In-Plane Lunar Injection Opportunities from an Orbiting Space Station

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Nomenclature

i = inclination of earth satellite orbit plane to earth's equatorial plane
J = coefficient of second harmonic of earth's gravitational potential
r = radial distance of vehicle from center of earth
R = equatorial radius of earth
t = time measured from the nominal
T = times for in-plane injection opportunities
δ_M = inclination of moon's orbit to earth's equatorial plane
ϕ = inclination of earth satellite orbital plane to the earth-moon plane
$\Delta\phi$ = regressional rate of earth satellite orbital plane about polar axis of earth
$\dot{\omega}$ = angular velocity of moon
Ω_E = angle measured in equatorial plane from moon's ascending node to node of earth satellite orbital plane and equatorial plane
Ω_M = angle measured in earth-moon plane from moon's ascending node to node of earth satellite orbital plane and earth-moon plane

THE manner in which a near-earth satellite orbital plane regresses about the polar axis has been well established, e.g., see Ref. 1. In the event that a lunar launch is to be made from an earth satellite orbit (the injection to be made in the plane of this earth satellite orbit), a delay in injection will cause the lunar trajectory, due to this regression, to be established in a plane having an orientation in space different from that originally planned. Consequently, the moon, in general, will not be at the nodal point of the earth satellite orbital and earth-moon planes at the time the vehicle arrives there. It is possible to correct for the effects of this regression by the application of a velocity increment to change the plane of the lunar trajectory, as discussed in Ref. 2. If the delays, however, are to be of the order of days, as might be the case for a space station, then it is of interest to know that there are certain discrete intervals of time, after the nominal, in which injection into the lunar trajectory can be accomplished in the plane of the earth satellite orbit. These opportunities correspond to times for which the moon and either nodal point of the earth satellite orbital and earth-moon plane line up. Since the two motions are in opposite directions, this alignment occurs whenever the total angular travel of the moon and the node of the earth-moon plane and earth satellite orbital plane add up to 180°. The determination of these injection opportunities for all nominal positions of the moon in its orbit (all nominal values of Ω_M in Fig. 1) is the object of this paper.

If Ω_E and Ω_M denote the location of the regressed node of the earth satellite orbital and the node of the earth-equatorial and earth-moon planes, respectively, measured from the moon's ascending node in the direction of regression, it is obvious from Fig. 1 that

$$\Omega_M = \cot^{-1}\left[\frac{\cos\delta_M \cos\Omega_E - \sin\delta_M \cot i}{\sin\Omega_E}\right] \quad (1)$$

where

$$0 \leq \Omega_E \leq 360 \qquad 0 \leq \Omega_M \leq 360$$

If at time $t = 0$ (the nominal time of injection from earth orbit) $\Omega_M = (\Omega_M)_0$ and $\Omega_E = (\Omega_E)_0$, then, after a time in earth orbit of $t = T$, the nodal point of the earth satellite orbital
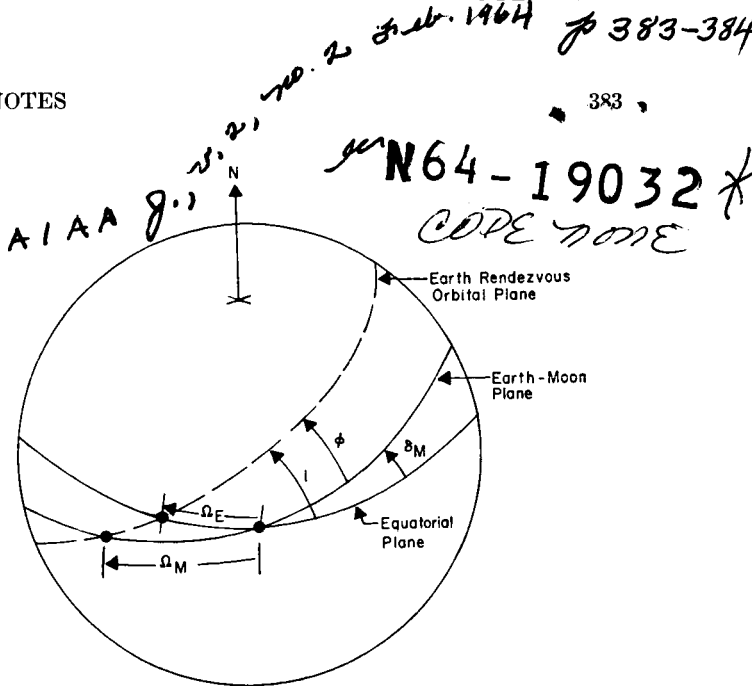
Fig. 1   Illustration of pertinent angles.

and earth-equatorial planes will have regressed an amount $\Delta\Omega_E$ along the earth-equatorial plane, and the nodal point of the earth satellite orbital and earth-moon planes will have regressed an amount $\Delta\Omega_M$ along the earth-moon plane. Therefore, at time $t = T$,

$$\Omega_M = (\Omega_M)_0 + \Delta\Omega_M \qquad \Omega_E = (\Omega_E)_0 + \Delta\Omega_E$$

The regression rate of the node along the earth-equatorial plane is constant and for circular orbits is

$$\Delta\phi = 2\pi J \cos i (R^2/r^2) \quad (2)$$

Then, after time T,

$$\Delta\Omega_E = T\Delta\phi \qquad \Omega_E = (\Omega_E)_0 + T\Delta\phi \quad (3)$$

The corresponding value of Ω_M, in terms of T, is obtained by substitution of Eq. (3) into Eq. (1). Therefore, at $t = 0$,

$$\Omega_M = (\Omega_M)_0 = \cot^{-1}\left[\frac{\cos\delta_M \cos(\Omega_E)_0 - \sin\delta_M \cot i}{\sin(\Omega_E)_0}\right]$$

and at time $t = T$

$$\Omega_M = \cot^{-1}\left(\frac{\cos\delta_M \cos[(\Omega_E)_0 + T\Delta\phi] - \sin\delta_M \cot i}{\sin[(\Omega_E)_0 + T\Delta\phi]}\right) \quad (4)$$

The determination of the time increments for the in-plane injection opportunities is accomplished by a solution for T of the equation

$$\Delta\Omega_M + \dot{\omega}T = 180 \quad \text{or} \quad \Omega_M - (\Omega_M)_0 + \dot{\omega}T = 180 \quad (5)$$

where $\dot{\omega}$ is the angular rate of the moon in its orbit (about 13.2 deg/day).

It is of considerable interest in such an operation as this to know the manner in which the inclination of the trajectory plane to the earth-moon plane varies for each injection opportunity. This inclination is designated by the angle ϕ in Fig. 1. From Fig. 1, it is obvious that

$$\phi = \sin^{-1}(\sin i \, \sin\Omega_E/\sin\Omega_M) \quad (6)$$

Then, if an injection opportunity occurs T days after the nominal, substitution of Ω_E from Eq. (3) and Ω_M from Eq. (4) into Eq. (6) will give the inclination of the trajectory plane to the earth-moon plane for this injection opportunity. This variation can be considerable, as will be pointed out below when a particular numerical example is illustrated.

To illustrate the significance of Eqs. (5) and (6), a numerical example is represented in Fig. 2 for the case of $\delta_M = 28.5°$, $i = 30°$, and $r = 315$ statute miles. Figure 2 gives the first few in-plane injection opportunities as a function of the
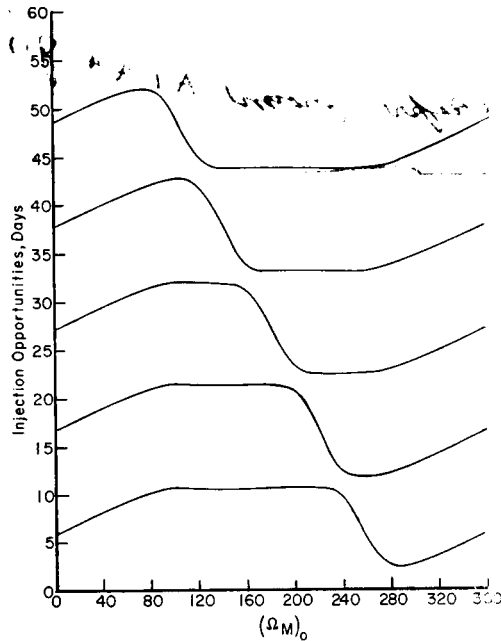
Fig. 2   In-plane injection opportunities for all nominal lunar positions.

nominal values of Ω_M. These curves show that for the majority of the nominal injection conditions the opportunities for injection, after the first opportunity, occur about every 10.5 days. The variations in ϕ at the injection opportunities for this case is between 1.5° and 58.5°.

References

[1] Blitzer, L., "Apsidal motion of an IGY satellite orbit," J. Appl. Phys. **28**, 1362 (1957).

[2] Wells, W. R., "The influence of precession of earth rendezvous orbits on lunar mission requirements," NASA TN D-1512 (1962).