SEP 9 1964

N5C-299

The Theory of Large Amplitude Oscillations in the One-Dimensional Low Pressure Thermionic Converter

N65 13224

by

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23 p.

Code 1
Cat. 03

ABSTRACT

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The large amplitude oscillations in a one-dimensional low pressure thermionic converter are analyzed. With the help of results obtained from computer calculations designed to simulate the operation of the converter, the current oscillations are explained in terms of the changing forms of the potential distribution in the system. The variations in the form of the potential function are explained by means of the concept of a "temporary dc state." Such a state differs from the ideal self-consistent state in that the electrons and the potential adopt new distributions while the ion distribution stays the same as in the self-consistent state. In fact, the ions are treated as too heavy to respond to the new potentials while the electrons adjust themselves rapidly to them. The existence of a temporary dc state is decided by solving a static problem only; but if such a state exists, then there is a possibility for relaxation type of oscillations in the system with frequencies characteristc to the ions.

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Introduction

The one-dimensional model of a low pressure thermionic converter consists of two parallel, infinitely extended plates separated by a distance "d" [Fig. 1]. A constant potential (V_d) is applied across these plates, one of which is at a higher temperature (T). The high temperature plate is capable of emitting both electrons and ions towards the colder plate with half-Maxwellian velocity distributions (the full Maxwellian distributions have temperature T), and with given saturation current densities (J_{si}, J_{se}). It is assumed that the plasma diode thus described works at such low pressures that all collisional effects between charged particles or charged particles and neutrals can be neglected; thus, the forces acting on the charged particles are due only to electric fields. The additional assumption is made that the boundary plates are non-reflecting, i.e. all particles that reach either plate are collected and lost for the system.

This diode would operate as an efficient thermionic converter if a small positive potential applied across it could cause electron saturation current to flow through it. Analyzing the dc states of this diode model, McIntyre[1] has found that if the emission is "ion-rich," meaning that the parameter α, defined by

$$\alpha = \frac{J_{si}}{J_{se}} \sqrt{\frac{m_i}{m_e}} \tag{1}$$

exceeds 1, and V_d is positive, then the potential is positive everywhere in the diode (see Fig. 2); and according to the static theory, no electrons are returned to the emitter. On the basis of these calculations, it was expected that the electron saturation current would flow in a thermionic converter as long as α is kept larger than 1; however, experiments with cesium diodes, that approximated the described theoretical model, well showed (see References 2, 3 for example) that electron saturation current does not flow in the converter at all times. These experiments showed in fact that the current through the low pressure thermionic converter is not constant in time, but it has large amplitude oscillations, and thus, the time average of this current remains well below the value calculated on the basis of dc analysis.

It is apparent now that dc analysis cannot sufficiently describe the operation of the low pressure thermionic converter. In order that we might gain some insight into its time-varying behavior, the operation of the low pressure thermionic converter was simulated on a computer. In the following, we describe the characteristics of the computer-simulated converter. The understanding gained from studying the large amplitude oscillations exhibited by the simulated system will enable us to draw conclusions about the stability of dc states in more general ion-electron systems.

Characteristics of a computer-simulated model of the low pressure thermionic converters

The behavior of a one-dimensional plasma diode (both static and time varying) can be understood from the study of the motion of charged particles between two infinitely extended parallel plates. In the one-dimensional formulation, charged particles become charged planes, or sheets that are also infinitely extended and are parallel to the boundary plates. The motion of each charged sheet (only motion perpendicular to the planes of these sheets is considered) is determined by its initial velocity at the emitter and by the electric fields it experiences throughout its motion in the diode. The electric field and potential distributions in the diode space as well as the current through the diode are determined from the combined effects of a large number of moving charged sheets located between the two boundary plates.

With the facilities of modern high-speed computers, it is possible to follow the motions of a large number of charged sheets in the assumed one-dimensional space of the converter. The computation of the trajectories of charged sheets is executed by increasing time in steps. During each time step, new charge sheets are started at the position of the boundary plane which takes the role of the emitter. The initial velocities of these injected sheets (both positively and negatively charged) are chosen randomly and according to the half-Maxwellian velocity distribution law of a thermionic emitter.

The sheets are advanced step-by-step according to their velocities and the electric field they experience until they reach either boundary plane when they are assumed to be lost for the system.

Similar sheet calculations have been used by many authors (References 4, 5, 6, 7, 8, 9). It is not our objective here to describe in detail these calculations or prove their credibility; for that the reader is directed to the cited references. We only use the results of these calculations in order to get an insight into the mechanism which causes large amplitude oscillations in the state of this one-dimensional system. In order to show that the oscillatory behavior of the particular example of a computer-simulated converter chosen is not caused by effects related to the computational methods themselves, it is necessary to point out the following:

1. There are large amplitude oscillations in the current of the computer-simulated diode regardless of the exact parameters chosen as long as $\alpha \gtrsim 1$, V_d is positive and is of the order of a few volts, and the length of the diode is larger than $20\lambda_{DB}$, where λ_{DB} is the electron Debye length corresponding to the full-Maxwellian electron distribution at the emitter.

$$\lambda_{DB} = \sqrt{\frac{\epsilon_o kT}{e\rho_{eo}}} \qquad (2)$$

where the electron density ρ_{eo} is related to the electron saturation current by the following expression:

$$\rho_{eo} = J_{se} \quad \frac{kT}{2\pi m_e} \tag{3}$$

2. The behavior of the simulated diode is independent of the size of the time step (the size of the time step used here is 1/50 of the average electron transit time), i.e. calculations made with smaller time steps showed exactly the same current vs. time curve as shown on Fig. 3.

3. The noted behavior of the computer-simulated diode is caused by collective effects of a large number of charged sheets (there are at least 3,000 sheets in the diode space at any instant) and this behavior does not change when we increase the number of sheets used for the calculations of the characteristics of the diode. Hence, the current vs. time curve is not influenced by shot-noise effects associated with the simple particle description of the system.

The foregoing observations suggest that large amplitude oscillations are genuinely inherent in the physical diode system.

In Fig. 3 we show some results of computer calculations that simulated the operation of a low pressure thermionic converter with the following parameters.

The normalized potential value was $\eta_d = 5$. The normalized distance between the boundary plates $\xi_d = d/\lambda_{DB} = 50$. The parameter α (see Eq. (1)) was chosen equal to 1.2. A mass ratio $m_i/m_e = 64$ was used for these calculations. The predicted dc potential distribution in this diode (see Fig. 2) is similar to that of McIntyre which suggests that saturation electron current should flow through the diode.

The actual current through the computer-simulated diode normalized to the electron saturation current is shown on Fig. 3 as function of time. Time is normalized to the average transit time of the electron. (We define an average electron transit time as the time that is taken by an electron traveling with the velocity $\sqrt{2kT/m_e}$ to cover the distance d. An average ion transit time would be $\sqrt{m_i/m_e}$ times or in this example, 8 times longer.)

The large amplitude oscillations in the diode current, i.e. in the number of electrons reaching the collector, can be related to the potential distribution in the diode at different times. The potential distributions at a few instances during one cycle of the oscillations are shown in Fig. 3. At the normalized time value $t = 30.4$, the potential in the diode looks very much like the expected dc potential distribution (see Fig. 2). We know that this is a self-consistent solution of the static equation, but the potential does not keep its shape and it changes rapidly to the potential form shown for $t = 31.2$. In other words, within a time interval of 0.8 times the average transit time of the electrons, the potential completely changes its shape. It is obvious that the diode potential cannot stay in this latter form since in due course, practically all the ions are returned to the emitter, and after the ions have left the center part of the diode, the potential has to collapse. This actually starts to happen at $t = 33.6$. The potential changes its shape again because ions leave the middle of the diode, but new ions cannot arrive yet from the emitter because of their slow speed. The lack of ions at the middle of the diode changes the form of the potential by depressing it, and a potential minimum is formed. This minimum turns back some electrons to the emitter, and limits the current through the diode (see potential distribution at $t = 36.4$). Consequently, the current has to decrease. It is seen from this diagram that the current indeed has a big dip reaching a minimum at $t = 37$. Now it depends only on the ion motion how long the potential stays below the axis, because when they fill up the potential minimum (an ion trap) at the front of the emitter, the potential at the middle will start to increase (see potential at $t = 39.2$). The form of the potential then again approximates the expected dc potential, and the whole cycle starts all over again.

The only apparent "mystery" in the described process is: why does the potential not stay at its expected dc value? This mystery can be resolved by re-examining the predicted dc solution of the ion-electron system in a new light.

The stability of dc states of ion-electron systems

Before we can show that the dc state of the low pressure thermionic converter is indeed unstable, we have to review the method by which this dc state was determined.

The usual way of calculating dc states for one-dimensional diodes is (see, for instance, Reference 1) to assume first the form of the dc potential distribution. If the form of the potential distribution is known, it is possible to express the space charge contributions of the emitted particles anywhere in the diode as the function of the potential. Knowing the exact form of the functional dependence of the total space charge on potential $[\rho(V)]$, Poisson's equation has to be satisfied $[d^2V/dx^2 = -\rho(V)/\epsilon_0]$. Poisson's equation can be integrated once directly to yield the stress balance, leaving a quadrature. The potential shown in Fig. 2 has an assumed form such that it is positive and monotonic for the whole diode space. In this case the space charge functions of electrons and ions normalized to ρ_{eo} (Eq. 3) have the following dependence on the normalized potential, η:

$$\rho_e(\eta) = -\frac{1}{2} F^-(\eta) \tag{4}$$

$$\rho_i(\eta) = \frac{\alpha}{2} e^{-\eta_d} F^+(\eta_d - \eta) \tag{5}$$

where the functions $F^-(\eta)$ and $F^+(\eta)$ are defined as:

$$F^-(\eta) = e^{\eta}(1 - \mathrm{erf}\sqrt{\eta}) \tag{6}$$

$$F^+(\eta) = e^{\eta}(1 + \mathrm{erf}\sqrt{\eta}) \tag{7}$$

using the error function $\mathrm{erf}(\eta) = 2/\sqrt{\pi}\int_0^{\eta} e^{-t^2}\,dt$. The final integral can be written in a normalized form (see Ref. 1):

$$\xi(\eta) = \int_0^{\eta} \frac{dt}{[G^-(t) + \alpha\, e^{-\eta_d}\, G^+(\eta_d - t) + G_0]^{1/2}} \qquad (8)$$

where ξ is x/λ_{DB} and x is the distance in the diode measured from the emitter. It is assumed that the potential is zero at the emitter; G_0 is an arbitrary constant yet to be determined. The functions $G^-(\eta)$ and $G^+(\eta)$ are defined as

$$G^-(\eta) = \frac{d\, F^-(\eta)}{d\eta} = F^-(\eta) + 2\sqrt{\eta/\pi} \qquad (9)$$

and

$$G^+(\eta) = \frac{d\, F^+(\eta)}{d\eta} = F^+(\eta) - 2\sqrt{\eta/\pi} \qquad (10)$$

(It has to be mentioned here that the mass ratio (m_i/m_e) does not enter into the dc analysis. Only the parameter $J_{si}\sqrt{m_i/m_e}$ is used for determining α; and as long as α stays constant, the same potential vs. distance curve is obtained for any value of m_i/m_e.) The arbitrary constant, G_0 , is chosen such that $\xi(\eta_d) = \xi_d = d/\lambda_{DB}$. Once this constant has been found, the dc potential distribution is determined, and it can be calculated numerically. The computed dc potential distribution of the diode with parameter $\xi_d = 50$, $\eta = 5$, $\alpha = 1.2$ is shown on Fig. 4 along with the normalized space charge functions of ions and electrons in the diode space for the same dc solution.

This dc solution is the self-consistent dc solution of this ion-electron system; therefore, this is the state in which the diode could stay in equilibrium. Let us assume that the dc state has developed in the converter so that the potential and the space charge functions are those shown on Fig. 4. We will show that starting from this dc solution the diode could by the rearrangement of only the electrons pass to another state which is in "equilibrium" for as long as the heavy ions can be treated as fixed. In other words, we look for such "temporary" equilibria by allowing only the electrons to move and keeping the distribution of the ions intact. This analysis is justified by the large mass-ration of ions and electrons.

We expect that the space charge distribution of the electrons is free to find any equilibrium state it desires within a time interval of a few average electron transit times; and since in a physical device $m_i/m_e > 100$, the distribution of ions is hardly affected during this interval. But does an alternative "temporary" equilibrium state exist? The problem of finding such a state is formulated mathematically as follows:

The normalized space charge density function of the ions for the original dc state can be expressed (or at least tabulated) as a function of normalized distance $(\rho_i(\xi))$. If the electrons are in equilibrium for an arbitrary potential function $(\eta(\xi))$ that satisfies the condition $\eta(\xi) > 0$ for $\xi > 0$, then their normalized space charge function has the form $\rho_e(\eta) = -1/2\ F^-(\eta)$. Poisson's equation now takes the form:

$$\frac{d^2\eta}{d\xi^2} = \frac{1}{2}\ F^-(\eta) - \rho_i(\xi) \qquad (11)$$

where $\rho_i(\xi)$ is the old ion distribution which is not in dc equilibrium with the new potential distribution. The second order differential equation (11) has a boundary condition $\eta(0) = 0$. With certain particular choices of the initial derivative $d\eta/d\xi$, one expects after integrating across the diode, to find that the specified terminal value $\eta = \eta_d$ at $\xi = \xi_d$. One choice of the initial derivative leads to the already known dc solution. But there may be another solution to Eq. (11) satisfying the condition $\eta(\xi_d) = \eta_d$ with a different initial value of $d\eta/d\xi$.

We have solved Eq. (11) numerically. For the original dc solution, the initial value of $d\eta/d\xi$ was 1.068; a second solution was found for $d\eta/d\xi = 2.58$. The second solution is shown on Fig. 5.

The importance of this second form of the potential function which we call the "temporary dc state" is the following. Once the basic static solution has developed in the diode, the electrons have a choice of redistributing themselves and find a different potential function in the diode which is also static as far as the electrons are concerned. This later static solution, on the other hand, is only temporary because it is not consistent with the distribution of ions and in a time interval of an average ion transit time it has to change. The speed of this latter change depends on the ion transit time, and this is the reason why the main oscillations are characteristic to the average ionic speed.

We supply one hypothesis for the question why the electrons find the temporary static state more desirable than the proper dc state. It is clear from observing the two potential curves on Fig. 4 and Fig. 5 that the transit time of the electrons is shorter for the temporary dc state than that for the real dc state because in the former case they are accelerated near the emitter, while in the latter case they travel along the plasma-like region with comparatively smaller velocities. In fact, if we **perturb the proper dc state** in such a manner that its form is more similar to the temporary state, then we find also that the electrons have a smaller average transit time. Consequently, the application of the principle of least action for this problem would find the temporary dc state to be the more probable state than the proper dc state. We point to the results of the computer-simulated diode to support the fact that the rearrangement of the electron distribution in this situation actually does take place.

The existence of a temporary dc state allows the electrons to stay in equilibrium as long as the distribution of ions stays comparatively constant. This state is thus responsible for the large amplitude, relaxation type of oscillations exhibited by one-dimensional ion-electron systems such as the low pressure thermionic converter. The characteristic frequency of these oscillations depends on the transit time of the ions.

A general conclusion regarding equilibria of electron-ion systems can be drawn from our specific analysis: when dc states of ion-electron systems are sought, it is not sufficient to find the self-consistent dc states only which disregard the difference between the masses of ions and electrons. Based upon the self-consistent dc states, a search for temporary dc states is required in order to tell whether a rapid rearrangement of the electrons could bring them into equilibrium in a state different than the self-consistent dc state. If such a temporary state is found, large amplitude relaxation oscillations are possible in the system, and the self-consistent dc state might appear only as a transitory state during the system's actual operation.

Conclusion

Because there is a large difference between the masses of ions and electrons, it is possible to re-examine the dc states of a physical system (containing ions and electrons) for a time period that is significant only for the motion of the electrons, but is insignificant for the ions. The conditions of this re-examination are to keep the distribution of ions unchanged but change the distribution of electrons in search of a new solution to the static equations. If the static equations can be satisfied for the electrons under these new conditions, then it is possible that in the laboratory the electrons will rapidly rearrange themselves when the original dc state is reached and cause the system to stay in the temporary stable state (which is not consistent with the distributions of ions). The temporary dc state will break up as soon as the effect of this new state starts to show in the motion of the ions. Through this "relaxation" mechanism ion-electron systems can exhibit oscillations that are characteristic of the motion of the ions.

The low pressure thermionic converter indeed has a well-defined temporary dc state which would account for the low frequency, large amplitude oscillations it exhibits in the laboratory.

FIGURE CAPTIONS

Fig. 1 The schematic model of a low pressure thermionic converter.

Fig. 2 The expected dc potential distribution in the converter for slightly ion-rich operation ($J_{si}/J_{se} > \sqrt{m_e/m_i}$) and a positive applied potential on the collector.

Fig. 3 Characteristics of the computer-simulated converter. Time is normalized to the average transit time of the electrons, current is normalized to their saturation current.

Fig. 4 The self-consistent dc states of the diode.

Fig. 5 The temporary dc state of the diode. The space charge density function of ions is the same as that on Fig. 4, but the space charge density function of electrons is rearranged so that the electron space charge is self-consistent with the new form of the potential function.

Acknowledgements

The author wishes to thank Dr. D. A Dun and Dr. O. Buneman for many helpful suggestions on this subject. The help of M. F. O'Neal in some stages of the numerical computations is also appreciated. This research was supported by a NASA grant (NSG 299-62).

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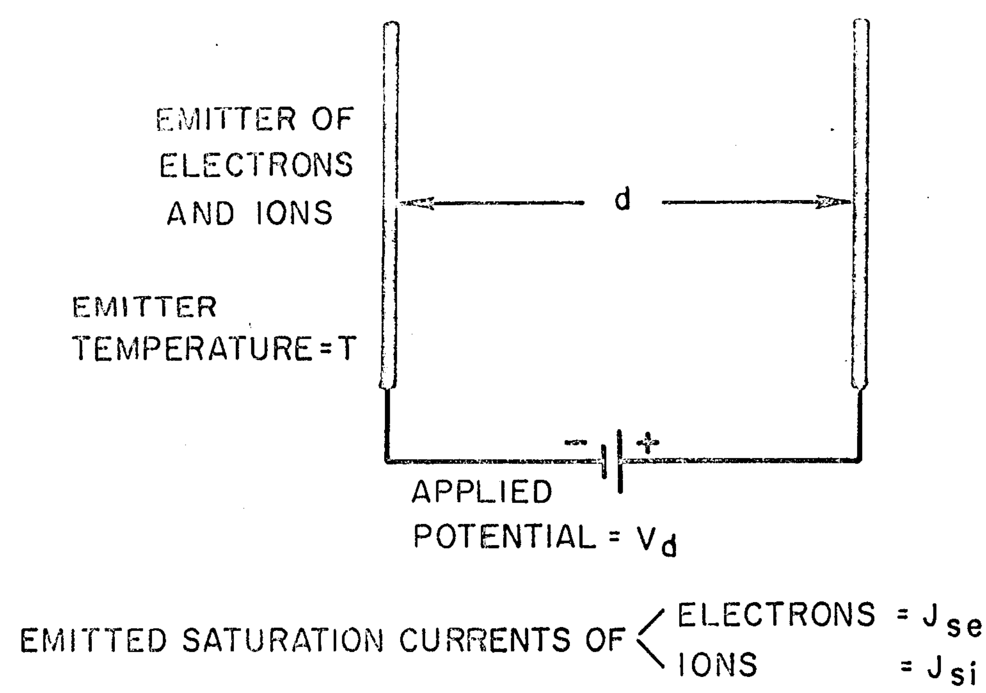
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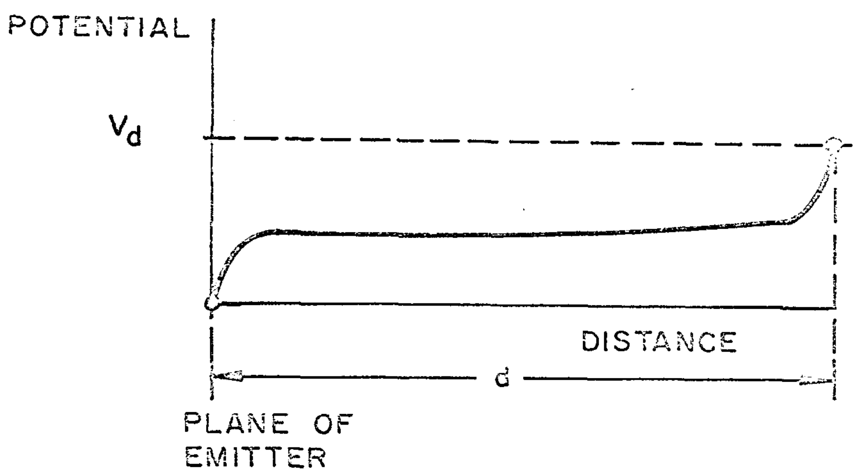
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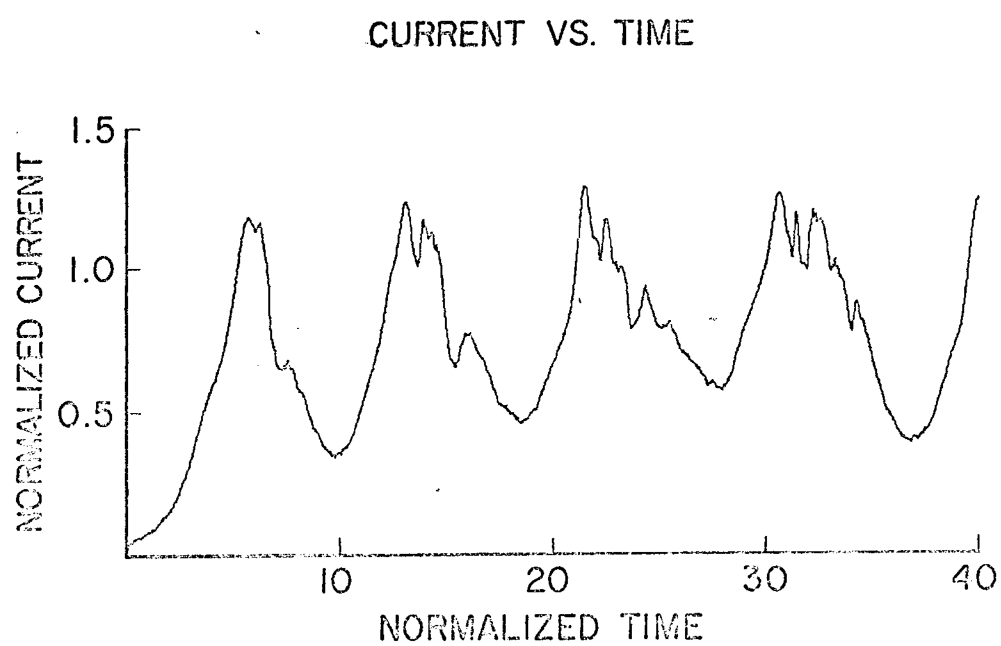
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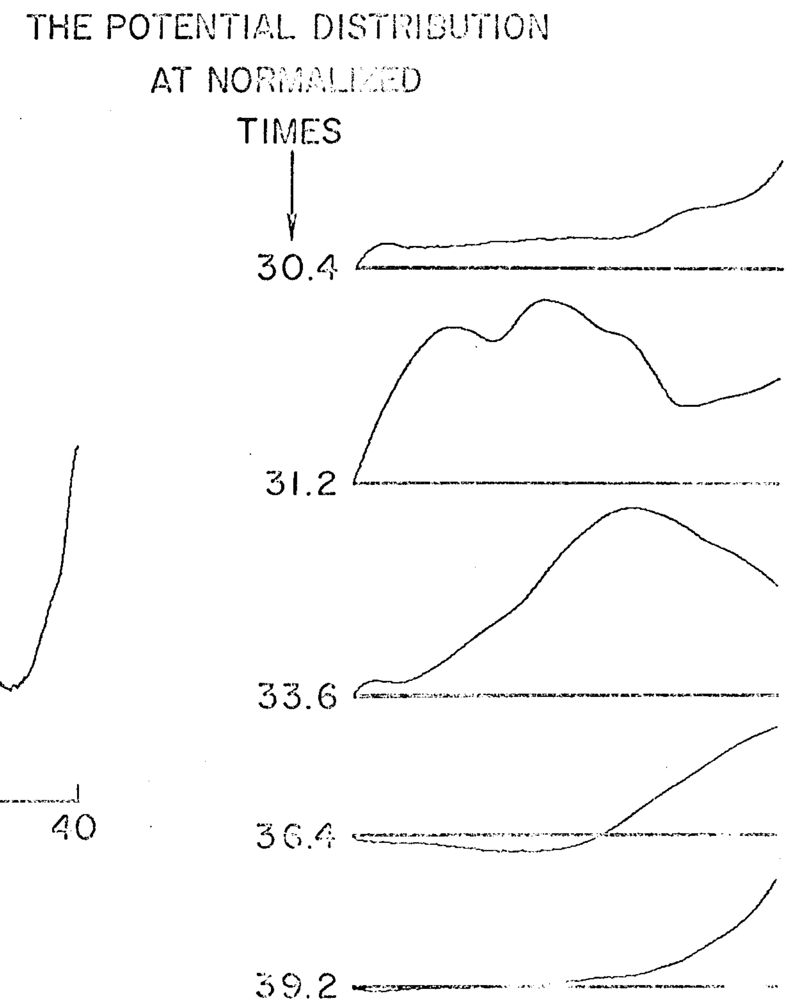
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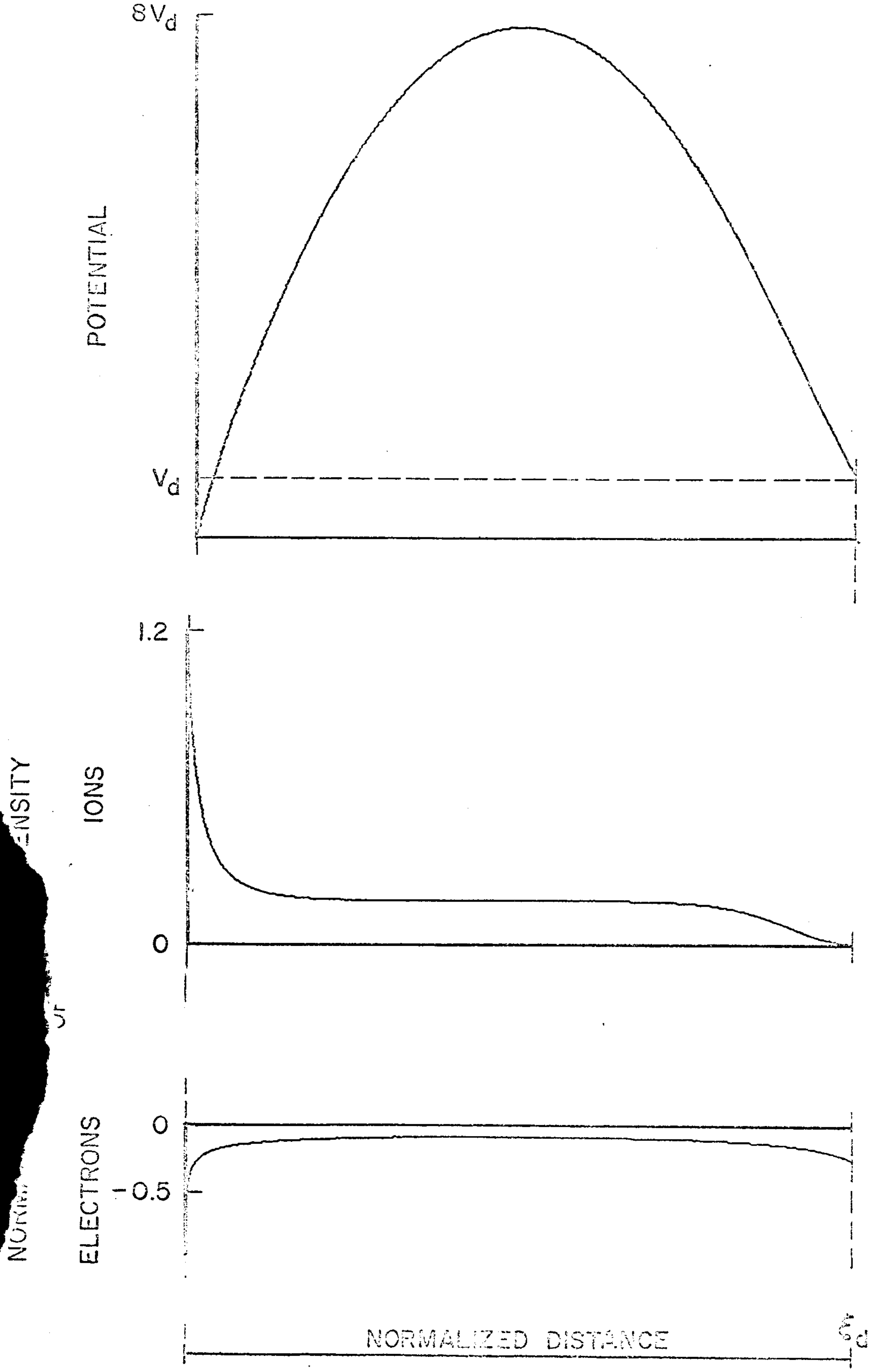
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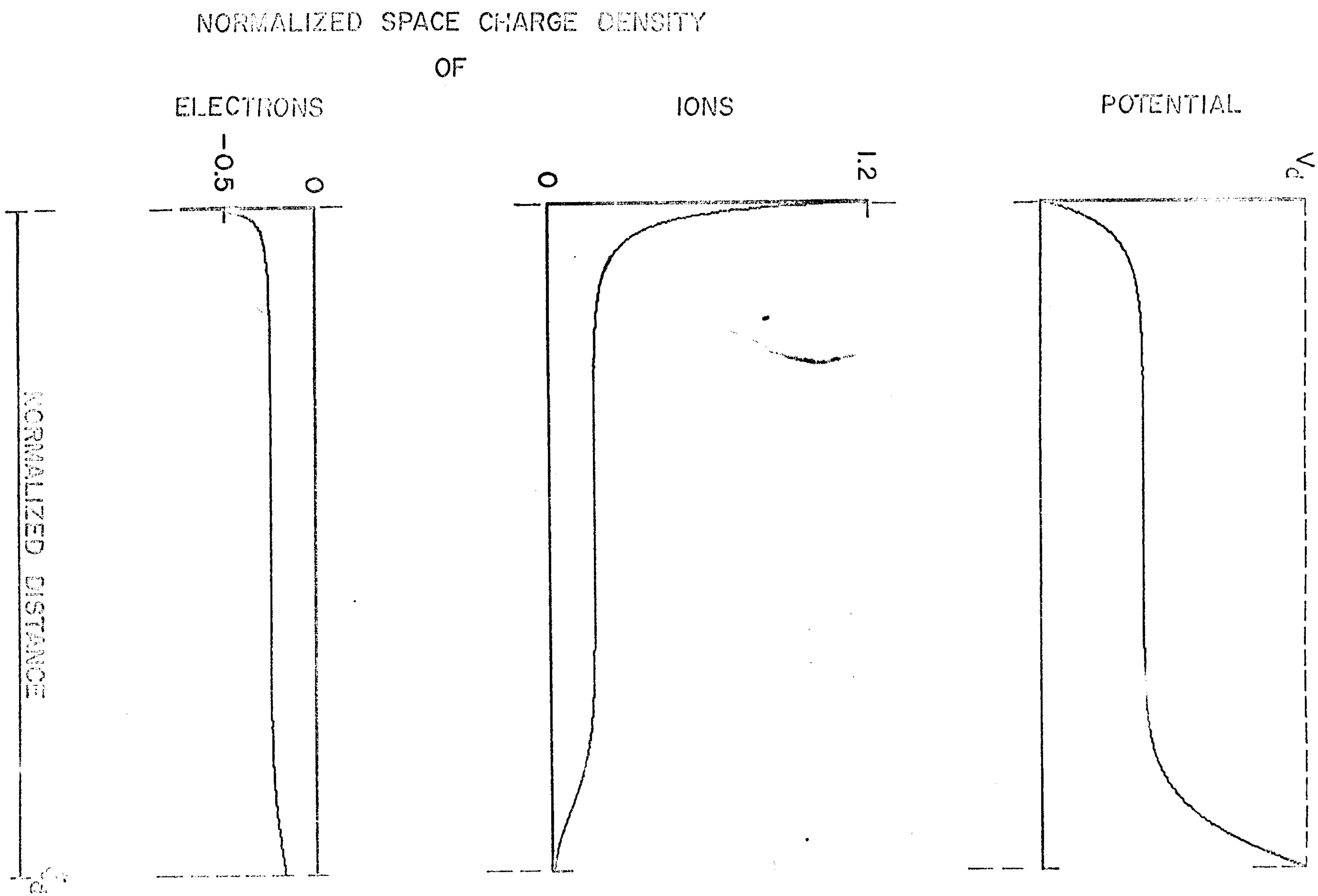
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