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THE DOUBLE-PERIPHERAL MODEL, AND ITS APPLICATION TO INELASTIC PION-NUCLEON COLLISIONS

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ABSTRACT

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The double-peripheral model is discussed and applied to four pion-nucleon reactions producing respectively a π^0, π^+, ω, and ρ^0. The results for the first two reactions are compared with a recent experiment at 4 GeV/c, and some additional predictions are given.

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ERRATUM

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The second sentence of the second paragraph of page 18 should read as follows :

"For the first, we get the curve of Fig. 3b, and for the latter, we get a contribution......"

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1. INTRODUCTION

For inelastic processes at energies sufficiently above threshold, the peripheral model [1] appears to be a useful description. Elaborating on this, Amati, Fubini, and Stanghellini developed the multiperipheral model [2], which they were able to solve in the asymptotic region of very high energies. The predictions of this model are in qualitative agreement with the available information from the highest accelerator energies as well as cosmic ray energies.

The double-peripheral model is the multiperipheral model, specialized to the exchange of exactly two virtual mesons. This is indicated by the graphs of Fig. 1. The "order of peripheralism" equals the number of virtual mesons in the graph. Among the virtual mesons, we shall allow not only pions, but also resonances like ρ and ω, and also the "vacuum (Pomeranchuk) trajectory".

The outgoing particles in Fig. 1 need not be stable, but may be resonances and even more general groups of particles. The only restriction is that the "invariant mass" of each group should be low enough to exclude additional peripheral structure <u>within</u> the group, since such interactions are in fact taken care of by a multiperipheral graph of the next higher order.

The double-peripheral graphs are calculated from Feynman's rules, the main additional feature being "form factors" of the momentum transfers. In section 2 we discuss our choice of form factors and coupling constants. In section 3 we develop the detailed expressions for the matrix elements and cross-sections for the following reactions :

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p \tag{1}$$

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n \tag{2}$$

$$\pi^- + p \rightarrow \pi^- + \omega^0 + p \tag{3}$$

$$\pi^- + p \rightarrow \pi^- + \rho^0 + p \tag{4}$$

Section 4 contains a comparison of our principal results with available experimental data on reactions (1) and (2), whereas for reactions (3) and (4), we restrict our discussion to qualitative remarks.

2. FORM FACTORS AND COUPLING CONSTANTS

Our double-peripheral graphs are calculated in lowest order perturbation theory, according to Feynman's rules, with real coupling constants at the vertices. Two additional rules appear to be necessary, however,

(i) each meson propagator includes a "universal form factor" $F(t)$, which is taken as

$$F(t) = F(0) \cdot e^{2.5t}, \qquad [t] = GeV^2 \tag{5}$$

(ii) whenever possible, a "vacuum state" is admitted among the virtual particles, with a "propagator"

$$e^{2.5t}, \qquad [t] = GeV^2 \tag{6}$$

Let us first discuss rule (i). The form factor (5) has been found to be useful for both pion and vector meson exchange in several K^+p interactions [3]. While it is possible that the pion form factor drops off somewhat faster than (5) at small values of $-t$, such a difference would have little, if any, influence on our quantitative calculations (cf. section 4) for reactions (1) and (2).

The form factor is understood to be normalized to 1 at the particle pole. Thus for pions, $F(0) = e^{-2.5\mu^2} = 0.954$. For heavier particles, with mass m, we shall still require $F(m^2) = 1$, but not necessarily $F(0) = e^{-2.5m^2}$, since the exponential behaviour is in fact a rough estimate for small negative values of t, rather than a precision fit which could be extrapolated far away from the physical region.

Rule (ii) is adopted to represent the effects of diffraction scattering, implying a large imaginary amplitude at small angles.

Again, the specific form (6) is more a guess than a proven statement, the exponent being borrowed from (5) and the purely real form (leading to a purely imaginary amplitude) being the simplest possible choice.

One more comment about the form factor. In a field theoretical treatment, such a factor would arise from the vertex functions as well as from modifications of the particle propagators,

$$F(t) = F_{vertex\,1}(t) \cdot F_{prop.}(t) \cdot F_{vertex\,2}(t) \qquad (7)$$

For the single peripheral model, the "universal" form (5) implies that all three functions entering (7) are independent of the type of particles (pions, nucleons, resonances) that participate. In the double-peripheral model, there is always one vertex where two virtual mesons join. The corresponding vertex function is a function of two momentum transfers, and the form (5) is possible only if

$$F_{vertex}(t,t') = F_{vertex}(t) \cdot F_{vertex}(t') \qquad (8)$$

A comparison of t distributions for different values of t' would provide a direct test of (8).

In section 4 we shall use the following coupling constants :

$$\frac{G^2_{\pi N}}{4\pi} = 14, \qquad \frac{G^2_{\rho\pi\pi}}{4\pi} = 2.76$$

(corresponding to a ρ decay width of one pion mass), and

$$\frac{G^2_{\pi\rho\omega}}{4\pi} = 0.35 \quad ^{4)}$$

when the $\pi\rho\omega$ vertex is taken in the form

$$\frac{1}{\mu} G_{\pi\rho\omega} \cdot \varepsilon^{\mu\nu\lambda\sigma} P_\mu(\rho) P_\nu(\omega) \varepsilon_\lambda(\rho) \varepsilon_\sigma(\omega) ,$$

and $\Gamma_\omega = 8$ MeV. Finally the ω NN coupling will be taken as a pure vector coupling with an "effective strength"

$$F_\omega^2(0) \cdot \frac{G_{\omega N}^2}{4\pi} = 3 .$$

The value found by Scotti and Wong [5], taking $F_\omega(t) = 1$, is

$$\frac{G_{\omega N}^2}{4\pi} = 2.77$$

Our value of the effective strength will serve as an order of magnitude estimate only.

3. MATRIX ELEMENTS AND CROSS-SECTIONS

All reactions discussed are of the form

$$P + \pi \longrightarrow P' + A + B$$

where we always have an incident proton and pion, P' represents the outgoing nucleon (either proton or neutron), and A and B represent two outgoing bosons. We shall use the same symbols for the associated four-momenta, except for the incident pion which has four-momentum denoted by K. In the c.m. frame of the AB system, these quantities have the following components

$$\begin{aligned} P &= (E, \underline{p}), \quad E = \sqrt{p^2 + M^2} \\ K &= (E_k, \underline{k}), \quad E_k = \sqrt{k^2 + \mu^2} \\ A &= (E_a, \underline{a}), \quad E_a = \sqrt{a^2 + m_a^2} \\ B &= (E_b, \underline{b}), \quad E_b = \sqrt{b^2 + m_b^2} \end{aligned} \qquad (9)$$

The double-peripheral graphs corresponding ~~to AB~~ are of the two types appearing in Fig. 2, where X, X_1, X_2 represent the exchanged objects. The associated four-momentum transfers are respectively

$$\begin{aligned} \Delta &= P - P' \\ \Delta_a &= K - A \\ \Delta_b &= K - B \end{aligned} \qquad (10)$$

Note also that the bracket (X, X_j) identifies the graph for a particular channel. We shall use it as a reference symbol.

We employ the same metric as Jauch and Rohrlich [6]. Our matrix element is Lorentz invariant and is related to the non-invariant $M_{fi}^{(3)}$, for which Jauch and Rohrlich provide the recipe, by

$$\mathcal{M}_{fi} = (2\pi)^2 (E E' E_k E_a E_b)^{1/2} \cdot \mathcal{M}_{fi}^{(3)}$$

The cross-section is given by

$$\sigma = \frac{1}{(2\pi)^2} \left((P \cdot K)^2 - \mu^2 M^2 \right)^{-\frac{1}{2}} \cdot \int \frac{d^3 p'}{E'} \int \frac{d^3 a}{E_a} \int \frac{d^3 b}{E_b} \, \delta(P + K - P' - A - B) \times$$

$$\times \frac{1}{2} \sum |\mathcal{M}_{fi}|^2 \qquad (11)$$

where $\sum$ denotes the sum over both initial and final spins.

There are in these channels nine final state momentum variables. We eliminate four, namely E_a and $\underline{b}$, by energy-momentum conservation so that

$$\sigma = \frac{1}{(2\pi)^2} \frac{1}{p^* \sqrt{s}} \int \frac{d^3 p'}{E'} \int d\cos\vartheta \int d\varphi \cdot \frac{a}{\sqrt{s'}} \cdot \frac{1}{2} \sum |\mathcal{M}_{fi}|^2 \qquad (12)$$

where p^* is the magnitude of the incident momentum in the incident over-all c.m. system, $s = -(P+K)^2$, and $s' = -(A+B)^2$. The polar angle ϑ and the azimuthal angle φ are the angles of $\underline{a}$ in the AB c.m. system with polar axis $\underline{k}$. Thus ϑ is the scattering angle between the incident pion and the outgoing boson A. Finally φ is the Treiman-Yang angle [7] and we take $\varphi = \frac{\pi}{2}$ along the direction $\underline{k} \times (\underline{p} + \underline{p}')$.

Of the remaining five final state variables, the azimuthal angle of the outgoing nucleon with respect to the incident proton in the over-all c.m. system is redundant. We choose s', $t = -\Delta^2$, $\cos\vartheta$, and φ, for the independent final state variables, and therefore we transform

$$\int \frac{d^3p'}{E'} = \frac{\pi}{2p^*\sqrt{s}} \int dt\, ds' \tag{13}$$

It follows that

$$\sigma = \frac{1}{8p^{*2}s} \int dt\, ds'\, d\cos\vartheta \frac{a}{\sqrt{s'}} \overline{\sum |M_{fi}|^2} \tag{14}$$

where the bar over the spin summed square of the matrix element denotes the average over the Treiman-Yang angle φ. The distributions in each one of the remaining variables, $\cos\vartheta$, s', and t, were obtained by means of the 7090 computer at CERN. We give the $\cos\vartheta$ distributions for reactions (1) and (2) in Fig. 3.

We proceed with the discussion of the matrix element for the reaction (1). Referring to Fig. 2 with A as the outgoing negatively charged pion, there are four contributing graphs. These graphs are (π^0, vac), (π^0, ρ^-), (ω, ρ^0), (ω, ρ^-), where vac denotes the vacuum exchange. Among these graphs, only the last two interfere. The corresponding contributions to M_{fi} are respectively

$$\mathcal{M}(\pi^0, vac) = \frac{G^2_{\pi vac}}{4\pi} \cdot \frac{G_{\pi N}}{\sqrt{4\pi}} \cdot \frac{F_\pi(t)}{t-\mu^2}\, i\, e^{2.5t_1} \cdot \frac{s'}{s_0} \times$$

$$\times M \bar{u}_{s'}(p') \gamma_5 u_s(p) \tag{15}$$

$$\mathcal{M}(\pi^0, \rho^-) = \frac{G^2_{\pi\pi\rho}}{4\pi} \frac{G_{\pi N}}{\sqrt{4\pi}} \cdot \frac{F_\pi(t)}{t-\mu^2} \cdot \frac{F_\rho(t_2)}{t_2 - m_\rho^2} (K+B)_\mu \times$$

$$\times \left(g^{\mu\nu} + \frac{\Delta_b^\mu \Delta_b^\nu}{m_\rho^2}\right)(A+\Delta)_\nu M \bar{u}_{s'}(p') \gamma_5 u_s(p) \tag{16}$$

and

$$\mathcal{M}(\omega;\rho^{0})+\mathcal{M}(\omega,\rho^{-}) = \frac{G_{\rho\pi\pi}}{\sqrt{4\pi}}\,\frac{G_{\pi\rho\omega}}{\mu\sqrt{4\pi}}\,\frac{G_{\omega N}}{\sqrt{4\pi}}\,\frac{F_{\omega}(t)}{t-m_{\omega}^{2}}\,\epsilon_{\mu\nu\lambda\sigma}\,\Delta^{\lambda}\times$$

$$\times\left[\frac{F_{\rho}(t_{1})}{t_{1}-m_{\rho}^{2}}(K+A)^{\mu}\Delta_{a}^{\nu}+\frac{F_{\rho}(t_{2})}{t_{2}-m_{\rho}^{2}}(K+B)^{\mu}\Delta_{b}^{\nu}\right]M\,\bar{u}_{s'}(p')\gamma^{\sigma}u_{s}(p) \tag{17}$$

where $t_1 = -\Delta_a^2$, $t_2 = -\Delta_b^2$, and $\epsilon^{\mu\nu\lambda\sigma}$ is the Levi-Cevita alternating symbol. The factor s'/s_0 in (15) is the spin factor of the vacuum trajectory at $t = 0$.

When we square the amplitudes (15) and (16) and sum over all spins, we obtain respectively

$$\sum|\mathcal{M}(\pi^{0},vac)|^{2} = \left(\frac{G^{2}_{\pi\,vac}}{4\pi}\right)^{2}\cdot\frac{G^{2}_{\pi N}}{4\pi}\cdot\frac{F_{\pi}^{2}(t)}{(t-\mu^{2})^{2}}\cdot e^{5t_{1}}\cdot\left(\frac{s'}{s_{0}}\right)^{2}\cdot\left(-\frac{t}{2}\right) \tag{18}$$

and

$$\sum|\mathcal{M}(\pi^{0},\rho^{-})|^{2} = \left(\frac{G^{2}_{\rho\pi\pi}}{4\pi}\right)^{2}\cdot\frac{G^{2}_{\pi N}}{4\pi}\,\frac{F_{\pi}^{2}(t)}{(t-\mu^{2})^{2}}\,\frac{F_{\rho}^{2}(t_{2})}{(t_{2}-m_{\rho}^{2})^{2}}\left(-\frac{t}{2}\right)\left(2s'-t+t_{2}-3\mu^{2}\right)^{2} \tag{19}$$

In order to discuss (17) and the other reactions it is convenient to introduce the symbols

$$\begin{aligned} V_{j} &= \frac{G_{\rho\pi\pi}}{\sqrt{4\pi}}\,\frac{G_{\pi\rho\omega}}{\mu\sqrt{4\pi}}\,\frac{G_{\omega N}}{\sqrt{4\pi}}\,\frac{F_{\omega}(t)}{t-m_{\omega}^{2}}\,\frac{F_{\rho}(t_{j})}{t_{j}-m_{\rho}^{2}} \\ e^{\lambda} &= \sqrt{\frac{2}{\Delta^{2}}}\,M\,\bar{u}_{s'}(p')\gamma^{\lambda}u_{s}(p) \\ \nu &= P+P' \end{aligned} \tag{20}$$

In terms of Δ and v we have

$$\sum e^{*\lambda} e^{\sigma} = g^{\lambda\sigma} - \frac{1}{\Delta^2}\left(\Delta^\lambda \Delta^\sigma - v^\lambda v^\sigma\right) \tag{21}$$

We may now express (17) in the more concise form

$$\mathcal{M}(\omega,\rho^0) + \mathcal{M}(\omega,\rho^-) = \sqrt{-\frac{t}{2}}\cdot 2\left(V_1 \det(K,A,\Delta,e) + V_2 \det(K,B,\Delta,e)\right) \tag{22}$$

Noting that $A+B = K+\Delta$ we obtain, with the aid of (21)

$$\sum |\mathcal{M}(\omega,\rho^0) + \mathcal{M}(\omega,\rho^-)|^2 = 2\,(V_1 - V_2)^2\left[-tQ^2 + (Q\cdot v)^2\right] \tag{23}$$

when $Q_\lambda = \epsilon_{\lambda\mu\nu\sigma}\, K^\mu A^\nu \Delta^\sigma$. In the AB c.m. system, Q has the components $(0, \sqrt{s'}\ \underline{k}\times\underline{a})$. Recalling that the Treiman-Yang angle assumes the value $\frac{\pi}{2}$ along the direction $\underline{k}\times\underline{v}$, we have finally

$$\begin{aligned}\sum |\mathcal{M}(\omega,\rho^0) + \mathcal{M}(\omega,\rho^-)|^2 = {} & 4\,\frac{G^2_{\rho\pi\pi}}{4\pi}\,\frac{G^2_{\pi\rho\omega}}{4\pi\mu^2}\,\frac{G^2_{\omega NN}}{4\pi}\,\frac{F_\omega^2(t)}{(t-m_\omega^2)^2}\times \\ & \times\left[\frac{F_\rho(t_1)}{t_1 - m_\rho^2} - \frac{F_\rho(t_2)}{t_2 - m_\rho^2}\right]^2\cdot\left(-\frac{t}{2}\right) a^2 \sin^2\vartheta\times \\ \times\Big\{ s' k^2 \sin^2\varphi + \cos^2\varphi & \left[\frac{1}{4}(2M^2 - 2s + \mu^2 + s' - t)^2 + \frac{1}{t}\cdot 4s' k^2 M^2\right]\Big\}\end{aligned} \tag{24}$$

The sum of (18), (19), and (24), with $\sin^2\varphi$ and $\cos^2\varphi$ replaced by 1/2 is inserted into (14) for $|M_{fi}|^2$.

We go on to the discussion of the matrix element for reaction (2). The final nucleon is now a neutron and B is a positively charged pion. The only contributing graphs are (π^+, vac) and (π^+, ρ^0). These graphs give precisely the same contributions as the first two graphs of reaction (1), except for an additional factor $\sqrt{2}$ in the matrix element, due to the relation $G_{\pi^+p} = \sqrt{2}G_{\pi^0p}$. The result is

$$\sum |\mathcal{M}_{fi}|^2 = 2\,\frac{G^2_{\pi N}}{4\pi}\,\frac{F_\pi^2(t)}{(t-\mu^2)^2}\left(-\frac{t}{2}\right) \times$$

$$\times \left\{\left(\frac{G^2_{\pi vac}}{4\pi}\right)^2 e^{5t_1}\left(\frac{s'}{s_0}\right)^2 + \left(\frac{G^2_{\rho\pi\pi}}{4\pi}\right)^2 \frac{F_\rho^2(t_1)}{(t_1-m_\rho^2)^2}\left(2s'-t+t_1-3\mu^2\right)^2\right\} \quad (25)$$

Before proceeding with the discussion of reactions (3) and (4), we discuss the polarization effects of vector meson production. The production and decay amplitudes are of the form

$$\mathcal{M}_{prod} = \mathcal{M}^{\mu}_{prod}\cdot \varepsilon_\mu(A) \quad (26)$$

and

$$\mathcal{M}_{decay} = \mathcal{M}\, D^{\lambda}\varepsilon_\lambda(A) \quad (27)$$

where $\varepsilon(A)$ is the polarization vector, and $D^2 = 1$. In the rest system of the decaying vector meson, D_λ has the components $(0, \hat{n})$, where $\hat{n}$ is a unit vector. In the case of ρ decay, $\hat{n}$ is directed along the decay axis. In the case of ω decay, $\hat{n}$ is normal to the decay plane. In either event $D\cdot A = 0$, whereby

$$\sum_{spin} \mathcal{M}_{prod}\cdot\mathcal{M}_{decay} = \mathcal{M}\cdot\mathcal{M}^{\lambda}_{prod}\cdot D_\lambda \quad (28)$$

In the A rest frame $M^{\lambda}_{prod} \cdot D_{\lambda} = \underline{M}_{prod} \cdot \hat{n}$ and the distribution for $\hat{n}$ is obtained by inserting $\underline{M}_{prod} \cdot \hat{n}$ for M_{fi} in (14), and the spin summation is replaced by taking three times the average over the solid angle $\Omega(\beta, \alpha)$ of $\hat{n}$ with respect to $\underline{k}_A$. The vector $\underline{k}_A$ is the momentum of the incident pion in the A rest frame, and β and α are the polar and azimuthal angles of $\hat{n}$ with respect to $\underline{k}_A$. In terms of invariants

$$k_A^2 = \frac{1}{4m_a^2}\left[t_1^2 - 2t_1(m_a^2+\mu^2) + (m_a^2-\mu^2)^2\right]$$

Referring again to Fig. 2, for the reaction (3), A is the ω. The contributing graphs are (π^0, ρ^-), (π^0, ρ^0), (ω^0, ρ^-) and (ω^0, vac). Only the first two graphs interfere, and we discuss these first. The corresponding matrix element is

$$\mathcal{M}(\pi^0, \rho^-) + \mathcal{M}(\pi^0, \rho^0) = (U_1 - U_2) \cdot 2 \det(\Delta, D, A, \Delta_a) \times \\ \times M\, \bar{u}_{s'}(p') \gamma_5 u_s(p) \tag{29}$$

where

$$U_j = \frac{G_{\rho\pi\pi}}{\sqrt{4\pi}} \cdot \frac{G_{\pi\rho\omega}}{\mu\sqrt{4\pi}} \cdot \frac{G_{\pi N}}{\sqrt{4\pi}} \cdot \frac{F_\pi(t)}{t-\mu^2} \cdot \frac{F_\rho(t_j)}{t_j - m_\rho^2} \tag{30}$$

Using energy-momentum conservation we have

$$\det{}^2(\Delta, D, A, \Delta_a) = \det{}^2(\Delta, D, A, K) = (D \cdot Q)^2 \tag{31}$$

where Q was previously defined [following (23)]. In the ω rest frame, the components of Q are again simple, namely $(0, m_\omega \cdot \underline{k}_A \times \underline{\Delta}_A)$, where $\underline{\Delta}_A$ is the three-vector part of Δ in this system. We fix the co-ordinates so that α assumes the value $\frac{\pi}{2}$ in the direction $\underline{k}_A \times \underline{\Delta}_A$. We also have the relation

$$m_\omega \underline{k}_A \times \underline{\Delta}_A = \sqrt{s'}\, \underline{k} \times \underline{a} \tag{32}$$

so that

$$\sum \left| \mathcal{M}(\pi^\circ, \rho^-) + \mathcal{M}(\pi^\circ, \rho^\circ) \right|^2 = 4 \frac{G^2_{\rho\pi\pi}}{4\pi} \frac{G^2_{\pi\rho\omega}}{4\pi\mu^2} \frac{G^2_{\pi N}}{4\pi} \frac{F_\pi^{\,2}(t)}{(t-\mu^2)^2} \times$$
$$\times \left[\frac{F_\rho(t_1)}{t_1 - m_\rho^2} - \frac{F_\rho(t_2)}{t_2 - m_\rho^2} \right]^2 \cdot \left(-\frac{t}{2}\right) \cdot s' k^2 a^2 \sin^2\vartheta \sin^2\beta \sin^2\alpha \tag{33}$$

We next consider the contributions from the (ω, ρ^-) graph, namely

$$\mathcal{M}(\omega, \rho^-) = \frac{G^2_{\pi\rho\omega}}{4\pi\mu^2} \cdot \frac{G_{\omega N}}{\sqrt{4\pi}} \cdot \frac{F_\omega(t)}{t - m_\omega^2} \cdot \frac{F_\rho(t_1)}{t_1 - m_\rho^2} \det(R, D, A, K) \tag{34}$$

where $R^\sigma = \sqrt{\frac{\Delta^2}{2}}\, \epsilon^{\sigma\lambda\mu\nu} e_\lambda \Delta_\mu B_\nu$. This matrix element involves rather lengthy expressions. We find it convenient to introduce a number of additional symbols to shorten them. The first among these is $L_\lambda = \epsilon_{\lambda\mu\nu\sigma} \cdot A^\mu R^\nu K^\sigma$, which in the ω rest frame has the components $(0, m_\omega \cdot \underline{R}_A \times \underline{k}_A)$, where $\underline{R}_A$ is the three-vector component of R above. Then we have

$$\det(R, D, A, K) = \hat{n} \cdot \underline{L} = m_\omega k_A \sin\beta \left(R_{1A} \sin\alpha + R_{2A} \cos\alpha \right) \tag{35}$$

If we average over α, use (21), and the relation

$$-\epsilon_{\sigma\lambda\mu\nu}\,\epsilon^{\sigma\alpha\beta\rho} = g_\lambda^\alpha\left(g_\mu^\beta g_\nu^\rho - g_\mu^\rho g_\nu^\beta\right) + g_\lambda^\beta\left(g_\mu^\rho g_\nu^\alpha - g_\mu^\alpha g_\nu^\rho\right) + g_\lambda^\rho\left(g_\mu^\alpha g_\nu^\beta - g_\mu^\beta g_\nu^\alpha\right) \tag{36}$$

we obtain

$$\int_0^{2\pi}\frac{d\alpha}{2\pi}\sum|\hat{n}\cdot L|^2 = \frac{m_\omega^2}{4}\sin^2\beta\,\times \times\left\{-t\left[2k_A^2\delta^4 - t_1(\underline{k}_A\times\underline{\Delta}_A)^2\right] + (\underline{k}_A\times\underline{W}_A)^2\right\} \tag{37}$$

In (37) we have introduced the symbol

$$\delta^4 = \frac{1}{4}\left(t^2+t_1^2+\mu^4\right) - \frac{1}{2}\left(tt_1 + t\mu^2 + t_1\mu^2\right)$$

and the $\underline{W}_A$ component of $W_\sigma = \epsilon_{\sigma\lambda\mu\nu}\cdot v^\lambda\Delta^\mu B^\nu$, in the rest system of the ω. The expression (37) displays the pure $\sin^2\beta$ distribution contributed by this graph. We have from (32) the expression for $m_\omega^2(\underline{k}_A\times\underline{\Delta}_A)^2$ in terms of the AB c.m. variables. The term not easily transformed is $(\underline{k}_A\times\underline{W}_A)^2$. We write this in the form

$$(\underline{k}_A\times\underline{W}_A)^2 = k_A^2 W^2 + k_A^2\left(W_{0A}^2 - W_{3A}^2\right) \tag{38}$$

Using (36) we get

$$k_A^2 W^2 = k_A^2(t-4M^2)\delta^4 - k_A^2 t\,(v\cdot B)^2 \tag{39}$$

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and the remainder is

$$k_A^2 \left(W_{0A}^2 - W_{3A}^2 \right) = -t_1 \, v_{1A}^2 \, (\underline{k}_A \times \underline{\Delta}_A)^2 \tag{40}$$

After some manipulations we obtain the result

$$(\underline{k}_A \times \underline{W}_A)^2 = \left\{ k_A^2 t \left(v_0^2 \frac{t}{k^2} - v_1^2 \cos^2\varphi \right) - \frac{t_1}{m_\omega^2} k^2 v_1^2 s' \sin^2\varphi \right\} a^2 \sin^2\vartheta$$
$$+ k_A^2 v_1^2 \delta^4 - 2 k_A^2 t v^0 \frac{a}{k} v_1 (\delta^4 - t a^2 \sin^2\vartheta)^{1/2} \sin\vartheta \cos\varphi \tag{41}$$

where

$$v_1 = \left(t - 4M^2 - t \frac{v_0^2}{k^2} \right)^{1/2}$$
$$v^0 = \frac{1}{2\sqrt{s'}} \left(2M^2 - 2s + \mu^2 + s' - t \right) \tag{42}$$

We note that the coefficient of the $\cos\varphi$ term in (41) is always positive (t is negative). Upon taking the average over φ, we obtain finally

$$\int_0^{2\pi} \frac{d\alpha}{2\pi} \overline{\sum |\mathcal{M}(\omega,\rho^-)|^2} = \left(\frac{G_{\pi\rho\omega}^2}{4\pi\mu^2} \right)^2 \frac{G_{\omega N}^2}{4\pi} \frac{F_\omega^2(t)}{(t-m_\omega^2)^2} \frac{F_\rho^2(t_1)}{(t_1-m_\rho^2)^2} \frac{\sin^2\beta}{4} \times$$
$$\times \left\{ -2m_\omega^2 t k_A^2 \delta^4 + t t_1 k^2 a^2 s' \sin^2\vartheta + \left[m_\omega^2 k_A^2 t \left(v_0^2 \frac{t}{k^2} - \frac{1}{2} v_1^2 \right) - \frac{1}{2} t_1 k^2 v_1^2 s' \right] a^2 \sin^2\vartheta + m_\omega^2 k_A^2 v_1^2 \delta^4 \right\} \tag{43}$$

There appears to be no way to make this rather tedious expression much more transparent. This is because both virtual mesons and the outgoing meson carry spin.

For this reaction there remains only the graph $(\omega$, vac). Apart from factors which require no further discussion, the spin summed square of the matrix element is proportional to

$$X = k_A^2 \left[-t + (\underline{v}_A \cdot \hat{n})^2 - (\underline{\Delta}_A \cdot \hat{n})^2 \right] \tag{44}$$

If $M(\omega$, vac) were considered alone, then due to the dominance of small Δ, the relevant direction in the $\hat{n}$ distribution X would be along the average nucleon momentum. However, for the other contribution previously discussed, the angle β is more suitable. We average over α and get

$$\frac{1}{2\pi} \int_0^{2\pi} X d\alpha = -t k_A^2 + \frac{1}{2}\sin^2\beta \left[(\underline{k}_A \times \underline{v}_A)^2 - (\underline{k}_A \times \underline{\Delta}_A)^2 \right] + \cos^2\beta \left[(\underline{k}_A \cdot \underline{v}_A)^2 - (\underline{k}_A \cdot \underline{\Delta}_A)^2 \right] \tag{45}$$

Equation (45) exhibits the contribution to the β distribution from this graph, and is the only part containing terms other than pure $\sin^2\beta$. We now average over β also and get

$$\frac{1}{4\pi k_A^2} \int d\Omega(\alpha,\beta) X = -t + \frac{2}{3}(t - 2M^2) + \frac{(s' - 2\mu^2 + t_1)^2}{12 m_\omega^2} - \frac{a^2}{3m_\omega^2} v_1^2 \sin^2\vartheta \cos^2\varphi + \frac{2}{3m_\omega^2} a v_1 v^0 E_a \sin\vartheta \cos\varphi - \frac{v_0^2}{3m_\omega^2}\left[E_a + \frac{a}{k}\frac{s' + t - \mu^2}{2\sqrt{s'}} \cos\vartheta \right]^2 \tag{46}$$

We observe again, that the coefficient of the $\cos\varphi$ term is never negative.

We consider finally the reaction (4). The particle A is a ρ^0. The contributing graphs are (ρ^0, π^-), (vac, π^-), and (vac, ρ^0). We state only the results :

$$\sum |\mathcal{M}(\rho^0,\pi^-)|^2 = 8\left(\frac{G_{\rho\pi\pi}^2}{4\pi}\right)^2 \frac{F_\rho^2(t)}{(t-m_\rho^2)^2}\frac{F_\pi^2(t_1)}{(t_1-\mu^2)^2} k^2 \cos^2\beta \left[\eta\left(\frac{G_T^2}{4\pi M^2}-\frac{G_V^2}{4\pi t}\right)-\frac{\delta^4}{4\pi}(G_V+G_T)^2\right]$$

$$\eta = t(t-4M^2+v_1^2\sin^2\varphi)a^2\sin^2\vartheta - \frac{t}{k^2}v_0^2\delta^4$$
$$-2\frac{t}{k}v_1 v^0(\delta^4-ta^2\sin^2\vartheta)^{1/2} a\sin\vartheta\cos\varphi \tag{47}$$

and

$$\sum|\mathcal{M}(vac,\pi^-)+\mathcal{M}(vac,\rho^0)|^2 = 2\,\frac{G_{\rho\pi\pi}^2}{4\pi}\,\frac{G_{vacN}^2}{4\pi}\,e^{5t}\left(\frac{s'}{s_0}\right)^2(4M^2-t)\times$$
$$\times\ \frac{\cos^2\beta}{\mu^2}\left[\frac{G_{\pi vac}}{\sqrt{4\pi}}\cdot\frac{F_\pi(t_1)}{t_1-\mu^2}\cdot k_A+\frac{G_{\rho vac}}{\sqrt{4\pi}}\cdot\frac{F_\rho(t_2)}{t_2-m_\rho^2}\cdot K\right]^2$$

$$K = k_A + \frac{kas'}{2k_A m_\rho^2}\sin\vartheta\tan\beta\cos\alpha$$
$$+\frac{1}{2k_A}\left[-tk_A^2+\frac{k_A^2}{4m_\rho^2}(s'-2\mu^2+t_1)^2-\frac{k^2}{m_\rho^2}s'a^2\sin^2\vartheta\right]^{1/2} \tag{48}$$

We note that the coefficient of $\cos\varphi$ is never negative and, that after taking the average over α, the distribution is pure $\cos^2\beta$.

4. COMPARISON WITH EXPERIMENT

In the present literature, few accurate experiments of the type we need, on inelastic πp collisions above 4 GeV/c are reported. In a large portion of all events, the outgoing particles can be grouped into one resonance plus one particle, or into two resonances. Such events are candidates for the single-peripheral model. With increasing experimental accuracy, the realm of the multi-meson resonances appears to expand, thereby limiting the regions where the double-peripheral model might apply. In addition, there appears to be a non-peripheral background among the remaining events. In the distribution $\frac{d\sigma}{dt}$, the background manifests itself as a long tail to the sharp peak near $t = 0$.

At present, the best reactions for the double-peripheral model are (1) and (2). They have recently been analyzed at 4 GeV/c [8], in a way which is suitable for comparison with the double-peripheral model. No π-N resonances appear to be present, and only events with $-t < 15\mu^2$ are taken. In order to exclude the observed ρ and f_0 resonances, we require $M^2_{\pi\pi} = s' > 1.175\ \mathrm{GeV}^2$ for reaction (1), and $s' > 1.9\ \mathrm{GeV}^2$ for reaction (2).

The resulting $\cos\vartheta$ distributions are shown in Fig. 3. In Fig. 3a, the theoretical curve is normalized to the number of events in the forward and backward peaks. The curve of Fig. 3b contains no free parameters. In discussing the theoretical curves, we first notice that the contribution of the (ω, ρ) graphs, Eq. (24), to reaction (1) is 0.016 mb only (using the coupling constants of section 2). The differential cross-section of these graphs is symmetric around $\cos\vartheta = 0$, with peaks at $|\cos\vartheta| \sim 0.85$. This means that ω exchange contributes barely 15% to the backward peak, and even less to the forward peak of Fig. 3a. We therefore omitted the ω exchange altogether.

The forward peak in Fig. 3a is due to the (π^0, vac) exchange, Eq. (18). Its shape seems to substantiate our conjecture regarding the exponential (6). From the total number of events in the forward peak, we find

$$G^2_{\pi\,vac}/4\pi s_0 = 5.4/\mathrm{GeV}^2.$$

Graph (π^o, ρ^-) finally gives the smaller backward peak. Application of (19) to the events in that peak gives $F_\rho^2(0) = 0.18$ (extrapolation of the exponential law (5) gives $F_\rho^2(0) = \exp(-5m_\rho^2) = 0.06$).

Having fixed our normalization in reaction (1), we can compute both differential and total cross-sections for reaction (2). For the first, we get a contribution of 0.10 mb from (π^+, vac) and 0.04 mb from (π^+, ρ^o), summing up to 0.14 mb, in good agreement with experiment.

At 16 GeV/c pion momentum [9], it has been suggested that the (π^+, vac) graph should explain the large forward peak of reaction (2). This is in accord with our analysis at 4 GeV/c. In principle, we could apply our formulae to calculate the ρ exchange contribution at 16 GeV/c. But if we regard our form factors (5) and (6) only as a first approximation which suppresses the energy dependence [10], such an extrapolation would exceed the scope of the model.

Next, we turn to reaction (3), the cross-section for which is obtained from Eqs. (33), (43), (46). Again, we have to omit from the experimental data all events in which the final pion is in resonance either with the final proton or with the ω. Unfortunately, due to the recently discovered $\pi-\omega$ resonance [11], only a few events remain. We therefore give just the main qualitative results :

(A) the $\cos\vartheta$ distribution exhibits peaks at forward and at backward angles. This agrees with the available experimental data [12]. The forward peak is due to graphs (π^o, ρ^-) and (ω, ρ^-), the backward peak is due to (π^o, ρ^o) and (ω, vac).

(B) since the vacuum exchange (ω, vac) is the only one having $\cos^2\beta$ terms, all events at $\cos\vartheta > 0$ should have a pure $\sin^2\beta$ distribution.

(C) from Eqs. (41), (46) it follows that the distribution in the Treiman-Yang angle φ is larger at $\varphi = 0$ than at $\varphi = \pi$.

Finally, for reaction (4), the double-peripheral model predicts, besides the usual peaking at small t, a $\cos\vartheta$ distribution with a forward peak, due to the graphs (vac, π^-), (ρ^0, π^-), and (ρ^0, ω), and a backward peak, due to (vac, ρ^0).

It is interesting to notice that in the primitive formulation of the peripheral model, graph (vac, π^-) belonged to two different processes, namely (a) pion diffraction dissociation (from the point of view of vacuum exchange), and (b) peripheral ρ production with off-shell π-N scattering (from the point of view of pion exchange). Such duplication does not occur in the double-peripheral model.

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FIGURE CAPTIONS

Figure 1 Peripheral graphs :

(a) single peripheral,

(b) double peripheral,

(c) general multiperipheral.

Figure 2 The two inequivalent graphs of the double-peripheral model. The letters denote the 4 momenta, the arguments π, X, X_1, X_2 denote the incident pion and exchanged particles respectively.

Figure 3 Distribution in $\cos\vartheta$, the angle between incident and outgoing π^- in the c.m. system of the final pions, for $\pi^- p$ collisions at 4 GeV/c, for $-t < 15\mu^2$:

(a) $\pi^- p \rightarrow \pi^- \pi^0 p$, for $s' > 1.175$ GeV^2,

(b) $\pi^- p \rightarrow \pi^+ \pi^- n$, for $s' > 1.9$ GeV^2.

Theoretical curves from double integration of Eqs. (18) and (19), experimental histogram from Ref. [8].

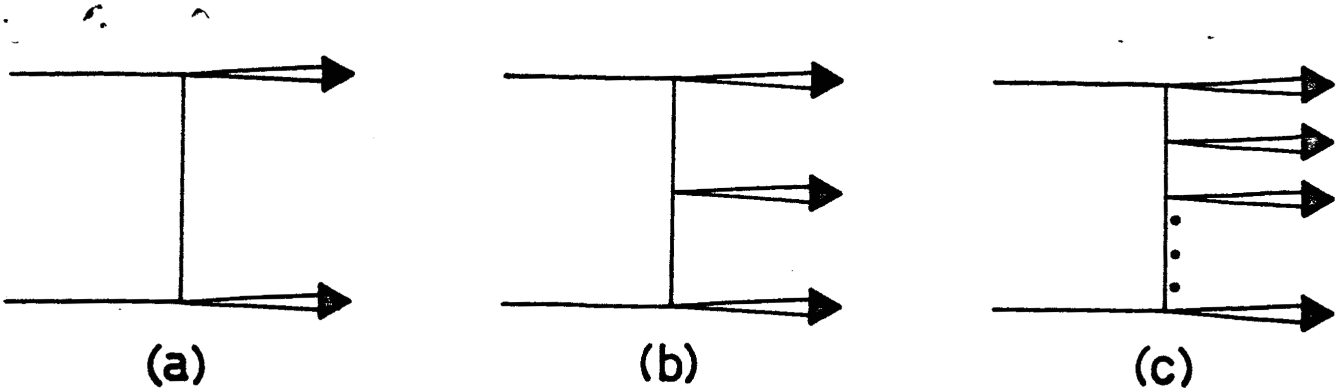

FIG. 1

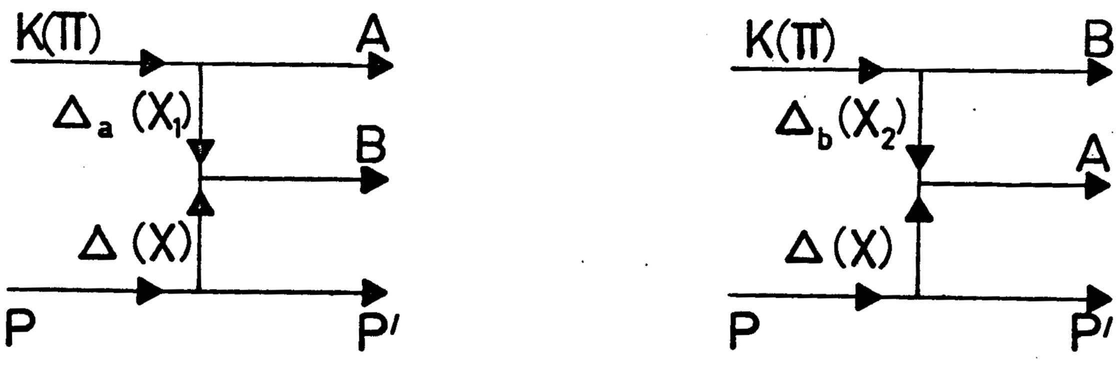

FIG. 2

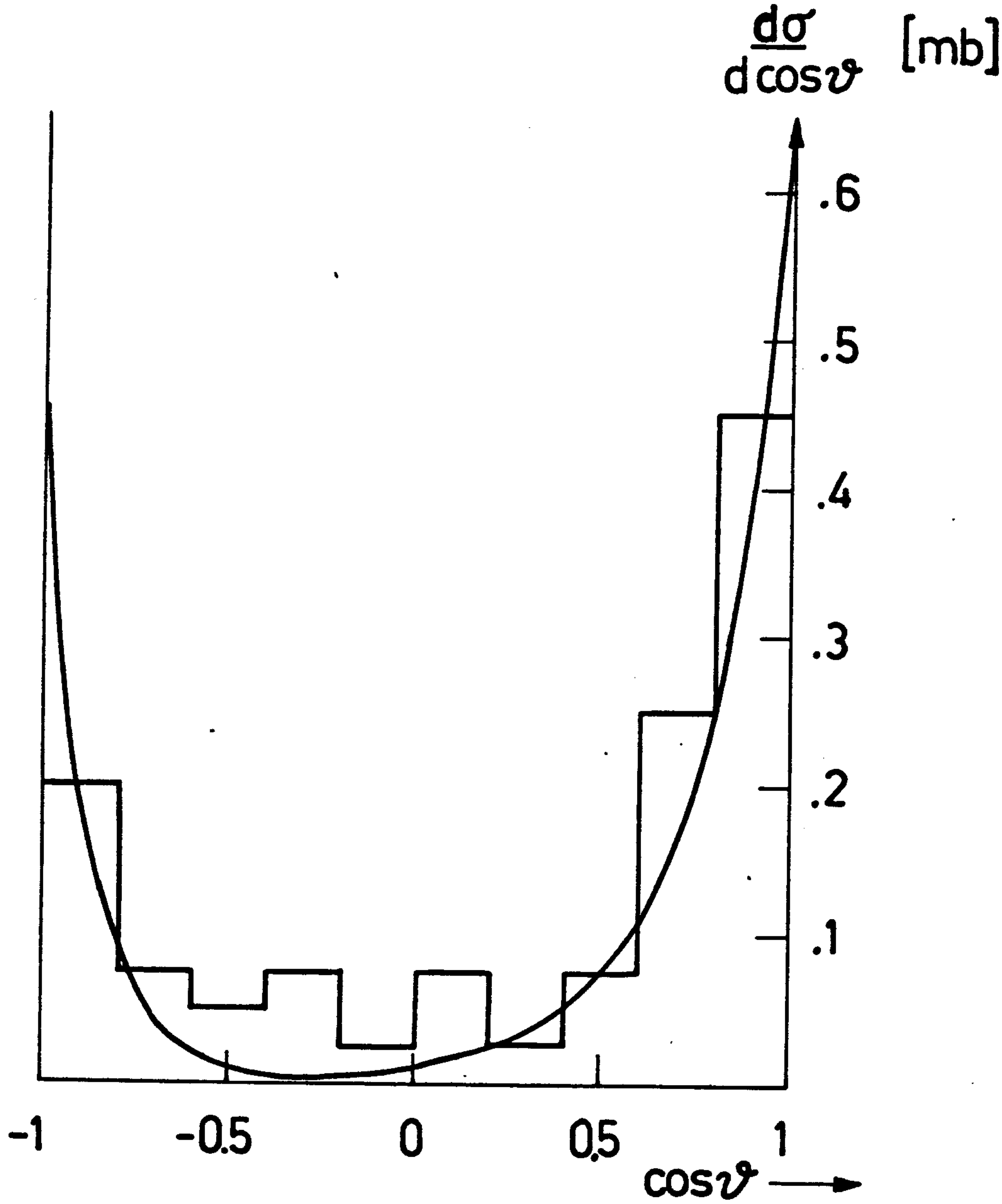

FIG. 3a

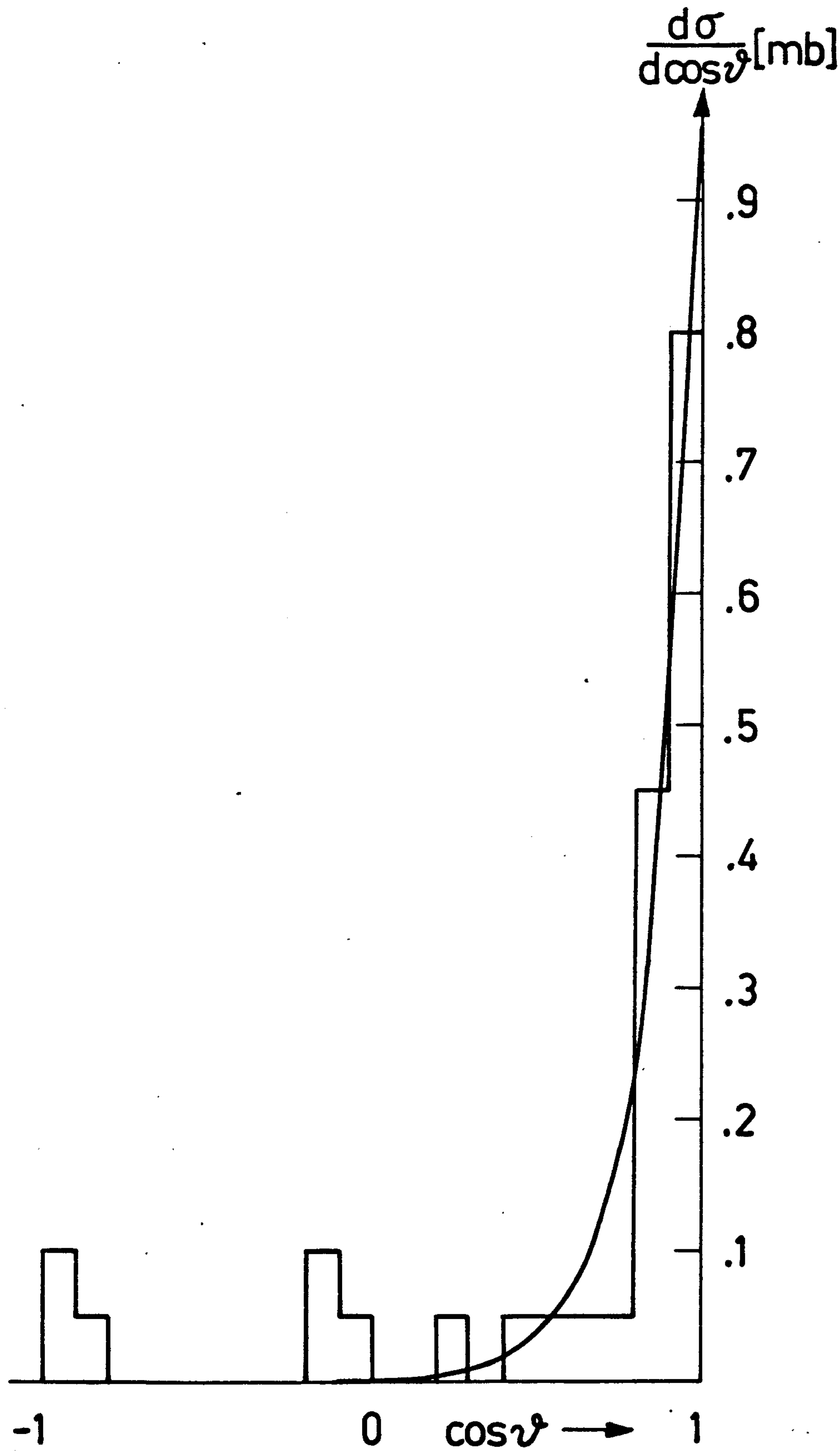

FIG. 3b