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AN ALGORITHM FOR THE COORDINATES OF VINTI'S ZONAL HARMONIC PERTURBATIONS OF AN ACCURATE REFERENCE ORBIT

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SUMMARY

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The procedure developed in an earlier paper for the computation of coordinates for Vinti's accurate reference orbit is slightly extended to include his treatment of the zonal harmonic perturbations of the reference orbit. Herein is discussed the method for obtaining the position and velocity coordinates due to the more important part of the residual potential.

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INTRODUCTION

Vinti (Ref. 1) has found a gravitational potential for an axially symmetric planet, given by the expression

$$V(\rho, \eta) = \frac{-\mu\rho}{\rho^2 + c^2\eta^2},$$

which accounts for more than 99.5 percent of the deviation of the Earth's potential from spherical symmetry. A method for correcting for the effects of any of the zonal harmonics of the residual potential has also been devised (Ref. 2). This includes the residual fourth harmonic with coefficient $J_4 + J_2^2$, and the third harmonic with coefficient J_3. The latter gives rise only to short periodic and long periodic effects without singularities. Because of the work of Kaula 1962, and King-Hele, Cook and Rees 1963, giving values of $J_4 + J_2^2$ for the earth ranging from $-(0.9)$ 10^{-6} to $(0.4)10^{-6}$, it has been decided to accept the value $J_4 + J_2^2 = 0$ given by Vinti for purposes of orbit computation.

ALGORITHM

1. δx, δy, and δz

From Ref. 1, we have

$$n = \mu^{\frac{1}{2}} a^{-\frac{3}{2}}$$

$$(v - \ell) = v_0$$

$$g_0 = \psi_s - M_s + 2\pi(\nu_1 - \nu_2) t$$

$$g = \psi_s - M_s$$

$$E = M_s + E_0 + E_1 + E_2, \text{ and}$$

$$v = M_s + v_0 + v_1 + v_2 .$$

From Ref. 2, we then compute;

the third harmonic short periodic quantities:

$$L_3, G_3, \ell_3, g_3 \text{ and } h_3;$$

the third harmonic long periodic quantities:

$$\tilde{G}_3, \tilde{\ell}_3, \tilde{g}_3, \text{ and } \tilde{h}_3.$$

With $\sigma_4 = J_4 + J_2^2 = 0$, then the variations in the Delaunay variables are given by:

$$\delta L = J_3 L_3, \quad \delta H = 0$$

$$\delta G = J_3 G_3 + \frac{J_3}{J_2}\tilde{G}_3$$

$$\delta\ell = J_3 \ell_3 + \frac{J_3}{J_2}\tilde{\ell}_3$$

$$\delta g = J_3 g_3 + \frac{J_3}{J_2}\tilde{g}_3$$

and

$$\delta h = J_3 h_3 + \frac{J_3}{J_2} \tilde{h}_3 .$$

The variations of the elements a, e, and η_0 are then,

$$\delta a = \frac{2}{an} \delta L$$

$$\delta e = \frac{pn}{\mu e} \delta L - (ae)^{-1} \left(\frac{p}{\mu}\right)^{\frac{1}{2}} \delta G, \text{ and}$$

$$\delta \eta_0 = \frac{1 - \eta_0^2}{\mu \eta_0 p} \delta G$$

The variations in the uniformising variables E, v, and ψ are;

$$\delta E = (\rho/a) \, (\delta \ell + \sin E \, \delta e)$$

$$\delta v = (1 - e^2)^{1/2} \, (\rho/a) \, \left[\delta E + (1 - e^2)^{-1} \sin E \, \delta e\right],$$

and

$$\delta \psi = \delta v + \delta g$$

The variations in the spheroidal coordinates ρ, η, and ϕ are;

$$\delta \rho = (\rho/a) \, \delta a - a \cos E \, \delta e + ae \sin E \, \delta E$$

$$\delta \eta = (\eta/\eta_0) \, \delta \eta_0 + \eta_0 \cos \psi \, \delta \psi,$$

and

$$\delta\phi = \delta h + \left(1 - \eta_0^2 \sin^2 \psi\right)^{-1} \left(1 - \eta_0^2\right)^{1/2} \left[\delta\psi - \frac{1}{2} (\mu p)^{-1/2} \sin 2\psi \, \delta G\right]$$

Then

$$\rho_{total} = \rho_{reference} + \delta\rho$$

$$\eta_{total} = \eta_{reference} + \delta\eta$$

and

$$\phi_{total} = \phi_{reference} + \delta\phi$$

Also;

$$x_{total} = \sqrt{\left(\rho^2_{total} + c^2\right) \left(1 - \eta^2_{total}\right)} \cos \phi_{total}$$

$$y_{total} = \sqrt{\left(\rho^2_{total} + c^2\right) \left(1 - \eta^2_{total}\right)} \sin \phi_{total}$$

and

$$z_{total} = \rho_{total} \, \eta_{total}$$

2. $\delta\dot{x}$, $\delta\dot{y}$, and $\delta\dot{z}$

Since, as explained in Ref. 2, secular terms are taken through order $J_2^2 = J_3$, and long periodic terms through order $J_3/J_2 = J_2$; then, in the determination of $\delta\dot{x}$, $\delta\dot{y}$, and $\delta\dot{z}$, all terms involving a variation multiplied by c^2 are ignored.

From the expressions;

$$\dot{\rho} = \frac{P_\rho}{h_\rho^2}$$

where

$$h_\rho^2 = \frac{\rho^2 + c^2\eta^2}{\rho^2 + c^2}$$

and

$$P_\rho = \frac{ae\sqrt{-2\alpha_1 \ (\rho^2 + A\rho + B)} \ \sin E}{(\rho^2 + c^2)}$$

We have,

$$\delta h_\rho = \frac{\rho\delta\rho \ \left[(\rho^2 + c^2) - (\rho^2 + c^2\eta^2)\right] + c^2\eta\delta\eta \ (\rho^2 + c^2)}{(\rho^2 + c^2\eta^2)^{1/2} \ (\rho^2 + c^2)^{3/2}} = 0$$

Also,

$$\delta P_\rho = P_\rho \left\{\frac{\delta a}{a} + \frac{\delta e}{e} + \cot \ E \, \delta E + \frac{\delta\alpha_1}{2\alpha_1}\right.$$

$$+ \frac{1}{2(\rho^2 + A\rho + B)} \ \left[(2\rho + A) \ \delta\rho + \rho\delta A + \delta B\right]$$

$$\left. - \frac{2\rho\delta\rho}{(\rho^2 + c^2)}\right\}$$

where

$$\delta\alpha_1 = \frac{\mu}{2} \ \frac{(\delta a + \delta b_1)}{(a + b_1)^2}$$

and

$$\delta b_1 = 0, \text{ and consequently, } \delta A = 0 .$$

Now,

$$\delta D = (ap - c^2)\ \left(a\delta p + p\delta a - 2c^2 \eta_0 \delta \eta_0\right)$$

$$+ \left(ap - c^2 \eta_0^2\right)\ (a\delta p + p\delta a) + 8c^2\ \left(a^2\eta_0 \delta\eta_0 + \eta_0^2\, a\delta a\right)$$

$$= 2ap\ (a\delta p + p\delta a)$$

and

$$\delta p = (1 - e^2)\ \delta a - 2ae\ \delta e$$

Also,

$$\delta B = \delta \left(c^2\eta_0^2 \frac{D'}{D}\right) = 0$$

and

$$\delta D' = 8ac^2\ \left[\delta a\ \left(1 - \eta_0^2\right) - a\, \eta_0\, \delta\eta_0\right] + \delta D = \delta D$$

Therefore,

$$\delta P_\rho = P_\rho \left\{\frac{\delta a}{a} + \frac{\delta e}{e} + \cot\ E\ \delta E + \frac{\delta \alpha_1}{2\alpha_1}\right.$$

$$\left. + \left(2\rho \left[\frac{1}{2(\rho^2 + A\rho + B)} - \frac{1}{(\rho^2 + c^2)}\right] + \frac{A}{2(\rho^2 + A\rho + B)}\right) \delta\rho\right\}$$

and then,

$$\delta\dot{\rho} = \frac{\delta P_\rho}{h_\rho^2}$$

Now

$$\delta\dot{\eta} = \delta \left(\frac{P_\eta}{h_\eta^2}\right) = \frac{h_\eta\, \delta P_\eta - 2\, P_\eta\, \delta h_\eta}{h_\eta^3}$$

Here,

$$\delta h_\eta = \frac{2\rho\,(1-\eta^2)\;\delta\rho + 2\rho^2\eta\,\delta\eta}{(1-\eta^2)^2}$$

and,

$$\delta P_\eta = P_\eta \left\{ \frac{\delta\eta_0}{\eta_0} - \mathrm{Tan}\;\psi\delta\psi + \frac{\delta\alpha_1}{2\alpha_1} + \frac{(\eta_2\delta\eta_2 - \eta\delta\eta)}{\left(\eta_2^2 - \eta^2\right)} \right.$$

$$\left. + \frac{2\eta\,\delta\,\eta}{(1-\eta^2)} \right\}$$

Since,

$$\eta_2^{-2} = \frac{\alpha_2^2 - 2\alpha_1 c^2}{2\left(\alpha_2^2 - \alpha_3^2\right)} \left\{ 1 - \left[1 + \frac{8\alpha_1 c^2\left(\alpha_2^2 - \alpha_3^2\right)}{\left(\alpha_2^2 - 2\alpha_1 c^2\right)^2} \right]^{1/2} \right\}$$

then,

$$\delta\eta_2 = 0,$$

and

$$\delta P_\eta = P_\eta \left\{ \frac{\delta\eta_0}{\eta_0} - \mathrm{Tan}\;\psi\delta\psi + \frac{\delta\alpha_1}{\alpha_1} + \left(\frac{2}{(1-\eta^2)} - \frac{1}{\left(\eta_2^2 - \eta^2\right)} \right)\eta\delta\eta \right\}$$

Also,

$$\delta\,\dot{\phi} = \delta\left(\frac{P_\phi}{h_\phi^2}\right) = \frac{-2\alpha_3\,\delta h_\phi}{h_\phi^3}$$

where,

$$\delta h_\phi = \frac{1}{h_\phi}\left\{ (1-\eta^2)\;\rho\delta\rho - (\rho^2 + c^2)\;\;\eta\delta\eta \right\}$$

Using the expressions,

$$x = \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \cos \phi$$

$$y = \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \sin \phi$$

and

$$z = \rho\eta$$

We then compute;

$$\delta x = \frac{\cos \phi}{\sqrt{(\rho^2 + c^2)(1 - \eta^2)}} \left((1 - \eta^2)\, \rho\delta\rho - (\rho^2 + c^2)\, \eta\delta\eta\right)$$

$$- \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \sin \phi\delta\phi$$

$$\delta y = \frac{\sin \phi}{\sqrt{(\rho^2 + c^2)(1 - \eta^2)}} \left((1 - \eta^2)\, \rho\delta\rho - (\rho^2 + c^2)\, \eta\delta\eta\right)$$

$$+ \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \cos \phi\delta\phi$$

and

$$\delta z = \rho\delta\eta + \eta\delta\rho$$

Then, from the quantities,

$$\dot{x} = x\left(\frac{\rho\dot{\rho}}{\rho^2 + c^2} - \frac{\eta\dot{\eta}}{1 - \eta^2}\right) - \frac{y\alpha_3}{h_\phi^2}$$

$$\dot{y} = y\left(\frac{\rho\dot{\rho}}{\rho^2 + c^2} - \frac{\eta\dot{\eta}}{1 - \eta^2}\right) - \frac{x\alpha_3}{h_\phi^2}$$

and

$$\dot{z} = \rho\dot{\eta} + \eta\dot{\rho},$$

$$\delta\dot{x} = \left(\frac{\rho\dot{\rho}}{\rho^2 + c^2} - \frac{\eta\dot{\eta}}{1 - \eta^2}\right)\delta x$$

$$+ x\left\{\frac{(\rho^2 + c^2)(\rho\delta\dot{\rho} + \dot{\rho}\delta\rho) - 2\rho^2\dot{\rho}\delta\rho}{(\rho^2 + c^2)^2}\right.$$

$$\left. - \frac{(1 - \eta^2)(\eta\delta\dot{\eta} + \dot{\eta}\delta\eta) - 2\eta^2\dot{\eta}\delta\eta}{(1 - \eta^2)^2}\right\}$$

$$- \frac{h_\phi(\alpha_3\delta y + y\delta\alpha_3) + 2y\alpha_3\delta h_\phi}{h_\phi^3}$$

$$\delta\dot{y} = \left(\frac{\rho\dot{\rho}}{\rho^2 + c^2} - \frac{\eta\dot{\eta}}{1 - \eta^2}\right)\delta y$$

$$+ y\left\{\frac{(\rho^2 + c^2)(\rho\delta\dot{\rho} + \dot{\rho}\delta\rho) - 2\rho^2\dot{\rho}\delta\rho}{(\rho^2 + c^2)^2}\right.$$

$$\left. - \frac{(1 - \eta^2)(\eta\delta\dot{\eta} + \dot{\eta}\delta\eta) - 2\eta^2\dot{\eta}\delta\eta}{(1 - \eta^2)^2}\right\}$$

$$- \frac{h_\phi(\alpha_3\delta x + x\delta\alpha_3) + 2x\alpha_3\delta h_\phi}{h_\phi^3}$$

and,

$$\delta\dot{z} = \rho\delta\dot{\eta} + \dot{\eta}\delta\rho + \eta\delta\dot{\rho} + \dot{\rho}\delta\eta$$

From these, we can write;

$$\dot{x}_{total} = \dot{x}_{reference} + \delta\dot{x}$$

$$\dot{y}_{total} = \dot{y}_{reference} + \delta\dot{y}$$

and

$$\dot{z}_{total} = \dot{z}_{reference} + \delta\dot{z}$$

REMARKS

This algorithm constitutes an addition to the orbit generator (Ref. 3) of the Vinti program. It is brought into use only after the Izsak elements have been determined by comparison with observation.

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