NASA TECHNICAL MEMORANDUM

NASA TM X-53053

JUNE 2, 1964

N64-29178
(ACCESSION NUMBER)
29
(PAGES)
TMX-53053
(NASA CR OR TMX OR AD NUMBER)
(THRU)
1
(CODE)
27
(CATEGORY)

FACILITY FORM 602

NASA TM X-53053

BOOSTER PARAMETRIC DESIGN METHOD FOR PERFORMANCE AND TRAJECTORY ANALYSIS

PART II *PROPULSION*

by V. VERDERAIME
Aero-Astrodynamics Laboratory

NASA

George C. Marshall Space Flight Center, Huntsville, Alabama

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XEROX $ 2.60 ph

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ABSTRACT

Approximate equations for large, liquid chemical, rocket engine mass and space envelope are presented in parametric form. Well known propulsion performance equations are given with modifications to admit programming of mixture ratio shifts and throttling of propellant mass flow rate. Parameters used in mass and space envelope equations were nominal input design parameters in common with the propulsion performance equations such that their interdependence could be manifested in a vehicle trajectory and performance optimization study. Though results are based on current type engines, it is expected that coefficients and exponents used may be readily modified to define mass and size of moderately advanced rocket engines.

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LIST OF SYMBOLS

Nominal Input Design Parameters

Symbol	Definition
A_e	nozzle exit area, m^2
A_t	nozzle throat area, m^2
$\bar{F}$	total stage thrust, N
g_o	earth gravitational constant, m/s^2
I_{sp}	specific impulse, s
n	number of rocket engines burning
$\bar{p}_a$	atmospheric pressure at sea level or vacuum, N/m^2
$\bar{p}_c$	combustion chamber pressure, N/m^2
$\bar{p}_e$	nozzle exit pressure, N/m^2
p_s	combined "net positive suction pressure" of oxidizer and fuel pumps, N/m^2
p_{s_f}	"net positive suction pressure" of fuel pump, N/m^2
p_{s_o}	"net positive suction pressure" of oxidizer pump, N/m^2
$\bar{r}_m$	propellant mass mixture ratio
β	engine maximum gimbal angle, radians
$\bar{\gamma}$	specific heat ratio
ϵ	nozzle expansion ratio

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
λ	thrust coefficient factor
ρ_f	fuel density, kg/m^3
ρ_O	oxidizer density, kg/m^3
$\bar{\rho}_8$	combined densities of oxidizer and fuel, kg/m^3
	Independent Variable Parameters
p_a	atmospheric pressure as a function of altitude, N/m^2
p_c	combustion chamber pressure, N/m^2
r_m	mixture ratio
τ_i	discrete time intervals, s
	Dependent Variable Parameters
C_F	thrust coefficient
d_e	nozzle exit diameter, m
F	total stage thrust during flight, N
L_N	nozzle length, m
L_R	total rocket engine length, m
$\dot{m}$	propellant mass flow rate per rocket engine, kg/s
p_e	nozzle exit pressure, N/m^2
W	mass, kg

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
	Dependent Variable Parameters (Continued)
$W_{4.1}$	rocket engine total mass, kg
W_8	total mainstage propellant mass consumption, kg
$\dot{W}_8$	total mainstage propellant mass flow rate, kg/s
γ	specific heat ratio
ξ_1	specific impulse coefficient
ξ_2	specific heat coefficient

NOTE: All measurement units are SI units.

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BOOSTER PARAMETRIC DESIGN METHOD FOR PERFORMANCE AND TRAJECTORY ANALYSIS

PART II: PROPULSION

SUMMARY

The influence of the rocket mass and size on the overall vehicle performance is of concern to the trajectory and performance analysts. Since the engine size and mass are affected by variations of nominal propulsion performance parameters, and since the engine geometry affects the stage mass, significant coupling parameters were sought.

Well known performance equations were modified to allow mixture ratio shifts and propellant mass throttling as might be required for a mission study. Equations of total propellant consumption were derived for determining the stage propellant tank volumes [1].

Engine space envelope was defined by parameters which described the rocket chamber length and nozzle exit diameter. Mass equations were limited to three major categories: turbopump, rocket chamber, and accessories. With the exception of a pump parameter (p_s), the engine size and mass equations were adequately expressed by nominal parameters common to the propulsion performance. Consequently, it was possible for variations in nominal performance parameters to be reflected as engine size and mass modifications.

1.0 INTRODUCTION

In any preliminary trajectory analysis the propulsion system of each vehicle stage is a primary consideration from the viewpoint of propulsion performance and of the propellant bulk and structural masses in support of the propulsion system. When existing expulsion systems are selected, the characteristics are known and fixed, and subsequent vehicle performance optimization studies are limited to determining number of engines, thrust duration, propellant loading, number of stages, configuration, and masses for a given mission. However, if a projected mission is known to require the development of a new rocket engine, the trajectory analysts may well influence the design at its conception by investigating each propulsion design parameter for its significance to the overall vehicle performance. Trends may be indicated such as types and range of propulsion controls desired for shifting propellant mixture ratio or flow rate at strategic intervals during flight.

To accomplish this type of investigation, the trajectory analysts must have a mathematical model of stage shape, mass, and propulsion system with sufficient interconnecting parameters to obtain expeditious and realistic preliminary results. This is the second in a series of reports describing a method for developing such a stage mathematical model as applied to trajectory analysis. When this series of reports was initiated, it was thought that a good approach to presenting the method was to derive parametric equations of a conventional booster stage relevant to a bipropellant, liquid chemical propulsion system familiar to the reader [1] (other configurations could also be adapted to these resulting equations through conversion factors). It is the purpose of this document to proceed with this conventional configuration by developing the related rocket engine performance, size, and mass parametric equations of a conventional type.*

Engine major component categories are rocket chamber, turbopump and engine accessories. Input design parameters common to these categories will be limited to those essential for a preliminary type of trajectory analysis and in no way will attempt to duplicate detailed thermodynamics and mechanics analysis normally conducted by rocket engine designers. Typical thermodynamics properties and performance values will be referenced where necessary, and mathematical simplicity is again emphasized.

An important point to remember in judging results from these equations is that they must indicate a realistic trend of size and mass variation with each variation of such pertinent engine parameters as specific impulse, chamber pressure, propellant density, throat area, and nozzle expansion ratio. Another trend to be considered, intuitively, is that engine performance improves through continued manufacturer's development with minor resulting changes in size and mass. Since early engines of a new type seldom meet the specified performance, conservative performance equations are recommended when vehicle launch schedules do not admit sufficient development time.

The results presented here have been significantly influenced by the author's former association with Rocketdyne and by discussions with many persons of the MSFC Propulsion and Vehicle Engineering Laboratory. Special thanks are extended to Mr. Helmut J. Horn for his suggestions and encouragement and to Mrs. Sarah Hightower, Mrs. Irene Dolin, and Mr. James Hackney for their assistance with the manuscript.

* Engines operating with chamber pressures in excess of 10^7 newtons per square meters, or low thrust with multi-hour life, represent a new generation employing advanced technology and new philosophies which may not conform with results presented.

There seems to have been very limited documentation on this subject in the past. It is hoped that this document will be of benefit to the reader and will perhaps arouse his interest to reprove or improve the technique or to present his own.

2.0 ROCKET CHAMBER PERFORMANCE

Of the three major assembly categories, the rocket chamber figures into the trajectory analysis from its propulsion performance characteristics in addition to its mass contribution. It consists of an "injector face" for introducing and mixing propellants into a "combustion chamber" where chemical energy release produces high pressures and temperature which are converted into kinetic energy of the exhaust jet by the "nozzle." Increasing the design chamber pressure may not only increase the propulsion performance but may likely increase the turbopump and rocket chamber mass. Varying the nozzle expansion ratio causes a variation in engine performance, mass, and geometry. This interdependence suggests that derivation of engine mass and size parametric equations must commence with a propulsion performance equation to seek common input design parameters such that a variation causing an increase in propulsion performance would reflect an engine mass bonus or penalty.

A propulsion input parameter most familiar to the trajectory analysts is the specific impulse (I_{sp}). It has the unit of time and may be thought of as the time required to consume a unit weight of propellant producing a unit of thrust. It is a characteristic of the propellant combination, mixture ratio, combustion temperature, and molecular weight of the gases.

For the condition of nozzle exit pressure (p_e) equal to ambient pressure (p_a), known as optimum expansion, the product of specific impulse and gravitational constant (g_o) is identically equal to the ideal velocity of the rocket exhaust gas. Though the specific impulse may be determined for this condition by calculating the ideal exhaust velocity in an accepted method [2], a more realistic value may be obtained experimentally or from a similarly experienced propellant combination and mixture ratio.

Using the specific impulse as a nominal input design parameter for a given propellant combination and nominal mixture ratio ($\bar{r}_m$)*, the propellant mass flow rate ($\dot{m}$) consumed per rocket engine is determined from the relationship

$$\dot{m} = \left(\frac{\bar{F}}{n}\right) \frac{1}{I_{sp}\ \xi_1\ g_o}\ , \tag{2.0a}$$

where (n) is the number of engines employed to produce the stage total nominal thrust ($\bar{F}$) at a specified nominal ambient pressure ($\bar{p}_a$), and (ξ_1) is the specific impulse coefficient which is a function of mixture ratio variation. If, for example, an oxygen/RP-1 propellant combination having an I_{sp} = 300.4 seconds for an $\bar{r}_m$ = 2.56 is selected from a table of calculated [4], or experimental values, then tabulated changes in specific impulse with variations in mixture ratio may be represented by a polynomial in the form

$$\xi_1 = 1 - 0.002\ (r_m - 2.56) - 0.084\ (r_m - 2.56)^2 + 0.046\ (r_m - 2.56)^3.$$

Another propulsion parameter familiar to the trajectory analyst is the booster stage total thrust (F) that varies with atmospheric pressure and is given by

$$\frac{F}{n} = p_c\ A_t\ \lambda C_F, \tag{2.0b}$$

where (p_c) is the combustion chamber pressure and (A_t) is the nozzle throat area. The thrust coefficient factor (λ) includes engine cant angle and mass discharge correction factors varying between 0.90 to .99. The thrust coefficient (C_F) is calculated from

$$C_F = \left\{\frac{2\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} + (p_e - p_a)\,\frac{\epsilon}{p_c}\ , \tag{2.0c}$$

---

* A bar fixed over a parameter symbol implies that this symbol is used to define two different values. When written with a bar it is a nominal value fixed by design conditions and must not be varied during flight. Without the bar, the parameter varies during flight, either because of ambient conditions or because of the analysts attempting to produce a desired flight result.

which contains the ambient (atmospheric) pressure parameter that varies with launch site environment and vehicle altitude during flight.

A shift in mixture ratio will cause a change in the specific heat ratio (γ) of equation (2.0c). This change may be conveniently expressed by

$$\gamma = \bar{\gamma}\, \xi_2, \tag{2.0d}$$

where ($\bar{\gamma}$) is the nominal specific heat ratio corresponding to the nominal mixture ratio. The specific heat ratio coefficient (ξ_2) is a function of the mixture ratio variation and is treated similarly to the specific impulse coefficient (ξ_1). If, for instance, an oxygen-gasoline propellant system is used [3] having $\bar{r}_m = 2.56$ and $\bar{\gamma} = 1.22$, then we write the specific heat ratio coefficient in the following form:

$$\xi_2 = 1 - 0.012\ (r_m - 2.56) + 0.028\ (r_m - 2.56)^2 - 0.025\ (r_m - 2.56)^3.$$

If the chamber pressure and nozzle expansion ratio (ϵ) are held constant, the nozzle exit pressure and thrust coefficient will vary with variations of specific heat ratio resulting from mixture ratio shifts. This is also true if the mixture ratio and nozzle expansion ratio are held constant and the chamber pressure is varied during flight. To determine this disturbed nozzle exit pressure due to imposed chamber pressure and mixture ratio changes, we use the classical nozzle expansion ratio equation

$$\epsilon = \frac{A_e}{A_t} = \left(\frac{p_c}{p_e}\right)^{1/\gamma} \frac{1}{\left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma-1}} \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}} . \tag{2.0e}$$

In practice, the nozzle expansion ratio is physically fixed by the nozzle throat and the nozzle exit (A_e) areas for selected nominal design parameters of chamber pressure ($\bar{p}_c$), exit pressure ($\bar{p}_e$), mixture ratio ($\bar{r}_m$), specific heat ratio ($\bar{\gamma}$), and engine thrust ($\bar{F}/n$) at a specified ambient pressure ($\bar{p}_a$). These selected nominal values must also obey the relationship of equation (2.0e) and may be written into a fixed nozzle expansion ratio equation as

$$\epsilon = \left(\frac{\bar{p}_c}{\bar{p}_e}\right)^{1/\bar{\gamma}} \frac{1}{\left(\frac{\bar{\gamma}+1}{2}\right)^{\frac{1}{\bar{\gamma}-1}} \left\{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}\left[1-\left(\frac{\bar{p}_e}{\bar{p}_c}\right)^{\frac{\bar{\gamma}-1}{\bar{\gamma}}}\right]\right\}^{1/2}} . \tag{2.0f}$$

For a conventional bell-type nozzle, the nozzle expansion ratio is determined from the nominal conditions stated in equation (2.0f) and is substituted into equations(2.0e) and (2.0c). Then, applying the nozzle exit pressure of equation (2.0e) into equation (2.0c), the analyst may determine the engine thrust from equation (2.0b) for variations of atmospheric pressure and for discrete variations of mixture ratio or chamber pressure.

The stage propellant flow rate is determined from equation (2.0a). Substituting equation (2.0b) and fixing the ambient pressure ($\bar{p}_a$) of the thrust coefficient to conform with the definition of ($\bar{F}$), we obtain the stage mass flow rate ($\dot{W}_8$):

$$\dot{W}_8 = \frac{A_t \lambda n p_c}{I_{sp} g_o (\xi_1)} \left[\left\{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} + (p_e - \bar{p}_a)\frac{\epsilon}{p_c}\right]. \tag{2.0g}$$

If the mass flow rate is not constant during stage flight due to thrust throttling, mixture ratio shifting, or engine-out conditions, then the total mainstage propellant mass (W_8)* consumed is

$$W_8 = \frac{A_t \lambda}{I_{sp} g_o} \sum_j \frac{(n)_j (\tau)_j (p_c)_j}{(\xi_1)_j} \left[\left\{\frac{2\gamma_j^2}{\gamma_j-1}\left(\frac{2}{\gamma_j+1}\right)^{\frac{\gamma_j+1}{\gamma_j-1}}\left[1-\left(\frac{p_{e_j}}{p_{c_j}}\right)^{\frac{\gamma_j-1}{\gamma_j}}\right]\right\}^{1/2}\right.$$

$$\left. + (p_{e_j} - \bar{p}_a)\frac{\epsilon}{p_{c_j}}\right], \tag{2.0h}$$

in which all parameters are held constant for discrete time intervals (τ_j).

* MSFC Standard [5] notation for mainstage propellant mass.

The total fuel and oxidizer tank sizes [1] are determined from total propellant consumption (W_8) of equation (2.0h) and from the weighted average mixture ratio given by

$$r_{m_{avg}} = \bar{r}_m \frac{\sum_j (\xi_1)_j (\tau)_j}{\sum_j (\tau)_j}, \tag{2.0k}$$

where ($\bar{r}_m$) is the nominal propellant mixture ratio corresponding to $\xi_1 = 1$.

Thus, the thrust performance of the rocket chamber is expressed by equation (2.0b) where the thrust coefficient is defined by equation (2.0c) for variations of pressures and mixture ratio. Imposed changes in chamber pressure effect changes in thrust coefficient of equation (2.0c) because of the disturbed exit pressure of the fixed nozzle expansion ratio which must satisfy equation (2.0e). Changes in propellant combinations and mixture ratio shifts are accounted for by a change in specific heat ratio of equation (2.0d) and the specific impulse coefficient in equation (2.0g). The propellant mass flow rate may be calculated from equation (2.0g) and the stage propellant container size [1] may be determined from the mainstage propellant consumption equation (2.0h) having an average mixture ratio expressed by equation (2.0k).

3.0 ROCKET CHAMBER DIMENSIONS

Rocket chamber dimensions are important to the trajectory analysts only because they may influence the engine and stage masses or vehicle aerodynamic drag. Clearly, the engine length, nozzle exit diameter, and cant or gimbal angle (β) of multi-engine boosters will often dictate the size of aerodynamic shrouds and even the stage diameter, both of which affect the stage mass and vehicle drag. In upper stages, the engine size not only influences the interstage length and mass, but it is also a major consideration in separation and control studies. Behind it all are three nominal input design parameters, the engine throat area, nozzle expansion ratio, and propellant specific impulse.

Though these parameters were initially introduced to define propulsion performance, they may now be applied directly to subsequent equations of nozzle exit diameter (d_e) and engine total length (L_R).

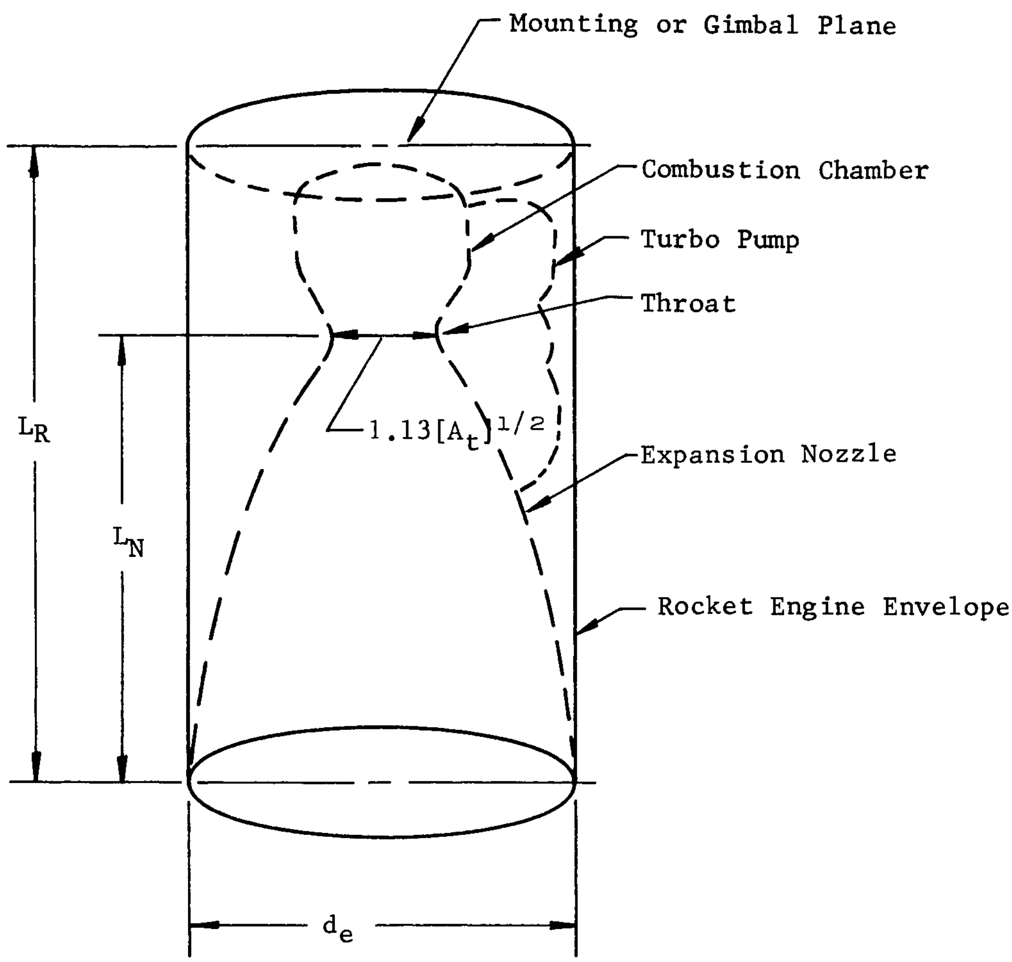

FIGURE 1

SCHEMATIC DIMENSIONAL DIAGRAM

By definition of equation (2.0e), the nozzle exit area is

$$A_e = A_t \epsilon$$

so that the nozzle exit diameter is given by

$$d_e = 1.14 \sqrt{A_t \epsilon} . \tag{3.0a}$$

In practice, the length (L_N) of a conventional bell-type nozzle is considerably shorter than the ideal resulting only in minor reduction of the thrust coefficient for a desirable mass saving. Based on a survey of several bell-type nozzles of existing liquid chemical engines, the nozzle length between throat and exit planes may be empirically expressed as

$$L_N = 1.38 \sqrt{A_t \epsilon} . \tag{3.0b}$$

The chamber length from the throat area to the bearing is composed of combustion chamber, dome, and engine attachment which are based upon current knowledge of combustion efficiency and stability and upon uniqueness of engine design. Therefore, this length is also expected to be expressed in empirical form. Propulsion parameters which influence the combustion chamber and dome lengths are those which are related to the propellant mass flow rate and the chemical reaction time of the propellant in the combustion chamber. These are throat area, specific impulse and chamber pressure. However, the chamber pressure influence is so small that it may be ignored. Until a better parametric equation can be established, the following simplified empirical expression is suggested:

$$L_R - L_N = 0.1 \, (A_t)^{1/3} \, (I_{sp})^{1/2} . \tag{3.0c}$$

Adding equation (3.0b) with (3.0c), we obtain

$$L_R = 0.1 \, (A_t)^{1/3} \, (I_{sp})^{1/2} + 1.38 \sqrt{A_t \epsilon} , \tag{3.0d}$$

which is the rocket engine length from the nozzle exit plane to the engine attachment plane just forward of the dome. These coefficients and exponents are only average values and may have to be adjusted with experience.

We shall now use equations (3.0a) and (3.0d) to define the rocket engine space envelope of Figure 1 with the assumption that the nozzle exit radius, or the gimbaled engine envelope of Figure 2, is larger than the radial extremity of the attached turbopump assembly. This is particularly true of long burning booster stages.

For an engine gimbaled a nominal angle (β) about two perpendicular axes, the maximum radial occupancy of the engine is at the corners and may be approximated by

$$\frac{d_e}{2} + 1.4\beta\, L_R. \tag{3.0e}$$

Figure 3 presents a suggested minimum diametrical dimension required for aerodynamic shielding of multi-engines in various fixed and gimbaled arrangements. If a shroud is used to totally enclose engines in extreme gimbaled position, add 0.1 d_e clearance to the diametrical dimension (D_e).

4.0 ROCKET ENGINE MASS DETERMINATION

Weight predictions of rocket engines, consisting of hundreds of complex and high performance parts, vary in scope and application. On the one hand, an average value of engine thrust-to-weight ratio may suffice for general discussions or for preliminary mission studies emphasizing modes rather than stage performance. On the other hand, an engine manufacturer's comprehensive mass analysis may be required for a propulsion uprating study based on specific propellant combinations, cooling characteristics, minimum and maximum pressures, structural materials, start system, heat exchangers, controls, and tolerances among other considerations.

The one presented is intended to optimize, or intelligently compromise, propulsion performance requirements with the engine and stage requirements in preliminary studies through equations consisting of parameters common to propulsion performance and stage structural mass. Whenever an engine mass parameter requires more than a cursory knowledge of turbopump machinery or combustion devices, this parameter is referred to a performance parameter on a "best fit" basis.

One would assume that an ideal engine mass equation is composed of the sum of all parametric mass equations of each subcomponent. But if an attempt is made to include the mass analysis of each subcomponent, the intricate composition may be better appreciated by investigating a simple subcomponent such as the oxidizer pump discharge duct. Assuming the material density to be a constant for such application, we proceed to

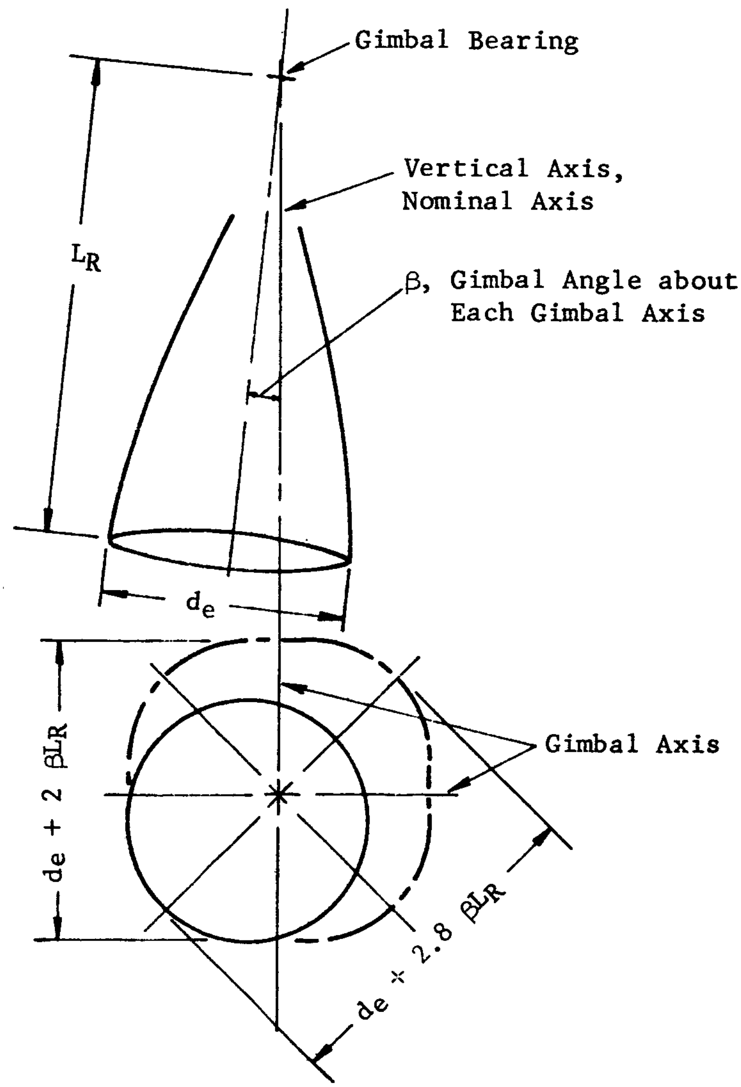

FIGURE 2. SQUARE GIMBAL PATTERN

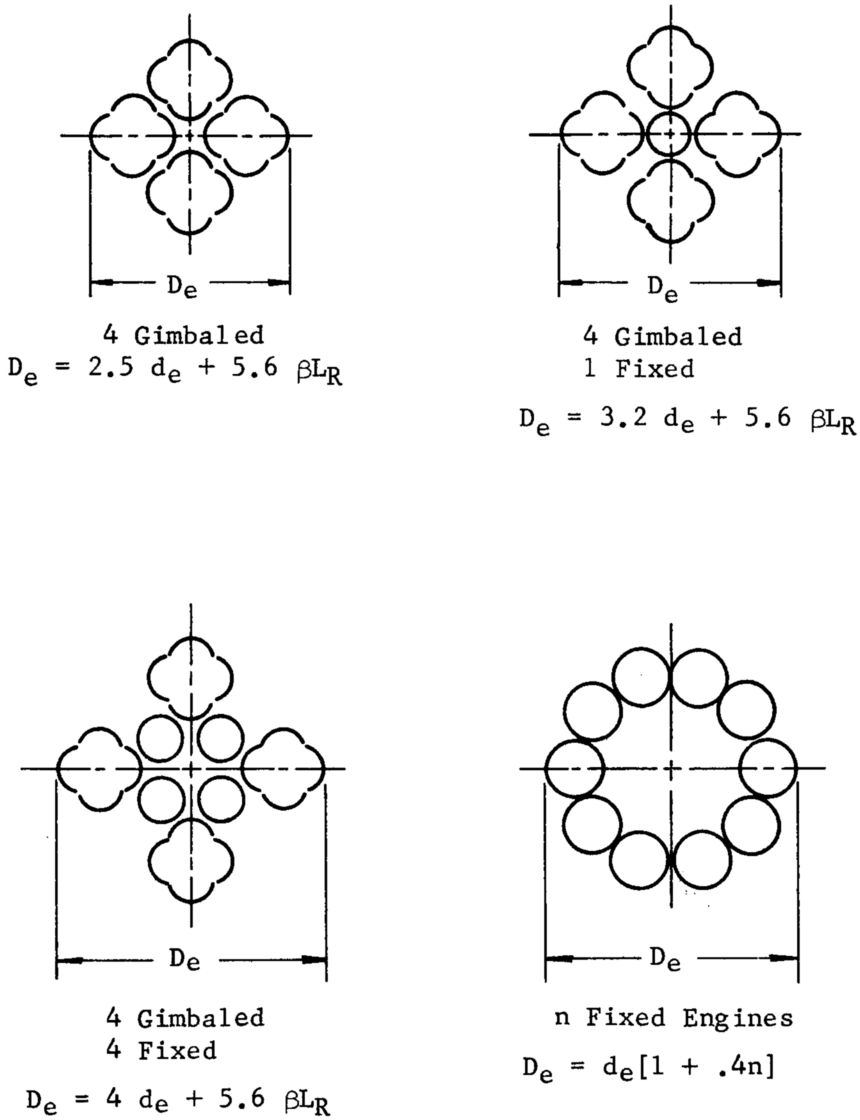

FIGURE 3. MULTI-ENGINE ARRANGEMENT

determine the duct material volume from its diameter, thickness and length. Reasoning that the duct diameter is determined from the propellant mass flow of equation (2.0a), mixture ratio, oxidizer density, and velocity, we conclude that the diameter is related to the "best fit" parameters in the form

$$d = C_1 \bar{p}_c^{x_1} \cdot A_t^{x_2} \cdot \bar{r}_m^{x_3} \cdot I_{sp}^{x_4} \cdot \rho_o^{x_5}$$

where C_1 and x_j are coefficients and exponents, respectively. The duct thickness is determined from the oxidizer pump discharge pressure, duct diameter already discussed, and material constants. Written in general form, the thickness is

$$t = C_2 \cdot d \cdot \bar{p}_c^{x_6}$$

where ($\bar{p}_c$) is the "best fit" for the pump discharge pressure. Relating the duct length to parameters of the combustion chamber geometry offered in Section 3 and then combining with the expressions above, we obtain the oxidizer discharge duct mass in the form

$$W = C_3 \cdot \bar{p}_c^{x_7} \cdot A_t^{x_8} \cdot \bar{r}_m^{x_3} \cdot I_{sp}^{x_9} \cdot \rho_o^{x_5}.$$

Since it is not likely that the mass of another engine component will contain parameters having identical exponents, it is expected that the engine total mass equation will contain as many terms as there are subcomponents. Such an equation would be unrealistic not only from the viewpoint that the effort is excessive compared to inputs based on approximate "best fit," but also from the viewpoint that the equation would be biased to a particular pump design and incapable of updating or modifying. A more rational derivation of the total mass equation for the purpose of stage trajectory and performance analysis would be based on empirical expressions of major assemblies. Hence, the pump discharge ducts would be logically grouped with the turbopump assembly as would be the pump mounts and piping valves. The gimbal bearing, dome, injector, and chamber would constitute another logical assembly, and accessories a third. This will be the philosophy adopted in the ensuing subsections.

4.1 Turbopump Mass

The turbopump is a very complex machine used to transfer propellants from the vehicle tanks to the rocket engine combustion chamber at the desired chamber pressure and mass flow rate. The assembly consists of an oxidizer and a fuel pump driven by a common or individual gas turbine through a gear train. Though the mass of each of these components varies with types, space restrictions, and propellant combinations, a parametric equation may be useful to budget a gross mass for the entire assembly to which it can be ultimately designed and optimized.

Parameters selected for predicting the turbopump assembly mass must be common to the propulsion or stage performance and to all conventional pumps. Therefore, observing that the turbopump mass increases with increasing nominal thrust of a rocket engine, we refer to equation (2.0b) and select the parameter (A_t) and use a nominal chamber pressure ($\bar{p}_c$). The pump performance is independent of the thrust coefficient factor (λ) and the thrust coefficient (C_F) of the thrust chamber, which consequently are excluded from the mass equation. Dividing by the specific impulse of the required bipropellant combination yields an expression allied to propellant mass flow rate of equation (2.0a) which is a pump and turbine characteristic. If we divide again by the propellant density, we obtain an expression related to the propellant volume flow rate which is definitely a measure of pump and turbine physical sizes and relevant to their masses. Since we are deriving one expression to represent the combined masses of fuel and oxidizer pump and turbine, we must also combine the propellant densities into one expression that will include the mixture ratio parameter. The combined propellant nominal density ($\bar{\rho}_8$) for a nominal mixture ratio was readily found to be

$$\bar{\rho}_8 = \frac{\rho_f \, \rho_o \, (1 + \bar{r}_m)}{\rho_f \, \bar{r}_m + \rho_o} \qquad (4.1a)$$

where ρ_f and ρ_o are fuel and oxidizer densities [1], respectively.

A parameter which has not appeared in the stage configuration nor in the propulsion performance is the minimum required "net positive suction pressure" (p_s). It is often referred to as NPSP and is not only significant to the turbopump mass prediction but also establishes the vehicle propellant tank pressures and influences the tank arrangement, ullage pressures, and masses as we shall see in subsequent parts of this series. Because fluid static pressures exist at local regions within the pump that are lower than the inlet static pressure, the inlet static pressure must be higher than the propellant vapor pressure to insure against cavitation at these low pressure regions within the pump. In brief, the net positive suction pressure is the minimum allowed pressure difference between pump inlet pressure and propellant vapor pressure. This difference is dependent upon the pump geometry and rotation (rpm). Increasing the value of net positive suction pressure increases the turbopump rpm which in turn decreases the turbopump assembly size and mass. However, increasing the minimum required net positive suction pressure would necessarily increase the vehicle tank pressures and mass, which is a more important consideration for booster stages. This is particularly true since the tank skin mass is directly proportional to the tank pressure while the turbopump mass is inversely proportional to the square root of the net positive suction pressure.

Inasmuch as the fuel and oxidizer vapor pressures are different and pump types vary, the net positive suction pressures are different for the fuel and oxidizer pump and must be treated as individual input design parameters for optimizing the propellant tank arrangement and masses. However, for the purpose of determining the turbopump mass, we may conveniently combine the net positive suction pressure of the oxidizer (p_{s_o}) and the fuel (p_{s_f}) in the approximate* form

$$p_s = \frac{p_{s_f}\, p_{s_o}\, (1 + \bar{r}_m)}{p_{s_f}\, \bar{r}_m + p_{s_o}} \,. \qquad (4.1b)$$

Having selected all the pertinent parameters and their respective relationships, we write the desired turbopump assembly mass as

$$W_{Tp} = 52 \times 10^3 \,\frac{1}{\sqrt{p_s}} \left[\frac{\bar{P}_c\, A_t}{I_{sp}\, \bar{\rho}_8}\right] . \qquad (4.1c)$$

* A more accurate expression would include products of net suction pressure with density of fluids pumped. However, the increased accuracy is neither warranted nor significant.

The coefficient is an average value for large rocket engines and includes the mass of piping, valves, turbopump and related accessories. In programming this equation, it must be remembered that, though the engine may be throttled or the mixture ratio varied during flight, the turbopump mass is a constant. Therefore, the pressure and density input parameters are fixed nominal values as indicated by the bar symbol. All other parameters of equation (4.1c) are assumed to be nominal fixed values throughout the text.

A probable range of net positive suction pressures are 60,000 to 220,000 newtons per square meters for LOX; 80,000 to 200,000 for kerosene; and 20,000 to 70,000 for liquid hydrogen.

4.2 Rocket Chamber Mass

The injector face, dome, combustion chamber and nozzle are components of the rocket chamber assembly that were previously cited with respect to propulsion performance. We shall now briefly review these same components with the objective of relating their structural masses to the already established propulsion parameters.

The dome and injector are either separate or integral components, whose size is related to the propellant flow rate and whose structural mass is dependent on chamber pressure loading. Because these component masses are small and are suspected of being allied to the combustion chamber and nozzle mass parameters, we shall compress them into one rocket chamber assembly mass equation as was done with the turbopump.

The combustion chamber and nozzle structure consists of contoured, adjacent tubes assembled longitudinally to contain and direct the desired flow of hot combustion gases under pressure. These tubes are regeneratively cooled by one of the propellants which flow through the tubes prior to entering the injector. The tube assembly mass may be calculated from its tube length, diameter, thickness, density, and number of tubes. While the tube diameter and number of tubes are selected on the heat and propellant flow requirements through the tubes, the tube length and number of tubes are dependent on the chamber size. The tube thickness is proportional to the chamber pressure. Knowing that the rocket chamber size increases with throat area and nozzle expansion ratio, but decreases with increasing chamber pressure, and that the regeneratively cooled tube mass increases with propellant mass rate, we would expect the tube assembly mass to assume the form

$$W = C_1 \frac{A_t}{\bar{p}_c} (\dot{m})^{x_1} (\epsilon)^{x_2}.$$

In addition to the tube mass, the chamber consists of hoop bands used to reinforce the chamber tubular walls subjected to the combustion gas pressure. The mass of hoop bands is also related to the chamber size and chamber pressure and may also be combined with the tubes, dome and injector in the form given above. Substituting only the pertinent parameters of equations (2.0a) and (2.0b) for the propellant flow rate and assigning realistic coefficients and exponents to the equation above, we obtain a manageable empirical mass equation for the rocket chamber assembly:

$$W_{RC} = 90 \times 10^{6} \left[\frac{A_t^3 \epsilon}{\bar{p}_c \ I_{sp}}\right]^{1/2} . \qquad (4.2a)$$

4.3 Rocket Engine Accessory Masses

A final category of rocket engine mass is the accessories, which may vary widely with each engine and possibly with each application of the same engine. This category is composed of pump inlet lines, primary and auxilary instrumentation, start system, controls, and interface connection systems, to mention but a few of the most common components. Because of the ambiguity of types, quantities, and sizes of accessories required during the preliminary phases of trajectory studies, the total mass allowance is best related to the propellant volume flow rate parameters and is given by

$$W_A = 90 \frac{\bar{p}_c \ A_t}{I_{sp} \ \bar{\rho}_8} . \qquad (4.3a)$$

4.4 Rocket Engine Total Mass

Having derived approximate mass parametric equations of the turbopump, rocket chamber and engine accessories, we combine equations (4.1c), (4.2a), and (4.3a) to express the total engine mass (W4.1)* as

$$W_{4.1} = \left[\frac{55 \times 10^{3}}{\sqrt{p_s}} + 90\right] \left[\frac{\bar{p}_c \ A_t}{I_{sp} \ \bar{\rho}_8}\right] + 90 \times 10^{6} \left[\frac{A_t^3 \epsilon}{\bar{p}_c \ I_{sp}}\right]^{1/2} . \qquad (4.4a)$$

* MSFC std. notation [5] for liquid rocket engine mass.

It is interesting to note that the total engine mass increases with increasing throat area, decreasing specific impulse and increasing nozzle expansion ratio. However, it is not obvious that an increase in chamber pressure will effect an increase or decrease in total engine mass without knowing relative values of associated mass parameters.

5.0 CONCLUDING REMARKS

The performance, size, and mass of a large, conventional, liquid chemical rocket engine were suitably defined by interconnecting parameters. The propellant flow rate and thrust performance equations were modified to include propellant mixture ratio shift and throttling during stage burning. This was accomplished through the engagement of the thrust coefficient and nozzle expansion ratio, in terms of specific heat ratio and pressures. It may be noted that for a fixed chamber pressure and nozzle expansion ratio, a shift in mixture ratio causes a change in the specific heat ratio which alters the thrust coefficient and thrust. A mixture ratio shift also shifts the specific impulse coefficient. In throttling the mass flow rate, the chamber pressure is reduced, the thrust coefficient is affected and the net results is a reduction of thrust.

An equation of total propellant mass consumption during flight time was presented which allowed for variations of pressure and mixture ratio shifts. An average mixture ratio shift for the flight history was also presented in order to calculate the stage propellant tank sizes [1].

The rocket engine space envelope was defined by its longitudinal length and nozzle exit diameter. The most significant parameters affecting the size were the throat area and nozzle expansion ratio.

The engine total mass equation appeared to be strongly dependent on those parameters which defined propellant mass flow rate and moderately dependent on the nozzle expansion ratio. An increase in the nominal chamber pressure increases the turbopump mass but decreases the rocket chamber mass for a fixed thrust value. The net increase or decrease depends on relative values of associated mass parameters.

Looking back for a moment, we recall the analyst's contentment in conducting his preliminary trajectory and performance studies with a few basic propulsion equations and a modestly estimated mass fraction. Minor refinements often resulted in great improvements. Now, more sophisticated refinements are possible than those presented, and the complexity is increasing more rapidly than the resulting improvements. If the reader is troubled as to the merit of incorporating this, or similar, bulk of equations into a program, he may be reminded that the added analytical labor is more rewarding now in that it can be used for optimizing stage parameters which lead to cost reduction, or added mission potential, of ever-increasing new vehicle proportions.

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APPROVAL NASA TM X-53053

BOOSTER PARAMETRIC DESIGN METHOD FOR PERFORMANCE AND TRAJECTORY ANALYSIS

PART II: PROPULSION

By V. Verderaime

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Helmut J. Horn
Chief, Dynamics and Flight Mechanics Division

E. D. Geissler
Director, Aero-Astrodynamics Laboratory

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