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MEASUREMENT OF ERRORS IN MANUALLY RECORDING TRANSIT TIMES OF STARS AND DISTANT PLANETS

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SUMMARY

An experimental investigation has been conducted to determine the accuracy with which celestial-body-transit times can be manually recorded. This investigation determined the effects on the timing-error variance of both celestial-body size and the apparent rate of motion of the celestial body through the observer's field of view. The celestial bodies simulated were Jupiter (as seen through a 12-power telescope of 2-inch aperture) and a first-magnitude star (as seen through a 60-power telescope of 2-inch aperture). The apparent rates of motion simulated varied between 56° and 224° per minute. Also an observer's ability to measure the time interval between successive body transits was determined.

Results of the investigation showed that the standard deviation in the timing error varied approximately inversely as the apparent rate of motion. Standard deviations were slightly less for the star than for the planet. Also the error variance in recording a time interval between body transits was approximately equal to the error variance in recording a single-body transit. From the data an empirical graph was derived which shows expected error in measuring the angle between two celestial bodies plotted against actual rate of rotation for various values of instrument magnification.

INTRODUCTION

Methods which make use of rotation have been proposed for navigation of spacecraft (for example, ref. 1). These methods fall into two general categories: the observer and his spacecraft may rotate with respect to some inertial axis, as in the case of a rotating manned space station, or an optical navigation device may be rotated within a nonrotating vehicle.

These navigation schemes would use the transit times of celestial bodies over a light-sensing device to measure space angles. In principle, such navigation measurements would be extremely accurate since they depend primarily upon accurate measurements of time intervals. However, when these transit times are manually recorded, some error is introduced into the system which is due to the individual observer. This problem of individual errors in manually recording transit times was first recognized by astronomers such as Bessel in the latter part of the eighteenth century. It was noticed then that various observers recorded different values of meridian transit times for the same star. These individual errors were called "personal equation."

Were this individual error to remain constant during any period of transit recordings, there would be no problem with respect to space navigation as it is the difference between transit times which determines the angle between two bodies. However, it is to be expected that this error will vary from observation to observation in some random fashion, and that this variance will be, in part, a function of the apparent velocity of the celestial body being observed. The amount of variance must be small compared with the period of rotation in order to give good accuracies in the angle measurements.

The work of astronomers in determining observation errors has been concerned mainly with the mean error while very little work has been reported concerning the error variance (refs. 2 and 3). Therefore, an experimental study was made to determine variance in personal equation and also to determine what parameters might cause this variance to change.

The first series of experiments to be reported investigated the abilities of observers to mark the time of passage of a single simulated celestial body across the vertical hairline of the reticle of a telescope. Rotation of the telescope was simulated by a rotating prism, and a photoelectric device was used to measure the exact time of transit.

The error made in timing the interval between two successive celestial-body transits may be greater or less than the error in timing a single-body transit, depending on the amount of correlation between the two individual timing errors. Accordingly, tests were made to determine the ability of an observer to measure such incremental transit times.

A theory was developed to explain the trend of the experimental results. In part, this theory indicated that the variance in the personal equation might approach the variance in neuromuscular response time as the apparent rate of motion of a transiting body increased. Experiments were therefore performed to determine the neuromuscular response time for each of the observers who participated in the present investigation.

Two different celestial bodies were simulated in the experiments to determine the effect of target body size on the timing-error variance. The larger of the two sources most nearly simulated the appearance of the planet Jupiter, at its nearest approach to the Earth, as it would be seen through a 12-power telescope of about 2-inch aperture. The smaller source was a reasonably good simulation of a first-magnitude star as seen through a telescope of 2.2-inch aperture with 60-power magnification.

The test subjects were engineering personnel with no previous experience at the particular tasks involved in these experiments.

SYMBOLS

M	magnification of observing instrument
r	mean value of timing errors
r_n	mean value of neuromuscular lag time
r_t	mean value of timing errors for single celestial-body transits
T	interval of time between celestial-body transits
Δt	timing error
Δt_i	timing error in judging reaction time
Δt_m	timing error due to error in judging angular separation between celestial body and vertical hairline
Δt_t	total timing error, $\Delta t_i + \Delta t_m$
ΔU	error in judging angular separation between celestial body and vertical hairline
W	actual rate of rotation
ρ	statistical coefficient of correlation
σ	standard deviation
σ_I	standard deviation of error in observed time interval
σ_i	standard deviation of timing error in judging reaction time
σ_n	standard deviation of neuromuscular response time
σ_t	standard deviation of total timing error
σ_u	standard deviation of angular error in centering celestial body on vertical crosshair
σ_θ	standard deviation of angular error in a single celestial-body transit
$\sigma_{\Delta\theta}$	standard deviation of error in the angle measured between two celestial bodies
φ	viewing angle, measured between axis of rotation and line of sight

ω apparent rate of rotation

Subscripts:

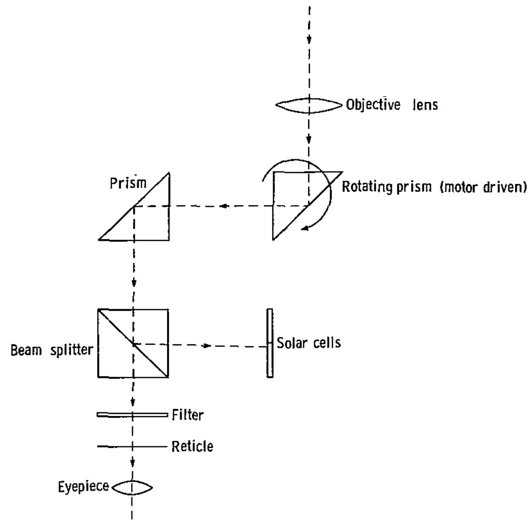
1 first celestial-body transit

2 second celestial-body transit

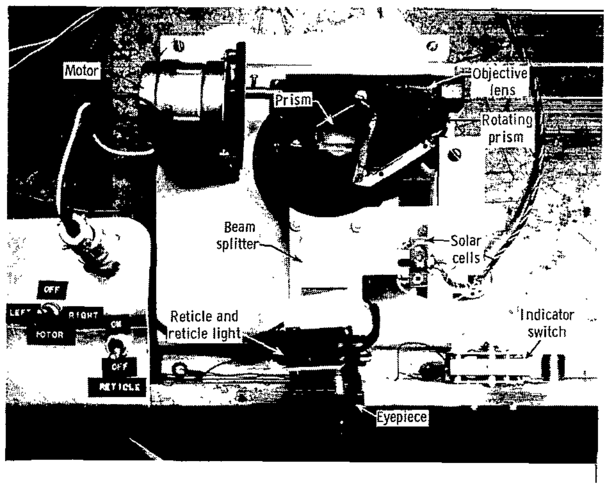
j,n integers

APPARATUS FOR CELESTIAL-BODY TRANSITS

The apparatus shown in figure 1 was used in the experiments to determine variance in an observer's ability to mark the times of transit of celestial bodies at various rates of motion. The operation of this device may be described as follows: The objective lens focused an image of a simulated celestial body through the system of prisms and filters onto the plane of the reticle, where it would be viewed by an eyepiece. A portion of the unfiltered light was diverted by the beam-splitting prism to form an image on the photoelectric sensing device. This device was constructed of two rectangular silicon photovoltaic cells placed side by side with their line of separation parallel to the vertical hairline of the reticle. With the celestial-body image centered statically on the vertical hairline, the photocell mount was adjusted until the indicated difference between the outputs of the two photocells was zero, whereupon the mount was clamped in place. To perform a test, the motor-driven prism was rotated to cause the celestial-body image to traverse the field of view. The observer closed a switch at the instant he observed the star transit.



(a) Diagram of apparatus.



(b) Apparatus. L-63-3966.1

Figure 1.- Apparatus used in tests.

Two methods were used for recording the data. In the first method, an oscillograph recorded the closing of the observer's switch and the photocell output on adjacent channels. Figure 2 shows samples of the records taken. It is evident that the timing error is readily measured if the chart speed is known. The actual speed was 4.6 cm per second.

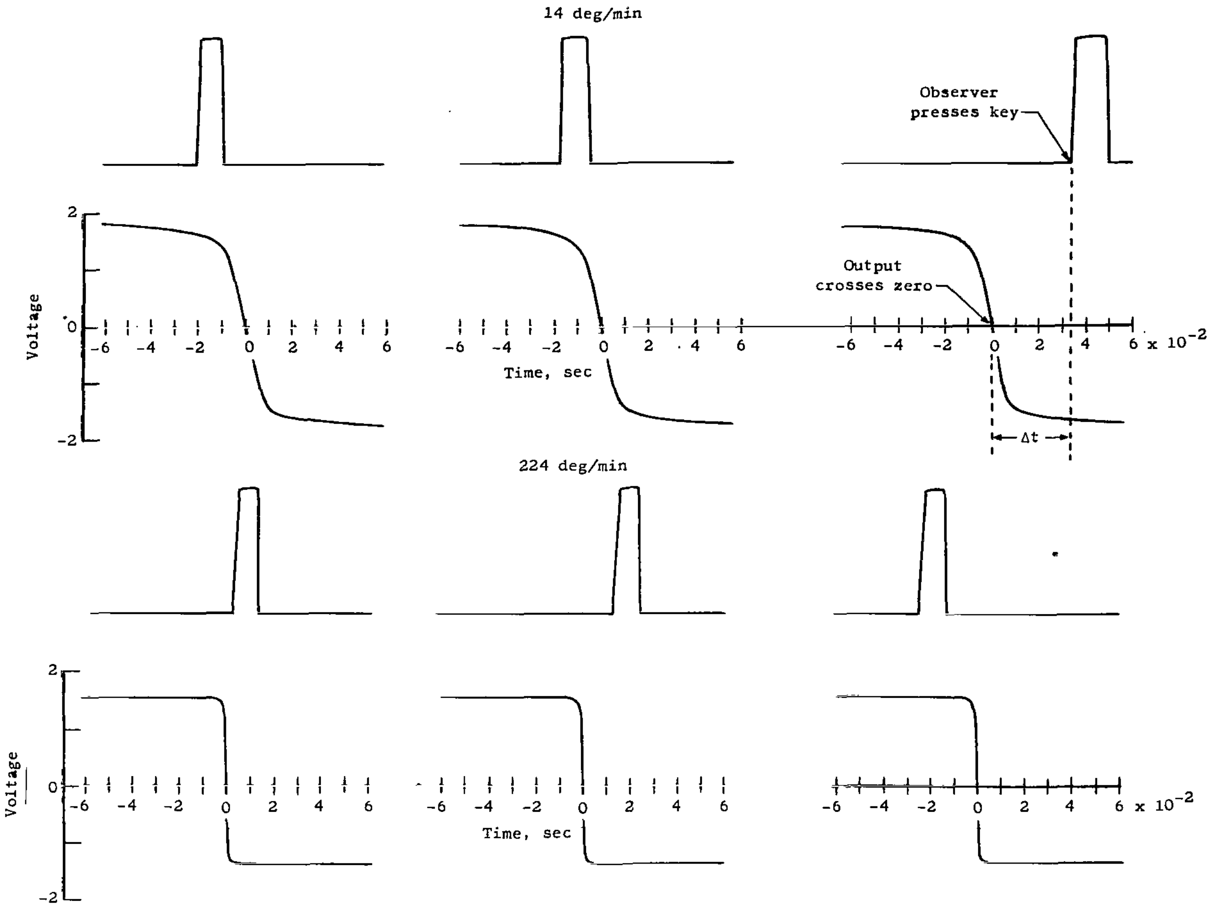


Figure 2.- Data from chart recorder.

The second method of recording data made use of an electronic interval timer. The differential photocell output started the counter as it passed through null, and the observer's switch stopped the counter after a delay of 0.45 second, furnished by a time-delay circuit. This delay was required in the observer's switching circuit to prevent the stopping signal from preceding the starting signal.

Two different celestial bodies were simulated by zirconium arc lamps of 25- and 100-watt ratings placed 12 feet from the objective lens. The nominal characteristics of the arc lamps are given in table I. More detailed characteristics are found in reference 4.

The filter used to reduce the apparent brightness of the lamp consisted of seven layers of neutral-density gelatin filter material; each layer had a density of 0.75. $\left(\text{Density} = \log_{10} \frac{\text{Incident flux}}{\text{Transmitted flux}}.\right)$ Total density was thus 5.25, and the light transmitted was 5.62×10^{-6} times the incident light. By taking into account this filter and allowing a reduction factor of 100 for the lens (relative aperture about f/8) and a further loss of 2/3 in the beam-splitting prism, the brightness of the observed image was calculated to 0.12 footlambert for the 25-watt lamp and 0.22 footlambert for the 100-watt lamp, based on the values of 21 candles per sq mm and 39 candles per sq mm specified for the lamps.

The image of the 100-watt lamp was about 0.0032 inch in diameter (calculated) and subtended a visual angle of about 10 arc minutes. This image was a reasonable simulation of the appearance of Jupiter at its nearest approach to the Earth, as seen through a 12-power telescope of about 2-inch aperture. The planet Mars would have about the same appearance at its nearest approach (in 1963) if viewed through a 45-power telescope of slightly greater aperture.

The image of the 25-watt lamp subtended a visual angle of about 5 arc minutes. This angle simulates the appearance of a star as viewed in a diffraction-limited telescope of magnification 27 times its objective diameter in inches. Magnification as high as 40 per inch are commonly used by astronomers (ref. 5), which would give a visual angle of 7.4 arc minutes. The amount of light contained in the image of the small source was about that which would be collected from a first-magnitude star by a telescope of 2.2-inch aperture.

(It may be of interest to note that the average typewritten period subtends an angle of 5 arc minutes at a distance of 18 inches.)

TEST PROCEDURES

Single Celestial-Body Transit Tests

Tests were run to determine an observer's timing-error variance in manually recording a single celestial-body transit for two different celestial bodies and at four different apparent rates of motion. In terms of the inertial rotation rate of a space vehicle W, the magnification M of the observing instrument, and the angle φ between the line of sight and the axis of rotation, the apparent rate of rotation ω is defined as

$$\omega = MW \sin \varphi \tag{1}$$

For these tests the instrument magnification was 7× and φ was held constant at 90°. The apparent rates produced were 14°, 56°, 112°, and 224° per minute.

Each of the eight test subjects repeated the task of manually recording the celestial-body transit time until he became fatigued or the time available ran out.

Measurement of Time Intervals Between Body Transits

In this series of tests the observer's task was to measure the time interval between two successive planet transits. To do this, the observer was required to record separately each successive body-transit time. The three observers, who took part in the test, were instructed to regard the transit-time recordings of the two planets as entirely separate events. That is, they were not to allow their accuracy in recording the transit of the first planet to influence their recording of the transit of the second planet.

The larger celestial body (100-watt lamp) was used in these tests in conjunction with time-between-transit intervals of 6 and 22 seconds along with apparent rotation rates of 56° and 224° per minute.

Neuromuscular Response Time

In this test the observer's task was to close a switch as soon as he saw a light. The operator closed a silent switch which simultaneously activated both the electronic counter and the light which the observer was watching. The closing of the observer's switch stopped the time-interval counting of the electronic counter. Thus the resulting time interval indicated by the counter represented the observer's neuromuscular response time. From 50 observations per observer, the variance in neuromuscular response time for each observer was computed.

RESULTS AND DISCUSSION

Results of the experimental tests are presented in figure 3 and tables II and III. The variance σ^2 is given by

$$\sigma^2 = \frac{1}{n - 1} \sum_{j=1}^{n} \left(\Delta t_j - r\right)^2 \tag{2}$$

where the sample consists of n timing errors or lag times Δt_1, Δt_2, . . ., Δt_n, the mean of which is r (ref. 6). The standard deviation σ is the square root of the sample variance.

Single-Transit Observations

Three typical plots of standard deviation of the error in timing a single celestial-body transit as a function of apparent angular rate of the simulated body are shown in figure 3. These plots all show a decrease of standard deviation, as apparent rate increases (indicating an inverse relationship) until some minimum is approached. The minimum is in all cases somewhat greater than the standard deviation σ_n of the neuromuscular response time. Somewhat better accuracy was obtained with the smaller of the two simulated celestial bodies.

The mean transit-timing error, listed in table II, is smaller than the neuromuscular-response time in most cases. This fact is an indication that the observer was anticipating and was attempting to make the visual stimulus of the transit coincide with the audible closing of the switch. Such anticipation requires a judgment of response time, a judgment of rate of motion of the

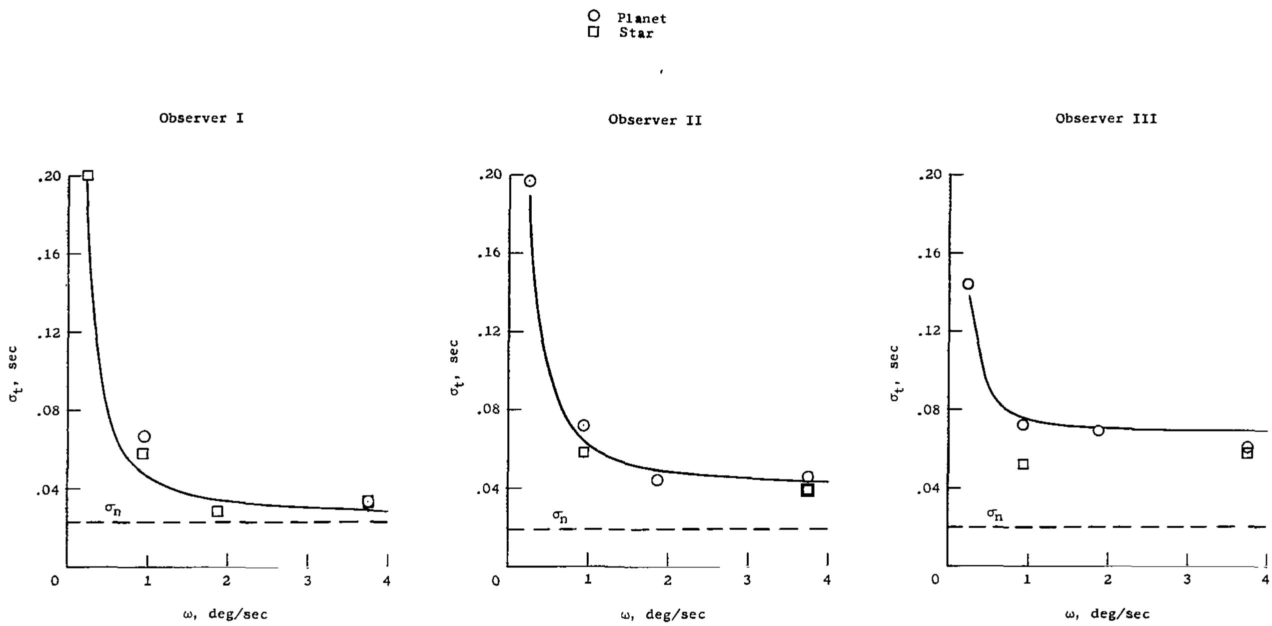


Figure 3.- Standard deviation in timing error plotted against apparent rate of celestial-body travel with empirical curves fitted by the method of least squares.

celestial body, and a judgment of its instantaneous position relative to the hairline. In other words, the observer estimates the relative separation which must exist between body and hairline at the instant he begins to close the switch in order for the actual transit and the closing of the switch to coincide.

If the observer's error in judging the angular position U is expressed as ΔU, its resulting contribution Δt_m to the timing error will be inversely proportional to the apparent angular rate ω of the celestial body. Thus,

$$\Delta t_m = \frac{\Delta U}{\omega} \tag{3}$$

If the error in judging reaction time is denoted by Δt_i, the total timing error for any given observation will be

$$\Delta t_t = \Delta t_i + \Delta t_m \tag{4}$$

or

$$\Delta t_t = \Delta t_i + \frac{\Delta U}{\omega} \tag{5}$$

If Δt_i and ΔU are independent random variables with standard deviations σ_i and σ_u, respectively, the variance of Δt_t will be given by

$$\sigma_t^2 = \sigma_i^2 + \frac{\sigma_u^2}{\omega^2} \tag{6}$$

Equation (6), which is one form of the equation of a hyperbola, was fitted to the experimental data in order to determine empirically the values of σ_i and σ_u for each observer. The fitted curves for the first three observers are shown in figure 3. Values of σ_i and σ_u for all observers are listed in table II. (Curves were only fitted when sufficient data were available to allow use of the method of least squares.) In all cases, σ_u was less than the calculated visual angle subtended by the simulated celestial body. The value of σ_i approached the value of σ_n for observer I, but for the other observers σ_i was more than twice σ_n.

Measurements of Intervals Between Transits

Each measurement of an interval between two consecutive transits yielded three errors: the error in each transit and the error in the interval, which is the difference of the two transit errors. The corresponding standard deviations were calculated from the individual errors and are listed in table III as $\sigma_{t,1}$, $\sigma_{t,2}$, and σ_I. In most cases, the standard deviation σ_I of the interval measurements is no greater than the standard deviations of the individual transit measurements σ_t and in some cases it is less.

This result indicates some degree of correlation, in the statistical sense, between consecutive transit measurements. The variance of the difference between two uncorrelated random variables is the sum of the individual variances. When there is a correlation between the variables, the relationship between the variances is

$$\sigma_I^2 = \sigma_{t,1}^2 + \sigma_{t,2}^2 - 2\rho\sigma_{t,1}\sigma_{t,2} \tag{7}$$

where ρ is the coefficient of correlation. Values of ρ calculated from the experimental results ranged from 0.307 to 0.999, as shown in table III.

Angular Errors

The results of the tests described may be used to predict the expected performance of an observer in measuring angles between celestial bodies from a rotating space vehicle.

Substitution of equation (1) into equation (6) gives

$$\sigma_t^2 = \sigma_i^2 + \frac{\sigma_u^2}{M^2 W^2 \sin^2\varphi} \tag{8}$$

The variance in angular error corresponding to σ_t^2 is simply obtained by multiplying equation (8) by the square of the true rate of rotation W.

Thus

$$\sigma_\theta^2 = W^2\left(\sigma_i^2 + \frac{\sigma_u^2}{M^2 W^2 \sin^2\varphi}\right) \tag{9}$$

$$\sigma_\theta^2 = W^2\sigma_i^2 + \frac{\sigma_u^2}{M^2 \sin^2\varphi} \tag{10}$$

Equation (10) is the variance in angular error in recording single celestial-body transits.

If the errors in timing successive star transits are assumed to be uncorrelated ($\rho = 0$), the standard deviation of the measured angle between the two bodies is

$$\sigma_{\Delta\theta} = \sqrt{2}\sigma_\theta \tag{11}$$

Figure 4 is a plot of the angular standard deviation $\sigma_{\Delta\theta}$ as a function of W for various constant values of magnification M. The empirical values of σ_i and σ_u for observer II observing the planet were used, and the viewing angle φ was fixed at 90°. For large values of magnification, $\sigma_{\Delta\theta}$ varies approximately linearly with the actual rate of rotation. The values of $\sigma_{\Delta\theta}$ in figure 4 may be considered conservative on the basis of the results in table III. It is important to note here that the apparent rate of rotation ω must always be kept less than 10 deg/sec, since faster rates will cause the star image to blur (ref. 7).

Limitations of Investigation

The results reported herein do not completely define the performance of a navigation system based on visual timing of transits from a rotating vehicle. As an example, the navigation measurements would necessarily include planets with considerably larger visual angles of subtense than those of the larger light source used in the investigation. Furthermore, it would be necessary in certain navigation procedures to keep the observed star or planet centered on a horizontal hairline while timing its transit in order to be able to determine the angle between the line of sight and the axis of rotation. The effects of this added task were not investigated. An additional factor that would enter

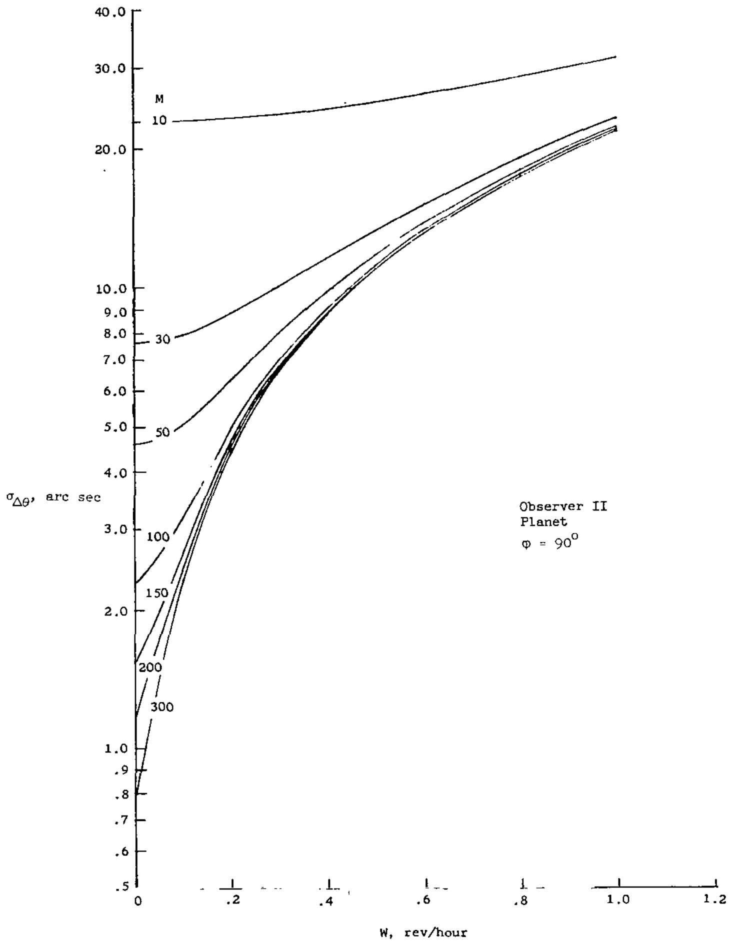


Figure 4.- Standard deviation in measuring space angles plotted against actual rate of vehicle rotation.

into the design of a navigation system is the constancy of the rate of rotation of the vehicle (or rate of rotation of the instrument inside the vehicle), an analysis of which is beyond the scope of this report.

CONCLUSIONS

Several general observations can be made from the results of the experimental investigation.

(1) The standard deviation of the transit timing error in manually recording celestial-body transits is approximately inversely proportional to the apparent rate of rotation.

(2) The error variance in manually recording the time interval between two successive body transits is approximately equal to the error variance in recording a single-body transit.

(3) For large values of magnification, the standard deviation of the error in measuring angles between two celestial bodies increases approximately linearly with actual rate of rotation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 7, 1964.

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TABLE I

SPECIFICATIONS OF CELESTIAL-BODY SIMULATORS

	Light source 1	Light source 2
Zirconium-arc lamp, watts	100	25
Diameter of light, in.	0.059	0.029
Candle power (nominal)	100	16
Brilliance, $\frac{\text{candles}}{\text{in.}^2}$	24,500	22,500
Apparent diameter, minutes of arc . .	10	5

TABLE II.- GENERAL SUMMARY OF DATA PERTAINING TO SINGLE-BODY TRANSITS

Observer	ω, deg/min	Celestial body	σ_t, sec	r_t, sec (a)	Number of observations for σ_t and r_t	σ_i, sec	σ_u, deg	σ_n, sec	r_n, sec	Number of observations for σ_n and r_n
I	224	1	0.032	-0.029	37			0.023	0.218	50
I	56	1	.067	-.061	25					
I	14	1	.020	-.231	43					
I	224	2	0.033	+0.029	40	0.027	0.038			
I	112	2	.028	+.041	30					
I	56	2	.058	+.010	25					
II	224	1	0.046	+0.073	32	0.043	0.045	0.019	0.203	50
II	112	1	.044	+.070	34					
II	56	1	.072	-.064	57					
II	14	1	.197	-.292	42					
II	224	2	0.039	-0.082	40					
II	56	2	.058	+.010	40					
III	224	1	0.060	+0.056	31	0.068	0.030	0.020	0.213	50
III	112	1	.069	-.022	42					
III	56	1	.072	-.064	57					
III	14	1	.144	+.254	42					
III	224	2	0.058	+0.008	45					
III	56	2	.052	-.040	38					
IV	224	1	0.045	+0.022	45	0.050	0.038	0.021	0.219	57
IV	112	1	.050	+.032	36					
IV	56	1	.073	-.081	73					
IV	14	1	.171	+.210	37					
V	224	1	0.054	-0.018	41	0.051	0.080	0.024	0.213	50
V	112	1	.068	+.007	42					
V	56	1	.100	+.044	40					
VI	224	1	0.043	-0.046	46	0.049	0.033	0.011	0.199	50
VI	112	1	.059	-.070	44					
VI	56	1	.059	-.179	31					
VII	224	1	0.056	+0.035	41	0.050	0.044	0.014	0.194	50
VII	112	1	.050	+.059	50					
VII	56	1	.070	-.078	65					

[a]Plus sign: observer was late in recording body transit.
Minus sign: observer was early in recording body transit.

TABLE III

TIME INTERVAL RECORDING ERRORS

Observer	ω, deg/min	T, sec	$\sigma_{t,1}$, sec	$\sigma_{t,2}$, sec	σ_I, sec	ρ	Number of observations
I	224	22	0.040	0.037	0.038	0.515	40
I	224	6	.037	.038	.031	.651	40
I	56	24	.047	.054	.044	.627	43
IV	224	22	.045	.060	.060	.377	40
IV	224	6	.039	.034	.037	.436	40
IV	56	24	.059	.060	.031	.862	40
VIII	224	22	.050	.067	.036	.851	40
VIII	224	6	.048	.049	.057	.307	40
VIII	56	24	.052	.072	.020	.999	40

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