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AIRFOIL CHARACTERISTICS IN A NONEQUILIBRIUM AMBIENT STREAM BASED ON LINEARIZED THEORY

by James J. Der

Ames Research Center
Moffett Field, Calif.

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SUMMARY

The aerodynamic characteristics of a lifting body in a nonequilibrium stream are studied for a simple case, namely, the case of a nonequilibrium oncoming stream past a thin airfoil at a small angle of attack. Analytical solutions are obtained by the use of nonequilibrium linearized theory. Results show that when the relaxation length on the windward side of the airfoil is not appreciably different from that on the leeward side, a nonequilibrium ambient condition will normally induce a higher pressure and lower drag, but will not affect the lift, pitching moment, and the center of pressure. The local nonequilibrium condition behind the leading-edge shock, on the other hand, will affect all of the aerodynamic characteristics in general. When the relaxation length on the windward side is significantly different from that on the leeward side, on the other hand, the nonequilibrium ambient condition will also affect the lift, pitching moment, and the center of pressure. In general, the higher suction on the leeward side gives higher lift and larger pitching moment because the condition on the leeward side is closer to frozen.

INTRODUCTION

The present study is motivated by the desire to learn something about the effects of a nonequilibrium condition in the ambient stream on the aerodynamic characteristics of a body. For an arbitrary body, such a study would normally require detailed numerical calculation of the flow around the body, with the free-stream condition out of equilibrium. Calculations of this type are, in general, tedious.

A case for which we can obtain a simple analytical solution is the nonequilibrium flow past a thin airfoil at a small angle of attack. From the results of this case we can discover some indication of what the ambient nonequilibrium would do to the characteristics of a lifting body, with or without the local flow condition in equilibrium.

In order to study problems whose solutions can be obtained analytically, we shall use a small-disturbance theory for nonequilibrium flow. This theory and its limitations have been described by Vincenti in reference 1, where he introduced a model for flow with nonequilibrium ambient conditions. The use of a small-disturbance theory limits the quantitative accuracy of the results. The results are not, therefore, intended to be accurate solutions to practical problems, but rather as qualitative information about the aerodynamic characteristics of airfoils in a nonequilibrium ambient stream.

In the following sections, the physical aspects of the problem are first described. A solution to the governing equation is then obtained. The result is applied to biconvex airfoils at angles of attack near zero where the relaxation time of the lower surface is close to that of the upper surface. Finally, the general case of an airfoil with different values of relaxation times on the upper and lower surfaces is discussed, and explicit expressions for the aerodynamic coefficients are given for the case where the condition on the upper surface is frozen (with the flow on the lower surface in nonequilibrium condition in general).

PRINCIPAL SYMBOLS

a	$\dfrac{\beta_e^2}{\beta_f^2}$
a_e	equilibrium sonic speed
a_f	frozen sonic speed
$c(\xi)$	a function defined by equation (4)
c.p.	center of pressure, $\dfrac{C_M}{C_L}$
C_D	drag coefficient, $\dfrac{\text{drag}}{(1/2)\rho_\infty U_\infty^2 \text{ chord length}}$
C_L	lift coefficient, $\dfrac{\text{lift}}{(1/2)\rho_\infty U_\infty^2 \text{ chord length}}$
C_m	moment coefficient, $\dfrac{\text{pitching moment}}{(1/2)\rho_\infty U_\infty^2 \text{ chord length}^2}$
C_p	pressure coefficient, $\dfrac{p - p_\infty}{(1/2)\rho_\infty U_\infty^2}$
G_∞	$-\beta_f\left(\dfrac{\partial h}{\partial q}\right)_\infty \left\{\rho_\infty(\beta_e^2 - \beta_f^2)\left[\left(\dfrac{\partial h}{\partial \rho}\right)_\infty + \left(\dfrac{\partial h}{\partial q}\right)_\infty \left(\dfrac{\partial q^*}{\partial \rho}\right)_\infty\right]\right\}^{-1}$

h	enthalpy
I_o	modified Bessel function of the first kind, zero order
K	$\tau_o U_\infty \left(\frac{\partial h}{\partial \rho}\right)_\infty \left[\left(\frac{\partial h}{\partial \rho}\right)_\infty + \left(\frac{\partial h}{\partial q}\right)_\infty \left(\frac{\partial q^*}{\partial \rho}\right)_\infty\right]^{-1}$
l	$\frac{\text{chord length}}{K}$
p	pressure
q	nonequilibrium variable (e.g., degree of dissociation)
s	Laplace transform variable
u,v	velocity in the x and y directions, respectively
U_∞	free-stream velocity
x,y	Cartesian coordinates parallel and normal to the free-stream direction, respectively
α	angle of attack (with respect of the free-stream direction)
β_e	$\sqrt{\left(\frac{U_\infty^2}{a_e^2}\right) - 1}$
β_f	$\sqrt{\left(\frac{U_\infty^2}{a_f^2}\right) - 1}$
$\delta(\xi)$	angle of the boundary at $\eta = 0$ with respect to the free-stream direction
ξ,η	skew coordinates; $\xi = \frac{x - \beta_f y}{K}$ and $\eta = \frac{\beta_f y}{K}$
θ	a function of $\xi = +\theta_u = -\theta_l$
ρ	density
μ	ratio of the relaxation lengths of the lower and the upper surfaces

σ $\left(\frac{a + s}{1 + s}\right)^{1/2}$

τ_o relaxation time behind the leading-edge shock

φ normalized perturbation velocity potential, defined by the relations $K\varphi_x = \frac{u - U_\infty}{U_\infty}$ and $K\varphi_y = \frac{v}{U_\infty}$, or $\varphi_\xi = \frac{u - U_\infty}{U_\infty}$ and $\varphi_\eta - \varphi_\xi = \frac{v}{\beta_f U_\infty}$

Φ Laplace transform of φ, $\int_0^\infty \exp(-s\xi)\varphi(\xi,\eta)d\xi$

Superscripts and Subscripts

$(\ )^*$ equilibrium value

$(\bar{\ })$ Laplace transformed quantity

$(\ )'$ derivative of ( ) with respect to its argument

$(\ )_u$ upper surface of the airfoil

$(\ )_l$ lower surface of the airfoil

$(\ )_\infty$ free-stream value

$(\ )_{nf}$ due to nonequilibrium free stream

$\left.\begin{matrix}(\ )_x, (\ )_y, \\ (\ )_\xi, (\ )_\eta\end{matrix}\right\}$ partial derivatives with respect to the subscripts

PHYSICAL DESCRIPTION

We shall consider an airfoil in a supersonic stream (fig. 1). The ambient stream is uniform but can be in a nonequilibrium condition. This can be expounded as follows: Let the state of the flowing gas be determined by the pressure p, density ρ, and a nonequilibrium variable q (e.g., degree of dissociation or vibrational energy). The enthalpy h, for example, is a function of these independent thermodynamic variables as $h = h(p,\rho,q)$. The state of the gas is said to be in equilibrium if q satisfies the relation $q = q^*(p, \rho)$, otherwise the state of the gas is out of equilibrium. If q takes a fixed value, that is, q equals a constant, the state of the gas is said to be in a frozen condition. In the problem we are considering, q is out of equilibrium everywhere including in the free stream, where

$$q_\infty \neq q^*(p_\infty,\rho_\infty) = q_\infty^*$$

The ambient flow is taken to be parallel to the x axis. It is assumed that the flow ahead of the airfoil is uniform with respect to both x and y. Thus, the relaxation time in the free stream, τ_∞, is assumed to be large. Behind the leading-edge shock, however, the relaxation time[1] of the fluid is reduced due to compression, and chemical reaction or vibrational relaxation will take place. For flow past an airfoil at an angle of attack, the amount of reduction of the relaxation time across the leading-edge shock on the windward surface is larger than that on the leeward surface because of higher compression on the lower side. We can classify the situation into three cases:

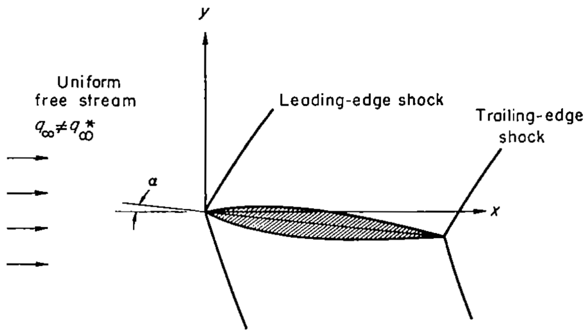

Figure 1.- Airfoils in a nonequilibrium free stream.

1. When the angle of attack α is much smaller than the leading-edge slope θ_o of the airfoil, the relaxation time on the upper (leeward) surface, τ_u, is approximately the same as that on the lower (windward) surface, τ_l. The deviation from equilibrium is the same on the two surfaces.

2. When α is smaller but of the same order as θ_o, compressive shocks still occur at the leading edge on both surfaces. Since $\tau_l < \tau_u$, however, the flow on the lower surface will be closer to equilibrium than that on the upper surface.[2]

3. When α is greater than θ_o, a compressive shock occurs at the leading edge on the lower surface, but the flow on the upper surface is under expansion, which means τ_u is larger than τ_∞, hence, is very large. Consequently, the flow on the upper surface is in a frozen condition.

We shall treat case 1 $(\tau_u = \tau_l)$ in detail, giving both explicit expressions and numerical results. Only analytical results will be presented for the other cases, which will be discussed qualitatively.

To attack this problem analytically, we shall restrict our study to a two-dimensional flow field that is perturbed slightly from the uniform

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[1]The relaxation time is a characteristic reaction time. It is essentially the time required for a gas to reach an equilibrium state.

[2]An exceptional case is the flow past a wedge. The degree of nonequilibrium in the free stream may be such that equilibrium on the upper surface is reached immediately behind the leading-edge shock, whereas it is not reached on the lower surface. This case is not treated here, however, since we are interested mostly in airfoils having closed profiles.

nonequilibrium free-stream condition. The relaxation time, however, is assumed to be reduced by an order of magnitude across the initial shock. Such an assumption for the relaxation time is hardly consistent with the assumption of small disturbances. When the disturbances are indeed small the relaxation time behind the leading-edge shock can be finite only if the free-stream relaxation time is finite so that nonuniform conditions exist in the free stream. When the disturbances are so large that the relaxation time is reduced by an order of magnitude, the assumption of small disturbances is violated. Nevertheless, it is expected that for either of these two cases the linear theory should give indications of the flow phenomena on the airfoil surface, at least qualitatively correct. The nonequilibrium variable can be any of the common ones, for example, degree of dissociation, but we assume that only one nonequilibrium mechanism is of importance at a time. This is the same flow model with which Vincenti (ref. 1) studied the flow over a wedge in a nonequilibrium oncoming stream.

AN ANALYTICAL SOLUTION

With the assumptions enumerated in the previous section we can describe the flow behind the leading-edge shock by a linear equation in terms of a perturbation velocity potential φ. This equation has been derived by Vincenti (ref. 1) for the upper plane, that is, for positive y. We shall obtain the pressure distribution on the upper plane, noting that the resulting expression is valid for the lower plane by simply replacing y by $-y$. In a linearized theory, the shock wave coincides with the initial Mach line. In the nonequilibrium linearized theory, the shock wave coincides with the initial frozen Mach line. Since $y = x/\beta_f$ is the initial frozen Mach line on the upper plane, all disturbances occur in $x > \beta_f y$. It will be convenient to use, in place of the Cartesian coordinates x and y, the skew coordinates $\xi \equiv (x - \beta_f y)/K$ and $\eta \equiv \beta_f y/K$. The flow quantity K is proportional to the relaxation length $\tau_0 U_\infty$ and is an important characteristic length in a nonequilibrium flow. To solve the resulting potential equation we shall use the method of Laplace transforms.

The Laplace transformation of the flow equation is given by Vincenti, who obtained the transformed solution for the upper plane, that is, positive η, (eq. (54) of ref. 1) as

$$\Phi(s,\eta) \equiv L[\varphi(\xi,\eta)] = A(s)e^{-(\sigma-1)s\eta} + B(s)e^{(\sigma+1)s\eta} - \frac{G_\infty(q_\infty - q_\infty{}^*)}{s^2\beta_f} \qquad (1)$$

The quantities $A(s)$ and $B(s)$ are to be determined from the boundary conditions. The quantity $G_\infty(q_\infty - q_\infty{}^*)/(s^2\beta_f)$ comes from the nonequilibrium ambient condition. It vanishes when $q_\infty = q_\infty{}^*$. Since G_∞ and $(q_\infty - q_\infty{}^*)$ are normally positive (ref. 1), the value of $G_\infty(q_\infty - q_\infty{}^*)/\beta_f$ is normally positive.

The boundary conditions are: (1) the disturbances are bounded, and (2) the stream line is parallel to the boundary at the airfoil surface, which can be taken to be at $y = 0$ (hence $\eta = 0$) for the small disturbance theory.

The first boundary condition means that the value of $B(s)$ must be identically zero; otherwise the disturbances grow without bound as $\eta \to \infty$. The second boundary condition means

$$\varphi_\eta(\xi,0) - \varphi_\xi(\xi,0) = \delta(\xi)/\beta_f$$

or

$$\Phi_\eta(s,0) - s\Phi(s,0) = \overline{\delta}(s)/\beta_f = L[\delta(\xi)]/\beta_f \tag{2}$$

where $\delta(\xi)$ is the slope of the boundary at $\eta = 0$ with respect to the free stream. This boundary condition gives the value of $A(s)$. As a result we obtain the transformed value of $\varphi_\xi(\xi,\eta) = (u - U_\infty)/U_\infty$ as

$$L[\varphi_\xi(\xi,\eta)] = s\Phi(s,\eta) = \left[\frac{G_\infty(q_\infty - q_\infty^*)}{\beta_f}\frac{1}{s\sigma} - \frac{\overline{\delta}}{\beta_f\sigma}\right] e^{-s(\sigma-1)\eta} - \frac{G_\infty(q_\infty - q_\infty^*)}{s\beta_f} \tag{3}$$

The inversion of this transformed relation will give the value of $(u - U_\infty)/U_\infty$, which also gives the value of the pressure and enthalpy since, within the first order,

$$C_p \equiv 2(p - p_\infty)/\rho_\infty U_\infty^2 = -2(u - U_\infty)/U_\infty$$

and

$$h - h_\infty = -(u - U_\infty)U_\infty$$

Similar expressions for other flow quantities can also be obtained.

The inverse transform of the last term in the right-hand side of equation (3) is simply $-G_\infty(q_\infty - q_\infty^*)/\beta_f$. No explicit inversion for the remaining terms in the right-hand side is known. Possibly they can be obtained by numerical means, (e.g., a method proposed by R. Bellman, et al. (ref. 2)).

Our present interest is, however, only the pressure distribution on the boundary, that is, at $\eta = 0$. For this case the exponential factor is unity. The inverse transforms of $1/(s\sigma)$ and $\overline{\delta}/\sigma$ can be obtained from a transform table (e.g., ref. 3). They are

$$L^{-1}\left[\frac{1}{s\sigma}\right] \equiv c(\xi) = \exp\left(-\frac{a+1}{2}\xi\right) I_0\left(\frac{a-1}{2}\xi\right) + \int_0^\xi \exp\left(-\frac{a+1}{2}t\right) I_0\left(\frac{a-1}{2}t\right) dt \tag{4}$$

where I_0 is the modified Bessel function of the first kind, zero order, and

$$L^{-1}\left[\frac{\overline{\delta}}{\sigma}\right] = \delta(0)c(\xi) + \int_0^\xi \delta'(t)c(\xi - t)dt \tag{5}$$

The pressure distribution on a boundary at $\eta = 0$ with slope $\delta(\xi)$ is, therefore, given by the expression

$$\beta_f \frac{p - p_\infty}{\rho_\infty U_\infty^2} = \delta(0)c(\xi) + \int_0^\xi \delta'(t)c(\xi - t)dt + G_\infty(q_\infty - q_\infty^*)[1 - c(\xi)]$$

which is quite general for any smooth $\delta(\xi)$.

The above equation is derived for y (hence η) = 0+. Therefore it gives the pressure on the upper surface. It is applicable on the lower surface (η = 0-) if one replaces y by $-y$ in the derivation. This would result in replacing δ by $-\delta$ in the above equation for the lower surface. Hence we can express the pressure on the upper surface, p_u, and lower surface, p_l, by the relations

$$\beta_f \frac{p_u - p_\infty}{\rho_\infty U_\infty^2} = \delta_u(0)c(\xi) + \int_0^\xi \delta_u'(t)c(\xi - t)dt + G_\infty(q_\infty - q_\infty^*)[1 - c(\xi)] \tag{6a}$$

$$\beta_f \frac{p_l - p_\infty}{\rho_\infty U_\infty^2} = -\delta_l(0)c(\xi) - \int_0^\xi \delta_l'(t)c(\xi - t)dt + G_\infty(q_\infty - q_\infty^*)[1 - c(\xi)] \tag{6b}$$

The first two terms in the right-hand side of equations (6a) and (6b) give the pressure distribution on the boundary when the ambient stream ahead of the leading-edge shock is in an equilibrium condition. The last term gives the effects of nonequilibrium free-stream conditions on the pressure. Within the first-order theory, therefore, the effect of nonequilibrium ambient condition is in the form of a simple additive quantity. The consequences of such weak coupling to the aerodynamic characteristics of an airfoil will be discussed in the next section. Because G_∞ and $q_\infty - q_\infty^*$ are normally positive, when the ambient stream is not in equilibrium, the pressure on the boundary is normally higher with the additive pressure proportional to $G_\infty(q_\infty - q_\infty^*)$.

CHARACTERISTICS OF AIRFOILS FOR THE CASE $K_u \cong K_l$

We shall now consider an airfoil at an angle of attack. First, we shall study thin airfoils in general. Later, we shall apply the results to a specific airfoil, namely, the biconvex airfoil. The case for which the relaxation times are approximately the same on the upper and lower surfaces will be treated presently. This is case 1 mentioned in the previous section. The case of $K_u \neq K_l$ will be treated in the next section.

The slope θ_u for the upper surface of an airfoil can be written, for convenience,

$$\theta_u = \theta(\xi) = \theta_0 f(\xi/l)$$

where θ_o is the leading-edge slope, $f(\xi/l)$ represents the variation along the chord line, and l is the chord length divided by K. (Recall that ξ is the streamwise distance from the leading edge divided by K.) At an angle of attack the slope of the upper surface with respect to the free-stream direction is $\delta_u = \theta - \alpha$. The slope of the lower surface with respect to the free-stream direction is, for a symmetric airfoil, $\delta_l = -(\theta + \alpha)$. By substitution of δ_u and δ_l into equations (6) we obtain the pressure coefficients for the upper surface, C_{p_u}, and that for the lower surface, C_{p_l}, as

$$\left.\begin{aligned} &\frac{\beta_f}{2\theta_o} C_{p_u} \\ &\frac{\beta_f}{2\theta_o} C_{p_l} \end{aligned}\right\} = \frac{G_\infty(q_\infty - q_\infty^*)}{\theta_o}[1 - c(\xi)] + \left[c(\xi) + \frac{1}{l}\int_o^\xi f'\left(\frac{t}{l}\right)c(\xi - t)dt\right] \mp \left[\frac{\alpha}{\theta_o} c(\xi)\right] \tag{7}$$

where the upper sign goes with C_{p_u} and the lower with C_{p_l}.

The various terms in equation (7) can be interpreted as follows: If the ambient stream is not in equilibrium (e.g., frozen in a certain degree of dissociation), then after the flow passes the leading-edge shock, the sudden reduction of the relaxation time (due to the compression) induces recombination, and the degree of dissociation tends toward an equilibrium value at a speed governed by the magnitude of the relaxation length behind the shock. This pressure rise induced by the recombination is given by the term inside the first bracket. It can also be, of course, due to other types of relaxation, for example, molecular vibration. The term inside the second bracket gives the pressure distribution on the surface due to the contour of the airfoil. This portion of the pressure distribution corresponds to that of the "thickness case" (ref. 4) in classical aerodynamics. Here, of course, in contrast to classical flows, the local conditions can be either equilibrium or nonequilibrium. Finally, the last term gives the pressure distribution due to the angle of attack, and corresponds to the pressure rise on a flat plate at an angle of attack in the classical case. With the expressions for C_{p_u} and C_{p_l} we can compute the aerodynamic coefficients by the following relations:

$$C_L = \frac{1}{l}\int_o^l (C_{p_l} - C_{p_u})d\xi \tag{8}$$

$$C_m = \frac{1}{l^2}\int_o^l \xi(C_{p_l} - C_{p_u})d\xi \tag{9}$$

$$C_D = \frac{1}{l}\int_o^l (\delta_u C_{p_u} - \delta_l C_{p_l})d\xi \tag{10}$$

while the center of pressure is

$$c.p. = C_m/C_L \tag{11}$$

Using equation (7) for C_{p_u} and C_{p_l} we obtain

$$\frac{\beta_f}{\alpha} C_L = \frac{4}{l} \int_0^l c(\xi)d\xi \tag{12}$$

$$\frac{\beta_f}{\alpha} C_m = \frac{4}{l^2} \int_0^l \xi c(\xi)d\xi \tag{13}$$

$$c.p. = \frac{\int_0^l \xi c(\xi)d\xi}{l \int_0^l c(\xi)d\xi} \tag{14}$$

$$\frac{\beta_f}{\theta_o^2} C_D = \frac{4}{l} \int_0^l \left\{\left(\frac{\alpha}{\theta_o}\right)^2 c(\xi) + f\left(\frac{\xi}{l}\right)\psi(\xi) + \frac{G_\infty(q_\infty - q_\infty{}^*)}{\theta_o} f\left(\frac{\xi}{l}\right)[1 - c(\xi)]\right\}d\xi \tag{15}$$

where

$$\psi(\xi) \equiv c(\xi) + \frac{1}{l} \int_0^\xi f'\left(\frac{t}{l}\right)c(\xi - t)dt$$

Equations (12), (13), and (14) show that, within the first-order accuracy, the lift, pitching moment, and the center of pressure of an airfoil are <u>not</u> affected by the nonequilibrium condition in the ambient stream. They are, however, affected by the nonequilibrium condition behind the leading-edge shock. The explanation of such rather unexpected phenomena is that the pressure induced by the nonequilibrium ambient condition is the same on the upper and lower surface, hence, there is no effect on the lift and pitching moment (and, therefore, center of pressure). Such is not the case, however, for the drag.

The drag coefficient consists of three parts: The first part is the same as a flat plate at an angle of attack and is given by the expression

$$\frac{\beta_f}{\theta_o^2} C_{D_{\text{flat plate}}} = \left(\frac{\alpha}{\theta_o}\right)^2 \frac{4}{l} \int_0^l c(\xi)d\xi \tag{15a}$$

The second part is the drag due the thickness of the airfoil and is given by the expression

$$\frac{\beta_f}{\theta_o^2} C_{D_{thickness}} = \frac{4}{l} \int_0^l f\left(\frac{\xi}{l}\right)\left[c(\xi) + \frac{1}{l} \int_0^\xi f'\left(\frac{t}{l}\right)c(\xi - t)dt\right] d\xi \tag{15b}$$

Finally, the effects of the nonequilibrium condition in the ambient stream to the drag is given by

$$\frac{\beta_f}{\theta_o^2} C_{D_{nf}} = \frac{G_\infty(q_\infty - q_\infty^*)}{\theta_o} \frac{4}{l} \int_0^l f\left(\frac{\xi}{l}\right)[1 - c(\xi)]d\xi \tag{15c}$$

The lift-drag ratio can be obtained, of course, by use of equations (12) and (15). In particular, for the flat plate in an equilibrium free stream, the lift-drag ratio is simply equal to $1/\alpha$. Thus, within first-order accuracy, the lift-drag ratio of a flat plate is not affected by the <u>local</u> nonequilibrium conditions.

The values of C_p and C_D depend on the shape of the airfoil. To obtain representative values for them we shall consider a specific airfoil, namely, the biconvex airfoil, which is given by

$$f = 1 - 2\frac{\xi}{l}$$

The corresponding C_p and C_D are

$$\left.\begin{array}{l} \dfrac{\beta_f}{2\theta_o} C_{p_u} \\[2em] \dfrac{\beta_f}{2\theta_o} C_{p_l} \end{array}\right\} = \frac{G_\infty(q_\infty - q_\infty^*)}{\theta_o} [1 - c(\xi)] + \left[c(\xi) - \frac{2}{l} \int_0^\xi c(\xi - t)dt\right] \mp \frac{\alpha}{\theta_o} c(\xi) \tag{16}$$

$$\frac{\beta_f}{\theta_o^2} C_D = \frac{4}{l} \int_0^l \left\{ \left(\frac{\alpha}{\theta_o}\right)^2 c(\xi) + \left(1 - 2\frac{\xi}{l}\right) \psi_1(\xi) \right.$$

$$\left. + \frac{G_\infty(q_\infty - q_\infty^*)}{\theta_o} \left(1 - 2\frac{\xi}{l}\right) [1 - c(\xi)]\right\} d\xi \tag{17}$$

where

$$\psi_1(\xi) \equiv c(\xi) - \frac{2}{l} \int_0^l c(\xi - t)dt$$

Figure 2 shows the variation of lift, pitching moment, and (for a biconvex airfoil in an equilibrium ambient stream) components of the drag coefficients versus l for $a = 1.5$. When the chord length is small compared with the relaxation length ($l \to 0$), the flow past the initial shock, not having sufficient time for relaxation toward the equilibrium condition, is essentially frozen. When the chord length is large compared with the relaxation length ($l \to \infty$), however, the flow behind the leading edge is mostly in an equilibrium condition. Direct computation will also verify that in the two limits, the coefficients take the following asymptotic values:

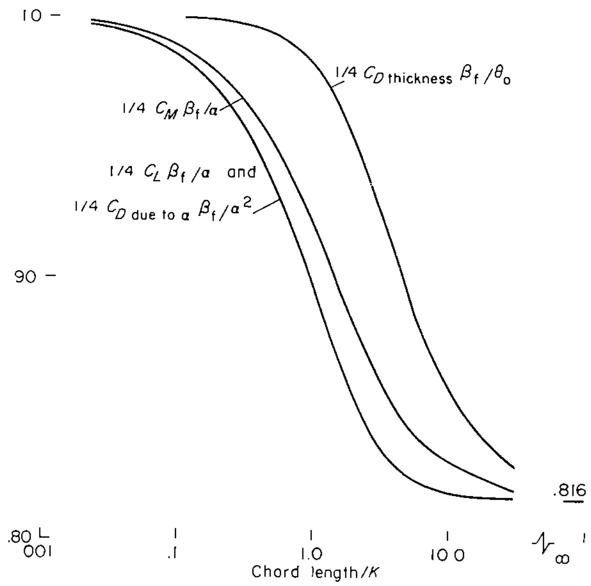

Figure 2.- Drag, lift, and pitching moment of a biconvex airfoil in a nonequilibrium flow field for $(M_e^2 - 1)/(M_f^2 - 1) = 1.5$.

equilibrium:

$$\lim_{l\to\infty} C_L = \frac{4}{\sqrt{a}} \frac{\alpha}{\beta_f}, \qquad \lim_{l\to\infty} C_m = \frac{2}{\sqrt{a}} \frac{\alpha}{\beta_f}$$

$$\lim_{l\to\infty} C_D = \frac{4}{\sqrt{a}\beta_f}\left(\alpha^2 + \frac{\theta_o^2}{3}\right)$$

frozen:

$$\lim_{l\to 0} C_L = \frac{4\alpha}{\beta_f}, \qquad \lim_{l\to 0} C_m = \frac{2\alpha}{\beta_f}$$

$$\lim_{l\to 0} C_D = \frac{4}{\beta_f}\left(\alpha^2 + \frac{\theta_o^2}{3}\right)$$

Thus the coefficients in the frozen limit are larger than the corresponding ones in the equilibrium limit by the factor of $\sqrt{a}$. For the case of $a = 1.5$, which is not unusual in modern high-temperature facilities, this means a difference of roughly 20 percent.

Figure 3 presents the change of C_D (ΔC_{Dnf}) of a biconvex airfoil when the condition in the ambient stream is out of equilibrium.

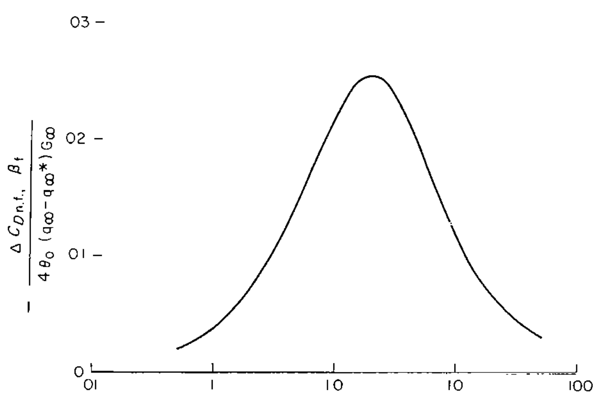

Figure 3.- Drag on a biconvex airfoil due to the nonequilibrium free stream for $(M_e^2 - 1)/(M_f^2 - 1) = 1.5$.

The drag induced by the nonequilibrium free stream is negative. The reason is that the pressure induced by the nonequilibrium ambient stream increases monotonically after the initial jump at the leading-edge shock; consequently, a net thrust results.

Note that $\Delta C_{D_{nf}}$ is zero when the flow is frozen behind the leading edge since no recombination (for dissociating case) occurs. When the condition behind the leading edge is equilibrium $\Delta C_{D_{nf}}$ is again zero because the pressure induced by the nonequilibrium stream is constant in the equilibrium limit.

We see that, within the first-order perturbation approximation, the ambient stream nonequilibrium actually has no effect on the lift and the pitching moment. The local stream nonequilibrium, however, does affect the lift and moment, as well as the drag coefficients. The drag coefficient, on the other hand, is affected by the nonequilibrium condition in the free stream by an amount of

$$\Delta C_{D_{nf}} = \frac{4\theta_o}{\beta_f} \frac{G_\infty(q_\infty - q_\infty^*)}{l} \int_o^l \left(1 - 2\frac{\xi}{l}\right) [1 - c(\xi)]d\xi$$

which is (1) proportional to $G_\infty(q_\infty - q_\infty^*)$, (2) dependent on the shape of the airfoil, and (3) not dependent on the angle of attack.

CASES WHERE $K_u \neq K_l$

So far we have treated the case where the relaxation length on the upper surface of the airfoil is nearly the same as that in the lower surface (i.e., $K_u \cong K_l = K$, which is approximately correct when $\theta_o \gg \alpha$). In general, due to the higher compression, the relaxation length is shorter on the lower surface, that is, $K_l < K_u$. If the angle of attack is larger than the leading-edge slope, $K_u > K_\infty$ due to expansion. Since K_∞ is assumed to be infinite, for this case, the condition is frozen on the upper surface.

The case where $K_u \neq K_l$ is more complicated than the case we have treated. We shall briefly describe such a more general case, however, and conclude by giving the explicit relations for the aerodynamic coefficients when $\theta_o \leq \alpha$, that is, $K_u \to \infty$.

For convenience, let

$$K_l = K\ , \qquad \text{and} \quad K_u = \frac{K_l}{\mu} = \frac{1}{\mu} K$$

then

$$l_l = l\ , \qquad \text{and} \quad l_u = \mu l$$

Note that $0 \leq \mu \leq 1$.

The relation in equation (6b) is still applicable directly for the lower surface since $\xi_l = \xi = x/K$ (for $\eta = 0$). For the upper surface the variable ξ must be replaced by $\xi_u = \mu\xi$ and l by $l_u = \mu l$. Thus we have

$$\beta_f \frac{p_l(\xi) - p_\infty}{\rho_\infty U_\infty^2} = -\delta_l(0)c(\xi) - \int_0^\xi \delta_l'(t)c(\xi - t)dt + G_\infty(q_\infty - q_\infty^*)[1 - c(\xi)] \tag{18a}$$

$$\beta_f \frac{p_u(\xi) - p_\infty}{\rho_\infty U_\infty^2} = \delta_u(0)c(\mu\xi) + \int_0^{\mu\xi} \delta_u'(t)c(\mu\xi - t)dt + G_\infty(q_\infty - q_\infty^*)[1 - c(\mu\xi)] \tag{18b}$$

where

$$\delta_u(\mu\xi) = \theta(\mu\xi) - \alpha = \theta_o f\left(\frac{\mu\xi}{\mu l}\right) - \alpha \tag{19a}$$

$$-\delta_l(\xi) = \theta(\xi) + \alpha = \theta_o f\left(\frac{\xi}{l}\right) + \alpha \tag{19b}$$

The expressions for C_p are, therefore,

$$\frac{\beta_f}{2\theta_o} C_{p_u}(\xi) = \frac{G_\infty(q_\infty - q_\infty^*)}{\theta_o}[1 - c(\mu\xi)] + c(\mu\xi)$$

$$+ \frac{1}{\mu l}\int_0^{\mu\xi} f'\left(\frac{t}{\mu l}\right)c(\mu\xi - t)dt - \frac{\alpha}{\theta_o}c(\mu\xi) \tag{20a}$$

$$\frac{\beta_f}{2\theta_o} C_{p_l}(\xi) = \frac{G_\infty(q_\infty - q_\infty^*)}{\theta_o}[1 - c(\xi)] + c(\xi) + \frac{1}{l}\int_0^{\xi} f'\left(\frac{t}{l}\right)c(\xi - t)dt + \frac{\alpha}{\theta_o}c(\xi) \tag{20b}$$

By direct substitution of C_p from equations (20) into equations (8) through (11), one can obtain the aerodynamic characteristics of an airfoil when $K_u \neq K_l$. An interesting case is where $\mu = 0$. For this case the flow on the upper surface is in a frozen condition. The value of C_{p_u} is given by, therefore,

$$\frac{\beta_f}{2\theta_o} C_{p_u}(\xi) = -\frac{\alpha}{\theta_o} + f\left(\frac{\xi}{l}\right) \tag{21a}$$

(Note that $f \leq 1$.) While C_{p_l} remains the same as before, that is,

$$\frac{\beta_f}{2\theta_o} C_{p_l} = \frac{G_\infty(q_\infty - q_\infty^*)}{\theta_o}[1 - c(\xi)] + c(\xi) + \frac{1}{l}\int_0^{\xi} f'\left(\frac{t}{l}\right)c(\xi - t)dt + \frac{\alpha}{\theta_o}c(\xi) \tag{21b}$$

The corresponding aerodynamic coefficients are

$$\frac{\beta_f}{\alpha} C_L = \frac{2}{l} \int_0^l \left\{\frac{G_\infty(q_\infty - q_\infty^*)}{\alpha} [1 - c(\xi)] + \frac{\theta_o}{\alpha} \left[\psi(\xi) - f\left(\frac{\xi}{l}\right)\right] + [c(\xi) + 1]\right\} d\xi \tag{22a}$$

$$\frac{\beta_f}{\alpha} C_m = \frac{2}{l} \int_0^l \xi \left\{\frac{G_\infty(q_\infty - q_\infty^*)}{\alpha} [1 - c(\xi)] + \frac{\theta_o}{\alpha} \left[\psi(\xi) - f\left(\frac{\xi}{l}\right)\right] + [c(\xi) + 1]\right\} d\xi \tag{22b}$$

$$\frac{\beta_f}{\alpha^2} C_D = \frac{\beta_f}{\alpha} C_L + \frac{2\theta_o}{\alpha} \frac{1}{l} \int_0^l f\left(\frac{\xi}{l}\right) \left\{c(\xi) - 1 + \frac{\theta_o}{\alpha} \left[\psi(\xi) + f\left(\frac{\xi}{l}\right)\right] + \frac{G_\infty(q_\infty - q_\infty^*)}{\alpha} [1 - c(\xi)]\right\} d\xi \tag{22c}$$

where

$$\psi(\xi) \equiv c(\xi) + \frac{1}{l} \int_0^\xi f'\left(\frac{t}{l}\right) c(\xi - t) dt \tag{23}$$

It is apparent, from equations (22), that in the present case, because of the difference between the relaxation lengths of the upper and lower surfaces ($K_u \neq K_l$), the aerodynamic coefficients are further affected. Consider C_L of equation (22a) as an example: Equation (22a) can be written as a sum of three terms, that is,

$$\frac{\beta_f}{\alpha} C_L = \frac{2}{l} \frac{G_\infty(q_\infty - q_\infty^*)}{\alpha} \int_0^l [1 - c(\xi)] d\xi + \frac{2\theta_o}{\alpha} \frac{1}{l} \int_0^l \left[\psi(\xi) - f\left(\frac{\xi}{l}\right)\right] d\xi + \frac{2}{l} \int_0^l [c(\xi) + 1] d\xi \tag{24}$$

Because the relaxation length is shorter on the lower surface ($K_l < K_u$), the pressure rise due to recombination (for dissociation equilibrium) is higher on the lower surface than that on the upper surface; the lift corresponding to this is given by the first term.

The local flow condition is also different on the lower and the upper surfaces. The condition on the lower surface is closer to equilibrium. This gives a higher pressure due to the thickness of the airfoil on the upper surface and a corresponding negative lift indicated in the second term.

Finally, the suction on the upper surface due to the angle of attack is higher since the condition on the upper surface is closer to frozen. This gives an increase of lift due to the angle of attack as indicated in the third term in equation (24).

CONCLUDING REMARKS

We may now make a few conjectures on the characteristics of lifting bodies in general, based on the observation of the preceding results.

When the relaxation length is relatively constant on the windward and the leeward sides, the effect of the nonequilibrium ambient condition on the lift and pitching moment is relatively small but will affect the drag and the pressure distribution. The local nonequilibrium conditions, however, will affect all of the aerodynamic characteristics. In fact, the aerodynamic coefficients are normally higher for bodies when the local stream is out of equilibrium.

In the more general case the relaxation length is shorter on the windward side than that on the leeward side. Thus, the condition on the windward side is closer to equilibrium, or, in other words, the leeward side is closer to frozen. In this case all of the aerodynamic coefficients are affected by both the ambient and the local nonequilibrium conditions. The ambient nonequilibrium condition will give an increase in lift.

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