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ANALYTICAL DETERMINATION OF THE TAKE-OFF PERFORMANCE OF SOME REPRESENTATIVE SUPERSONIC TRANSPORT CONFIGURATIONS

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SUMMARY

The take-off performance characteristics of typical supersonic transport configurations have been analytically determined with aerodynamic characteristics representative of both delta-wing and variable-geometry configurations. The investigation considered conditions where the thrust was assumed constant and where the thrust decreased with increasing velocity. Optimum full-power take-off distances and Civil Air Regulation runway lengths were obtained, and the results agree, generally, with previous similar studies. First-order empirical relations were determined which correlate the data well. Design nomograms derived from these empirical relations are presented. The results indicate, in general, that the take-off performance of the typical supersonic transports considered can be comparable to or better than present subsonic jet transports.

INTRODUCTION

One of the important areas for consideration in the design of the supersonic commercial transport is the take-off requirements. It is generally agreed that little, if any, deterioration from take-off performance levels of current subsonic jet transports will be allowed. Improvements in performance over subsonic jet transports would, of course, be desirable.

Several analytical studies have been made of the take-off characteristics of various types of configurations (for example, refs. 1, 2, and 3). However, no simple method of predicting take-off performance of supersonic transports is currently available. Also, little attention has been given to the effect of thrust decrease during acceleration, which could be particularly significant with turbofan engines.

The purpose of this investigation was to provide an assessment of the take-off distance requirements of some typical supersonic transport configurations and to correlate the results so that they may be applied to similar configurations. The take-off distance requirements were determined by analytical and numerical integration of the two-degree-of-freedom equations of motion on an electronic data processing machine. The aerodynamic characteristics, propulsion characteristics, and wing loadings assumed for the present study are representative of those associated with supersonic transport configurations currently of interest. The effects of these parameters on the minimum full-power take-off distance and on the Civil Air Regulation runway length are presented and discussed.

SYMBOLS

C_D	drag coefficient, $\frac{D}{qS}$
$C_{D,min}$	minimum drag coefficient
C_L	lift coefficient, $\frac{L}{qS}$
$C_{L,ma}$	maximum available lift coefficient associated with α_{ma}
$C_{L,min}$	lift coefficient associated with $C_{D,min}$
D	drag, lb
F	thrust, lb
F_{st}	static thrust, lb
g	acceleration due to gravity, ft/sec^2
h	altitude, ft
L	lift, lb
q	dynamic pressure, lb/sq ft
S	wing area, sq ft
s	horizontal distance, ft
s_{min}	minimum horizontal distance, ft
V	airplane forward speed, knots
V_r	rotation speed (speed at which rotation maneuver is initiated), knots
$(V_r)_{FP,opt}$	rotation speed corresponding to minimum full-power take-off distance, knots

V_1	critical engine failure speed, knots
W	airplane weight at take-off, lb
W/S	wing loading, lb/sq ft
α	angle of attack or rotation angle, deg
α_{ma}	maximum angle of attack available due to limits of airplane geometry, deg
γ	flight-path angle, radians
μ	coefficient of rolling friction
ρ	density of air, slugs/cu ft

METHOD OF ANALYSIS

For this study, the take-off procedure was divided into three segments: (1) acceleration from zero velocity to initiation of airplane rotation, (2) rotation to lift-off, and (3) lift-off to 35-foot altitude. The equations which were used for these various phases of the take-off are as follows:

Prior to rotation:

$$s = \frac{W/S}{\rho g\left(C_{D,min} - \mu C_{L,min}\right)} \log_e\left[\frac{1}{1 - \dfrac{\dfrac{\rho V_r^2}{2}\left(C_{D,min} - \mu C_{L,min}\right)}{\dfrac{W}{S}\left(\dfrac{F}{W} - \mu\right)}}\right] \tag{1}$$

During rotation:

$$\frac{dq}{ds} = \rho g\left[\frac{F}{W}\cos\alpha - \frac{C_D q}{W/S} - \mu\left(1 - \frac{C_L q}{W/S}\right)\right] \tag{2}$$

After lift-off:

$$\left.\begin{aligned} \frac{dq}{ds} &= \rho g\left(\frac{F}{W}\cos\alpha - \frac{C_D q}{W/S} - \gamma\right) \\ \frac{d\gamma}{ds} &= \frac{\rho g}{2q}\left(\frac{F}{W}\sin\alpha + \frac{C_L q}{W/S} - 1\right) \end{aligned}\right\} \tag{3}$$

These rigid-body equations of motion were formulated with the following assumptions: the thrust axis was parallel to the aircraft reference axis and the flight-path angle γ was small so that $\sin\gamma \approx \gamma$ and $\cos\gamma \approx 1$. The calculations were made for standard day conditions and incorporate the assumption that the wing loading, coefficient of friction, and rate of rotation remain constant during the applicable phases of each take-off. Values of thrust were selected such that the airplane neither decelerated nor lost altitude during take-off. The performance of both delta-wing (low-aspect-ratio) and variable-geometry (high-aspect-ratio) configurations was studied by using the representative aerodynamic characteristics presented in figure 1. Ground effects on the aerodynamic characteristics were not considered in the present study. A summary of the range of pertinent variables is presented in table I.

For each set of design conditions for the two types of airplanes, an optimization procedure was necessary to determine the rotation speed V_r which resulted in the minimum take-off distance. (See ref. 1 for additional details.) The variation of velocity, angle of attack, and altitude with distance for a typical take-off is presented in figure 2. This typical take-off consists of the following three steps:

(1) An acceleration on the ground with the configuration in a low-lift low-drag attitude (eq. (1))

(2) A constant-rate-of-rotation segment (eq. (2))

(3) A constant-angle-of-attack climbout segment (eqs. (3))

The feasibility of such a maneuver has been demonstrated in reference 4.

Most of the results are presented for a constant thrust-weight ratio. However, since thrust does vary with speed, the effect of this variation has been examined. A thrust decrease given by the empirical relation

$$\frac{F}{F_{st}} = C_1 + \frac{C_2}{C_3 + q} \qquad (4)$$

was used prior to the initiation of rotation. The constants C_1, C_2, and C_3 are presented in table II. The thrust was assumed constant after the start of rotation, since it changes relatively little from then until the 35-foot altitude is reached. The thrust variations which were used are presented in figure 3 and are compared with curves which are typical of advanced nonafterburning turbojet and turbofan engines.

RESULTS AND DISCUSSION

The results of the present investigation are presented in figure 4 for the representative combinations of wing loading, constant thrust-weight ratio, and aerodynamic characteristics. The minimum horizontal distance (neglecting Civil

Air Regulations (CAR)) in which the airplane can take off and reach a 35-foot altitude is shown as a function of the wing loading divided by maximum available lift coefficient, $\frac{W/S}{C_{L,ma}}$. The distance presented is that which may be attained if rotation is initiated at the optimum speed, as discussed in reference 1. Generally, the data indicate that take-off distance varies linearly with wing loading and almost inversely with thrust-weight ratio and maximum available lift coefficients.

A comparison of the present results with those of other references is presented in figure 5. The minimized distance is presented as a function of wing loading divided by the maximum available lift coefficient multiplied by thrust-weight ratio, $\frac{W/S}{C_{L,ma}(F_{st}/W)}$. The circular symbols represent the data from this study, whereas the other symbols represent data from similar studies. The solid line is an empirical relation which has been used for some time. Generally, the results obtained here agree very well with those of similar studies. Further, it is apparent that, for the range of variables considered, the empirical relation provides a good approximation of the take-off distance. For design purposes, a nomogram of the empirical relation is presented in figure 6.

The preceding results have been presented for constant thrust and weight throughout the take-off maneuver. Actually, the weight will, generally, vary by less than 1 percent during the portion of the take-off up to a 35-foot altitude. However, the thrust decreases by several percent, particularly prior to the initiation of rotation and when afterburning is not used. The effect of this thrust decrease on distance is presented in figure 7. Optimum take-off distance to a 35-foot altitude is shown plotted against wing loading for two types of thrust conditions: constant thrust equal to the static value and thrust which decreases (prior to rotation) as shown by the empirical relations in figure 3. The increase in distance due to thrust decrease amounts to between 4 and 18 percent of the distance for a constant thrust equal to the static value.

The runway distance which an airplane requires is presently defined by Special Civil Air Regulations (ref. 5). The purpose and overall effect of this regulation is to insure safe operation of commercial airplanes during take-off. (See, also, ref. 1.) The required runway distance is defined as the distance to accelerate to a speed V_1, experience an engine failure, and either continue the take-off to the 35-foot altitude or stop on the runway. The speed V_1 is determined such that the additional runway distance to reach a 35-foot altitude (one engine out) is equal to the distance required to stop the airplane on the runway. In no case may the required runway length be less than 115 percent of the full-power take-off distance.

Another condition of CAR is that V_r may not be less than V_1. In the case of high maximum available lift coefficients, the minimum full-power take-off distance occurs with $(V_r)_{FP,opt} < V_1$, as shown in figure 8. The effect of applying the condition that $V_r \geqq V_1$ is to increase the take-off distance, as shown in figure 9. As the figure indicates, however, this effect is small.

In figure 10 the CAR runway lengths for four-engine airplanes are compared with full-power distances for two configurations. For full power, constant thrust is assumed and $(V_r)_{FP,opt} \geqq V_1$. The runway lengths are presented both with and without thrust variation. These results include a reverse thrust deceleration of about one-third the maximum thrust remaining after engine failure. However, other results not included in this report indicate that the effect of including thrust reversal is relatively small. The percentage increase in CAR field length over full-power take-off distance tends to remain about constant with variations in lift coefficient and tends to become larger with wing loading.

The CAR runway lengths which have been calculated for a constant thrust-weight ratio can be approximated by the empirical relation as shown in figure 11. The empirical relation is 115 percent of the full-power distance plus 750 feet. The data include those of figure 10 as well as other computations; again, reverse thrust is included. The results of references 1 and 3 also show reasonable agreement with the empirical relation. A design nomogram based on the empirical relation of figure 11 is presented as figure 12.

The CAR runway lengths have included a 2.0-second delay to allow the pilot a period to decide, after an engine failure, whether to continue take-off or to stop. The typical curves in figure 13 indicate the CAR runway length which is attributable to the time delay, both for a 2.0-second delay and for a 2.5-second delay. Obviously, the time delay is a significant portion of the take-off procedure. However, the increase in distance corresponding to the use of a 2.5-second delay as opposed to the 2.0-second delay is relatively small. Thus, small variations in time delay would be expected to produce only a small effect on the empirical relation in figures 11 and 12.

The variation of CAR runway length with lift-off speed is presented in figure 14 for typical supersonic transports with thrust-weight ratios between 0.3 and 0.4. The aerodynamic characteristics are those presented in figure 1 and the wing loadings are noted in figure 14. A thrust decrease corresponding to turbofan engines is included, and the take-off performance of typical subsonic jet transports is also presented. The figure shows the region in which the supersonic transport will probably operate and the relations of the supersonic transport performance to that of the subsonic jet transports. It is apparent from the figure that take-off performance equivalent to or better than the present subsonic jet transports is feasible.

CONCLUDING REMARKS

The take-off performance characteristics of typical supersonic transport configurations have been analytically determined with aerodynamic characteristics representative of both delta-wing and variable-geometry configurations. The investigation considered conditions where the thrust was assumed constant and where the thrust decreased with increasing velocity. Optimum full-power take-off distances and Civil Air Regulation runway length were obtained, and the results agree, generally, with previous similar studies. First-order empirical relations were determined which correlate the data well. Design nomograms derived from these empirical relations are presented. The results indicate, in general, that the take-off performance of the typical supersonic transports considered can be comparable to or better than present subsonic jet transports.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., February 26, 1964.

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1. Hall, Albert W.: Take-Off Distances of a Supersonic Transport Configuration as Affected by Airplane Rotation During the Take-Off Run. NASA TN D-982, 1961.

2. Perkins, Courtland D., and Hage, Robert E.: Airplane Performance Stability and Control. John Wiley & Sons, Inc., c.1949.

3. O'Connor, John J.: Generalized Curves for Aircraft Field Length Prediction Based on Analog Computer Simulation. Tech. Inf. Ser. No. R61-SE87, Small Aircraft Engine Dept., Gen. Elec. Co., Aug. 11, 1961.

4. Hall, Albert W., and Harris, Jack E.: A Simulator Study of the Effectiveness of a Pilot's Indicator Which Combined Angle of Attack and Rate of Change of Total Pressure as Applied to the Take-Off Rotation and Climbout of a Supersonic Transport. NASA TN D-948, 1961.

5. Anon.: Turbine-Powered Transport Category Airplane of Current Design. Special Civil Air Regulation No. SR-422B, FAA, July 9, 1959.

TABLE I.- VALUES OF VARIABLES CONSIDERED

Wing loading, W/S	80 to 120 (increments of 10)
Thrust-weight ratio, F/W	0.3 to 0.6 (increments of 0.1)
Maximum available lift coefficient, $C_{L,ma}$	1.14 to 2.2 for variable-geometry configuration; 0.6 to 1.0 for delta-wing configuration
Lift-curve slope per degree, C_{L_α}	0.10 for variable-geometry configuration; 0.05 for delta-wing configuration
Coefficient of friction, μ	0.02 for take-off; 0.20 for braking
Rate of change of angle of attack, $\frac{d\alpha}{dt}$, deg/sec	3
Maximum available angle of attack, α_{ma}, deg	12 for variable-geometry configuration; 10 and 14 for delta-wing configuration
Time delay for CAR calculation, sec	2.0
Density of air, ρ, slug/cu ft	0.002377
Acceleration of gravity, g, ft/sec^2	32.2

TABLE II.- CONSTANTS FOR EMPIRICAL THRUST VARIATION

Engine	C_1	C_2	C_3
Turbojet	0.940	0.957	16.0
Turbofan	.865	3.64	27.0

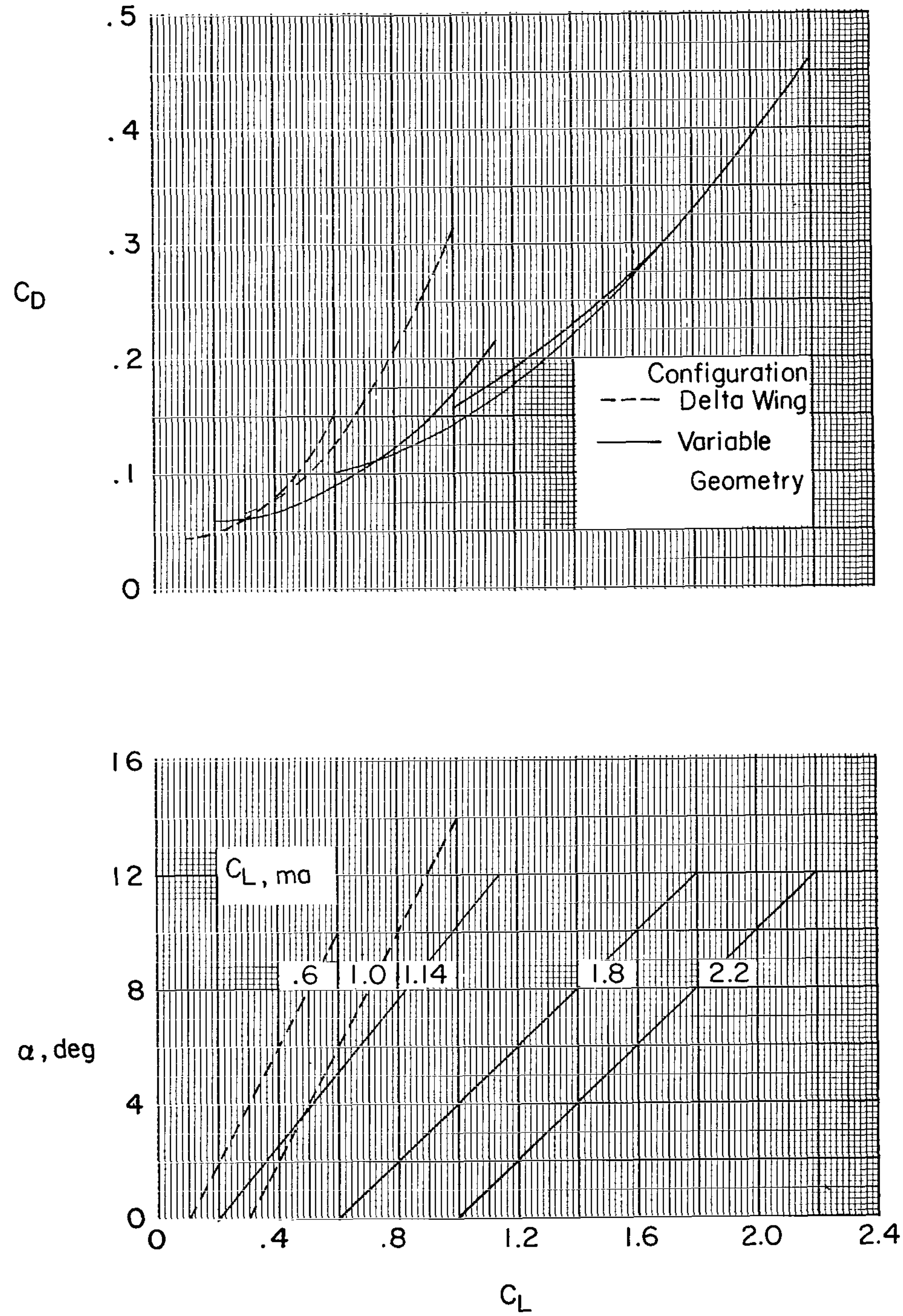


Figure 1.- Assumed aerodynamic characteristics of the delta-wing and variable-geometry supersonic transport configurations.

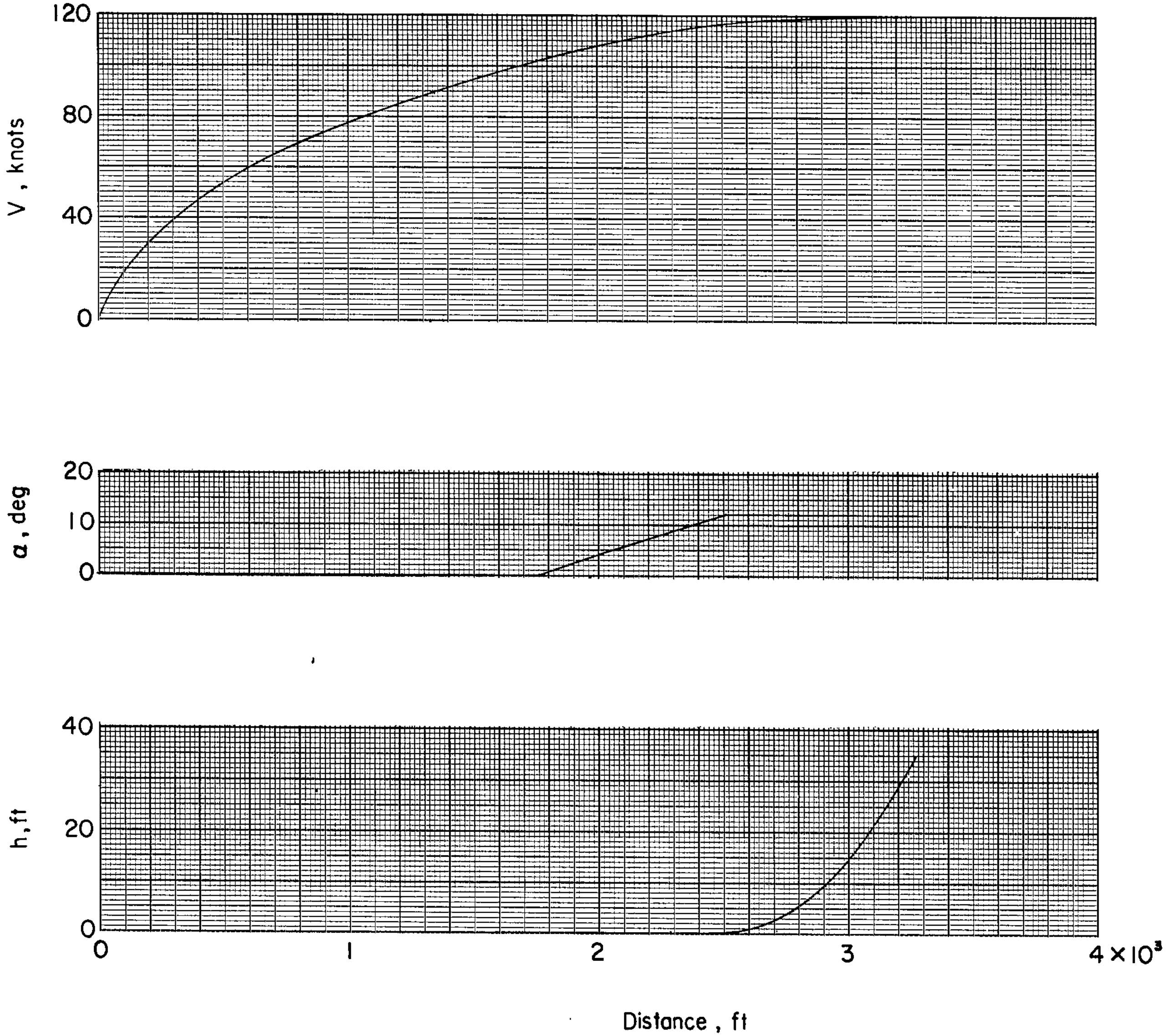


Figure 2.- Variation of altitude, angle of attack, and velocity with distance during a take-off with a constant thrust-weight ratio. $C_{L,ma} = 1.8$; $F_{st}/W = 0.3$; $W/S = 80$ lb/sq ft; $V_r = 102.5$ knots.

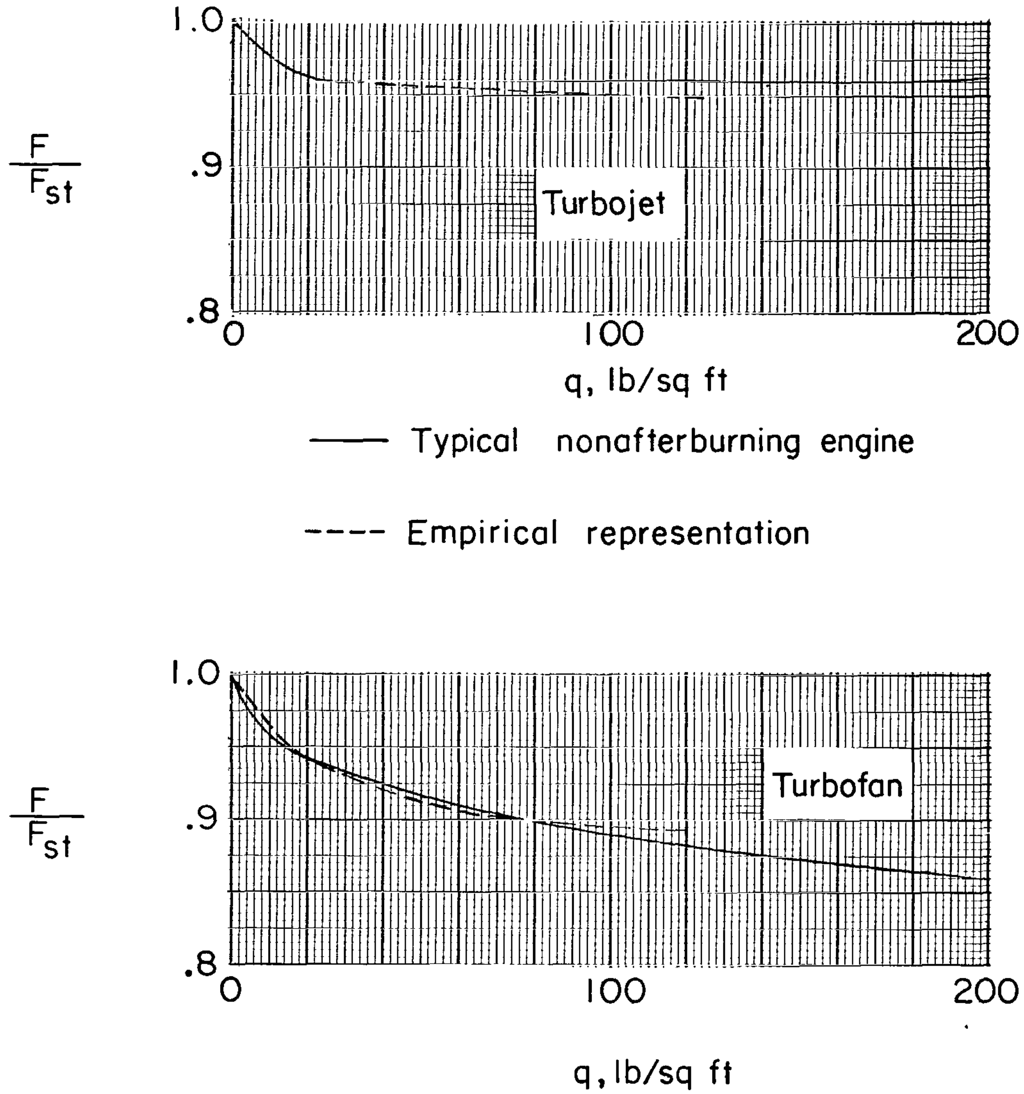


Figure 3.- Comparison of empirical engine characteristics with typical supersonic transport, nonafterburning engines. Sea level, standard day.

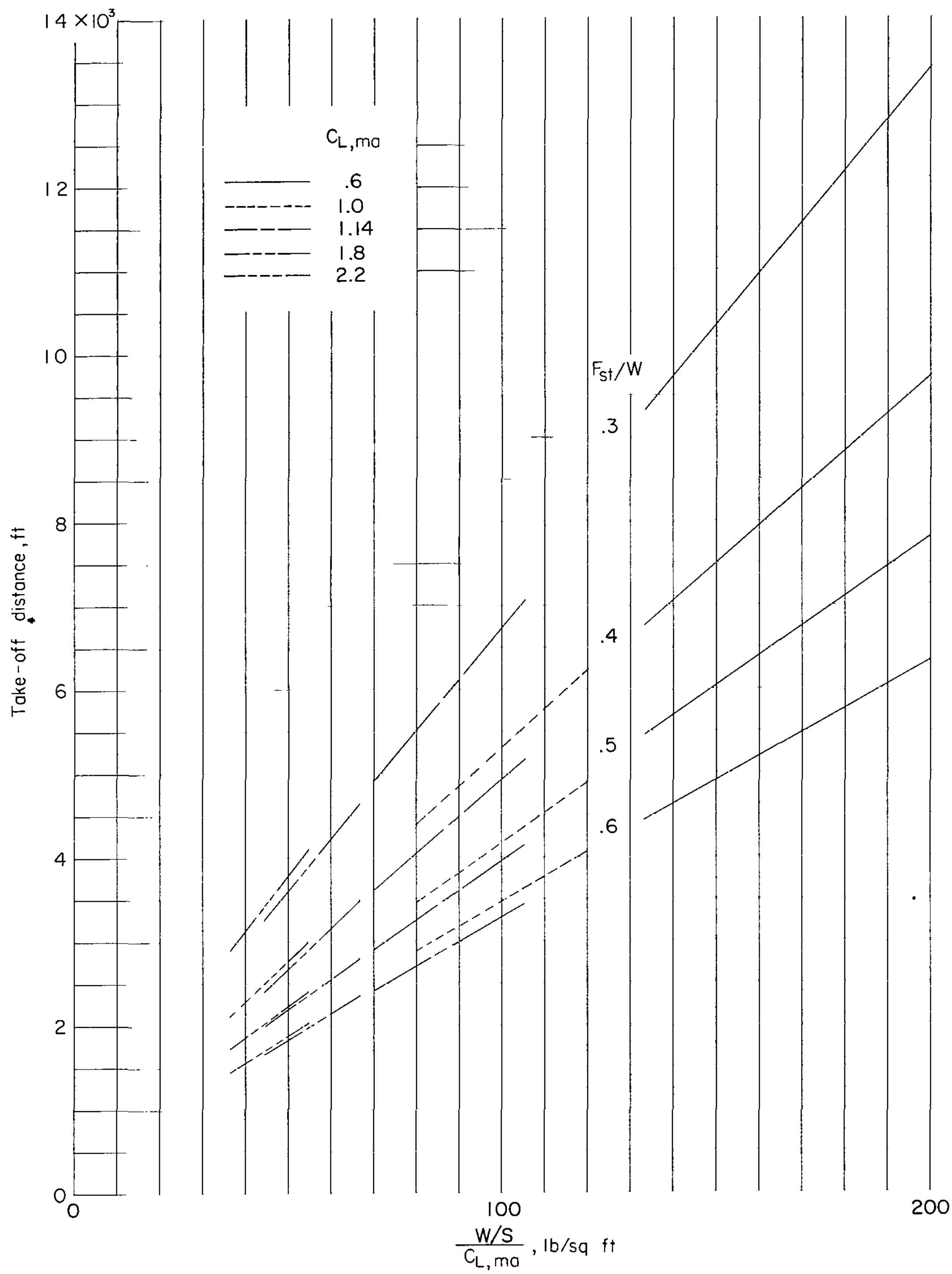


Figure 4.- Results of the analytical investigation of supersonic transport take-off performance. Constant thrust-weight ratio.

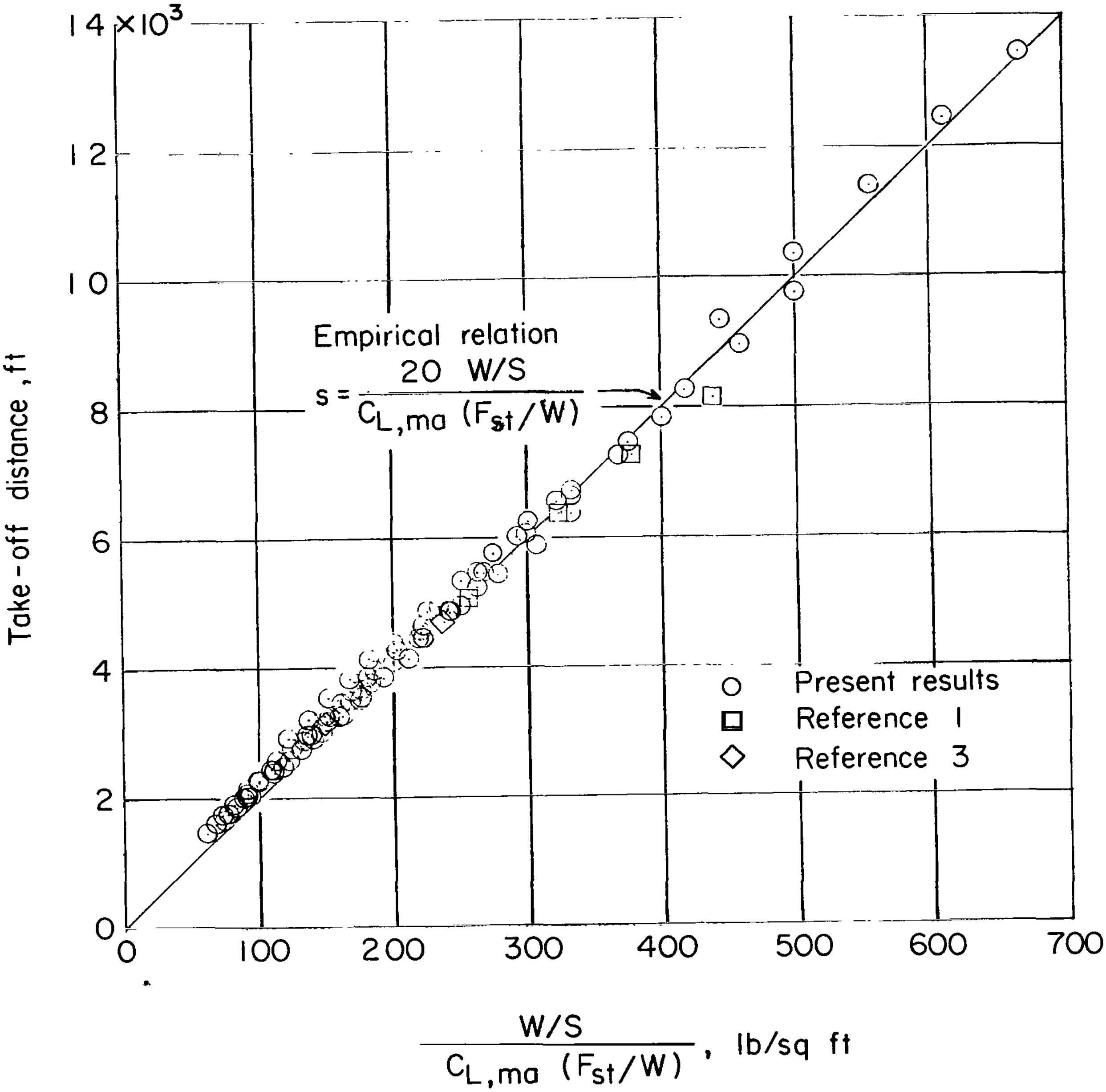


Figure 5.- Comparison of present results with those of previous investigations.

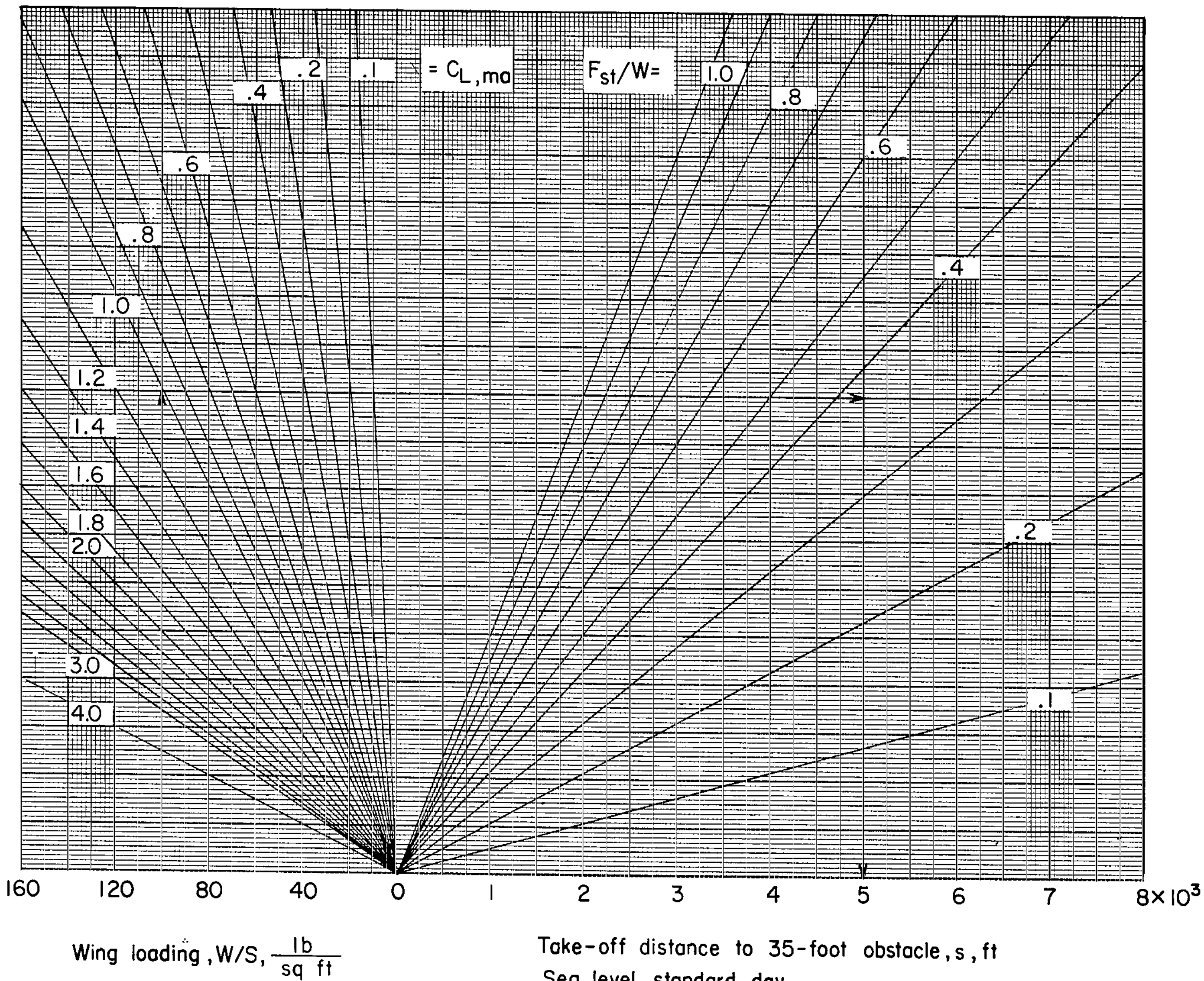
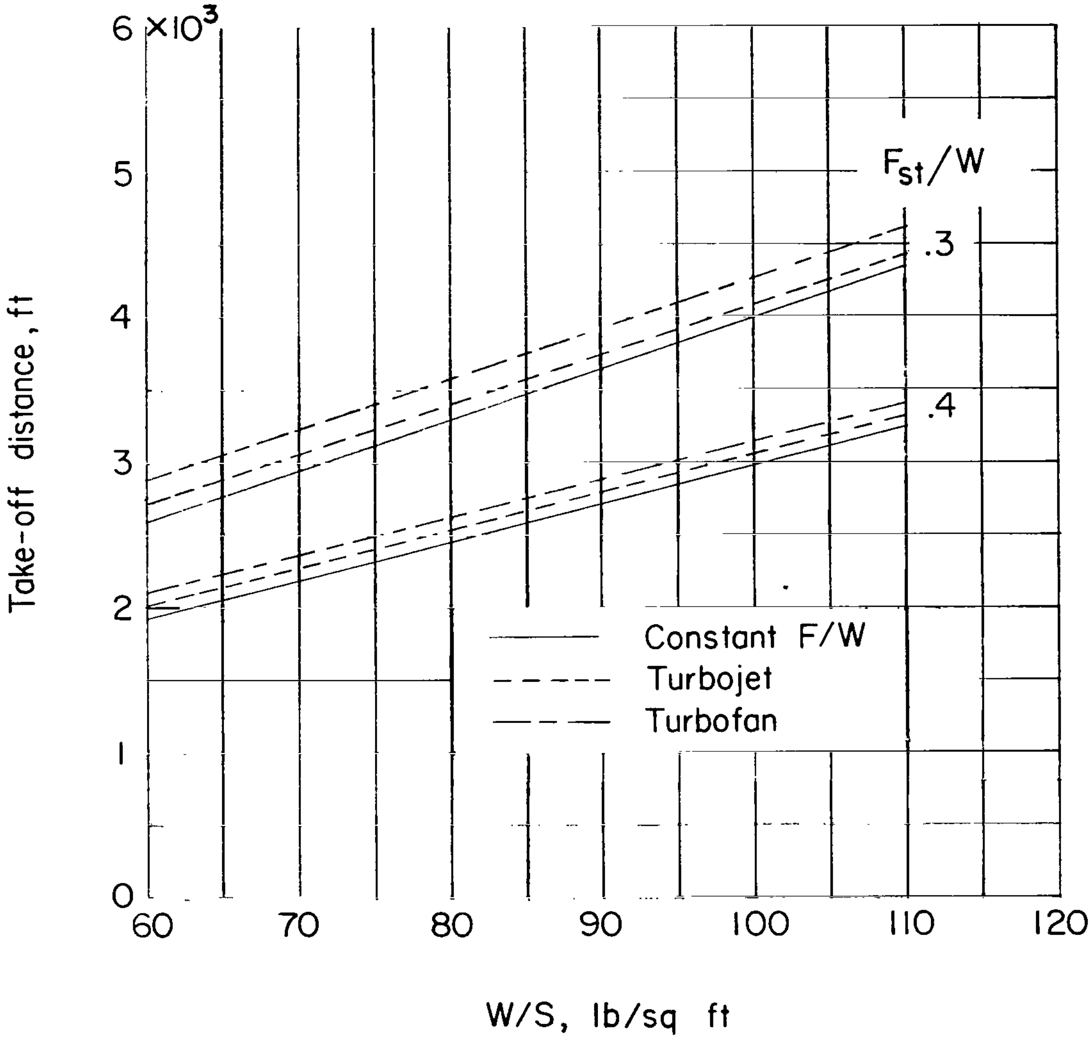
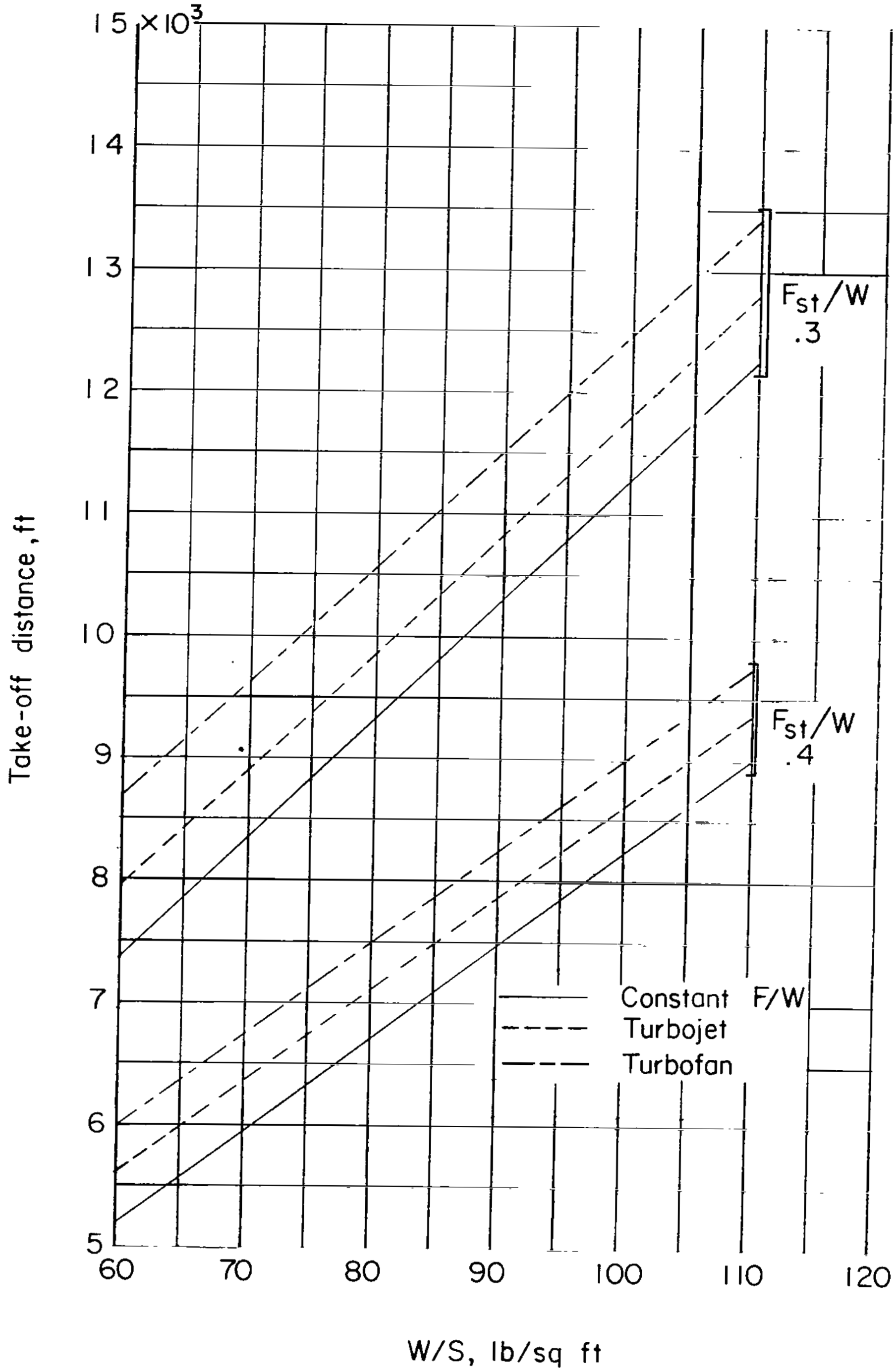


Figure 6.- Design nomogram of full-power take-off distances as expressed by $s = \dfrac{20\ W/S}{C_{L,ma}(F_{st}/W)}$.



(a) $C_{L,ma} = 1.8$.

Figure 7.- The effect on take-off distance of thrust variation during take-off.



(b) $C_{L,ma} = 0.6$.

Figure 7.- Concluded.

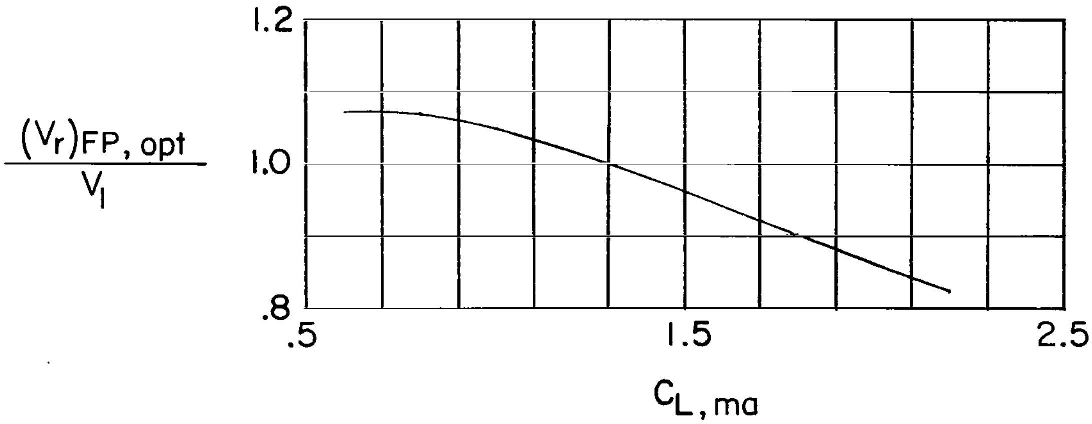


Figure 8.- Variation of $\frac{(V_r)_{FP,opt}}{V_1}$ with maximum available lift coefficient. W/S = 90 lb/sq ft; $F_{st}/W = 0.4$.

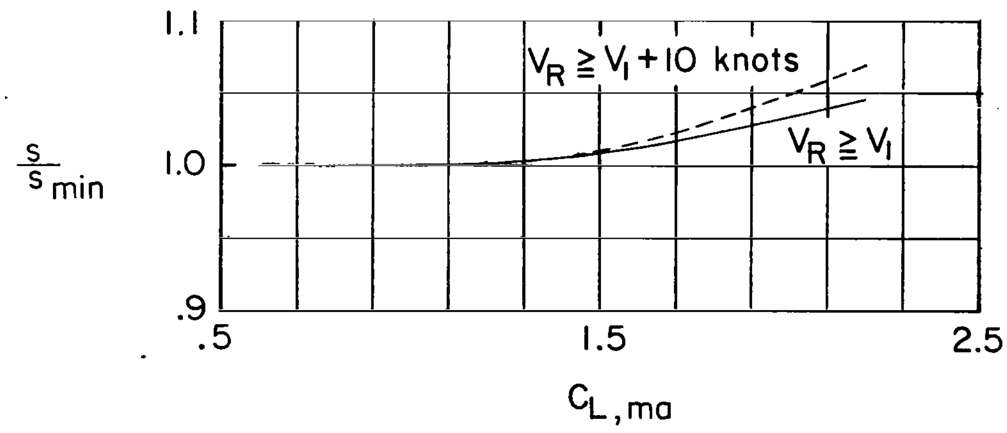
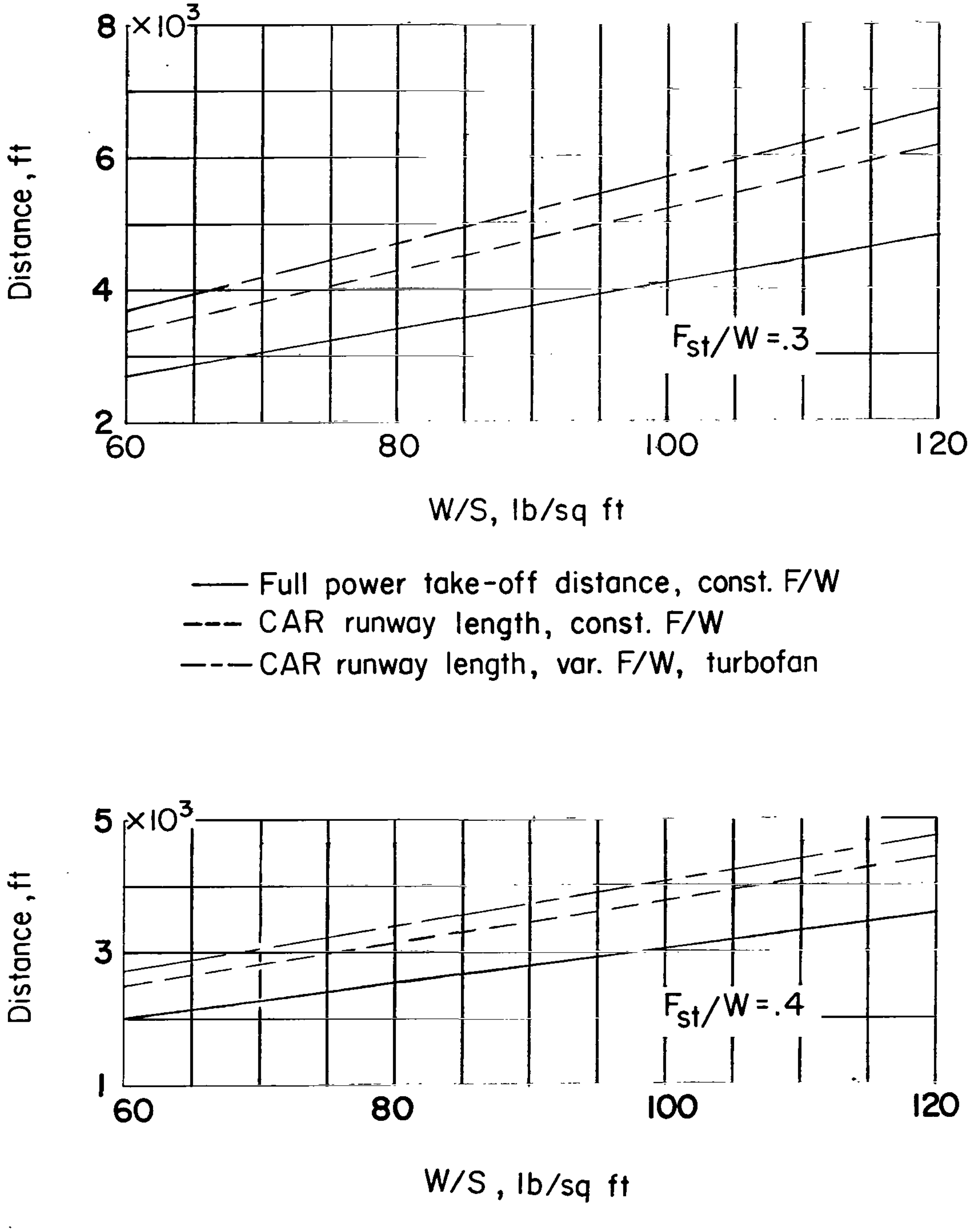
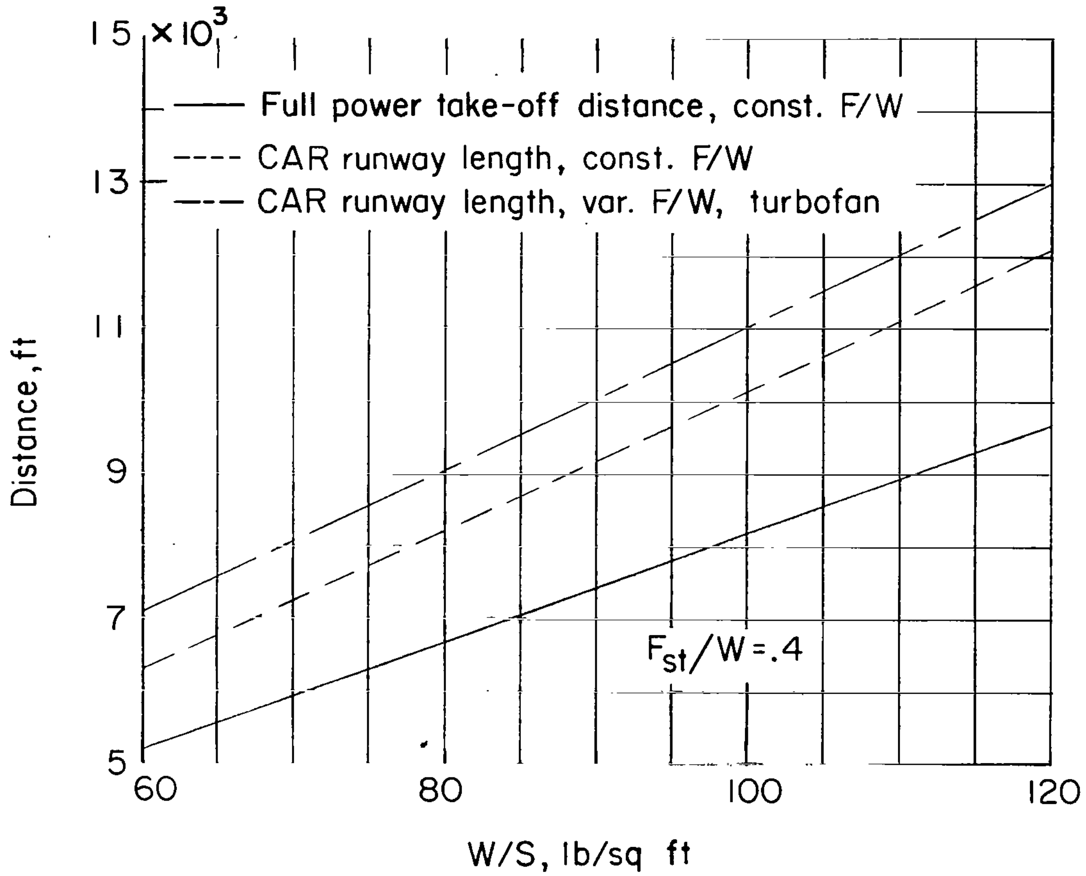


Figure 9.- The effect on take-off distance of requiring $V_r \geqq V$. W/S = 90 lb/sq ft; $F_{st}/W = 0.4$.



(a) $C_{L,ma} = 1.8$.

Figure 10.- CAR runway length for typical four-engine supersonic transport configurations.



(b) $C_{L,ma} = 0.6$.

Figure 10.- Concluded.

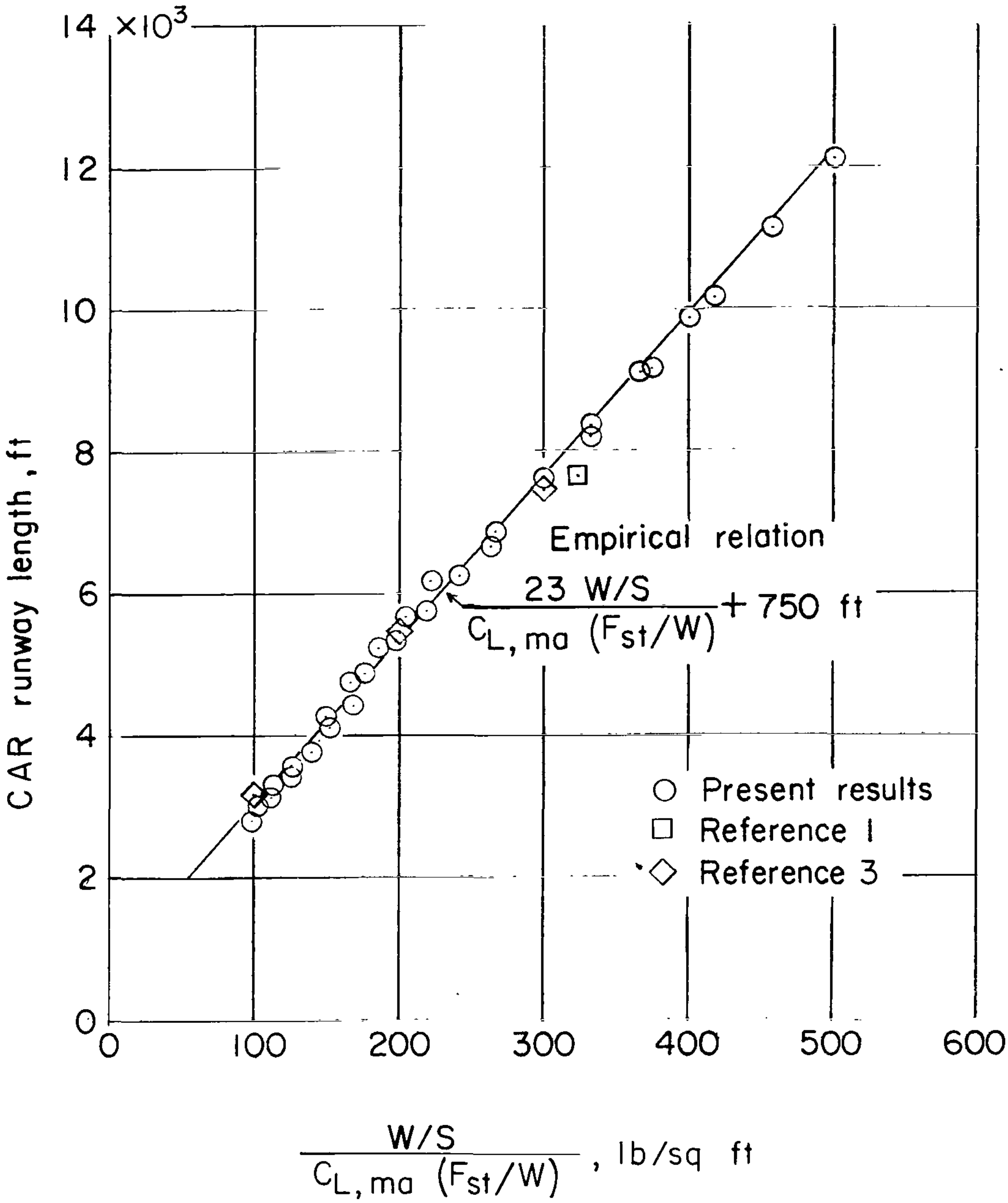


Figure 11.- Correlation of CAR runway length with empirical relation. Constant thrust-weight ratio.

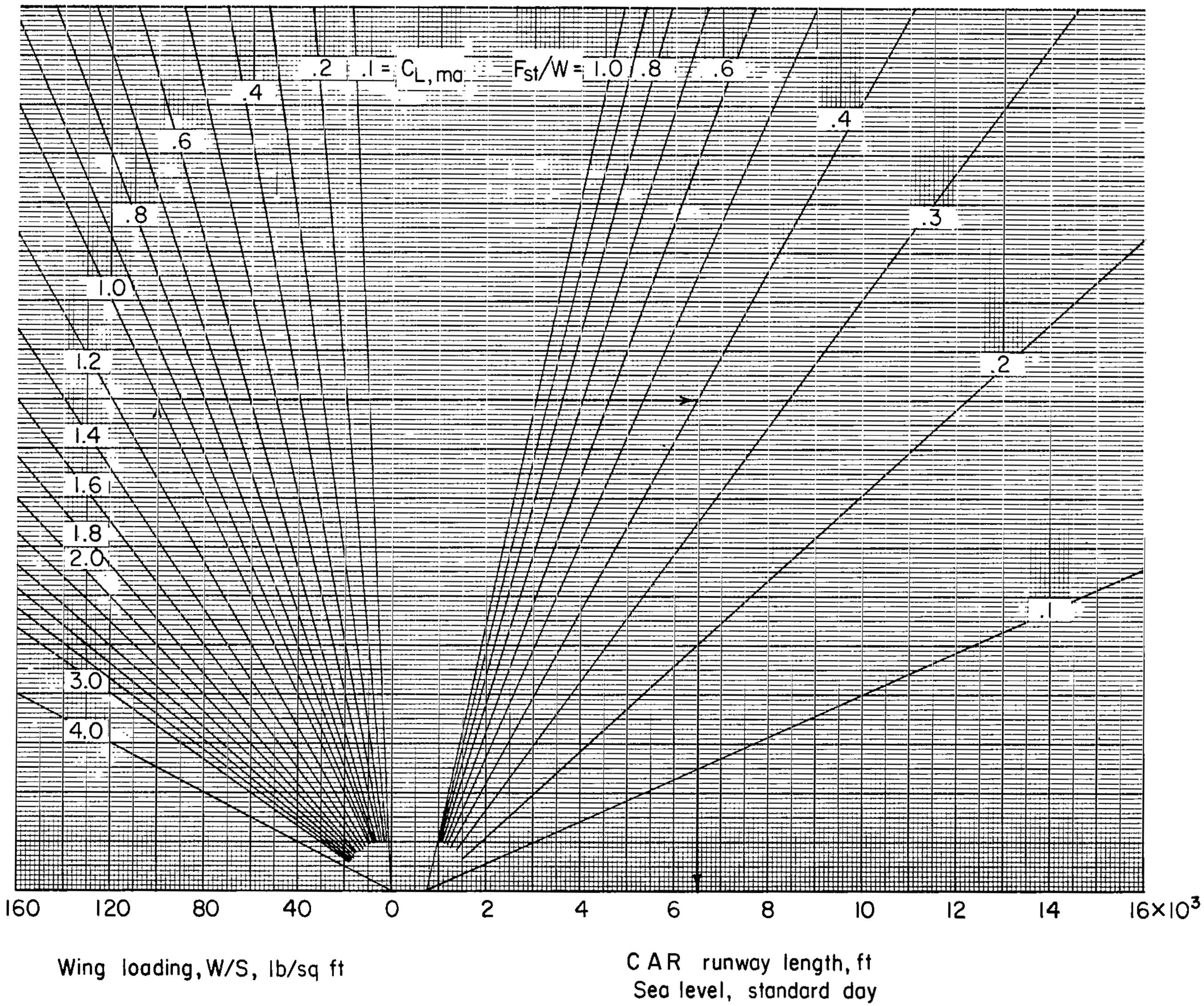


Figure 12.- Design nomogram of CAR runway length based on the empirical relation $s = 750 + \frac{23\ W/S}{C_{L,ma}(F_{st}/W)}$.

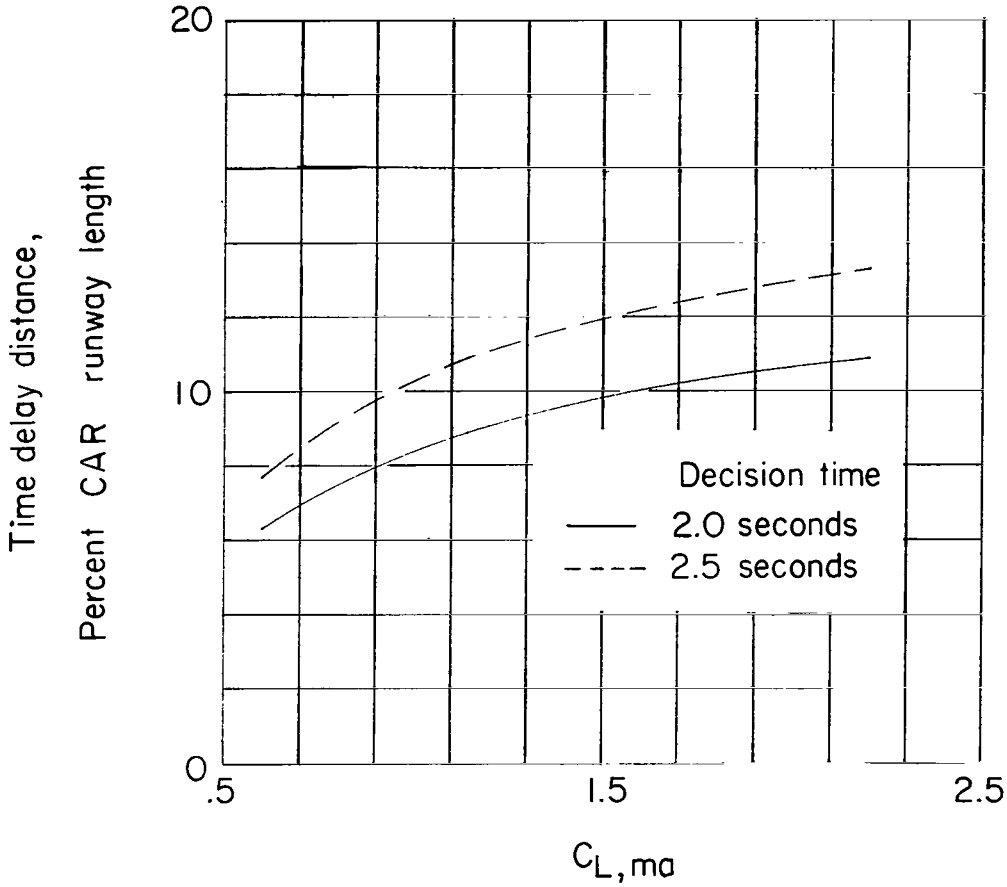


Figure 13.- CAR runway length attributable to pilot decision time. Constant thrust-weight ratio; $W/S = 90$ lb/sq ft; $F_{st}/W = 0.4$.

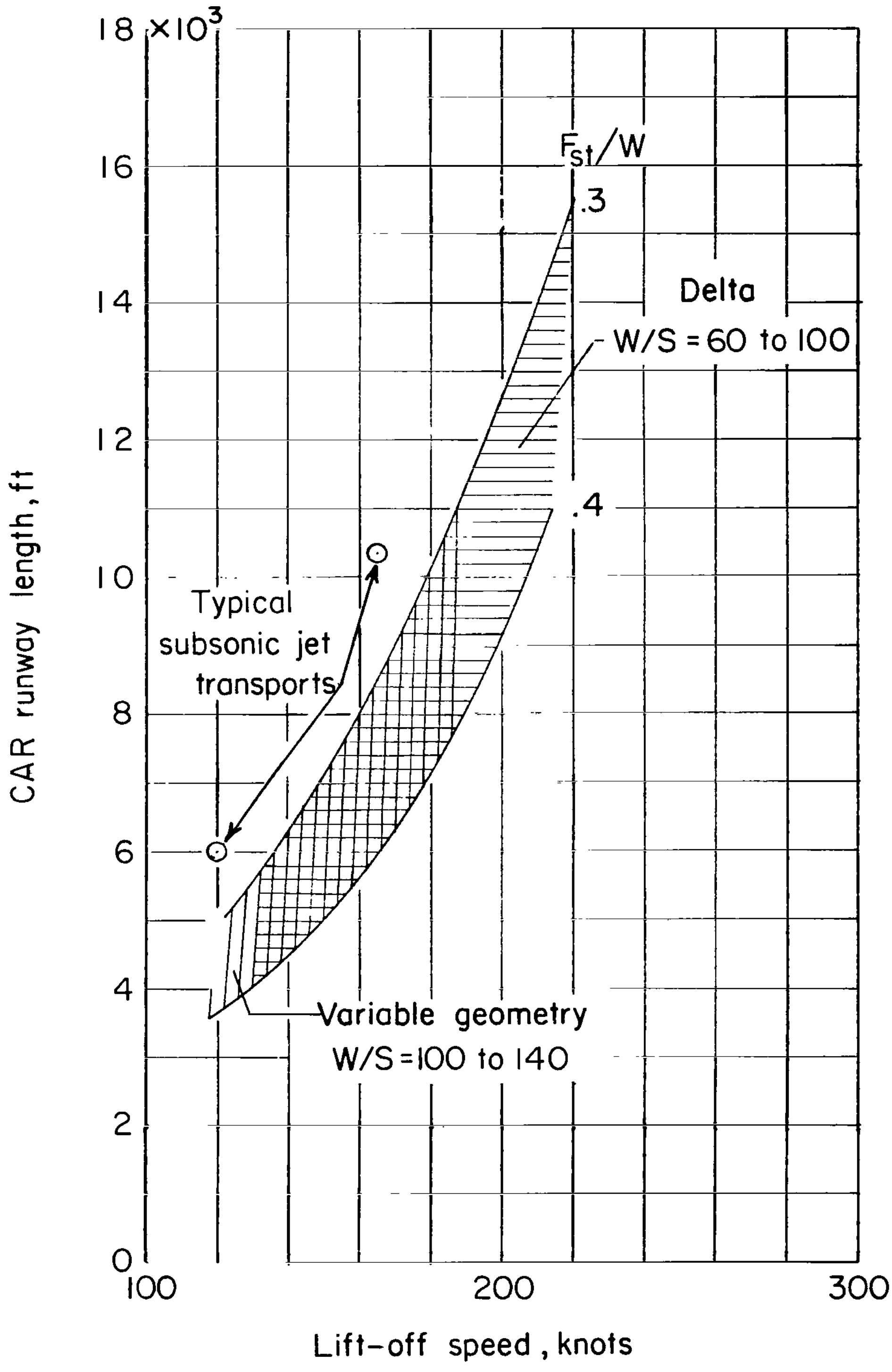


Figure 14.- Variation in CAR runway length with lift-off velocity.

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