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ESCAPE OF STARS DURING THE COLLISION OF TWO GALAXIES

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ABSTRACT

An approximate theory is developed to find out whether a star, moving initially on the plane of symmetry of a galaxy, escapes during the collision with another galaxy of small dimensions. The escaping stars are inside an ellipse on the plane of symmetry of the galaxy. Numerical calculations in a Bottlinger model of a galaxy give results of the same order of magnitude. The total amount of the escaping matter is only a few per cent of the total mass of the galaxy, if the collision velocity is of the order of 2000 km/sec. The behavior of the escaping and non-escaping stars is described. The escaping stars do not follow the colliding galaxy and do not form a bridge or link between the two galaxies.

I. ESCAPING STARS

The purpose of this paper is to consider the effects of the collision of two galaxies on stars moving initially on the plane of symmetry of one galaxy.

If we assume a relative velocity of collision of about 2000 km/sec (Spitzer and Baade 1951), the collision lasts about 10^7 years. During that time the perturbations due to the ellipticity of the first galaxy are insignificant for stellar orbits close to its plane of symmetry (Contopoulos and Bozis 1962; hereafter referred to as "Paper I"). Therefore in order to study approximately the effects of the collision on these orbits we may use a spherical model of a galaxy. The colliding galaxy is also assumed to be spherical but of small dimensions. We will see that, if its diameter is less than 2 kpc, its effects are essentially the same as those of a point mass. In the numerical examples below the mass of both galaxies is assumed to be about $10^{11} M_\odot$, and the radius of the first galaxy is taken 10 kpc.

In Paper I it has been proved that, if two such galaxies collide with a relative velocity $v = 2000$ km/sec, their relative motion may be considered as uniform; in fact the change of velocity is of the order of 30 km/sec and the change of the direction of the velocity is less than 5°.

Further, during the collision, a star moving in a circular orbit on the plane of symmetry of the first galaxy O with radius 8 kpc describes only about 1/20 of a revolution, i.e., about 18°; therefore, we may consider, in a rough approximation, the unperturbed orbit of S to be a straight line, $x' = x_0'$, perpendicular to the axis x', which is directed away from the center O (Fig. 1), and parallel to the axis y'. Under these assumptions the perturbation may be considered as a two-body encounter that lasts as long as the collision itself.

The galaxy O' is assumed to move along the axis z' with uniform velocity $\bar{v}$, reaching the point O_1 on the plane xOy at $t = t_0$. The initial velocity of the star is $\bar{v}_c$; if the perturbing galaxy was not present, S would reach the axis x' at $t = t_1$.

The velocities v and v_c are of the order of 2000 km/sec and 200 km/sec, respectively; therefore, during 10^7 years the galaxy O' moves approximately from $z' = 10$ kpc (initial position) to $z' = -10$ kpc (final position), while S moves about 2 kpc along a parallel to the y'-axis. The influence of O' on S when the distance OO' is larger than 10 kpc is very small. In fact, as we will see below, the escaping stars are inside an ellipse whose dimensions are of the order of 4 kpc. Therefore, their minimum distance from O' is about 2 kpc or less. When a point comes from infinity to a distance of 10 kpc the relative velocity changes only by about 1 per cent and the change of its direction is of the order of 2 per cent of the angle between the asymptotes. In fact, if v_∞ is the relative velocity before the encounter and v the relative velocity at a distance r from O', then

$$v^2 = \frac{2Gm}{r} + v_\infty^{\,2},$$

hence

$$\Delta v = v - v_\infty \simeq \frac{Gm}{r\,v_\infty},$$

and for $r = 10$ kpc, $v_\infty = 2000$ km/sec, and $m = 10^{11}\,M\odot$,

$$\Delta v = 22\,\text{km/sec}.$$

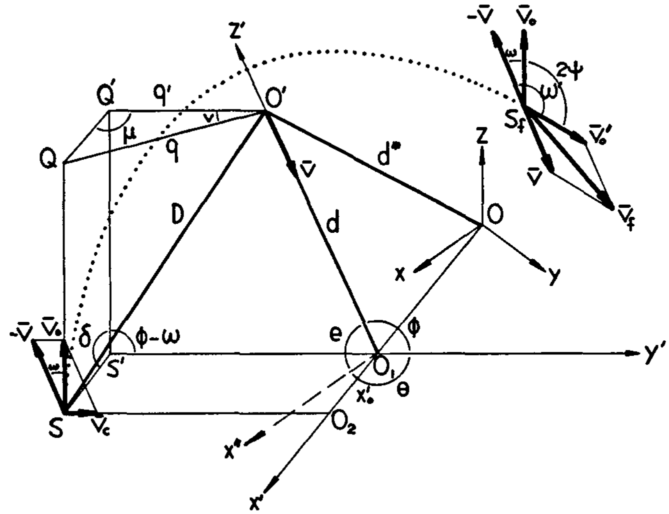

FIG. 1.—A galaxy O' of mass m is moving downward along the axis z' toward the point O_1 on the plane of symmetry of the galaxy O, of mass M. The axis Ox is parallel to the projection O_1x'' of O_1z' on the plane of symmetry. The distances of O' from O, O_1, and S are d^*, d, and D, respectively.

The angle 2ψ between the initial and final velocities is given by $\tan\psi = p/q$, where $x_1^2/p^2 - y_1^2/q^2 = 1$ is the equation of the relative orbit. As q is of the order of 2 kpc, and $p = Gm/v_\infty^2 \simeq 0.1$ kpc, we have $p/q \simeq 0.05$.

The tangent at a point (x_1, y_1) whose distance from the focus is 10 kpc makes an angle ψ' with the axis y_1, where

$$\tan\psi' = \tan\psi\,\sqrt{\left(1 - \frac{p^2}{x_1^2}\right)},$$

and $[x_1 - \sqrt{(p^2 + q^2)}]^2 + y_1^2 = 100$.

Then the deviation $(\psi - \psi')$ is given by

$$\tan(\psi - \psi') = \frac{(p/q)\,[10 + p - \sqrt{(100 + 20p - q^2)}\,]}{10 + p + (p^2/q^2)\sqrt{(100 + 20p - q^2)}} \simeq \frac{p}{q}\cdot\frac{0.05q^2}{10} = 0.005pq\,.$$

Hence

$$\frac{\psi - \psi'}{\psi} \simeq 0.005q^2,$$

and for $q = 2$, this is 2 per cent. Therefore we may consider the initial and final relative velocities of S with respect to O as being equal to the corresponding velocities in a two-body encounter at infinite distances.

The initial relative velocity of the star S with respect to the galaxy O' is $\bar{v}_0 = \bar{v}_c - \bar{v}$; the corresponding final velocity $\bar{v}_0'$ is equal to $\bar{v}_0$ in measure but not in direction. The final velocity of S with respect to the galaxy O is $\bar{v}_f = \bar{v}_0' + \bar{v}$; if v_f is greater than the velocity of escape v_e, the star escapes from the galaxy O.

In what follows we use a frame of reference where O' is at rest. The galaxy O is moving with respect to O' with velocity $-\bar{v}$ along the axis z' that makes an angle e with the plane xOy (Fig. 1). The projection of z' on the plane xOy makes an angle θ with O_1y', and φ is the angle $z'O_1y'$. Then

$$\cos\varphi = \cos e \cos\theta . \tag{1}$$

At the initial time t we have

$$y' = O_1S' = v_c(t - t_1) \tag{2}$$

and

$$z' = O_1O' = v(t_0 - t). \tag{3}$$

If ω is the angle between $\bar{v}_0$ and $-\bar{v}$, we have

$$\frac{\sin\omega}{v_c} = \frac{\sin\varphi}{v_0} = \frac{\sin(\varphi - \omega)}{v} \tag{4}$$

and

$$v_0^2 = v^2 + v_c^2 + 2 v v_c \cos\varphi . \tag{5}$$

The corresponding angle between $\bar{v}_0'$ and $-\bar{v}$ is ω', and the angle between $\bar{v}_0$ and $\bar{v}_0'$ is 2ψ, which is approximately equal to the angle between the asymptotes of the hyperbolic orbit. Therefore

$$\tan\psi = \frac{Gm}{q v_0^2}, \tag{6}$$

where q is the impact parameter.

The impact parameter is found as follows. Let SS' be parallel to the axis x' and $S'Q'$ parallel to $\bar{v}_0$, $O'Q$ and $O'Q'$ perpendicular to SQ and $S'Q'$, respectively; $S'Q'$ is on the plane $S'O_1O'$ and QQ' is perpendicular to SQ and $S'Q'$. The angle between the planes $QQ'S'$ and $O'Q'S'$ is μ, and the angle between the planes $QO'S$ and $Q'O'S'$ is ν. The angle $Q'S'O_1$ is equal to $\varphi - \omega$, therefore

$$\begin{aligned} q' &= -y'\sin(\varphi - \omega) - z'\sin\omega = -\left(y' + z'\frac{v_c}{v}\right)\sin(\varphi - \omega) \\ &= v_c(t_1 - t_0)\sin(\varphi - \omega) = -y_0'\sin(\varphi - \omega), \end{aligned} \tag{7}$$

where

$$y_0' = v_c(t_0 - t_1) \tag{8}$$

is the y' coordinate of S at $t = t_0$. The term q' is positive if $y_0' < 0$, and negative if $y_0' > 0$. Further

$$S'S = O_1O_2 = x_0', \quad Q'Q = x_0'\sin\delta, \quad \text{and} \quad q^2 = q'^2 + x_0'^2\sin^2\delta - 2q'x_0'\sin\delta\cos\mu . \tag{9}$$

From the trihedral angle $S'Q'SO_1$ we find

$$\cos\delta = \sin(\varphi - \omega)\cos I \tag{10}$$

and

$$\sin\delta\cos\mu = -\cos(\varphi - \omega)\cos I \tag{11}$$

where I is the angle between the planes $y'O_1z'$ and $y'O_1x'$, hence

$$\cos I = \cot\varphi \tan\theta = \frac{\cos e \sin\theta}{\sin\varphi}. \tag{12}$$

Therefore,

$$q^2 = x_0'^2[1 - \sin^2(\varphi - \omega)\cos^2 I] + y_0'^2 \sin^2(\varphi - \omega) - 2x_0'y_0' \sin(\varphi - \omega)\cos(\varphi - \omega)\cos I . \tag{13}$$

A star escapes if

$$v_f^2 = v^2 + v_0^2 - 2 v v_0 \cos\omega' \geq v_e^2 . \tag{14}$$

This is possible only if $v + v_0 \geq v_e$. Then we can set

$$-1 \leq \frac{v^2 + v_0^2 - v_e^2}{2 v v_0} = \cos\omega_0' < \frac{v^2 + v_0^2 - v_c^2}{2 v v_0} = \frac{v + v_c \cos\varphi}{v_0} = \cos\omega < 1 , \tag{15}$$

because $v_c < v_e$. Therefore $O < \omega_0' \leq \pi$, and if a star escapes we have

$$\cos\omega' \leq \cos\omega_0' . \tag{16}$$

The angle ω' can be found from the trihedral angle $S_f\bar{v}_0\bar{v}_0'(-\bar{v})$;

$$\cos\omega' = \cos 2\psi \cos\omega + \sin 2\psi \sin\omega \cos(\pi - \nu) , \tag{17}$$

where $(\pi - \nu)$ is the angle between the plane $S_f\bar{v}_0\bar{v}_0'$ (i.e., the plane $O'QS$ of the relative orbit) and the plane $S_f\bar{v}_0(-\bar{v})$ (which is parallel to the plane $O'Q'S'$); thus $(\pi - \nu)$ is the complement of the angle $QO'Q'$, and

$$\cos\nu = \frac{q' - x_0' \sin\delta \cos\mu}{q} = \frac{1}{q}[- y_0' \sin(\varphi - \omega) + x_0' \cos(\varphi - \omega)\cos I] . \tag{18}$$

If we use relations (6) and (17) in inequality (16) we get

$$\{x_0'^2[1 - \sin^2(\varphi - \omega)\cos^2 I] + y_0'^2 \sin^2(\varphi - \omega) - 2x_0'y_0' \sin(\varphi - \omega)\cos(\varphi - \omega)\cos I\}$$
$$(\cos\omega - \cos\omega_0') + \frac{2Gm}{v_0^2} \sin\omega \sin(\varphi - \omega) y_0' - \frac{2Gm}{v_0^2} \sin\omega \cos(\varphi - \omega)\cos I x_0' \tag{19}$$
$$\leq \frac{G^2m^2}{v_0^4}(\cos\omega + \cos\omega_0') ,$$

i.e., the escaping stars are inside the ellipse

$$x_0'^2(1 - \sin^2(\varphi - \omega)\cos^2 I) + (y_0' - y_c')^2 \sin^2(\varphi - \omega) - 2x_0'(y_0' - y_c')\sin(\varphi - \omega)\cos(\varphi - \omega) \cos I = \frac{G^2m^2 \sin^2\omega_0'}{v_0^4(\cos\omega - \cos\omega_0')^2} , \tag{20}$$

whose center is ($x_0' = 0$, y_c'), where

$$y_c' = -\frac{Gm \sin\omega}{v_0^2 \sin(\varphi - \omega)(\cos\omega - \cos\omega_0')} = -\frac{2Gm\, v_c}{(v_e^2 - v_c^2)\, v_0} . \tag{21}$$

The axes of this ellipse form an angle γ with the axes x' and y', where

$$\tan\gamma = -\tan(\varphi - \omega)\cos I = \frac{-\tan(\varphi - \omega)\cos e \sin\theta}{\sin\varphi} . \tag{22}$$

For $0 < \theta < \pi$, the angle γ is positive (measured counterclockwise) if $\varphi - \omega > \pi/2$ and negative if $\varphi - \omega < \pi/2$. For $-\pi < \theta < 0$, i.e., if the projection of z' on the plane $x'O_1y'$ is on the negative side of the axis x', the angle γ is positive if $\varphi - \omega < \pi/2$ and negative if $\varphi - \omega > \pi/2$. For $\theta = 0$ or $\theta = \pi$, the angle $\gamma = 0$.

If $\theta = \pi/2$ then $\varphi = \pi/2$, $I = e$, and equations (20)–(22) become

$$x_0'^2(1 - \cos^2\omega\cos^2 e) + \cos^2\omega(y_0' - y_c')^2 - 2x_0'(y_0' - y_c)\sin\omega\cos\omega\cos e = \frac{G^2m^2\sin^2\omega_0'}{v_0^4(\cos\omega - \cos\omega_0')^2}, \tag{23}$$

$$y_c' = -\frac{Gm\tan\omega}{v_0^2(\cos\omega - \cos\omega_0')}, \tag{24}$$

and

$$\tan\gamma = -\cot\omega\cos e\ . \tag{25}$$

From equation (20) we find easily the area of the ellipse

$$E = \pi a_x' a_y' = \frac{\pi G^2 m^2 \sin^2\omega_0'}{v_0^4(\cos\omega - \cos\omega_0')^2\sin(\varphi - \omega)\sin I}. \tag{26}$$

TABLE 1

r (kpc)	a_x (kpc)	a_y (kpc)	E (kpc)2	y_c' (kpc)	ω (degrees)
4.	1.41	1.42	6	−0.7	6.22
6.	1.94	1.95	12	−1.2	6.55
8.	2.41	2.42	18	−1.6	6.19
10.	2.80	2.81	25	−1.9	5.74

When $v \to \infty$, then $E \to 0$. If $I = \pi/2$, then the ellipse becomes

$$x_0'^2 + (y_0' - y_c')^2\sin^2(\varphi - \omega) = \frac{G^2m^2\sin^2\omega_0'}{v_0^4(\cos\omega - \cos\omega_0')^2}. \tag{27}$$

The semi-axes of the ellipse are then

$$a_x = \frac{Gm\sin\omega_0'}{v_0^2(\cos\omega - \cos\omega_0')} = \frac{2Gm\sin\omega_0'\sin(\varphi - \omega)}{(v_e^2 - v_c^2)\sin\varphi}, \tag{28}$$

and

$$a_y = \frac{Gm\sin\omega_0'}{v_0^2(\cos\omega - \cos\omega_0')\sin(\varphi - \omega)} = \frac{2Gm\sin\omega_0'}{(v_e^2 - v_c^2)\sin\varphi}. \tag{29}$$

If $v \to \infty$, then $a_x \to 0$ and $a_y \to 0$.

The ratio of the axes is

$$a_x/a_y = \sin(\varphi - \omega) = v\sin\varphi/v_0\ , \tag{30}$$

and the area of the ellipse is

$$E = \pi a_x a_y = \frac{4\pi G^2m^2\sin^2\omega_0'\sin(\varphi - \omega)}{(v_e^2 - v_c^2)^2\sin^2\varphi}. \tag{31}$$

Table 1 gives the values of a_x, a_y, E, y_c', and ω for the model considered in Section II with $e = \pi/2$, $\varphi = \pi/2$, $v = 2000$ km/sec $= 2.0454$ kpc/10^6 years, and different minimum distances $r = \sqrt{(\alpha^2 + \beta^2)}$ of the colliding galaxy from the center.

If the effective radius of the galaxy R_G is about 10 kpc, the above area is between 2 per cent (for $r = 4$ kpc) and 8 per cent (for $r = 10$ kpc) of the total area.

The proportion of the escaping stars is roughly $E\rho/\pi R_G \bar{\rho}^2$ where $\bar{\rho}$ is the mean density of the galaxy and ρ is the mean density in the region of escape.

Table 2 gives the values of φ, E, y_c', ω for ($e = 75°$, $\theta = 0°$), ($e = 45°$, $\theta = 0°$) and ($e = 45°$, $\theta = 45°$).

The value of γ in the first two cases is zero, and in the last case varies between $-39^\circ\!.8$ and $-39^\circ\!.1$.

It is seen that the values of E and y_c' in all three cases are quite near the corresponding values of Table 1. Therefore, the amount of escaping matter from the plane of symmetry of a galaxy does not depend very much on the direction of the collision.

We do not give the values of E, etc., for $r < 4$ because near the center the assumption of a binary collision between the star and the approaching galaxy is not even approximately satisfied.

If the diameter of the colliding galaxy is 2 kpc (or less), its cross-section is small, in general, in comparison with the area E of the region of escape; therefore, the amount of escaping matter is essentially the same as in the case of a point mass. If the diameter is larger than 2 kpc, the amount of escaping matter will be smaller.

TABLE 2

r (kpc)	$e=75°$, $\theta=0°$, $\varphi=75°$			$e=45°$, $\theta=0°$, $\varphi=45°$			$e=45°$, $\theta=45°$, $\varphi=60°$		
	E (kpc)^2	y_c' (kpc)	ω (degrees)	E (kpc)^2	y_c' (kpc)	ω (degrees)	E (kpc)^2	y_c' (kpc)	ω (degrees)
4	6	−0.7	5.85	7	−0.7	4.10	7	−0.7	5.12
6	11	−1.2	6.15	11	−1.1	4.29	13	−1.2	5.37
8	17	−1.6	5.82	17	−1.5	4.08	20	−1.5	5.09
10	23	−1.9	5.41	23	−1.8	3.80	28	−1.9	4.74

II. NUMERICAL CALCULATIONS

In order to calculate numerically the orbits of stars during the collision of two galaxies we have chosen a simple model for the first galaxy, in which the forces per unit mass are given by Bottlinger's formula

$$F = \frac{a r}{1 + b r^3}. \tag{32}$$

This formula has been applied by Lohmann (1953; 1954*a*, *b*) to our Galaxy, as well as to M31 and M33, and by Kerr and de Vaucouleurs (1956) to the Large Magellanic Cloud. It is discussed, together with other models, by de Vaucouleurs (1959) and Perek (1962).

The values of a and b were chosen to represent approximately the force function F given by Schmidt (1956) for the plane of symmetry of our Galaxy, namely,

$$a = 0.0055\,(10^6 \text{ years})^{-2}, \qquad b = 0.012 \text{ kpc}^{-3}.$$

The corresponding potential function is

$$U = -\frac{a}{b k}\left\{\frac{1}{6} \ln \left[\frac{r^2 - k r + k^2}{(r + k)^2}\right] - \frac{1}{3} \cot^{-1}\left(\frac{2r - k}{k\sqrt{3}}\right)\right\}, \tag{33}$$

where $k = b^{-1/3}$ and $U_\infty = 0$.

In this model the mass of the Galaxy is

$$M=\frac{a}{bG}\simeq 1.02\times 10^{11}M_{\odot}. \tag{34}$$

The circular velocity is

$$v_c=\sqrt{(Fr)}, \tag{35}$$

and the escape velocity is

$$v_e=\sqrt{(2U)}. \tag{36}$$

Table 3 gives the values of v_c and v_e for different distances r from the center in the above field.

Now we consider the following situation (Fig. 1): A point mass $O'(m)$ goes through a spherical galaxy $O(M)$, represented by the force function (32). This force function is assumed to be stationary; this is justified if the colliding galaxy does not come very close

TABLE 3

r (kpc)	v_c (kpc/10^6 years)	v_e (kpc/10^6 years)	r (kpc)	v_c (kpc/10^6 years)	v_e (kpc/10^6 years)
4.........	0.2231	0.4294	14.........	0.1783	0.2549
6.........	.2348	.3750	16.........	.1676	.2388
8.........	.2220	.3321	18.........	.1584	.2253
10.........	.2057	.2997	20.........	0.1506	0.2138
12.........	0.1909	0.2748			

to the center of the galaxy O, because then the amount of escaping matter is rather small. In fact the density corresponding to the force function (32) is

$$\rho=\frac{3a}{4\pi G(1+br^3)^2},$$

and the mean density is

$$\bar{\rho}=\frac{3a}{4\pi G bR_G{}^3},$$

so that if we take a mean radius of the galaxy $R_G = 10$ kpc, then the proportion of the escaping stars is

$$\frac{E\rho}{\pi R_G{}^2\bar{\rho}}\simeq\frac{0.04E}{(1+0.012r^3)^2};$$

for $r = 10$ kpc this is about 0.6 per cent while for $r = 4$ kpc this is 8 per cent.

The motion of O' is assumed to be uniform along the line $O'O_1$, where $O_1(\alpha, \beta)$ lies on the plane xOy. The plane xOz is taken parallel to the direction $O'O_1$ and e is the angle between this line and the plane xOy. The coordinates of the star S are x, y, z, and d, d^*, D are the distances of O' from O_1, O, and S, respectively.

The unperturbed orbit of S is, in general, a rosette on the plane xOy. In this paper, however, we consider only the case of circular orbits around O.

The perturbation acting on the star is equal to the acceleration of S due to the galaxy O' minus the acceleration of the galaxy O, taken as a whole. As the attraction between O and O' is $mF(d^*)$, the acceleration of O is $(m/M)F(d^*)$. Thus the equations of motion of S are

$$\frac{d^2x}{dt^2}=-F(r^*)\frac{x}{r^*}+\frac{Gm}{D^3}(\alpha+d\cos e-x)-F(d^*)\frac{m}{M}\frac{(\alpha+d\cos e)}{d^*}, \tag{37}$$

$$\frac{d^2y}{dt^2}=-F(r^*)\frac{y}{r^*}+\frac{Gm}{D^3}(\beta-y)-F(d^*)\frac{m}{M}\frac{\beta}{d^*}, \tag{38}$$

and

$$\frac{d^2z}{dt^2} = -F(r^*)\frac{z}{r^*} + \frac{Gm}{D^3}(d\sin e - z) - F(d^*)\frac{m}{M}\frac{d\sin e}{d^*}, \tag{39}$$

where

$$r^* = \sqrt{(x^2+y^2+z^2)}, \tag{40}$$

$$d^* = \sqrt{[(\alpha+d\cos e)^2+\beta^2+(d\sin e)^2]}, \tag{41}$$

$$D = \sqrt{[(\alpha+d\cos e-x)^2+(\beta-y)^2+(d\sin e-z)^2]}, \tag{42}$$

and

$$d = d_0 - vt. \tag{43}$$

The numerical values used are the following: $m/M = 1$, $MG = a/b$, $v = 2000$ km/sec = 2.0454 kpc/10^6 years, $d_0 = 100$ kpc, $\alpha = 8$ kpc, $\beta = 0$, and $e = 90°$ (case A), or

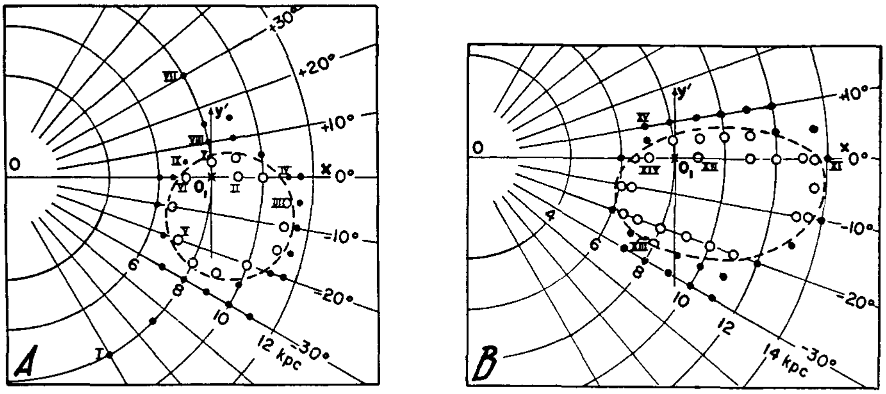

FIG. 2.—The position (r, g in polar coordinates) of escaping (*open circles*) and non-escaping (*solid circles*) stars in their unperturbed orbits when the colliding galaxy is going through the point $O_1(\alpha = 8$ kpc, $\beta = 0)$. In case A, $e = 90°$; in case B, $e = 45°$.

$e = 45°$ (case B). The initial conditions for $t = 0$ were taken in such a way that the star, moving in the unperturbed circular orbit, would reach an axis forming an angle g with the positive x-axis (g is positive if measured counterclockwise), when the colliding galaxy reaches the point O_1 (α, β), namely, at about $t = 48.89 \times 10^6$ years.

The greatest part of the calculations was made with the IBM 7090 computer of NASA's Institute for Space Studies, New York. The Runge-Kutta integration method was used with a step 0.1 (10^6 years) and the orbits were calculated for 200×10^6 years. At $t = 200 \times 10^6$ years the colliding galaxy is at a distance of about 300 kpc from the galaxy O and its influence is then negligible. In some cases, when the stars escaped rather fast from the galaxy O, the calculations were stopped earlier.

The initial z coordinates and velocities were all assumed to be zero. Figure 2*A* shows the positions that the stars would have in case A ($e = 90°$) at $t = 48.89 \times 10^7$ years (when the colliding galaxy reaches the point O_1), if there was no perturbation. The polar coordinates of these positions are (r, g); therefore the initial polar coordinates, for $t = 0$, are

$$r_0 = r$$

and

$$g_0 = g - 48.89\frac{v_c}{r} \times 57.^\circ 2957795$$

The dots represent stars that are not escaping. The small circles represent escaping stars. The region of escaping stars is approximately a circle of diameter 5 kpc. From Table 1 we see that the ellipse of escaping stars for $r = \sqrt{(\alpha^2 + \beta^2)} = 8$ kpc is very nearly a circle of diameter 4.8 kpc; the agreement with the calculations is very good in this case.

The center of the circle in Figure 2A is not at $x_c' = 0$, as it was expected theoretically, but at about $x_c' = 1$ kpc, i.e., further out than the point O_1. On the other hand, the y' coordinate of the center is about $y_c' = -1.5$ kpc, i.e., very close to the calculated value. Therefore, the only discrepancy between theory and calculations is that the actual region of escape is further out than expected; this is due, of course, to the fact that the unperturbed orbits are curved and not straight lines as they were assumed in Section I. The only consequence of this effect is that the region of escape actually includes less dense areas, therefore, the amount of escaping matter is smaller.

In case B ($e = 45°$) the calculated orbits are marked in Figure 2B. The region of escape is approximately an ellipse of dimensions $2a = 8$ kpc, $2b = 5$ kpc, and the area $E = 31$ kpc^2. This is bigger than expected on the basis of the simple theory developed in Section I. However, even in this case the area E is only 10 per cent of the effective area in the plane of symmetry of the galaxy O, and as it includes mainly the outer parts of the galaxy the mass of the escaping matter is less than 2 per cent of the total matter near the plane xOy.

The individual orbits of stars during the collision show a number of interesting characteristics. Figures 3 and 4, copied from plots made by the NASA computer, show the main types of orbits encountered, in the case $e = 90°$.

A common characteristic of all the orbits is that the perturbations are very small until the colliding galaxy goes through the galaxy O. Until that moment the orbits are practically circles on the xy-plane. After the collision the orbits may be divided into three types: (a) orbits of stars that remain far from the region of escape; (b) "escaping" orbits; and (c) orbits of stars near the region of escape.

If the star is far from the region of escape, its orbit is not changed much by the collision. It remains almost circular, and the z-motion is very small. Such is the case of orbit I ($r = 8$ kpc, $g = -60°$).

The orbits of stars inside or near the region of escape can be divided into the following groups (cf. Figs. 2A and 3A–3D): (i) stars with $g \leq 0°$ and $r \geq 8$, or r smaller than 8, but g not near 0°; (ii) stars with $r < 8$ and $g \leq 0°$ but g near 0°; and (iii) stars with $g > 0°$. We discuss these groups separately.

i) The stars with $g \leq 0°$ that are inside or near the region of escape are accelerated toward the positive y'-axis as the colliding galaxy goes through the galaxy O. If $r > 8$ kpc the orbits are deflected to the left. This deflection is bigger whenever the approach to the colliding galaxy is closer. For example, in case II ($r = 9$, $g = 0°$) the star is ejected abruptly to the left and down, while in case III ($r = 11$, $g = -5°$) the star escapes but with smaller velocity. In case IV ($r = 11$, $g = 0°$) the star does not escape, but its orbit does not differ much from the escaping orbits for $t = 200 \times 10^6$ years. Only the fact that its velocity is less than the escape velocity makes it sure that it does not escape, but forms a very elongated orbit around the center of the galaxy O.

All orbits near the boundary of the region of escape from ($r = 11$, $g = 0°$) to ($r = 8$, $g = -25°$) are similar to cases III and IV. The y component of the final velocity is negative for large r, while it is positive for r near 8.

The same general behavior is shown by the orbits with $r < 8$ but g not close to 0°. For example, case V ($r = 7$, $g = -20°$) represents an escaping orbit; the star escapes toward the positive y direction and downward.

ii) The orbits with $r < 8$ and g near 0° show an abrupt deviation to the right. This is most marked in case VI ($r = 7$, $g = 0°$) when the star is ejected abruptly to the right and downward.

iii) The orbits with positive g are usually not escaping. In general, the additional attraction of the colliding galaxy makes the orbits more elongated. Such is the case VII ($r = 8$, $g = +30°$), that is rather far from the region of escape. In case VIII ($r = 8$, $g = +10°$), however, the deviation is so large, that the orbit reverses the sense of rotation, although the star does not escape. A large deviation, but to the right, is shown in case IX ($r = 7$, $g = +5°$). Finally, case X ($r = 8$, $g = +5°$) shows an abrupt reversal of the motion of the star and ejection toward the negative y-axis. Such a behavior, however, is quite exceptional.

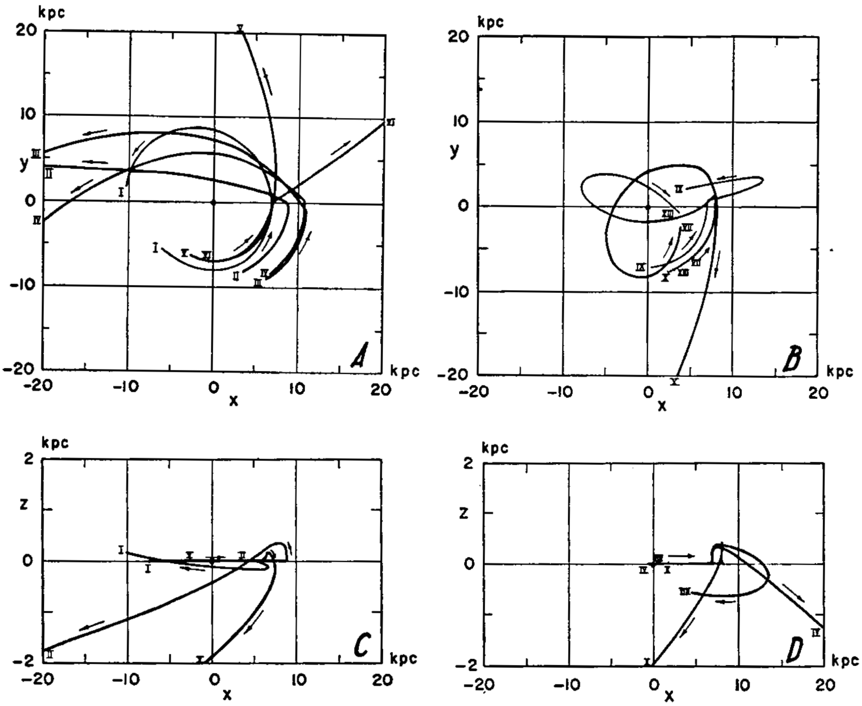

FIG. 3.—Characteristic cases of orbits during a collision through O_1($a = 8$ kpc, $\beta = 0$) with $e = 90°$: I ($r = 8$ kpc, $g = -60°$; non-escaping); II ($r = 9$ kpc, $g = 0°$; escaping); III ($r = 11$ kpc, $g = -5°$; escaping); IV ($r = 11$ kpc, $g = 0°$; non-escaping); V ($r = 7$ kpc, $g = -20°$; escaping); VI ($r = 7$ kpc, $g = 0°$; escaping); VII ($r = 8$ kpc, $g = +30°$; non-escaping); VIII ($r = 8$ kpc, $g = +10°$; non-escaping); IX ($r = 7$ kpc, $g = +5°$; non-escaping); X ($r = 8$ kpc, $g = +5°$; escaping). A, B give the projections of the orbits on the xy-plane, while C, D give the projections of some orbits on the xz-plane.

From the above discussion we conclude that the greatest proportion of the ejected stars escapes toward the left (between the positive y-axis and the negative x-axis) and downward. The stars do not follow the colliding galaxy after the collision and they do not form a bridge or link between the two galaxies.

In case B ($e = 45°$) the orbits are similar to those of case A, except as regards the z coordinate (cf. Figs. 2B and 4).

i) The stars with $r \geq 8$, $g \leq 0°$ have comparatively large motions upward, as the colliding galaxy approaches, which are changed to downward motions later, as in case XI ($r = 14$, $g = 0°$). This is because in these cases the approaching galaxy comes rather close to these stars before it reaches the plane xOy at O_1.

If r is near 8 kpc but $r > 8$, and $g \simeq 0°$ the upward motions are so fast that the stars eventually escape upward and to the left (case XII, $r = 9$, $g = 0°$). If r is near 8 kpc but g not near 0° the upward motions are small (case XIII, $r = 8$, $g = -25°$), and if $r < 8$ kpc the stars go downward, showing only a small upward kink, as in case V. The downward motions in this case can be explained easily, because as the approaching galaxy comes from the right, it does not influence appreciably the stars to the left of O_1 until it goes beyond the point O_1 and comes rather close to them.

ii) The stars with $r < 8$ but close to 8 and g near 0° show an abrupt motion downward (case XIV, $r = 7$, $g = 0°$).

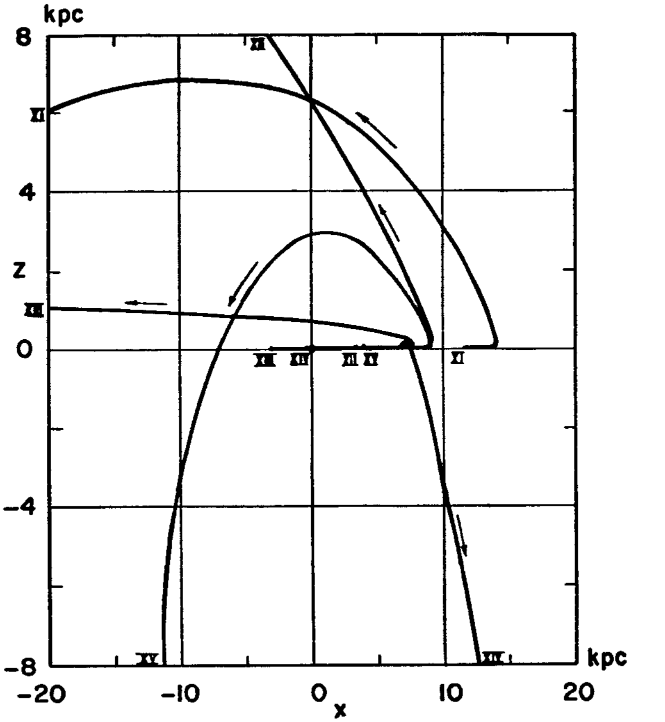

FIG. 4.—Characteristic cases of orbits during a collision through $O_1(\alpha = 8$ kpc, $\beta = 0)$ with $e = 45°$: XI ($r = 14$ kpc, $g = 0°$; escaping); XII ($r = 9$ kpc, $g = 0°$; escaping); XIII ($r = 8$ kpc, $g = -25°$; escaping); XIV ($r = 7$ kpc, $g = 0°$; escaping); XV ($r = 9$ kpc, $g = +10°$; non-escaping). Only the projections on the xz-plane are given.

iii) The orbits of stars with $g > 0°$ show all the peculiarities mentioned in case A. The z motions are of the form of case XV ($r = 9$, $g = +10°$) if $r > 8$, and of the form of case XIV if $r < 8$, but with less steep descent.

We conclude that if a galaxy goes through the plane of symmetry of another galaxy at an angle different from 90° it causes the stars to acquire appreciable z velocities. The escaping stars move usually downward, but they do not follow the colliding galaxy in its course.

The final conclusion of this paper is that, during a collision of two galaxies with rather large relative velocity, only a small fraction of matter escapes and this does not form any bridge or link between the galaxies. Therefore the observed bridges between pairs of galaxies probably are not due to collisions with relative velocities of the order of 2000 km/sec. It is possible, however, that much slower collisions, not considered in this paper, may cause much larger changes in the colliding galaxies and eventually explain the formation of bridges between them.

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