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ABSTRACT

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In many recent papers concerned with providing an explanation for the geomagnetic anomaly, agreement with measured data has been obtained from the equations of motion for electrons and ions when used with an empirical boundary condition, whereas poor agreement has resulted from attempts to numerically integrate the commonly employed form of the continuity equation. We have been able to explain this discrepancy by demonstrating that the assumptions used to derive this form of the continuity equations do not agree with observation.

Since the equations of motion do provide a favorable description for the geomagnetic anomaly, we have studied the possible physical models leading to the form of the equations used, and found that although field aligned diffusive equilibrium provides the correct form, a more reasonable assumption concerning electron and ion collisions with neutrals also leads to the same result. We have then been able to provide a more realistic theoretical description of the geomagnetic anomaly by employing an analytic form for the boundary condition which is in better agreement with measurement than those previously used.

Finally, by combining the equations of motion for neutrals, electrons and ions, we have been able to indicate geomagnetic control for the neutral atmosphere in the lower F region of the ionosphere, although the exact shape of this distribution is unknown.

AUTHOR

INTRODUCTION

In recent months, it has become increasingly evident that some confusion exists in the understanding of the basic physical mechanisms governing diffusion and the existence of the geomagnetic anomaly in the ionosphere. This apparent confusion arises by comparison of the work of Chandra (1964), (to be referred to as C-I), in which it is shown that the assumption of ambipolar diffusion along a field line cannot lead to geomagnetic control of the charged particles, and such papers as Goldberg and Schmerling (1962, 1963), (to be referred to as GS-I and GS-II), and Goldberg, Kendall, and Schmerling (1964), (to be referred to as GKS), in which this process does appear to produce geomagnetic control of the charged particle density in the ionosphere.

The purpose of this paper is to describe and resolve the confusion which exists in the field at this moment, and then to point out the new problems with which we must contend in order to derive and apply the diffusion equation to ionospheric problems correctly. In addition, a section will be devoted to an improved theoretical description of the geomagnetic anomaly by using an analytic expression for the vertical electron density distribution at the equator which is more in accordance with measurement than the simple Chapman type distribution employed in GKS.

FUNDAMENTAL EQUATIONS AND DEFINITIONS

The major cause of confusion appears to lie in the application of two phrases, viz. ambipolar diffusion and diffusive equilibrium. Let us investigate and discuss each of these terms to determine how loose usage of them has led to some of the current problems of misunderstanding.

In the normal sense, ambipolar diffusion refers to a plasma in which the negative (electrons) and positive (ions) charges do not move independently due to the influence of the electric field caused by their Coulomb interactions. In this medium, the electrons and ions drift in pairs and this motion of electron-ion pairs is referred to as ambipolar diffusion. The condition for ambipolar diffusion in a neutral plasma is thereby

$$\vec{v}_e = \vec{v}_i = \vec{v} \tag{1}$$

where $\vec{v}$ is macroscopic velocity and the subscripts e and i refer to electrons and ions respectively. When

$$\vec{v} = 0 \tag{2}$$

the condition for diffusive equilibrium is satisfied.

The implications of (1) are quite straightforward, as shown in C-I. In an isothermal atmosphere and in the presence of a magnetic field, this requires $\nabla x(\vec{v}x\vec{B}) = 0$. In particular, the assumption of field aligned plasma diffusion ($\vec{v}x\vec{B} = 0$) can only be satisfied for the trivial case, $\vec{v} = 0$, resulting in a hydrostatic distribution of electron density independent of geomagnetic latitude.

On the other hand, favorable comparison between Alouette topside sounder measurements and theoretical calculations of the geomagnetic anomaly has been obtained in GKS by assuming conditions of ambipolar diffusion and diffusive equilibrium along field lines, thereby indicating a possible conflict with the results in C-I. The problem resolves itself once one investigates the meaning of ambipolar diffusion and diffusive equilibrium in the GKS sense.

Let us first write the general equations of motion for neutrals, electrons and ions, respectively, where the subscript n refers to neutrals. Following C-I:

$$\frac{n_e m_e m_n}{m_e + m_n} \nu_{en} (\vec{v}_n - \vec{v}_e) + \frac{n_i m_i m_n}{m_i + m_n} \nu_{in} (\vec{v}_n - \vec{v}_i) = -\nabla p_n + n_n m_n \vec{g} \tag{3}$$

$$\frac{n_i m_e m_i}{m_e + m_i} \nu_{ei} (\vec{v}_e - \vec{v}_i) + \frac{n_e m_e m_n}{m_e + m_n} \nu_{en} (\vec{v}_e - \vec{v}_n) = -\nabla p_e + n_e m_e \vec{g} - e\, n_e (\vec{E} + \vec{v}_e \times \vec{B}) \tag{4}$$

$$\frac{n_i m_i m_e}{m_e + m_i} \nu_{ei} (\vec{v}_i - \vec{v}_e) + \frac{n_i m_i m_n}{m_i + m_n} \nu_{in} (\vec{v}_i - \vec{v}_n) = -\nabla p_i + n_i m_i \vec{g} + e\, n_i (\vec{E} + \vec{v}_i \times \vec{B}) \tag{5}$$

where n is number density, $\nu_{k\ell}$ is the collision frequency between the k^{th} and ℓ^{th} particle, m is mass, p is pressure, $\vec{g}$ is gravitational acceleration, e is the absolute value of electron charge, $\vec{E}$ is electric field, and $\vec{B}$ is magnetic field. In writing equations (3) - (5) it is assumed that $\frac{\nu_{\ell k}}{n_k} = \frac{\nu_{k\ell}}{n_\ell}$.

In the following we assume that the plasma is in a quasi-neutral state

$$n_e \approx n_i = N \tag{6}$$

and that the electrons, ions and neutrals obey the ideal gas law in the ionosphere,

$$p_j = n_j k\, T_j \tag{7}$$

where k is Boltzmann's constant and T is temperature. Furthermore, we assume thermal equilibrium, i.e.

$$T_e = T_i = T \qquad (8)$$

Then,

$$p_e = p_i = p \qquad (9)$$

In addition, we assume for simplicity that

$$\vec{v}_n \approx 0 \qquad (10)$$

Then summation of (4) and (5) provides

$$\frac{m_n m_e N}{m_e + m_n} \nu_{en} \vec{v}_e + \frac{m_n m_i N}{m_i + m_n} \nu_{in} \vec{v}_i = -2\nabla p + N(m_e + m_i)\vec{g} + \vec{J} \times \vec{B} \qquad (11)$$

where $\vec{J}$ is current density, defined as

$$\vec{J} = Ne\ (\vec{v}_i - \vec{v}_e) \qquad (12)$$

Since we are investigating ambipolar diffusion and diffusive equilibrium in the GKS sense, it is desirable to write this equation in component form along a field line as

$$\frac{m_n m_e}{m_e + m_n} \nu_{en} \vec{v}_e \cdot \vec{h} + \frac{m_n m_i}{m_i + m_n} \nu_{in} \vec{v}_i \cdot \vec{h} = \left[\frac{-2kT\nabla N}{N} + (m_e + m_i)\vec{g}\right] \cdot \vec{h} \qquad (13)$$

where $\vec{h}$ is a unit vector in the direction of the magnetic field. Let us write (13) in more familiar form by using

$$m_e \ll m_i,\ m_n \tag{14}$$

and defining the scale height of the ionizable constituent as H_i, where

$$H_i = \frac{kT}{m_i g} \tag{15}$$

Also, for convenience, we make the approximation

$$m_n \approx m_i \tag{16}$$

Then

$$\frac{m_e \nu_{en}}{2} \vec{v}_e \cdot \vec{h} + \frac{m_i}{4} \nu_{in} \vec{v}_i \cdot \vec{h} = -kT\left(\frac{\nabla N}{N} + \frac{\vec{i}_r}{2H_i}\right) . \vec{h} \tag{17}$$

Finally, we write

$$\vec{h} = -(\vec{i}_r \sin I + \vec{i}_\theta \cos I) \tag{18}$$

where $\vec{i}_r$ and $\vec{i}_\theta$ are unit vectors in the r and θ directions and I is the magnetic dip angle, reckoned positive when the north seeking pole of the needle points downward. Now, if we treat ambipolar diffusion in the GKS sense, we simply imply that the electron and ion velocity components in the field direction are equal, i.e.

$$\vec{v}_e \cdot \vec{h} = \vec{v}_i \cdot \vec{h} = v_{11} \tag{19}$$

Applying (18) and (19) in (17), we obtain

$$v_{11} = \frac{-kT}{\mu\nu} \left[\sin I \left(\frac{1}{N}\frac{\partial N}{\partial r} + \frac{1}{2H_i}\right) + \frac{\cos I}{Nr} \frac{\partial N}{\partial \theta}\right] \tag{20}$$

where

$$\mu\nu = \frac{m_e \nu_{en}}{2} + \frac{m_i \nu_{in}}{4} \tag{21}$$

Assuming that

$$m_e \nu_{en} << m_i \nu_{in} \tag{22}$$

because of (14), we may write

$$\mu\nu \approx \frac{m_i \nu_{in}}{4} \tag{23}$$

Equation (20) is a familiar result derived in such papers as Kendall (1962) and GS-II. However, it is clearly not the result of ambipolar diffusion, which is given by (1), but instead, the result of a statement concerning the field line components of electron and ion velocities given by (19).

If we now demand

$$v_{\parallel} = 0 \tag{24}$$

which is the statement implying diffusive equilibrium along a field line in the GKS sense, we obtain the familiar equation

$$\sin I \left(\frac{1}{N}\frac{\partial N}{\partial r} + \frac{1}{2H_i}\right) + \frac{\cos I}{Nr}\frac{\partial N}{\partial \theta} = 0 \tag{25}$$

which can also be written as

$$\frac{1}{N}\frac{dN}{dr} + \frac{1}{2H_i} = 0 \tag{26}$$

provided we recognize that r and θ are not independent in (25) but related by the dipole field conditions

$$r = r_o \sin^2 \theta \tag{27}$$

and

$$\tan I = 2 \cot \theta \tag{28}$$

It is evident that (26) can only be treated in total derivative form if the integration is carried out along the field line.

Statements concerning the components of vectors in a particular direction, such as (19), do not imply any conditions on the total vector. As a result, (25) has not required the assumption of any restrictions on the behavior of the velocity components normal to the field lines.

Equation (26) has been the basis of describing geomagnetic control in the upper F-region in GKS paper. Although (26) has been derived assuming diffusive equilibrium along a field line, it is undersirable to apply this concept because it is of purely hypothetical nature. We now investigate other assumptions to find a more realistic justification for (26).

Let us rewrite (17) as

$$\frac{m_e \nu_{en}}{2} v_{e_{11}} + \frac{m_i \nu_{in}}{4} v_{i_{11}} = -kT\left(\frac{\nabla N}{N} + \frac{\vec{i}_r}{2H_i}\right).\vec{h} \tag{29}$$

We find two ways for the right hand side of (29) to approach zero. The first imposes a new condition on the velocities, viz:

$$\vec{v}_e = - \frac{m_i \nu_{in} \vec{v}_i}{2m_e \nu_{en}}$$

or

$$v_{e_{\parallel}} = - \frac{m_i \nu_{in} v_{i_{\parallel}}}{2m_e \nu_{en}} \tag{30}$$

a result which, although possible, would require a very special condition that the electron velocity be of the order of 10^3 times greater than and in the opposite direction of the ion velocity.

However, if we can demand that the collision frequencies between electrons and neutrals and between ions and neutrals be sufficiently small so that the drag forces arising due to collision be negligible as compared to the pressure gradient, gravity and Lorentz forces, it is possible to derive equation (26) without imposing any restriction on the velocities of electrons and ions. We believe that this assumption is more realistic in the upper F-region where the gyro-frequencies of electrons and ions are much greater than their corresponding collision frequencies.

Although the collision frequency assumption is physically more desirable, it prevents us from obtaining a simple expression for $v_{e_{\parallel}}$ or $v_{i_{\parallel}}$. Instead, we must return to the original equations of motion, (4) and (5), and solve for $\vec{v}_e$ and $\vec{v}_i$ explicitly, as has been carried out in the appendix in C-I. Unfortunately this introduces a very serious complication in the work because of the difficulty in eliminating electric field from the expressions of $\vec{v}_e$ and $\vec{v}_i$ without making specific assumptions about the relationship between $\vec{v}_e$ and $\vec{v}_i$. The implications of these assumptions will be discussed in the latter part of this paper. In the following section we proceed to discuss the physical implications of equation (26).

THE ELECTRON DENSITY DISTRIBUTION WITH A VARIABLE SCALE HEIGHT

Equation (26) can be integrated along a field line to provide the general solution

$$N(r,\theta) = f(r_o,\pi/2)e^{\frac{r\cot^2\theta}{2H_i}} \tag{31}$$

However, if we treat T and m_i constant but recognize that g is proportional to $1/r^2$, H_i is then proportional to r^2, and we obtain

$$N(r,\theta) = f(r_o,\pi/2)e^{\frac{r\cos^2\theta}{2H_i(r)}} \tag{32}$$

In both cases, $f(r_o,\pi/2)$ is an arbitrary function of height at the equator which cannot be determined by the equations of motion from which (31) or (32) are derived. The function $f(r_o,\pi/2)$ must therefore be given as a boundary condition in this problem and can only be determined empirically or by use of additional equations governing the physics of the problem.

Since (31) or (32) depend exclusively upon the equations of motion, it appears that an additional equation, such as the continuity equation, should lead to the desired boundary condition. Unfortunately, as we will show in the next section, the derivation and solution of the continuity equation depend upon a knowledge of $\vec{E}$. Thus, the complexity of the problem becomes quite formidable and it is difficult to anticipate a simple method of solution at this time.

Instead we depend upon an empirical type boundary condition, which may very well be the solution of the correct continuity equation, to derive the explicit form of the electron density distribution.

The incorporation of a Chapman distribution for the boundary condition in (31) leads to the results obtained in GKS. Since such a boundary condition can only be considered as a rough approximation to the shape of the actual vertical electron density distribution at the equator, it is desirable to employ an analytic boundary condition which more closely resembles the true height profiles. Chandra (1963) has proposed a modified form of the Chapman function which includes the effect of variable scale height and which is found to fit the measured vertical distribution for electron density at mid-latitudes far more accurately than the simple Chapman form. We assume here that such a function also describes the vertical electron density distribution at the equator. We can then write

$$f(r_o,\pi/2) = N_{r_{mo}} \exp \tfrac{1}{2} \left\{ 1 - \frac{r_o - r_{mo}}{H_o\left[1 - \alpha \exp\left(-\alpha \dfrac{(r_o - r_{mo})}{2H_o}\right)\right]} - \exp\left[- \frac{r_o - r_{mo}}{2H_o\left[1 - \alpha \exp\left(- \dfrac{\alpha (r_o - r_{mo})}{2H_o}\right)\right]}\right]\right\} \tag{33}$$

where H_o is the scale height of the ionic constituent and $N_{r_{mo}}$ is the value of electron density at the equatorial height r_{mo}. The parameter α, which is a measure of departure from the simple Chapman function, and thereby a shape factor, is defined as

$$\alpha = \frac{H_o - H(r_{mo})}{H_o} \tag{34}$$

where $H(r_{mo})$ is that value of $H(r_o)$ at $r_o = r_{mo}$. Also, r_o is understood to be the radial height specifically at $\theta = \pi/2$.

Although it will not be shown here, (31) and (32) produce nearly identical results in the equatorial region because the small variation of r in the height region of our interest. Furthermore, the simplified form given by (31) is more convenient for comparison with the results of the GKS paper. We therefore substitute (33) into (31) and obtain

$$N(r,\theta) = N_{r_{mo}} \exp \tfrac{1}{2} \left\{ 1 - \frac{r \csc^2\theta - r_{mo}}{H_o\left[1 - \alpha \exp\left(-\frac{\alpha(r \csc^2\theta - r_{mo})}{2H_o}\right)\right]} + \frac{r \cot^2\theta}{H_i} \right.$$

$$\left. - \exp\left[- \frac{r \csc^2\theta - r_{mo}}{2H_o\left[1 - \alpha \exp\left(-\frac{\alpha(r \csc^2\theta - r_{mo})}{2H_o}\right)\right]} \right] \right\} \tag{35}$$

Equation (35) then provides a general expression for the electron density at all heights and colatitudes provided that we are in a region where the effects of collision can be neglected.

The variation of $N(h,\theta)/N_{h_{mo}}$ with colatitude at constant height is shown in Figures 1-4 for various values of α, h_{mo} and H_o. In these figures, we have converted radial height r to altitude h by taking the earth's radius as 6370 km. We have also assumed that $H_o = H_i$ because the effective scale height in (33) approaches H_o at high altitudes and Chandra (1963) has indicated that H_o becomes equal to H_i at these altitudes.

We first note that the basic features of the theoretical electron density distribution are unaltered from those first obtained in GKS to describe the geomagnetic anomaly in the vicinity of the equator. Once again the theoretical description breaks down in the bottomside but this is precisely the region where the neglect of momentum transfer terms becomes in-valid. Furthermore, comparison of Figures 3 and 4 clearly shows the insensitivity of the topside results to the parameters h_{mo} and H_o (except for shifting the constant height profile scale vertically). We therefore conclude the principal properties of the curves can be studied quite extensively by simply altering the shape factor α.

The changes due to variations in α are shown by comparison of Figures 1, 2 and 3. We have also provided a more detailed comparison for one particular height profile in Figure 5. Although we have included values up to $\alpha = 0.6$ to demonstrate the trend of the curves, the highest values are extreme and not likely to be representative of ionospheric conditions. On the other hand, $\alpha = 0.1$ to $\alpha = 0.4$ are very reasonable values for us to expect under normal conditions representing diurnal and solar cycle variations.

Finally, in equation (35), if we identify the term $H_o\left[1-\alpha \exp(-\alpha(r \csc^2\theta - r_{mo})/2H_o)\right]$ with $1/k$ of the GKS paper, we see that since α is positive, $kH_i>1$ is true for all heights. In particular, if $\alpha = 0$, we generate curves which are identical to those in GKS for $kH_i=1$. This explains why $kH_i>1$ provides the closest fit with experimental data in that paper.

PROBLEMS INVOLVED IN THE DERIVATION OF THE DIFFUSION EQUATION

In the previous sections we have seen how the equations of motion for electrons and ions are sufficient to obtain a theoretical description of the electron density distribution in the topside equatorial region of the ionosphere under equinox conditions. This has required us to make certain assumptions concerning collision frequencies or velocity components along field lines and also forced the application of an empirical boundary condition at the equator. In order to produce the empirical boundary condition theoretically and also obtain a solution which is valid in both the topside and bottomside equatorial F region, it is necessary to turn to the continuity equation for additional information. Using the explicit expressions for velocity which are derivable from the equations of motion, it is then possible to derive the diffusion equations associated with the ionosphere.

If we simply require total ambipolar diffusion (equation 1) to occur in the ionosphere so that $\vec{v}$ is independent of the electric field explicitly, and also demand that, in all regions concerned, the motion along field lines are much larger than the drifts normal to field lines, we must then invoke $\vec{v} \times \vec{B} = 0$ which, using (4), (5) and (28), gives the constraint equation

$$\frac{1}{N}\frac{\partial N}{\partial r} + \frac{1}{2H} = \frac{2}{N}\frac{\partial N}{r\partial\theta}\cot\theta \tag{36}$$

This leads to a hydrostatic distribution of electron density which does not agree with measured results, as has been demonstrated in C-I. A second approach (Kendall, 1962, and GS-II) is the assumption that ambipolar diffusion exists only along field lines (see equation (19)). Thus, if we assume that the parallel velocity components of electron and ion velocity are equal and much greater than either of the unequal perpendicular velocity components, we can write (20) as a good approximation for the entire velocity. In mathematical notation, we have

$$\vec{v} \cdot \vec{h} = \left|\vec{v}_{\parallel}\right| = v_{\parallel} \gg v_{e_{\perp}},\ v_{i_{\perp}} \tag{37}$$

where $v_{e_\perp}$ and $v_{i_\perp}$ are the perpendicular components of electron and ion macroscopic velocities respectively. This implies that

$$\vec{v}_{11} \approx \vec{v} \tag{38}$$

and provides us with a velocity expression independent of electric field. The general contention has been that (38) allows us to write the steady-state continuity equation in the following form:

$$Q - L = \nabla \cdot N \vec{v} \approx \nabla \cdot N \vec{v}_{11} \qquad (39$$

where Q and L are production and loss respectively. The procedure has been to substitute (20) into (39) and obtain the well known form of the two dimensional diffusion equation without invoking the equation of constraint (36).

We wish to discuss this approach by first questioning the validity of (37), and then demonstrating that even if it were true, (38) cannot in general imply (39) without the additional inclusion of the constraint equation. This will demonstrate that the field line ambipolar diffusion approach with neglect of the perpendicular velocity components is identical to the total ambipolar diffusion case in which velocities are assumed to lie along field lines. Thus, the results of the two approaches are identical, leading to the conclusion that ambipolar diffusion in which the macroscopic velocity lies along a field line, cannot be the correct physical model to describe the equatorial electron density distribution in the F region of the ionosphere.

Let us first consider (37). We have already seen that the assumption of diffusive equilibrium along a field line (v_{11} = o) leads to a correct description of the electron density in at least the topside region of the ionosphere. If this is the true model of the physical situation, then it is inconsistent with (37) and we cannot expect any results obtained using (37) to provide us with correct results concerning this region. If, on the other hand, the neglect of momentum transfer terms can be attributed to small collision frequencies instead of diffusive equilibrium, (37) need not be violated. This might be a further justification for validating the collision frequency assumption instead of the diffusive equilibrium model. Unfortunately, as we approach the equator, we see from (20) that $v_{\shortparallel}$ approaches zero since both sin I and $\partial N/\partial\theta$ approach zero. The latter condition is based strictly upon the empirical condition of symmetry about the equator. We therefore find that no matter how small $v_{e_\perp}$ and $v_{i_\perp}$ may be, there will always be a region about the equator in which (37) does not apply unless

$$v_{e_\perp} = v_{i_\perp} = o \tag{40}$$

which is identical to the equation of constraint, (36).

We now return to the second question. That is, even if the parallel components of electron and ion velocities are much greater than the perpendicular component, which could still be possible provided $v_{e_{\shortparallel}} \neq v_{i_{\shortparallel}}$, is it possible to describe the electron density distributions in the entire region of the ionosphere by (39)? We note that

$$\nabla \cdot N\vec{v} = \vec{v} \cdot \nabla N + N\nabla \cdot \vec{v} = (\vec{v}_{\shortparallel} + \vec{v}_{\perp}) \cdot \nabla N + N\nabla \cdot (\vec{v}_{\perp} + \vec{v}_{\shortparallel}) \tag{41}$$

where $\vec{v}$ is now either the electron or ion velocity and

$$\vec{v} = \vec{v}_{\shortparallel} + \vec{v}_{\perp} \tag{42}$$

Now, in order to write (39), we must demand that

$$\vec{v}_{11} \cdot \nabla N + N\nabla \cdot \vec{v}_{11} >> \vec{v}_{\perp} \cdot \nabla N + N\nabla \cdot \vec{v}_{\perp} \tag{43}$$

Although (43) could be true for certain special cases, there is no a priori guarantee that (43) will be implied by (37) in general without the additional condition that $v_{\perp} = o$. Thus, if we are to write (39) as a direct and general implication of (37), we are once again forced to employ the constraint equation.

We cannot state that (40) holds in a very small region about the equator so that its effect outside this region can be neglected. The geomagnetic anomaly itself is a second order effect and we cannot expect to reproduce it by neglecting second order terms which are responsible for its existence.

We therefore find that if ambipolar diffusion exists in the ionosphere, and if it is restricted to the field line direction, we cannot assume (37) without imposing an additional constraint equation. Furthermore, (37) does not generally imply (39) in the ionosphere with or without ambipolar diffusion unless the constraint equation is also employed. However, since (37), (39), and the assumption of ambipolar diffusion along a field line do not provide to the correct description of equatorial electron density, we must conclude that these assumptions are not valid in a theory leading to a description of the electron density distribution in the equatorial ionosphere.

Kendall (1962), and Rishbeth, Lyon and Peart (1963), have attempted to numerically integrate (39) derived from (20) and (37), without invoking the equation of constraint. They have been unable to obtain the correct description of the geomagnetic anomaly and have therefore concluded that diffusion may not be a very important physical process governing the measured distribution of electron density. However, on the basis of the discussion presented in this section, it now appears that the physical assumptions used in deriving the form of the continuity equation used in their work may not be valid, which simply implies that the diffusion equation is far more complicated than originally believed. Since (19) and (37) are no longer valid, we can no longer equate electron and ion velocities to eliminate electric field. Instead, we must write separate continuity equations for electrons and ions and describe the behavior of electric field before it is possible to obtain the correct theoretical description of the geomagnetic anomaly.

It may appear that the results presented in GS-II are also not valid for the reasons discussed above. However, a closer inspection of GS-II shows that no new information was obtained from the solution of continuity equation than that already available from the equations of motion. The equation discussed in GS-II was simply

$$\nabla \cdot N\vec{v}_{11} = 0 \tag{44}$$

where the explicit production and loss terms were neglected in obtaining the series solution. Furthermore, as shown in GKS, the equation of motion leading to (26), whether derived assuming $\vec{v}_{11} = 0$, or by making assumptions concerning collision terms, has the identical solution to that obtained from (44) in GS-II. For the case $\vec{v}_{11} = 0$, (44) obviously cannot give any new information. This explains why the empirical boundary condition was necessary to obtain a non-arbitrary solution from (44) in GS-II. We should point out, however, that solutions of (39) making use of explicit production and loss terms should not give correct results in the equatorial regions of the ionosphere for the reasons discussed in this section.

THE DISTRIBUTION OF THE NEUTRAL ATMOSPHERE

In this section we will show that when the drag forces are not negligible, as might be the case in the lower F-region and E-region, it is possible to study the behavior of the neutral atmosphere without imposing any restriction on the velocities of the various constituents. To obtain the necessary starting equation, we first sum (3), (4) and (5):

$$-\nabla(p_e + p_i + p_n) + (n_n m_n + Nm_i)\vec{g} + \vec{J} \times \vec{B} = 0 \qquad (45)$$

where we have once again used (14). The component of (45) along the direction of magnetic field is then

$$[-\nabla(p_e + p_i + p_n) + (m_n n_n + m_i N)\vec{g}] \cdot \vec{h} = 0 \qquad (46)$$

Comparison of (45) and (46) shows that the net force due to pressure gradient and gravity of all particles is perpendicular to the magnetic field and balanced by a current flow force. Next, using (7), (8), (9), (18) and (28), we have

$$\frac{\partial(n_n + 2N)}{\partial r} + \frac{\tan\theta}{r}\frac{\partial(n_n + 2N)}{\partial\theta} + \frac{n_n}{H_n} + \frac{N}{H_i} = 0 \qquad (47)$$

where H_n is the scale height of the neutral atmosphere.

Since $N << n_n$ and $H_i \approx H_n$, we can write

$$\frac{n}{H_n} + \frac{N}{H_i} \approx \frac{n}{H_n} + \frac{N}{H_n} \approx \frac{n + 2N}{H_n} \qquad (48)$$

where the suffix on n has been dropped for simplicity. Then (47) becomes, in total derivative form,

$$\frac{d(n + 2N)}{dr} + \frac{n + 2N}{H_n} = 0 \qquad (49)$$

Integration of (49) along the field line gives

$$n(r,\theta) \approx n + 2N = g(r_o,\pi/2)e^{-\int_{r_o}^{r} \frac{dr}{H_n}} \tag{50}$$

where $g(r_o,\pi/2)$ is an arbitrary function of height at the equator and r_o is defined in (27). If we now demand that the radial distribution of the neutrals obey the normal hydrostatic law at the equator, so that

$$g(r_o,\pi/2) = n_{oo}\, e^{-\int_{r_{oo}}^{r_o} \frac{dr}{H_n}} \tag{51}$$

where n_{oo} is the neutral number density at height r_{oo} on the equator, then

$$n(r) = n_{oo}\, e^{-\int_{r_{oo}}^{r} \frac{dr}{H_n}}, \tag{52}$$

a result which is entirely independent of θ. If, on the other hand, $g(r_o,\pi/2)$ is perturbed <u>in any manner</u> from the exact hydrostatic equilibrium case, we will obtain a distribution for n which does depend on θ. The origin of this angular dependence on the neutrals may seem somewhat surprising until we realize that in selecting a functional form for $g(r_o,\pi/2)$, any deviation in the equatorial neutral distribution from hydrostatic equilibrium must arise due to collisions between neutrals and geomagnetically controlled charged particles. Thus, if the collisions between neutrals and charged particles are sufficiently large to make the momentum transfer forces between charged and neutral particles important, the neutrals will begin to tend toward the angular distribution of the geomagnetically controlled particles. This can also be seen from (3), where it is obvious that we will not obtain the exact hydrostatic distribution in a region when the terms on the left hand side become important.

On this basis, we might expect to observe angular variations of the neutral distribution in the bottomside regions of the ionosphere where charged-neutral particle interactions become important.

CONCLUSIONS

From the discussion and results of this paper, we have shown the following:

1. From the equations of motion, it is possible to derive an expression for the electron-density distribution along a field line either by assuming diffusive equilibrium along the direction of the magnetic field or by neglecting the drag forces arising from collisions. The latter assumption appears to be more realistic in the topside of the ionosphere. In either case, it is necessary to assume a radial distribution at the equator to obtain the electron density distribution.

2. We have provided a more accurate formula for the representation of the equinox geomagnetic anomaly than that produced in GKS. Since the empirical boundary condition equation used herein has been shown by Chandra (1963) to fit nearly all vertical profiles of electron density measured to date, we can safely assume that the proper selection of parameters in this formula will lead to a reasonable reproduction of the anomaly in any equatorial region of the ionosphere where interactions of neutrals with charged particles are small because of infrequent collisions.

3. The theory discussed above is semiphenomenological; i.e., it is based on effect and not cause. It does not require a knowledge of the complicated array of physical effects and mechanisms which combine to form the geomagnetic anomaly but, instead, uses an empirical boundary condition which is the accumulated effect of all these causes.

Naturally, if we are to increase our knowledge of the basic mechanisms causing the anomaly and thereby replace the empirical boundary condition by one based on more fundamental considerations than measurement, we must turn to the equations of continuity. Unfortunately, the derivation of the correct continuity equations requires knowledge concerning the electron and ion velocities and/or the electric fields acting on these particles. Currently, most derivations of the continuity equations employ simplifying assumptions, such as $v_{e_{\parallel}} = v_{i_{\parallel}} = v_{\parallel}$; $v_{\parallel} >> v_{e_{\perp}}, v_{i_{\perp}}$. The equation derived in the literature under the above assumptions has been numerically integrated by several workers to obtain a theoretical electron density distribution near the equator under steady state conditions. The results obtained by these workers have been unable to account for the gross features of the geomagnetic anomaly, at least to the correct order of magnitude. This has lead them to believe that diffusion is of minor importance in governing the geomagnetic anomaly.

We have been able to demonstrate that the velocity assumptions described above do not lead to the proper description of the geomagnetic anomaly. We therefore feel that the assumption about velocities used in the continuity equation rather than the ineffectiveness of motions are responsible for the unsatisfactory description of the geomagnetic anomaly obtained by others.

4. A study of the neutral atmosphere distribution has led us to the conclusion that geomagnetic control of neutrals occurs in any region of the ionosphere where interactions of neutrals with charged particles become important. Since this is most likely to occur in the lower F region of the ionosphere we suggest that such geomagnetic control of the neutrals might be observable in this region.

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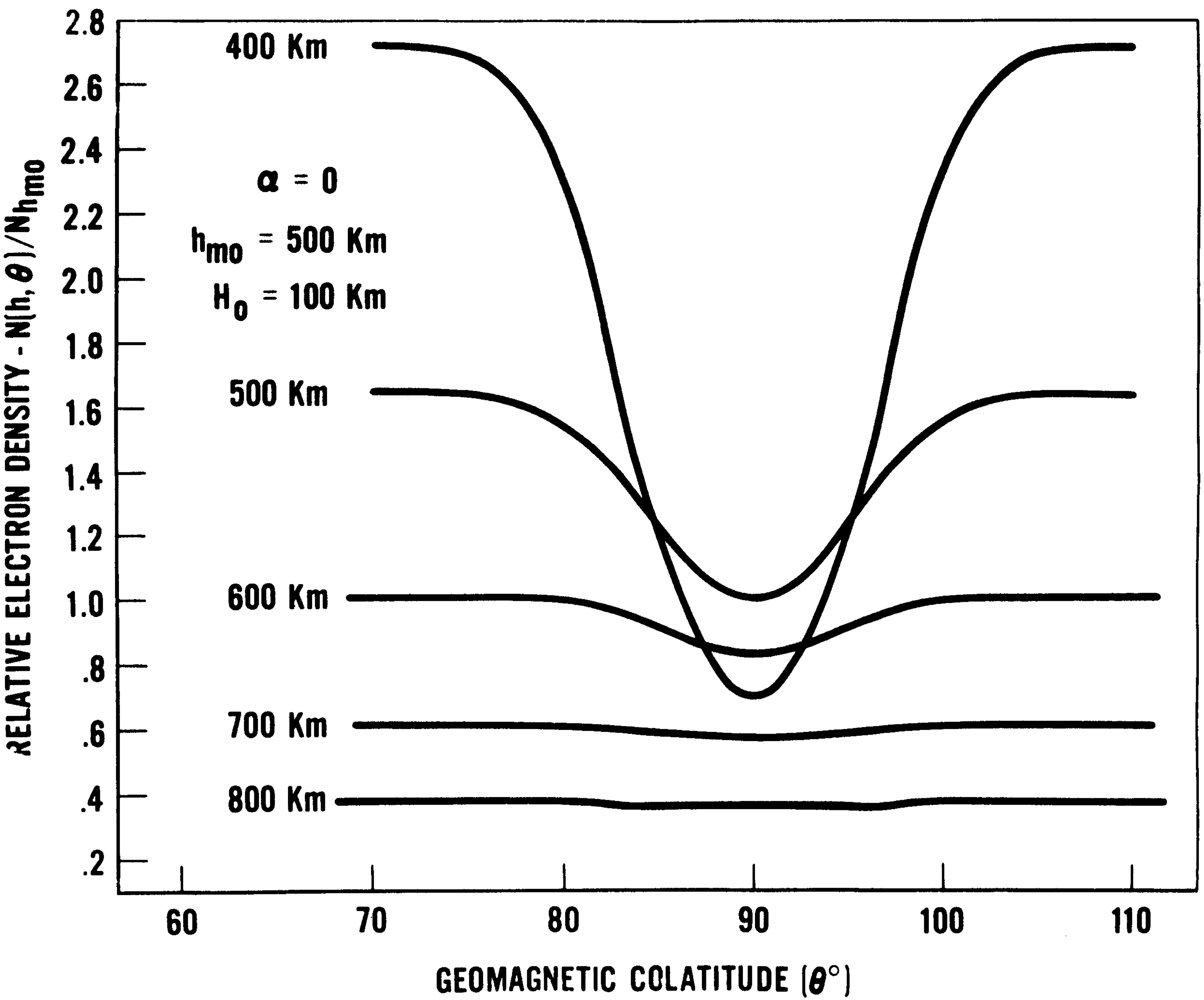
LIST OF FIGURES



2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
RELATIVE ELECTRON DENSITY - $N(h, \theta)/N_{h_{mo}}$
400 Km
500 Km
600 Km
700 Km
800 Km
$\alpha = 0$
$h_{mo} = 500$ Km
$H_0 = 100$ Km
60
70
80
90
100
110
GEOMAGNETIC COLATITUDE ($\theta°$)

Figure 1

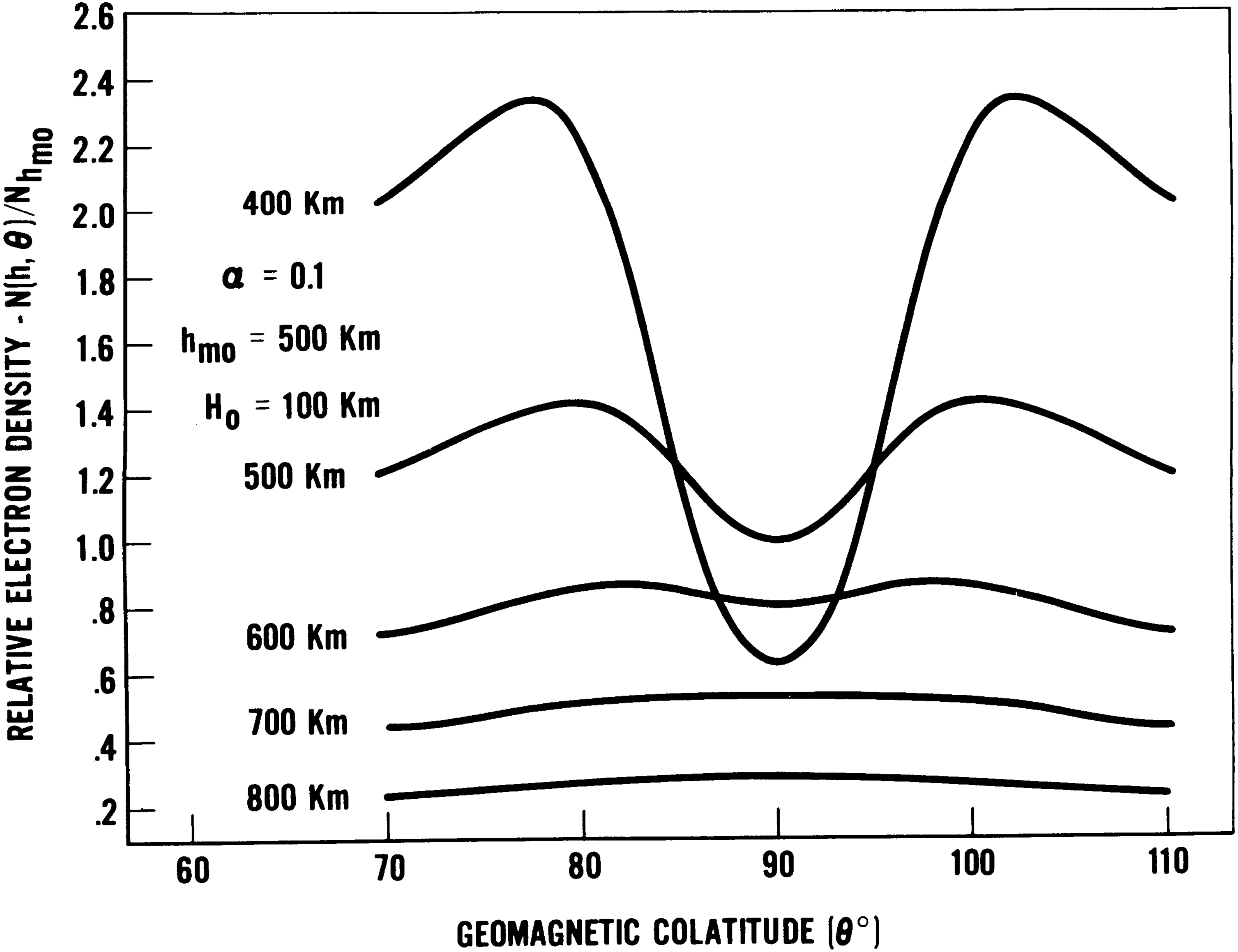
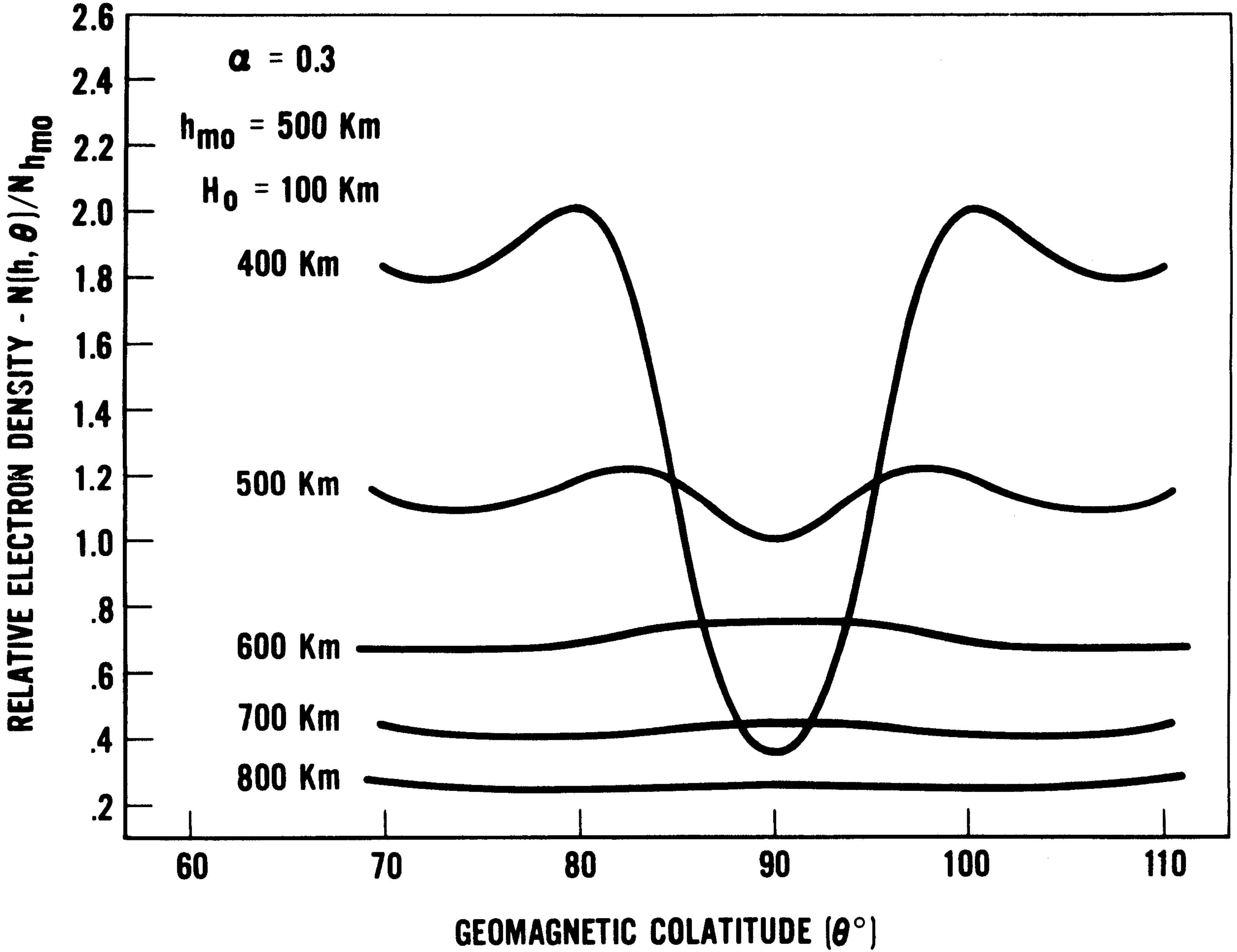

RELATIVE ELECTRON DENSITY - N(h, θ)/N_{h_mo}
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
400 Km
α = 0.1
h_mo = 500 Km
H_o = 100 Km
500 Km
600 Km
700 Km
800 Km
60
70
80
90
100
110
GEOMAGNETIC COLATITUDE (θ°)

Figure 2



α = 0.3
h_mo = 500 Km
H_0 = 100 Km
400 Km
500 Km
600 Km
700 Km
800 Km
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
60
70
80
90
100
110
RELATIVE ELECTRON DENSITY - N(h, θ)/N_hmo
GEOMAGNETIC COLATITUDE (θ°)

Figure 3

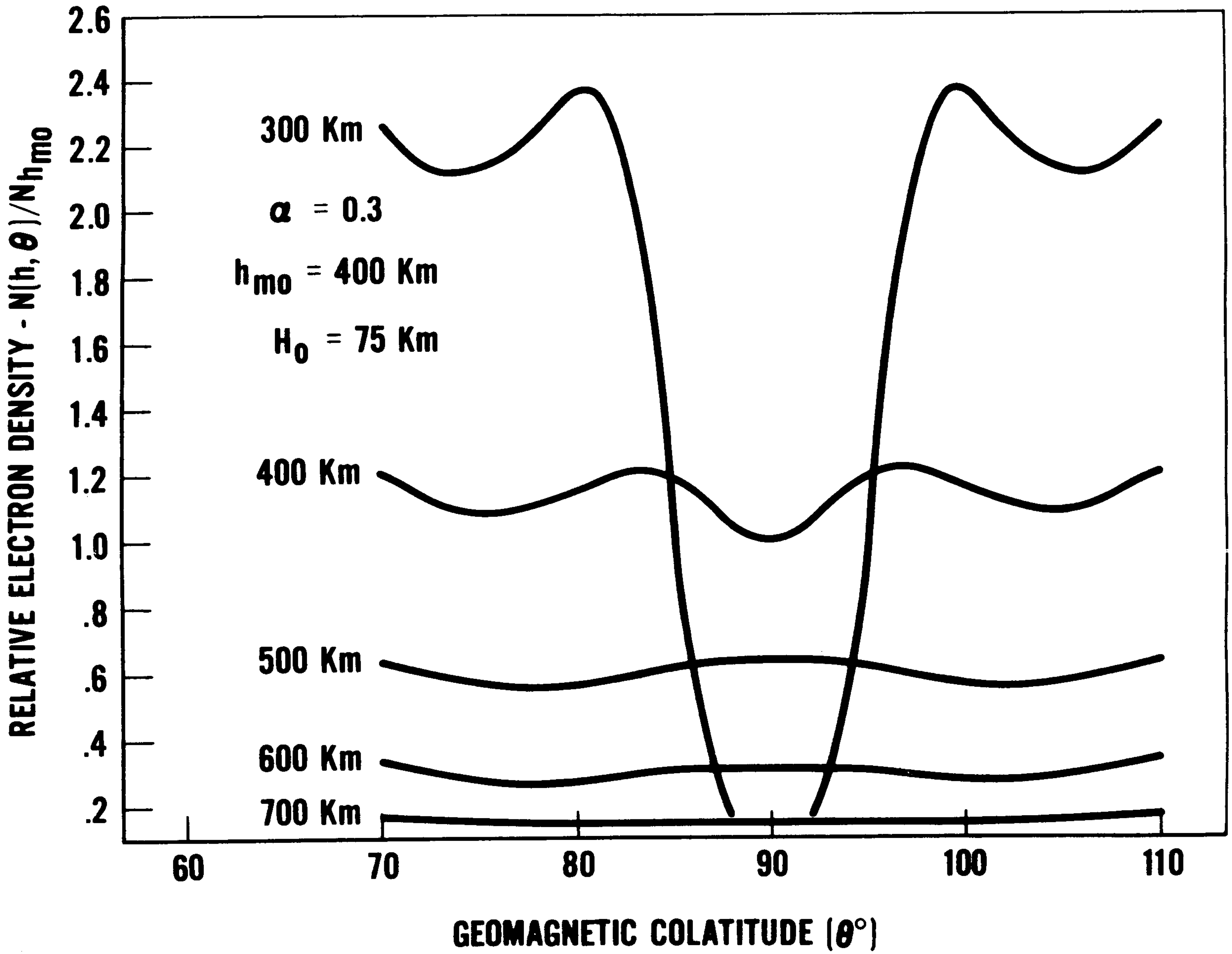
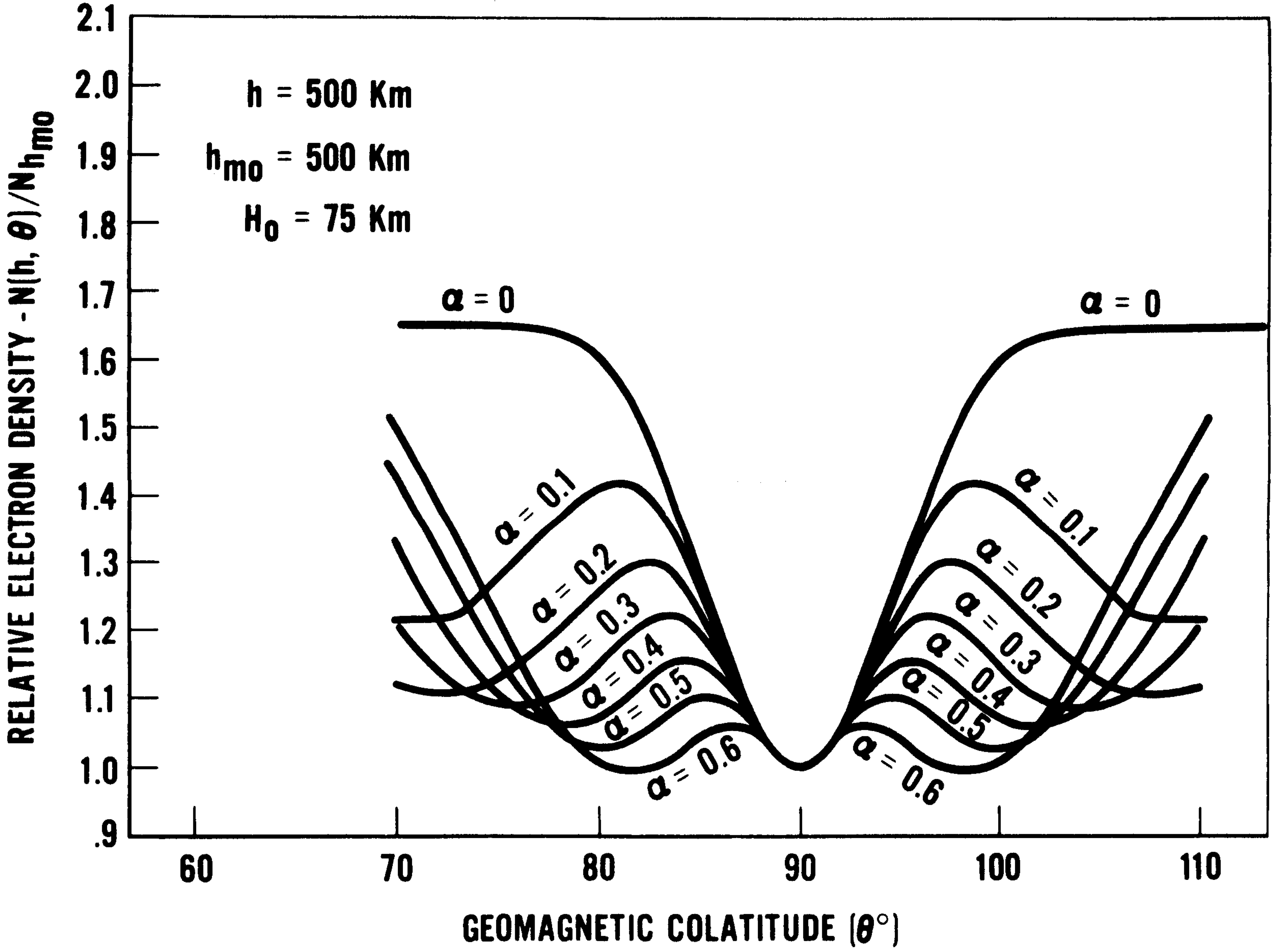

2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
RELATIVE ELECTRON DENSITY - N(h, θ)/N_{h_{mo}}
300 Km
α = 0.3
h_mo = 400 Km
H_o = 75 Km
400 Km
500 Km
600 Km
700 Km
60
70
80
90
100
110
GEOMAGNETIC COLATITUDE (θ°)

Figure 4



h = 500 Km
h_{mo} = 500 Km
H_0 = 75 Km
α = 0
α = 0.1
α = 0.2
α = 0.3
α = 0.4
α = 0.5
α = 0.6
RELATIVE ELECTRON DENSITY - $N(h, \theta)/N_{h_{mo}}$
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
60
70
80
90
100
110
GEOMAGNETIC COLATITUDE ($\theta°$)

Figure 5