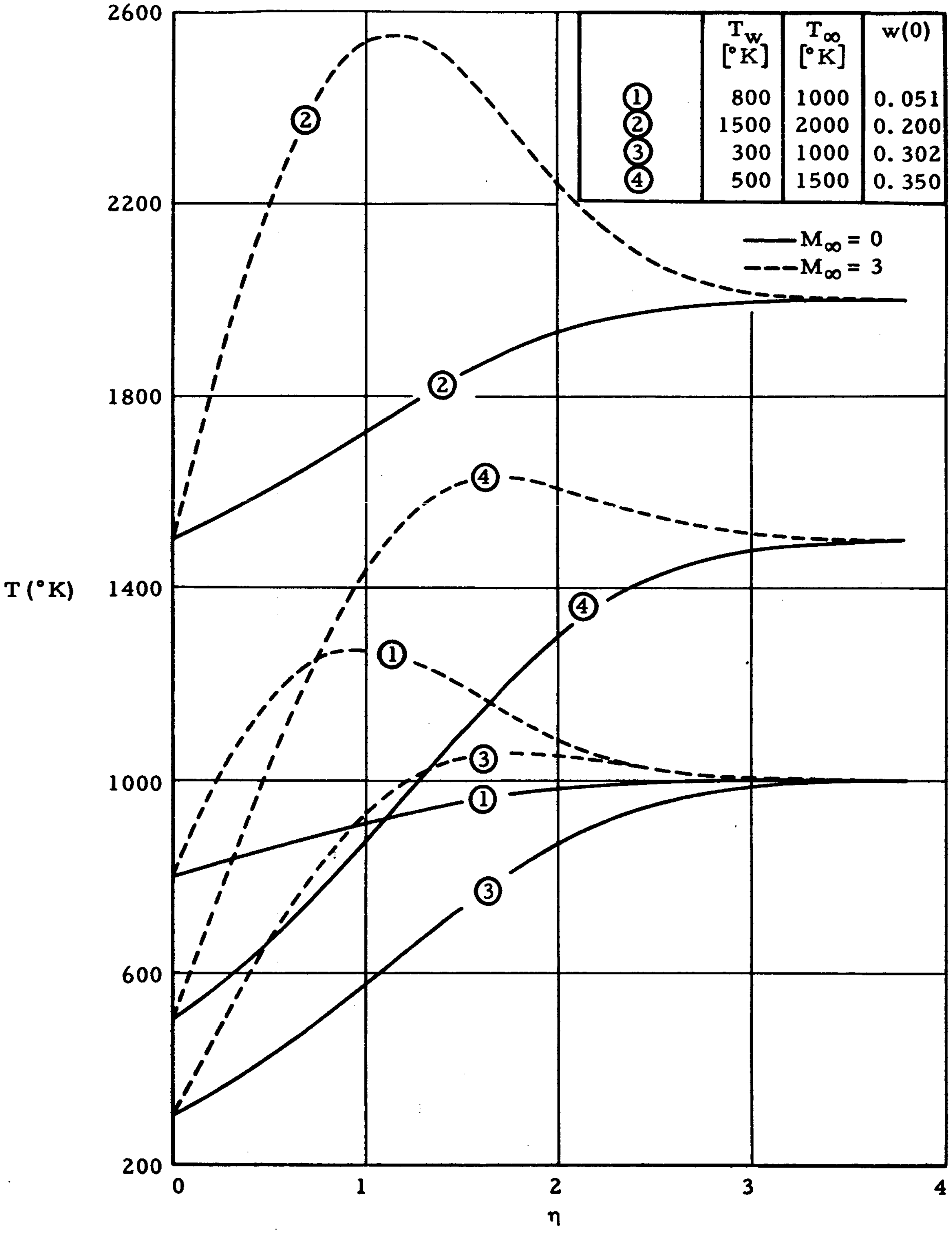


FIG. 13. TEMPERATURE PROFILES

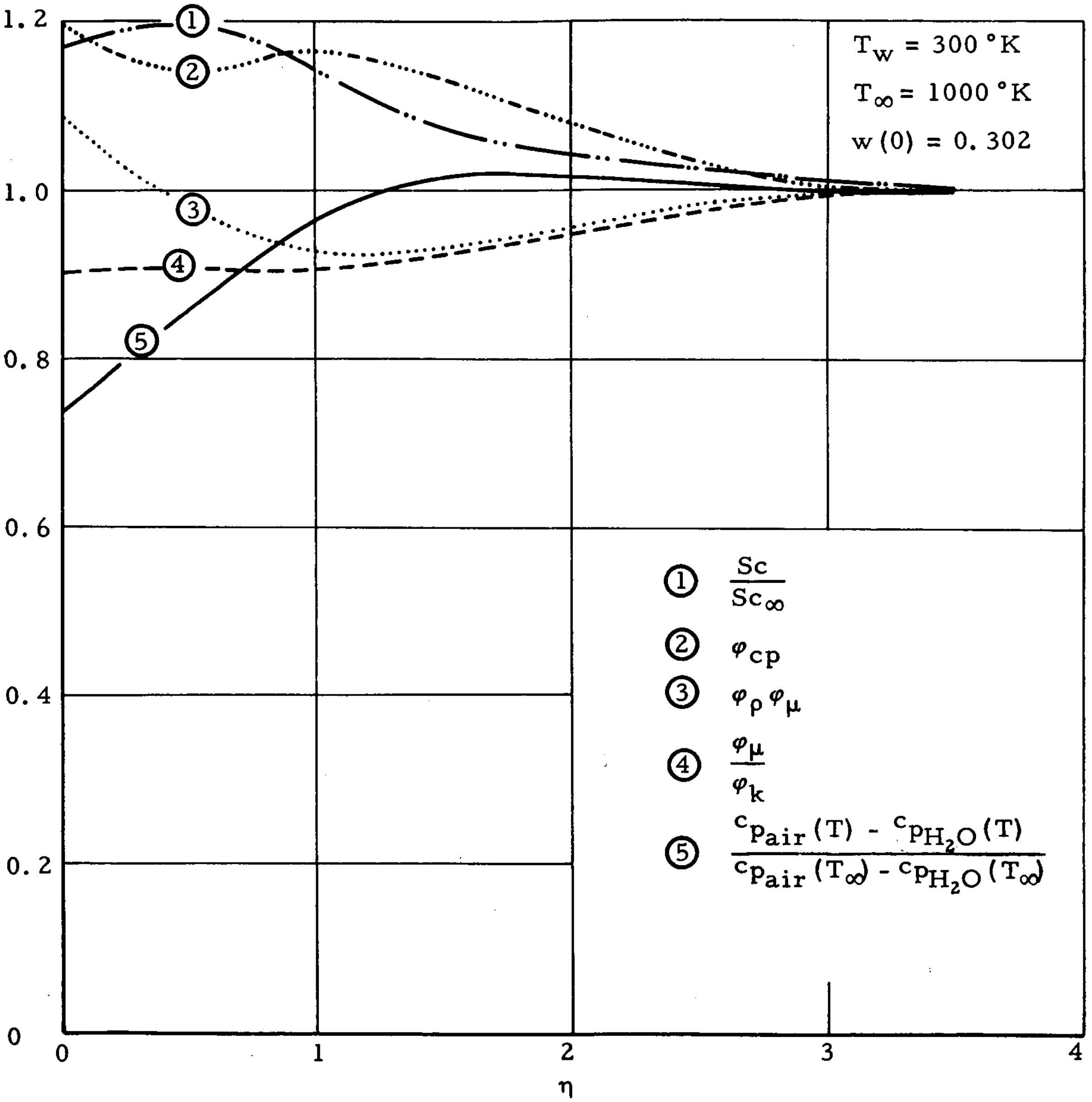
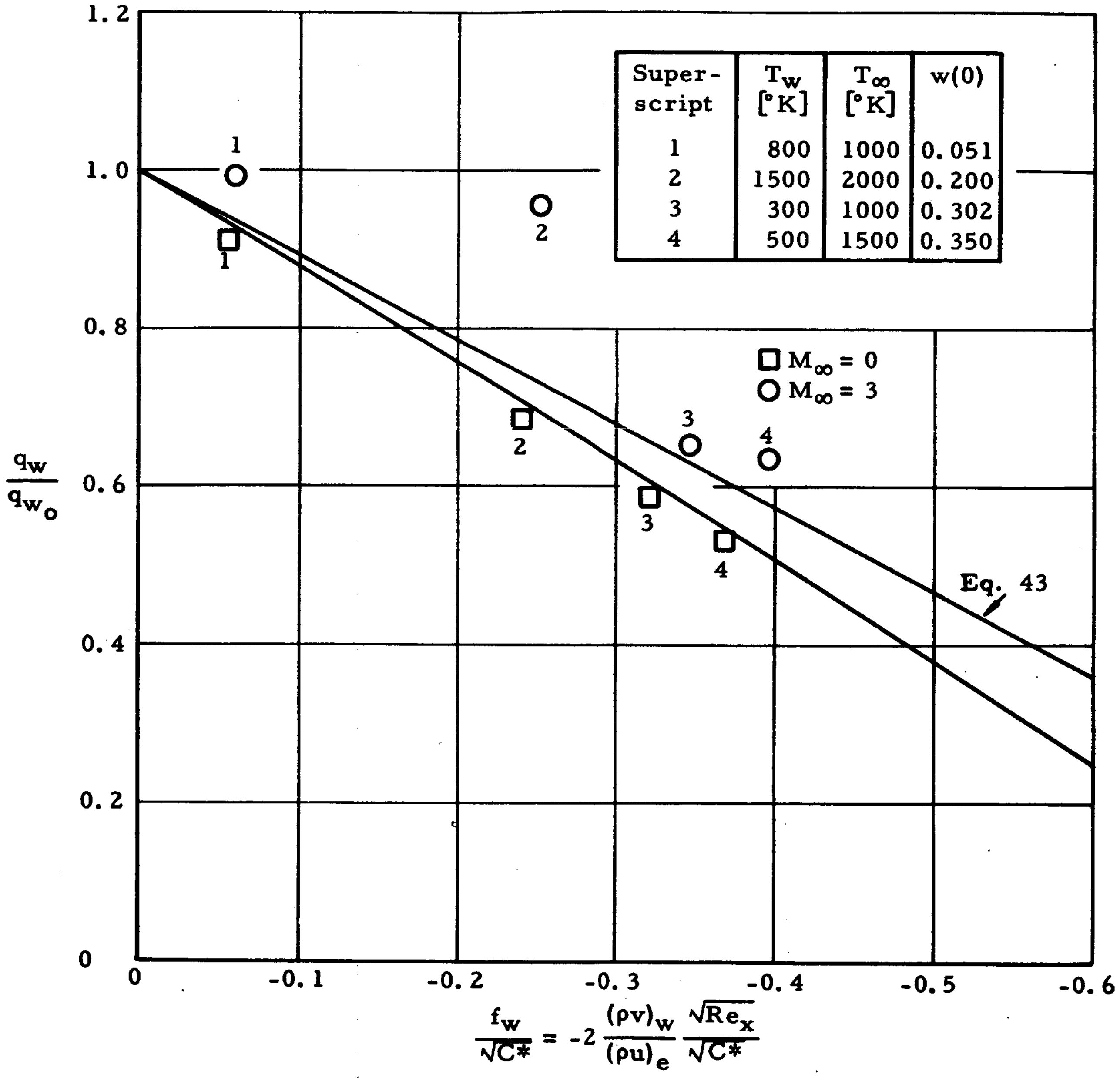


FIG. 14. VALIDITY OF THE PROPERTY ASSUMPTIONS (19) - (23) FOR ONE CASE



Super-script	T_w [°K]	T_∞ [°K]	w(0)
1	800	1000	0.051
2	1500	2000	0.200
3	300	1000	0.302
4	500	1500	0.350

FIG. 15. DIMENSIONLESS WALL HEAT TRANSFER RATE VS. BLOWING RATE FOR INJECTION OF WATER VAPOR INTO AIR

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HEAT AND MASS TRANSFER IN BINARY INERT GAS FLOW FOR DISTRIBUTIONS OF TEMPERATURE AND CONCENTRATION RENDERING THE PROPERTIES NEARLY CONSTANT

By Ernst W. Adams, John D. Warmbrod, C. Lee Fox, and Robert M. Huffaker

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