(91)

E

**NASA TECHNICAL NOTE**

**NASA TN D-2426**

**NASA TN D-2426**

N64-27943
(ACCESSION NUMBER)
36
(PAGES)
(NASA CR OR TMX OR AD NUMBER)
(THRU)
1
(CODE)
29
(CATEGORY)
FACILITY FORM 602

# STUDY OF POWERED-DESCENT TRAJECTORIES FOR MANNED LUNAR LANDINGS

*by Floyd V. Bennett and Thomas G. Price*

*Manned Spacecraft Center*

*Houston, Tex.*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1964

# STUDY OF POWERED-DESCENT TRAJECTORIES FOR MANNED LUNAR LANDINGS

By Floyd V. Bennett and Thomas G. Price

Manned Spacecraft Center
Houston, Tex.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Office of Technical Services, Department of Commerce,
Washington, D.C. 20230 -- Price $1.00

# STUDY OF POWERED-DESCENT TRAJECTORIES FOR MANNED LUNAR LANDINGS

By Floyd V. Bennett and Thomas G. Price

## SUMMARY

27943

The design powered-descent trajectory for the landing mission of the lunar excursion module is divided into three operational phases: an initial fuel-optimum phase, a landing-approach transition phase, and a final translation and touchdown phase. This paper contains an analysis of the operational tradeoffs available in these phases of the descent. Several trajectories were found that yield satisfactory operational features which allow adequate pilot control of the final approach and at the same time satisfy the abort and fuel economy criteria.

Author

## INTRODUCTION

The powered descent and landing on the lunar surface from lunar orbit is perhaps the most critical phase of the lunar-landing mission. Because of the large effect of weight upon the booster requirements of the earth launch and upon the payload delivered to the lunar surface, the weight of the fuel expended during powered descent and landing must be minimized. However, the crew is expected to control a major part of the maneuver, particularly the final landing approach. Control by the crew will logically add to the mission reliability because their faculties for judging can be used to assess the suitability of the surface for landing and their adaptive control capabilities can be applied to provide a degree of flexibility that is impossible with an automatic system. In order for the crew to be able to perform these functions properly, such factors as the trajectory characteristics of the landing approach, the attitude of the spacecraft, abort considerations, and the visibility limits of the spacecraft windows must be accommodated. Because some or all of these factors may be in conflict with the need for minimum fuel expenditure, it is important that the tradeoffs be well understood. The final selection of the mission design requires an understanding of the operational aspects of the maneuver.

Future fixed-base and free-flight piloted simulations are expected to provide a better understanding of the operational aspects of the maneuver. However, before these simulations are undertaken an analytical examination of the initially evident tradeoffs, coupled with a logical development of criteria to be applied to the landing maneuver, is thought to be an important and necessary step.

The purpose of this paper is to present an analytical study of the complete lunar-landing maneuver and to examine the maneuver characteristics in the light of assumed operational criteria.

## SYMBOLS

| | |
|---|---|
| $F$ | throttle setting, $\frac{T}{T_{max}}$ |
| $g$ | acceleration due to gravity |
| $g_e$ | acceleration due to gravity at earth sea level |
| $h$ | altitude, ft |
| $h_p$ | pericynthion altitude of descent transfer, ft |
| $h_t$ | initial altitude of landing-approach phase, ft |
| $\dot{h}$ | vertical descent rate, ft/sec |
| $I_{sp}$ | specific impulse, sec |
| $r$ | radius of moon, ft |
| $T$ | thrust, lb |
| $t$ | time, sec |
| $\frac{T}{W}$ | thrust-to-weight ratio of engine (earth weight) |
| $V$ | velocity, ft/sec |
| $V_c$ | characteristic velocity, $V_c = g_e I_{sp} \ln \mu$, ft/sec |
| $W$ | weight, lb |
| $\dot{x}$ | horizontal velocity at lunar surface, $\dot{x} = \frac{r}{r + h} V \cos \gamma$, ft/sec |
| $\beta$ | look angle to landing site with respect to vehicle thrust axis, deg (fig. 5) |
| $\gamma$ | flight-path angle with respect to local horizon, deg (fig. 5) |
| $\theta$ | pitch attitude with respect to local horizon, deg (fig. 5) |
| $\mu$ | ratio of initial-to-final mass |
| $\phi$ | out-of-plane heading angle, deg (fig. 15) |

Subscripts:

| | |
|---|---|
| f | total fuel |
| max | maximum |
| min | minimum |
| o | initial condition |
| t | condition for landing-approach transition |

## PHASES OF POWERED DESCENT

The powered-descent portion of the lunar-landing mission is a continuous-thrust maneuver of several minutes duration and is initiated at or near the pericynthion of the descent transfer orbit (fig. 1). This maneuver may logically be described in three phases: fuel-optimum descent, landing-approach transition, and final translation and touchdown.

In the initial phase, far from the landing site, the important consideration is optimum fuel performance. This fuel-optimum-descent phase is continued to a point where a modification to the trajectory is necessary to allow the crew to assess the approach to the landing site visually. This latter point is as yet undefined and is subject to tradeoffs which are examined in this paper.

The second phase, landing-approach transition, succeeds the fuel-optimum-descent phase and continues down to the initiation of the final landing approach. It is during this second phase that the visual assessment of the landing area is made by the crew.

The terminal phase of the descent trajectory from the end of the landing-approach transition to touchdown is the final translation and touchdown. This phase takes place close to the lunar surface, and because of abort considerations, involves relatively low velocities and conservative attitude deviations to translate and descend to the final touchdown point.

A sketch of the powered descent is given in figure 2.

Although the lunar-landing descent maneuver is still in the planning stage, considerable thought has been given to determining the important operational factors of each of the three phases. These factors are discussed in the next section.

## OPERATIONAL FACTORS

The major criteria for the entire powered descent are flight safety, in cluding abort considerations, and fuel economy. The important operational factors for establishing these criteria are presented for each phase of flight.

### Fuel-Optimum Descent

During fuel-optimum descent, primary concern is focused on fuel consumption. Two factors which influence the fuel performance most are the altitude at initiation of this phase and the thrust level used. The thrust is assumed to be constant for this phase.

The initial altitude should be as low as possible from the standpoint of fuel consumption (see fig. 3). The results shown in figure 3 are based on a calculus of variation technique reported in reference 1. Also, the initial altitude should be as low as possible in order to keep the time of flight during which the lunar excursion module is on a surface collision course as short as possible. However, consideration of lunar mountains which extend up to 20,000 feet (see ref. 2), and consideration of a margin of safety for guidance errors put a lower constraint on the initial altitude. Thus, a good compromise between fuel and safety requirements for the initial altitude appears to be about 50,000 feet. One further consideration on the initial altitude is the ability of the crew to survey the landing site in the event that orbital reconnaissance prior to the landing maneuver is desired. Here again the requirement would be for a reasonably low altitude.

After the initial altitude has been established, the initial thrust-to-weight ratio $T/W_o$ which yields minimum fuel consumption can also be established from figure 3. It can be seen that the value of the fuel optimum $T/W_o$ is about 0.7. However, consideration of the thrust level used in this phase must also take into account the throttling capability of the engine in order to produce the minimum thrust level desired for hover and translation. For the present analytical study it is assumed that the engine is operated at maximum thrust level during this phase; however, operationally, it might be desirable to operate slightly under this value in order to have some reserve capability for abort situations. A satisfactory value for the minimum thrust level at which control can be maintained during the translation phase is the thrust level necessary to support three-fourths of the lunar weight of the lunar excursion module at that point. Preliminary calculations indicate that the weight of the lunar excursion module will be reduced by about one-half during the powered descent; hence, it can be seen from figure 4 that for $T/W_o = 0.7$ (for fuel optimum) the throttle range $T_{max}/T_{min}$ is about 11 to 1. However, present state-of-the-art for engine design indicates that this range is too high. A range of about 9 to 1 is the maximum that should be planned. Thus, in order to reduce the throttle range, it is evident from figure 4 that the ratio $T/W_o$ must be reduced. It was shown in figure 3, however, that the fuel requirements increase with decreasing $T/W_o$; for $T/W_o$ below 0.7, therefore, a compromise between $T/W_o$ and throttle range must be made. From figure 3 it can be seen that only a slight fuel penalty comes from decreasing the $T/W_o$ to 0.4, whereas for reductions below this level, drastic penalties are incurred. For a $T/W_o$ of 0.4 the throttle range is only 6.5 to 1 which is well within the state-of-the-art capability. Hence, a value of 0.4 for $T/W_o$ appears to be a satisfactory compromise for this phase of the descent.

## Landing-Approach Transition

The factors important to the landing-approach-transition phase of the descent are those which relate the pilot's ability to assess the general suitability of the landing area visually and those that may cause his controlling task to be critical. The full extent of these problems will be known only after operational experience is obtained from fixed-base and free-flight piloted simulation tests. The factors known to have some importance during this phase are the ability of the pilot to view the landing site adequately, the time that he has for viewing the landing site, and the complexity of his control task. The latter factor would indicate the desire to have a minimum of required attitude changes and to have the approach velocities, particularly rate-of-descent, such that the control problem of the pilot does not require an undue amount of attention. On this basis, then, it is believed reasonable to constrain the trajectory during this phase by holding the attitude and throttle settings constant. Thus, the three parameters that will directly influence the operational factors are (1) spacecraft attitude, (2) throttle settings, and (3) the altitude at which this phase of the descent is initiated.

## Final Translation and Touchdown

During final translation and touchdown the final selection of the landing point is made and the landing completed. Generally speaking, this phase should avoid radical maneuvering and should be compatible with the requirements associated with an abort. Thus, the important operational factors for this phase are the initial altitude, the spacecraft attitude, horizontal velocity, vertical descent rate, and flight time. Limitations must be imposed on the spacecraft attitude, horizontal velocity, and vertical descent rate for reasons of flight safety and ease of control. Likewise, a limitation must be imposed on flight time for reasons of fuel consumption. However, within these limitations it is desired that the obtainable areas for landing be made as large as is feasible.

This phase of the descent is included primarily for completeness; hence, no parametric study of the important factors is intended. The section entitled Scope of Calculations contains limitations used in this phase.

## DESIGN LANDING SEQUENCE

The following description of the landing sequence should provide a better understanding of elements of the descent maneuver as they are discussed in the following sections.

The maneuver is initiated approximately at pericynthion altitude at a preselected position about 200 miles from the intended landing point. A constant thrust near the maximum capability of the descent engine will be utilized throughout this part of the trajectory. The trajectory will be shaped by the guidance logic to follow a near-fuel-optimum path to certain altitude, position, and velocity conditions predetermined as the desired initial conditions for the landing-approach transition. Upon reaching the desired conditions for the start of the landing-approach transition, the spacecraft attitude and throttle setting will be changed in accordance with preselected values. Closed-loop guidance could call for some modifications to the preselected values but these changes are not expected to be radical. During the landing-approach transition the pilot assesses the landing area and continues to update this assessment as the range is decreased. In addition, the pilot will judge the suitability of the landing-approach trajectory to assure himself that the approach is safe and the guidance system is working (a qualitative evaluation). If the pilot is dissatisfied with the approach, he may either abort or take over control of the spacecraft and modify the trajectory to a suitable one. If the predicted landing area appears generally suitable, the pilot will continue the approach to attain the desired initial conditions of the final translation and touchdown phase. Upon reaching the initial condition of this final phase, the pilot must decide upon the final landing position and control the trajectory to obtain this position within the time allotted for the maneuver. Some flexibility for minor changes in the landing position remain until a hover position is reached at a low altitude just prior to touchdown.

## SCOPE OF CALCULATIONS

### General

In an effort to keep the results of this study independent of the orbital altitude of the command module and of the type of orbital descent transfer, the initial conditions of the powered descent were circular orbital conditions at 50,000 feet ($V = 5{,}483.74$ ft/sec, $\gamma = 0°$). (The choice of 50,000 feet for the initial altitude was established in the section on Operational Factors.) The primary effect of different transfer trajectories on the powered descent would be only to change the magnitude of the initial velocity at 50,000 feet and, consequently, to change the characteristic velocity required by an equal amount. For example, for a Hohmann transfer from the command module circular orbit of 80 nautical miles, the initial velocity would be increased by 98 feet per second over circular orbit speed at 50,000 feet, and an equiperiod transfer from the same altitude would require an initial increase in velocity of 190 feet per second. The axis system used is shown in figure 5.

### Fuel-Optimum Phase

The fuel-optimum portion of this descent is based on the calculus of variations technique reported in reference 1 for two-dimensional motion, a circular gravitational body, and a constant level of thrust. The value of $T/W_o$ is 0.40, as established in the section on Operational Factors. The specific impulse $I_{sp}$ of 315 seconds (considered to be a reasonable design value) is assumed throughout the descent. The final conditions for the fuel-optimum phase are specified by the desired conditions at the transition altitude for the landing-approach phase.

### Landing-Approach Transition

For the landing-approach phase, the three major parameters were varied in the following manner. Three values for the transition altitude $h_t$ were considered: $h_t$ = 5,000 feet, 10,000 feet, and 15,000 feet. The attitude $\theta_t$ was varied from 90° (vertical attitude) for best visibility to 140° which is approaching the condition for the fuel-optimum descent to the surface. The throttle setting was varied from 0.75 to as low as 0.30 (throttle setting was 1.00 for the fuel-optimum phase). The equations of motion for this phase are based on the same assumptions that were used in the fuel-optimum phase.

### Final Translation and Touchdown

The final translation and touchdown phase of the descent is included primarily for completeness; hence no parametric study of the important factors is intended. Instead, a set of numbers and limitations were chosen to be generally compatible with the task. The initial conditions for this phase are assumed to be: altitude = 1,000 feet; velocity = 75 ft/sec; and flight-path angle = 0°. Flight time is limited to a maximum of 2 minutes; attitude excursions in pitch are limited to the range 60° to 120°; and horizontal velocity is limited to a maximum of 75 ft/sec. The vertical descent rate is limited to a maximum of 20 ft/sec. For purposes of standardizing the calculations, an altitude profile of the required descent rates is used for all descents (fig. 6). A minimum of 15 seconds of the allotted 2-minute flight time is allowed for descending from the 50-foot hover point shown in figure 6. Finally, in order to stay reasonably compatible with the abort situation the lunar excursion module is not allowed to have motion in the direction opposite to that of the command module; however, it is allowed to establish a heading angle $\phi$ up to 90° out of the plane of motion of the command module. Alleviation of any or all of the limitations cited herein may well be possible after experience is gained through simulation, but at the present time these limitations are considered reasonable.

The equations of motion for this phase are based on three-dimensional motion, a flat gravitational body, a variable level of thrust, and a constant spacecraft mass.

## RESULTS AND DISCUSSION

The discussion of the powered-descent phases of a lunar-landing mission is divided into two parts. The first part concerns the descent from the initial 50,000-foot altitude down to 1,000-foot altitude and includes the fuel-optimum phase and the landing-approach-transition phase. The second part is concerned with the final translation and touchdown from 1,000 feet to the surface.

### Descent to an Altitude of 1,000 Feet

The initial conditions for this portion of the descent are circular orbit conditions at 50,000 feet, and the terminal conditions are a 1,000-foot altitude and a velocity of 75 ft/sec directed along the horizontal.

Fuel requirements.- In discussing the results of the calculations for the design powered descent, the first consideration is to determine the characteristic velocity which is a measure of fuel consumption. For reference purposes, a constant-thrust fuel-optimum descent (to 1,000 ft) was calculated and found to require a characteristic velocity of 5,627 ft/sec. The time history of this fuel-optimum descent is given in figure 7. The characteristic velocity requirements for the design landing technique is presented in figure 8 as a function of the major parameters for the landing approach transition maneuver, $h_t$, $\theta_t$, and F. It is evident from this figure that in order to attain the value for each of the three parameters which yield the best operational feature, the fuel requirement would be prohibitive. For example, it was desired to make the transition maneuver at as high an altitude as possible in order to provide adequate time to assess the landing area; however, for corresponding values of F and $\theta_t$, the fuel requirements increase with $h_t$. In a similar manner the ideal attitude for maximum visibility (90° or vertical) and the ideal low value of F for establishing low descent rates and improving the abort situation are both very costly in fuel. These results are not too surprising, however, since the pitch angle for the fuel-optimum case in which the throttle is full open (F = 1.00) was found to vary from 164° to 154° in the final approach (see fig. 7). Thus, since it is too costly in fuel to obtain the ideal conditions, trajectories which can be obtained with only a moderate increase in characteristic velocity over the fuel-optimum case, 200 ft/sec to 300 ft/sec, must be investigated. The conditions for six such trajectories chosen for further investigation are listed in table I.

Visibility.- For considering the operational problem of landing-site visibility, time histories of the look angle $\beta$, defined as the angle between the thrust attitude axis and the line of sight to the landing site, are shown in figure 9. The landing site is assumed to be at a point 3,000 feet downrange of the 1,000-foot-altitude point (see section on Final Translation and Touchdown). It is apparent from figure 9 that all of these trajectories except trajectory (a), which has a high throttle setting (F = 0.75), yield considerably more visibility in both magnitude and time than does the fuel-optimum descent which is shown for reference. Trajectory (a) has very poor visibility up to only 38 seconds prior to reaching the 1,000-foot-altitude point. Trajectory (f) yields the best visibility of the six; however, since it has the lowest throttle ratio (0.30) and an attitude nearest the vertical (120°) it also requires more fuel than the others.

Vertical descent rate.- As stated in the section on Operational Factors, it is desirable to have a low rate of descent in order to ease the pilot's control problem. Time histories of the vertical velocity for the six trajectories are presented in figure 10. Once again, improvement over the fuel optimum case is found in all of these trajectories except trajectory (a). For trajectory (a) the pitch attitude of 140° and the throttle setting of 0.75 combined to produce vertical descent rates higher than the fuel optimum. For trajectories (b) to (f), considerable reductions in the vertical velocity are realized with the greatest reductions found in trajectory (f). For comparison, it should be mentioned that jet aircraft instrument landings have a descent rate of about 4,000 ft/min. Trajectories (b) to (e) have a rate of about 100 ft/sec (or 6,000 ft/min) at a 5,000-foot altitude and for trajectory (f) the rate is about 4,000 ft/min at 5,000 feet. However, it must be remembered that trajectory (f) requires more fuel than the others.

Time to assess landing area.- Another operational factor to be considered is the time available to assess the landing area. It can be seen from the time histories of figure 10 that trajectories (b) to (e) yield a slightly longer time at low altitudes than does the fuel optimum (60 sec compared to 45 sec to descend from 5,000 ft). Again, the more expensive trajectory (f) is greatly improved over the fuel optimum, requiring 2 minutes to descend from 5,000 feet.

Horizontal velocity.- Time histories of horizontal velocity are presented in figure 11. This velocity for the fuel-optimum trajectory is quite high (1,000 ft/sec at an altitude of only 5,000 feet). The operationally designed trajectories, however, yield a substantial reduction. (For example, trajectories (d) and (e) have a horizontal velocity of only 450 ft/sec at 5,000 ft.) Thus, by reducing the horizontal velocity as well as the vertical velocity, these descent trajectories not only make the landing safer, but also improve initial conditions in the event that it becomes necessary for the crew to control the lunar excursion module on the backup or abort guidance mode.

Design powered descent.- From the preceding discussion it is evident that several trajectories meet the operational requirements set up for the design of the powered descent. Profiles of the landing-approach-transition phase for a few of these descents are shown in figure 12. The descents from 50,000 feet are tabulated in tables II to V. Trajectory (a) was not included because of poor visibility and trajectory (f) was not included because of the additional fuel requirement. It should not be assumed that the trajectories of figure 12 are the only desirable descents, but any descents chosen should have operational features at least as good as these while maintaining low fuel requirements.

## Final Translation and Touchdown

The initial conditions for this final phase of the descent are a 1,000-foot altitude with a velocity of 75 ft/sec directed along the horizontal. Terminal conditions are touchdown on the lunar surface with an impact velocity of 6.7 ft/sec or less.

Descent trajectories.- Based on the limitations given in the section on Scope of Calculations, the minimum range, nominal, and maximum range descent trajectories were calculated. The minimum range descent is based on the minimum range required for reducing the forward velocity to zero. The nominal descent is based on achieving a range of about 3,000 feet for normal maneuvering. And finally, the maximum range descent is based on holding the maximum forward velocity for as long as possible. These descents are illustrated in figure 13. The parameters for these descents are tabulated in table VI. The minimum and maximum ranges were found to be about 1,100 feet and 6,800 feet, respectively. The characteristic velocity for these descents is given in table VII.

Maximum footprint.- One of the desired operational features was to obtain a large landing area or footprint. The maximum landing footprint, subject to the limitations given in the section on Scope of Calculations, was established by rotating the velocity vector of 75 ft/sec to the desired direction and holding it as long as possible. In order to rotate the velocity to the desired direction, the thrust is applied in the direction $90° + \phi/2$, where $\phi$ is the desired out-of-plane heading angle. (This is not an impulsive velocity change.) For abort considerations, this out-of-plane angle is limited to 90°. No piloting errors were assumed in the calculation of this footprint, hence it is expected to be slightly larger than an operationally obtainable limit (see fig. 14). The imposed limitations did not restrict the out-of-plane range appreciably, since 5,370 feet was obtained at the maximum out-of-plane angle of 90°. The maximum range, as shown previously, was 6,800 feet for the in-plane case. The 90° out-of-plane maneuver required the maximum characteristic velocity, 693 ft/sec (see table VII). A three-dimensional view of the maximum footprint with associated trajectories is shown in figure 15. (Only half of the footprint is shown; it is symmetrical about the forward range axis.)

## CONCLUDING REMARKS

An analysis of the powered-descent portion of the landing mission of the lunar excursion module is presented with special emphasis on the compromises imposed by various operational considerations. The design landing trajectory was divided into three operational phases. The initial phase is primarily concerned with fuel economy; the second phase, the landing-approach transition, emphasizes pilot control; and the final phase, the translation and touchdown, is concerned with obtaining as large a landing area as possible. Flight safety, including abort considerations, and fuel economy are overriding criteria throughout all phases of the descent. This design landing technique was found to yield several trajectories with satisfactory operational features which allow the pilot adequate control of the final approach and at the same time satisfy the abort and fuel economy criteria. It was also shown that the out-of-plane range capability during translation is nearly as great as the in-plane capability.

## REFERENCES


1. Miele, Angelo: General Variational Theory of the Flight Paths of Rocket-Powered Aircraft, Missiles and Satellite Carriers. AFOSR-TN-58-246 (AD-154 148).

2. Blanco, V. M., and McCuskey, S. W.: Basic Physics of the Solar System. Addison-Wesley Pub. Co., Inc., Reading, Mass., 1961.

TABLE I.- CONDITIONS FOR SAMPLE OPERATIONAL TRAJECTORIES

| Trajectory | F | $h_t$, ft | $\theta_t$, deg | $V_c$, ft/sec |
|---|---|---|---|---|
| a | 0.75 | 5,000 | 140 | 5,664 |
| b | .50 | 15,000 | 140 | 5,800 |
| c | .50 | 10,000 | 140 | 5,761 |
| d | .40 | 15,000 | 130 | 5,910 |
| e | .40 | 10,000 | 130 | 5,854 |
| f | .30 | 5,000 | 120 | 5,944 |

TABLE II. - TRAJECTORY (b) DESCENTS[a]

[Initial conditions: Circular orbit at 50,000 ft; Final conditions: $h$ = 1,000 ft, $V$ = 75 ft/sec, $\gamma = 0°$; $F$ = 0.50, $h_t$ = 15,000 ft, $\theta_t$ = 140°.]

| Time to go, sec | h, ft | Range to go, ft | θ, deg | V, ft/sec | -γ, deg | $-\dot{h}$, ft/sec | $\dot{x}$, ft/sec |
|---|---|---|---|---|---|---|---|
| 0 | 1,000 | 0 | 140.00 | 75.00 | 0 | 0 | 75.00 |
| 10.00 | 1,106 | 1,193 | 140.00 | 164.85 | 7.356 | 21.11 | 163.46 |
| 20.00 | 1,419 | 3,266 | 140.00 | 254.38 | 9.373 | 41.43 | 250.92 |
| 30.00 | 1,932 | 6,208 | 140.00 | 342.95 | 10.248 | 61.02 | 337.37 |
| 42.50 | 2,843 | 11,093 | 140.00 | 452.21 | 10.775 | 84.54 | 444.02 |
| 57.50 | 4,315 | 18,701 | 140.00 | 581.17 | 11.059 | 111.48 | 569.95 |
| 77.50 | 6,888 | 31,751 | 140.00 | 749.56 | 11.188 | 145.43 | 734.43 |
| 97.50 | 10,119 | 48,052 | 140.00 | 914.03 | 11.194 | 177.45 | 895.05 |
| 117.50 | 13,975 | 67,528 | 140.00 | 1,074.76 | 11.149 | 207.82 | 1,051.89 |
| 122.35 | 15,000 | 72,722 | [b]163.23 | 1,113.19 | 11.135 | 214.98 | 1,089.37 |
| 127.35 | 16,084 | 78,410 | 163.64 | 1,209.12 | 10.414 | 218.57 | 1,185.86 |
| 152.35 | 21,673 | 113,980 | 165.66 | 1,677.75 | 7.732 | 225.73 | 1,656.20 |
| 202.35 | 32,490 | 219,190 | 169.45 | 2,562.73 | 4.470 | 199.72 | 2,540.46 |
| 262.35 | 42,523 | 401,280 | 173.57 | 3,543.06 | 2.119 | 131.03 | 3,514.43 |
| 302.35 | 46,747 | 554,050 | 176.07 | 4,153.04 | 1.112 | 80.57 | 4,118.49 |
| 342.35 | 49,064 | 730,340 | 178.36 | 4,731.49 | 0.450 | 37.12 | 4,690.98 |
| 382.35 | 49,911 | 928,940 | 180.45 | 5,280.60 | .089 | 8.23 | 5,234.77 |
| 397.00 | 50,000 | 1,014,000 | 181.10 | 5,483.47 | 0 | 0 | 5,435.00 |

[a]Tabulated in reverse.

[b]Instantaneous change in θ for landing-approach transition.

TABLE III. - TRAJECTORY (c) DESCENTS[a]

[Initial conditions: Circular orbit at 50,000 ft;
Final conditions: $h$ = 1,000 ft, $V$ = 75 ft/sec, $\gamma = 0°$;
$F$ = 0.50, $h_t$ = 10,000 ft, $\theta_t$ = 140°.]

| Time to go, sec | $h$, ft | Range to go, ft | $\theta$, deg | $V$, ft/sec | $-\gamma$, deg | $-\dot{h}$, ft/sec | $\dot{x}$, ft/sec |
|---|---|---|---|---|---|---|---|
| 0 | 1,000 | 0 | 140.00 | 75.00 | 0 | 0 | 75.00 |
| 10.00 | 1,105 | 1,192 | 140.00 | 164.50 | 7.279 | 20.84 | 163.15 |
| 21.25 | 1,467 | 3,579 | 140.00 | 264.76 | 9.426 | 43.36 | 261.12 |
| 31.25 | 1,997 | 6,621 | 140.00 | 352.86 | 10.220 | 62.61 | 347.14 |
| 43.75 | 2,925 | 11,626 | 140.00 | 461.35 | 10.703 | 85.72 | 453.27 |
| 53.75 | 3,872 | 16,578 | 140.00 | 547.30 | 10.901 | 103.50 | 537.06 |
| 78.75 | 6,992 | 32,582 | 140.00 | 757.32 | 11.079 | 145.53 | 742.29 |
| 88.75 | 8,527 | 40,408 | 140.00 | 839.61 | 11.088 | 161.47 | 822.70 |
| 97.51 | 10,000 | 47,920 | [b]160.56 | 910.91 | 11.081 | 175.07 | 892.36 |
| 107.51 | 11,834 | 57,816 | 161.48 | 1,105.23 | 9.945 | 190.87 | 1,086.37 |
| 132.51 | 16,952 | 90,909 | 163.73 | 1,577.30 | 7.840 | 215.17 | 1,557.93 |
| 152.51 | 21,335 | 125,730 | 165.49 | 1,941.65 | 6.548 | 221.41 | 1,921.79 |
| 192.51 | 30,015 | 216,630 | 168.87 | 2,637.95 | 4.513 | 207.56 | 2,616.00 |
| 252.51 | 40,808 | 402,980 | 173.61 | 3,610.39 | 2.333 | 146.94 | 3,581.76 |
| 292.51 | 45,685 | 558,360 | 176.55 | 4,216.00 | 1.315 | 96.73 | 4,181.39 |
| 352.51 | 49,380 | 834,820 | 180.59 | 5,066.92 | 0.338 | 29.90 | 5,023.33 |
| 383.00 | 50,000 | 1,000,000 | 182.50 | 5,483.47 | 0 | 0 | 5,435.00 |

[a]Tabulated in reverse.

[b]Instantaneous change in $\theta$ for landing approach transition.

TABLE IV. - TRAJECTORY (d) DESCENTS[a]

[Initial conditions: Circular orbit at 50,000 ft; Final conditions: $h = 1{,}000$ ft, $V = 75$ ft/sec, $\gamma = 0°$; $F = 0.40$, $h_t = 15{,}000$ ft, $\theta_t = 130°$.]

| Time to go, sec | h, ft | Range to go, ft | $\theta$, deg | V, ft/sec | $-\gamma$, deg | $-\dot{h}$, ft/sec | $\dot{x}$, ft/sec |
|---|---|---|---|---|---|---|---|
| 0 | 1,000 | 0 | 130.00 | 75.00 | 0 | 0 | 75.00 |
| 10.00 | 1,093 | 1,051 | 130.00 | 136.38 | 7.796 | 18.50 | 135.09 |
| 20.00 | 1,368 | 2,700 | 130.00 | 198.05 | 10.583 | 36.37 | 194.64 |
| 30.00 | 1,818 | 4,942 | 130.00 | 259.31 | 11.940 | 53.65 | 253.62 |
| 42.50 | 2,620 | 8,569 | 130.00 | 335.08 | 12.835 | 74.44 | 326.56 |
| 57.50 | 3,917 | 14,116 | 130.00 | 424.74 | 13.380 | 98.29 | 412.93 |
| 77.50 | 6,186 | 23,511 | 130.00 | 542.13 | 13.694 | 128.34 | 526.14 |
| 97.50 | 9,038 | 35,148 | 130.00 | 657.10 | 13.787 | 156.59 | 637.16 |
| 117.50 | 12,439 | 48,983 | 130.00 | 769.73 | 13.769 | 183.20 | 745.98 |
| 127.50 | 14,335 | 56,711 | 130.00 | 825.20 | 13.736 | 195.94 | 799.59 |
| 130.86 | 15,000 | 59,428 | [b]162.62 | 843.71 | 13.723 | 200.15 | 817.48 |
| 150.86 | 19,175 | 79,708 | 164.23 | 1,231.34 | 10.063 | 215.16 | 1,208.34 |
| 195.86 | 28,913 | 153,040 | 167.63 | 2,061.41 | 5.849 | 210.06 | 2,040.33 |
| 245.86 | 38,405 | 276,720 | 171.08 | 2,919.75 | 3.237 | 164.86 | 2,895.59 |
| 305.86 | 46,097 | 479,200 | 174.79 | 3,872.13 | 1.343 | 90.73 | 3,840.02 |
| 345.86 | 48,788 | 644,650 | 177.01 | 4,465.30 | 0.581 | 45.27 | 4,427.19 |
| 385.86 | 49,891 | 832,980 | 179.03 | 5,028.06 | .143 | 12.56 | 4,984.44 |
| 419.00 | 50,000 | 1,005,000 | 180.40 | 5,483.47 | 0 | 0 | 5,435.00 |

[a]Tabulated in reverse.

[b]Instantaneous change in $\theta$ for landing approach transition.

TABLE V.- TRAJECTORY (e) DESCENTS[a]

[Initial conditions: Circular orbit at 50,000 ft;
Final conditions: $h$ = 1,000 ft, $V$ = 75 ft/sec, $\gamma = 0°$;
$F = 0.40$, $h_t$ = 10,000 ft, $\theta_t = 130°$.]

| Time to go, sec | h, ft | Range to go, ft | θ, deg | V, ft/sec | -γ, deg | -ḣ, ft/sec | ẋ, ft/sec |
|---|---|---|---|---|---|---|---|
| 0 | 1,000 | 0 | 130.00 | 75.00 | 0 | 0 | 75.00 |
| 10.00 | 1,091 | 1,049 | 130.00 | 136.00 | 7.652 | 18.11 | 134.77 |
| 20.00 | 1,360 | 2,694 | 130.00 | 197.28 | 10.397 | 35.60 | 193.99 |
| 30.00 | 1,801 | 4,927 | 130.00 | 258.13 | 11.734 | 52.50 | 252.66 |
| 42.50 | 2,585 | 8,540 | 130.00 | 333.41 | 12.617 | 72.83 | 325.21 |
| 57.50 | 3,854 | 14,064 | 130.00 | 422.50 | 13.152 | 96.13 | 411.14 |
| 77.50 | 6,073 | 23,417 | 130.00 | 539.15 | 13.459 | 125.49 | 523.78 |
| 97.50 | 8,861 | 35,000 | 130.00 | 653.41 | 13.546 | 153.04 | 634.25 |
| 104.72 | 10,000 | 39,719 | [b]160.24 | 694.07 | 13.547 | 162.58 | 673.58 |
| 124.72 | 12,639 | 52,052 | 161.59 | 989.72 | 10.937 | 194.27 | 1,066.66 |
| 149.72 | 18,744 | 89,769 | 164.20 | 1,560.02 | 7.900 | 214.43 | 1,540.15 |
| 199.72 | 29,500 | 189,360 | 168.31 | 2,452.76 | 4.842 | 207.02 | 2,431.43 |
| 239.72 | 37,139 | 300,040 | 171.40 | 3,120.89 | 3.158 | 171.94 | 3,095.98 |
| 299.72 | 45,340 | 513,990 | 175.67 | 4,055.99 | 1.410 | 99.83 | 4,022.78 |
| 339.72 | 48,379 | 686,550 | 178.28 | 4,639.12 | 0.658 | 53.27 | 4,599.78 |
| 379.72 | 49,745 | 881,590 | 180.68 | 5,192.66 | .193 | 17.49 | 5,147.72 |
| 402.50 | 50,000 | 1,004,000 | 182.00 | 5,483.47 | 0 | 0 | 5,43[illegible].00 |

[a]Tabulated in reverse.

[b]Instantaneous change in θ for landing approach transition.

TABLE VI. - PARAMETERS FOR DESCENTS FROM 1,000 FEET

$\left[60° \leq \theta \leq 120°;\ \dot{h}_{max} = 20\ \text{ft/sec}\right]$

| Descent | t, sec | $\dot{h}$, ft/sec | $\dot{x}$, ft/sec | h, ft | θ, deg | T/W |
|---|---|---|---|---|---|---|
| Minimum range | 0 | 0 | 75 | 1,000 | | |
| | | | | | 120 | 0.14 |
| | 15 | 20 | 40.5 | 850 | | |
| | | | | | 120 | .20 |
| | 27.6 | 16.5 | 0 | 620 | | |
| | | | | | 90 | .17 |
| | 52 | 10 | 0 | 300 | | |
| | | | | | 90 | .17 |
| | 79 | 5 | 0 | 100 | | |
| | | | | | 90 | .17 |
| | 98 | 0 | 0 | 50 | | |
| | | | | | 90 | .15 |
| | 113 | 6.7 | 0 | 0 | | |
| Nominal | 0 | 0 | 75 | 1,000 | | |
| | | | | | 105 | 0.13 |
| | 15 | 20 | 59 | 850 | | |
| | | | | | 100 | .18 |
| | 52 | 10 | 23 | 300 | | |
| | | | | | 94.5 | .17 |
| | 79 | 5 | 10 | 100 | | |
| | | | | | 94.4 | .17 |
| | 99 | 0 | 0 | 50 | | |
| | | | | | 90 | .15 |
| | 114 | 6.7 | 0 | 0 | | |
| Maximum range | 0 | 0 | 75 | 1,000 | | |
| | | | | | 90 | 0.12 |
| | 15 | 20 | 75 | 850 | | |
| | | | | | 90 | .17 |
| | 52 | 10 | 75 | 300 | | |
| | | | | | 90 | .17 |
| | 79 | 5 | 75 | 100 | | |
| | | | | | 118.5 | .20 |
| | 103 | 0 | 0 | 50 | | |
| | | | | | 90 | .15 |
| | 118 | 6.7 | 0 | 0 | | |

TABLE VII. - CHARACTERISTIC VELOCITY REQUIREMENTS FOR FINAL TRANSLATION AND TOUCHDOWN

[Initial conditions: $h = 1{,}000$ ft; $V = 75$ ft/sec; $\gamma = 0°$]

| Out-of-plane angle, $\phi$, deg | Type of descent | $V_c$, ft/sec |
|---|---|---|
| 0 | minimum range | 636 |
| 0 | nominal | 621 |
| 0 | maximum range | 663 |
| 30 | maximum range | 674 |
| 45 | maximum range | 679 |
| 60 | maximum range | 684 |
| 90 | maximum range | 693 |

NASA-S-64-875

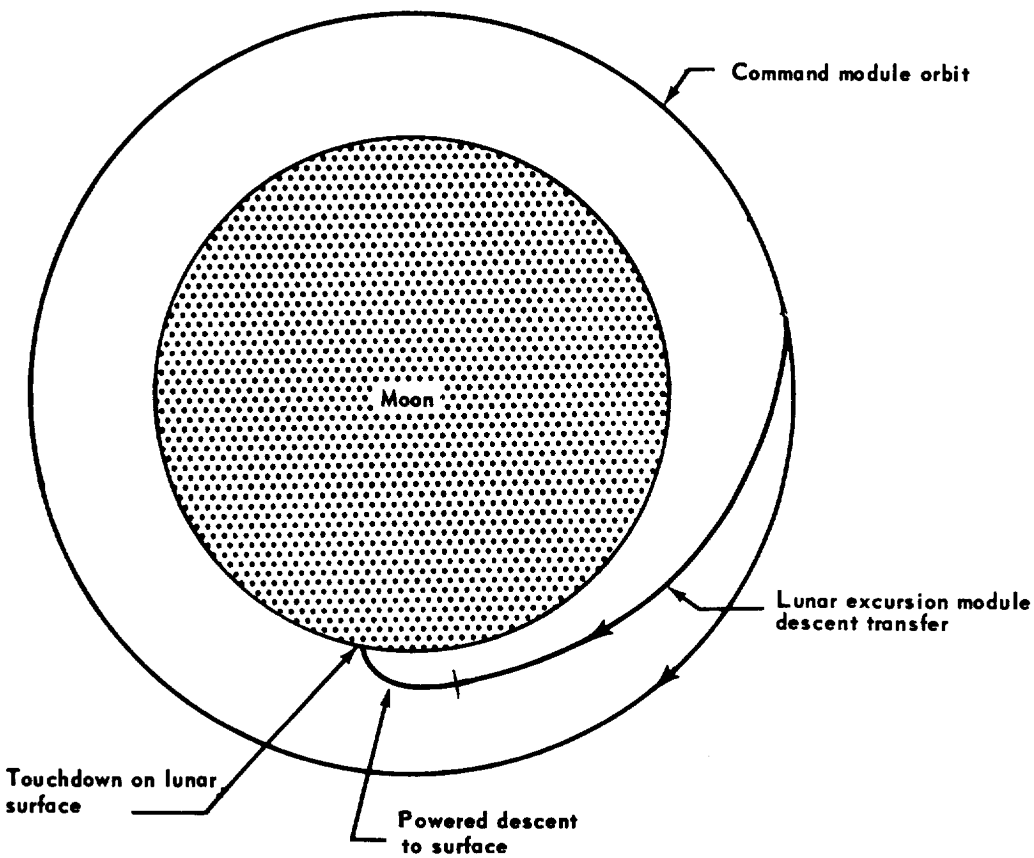



Figure 1.- Sketch of lunar excursion module descent.

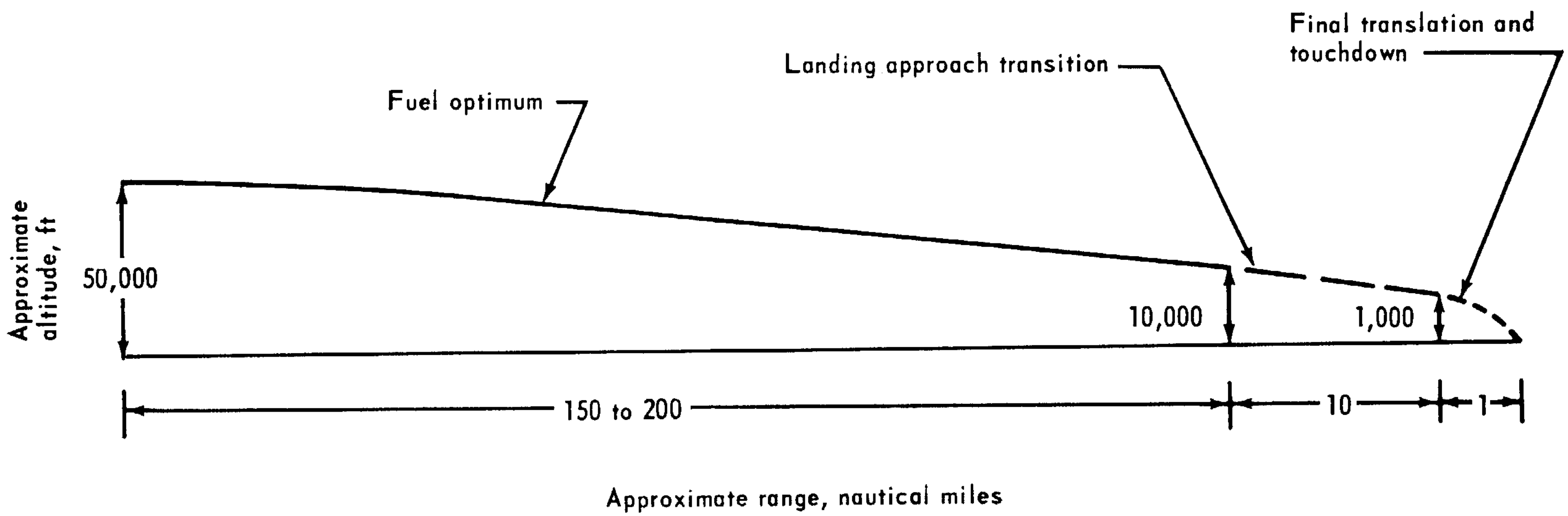



Figure 2.- Three phases of powered flight.

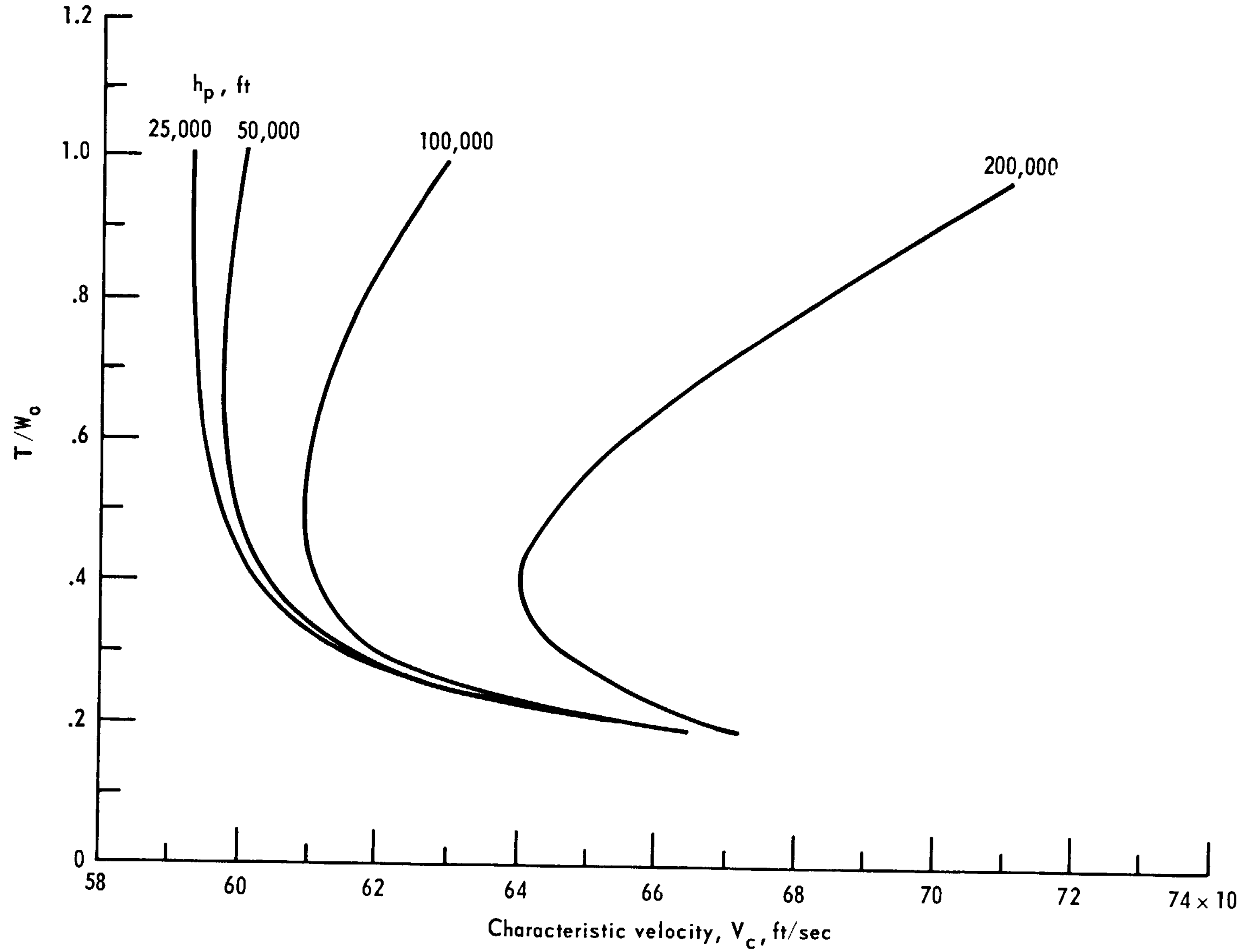



Figure 3.- Characteristic velocity for fuel-optimum powered descents from pericynthion altitude of descent transfer with apocynthion altitude of 100 nautical miles.

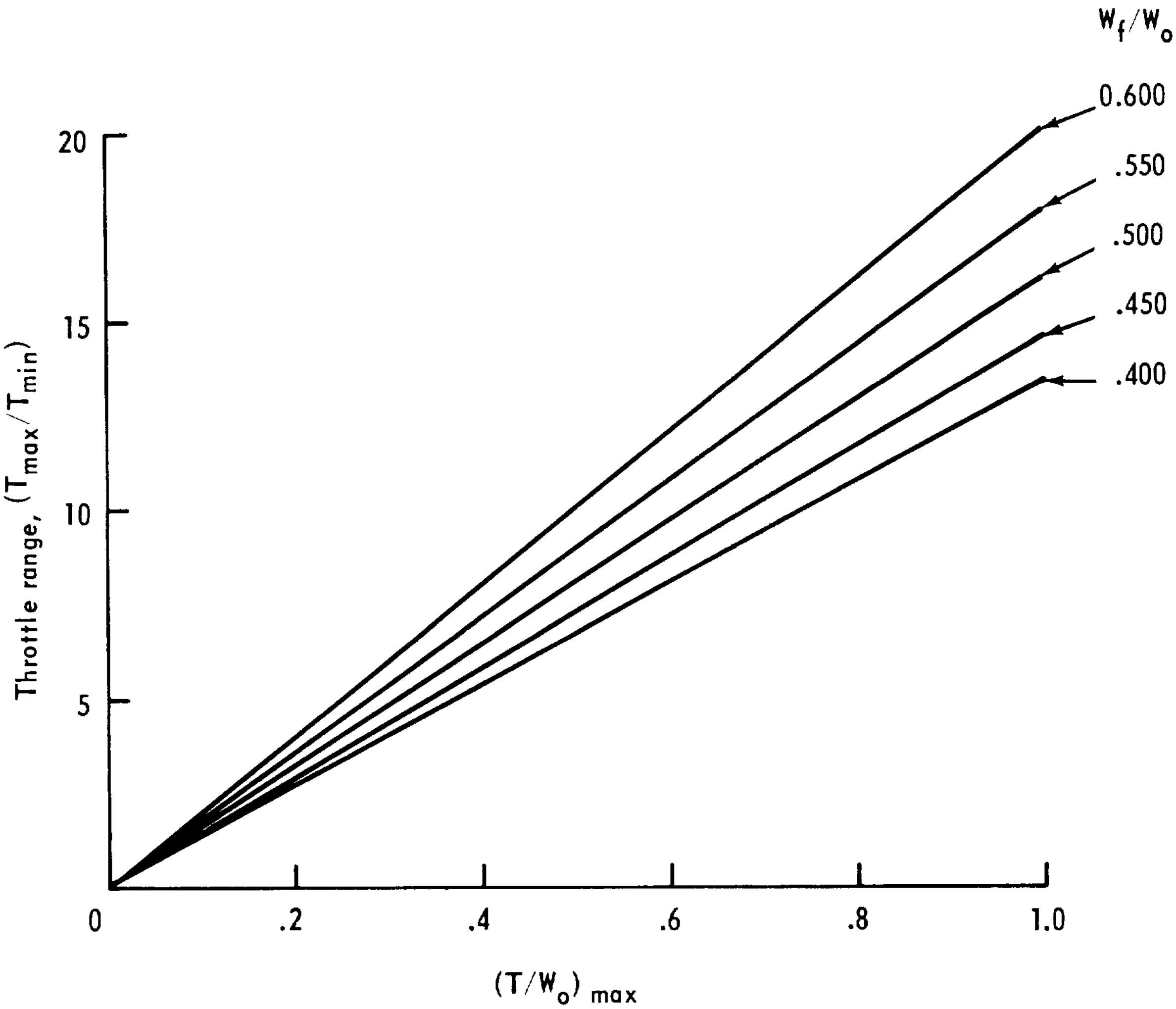



Figure 4.- Variation of throttling range with maximum $T/W_o$ for several fuel consumption ratios. Minimum thrust ≈ 0.75 lunar g.

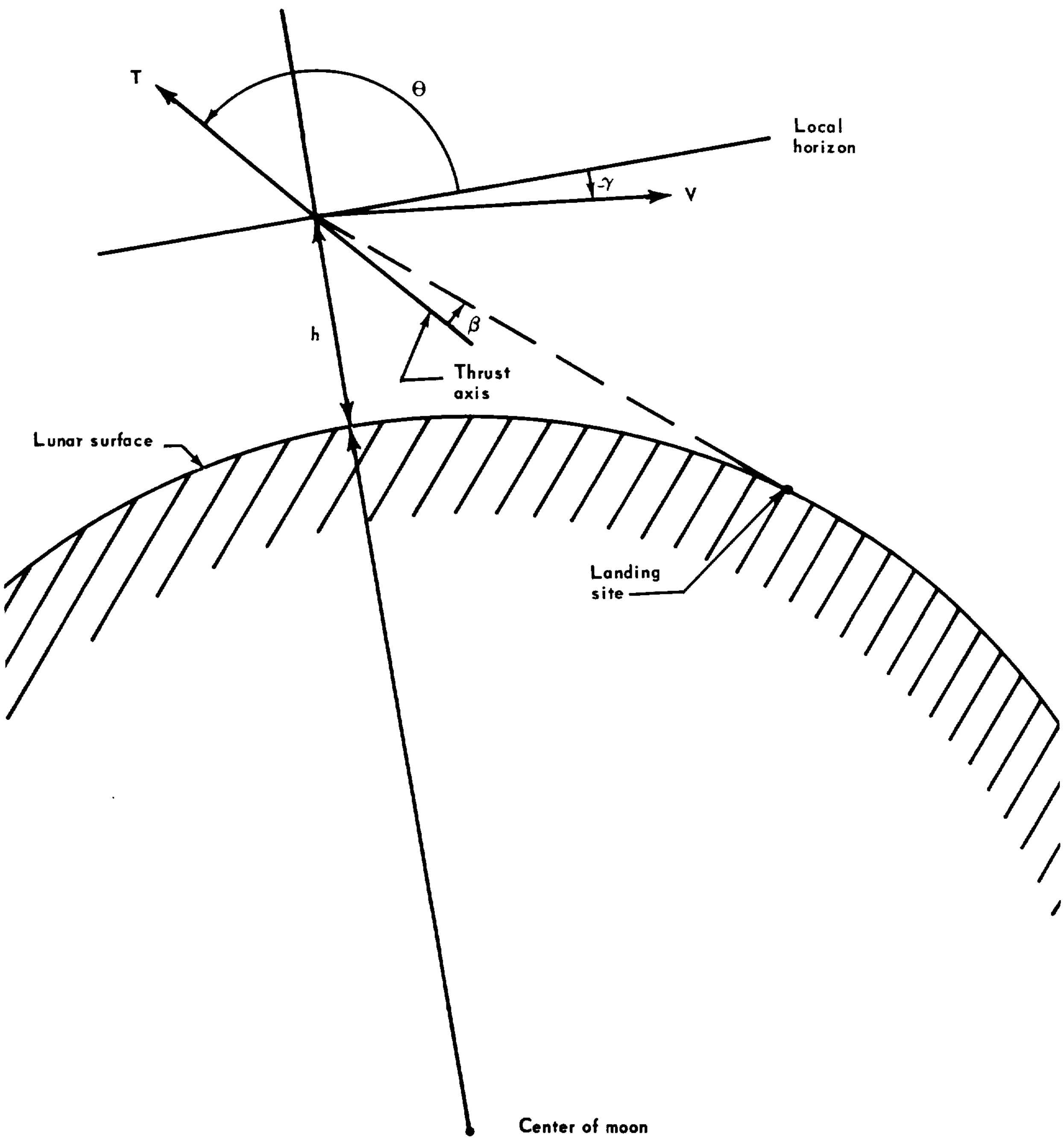



Figure 5.- Sketch of axis system.

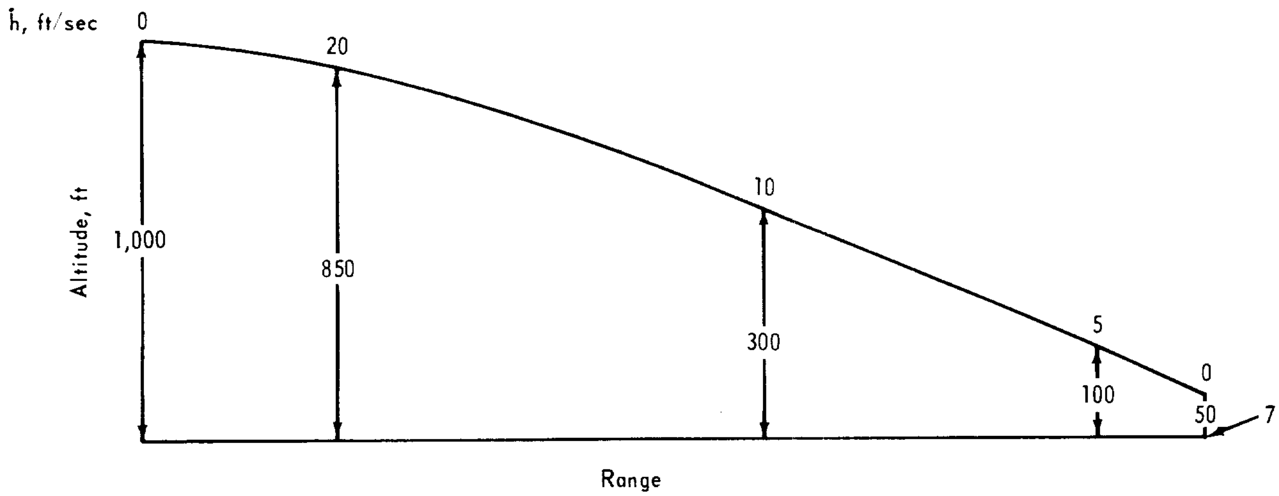



Figure 6.- Profile of vertical descent rates for final translation and touchdown phase. $\dot{x}_o$ = 75 ft/sec; maximum descent time = 2 min.

NASA-S-64-877

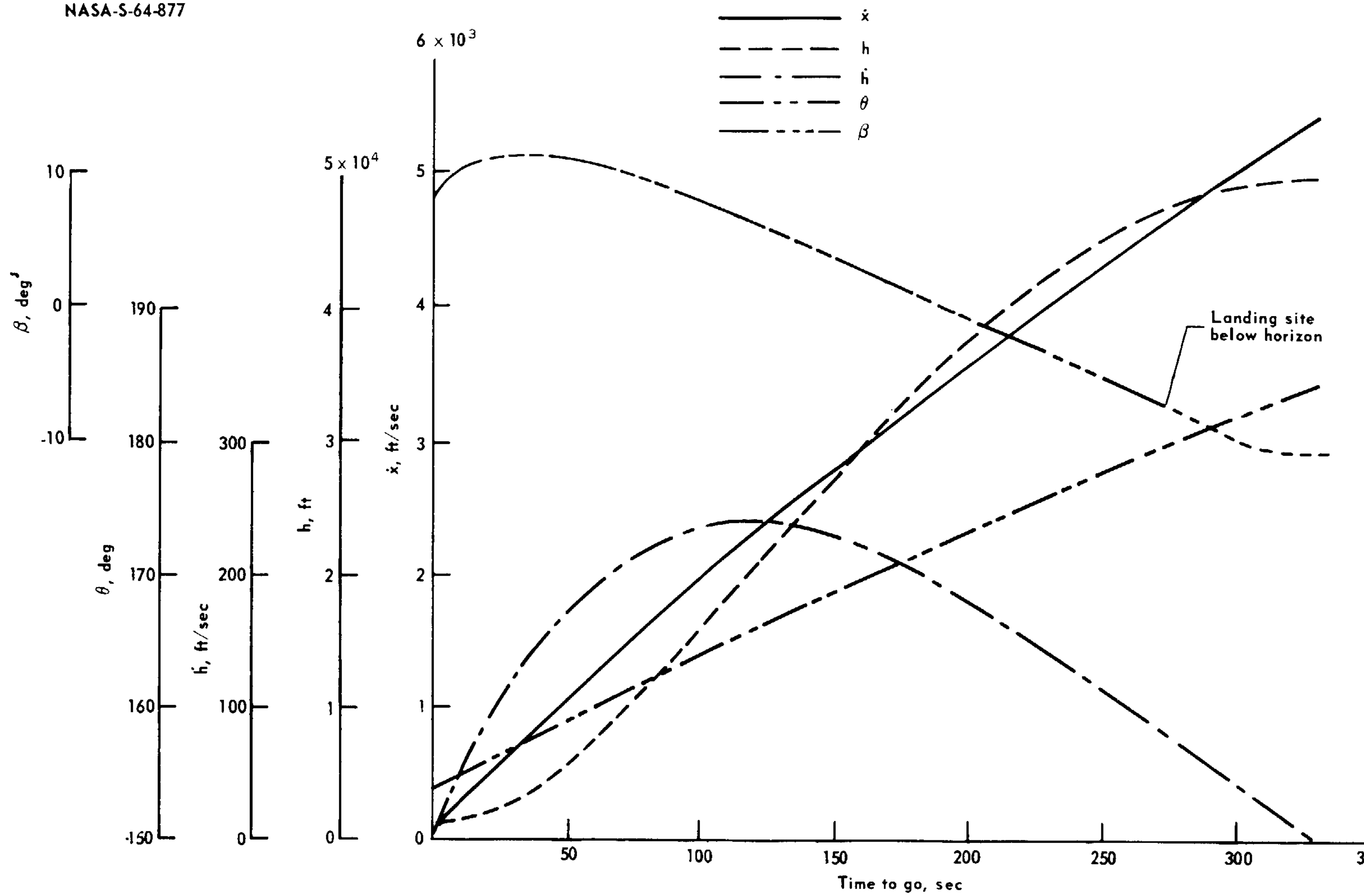



Figure 6.- Time history of fuel optimum landing from 50,000-foot circular orbit. $T/W_o = 0.4$; $I_{sp} = 315$ sec. Final conditions: $h = 1,000$ ft; $\dot{x} = 75$ ft/sec; $\dot{h} = 0$ ft/sec.

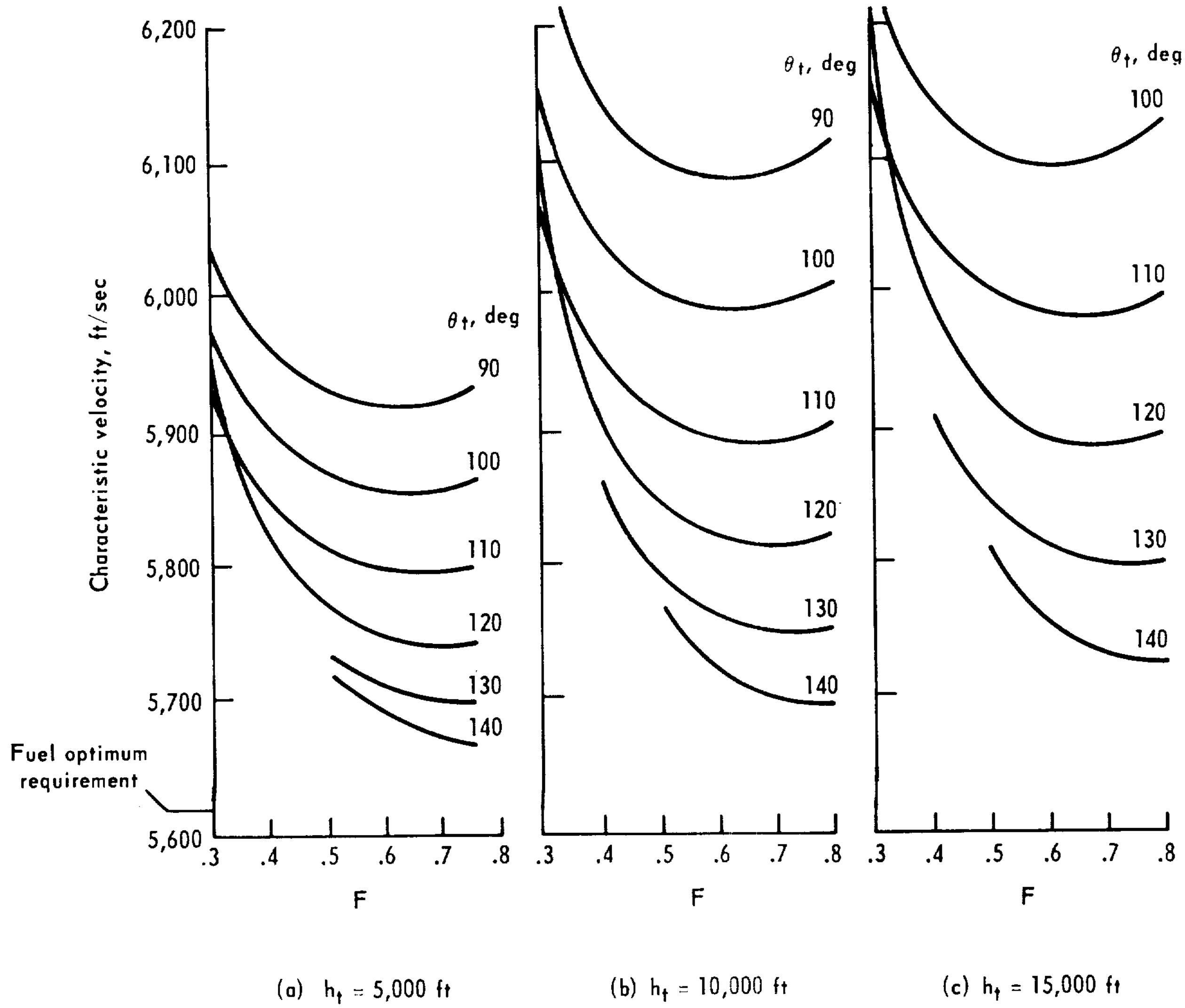



Figure 8.- Characteristic velocity for design powered descent. Initial conditions: 50,000-foot circular orbit; final conditions: $h = 1{,}000$ ft, $V = 75$ ft/sec, $\gamma = 0°$

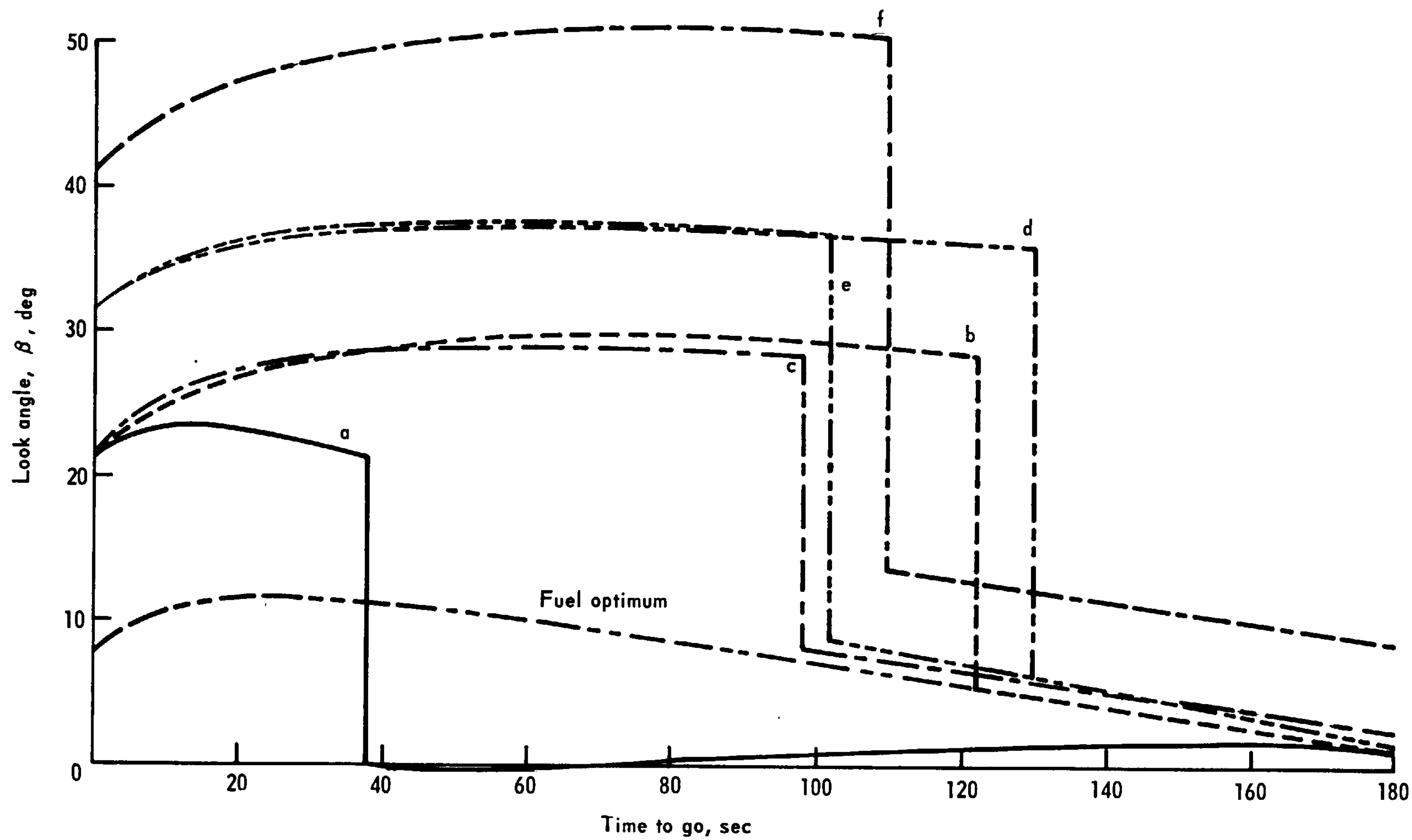



Figure 9.- Time history of look angle to nominal landing site. (3,000 feet downrange of 1,000-foot altitude point.)

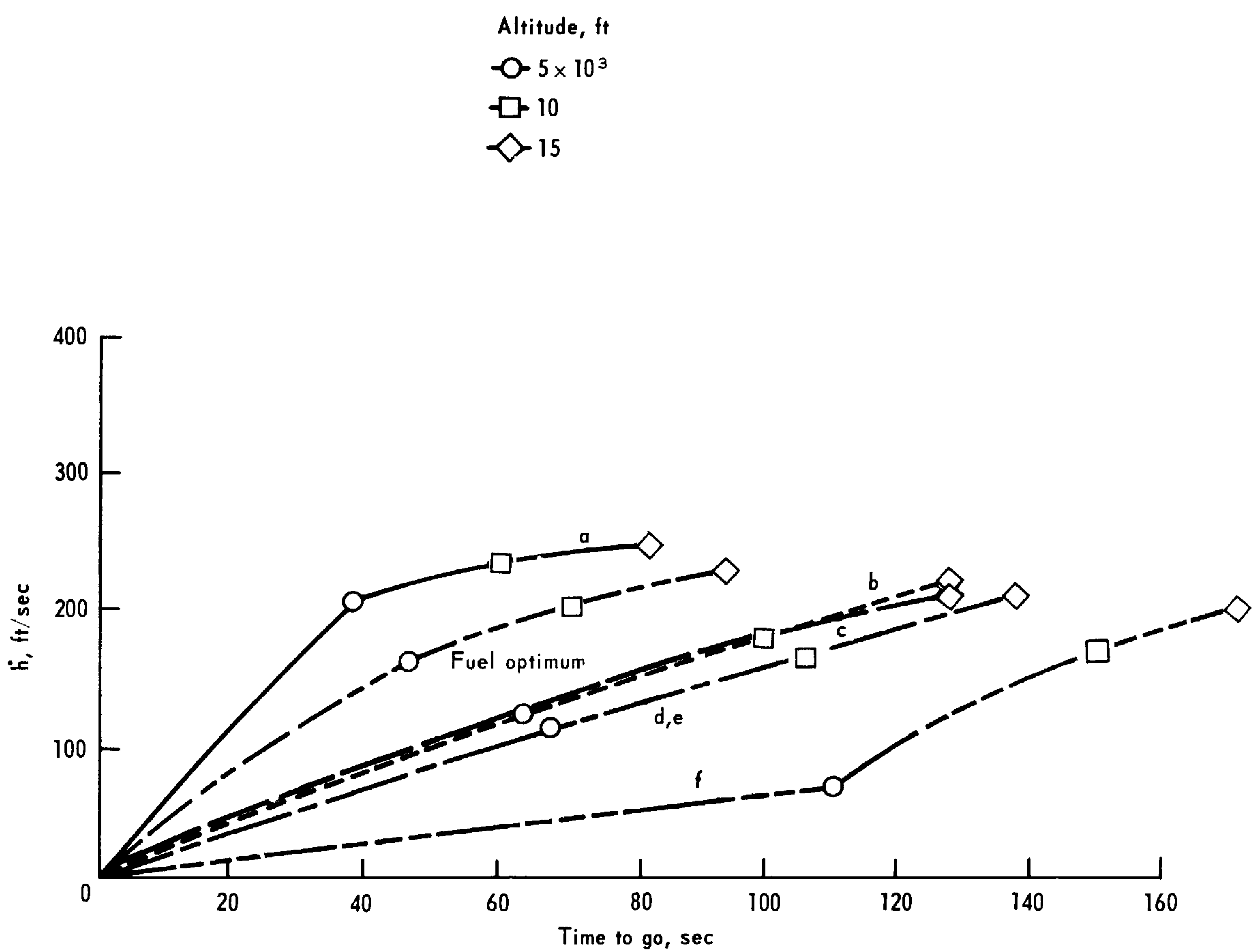



Figure 10.- Time history of vertical velocity.

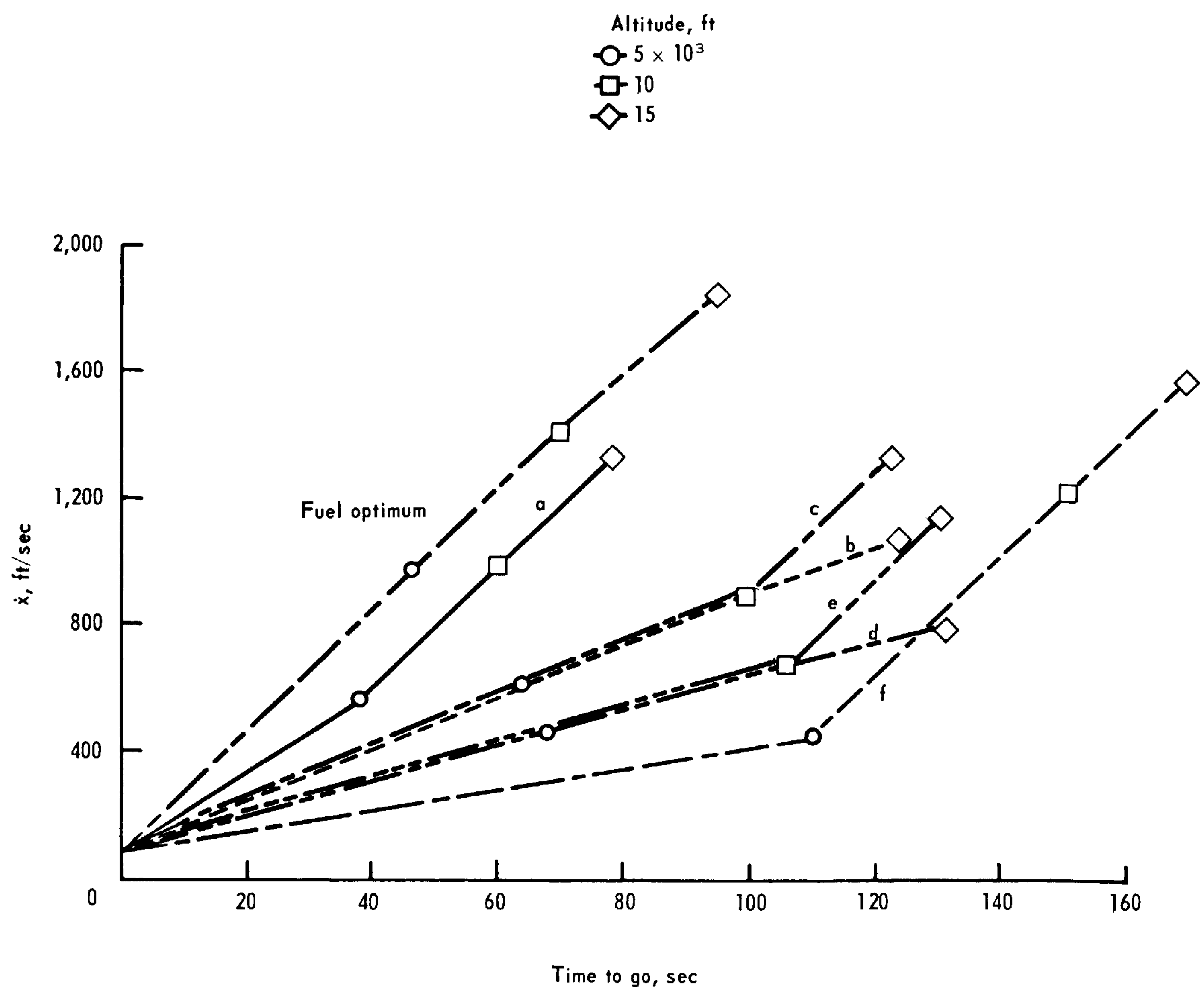
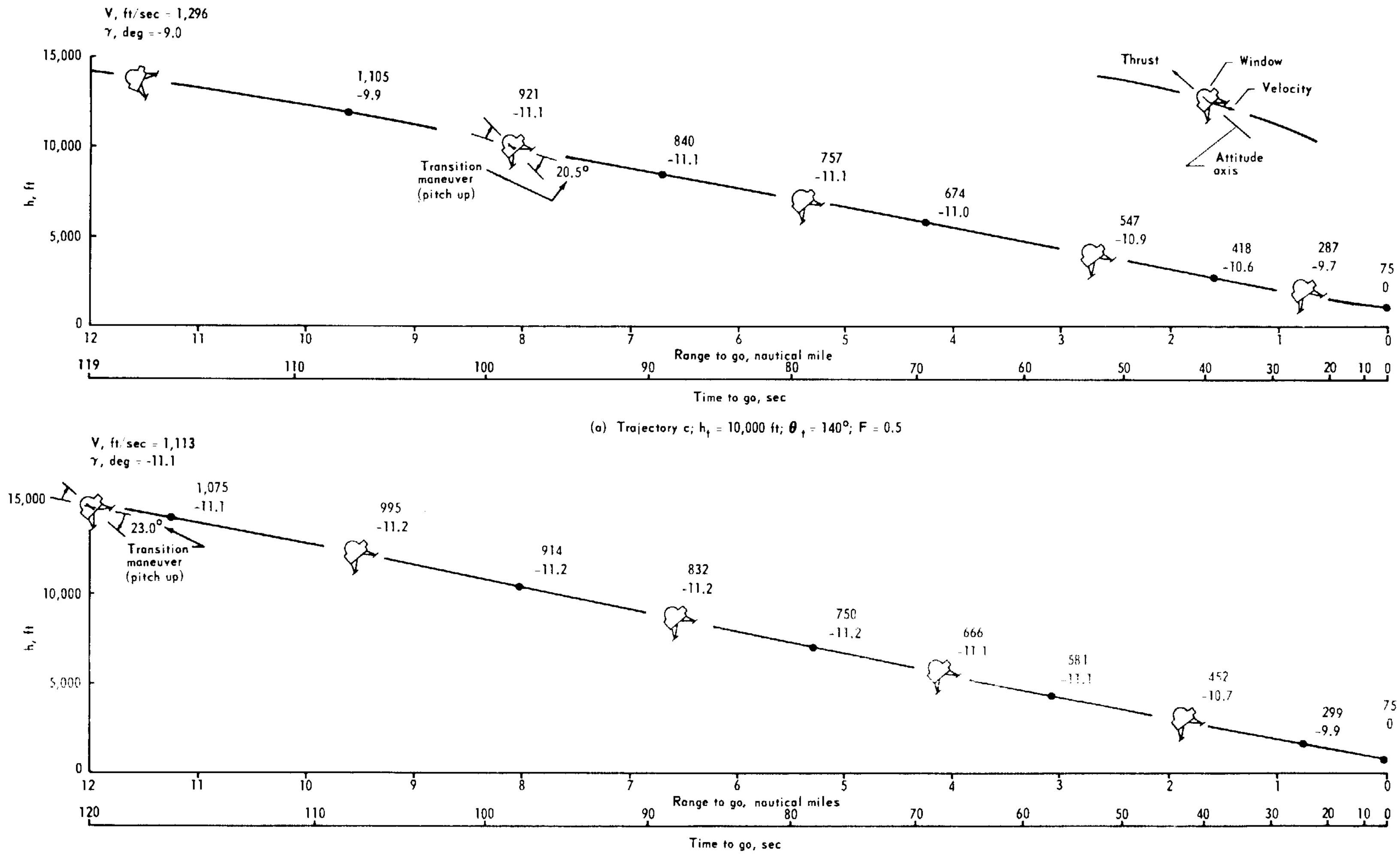


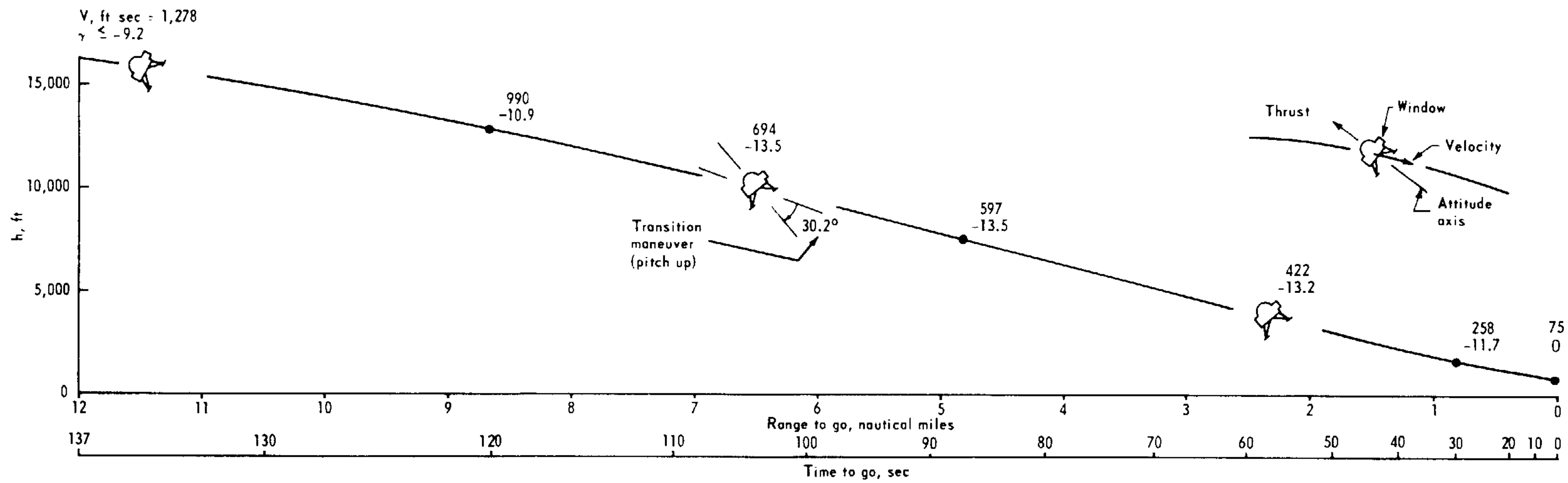

Figure 11.- Time history of horizontal velocity.



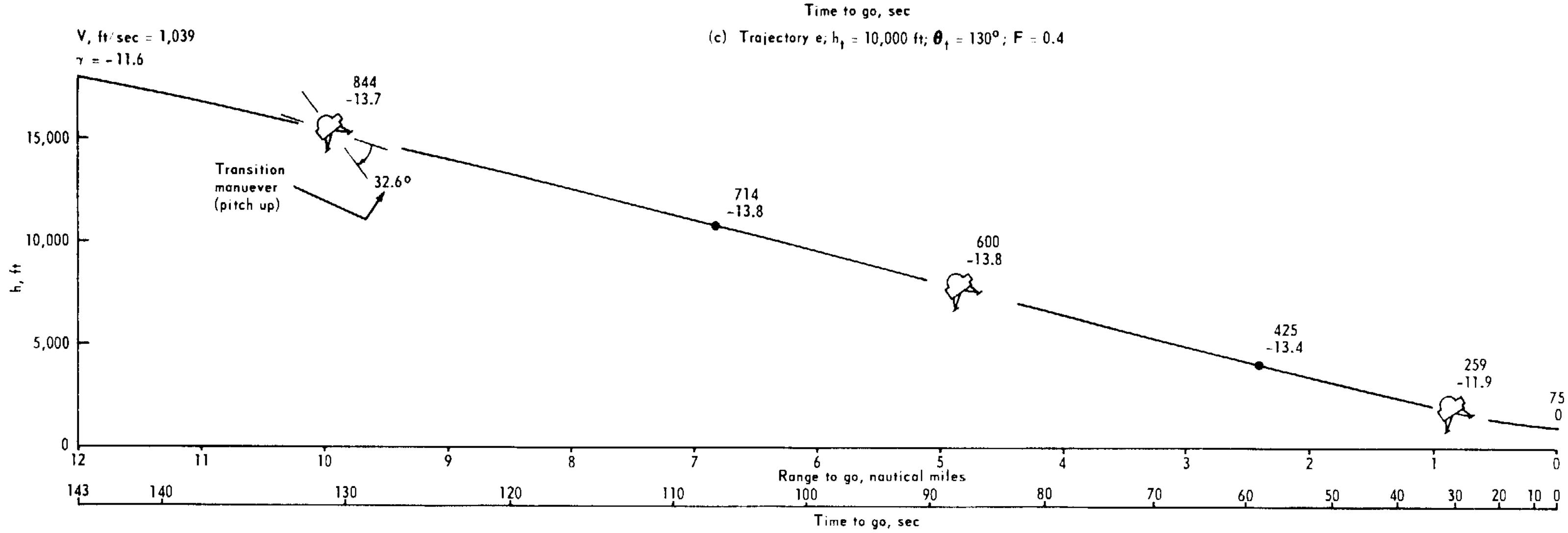

(a) Trajectory c; $h_t$ = 10,000 ft; $\theta_t$ = 140°; F = 0.5

(b) Trajectory b; $h_t$ = 15,000 ft; $\theta_t$ = 140°; F = 0.5

Figure 12.- Profiles of several landing approach transition trajectories

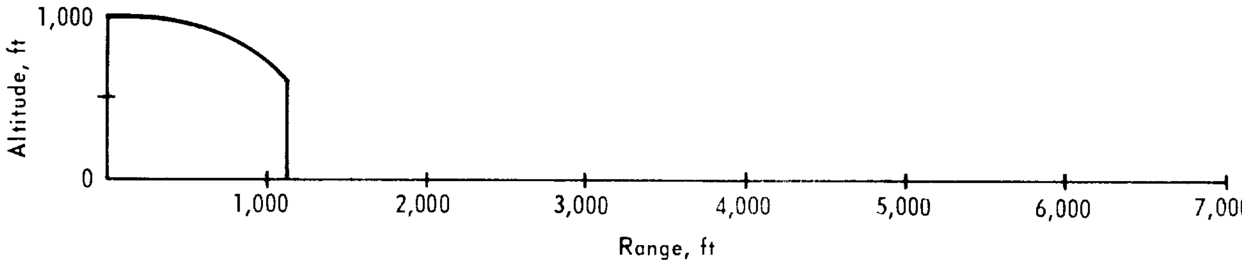



(c) Trajectory e; $h_t$ = 10,000 ft; $\theta_t$ = 130°; F = 0.4

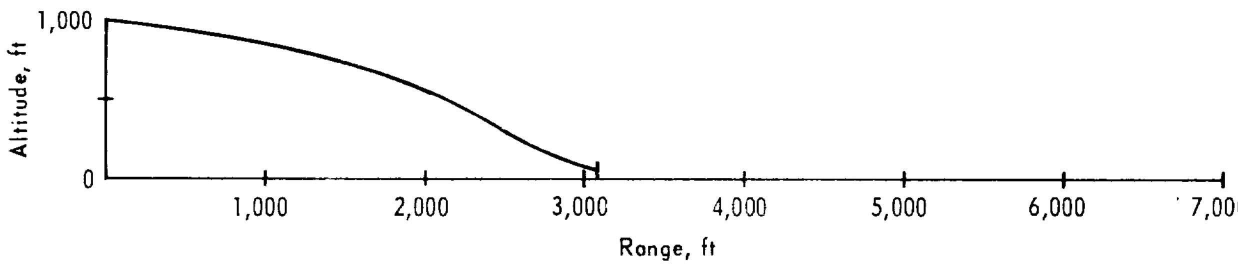



(d) Trajectory d; $h_t$ = 15,000 ft; $\theta_t$ - 130°; F - 0.4

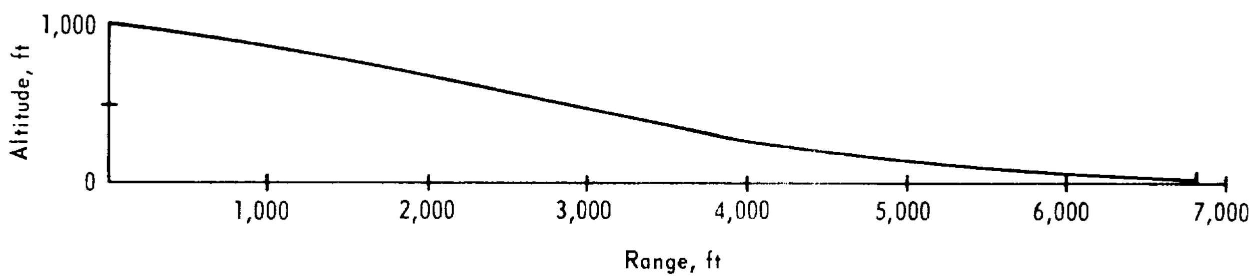
Figure 12. - Concluded.




(a) Minimum range descent




(b) Nominal descent




(c) Maximum range descent

Figure 13.- Lunar excursion module descents from 1,000 feet.
Limitations: $60° \leq \theta \leq 120°$; $\dot{h}_{max} = -20$ ft/sec.

NASA-S-64-873

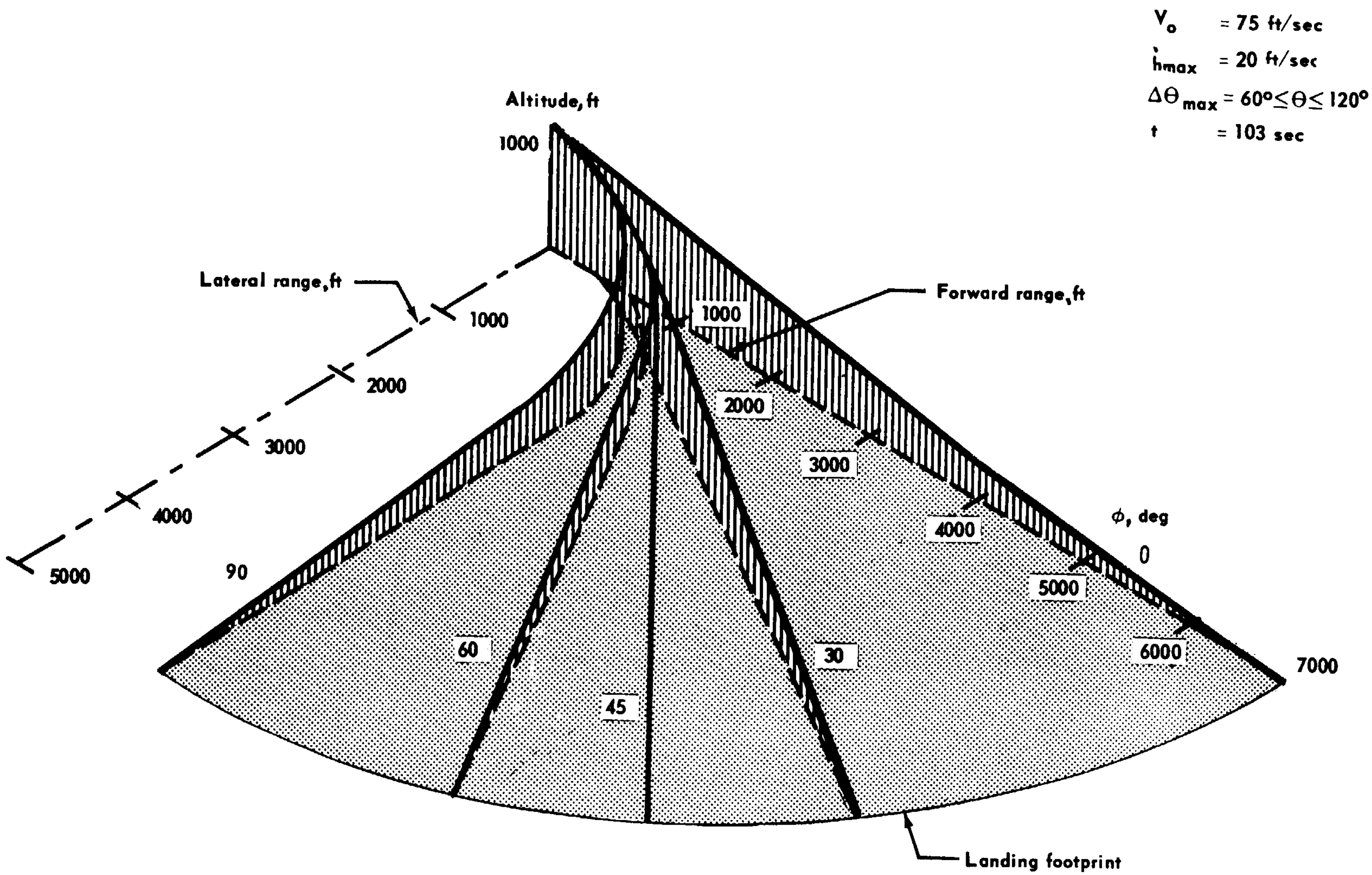



Figure 15.- Maximum footprint for lunar excursion module descent from 1,000 feet. (Only half of footprint is shown; footprint is symmetrical about forward axis.)

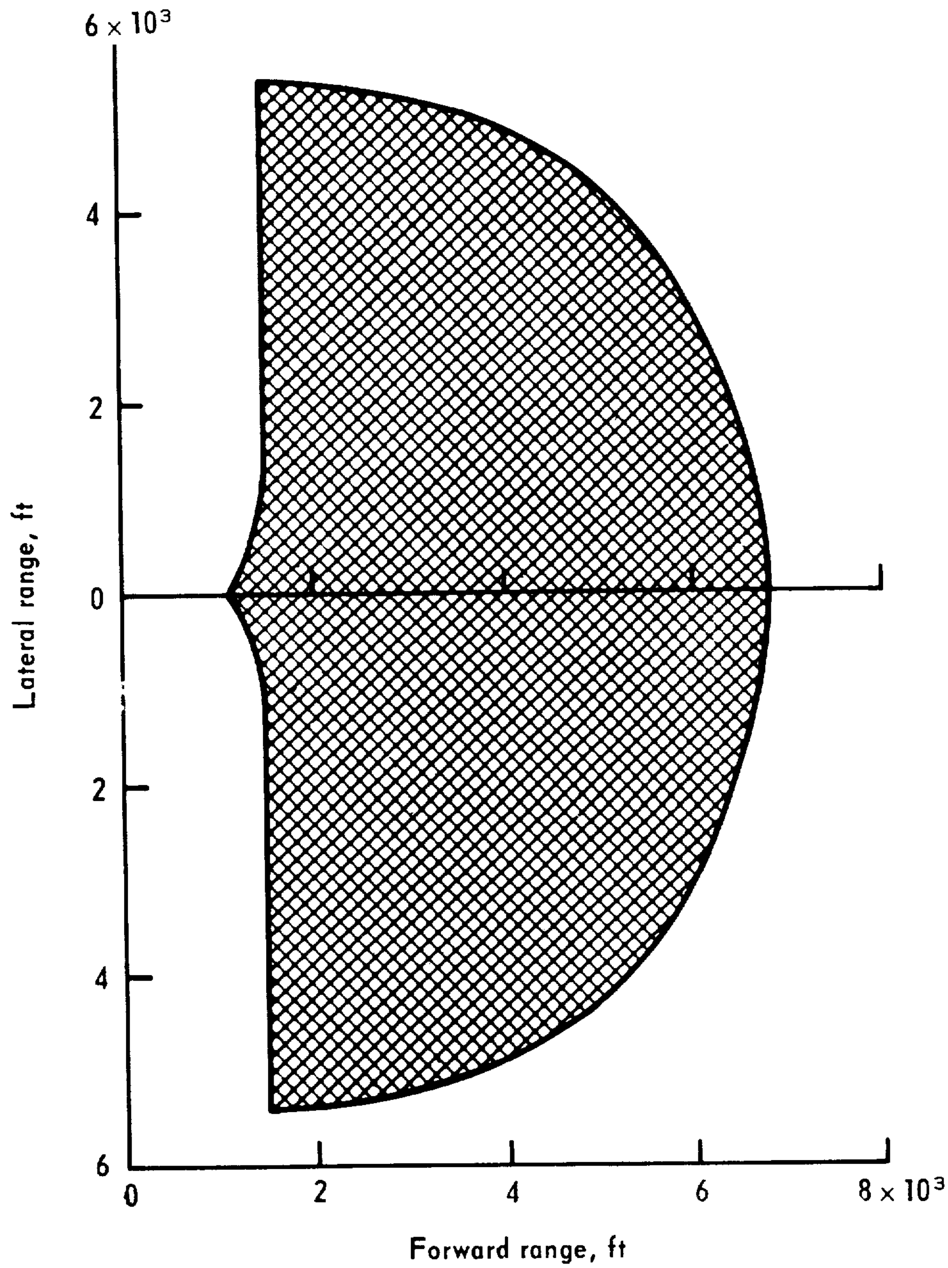



Figure 14.- Maximum footprint for lunar excursion module descent from 1,000 feet. $V_o$ = 75 ft/sec; $\gamma_o$ = 0°.