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NASA, *Lewis Research Center, Cleveland, Ohio*

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Cleveland, Ohio

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SUMMARY

As a part of an overall investigation of the behavior of rocket engine propellants stored in space vehicle tanks while exposed to weightlessness, a study was conducted in a 1-g acceleration field to determine the hydrostatic stability of the liquid-vapor interface in a cylindrical container.

The Bond number criterion, consisting essentially in the ratio of acceleration to capillary forces, was found to be valid for predicting the regions of hydrostatic stability of the liquid-vapor interface. The absolute value of the Bond number varied from 0.84 when the cylinder was at a 180° orientation to the acceleration field to essentially infinity for a 0° orientation. The factors that affect the magnitude of the Bond number are the specific surface tension of the liquid and the orientation of the interface with respect to the acceleration field.

When the liquid-vapor interface was positioned at the edge of the cylinder, a significant increase in Bond number was obtained over that obtained when the interface was located in the interior of the cylinder.

INTRODUCTION

The NASA Lewis Research Center is currently conducting a study of the problems associated with the behavior of rocket engine propellants stored in space vehicle tanks while exposed to weightlessness (zero gravity) during coasting periods. A knowledge of the equilibrium liquid-vapor interface configuration will be needed to solve the problems of effective tank venting, pump inlet design, orientation control of space vehicles, and effective long-term propellant storage. The liquid and vapor could, of course, be properly positioned by means of acceleration fields, created, for example, by spinning the tank or firing (ullage) rockets, but these methods might require relatively high energy levels or could be otherwise undesirable, especially for large vehicles. More desirable would be the proper employment of the surface tension of the liquid itself through the use of proper tank geometry.

The behavior of typical wetting and nonwetting liquids in spherical, cylindrical, and conical glass tanks has been studied in a drop-tower zero-gravity facility. The results of these studies, presented in references 1 to 3, allow the prediction of the equilibrium liquid-vapor interface configuration during weightlessness as a function of container geometry, liquid properties, and contact angle. Also, an investigation into the capillary rise in tubes during weightlessness (ref. 4) led to a verification of the theory that solid-liquid-vapor systems tend toward a minimum-surface-energy configuration when the force of gravity is removed from the system. As a result of these studies, a set of criteria was established for the prediction of the static equilibrium configuration of the liquid-vapor interface in a propellant tank as a function of liquid properties and tank geometry. Conversely, the tank geometry could be changed to provide alternative interface configurations, if they were more desirable.

These studies determined the configuration of the interface under the conditions wherein no external accelerations disturbed the system. Vehicles will be subjected to a number of acceleration disturbances, however, as missions grow more complex, such as those resulting from orientation maneuvers, docking jolts, crew or equipment movement, etc. These disturbances will occur at all angles to the vehicle thrust axis and will tend to break the established liquid-vapor interface and cause the vapor to move in the direction of the acceleration. The magnitude of the acceleration that will disrupt the interface is expected to be determined by the Bond number criterion, which is a dimensionless parameter consisting essentially of the ratio of acceleration to capillary forces.

Reynolds (ref. 5) has reformulated the Bond number criterion as follows:

$$Bo = \frac{\rho L^2 a}{\sigma}$$

where ρ is the density of the liquid, L is a characteristic dimension of the system, a is the acceleration, and σ is the surface tension of the liquid.

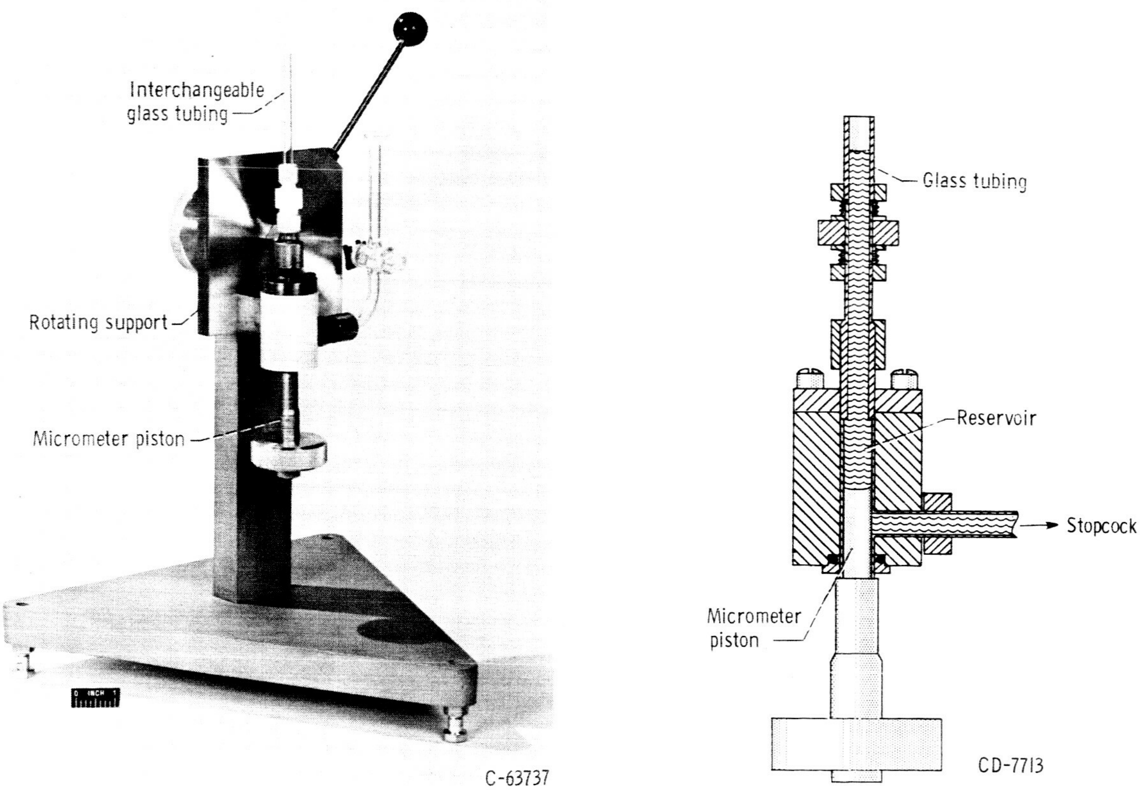
It is expected that below some critical value of Bond number the surface energy forces will be able to maintain control of the interface (the interface will be stable), whereas above this value the acceleration forces will break the interface. It is further expected that this critical value of Bond number will be a function of the configuration of the system and the direction of the acceleration with respect to the liquid-vapor interface. Conflicting values of the critical Bond number have resulted from both refined mathematical analysis and extrapolation of known capillary phenomena (see appendix). Experimental determination of the critical Bond number has not been conducted to resolve the conflict or to establish the validity of the criterion itself.

The purpose of this report is to present the results of an experimental investigation of the hydrostatic stability of a column of liquid conducted in the acceleration field due to gravity (1) to verify the Bond number criterion for a cylindrical tube, (2) to verify the functional relation of the liquid properties that are involved in the criterion, and (3) to establish the numerical value of the critical Bond number at which instability of the liquid-vapor interface occurs.

APPARATUS AND PROCEDURE

Apparatus

The experimental apparatus used to obtain the data presented herein is shown in figure 1(a) and consists essentially of a cylindrical test section mounted so that it could be rotated to any angular position with respect to the acceleration field. A schematic cross section of the apparatus (fig. 1(b)) illustrates the system configuration. When the stopcock is closed, cavitation will not occur because of the small height of the liquid column. Stability of the interface (liquid remaining in the cylinder) is then solely a function of the characteristic dimension of the cylinder and the liquid properties. The test section consisted of a sharp-edged 9-centimeter-long precision-diameter borosilicate glass tube. The tubing used had bore diameters uniform to ±0.01 millimeter and ellipticity of less than 4 percent as determined by optical-comparator techniques. The precision tube was attached through a short length of glass tubing to a small reservoir by means of union reducing tube fittings. The volume of the reservoir and the position of the liquid in the test section were controlled by a stainless-steel micrometer piston. Attached to the res-



(a) Photograph showing location of components.

(b) Schematic cross section.

Figure 1. - Experimental apparatus.

ervoir are a glass fill line and stopcock that ensured a positive seal. The reservoir and tube fittings were machined polytetrafluoroethylene. The entire assembly was mounted on a plate that could be smoothly hand rotated to any desired angular position.

Test Liquids

The purity certified liquids employed in the investigation were chosen to provide approximately a decade range of specific surface tension β (the ratio of surface tension to density). Most of the test liquids were chosen to have a 0° contact angle with glass. Two liquids with finite contact angles were also used to obtain limited data on the relative change in Bond number with contact angle. The pertinent physical properties of the test liquids in this study are shown in table I. The surface tensions of the liquids were carefully measured

TABLE I. - PROPERTIES OF TEST LIQUIDS

Liquid	Density at 20° C, ρ, g/cm^3	Surface tension at 20° C in air, σ, dynes/cm	Contact angle with glass in air, θ, deg	Specific surface tension, $\sigma/\Delta\rho$, cm^3/sec^2	Ratio of specific surface tension to acceleration[b], $\sigma/\Delta\rho a$, cm^2
Trichlorotrifluoroethane	1.579	18.6	0	11.79	1.20×10^{-2}
Carbon tetrachloride	1.595	26.8	0	16.81	1.71
Sym. tetrabromoethane	2.964	49.67	40	16.76	1.71
Methanol	.7928	22.6	0	28.54	2.91
Ethanol, anhydrous	.7893	22.3	0	28.28	2.88
80 Percent ethanol[a]	.8688	24.91	0	28.70	2.93
34 Percent ethanol[a]	.9570	33.24	0	34.77	3.55
Mercury	13.55	476.1	130	35.15	3.59
30 Percent ethanol[a]	.9621	34.76	0	36.17	3.69
20 Percent ethanol[a]	.9735	39.8	0	40.92	4.17
10 Percent ethanol[a]	.9847	49.6	0	50.42	5.14
Water	.9982	72.75	0	72.95	7.44

[a]Percentage compositions by volume with water.

[b]Acceleration due to gravity, 980.2 cm/sec^2.

by the ring method with a Du-Noüy tensiometer, the ring correction being obtained from an equation derived from the curves of reference 6. Correlation with available published surface tension data was 0.2 dyne per centimeter. Inasmuch as the surface tension of solutions changes with time until absorption equilibrium occurs, especially so when dilute solutions are employed (refs. 7 and 8), several hours were allowed for equilibrium to occur before measurements were made. Measured surface tensions of the 10-, 20-, and 30-percent dilute solutions of ethanol and water were reproducible to ±0.5 dyne per centimeter.

The contact angles of symmetrical tetrabromoethane, trichlorotrifluoroethane, and mercury in air with borosilicate glass were obtained by the tilting-plate method. The widely reported hysteresis phenomenon (ref. 9) associated with contact-angle measurement was noted; however, the deviation of the values of the advancing and receding angles from the static angle was less than the reproducibility of the static angle measurements (4°) and, hence, does not alter the stated values.

Procedure

Contamination of the glass surfaces and liquids, which could alter the surface tensions and contact angles of the test liquids, was carefully avoided. All data were taken and liquid property measurements were made in a controlled environment. The precision glass cylinders and all glassware used in the investigation were immersed in heated chromic acid, cleaned ultrasonically in a detergent and distilled water solution, thoroughly rinsed with a solution of four parts methanol and six parts distilled water, and finally warm-air dried. The remaining components were similarly cleaned with a carbon tetrachloride immersion for the polytetrafluoroethylene materials and a trichloroethylene vapor degreasing for the metals being used in place of the chromic acid treatment used for glass.

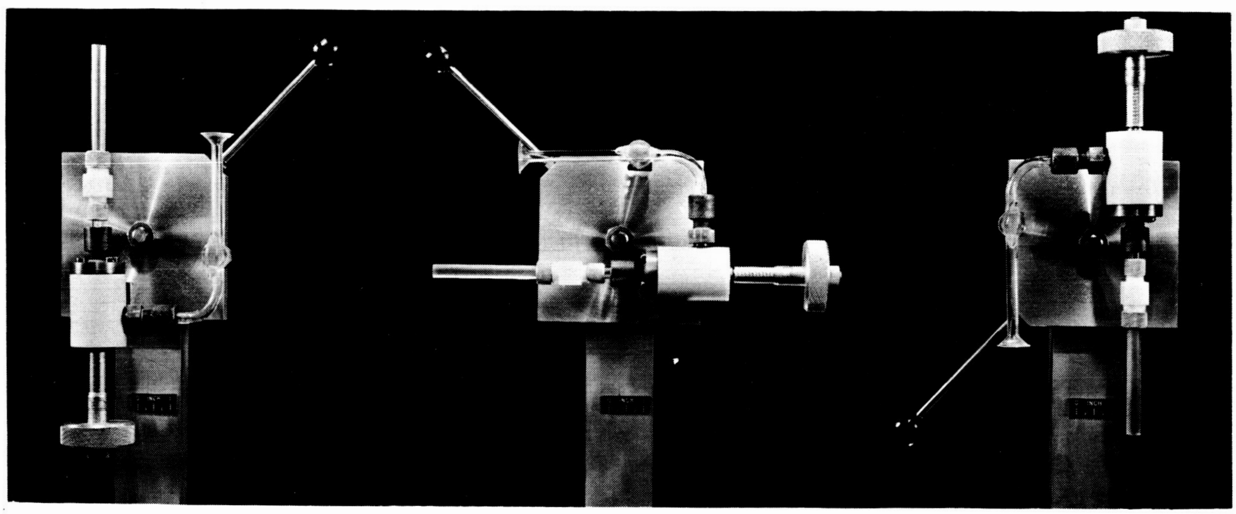
After the cleaning process, the apparatus was assembled, and components were leveled with respect to the rotating support plate. The glass cylinder was alined parallel to the plane of the support. The apparatus was filled to approximately the desired level in the cylinder. Thirty minutes were allowed for temperature equilibrium to be obtained. Any vapor bubbles that may have formed were removed by oscillating the liquid by means of a dropping pipette at the fill-line opening. The stopcock was closed, and the micrometer piston (fig. 1(b)) was used to position the liquid level in the test section. To avoid introducing an acceleration force other than that due to gravity, it was necessary to rotate the supporting plate slowly to the desired angular orientation. Following a prescribed time interval, stability, or nonstability, was recorded. It appeared that the stable configurations would remain so regardless of the time interval involved, provided, of course, that evaporation was negligible. Initial trials remained unchanged after periods as long as 48 hours; later trials were examined generally for 10 minutes.

EXPERIMENTAL RESULTS

The experimental apparatus used in obtaining the data resulted in a range of tube diameters in which stability or instability (generally characterized by breakage of the interface and liquid running from the tube) was observed for each value of the liquid specific surface tension investigated. Two data points were obtained for each test liquid at each angular position: the largest available diameter at which the liquid-vapor interface was stable, and the next available successive diameter at which the liquid-vapor interface was unstable. Hence, the critical radius for each liquid was not obtained directly but is the result of a curve fitted to the entire series of data points at each defined angular orientation. The maximum difference in diameter recorded for any particular test liquid was 1.3 millimeters, and the average diameter deviation between the stable and nonstable points was 0.6 millimeter. Angular positions were referenced to the fill position defined to be 0° (fig. 1(c)).

Vertical Cylinders

The data for the vertical cylinders were obtained by positioning the liquid-vapor interface during the fill procedure approximately 15 millimeters from the exposed edge of the cylinder, the criterion being merely to keep the liquid a noticeable distance from the ground edge. The cylinder was then rotated 180° (fig. 1(c)). The data points indicating the stable and unstable



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Fill position, 0° Horizontal position, 90° Vertical position, 180°

(c) Angular orientations of experiments.

Figure 1. - Concluded. Experimental apparatus.

diameters as a function of liquid properties are shown in figure 2 for a range of liquids with contact angles of 0° and two liquids with contact angles of 40° and 130°. When instability did occur, the liquid-vapor interface either moved to the exposed ground edge of the cylinder and restabilized with a vapor

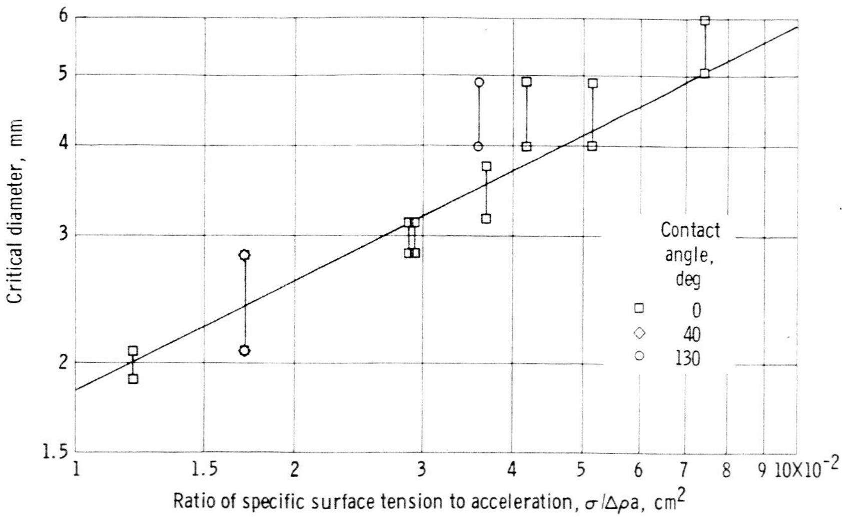


Figure 2. - Stability characteristics in vertical cylinder.

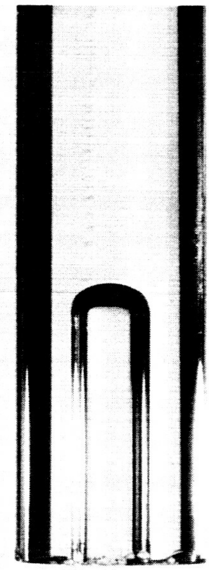


Figure 3. - Hydrostatic stability in vertical cylinder. Carbon tetrachloride; tube inside diameter, 0.208 centimeter.

bubble equal to the volume displaced by the liquid occupying a portion of the fill line, or the liquid left the cylinder completely. A representative photograph of the stable interface configuration is shown in figure 3. The stable interface configuration appears to be nearly spherical and symmetrical about the axis of the cylinder.

Symmetrical tetrabromoethane, which has a contact angle with glass in air of 40°, has the same limit of stability as that of carbon tetrachloride. Both liquids possess nearly identical specific surface tensions. The profile of the stable liquid-vapor interface configurations for tetrabromoethane was similar to that of carbon tetrachloride, with the addition of a well-defined contact angle.

Mercury, a nonwetting liquid, (contact angle of 130° with glass in air) is stable at a larger diameter than a comparable 0° contact-angle liquid with a similar value of specific surface tension. The stable interface configuration for mercury was necessarily convex to satisfy the obtuse contact angle.

Horizontal Cylinders

The data for the horizontal cylinders were obtained by positioning the liquid approximately 20 millimeters from the ground edge of the cylinder (again merely to keep the liquid-vapor interface away from the ground edge) and smoothly rotating the cylinder 90° (fig. 1(c)). The stability characteristics for the horizontal cylinders are shown in figure 4 for a range of liquids with a contact angle of 0° and two liquids with contact angles of 40° and 130°. In-

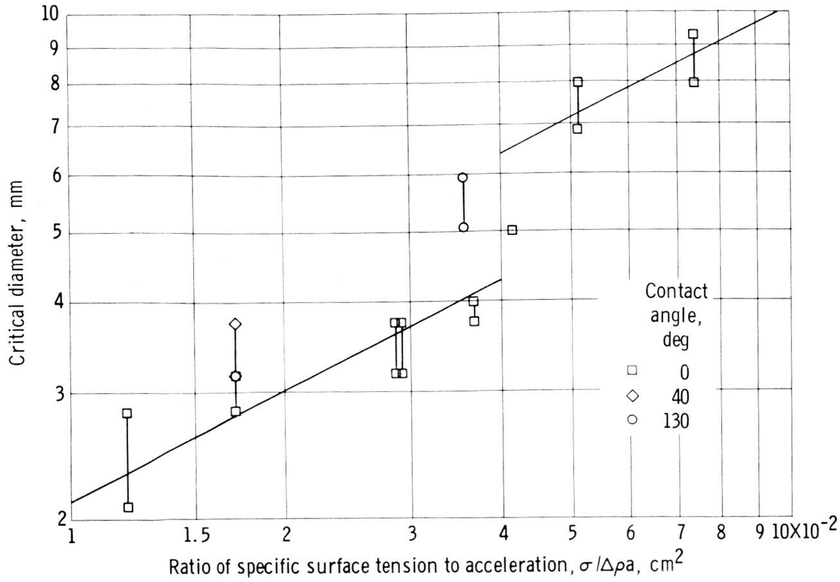
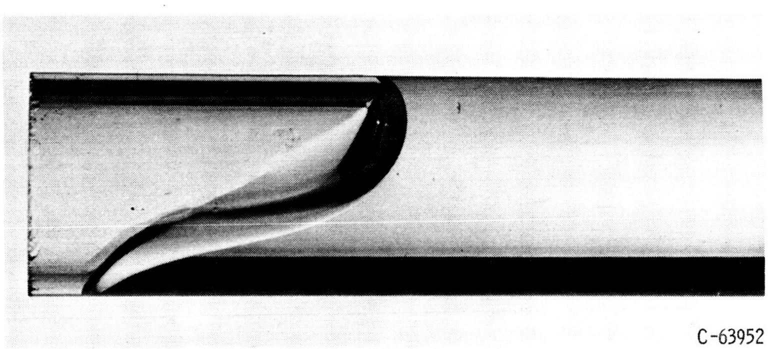
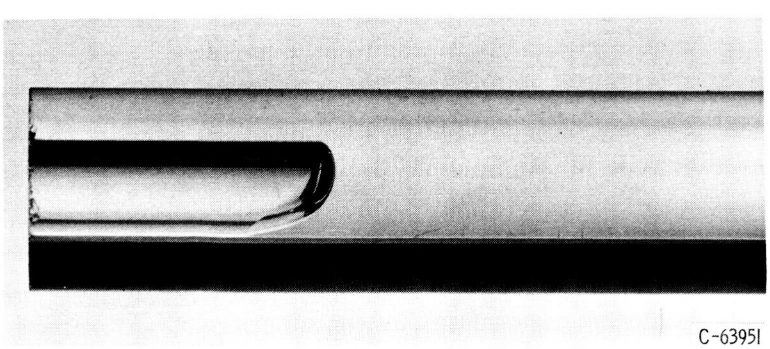


Figure 4. - Stability characteristics in horizontal cylinder.



(a) Water; tube inside diameter, 0.798 centimeter.



(b) Carbon tetrachloride; tube inside diameter, 0.284 centimeter.

Figure 5. - Hydrostatic stability in horizontal cylider.

stability in the horizontal cylinders was characterized by the liquid leaving the cylinder after breakage of the interface. As may be seen from the figure, there is a discontinuity at a specific surface tension of approximately 43 cubic centimeters per second squared. The stable interface configurations of water and carbon tetrachloride typically representative of these two regions are shown in figures 5(a) and (b), respectively.

The effect of contact angle in the horizontal position is to increase the largest stable diameter when compared with liquids that have a 0° contact angle and nearly identical liquid parameters.

Ground Edge of Cylinders

The data for the stability characteristics at angular orientations of 70°, 90°, 110°, and 180° at the ground edge of a cylinder are shown in figures 6 and 7 for a range of liquids with contact angles of 0° and one liquid with a contact angle of 40°. The liquid was positioned during the fill procedure to place the leading edge of the meniscus directly at the ground edge of the cylinder. The resultant liquid-vapor-interface configuration was nearly flat; that is, the radii of curvature of the interface were very large. The cylinder was then rotated to the desired angular orientation, and stability or instability was observed.

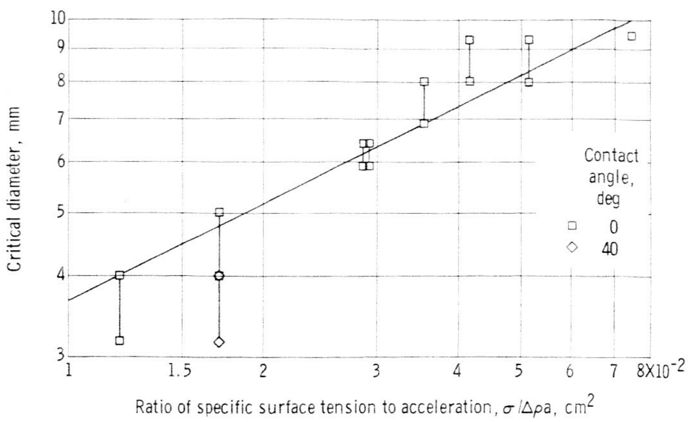


Figure 6. - Stability characteristics at ground edge of vertical cylinder at 180°.

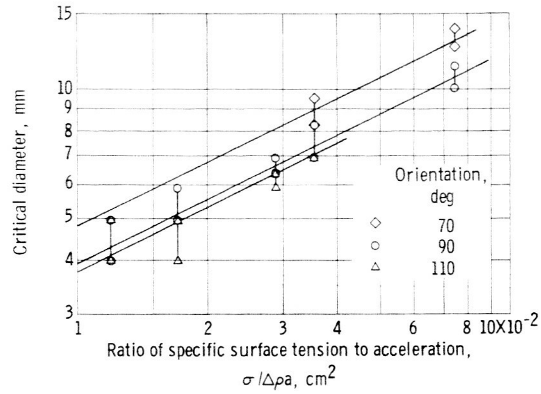


Figure 7. - Stability characteristics at ground edge of cylinder at several angular orientations.

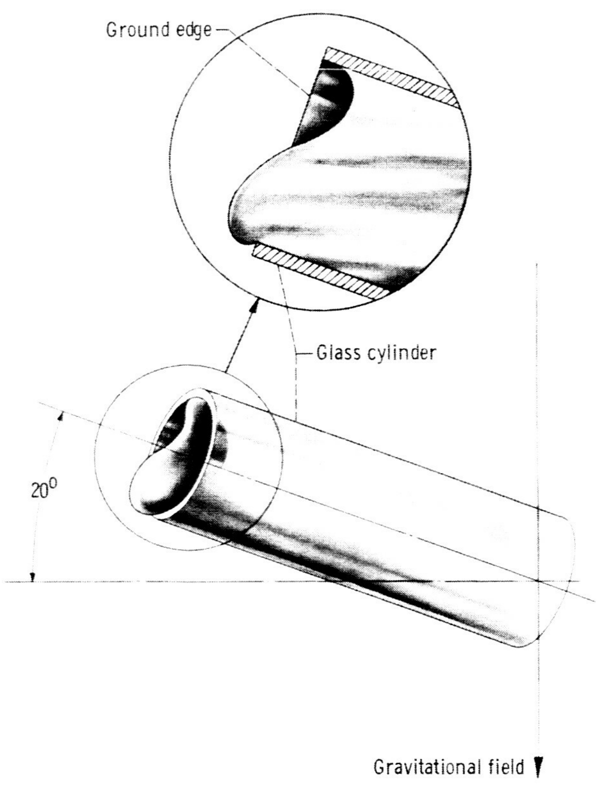


Figure 8. - Schematic diagram showing configuration of liquid-vapor interface at 70° orientation.

The liquid-vapor-interface configurations at 90°, 110°, and 180° were nearly identical to that at the 0° fill position. Instability completely drained the cylinder. The interface configuration for the 70° orientation is shown in figure 8. Instability was characterized by breakage of the interface. The 40° contact angle of tetrabromoethane at an orientation of 180° resulted in a lower range of stable diameter limits than a comparable 0° contact-angle liquid.

DISCUSSION OF RESULTS

Verification of Bond Criterion for Stability

Liquid in interior of cylinder. - The results of the experimental study of the stability characteristics of a liquid-vapor interface in an inverted closed-ended cylinder (fig. 2), establish a functional dependence of the critical radius on the pertinent liquid parameters expected to influence the stability of the interface. The form of the equation that results from the functional dependence of the critical radius on the liquid parameters, as determined from these data, verifies the Bond number criterion for stability under the acceleration field due to gravity is

$$R = 0.92\left(\frac{\sigma}{\Delta\rho a}\right)^{1/2}$$

where σ is the liquid-vapor surface tension, $\Delta\rho$ is the density difference between the liquid and the vapor, and a is the acceleration field. The constant and the exponent were obtained from the slope and intercept of the curve in figure 2. The equation obtained from the data is accurate within 0.5 millimeter over the range of liquid parameters investigated. The Bond number (based on radius) obtained from the data and valid only for contact angles of 0° is 0.84. This value agrees with the results of an analytic investigation by Bretherton (ref. 10).

With the acceleration imposed parallel (at a 90° orientation) to the interface, a rather sharp rise in critical diameter occurs at a specific surface tension of approximately 43 cubic centimeters per second squared (see fig. 4). Photographs of the liquid-vapor interface of representative 0° contact-angle liquids on glass above and below this value of specific surface tension are shown in figures 5(a) and (b). Liquids of a higher specific surface tension distinctly possess two points of inflection compared with one for liquids of lower specific surface tension (excluding the boundaries). Again, the form of the equation based on curves fitted to the two regions verifies the Bond criterion and results in Bond numbers of 2.5 and 1.12 for the upper and lower regions, respectively. Interestingly enough, when the diameter of the upper portion of the interface was used, as obtained from figure 5(a), rather than the radius of the cylinder, the new point obtained fell very close to the curve fitted to the lower region of the stability characteristics. It is not understood, however, why the liquid-vapor interface should assume this formation above specific surface tensions of 43 cubic centimeters per second squared. The formation, as shown in figure 5(b), would be the more probable configuration. It is to be noted that similar formations of "overstability" in inverted vertical cylinders have been reported (ref. 11).

Quite obviously, if the cylinder were in an upright or 0° position, the liquid-vapor interface would always be stable regardless of the magnitude of acceleration. In terms of the stability criterion, the Bond number, effectively, would be infinite. Intermediate to this maximum and the previously established Bond number (occurring at the 180° position), other regions of stability are apparent, with the Bond number dependent on the angular orientation of the interface with respect to the acceleration field. Table II presents a summary of experimental Bond numbers against the orientation of the liquid-vapor interface with respect to the acceleration field for the interface located in the interior of the cylinder.

Liquid at edge of cylinder. - The Bond number of 3.37 obtained from an analysis of the data presented in figure 6 shows that the edge effect doubled the critical diameter for interface stability obtained when the interface was in the interior of the cylinder. The Bond number for the stability characteristics at the edge of a vertical cylinder predicts the critical radius to within 1 millimeter for respective liquid parameters. It is noted that a con-

TABLE II. - EXPERIMENTAL BOND NUMBERS

Position of liquid-vapor interface	Orientation of cylinder with respect to acceleration field, deg	Bond number, $Bo = \frac{R^2 \Delta\rho a}{\sigma}$
Interior of cylinder	0	∞
	90	[a]2.53
	90	[b]1.12
	180	.84
Edge of cylinder	0	∞
	70	5.3
	90	3.8
	110	3.5
	180	3.37

[a]Specific surface tension, >43 cm^3/sec^2.

[b]Specific surface tension, <43 cm^3/sec^2.

tact angle of 40° lowers the critical diameter. An angular orientation of the cylinder in order to vary the direction of an acceleration field imposed on the liquid-vapor interface (fig. 7) resulted in progressively increasing Bond numbers as the angle was decreased. Table II gives the Bond numbers for the five angular orientations studied. Relatively little change is noted as the angle varies from 180° to 90°; however, as the angle varies from 90° to 0°, a large increase is noted. At a 90° orientation, the data did not exhibit the sharp discontinuity at a specific surface tension of 43 cubic centimeters per second squared as was observed for the 90° orientation in the interior of the cylinder. This observation lends support to the possibility that overstability is exhibited for the 90° orientation (see fig. 4).

Effect of Contact Angle

As reported previously (ref. 12), investigations have indicated that increasing contact angles will cause the Bond number to increase monotonically, reaching a maximum at a 90° contact angle, and ideally decreasing to the Bond number for 0° contact angles at 180°. If the exponent of the radius term in the Bond number criterion remains constant over a range of liquid parameters for finite contact angles, it may be seen from figure 2 that the Bond number for nonwetting liquids (mercury) would indeed be larger than that obtained for wetting liquids. No definite conclusion can be made, however, as to the effect of contact angle on the stability of the interface. Additional liquids with identical contact angles and differing liquid parameters are needed to verify the theoretical contention. Since most rocket engine propellants have 0° contact angles on the types of tank materials currently being employed in rocket vehicle design, no further attempt was made to extend these results to liquid-solid-vapor systems possessing contact angles other than zero. Judging from the available data, however, it appears that finite contact angles do affect the regions of stability.

SUMMARY OF RESULTS

An experimental investigation of the hydrostatic stability of the liquid-vapor interface in a cylindrical container was conducted in a 1-g acceleration field and yielded the following results:

1. The Bond number criterion, consisting essentially of the ratio of acceleration to capillary forces, is valid for predicting the regions of hydrostatic stability.

2. A Bond number of 0.84 based on radius was obtained when the cylinder was alined at 180° to the acceleration field.

3. The Bond number for hydrostatic stability at the ground edge of a cylinder is larger than that obtained when the interface is positioned in the interior of the cylinder.

4. Angular orientation of the acceleration field with respect to the liquid-vapor interface and contact angle affect the Bond number.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, January 8, 1964

APPENDIX - FORMAL INVESTIGATIONS OF HYDROSTATIC STABILITY

The classical approach to the problem of hydrostatic stability has consisted in examining the character of the solution of the equations of disturbances applied to the interface. If the exponents of the series solution are pure imaginary, the disturbed interface will oscillate in time, and the undisturbed state of the interface is considered stable. If the exponents are real, the disturbance is unbounded in time, and the undisturbed state is considered unstable. The required boundary condition is a fixed contact angle at the wall. In the case of disturbed equilibrium, this leads algebraically to the usual criterion of minimum potential energy as a necessary condition for stability. Variational techniques have been used in minimizing the sum of the gravitational and free surface potentials subject to the constraint of constant liquid volume. The resultant Euler differential equation is solved consistent with the required boundary conditions. The solutions are necessarily approximate, largely because of the nonlinearity of the differential equations and the difficulty encountered with a rigorous mathematical formulation for a 0° contact-angle boundary condition.

The first formal analysis of hydrostatic stability was conducted by Maxwell (ref. 13), who found that the Bond number (based on radius) was 14.68 for a circular orifice. Maxwell quoted agreement with Duprez, who experimentally obtained a Bond number of 15.04 by observing the disruption of the interface formed by two immiscible liquids (olive oil and a mixture of water and ethanol). The disruption or instability was brought about by increasing the density of the upper liquid. This large value of Bond number appears to be a result of linearizing the curvature of the interface by assuming the vertical height of the meniscus to be small. The effect of the contact angle was neglected in the analysis.

Employing the classical approach to capillary wave phenomena with gravity taken into account, Lamb (ref. 14) found that the liquid-vapor interface would be unstable in the two-dimensional case for Bond numbers greater than 2.47. Again, the effect of contact angle was neglected in the discussion.

The growth of a sinusoidal deformation of the interface between two fluids of different densities under an imposed acceleration directed perpendicular from the less-dense to the more-dense fluid has been studied analytically by Taylor (ref. 15) and experimentally verified (ref. 16). The analysis of this phenomenon, commonly known as Taylor instability, was later refined (refs. 17 and 18) to include the effects of surface tension and viscosity. Surface tension was found to have a definite stabilizing effect on the amplitude growth of the initial disturbance. Viscosity, while not removing the instability, was found to reduce the rate of growth of the disturbance. More recent analyses (refs. 19 and 20) based on higher order approximations predict overstability beyond the cutoff value given by the linearized model of Taylor. It should be noted that a contact angle of 90° is generally implied in theoretical discussions of Taylor instability.

In another example of the classical approach to the problem, Reynolds (ref. 5) employed the dynamic method (comparable to the analysis of Taylor instability) and examined the boundedness of the amplitudes of infinitesmal disturbances of the interface in time with a fixed 90° contact angle at the walls of the two-dimensional model. His linearized approach indicated a stable liquid-vapor interface for Bond numbers less than 2.5.

The static method of reference 21 assumed a hemispherical interface configuration for liquids with 0° contact angles in zero gravity, and a Bond number of 0.72 was obtained for an inverted channel closed at one end. The method of variational techniques in reference 12 indicates that the static and dynamic methods will yield identical stability criteria. The Bond number will be 0.72 at a contact angle of 0° and should increase monotonically to 2.47 for a 90° contact angle.

A Bond number of approximately 3 was obtained (ref. 11) by equating the buoyant forces of a hemispherical vapor bubble (implying a 0° contact angle) to the surface tension forces. Although this method is a rather informal approach to the problem, it does provide an alternative view of the criterion of stability: rather than consider the liquid as leaving the cylinder, a vapor bubble can easily be regarded as entering the cylinder. The latter is frequently employed in descriptions of Taylor instability in which fingers of vapor penetrate the interface and replace the spikes of liquid deforming the interface. This alternative viewpoint is particularly advantageous because of the extensive analytical and experimental treatment developed in determining the profile of menisci and sessile drops.

The equation governing the profile may be formulated by substituting the differential expressions for the radii of curvature into the Laplace equation for describing the pressure difference across an interfacial surface. The detailed shape of the profile when applied to arbitrary contact angles has not been solved in closed form, but various approximate solutions, summarized in reference 9, have been formulated with verified high degrees of accuracy. Bashforth and Adams (ref. 22) have computed and compiled numerical approximations suitable for the determination of profiles of figures of revolution, such as sessile drops and menisci in cylindrical tubes. Their results have since been extended and refined (ref. 23). Since the two profiles are symmetrical, the case of the sessile drop can be regarded as identical to that of a sessile bubble. The problem of hydrostatic stability can be restated as determining the largest radius of a cylinder that will fit over a given sessile bubble while satisfying the required contact angle.

In attempting to relate bubble velocity with bubble size in a cylinder, the analysis of reference 24 appears to be the first to report the existence of a critical radius at which no bubble rise will occur. The Bond number for this stationary bubble based on the analysis is approximately 1. A critical Bond number of 0.15 was obtained when this phenomenon was observed in an air bubble viscosimeter (ref. 25). A numerical value of 1.27 is further suggested in reference 10. Recently, Bretherton (ref. 10) calculated a Bond number criterion of 0.842 by digital techniques. The point of inflection of the bubble profile at which the tangent plane is vertical (and thereby satisfies a 0° contact angle) was established as the reference point for the critical radius.

Except for a few additional articles of related interest (refs. 26 to 28) the disparity of Bond numbers and the lack of experimental data are surprising in view of the fact that the phenomenon appears to be well known.

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