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DESIGNING TOROIDAL INDUCTORS WITH DC BIAS

by G. D. Smith

Goddard Space Flight Center
Greenbelt, Md.

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SUMMARY

A practical method of designing toroidal inductors with dc bias is presented. The toroidal core equations for the inductance, magnetizing force, and core winding factor are used to derive two equations for selecting the smallest possible core. Acceptable requirements are established for the variation of Q with frequency and for the variation of inductance with temperature. The inductor equations and requirements indicate the desirable combinations of core constants and from these the smallest core may be determined.

A step-by-step procedure for establishing the inductor requirements, selecting the smallest core, and designing the inductor is given, explained, and illustrated. Tables and curves employed in this method of design are presented.

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INTRODUCTION

Selection of the best magnetic core for an inductor frequently involves trial-and-error type calculations. Comparison of the physical parameters of the core with the basic inductor equations and requirements reveals a clear, accurate, and concise method of designing inductors with dc bias. Equations derived from the basic inductor equations can be used to select the smallest core and auxiliary inductor requirements can be used to obtain an optimum design.

The design of an inductor frequently involves consideration of its effect upon the magnetic field in which it is placed. This is especially true in the design of high-current inductors for converters and regulators used in satellites which employ sensitive magnetic field detectors. For this type of design problem it is frequently imperative that a toroidal core be used, because of the low magnetic effect external to the core. The core should be small to minimize the amount of magnetic material in the inductor. A large number of air gaps should be spaced evenly around the circumference and across the cross-section of the core.

The molybdenum permalloy toroidal powder cores are well suited to this type of design problem. A large variety of sizes have high permeability with high Q at high frequency. A discussion of the characteristics of these cores will be used to illustrate this method of designing toroidal inductors. All curves and data are for molybdenum permalloy powder cores. They are not intended to cover all commercially available cores,* but only to augment the design discussion.

DERIVATION OF EQUATIONS

The smallest core which fulfills the inductor requirements of inductance and inductance tolerance at peak current can be determined from the basic equations of the core. The inductance of a toroidal inductor is

$$L = \frac{0.4\pi N^2 \mu A}{10^8 l} , \tag{1}$$

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*All magnetic core data are taken from Magnetics, Inc., Catalog No. PC-303.

where

L = the inductance in henries,

N = the number of turns of wire,

μ = the relative permeability,

A = the cross-sectional area of the core in square centimeters,

l = the mean magnetic path length in centimeters.

In selecting the smallest core to meet the requirements of Equation 1, the maximum number of turns of wire which can be placed on the core must be known. This is a function of the ratio of the core window area (W) to the wire cross-sectional area (A_w). Defining a winding factor (K_w) as the ratio of the total wire area (NA_w) to the core window area (W), we can write

$$K_w = \frac{NA_w}{W} , \tag{2}$$

where A_w and W are in circular mils. Experience in winding many cores of various sizes shows that the maximum practicable winding factor is approximately 0.4. This means that approximately 60 percent of the window area is required for wire insulation and space between the wires, leaving only 40 percent ($K_w = 0.4$) for the wire. The smallest toroidal core which will meet the requirements of Equations 1 and 2 can be selected by combining these equations. Solving Equations 1 and 2 for N^2 and setting them equal gives

$$LA_w^2 = \frac{0.4\pi\mu AK_w^2 W^2}{10^8 \, l} , \tag{3}$$

where the left side contains the inductor requirements and the right side the core and winding factor contant. Thus, the smallest core which meets the requirements of Equations 1 and 2 is one which has a core and winding factor constant equal to or next greater than the inductor requirements on the left of Equation 3.

The relative permeability and inductance of an inductor vary with the magnetizing force. This relationship is shown in Figure 1 for some powder permalloy cores of different relative permeabilities. This family of curves reveals that excessive variations in the inductance may occur unless the magnetizing force is limited. For this reason an inductance tolerance is imposed; it is transformed into a maximum magnetizing force requirement by use of manufacturer curves similar to those in Figure 1. The magnetizing force is related to the inductor current by Ampere's law:

$$H = \frac{0.4\pi NI}{l} , \tag{4}$$

where H is the magnetizing force in oersteds and I is the current in amperes. Equation 4 is employed to impose maximum requirements; therefore, $H_{\pm\%}$ denotes the maximum value of the magnetizing force for the inductance tolerance indicated in percent from the nominal and I_p denotes

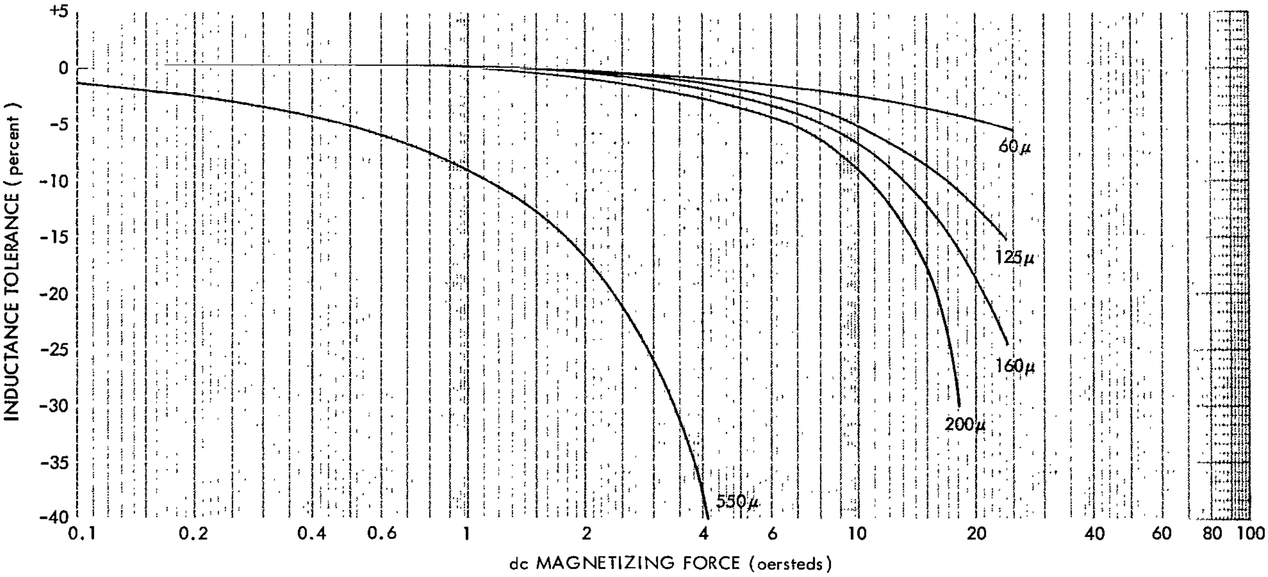

Figure 1—Inductance tolerance vs. dc magnetizing force for several powder permalloy cores.

the corresponding peak current. The number N in Equation 4 cannot be determined until the core is selected. For this reason N is replaced by its equivalent from Equation 1, and the core constants are placed on the right:

$$LI_p^2 = \frac{\mu A l H_{\pm\%}^2}{0.4\pi 10^8} \; . \qquad (5)$$

Therefore, the core chosen must have a constant equal to or next greater than the inductor requirements on the left of this equation.

The smallest core which meets the requirements of Equations 1, 2, and 4 is one with the core constants on the right of Equations 3 and 5 equal to or greater than the respective inductor requirements on the left.

DESIGN PROCEDURE

A detailed procedure for selecting the core and designing the inductor follows:

1. Form or obtain a table, similar to Table 1, of all available cores for a suitable inductance tolerance. Table 1 was made for 10 percent inductors since the magnetizing force $(H_{\pm 10})$ was taken from Figure 1 where the inductance is down 10 percent. The core number in column 1 of Table 1 indicates the physical size and permeability of the core.

2. Select the wire size to be used for winding the core. The maximum rms current shown in Table 2 should be used as a guide for this selection. In case a very low dc resistance or very high Q is desired, a slightly large wire size should be selected.

Table 1

Molybdenum Permalloy Powder Core Data.

Core Number	μ	$H_{\pm 10}$ (oersteds)	Inside Diameter (in.)	Outside Diameter (in.)	Height (in.)	A (cm^2)	l (cm)	W (cir mil)	$\frac{0.4\pi\mu A K_w^2 W^2}{10^8 l}$ $[cm\,(cir\,mil)^2]$	$\frac{\mu A l H_{\pm 10}^2}{0.4\pi\, 10^8}$ $(cm^3\,oersteds^2)$	Frequency Range (kc)	Inductance vs. Temperature (see Table 4)	K_1 (ft/turn)
55894	60	45	0.555	1.09	0.472	0.635	6.35	320,000	1.23×10^3	3880×10^{-6}	10-50	A2, B4, D4, W4	0.135
55930	125	17.5	0.555	1.09	0.472	0.635	6.35	320,000	2.58×10^3	1230×10^{-6}	0-20	B4, D4, E4, W4	0.135
55928	160	13	0.555	1.09	0.472	0.635	6.35	320,000	3.30×10^3	870×10^{-6}	0-10	A2, D4, W4	0.135
55927	200	11	0.555	1.09	0.472	0.635	6.35	320,000	4.12×10^3	774×10^{-6}	0-7	A2	0.135
55926	550	1.2	0.555	1.09	0.497	0.635	6.35	320,000	11.3×10^3	25.6×10^{-6}	0-4	A2	0.135
55071	60	45	0.760	1.332	0.457	0.655	8.10	590,000	3.395×10^3	5120×10^{-6}	10-50	A2, B4, D4, W4	0.146
55548	125	17.5	0.760	1.332	0.457	0.655	8.10	590,000	7.06×10^3	1620×10^{-6}	0-20	A2, B4, D4, W4	0.146
55546	160	13	0.760	1.332	0.457	0.655	8.10	590,000	9.04×10^3	1145×10^{-6}	0-10	A2, D4, W4	0.146
55545	200	11	0.760	1.332	0.457	0.655	8.10	590,000	11.3×10^3	1030×10^{-6}	0-7	A2	0.146
55585	125	17.5	0.888	1.385	0.387	0.449	8.91	810,000	8.3×10^3	1220×10^{-6}	0-20	A2, D4, W4	0.137
55583	160	13	0.888	1.385	0.387	0.449	8.91	810,000	10.6×10^3	864×10^{-6}	0-10	A2, D4, W4	0.137
55582	200	11	0.888	1.385	0.387	0.449	8.91	810,000	13.3×10^3	776×10^{-6}	0-7	A2	0.137
55076	60	45	0.848	1.445	0.444	0.670	8.99	730,000	4.8×10^3	5800×10^{-6}	10-50	A2, B4, D4, W4	0.151
55324	125	17.5	0.848	1.445	0.444	0.670	8.99	730,000	10.0×10^3	1835×10^{-6}	0-20	A2, B4, D4, W4	0.151
55322	160	13	0.848	1.445	0.444	0.670	8.99	730,000	12.8×10^3	1300×10^{-6}	0-10	A2, D4, W4	0.151
55321	200	11	0.848	1.445	0.444	0.670	8.99	730,000	16.0×10^3	1170×10^{-6}	0-7	A2	0.151
55083	60	45	0.918	1.602	0.605	1.056	9.87	860,000	9.55×10^3	$10{,}000 \times 10^{-6}$	10-50	A2, D4, W4	0.188
55254	125	17.5	0.918	1.602	0.605	1.056	9.87	860,000	19.8×10^3	3180×10^{-6}	0-20	A2, D4, W4	0.188
55252	160	13	0.918	1.602	0.605	1.056	9.87	860,000	25.5×10^3	2250×10^{-6}	0-10	A2	0.188
55251	200	11	0.918	1.602	0.605	1.056	9.87	860,000	31.8×10^3	2020×10^{-6}	0-7	A2	0.188

Table 2

Resistance, Area, and Current Capacity of Various AWG Wire Sizes.

AWG Wire Size	Resistance/Length (ohm/ft)	Wire Area* (cir mil)	Current Capacity† (amps)
8	0.00063	23,000	16.5
9	0.00079	16,500	13.1
10	0.00100	11,700	10.2
11	0.00126	8890	821
12	0.00159	7100	6.52
13	0.00200	5660	5.18
14	0.00252	4520	4.12
15	0.00318	3610	3.26
16	0.00402	2900	2.58
17	0.00505	2330	2.05
18	0.00639	1860	1.63
19	0.00805	1490	1.29
20	0.01013	1197	1.024
21	0.0128	961	0.8123
22	0.0162	767	0.6401
23	0.0203	620	0.5108
24	0.0257	497	0.4040
25	0.0324	400	0.3204
26	0.0410	320	0.2528
27	0.0514	259	0.2016
28	0.0653	207	0.1588
29	0.0822	169	0.1277
30	0.104	134	0.1000
31	0.131	108	0.0792
32	0.162	88.3	0.0640
33	0.206	70.5	0.0504
34	0.261	56.2	0.0397
35	0.331	44.9	0.0314
36	0.415	36.0	0.0250
37	0.512	29.1	0.0202
38	0.648	23.0	0.0160
39	0.847	17.6	0.0122
40	1.074	14.4	0.0096
41	1.32	11.6	0.0078
42	1.66	9.0	0.0063
43	2.14	7.3	0.0048
44	2.59	6.2	0.0040

*Based on nominal diameter of wire with heavy Formvar insulation.

†Based on 1000 cir mil per amp.

Table 3

Frequency Range of Cores.

Core Permeability	Frequency Range (kc)
60	10-50
125	0-20
160	0-10
200	0-7
550	0-4

3. Compute the product (Equation 3) of the inductance and the square of the wire cross-sectional area (LA_w^2).

4. Compute the product (Equation 5) of the inductance and the square of the peak current (LI_p^2).

5. Determine the acceptable core permeabilities for the maximum inductor frequency. The approximate frequency ranges for core permeabilities are shown in Table 3. In view of the fact that the frequency ranges are approximations, they should not be followed rigidly. Table 3 is based on the variations of Q with frequency as shown in Figures 2-5 for several different core sizes and permeabilities. Comparison of these curves reveals that Q varies with core size for a certain core permeability, frequency, and inductance. For this reason, these more exact curves cannot be used readily until the core size is determined. Because of the inaccuracies introduced in making the approximations for Table 3, it is frequently more convenient to omit this step and consider the permeabilities of the cores (step 7).

6. Determine the acceptable variation in inductance as a function of temperature. A list of inductance tolerances over specific temperature ranges, similar to Table 4, is usually available from the core manufacturer.

7. Select the core on the basis of the information obtained in steps 3, 4, 5, and 6. That is, select the smallest core from a table similar to Table 1 which: (1) has a constant for the core and winding factor, $0.4\pi\mu AK_w^2 W^2/10^8 l$, that is equal to or next greater than the inductor requirements LA_w^2 ; (2) has a constant for the core, $\mu AlH_{\pm 10}^2/0.4\pi 10^8$, that is equal to or next greater than the inductor requirements LI_p^2 ; (3) and meets the requirements established in steps 5 and 6. The frequency range from Table 3 (step 5) and the inductance tolerance vs. temperature characteristic available from Table 4 (step 6) are also included in Table 1. Thus, these four columns (10, 11, 12, and 13) of Table 1 should be used for selecting the core.

In view of the knowledge obtained about the core size, the acceptable core permeabilities, determined in step 5, should be reconsidered by use of curves similar to those of Figures 2-5 for the individual cores.

8. Compute the number of turns of wire for the selected core to give the required inductance by using Equation 1.

9. It is frequently desirable to use the largest wire possible on the core to decrease the dc resistance and power loss. This can be accomplished by using Equation 2 with the winding factor K_w set equal to 0.4. The dc resistance of the inductor should be calculated from

$$R_{dc} = K_1 NK_2 \quad , \qquad (6)$$

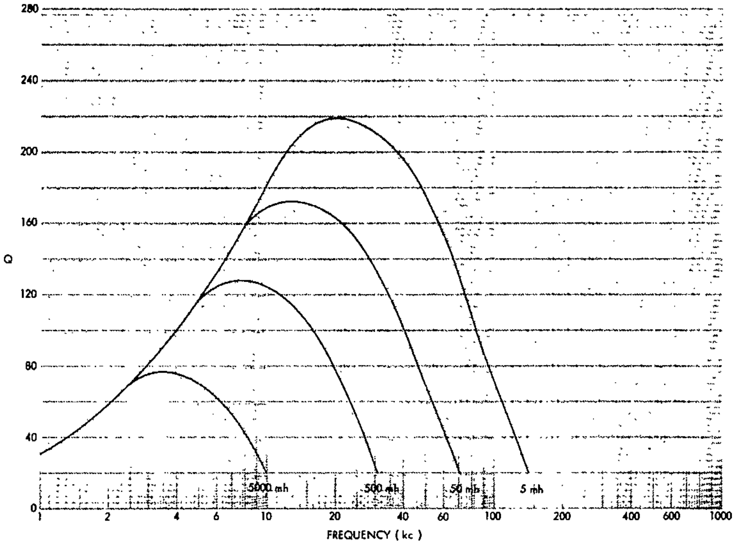

Figure 2—Curves of Q vs. frequency for core 55894; μ = 60,
A = 0.635 cm², l = 6.35 cm, W = 320,000 cir mil.

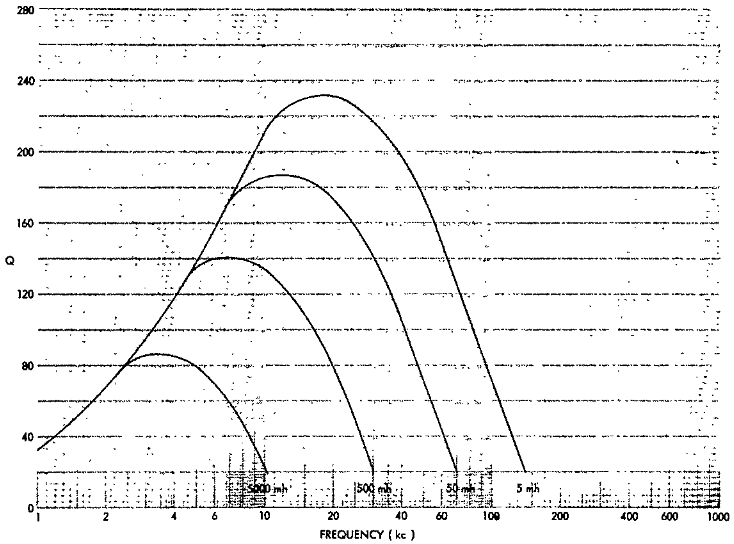

Figure 3—Curves of Q vs. frequency for core 55071; μ = 60,
A = 0.655 cm², l = 8.1 cm, W = 590,000 cir mil.

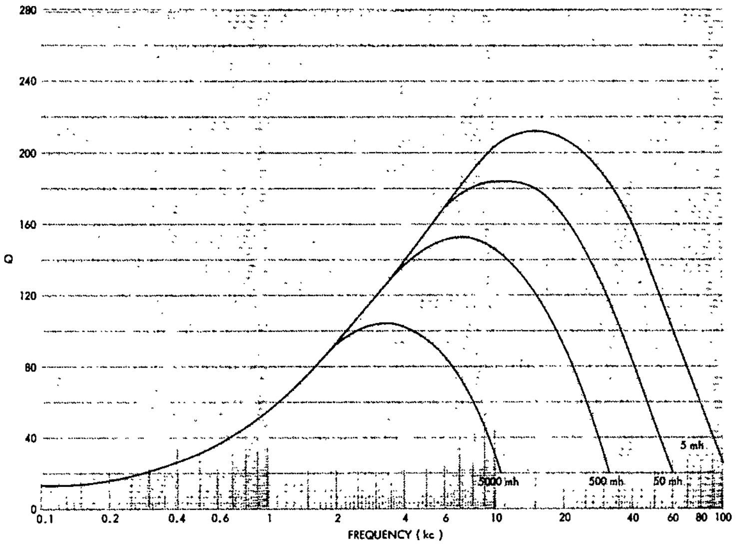

Figure 4—Curves of Q vs. frequency for core 55930; μ = 125,
A = 0.635 cm², l = 6.35 cm, W = 320,000 cir mil.

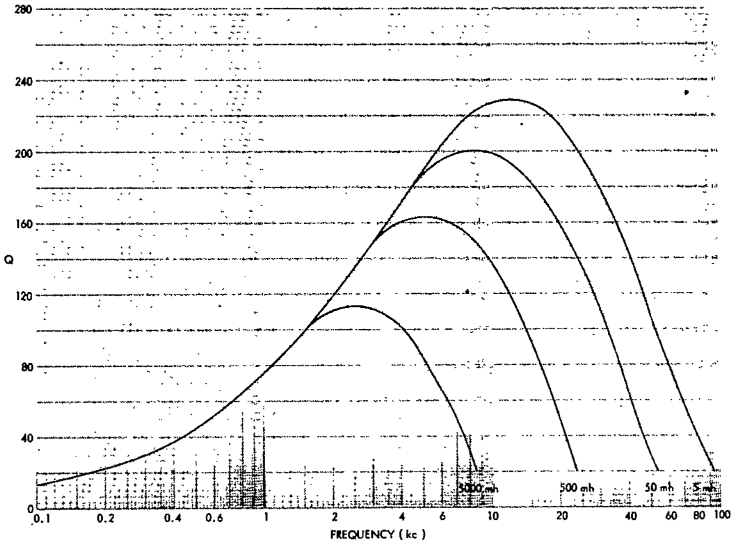

Figure 5—Curves of Q vs. frequency for core 55548; μ = 125,
A = 0.655 cm², l = 8.1 cm, W = 590,000 cir mil.

Table 4

Variation of Inductance with Temperature

Core Identification (commercial designation)	Inductance Tolerance vs. Temperature
A2	Not specified
B4	±0.1 percent from 13° to 35°C
D4	±0.1 percent from 0° to 55°C
E4	±0.15 percent from 0° to 55°C
W4	±0.25 percent from -55° to +85°C

where

R_{dc} = the dc resistance of the inductor in ohms,

K_1 = the average length of wire per turn in ft (Table 1),

K_2 = the resistance per unit length of the wire in ohms/ft (Table 2).

10. Using Equation 4, compute the maximum magnetizing force $\left(H_{max}\right)$ for peak current. Refer to a curve similar to Figure 1 and determine the maximum change in inductance due to the maximum magnetizing force.

EXAMPLE

A toroidal inductor will be considered which has an inductance of 5 mh with a peak current of 0.55 amp, an rms current of 0.5 amp, and a maximum frequency of 10 kc. From the previous section, the following steps are performed:

1. Table 1 may be used in this example.

2. From Table 2, select No. 23 AWG wire for which $A_w = 620$ cir mil.

3. Solve $LA_w^2 = 5 \times 10^{-3} (620)^2 = 1920$ henries (cir mils)^2

4. Solve $LI_p^2 = 5 \times 10^{-3} (0.55)^2 = 1.51 \times 10^{-3}$ henries $\left(\text{amps}^2\right)$

5. From Table 3, permeabilities of 60 and 125 are considered suitable.

6. Since a wide temperature range is desired, the W4 cores from Table 4 should be used.

7. Two cores of the same size in Table 1 meet the requirements of steps 3, 4, 5, and 6—cores 55071 and 55548. The curves of Q vs. frequency for a 5 mh inductance using these two cores are shown in Figures 3 and 5. These curves indicate a high Q well beyond the maximum frequency for this inductor. The highest permeability core (55548) is selected to permit the use of large wire having a low dc resistance.

8. From Equation 1

$$N^2 = \frac{Ll\ 10^8}{0.4\pi\mu A} = \frac{5 \times 10^{-3}\,(8.10)\,10^8}{0.4\pi\,(125)\,(0.655)}$$

$$= 3.94 \times 10^4$$

$$N = 198 \text{ turns.}$$

9. The largest wire which can be used can be determined from Equation 2,

$$A_{w,\max} = \frac{K_w W}{N} = \frac{0.4\,(590,000)}{198}$$

$$= 1190 \text{ cir mil.}$$

From Table 2, No. 20 wire can be used. The dc resistance is given by

$$R_{dc} = K_1 N K_2 = (0.146)\,(198)\,(0.01013)$$

$$= 0.292 \text{ ohm.}$$

10. The magnetizing force at peak current is given by Equation 4:

$$H = \frac{0.4\pi N I_p}{l} = \frac{0.4\pi\,(198)\,(0.55)}{8.10}$$

$$= 16.9 \text{ oersteds.}$$

Reference to Figure 1 reveals the inductance tolerance to be 9 percent.

The design results for cores 55071 and 55548 are shown in Table 5. This table reveals that the high permeability core yields the lowest number of turns of wire, which allows larger wire to be used and in turn yields a considerably lower dc resistance. The high permeability core also gives a higher Q at maximum frequency and a greater inductance change.

Table 5

Inductor Design Using Cores of the Same Size.

Core Number	55071	55548
Permeability	60	125
N (turns)	286	198
Q at 10 kc	210	230
Wire Size	No. 22	No. 20
dc Resistance (ohm)	0.675	0.292
Inductance Tolerance (percent)	5	9

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>
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