A FORTRAN PROGRAM TO CALCULATE AN ENGINEERING ESTIMATE OF THE THERMAL RADIATION TO THE BASE OF A MULTI-ENGINE SPACE VEHICLE AT HIGH ALTITUDES

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ABSTRACT

A method of estimating the radiant heat flux in a base of arbitrary shape from intersection regions caused by the interaction of hydrogen-oxygen engine exhaust jets is presented. An approximate method of generating the intersection region shape and temperature-pressure profiles is discussed. A computer program incorporating both of the above is described and instructions are given for its loading and use. The accuracy of this program is expected to yield within an order of magnitude of the true value of thermal radiation.

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SUMMARY

A method of estimating the radiant heat flux in a base of arbitrary shape from intersection regions caused by the interaction of hydrogen-oxygen engine exhaust jets is presented. An approximate method of generating the intersection region shape and temperature-pressure profiles is discussed. A computer program incorporating both of the above is described and instructions are given for its loading and use. The accuracy of this program is expected to yield within an order of magnitude of the true value of thermal radiation.

I. INTRODUCTION

A contributing factor to the base heating rate on upper stage vehicles is radiation from the exhaust plumes. Limiting the discussion to upper stage vehicles makes it unnecessary to consider the afterburning problem. With this simplification, this radiant heating problem lends itself to division into three distinct radiating regions: the supersonic cores, the first order plume impingement regions, and the extremely complex flow field downstream from the base where the supersonic cores, first order regions, and higher order regions merge. The term "first order plume impingement region" is used to describe the region created by the shock formed when two exhaust plumes intersect. This region is an area of high temperature and pressure. A second order region is formed by the intersection of two first order regions. This terminology can be extended to describe any number of intersecting regions and will be used throughout this report.

Of the three types of radiating regions, the first two yield to analysis while the third, being virtually undefinable, does not. It was decided early in the analysis to consider the supersonic cores and first order regions as independent primary sources of radiation. To take account of the complex downstream flow field, it was decided to estimate a "lump sum" addition to the final overall radiative flux, this estimate being based on the numbers obtained for the primary radiation sources. Further analysis has revealed that of the two remaining regions, the supersonic cores are often insignificant; thus we have limited ourselves in the computer program described here to a calculation of the radiation from the first order intersection regions only.
The supersonic cores in general are at a considerably lower calculated temperature than the intersection region and, in addition, are located closer to the base. Because of this, radiation is less intense and more completely blocked by components in the base region. However, for the user who feels that the supersonic cores will contribute materially to his particular problem, a program has been developed to calculate the additional radiation from these regions using methods identical to those described in this report.

Proper use of the program discussed here will thus provide the user with an estimate of the incident radiation in a minimum of time and will show if a more detailed analysis is needed.

II. CAPABILITIES AND LIMITATIONS

A. Capabilities

The program will estimate the radiation incident on a base of arbitrary configuration from any number of first order regions and will predict how much of this radiation will be blocked by intervening structures in the base region. The user has the option of choosing the method of calculation of the temperature and pressure fields in the intersection region from the three methods programmed. In addition, the actual radiation is calculated by two different methods to allow comparison of two theoretical approaches to the problem. Of these methods, the average temperature and pressure method is more conservative since it assumes that the radiation passes through a transparent space from the radiating region to the base, when in fact it is passing through an absorbing medium. This method thus provides a probable upper limit to the heat flux rate. The other approach accounts for the absorption by an approximate method outlined by Hottel in Reference 1. Each method is presented with and without a blockage correction factor.

B. Limitations and Assumptions

As mentioned earlier, this program computes only the radiation contribution from the first order intersection regions. These regions are assumed to have approximately elliptical cross sections and their temperature and pressure profiles are radially invariant. That is, temperature and pressure are assumed to vary in an axial direction only. These profiles and the geometry of the region are generated from input taken from a method of characteristics printout for a free jet expansion into quiescent air. The flow model for the first order regions is generated by assuming that this expanding plume impinges on a flat plate placed in the flow at a distance equal to one half the distance between nozzle centerlines. Oblique shock theory is then used to produce the geometry as described in Section IIIA of this report.
Both radiation methods use the emissivity data of Hottel given in Reference 1 without the correction for partial pressure. These data are used in the form of a polynomial curve fit with a maximum error in fit of 3 percent. Radiation from each intersection region is assumed to pass through a transparent medium until it impinges on the base or is blocked by one of the solid components in the base region (such as motors and heat shield).

Therefore, the program is restricted to use in cases where the only significant radiating species in the exhaust is water vapor. If the curves were fit to new emissivity data, the program could be changed to treat other radiating species.

III. THEORETICAL BACKGROUND

The gas radiation is treated with a comparatively crude approximation. The emission and absorption coefficients, which actually vary in more or less unknown fashion through the emission spectrum of the gas, are replaced by a lumped emissivity representing an integrated effect over the whole emission spectrum. This lumped or total emissivity was obtained from experiments on comparatively small samples of a homogeneous gas and extrapolated versus sample size, pressure, and temperature. The extrapolation becomes progressively more uncertain the further one moves away from the actual test conditions. Use of these data for estimating radiative heating of full scale rocket configurations should normally involve extrapolation of these data to larger beam lengths and lower pressures than those used in the basic experiments. Further, these measurements from homogeneous gas samples are used to estimate radiative emission from nonhomogeneous gas volumes. This may produce additional uncertainties.

Two methods are proposed for this estimate: the average temperature method and the absorption method. In both cases, the radiating gas volume is subdivided into conical pencil-like volume elements, extending along lines of sight from the base surface element in question. Temperature, density, and partial pressure of the radiating species vary along the gas pencil.

For the average temperature method, the properties of the gas, i.e., $T^4$ and partial pressure, are averaged over the length of the pencil. The "pencil" is then treated as a homogeneous gas sample, yielding a corresponding emissivity with a corresponding view factor. From these, the energy flux contribution that impinges on the base surface element under investigation is computed.
In the absorption method, the "gas pencil" is divided longitudinally into individual elements of different pressures and temperatures. The energy radiated from each longitudinal element toward the base surface element under investigation is subject to absorption losses in the pencil elements it has to pass through. This absorption loss is estimated by the assumption that the whole gas pencil between the radiating element and the surface of the intersection region has the same gas properties (T, P_{H_2O}) as the radiating element. While this model admittedly loses any effects of property variation on the absorption, it significantly reduces the complexity of the estimate by eliminating one integration.

To simplify the presentation, the following discussion will consider the program as though it were divided into four separate programs.

A. Geometry and Temperature-Pressure Profiles

From a method of characteristics analysis of a free plume expansion into quiescent air, one may obtain values for Mach number (M) and flow direction angle (θ) at various points in the exhaust plume. Values of M and θ along a line of constant radius are interpolated from this output (Figure 1). In addition, one obtains the plume expansion radius (radial distance from nozzle centerline to jet boundary streamline) as a function of axial distance. Using these input data, the initial region geometry and temperature-pressure profiles are generated using the following procedure.

1. Geometry of the Intersection Region

In Figure 2, a representative cross section is drawn. The semi-major axis of the cross section, ADIS, was determined by connecting the overlap of circles whose diameters were established by the free plume boundaries from the method of characteristics. Determination of the semi-minor axis was more difficult. It was assumed that the flow field from two adjacent streams deflects by the same amount. This assumption preserves symmetry about the major axis of the region and is compatible with a hypothetical model consisting of a plane placed half way between two adjacent engines. This model is sketched in Figure 3. At points 0, 1, 2, 3, etc., the Mach number and flow angles are known. The flow impinging on the plate at point 0 produces an oblique shock at an angle $\delta_0$ given by the equation
\[ \cot \theta = \tan \theta_0 \left( \frac{\gamma + 1}{2} \frac{M_1^2}{M_1^2 \sin^2 \theta - 1} - 1 \right) \]  \hspace{1cm} (1)

where \( M_1 \), the Mach number before the shock, is the Mach number \( M_0 \) which we have obtained from the method of characteristics analysis. This shock is shown as OB-1. It is clear that it is at an angle of \( (6 - \theta) \) from the hypothetical flat plate. The mass flow between streamline SL-0 and SL-1 is now assumed to be turned at the shock so that it is flowing downstream parallel to the wall. Because of continuity, the flow occupies a certain distance from the plate to some streamline SL-1'. Next, the Mach number and flow angle known to exist at point 1 are transferred radially to point 1'. This transfer is justified on the grounds that the temperature and pressure are assumed constant in a radial direction and that thus the flow angle and Mach number must be constant also. At point 1' a new shock angle \( \theta_1 \) is calculated as before and another section of the intersection region boundary is determined. The process is repeated until the entire region geometry is obtained. It is immediately obvious that this approach ignores mass flowing between point A and point 1'. This assumption should be reasonable if the value of \( (6 - \theta)_0 \) is small and the value of \( (\theta)_1 \) is large. This case corresponds to the highly expanding plume, which is the case normally encountered in high altitude operation.

Equation (1) is limited in that it can only generate shock angles for values of flow angle less than a certain value. Figure 12 shows a parametric plot of this equation for a \( \gamma \) value of 1.23 and for Mach numbers ranging from 1.1 to 20. It is clear that since plume Mach numbers seldom exceed 10, flow angles of 55 degrees and above will not yield solutions to equation (1). In these cases, a shock angle of 65 degrees was assumed since this is approximately the maximum shock angle that can be produced by equation (1) for Mach numbers in the range of those encountered in jet plumes. (Note that there are two solutions to equation (1) for any flow angle. The highest, or "strong shock" solution, is not used in the analysis.)

The process outlined above is repeated until a complete geometrical profile of the region is developed. Figure 4 illustrates the application of the procedure for several increments. Referring to the Figure, we can see that the geometrical relation which links any one radial distance to the preceding radial distance is

\[ (B/D)_n = (B/D)_{n-1} + \tan (6 - \theta)_{n-1} \frac{X}{D} \tan (X/D) - (X/D)_{n-1}. \]  \hspace{1cm} (2)
2. **Pressure-Temperature Profiles**

With the geometry of the intersection region computed, the program returns to the input data. The pressure-temperature profiles are generated using the same model as before. Postulating as before the information of an oblique shock whose angle is determined by the input flow angle for that particular axial distance, the program resolves itself into the determination of static temperatures and pressures behind a shock from the total conditions before the shock. Three methods of calculating these temperatures and pressures are programmed and may be selected by the user. In general, the oblique shock temperature and pressure equations should be used since they are more conservative than the other two models and because their use maintains consistency with the equations used to generate the region geometry.

Using the subscripts 1, 2, and 0 to denote, respectively, static conditions before the shock, static conditions after the shock, and total conditions, the following isentropic perfect gas relation holds:

\[
\frac{P_2}{P_0} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-\frac{\gamma}{\gamma-1}}. \tag{3}
\]

Also the adiabatic perfect gas relation is

\[
\frac{T_2}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-1}. \tag{4}
\]

The oblique shock relations used are taken from NACA Report 1135 as

\[
\frac{T_2}{T_1} = \frac{\left[2\gamma M_1^2 \sin^2 \delta - (\gamma - 1)\right][\gamma - 1] M_1^2 \sin^2 \delta + 2}{(\gamma + 1)^2 M_1^2 \sin^2 \delta} \tag{5}
\]

and

\[
\frac{P_2}{P_1} = \frac{2\gamma M_1^2 \sin^2 \delta - (\gamma - 1)\gamma}{\gamma + 1}. \tag{6}
\]
As mentioned in the geometry section, cases exist where the shock angle $\delta$ cannot be determined. In these situations, the program automatically switches to another method of calculation using the Rankine-Hugoniot equation

$$T_2 = T_0 \left[ \frac{(1 + \beta)(1 + \alpha) + \alpha (\beta + \beta^2)}{(1 + \gamma - \frac{1}{2} M_1^2)(1 + \alpha) + \beta + \gamma - \frac{1}{2} M_1^2 \beta^2} \right], \quad (7a)$$

where

$$\alpha = \frac{\gamma - 1}{\gamma + 1} \quad \text{and} \quad \beta = \gamma M_1^2 \sin^2 \theta \quad (7b)$$

for the temperature calculation.

For the pressure calculation, the modified Newtonian Impact Theory equation

$$\frac{P_2}{P_1} = C_p \left( \frac{\gamma}{2} \right) M_1^2 + 1 \quad (8)$$

and

$$C_p = 2 \sin^2 \theta \quad (9)$$

is used. These equations may also be called for by the user if desired by inputing a control value as described in Section VIII A of this report.

In addition, the isentropic flow equation

$$\frac{T_2}{T_1} = \left( \frac{C_p \left( \frac{\gamma}{2} M_1^2 + 1 \right)}{\gamma} \right)^{\frac{\gamma - 1}{\gamma}} \quad (10)$$
is programmed and may be requested by use of the control variable. The pressure calculation is made using equation (8).

These calculations are repeated progressing in an axial direction until the entire profile has been generated. Due to the assumption that the static temperature before the shock is constant, it follows that the static temperature behind the shock does not vary from point 0 to point 1' in Figure 3 and thus also from 0 to 1. Then, at point 1, a step discontinuity will exist and the temperature from 1 to 2 will be the temperature calculated from the flow angle at point 1. If a plot of temperature vs axial distance were obtained, these discontinuities would cause the curve to resemble a "step" function. To smooth the curve, linear temperature gradients and not constant temperatures are assumed to exist between analyzed axial points.

B. The Average Temperature Method

The Average Temperature Method is an analysis of the problem which uses the fundamental Stephan-Boltzman Law

\[ Q = \sigma T^4. \]  

(11)

Obviously, a sort of mean temperature and pressure for each radiating increment must be developed. The method of calculating these mean values is described below.

The intersection region was first divided into a number of increments. A line of sight from a point on the base to the lower edge of each increment was established and extended through to the rear face of the plume. A view factor, F, to account for the fact that the point on the base only "sees" one side of each radiating increment, was calculated following the analysis of Sparrow [2]. This derivation is performed in detail in Appendix A. This factor is calculated for each individual increment radiating to a point (Figure 5).

The mean temperature along the beam length itself is

\[ T_{AV} = \frac{x_1}{x_2 - x_1} \int_{x_1}^{x_2} (T(x))^4 \, dx \]

(12)
where \( x_1 \) and \( x_2 \) are the axial coordinates of the points where the beam enters and leaves the intersection region, respectively. A similar formula is applied for the evaluation of the average pressure along the beam length. Using this average pressure, the partial pressure of the water vapor in the exhaust is

\[
P_{\text{H}_2\text{O}} = (F_{X1}) P_{\text{AV}},
\]

where \( F_{X1} \) is the mole fraction of \( \text{H}_2\text{O} \) vapor in the exhaust plume.

With the partial pressure, temperature, and beam length determined, the emissivity is obtained from the data of Hottel [1]. These emissivity data have been curve-fitted with a series of polynomial expressions such that the maximum error is \( \pm 3 \) percent in fit. From this the emissivity may be determined and used in the final expression for \( Q \), the heat flux

\[
Q = \sigma F \varepsilon \frac{T^4_{\text{AV}}}{4}\frac{\pi}{X} \cos \theta \cos \omega \ dX.
\]

C. The Absorption Method

The absorption method employs the equation developed by Hottel [1] to account for the absorption in the intersection region. The equation employed is

\[
Q = \frac{1}{\pi} \int_{\omega} \int_{X} \sigma T^4 (d\varepsilon/dX) \cos \theta \cos \omega \ dX.
\]

In this expression, \( X \) is the optical beam length and is evaluated as

\[
X = \frac{P_{\text{H}_2\text{O}}(L)}{P_{\text{H}_2\text{O}}}.
\]
and

$$\frac{1}{\pi} \int_{\omega} \cos \theta d\omega$$  \hspace{1cm} (17)

is a solid angle shape factor previously called $F$. Then

$$Q = F_0 \int_{P_{wL}} T^4 \frac{dc}{d(PL)} d(PL).$$  \hspace{1cm} (18)

The plume is divided as before into increments and a pencil ray (line of sight) erected to the lower edge of each increment. This line of sight is now the optical beam along which equation (18) must be evaluated. To do this, we must first evaluate the pressure and temperature at various points along the beam. This is done using a straight line interpolation method which is quite valid because of the smooth nature of the temperature and pressure profiles and to the small distance between points at which the evaluation is made.

The emissivity slope is easily evaluated since the curve fits mentioned in Section IIIB are functions of (PL) with $T$ as a parameter. Thus, an explicit differentiation of these polynomial curve fits is possible without resorting to numerical techniques. Since all the variables are functions of the position variable $L$, we may now proceed to simplify equation (18).

$$Q = F_0 \int_{PL} T_1^4 \frac{1}{P} \frac{dc}{dL} PdL,$$  \hspace{1cm} (19)

since

$$d(PL) = PdL$$  \hspace{1cm} (20)
by the assumption that temperature and pressure are constant in the particular increment of length under analysis. Then

\[ Q = F_\sigma \int_{PL_1}^{PL_2} T_1^4 \frac{d\varepsilon}{dL} \, dL. \tag{21} \]

Equation (21) is evaluated using Simpson's rule and the value for F already computed in Section IIIB.

D. Blockage

In certain base configurations, a great deal of the radiation from the exhaust plumes will impinge upon the motors, heat shield, and other obstructions in the base, and will not therefore contribute to the heat flux rate to the base. To evaluate how much of the radiation is blocked, a section of the program generates a blockage factor and applies it to each intersection region. This blockage factor is not related to the form factor F computed in Section IIIB. The form factor expresses what portion of the intersection region the point "sees" assuming a transparent space between radiating region and point. The blockage factor increases the realism of the model by removing the "transparent space" assumption and substituting the actual base obstructions.

It was decided that the easiest method of generating a blockage factor was to determine if each individual beam was blocked and if it were to consider the radiation from the increment associated with that beam to be 0. Since the increments have widths which in some cases are appreciable, three lines of sight are erected to each increment, one to each side and one to the center. Each line is assumed to represent one-third of the radiation incident from the region. Thus, blockage of each beam reduces the incident radiation by one-third.

The obstructions are first represented as circles in a three-dimensional, Cartesian coordinate system located as follows.

1. The xy plane is located at the exit plane of the motors such that the z coordinates of all obstructions and the points to be analyzed are negative or zero.
2. The xz plane passes through the center of the heat shield and through the point being analyzed with the negative axis in the direction of the point being analyzed such that "x" and "z" coordinates of the point are negative and the "y" coordinate is always 0.

3. The yz plane is oriented as usual in a right-handed Cartesian coordinate system. See Figure 6 for a graphical illustration of this system.

The obstructions are assumed to be circles located in, or parallel to, the xy plane. In this way, a three-dimensional nozzle can be fairly well represented by, for example, three circles with radii equaling the radii of the nozzle at three different heights. The model expressing this nozzle would then become three circular areas in space given by the relations (in our coordinate system)

\[ (x - h)^2 + (y - k)^2 = r_1^2 \quad z = z_1 \]  \hspace{1cm} (22)

\[ (x - h)^2 + (y - k)^2 = r_2^2 \quad z = z_2 \]  \hspace{1cm} (23)

\[ (x - h)^2 + (y - k)^2 = r_3^2 \quad z = z_3 \]  \hspace{1cm} (24)

If the obstruction has its axis parallel to the axis of the vehicle (which is our z-axis) then "h" and "k" are constant for the three equations. The "h" and "k" coordinates are the "x" and "y" coordinates of the center of the obstruction. If the axis of the body is tilted with respect to the vehicle axis then "h" and "k" vary. They are then the coordinates of the center of the obstruction at that particular height. A slight error is thus introduced because of the assumption that the circles lie parallel to the xy plane when, in fact, they are tilted to be perpendicular to the obstruction axis; but this error will not be appreciable unless the tilt is considerable.

With equations (22), (23), and (24) defining three areas in space, it is clear that, for a line of sight to avoid intersecting one of these areas, yet in fact to be blocked by the nozzle, the line of sight must be almost parallel to the xy plane. Since in practice the intersection region must begin at some finite distance above the nozzle exit plane, and the point to be analyzed must be some distance below the nozzle exit plane, the line of sight angle should be sufficient to allow these three circles to adequately describe the blockage. In many cases, in fact, one circle suffices. (In theory, a large number of circles used as above could completely describe an axisymmetric obstruction. In practice, however, only forty equations may be used in the program.)
to describe the whole base or a portion of it if so desired. Therefore, one should not use more circles than are necessary on any one obstruction.

Once the entire base picture has been presented in the simplified terms shown above, the coordinate system is shifted so that it is now centered at the point to be analyzed, the orientation of the axes being as before. Now it is clear that all the obstructions are at positive "z" coordinates and most are also at positive "x" coordinates.

To illustrate this transformation, assume a point \( P \) located at \( (x', 0, z') \) as shown in Figure 6 where \( z = 0 \) is defined to exist at the nozzle exit plane. Assume also a point in an intersection region located at \( (x_i, y_i) \). Its \( z \) coordinate is unimportant. Assume also a circular blockage located at \( (h, k, z_o) \) with a radius \( r \). The equations for this blockage region are

\[
(x - h)^2 + (y - k)^2 \leq r^2, \quad z = z_o
\] (25)

To transfer these equations to a coordinate system centered at \( P \), the following transformation equations are needed. (The subscript 2 refers to the new system.)

\[
x_2 = x - x', \quad y_2 = y, \quad z_2 = z - z'.
\] (26)

Then, the transformed blockage equation is

\[
(x_2 + x' - h)^2 + (y - k)^2 \leq r^2, \quad z_2 = z_o - z'.
\] (27)

Given a line of sight with an angle, \( \beta \), from the horizontal (the new \( xy \) plane) and knowing the \( z \) coordinate of the obstruction (Figure 7), we may calculate the distance (HYP) from \( P \) to the point where the line of sight passes through the plane of the obstruction (\( z_2 = z_o - z' \)).

\[
HYP = \frac{z_2}{\tan (\beta)}.
\] (28)
Also, from the coordinates of the center of the intersection region which must be crossed by the beam, an angle $\theta$ may be calculated as follows:

$$\theta = \tan \frac{|y_i|}{x_i}. \quad (29)$$

With $\theta$ and HYP, the coordinates of the point where the line of sight passes through the intersection region may be calculated.

$$x_o = (\cos \theta) \text{ HYP} \quad (30)$$

$$y_o = (\sin \theta) \text{ HYP.} \quad (31)$$

The sign of $y_o$ is the same as the sign of $y_i$ since P is on the x-axis.

Now, if the calculated values of $x_o$ and $y_o$ satisfy equation (25) the line of sight is blocked. If not, the process is repeated with new parameters. Finally, a new summation of all the unblocked radiation is made to give a better estimate of the environment.

IV. PROGRAM DESCRIPTION

The program is written in Chain Fortran for the GE 225 computer. The use of Chains was caused by the large number of subscripted variables and makes the program a compendium of four smaller programs. Each of these smaller programs contains the coding of the equations contained in one of the four sections just discussed.

This segmenting makes checkout and error investigation quite simple since one can immediately isolate a given section as the one in which the error occurred and restrict his investigation to it.

Each of the four sections follows closely the pattern laid down in the preceding portion of this report. Numerous interpolation, approximation, and averaging methods which are similar to techniques developed in the standard texts [3] are used as needed in the program. No unusual numerical differentiation and integration techniques are employed, and nothing other than simple geometry is used in the various form factor and beam length calculations. In short, the theory presented in the preceding portion of this report is followed closely with little modification.
Figure 8 contains a printout of the program as it is read into the machine. Notice the marginal notations separating the four sections of the program. If each of these sections is compared with the corresponding theory section of this report by a person familiar with Fortran II, there is no reason why he should not be able to follow the programming and to verify his results in any way he wishes.

Figure 9 contains a flow diagram to completely illustrate the flow of control throughout the program.

V. NOTATION

<table>
<thead>
<tr>
<th>Program Symbol</th>
<th>Description and Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J)</td>
<td>Numerical subscript where J has a value from one number to another (i.e., 1 to L).</td>
</tr>
<tr>
<td>(or a similar subscript)</td>
<td></td>
</tr>
<tr>
<td>A(I, J)</td>
<td>Table of input from the method of characteristics.</td>
</tr>
<tr>
<td>(J = 1 to 4)</td>
<td></td>
</tr>
<tr>
<td>A(I, 1)</td>
<td>Distance, nondimensionalized by DIA, measured along the nozzle axial line from the exit plane of that nozzle. The first value must be the initial intersection point.</td>
</tr>
<tr>
<td>A(I, 2)</td>
<td>Mach number corresponding to A(I, 1) but located on the axial line of the intersecting region (not to be confused with critical condition Mach number, ( M^c )).</td>
</tr>
<tr>
<td>A(I, 3)</td>
<td>Flow direction angle corresponding to A(I, 1) but located on the axial line of the intersecting region, in radians.</td>
</tr>
<tr>
<td>A(I, 4)</td>
<td>Radius, nondimensionalized by DIA, of the circle formed by a plane drawn perpendicular to the nozzle axial line and cutting the exhaust plume for a corresponding A(I, 1).</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Description and Units</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>A(I, 1)</td>
<td></td>
</tr>
<tr>
<td>A(I, 2)</td>
<td>Center and radii of the blockage regions located in the (x, y, z) coordinate system illustrated in Figure 6, ft.</td>
</tr>
<tr>
<td>A(I, 3)</td>
<td></td>
</tr>
<tr>
<td>A(I, 4)</td>
<td></td>
</tr>
<tr>
<td>ABSIS</td>
<td>An increment length, nondimensionalized by DIA, that is $1/50$ that of the beam length, FLE(J).</td>
</tr>
<tr>
<td>ADIS(J)</td>
<td>One-half the length, nondimensionalized by DIA, of the major axis of the elliptical plane for the intersecting region.</td>
</tr>
<tr>
<td>ANGLE(J)</td>
<td>The angle formed by the beam and its projection in the base plane, radians.</td>
</tr>
<tr>
<td>AVTEM(J)</td>
<td>Average temperature for the beam length, FLE(J), °R.</td>
</tr>
<tr>
<td>AVP(J)</td>
<td>Average pressure for the beam length, FLE(J), ATM.</td>
</tr>
<tr>
<td>BDIS(J)</td>
<td>One-half the length, nondimensionalized by DIA, of the minor axis of the elliptical plane for the intersecting region.</td>
</tr>
<tr>
<td>BLED(J)</td>
<td>Dimensional value for the beam length, ft.</td>
</tr>
<tr>
<td>C</td>
<td>The angular difference between the shock angle, DELTA, and a corresponding flow direction angle), radians.</td>
</tr>
<tr>
<td>DELI</td>
<td>Minimum shock angle for a given GAMMA and Mach number, FM(J) (zero flow direction angle), radians.</td>
</tr>
<tr>
<td>DELM</td>
<td>Maximum shock angle for a given GAMMA and Mach number, FM(J), radians.</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Description and Units</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>DELTA</td>
<td>Shock angle for a given GAMMA, Mach number, FM(J), and flow direction, T(J), radians.</td>
</tr>
<tr>
<td>DIA</td>
<td>Diameter of the nozzle exit, ft.</td>
</tr>
<tr>
<td>EMIS</td>
<td>Emissivity of the beam length, FLE(J).</td>
</tr>
<tr>
<td>F</td>
<td>Shape factor corresponding to a given beam. (F is the view factor associated with each beam length that radiates energy back to the viewpoint.)</td>
</tr>
<tr>
<td>FINTX</td>
<td>x-coordinate of intersection region center in coordinate system of Figure 6.</td>
</tr>
<tr>
<td>FINTY</td>
<td>y-coordinate of intersection region center in coordinate system of Figure 6.</td>
</tr>
<tr>
<td>FK</td>
<td>The distance, nondimensionalized by DIA, in the base plane from the point of view to the axial line for the intersecting region. (The value of FK is always negative.)</td>
</tr>
<tr>
<td>FLAG</td>
<td>Value, either +1 or 0, instructing computer on how to proceed.</td>
</tr>
<tr>
<td>FLE(J)</td>
<td>Length, nondimensionalized by DIA, of the beam within the bounds of the intersecting region.</td>
</tr>
<tr>
<td>FM(J)</td>
<td>Mach number on the axial line of the intersecting region corresponding to XD(J) (not to be confused with critical condition Mach number, M*).</td>
</tr>
<tr>
<td>FXI</td>
<td>Mole fraction of each of the gas components in the exhaust plume.</td>
</tr>
<tr>
<td>GAMMA</td>
<td>Ratio of specific heats for the exhaust components.</td>
</tr>
<tr>
<td>L</td>
<td>Number of desired data values for XD(J).</td>
</tr>
<tr>
<td>LL</td>
<td>Number of data values input for each A(I, 1), A(I, 2), A(I, 3), and A(I, 4) in Chain I.</td>
</tr>
<tr>
<td>LLLLL</td>
<td>Number of blockage region circles.</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Description and Units</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>MCON</td>
<td>If +1: oblique shock theory temperature and pressure used.</td>
</tr>
<tr>
<td></td>
<td>0: Rankine-Hugonot temperature and Newtonian impact theory pressure used.</td>
</tr>
<tr>
<td></td>
<td>-1: isentropic temperature and Newtonian impact theory used.</td>
</tr>
<tr>
<td>PBPX</td>
<td>Ratio of base pressure to exit pressure.</td>
</tr>
<tr>
<td>PHI</td>
<td>View angle formed by one-half the minor axis, BDIS(J), of the elliptical plane for the intersecting region and a corresponding projection of part of the beam into the same elliptical plane (Figure 6), radians.</td>
</tr>
<tr>
<td>PH2O</td>
<td>Partial pressure for the beam length, FLE(J), ATM.</td>
</tr>
<tr>
<td>PO</td>
<td>Chamber pressure of the nozzle, ATM.</td>
</tr>
<tr>
<td>P(J) and PW</td>
<td>Static pressure for a location in the exhaust plume, ATM.</td>
</tr>
<tr>
<td>PX</td>
<td>x-coordinate of point being analyzed in coordinate system of Figure 5.</td>
</tr>
<tr>
<td>PY</td>
<td>y-coordinate of point being analyzed.</td>
</tr>
<tr>
<td>PZ</td>
<td>z-coordinate of point being analyzed.</td>
</tr>
<tr>
<td>QRAD</td>
<td>Radiant heat flux by the average temperature and pressure method for the beam length, FLE(J), BTU/ft²-sec.</td>
</tr>
<tr>
<td>QRADS</td>
<td>The total accumulated heat flux contribution by average temperature and pressure method for two or more beam lengths, BTU/ft²-sec.</td>
</tr>
<tr>
<td>QRDA</td>
<td>Radiant heat flux by absorption method for a single beam length, FLE(J), BTU/ft²-sec.</td>
</tr>
<tr>
<td>QRDAS</td>
<td>Total accumulated radiant heat flux by absorption method for two or more beam lengths, BTU/ft²-sec.</td>
</tr>
<tr>
<td>R</td>
<td>One-half the distance, nondimensionalized by DIA, between two adjacent nozzle center lines.</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Description and Units</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>RC(J)</td>
<td>One-half the length, nondimensionalized by DIA, of an axis in the elliptical plane for the intersecting region that is formed by a beam where it pierces this region and a point on the axial line of the same region.</td>
</tr>
<tr>
<td>RD(J)</td>
<td>Radius, nondimensionalized by DIA, measured in the same manner as that of A(1, 4) and corresponding to XD(J).</td>
</tr>
<tr>
<td>SABSIS</td>
<td>Total accumulated ABSIS increments to point in question.</td>
</tr>
<tr>
<td>SSF</td>
<td>The total accumulated shape factor for two or more beam lengths.</td>
</tr>
<tr>
<td>SXD</td>
<td>Perpendicular distance, nondimensionalized by DIA, between the base plane and the nozzle exit plane.</td>
</tr>
<tr>
<td>TAB</td>
<td>Table of flow direction angles, radians, obtained from a plot of flow direction angle, shock angle, and Mach number (Figure 13).</td>
</tr>
<tr>
<td>TABL</td>
<td>Table of Mach number values obtained from a plot of flow direction angle, shock angle, and Mach number (Figure 13).</td>
</tr>
<tr>
<td>TDIS(J)</td>
<td>One-half the length, nondimensionalized by DIA, of an axis perpendicular to RC(J) and in the same elliptical plane for the intersecting region.</td>
</tr>
<tr>
<td>TEMP(J) and TW</td>
<td>Static temperature for a location in the exhaust plume, °R.</td>
</tr>
<tr>
<td>THETA</td>
<td>Flow direction for a given GAMMA, Mach number, FM(J), and shock angle, DELTA, radians.</td>
</tr>
<tr>
<td>T(J)</td>
<td>Flow direction angle on the axial line of the intersecting region corresponding to XD(J), radians.</td>
</tr>
<tr>
<td>TO</td>
<td>Chamber temperature of the nozzle, °R.</td>
</tr>
<tr>
<td>Program Symbol</td>
<td>Description and Units</td>
</tr>
<tr>
<td>----------------</td>
<td>---------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>XD(J)</td>
<td>Arbitrary distance, nondimensionalized by DIA, that is measured in the same manner as is A(I, 1). (Simply numbered values for XD(J) make it easier for the user to plot the results. XD(J) can be set equal to A(I, 1) if so desired.)</td>
</tr>
<tr>
<td>XD(L + 1)</td>
<td>Selected value, nondimensionalized by DIA, that is slightly larger than the last input value for XD(J). (The value, XD(L + 1), is necessary so that the solution of shape factor can be completed for the last increment.)</td>
</tr>
<tr>
<td>XO</td>
<td>x-coordinate of point at which line of sight to I.R. pierces plane of obstruction.</td>
</tr>
<tr>
<td>YO</td>
<td>y-coordinate of point at which line of sight to I.R. pierces plane of obstruction.</td>
</tr>
<tr>
<td>ZO</td>
<td>z-coordinate of point at which line of sight pierces plane of obstruction.</td>
</tr>
</tbody>
</table>

VI. INPUT

The program requires as input certain characteristics of the rocket engines on the missile in question, the base geometry and a flow field. Since the input required is quite extensive, it will be treated as three separate topics. In addition, the discussion will be restricted to the input and how it is obtained leaving the actual load format for a separate section.

A. Flow Field Description

The program requires extensive input data to describe the downstream flow field. This input is received in the form of tables of the free exhaust plume properties, Mach number, M, flow angle, θ, and plume radius, R, as functions of axial distance. The axial distance and the plume radius are nondimensionalized by the exit diameter and will hereafter be referred to by their symbolic names X/D and R/D. The Mach number is not the characteristic Mach number $M^*$ but is related to it by

$$M = \sqrt{\frac{2M^*^2}{(\gamma + 1) - (\gamma - 1) M^*^2}}$$  \hspace{1cm} (32)
The angle $\theta$ is expressed in radians. These tables are obtained from a method of characteristics analysis as follows.

In Figure 1 it may be seen that by a two-way interpolation into the method of characteristics output, values of $M$ and $\theta$ may be found along a line of given radial distance from the nozzle centerline. In addition, at the specific $X/D$ location at which the $M$ and $\theta$'s are found, the free plume radius must also be input as shown in the figure. The first $X/D$ chosen must be at the initial intersection point and additional $X/D$'s down the plume may be chosen arbitrarily; however, the bottom number must not exceed 60.

Given now this set of tables of $M$, $\theta$, and $R/D$ as functions of $X/D$, the user next inputs the $X/D$ distances at which he wants his lines of sight to pierce the intersection region. The input fixes the number of increments into which the intersection region will be divided; thus, it will generally be advantageous to input as many as practical. The number of these points is in no way related to the number of points in the above tables except that no more than 60 may be input, and the highest and lowest value must be within the range of the previously input tables.

A review of the oblique shock equations will show that the equation relating the shock angle $\delta$ to the flow direction angle $\theta$ does not allow a $\delta$ to exist for all $\theta$ values. In particular, for a given specific heat ratio and Mach number, there exists a maximum $\theta$ beyond which a corresponding shock angle does not exist. The program is made aware of these limits by inputting a table of Mach numbers and the corresponding maximum $\theta$ values so that it will not try to solve for a nonexistent angle. This table consists of ten values and must be obtained for the correct specific heat ratio as it varies strongly with that parameter.

B. Rocket Motor Input

The given rocket motor must be characterized for the program by inputting its total temperature, total pressure, its exit diameter, the specific heat ratio of its exhaust gases and the pressure ratio (exit pressure to ambient pressure) at which the method of characteristics analysis was run.

C. Base Geometry

Referring to Figure 10, the program needs the distances $SXD$ and $FK$, both in the nondimensionalized (divided by the exit diameter) mode. In addition, the location of the blockage regions and intersection
regions in a coordinate system oriented as described in Section IIID are needed. The dimensions of this coordinate system must be in feet.

In addition to the above input, a number of constants which effect control of the execution cycle are needed and are described later in the load format section.

VII. OUTPUT

The output is limited since execution time is already quite long and excessive print statements only make it longer. As may be seen in Figure 11, the output is arranged in the same logical order as was followed in Section III. The first section is devoted to the temperature and pressure profiles; the second to one radiation method; the third to the absorption method; and the fourth to the blockage correction factors. As may be seen, the total cumulative radiation, as well as the radiation from the individual increments, is summed in each case.

Note that in the blockage correction section each increment is divided into left, middle and right sides. The coordinates at which the line of sight to each of these three points pierces the planes of the various obstructions \((x_0, y_0)\) are printed out. If the line of sight is blocked, the last set of \(x_0, y_0\) coordinates printed out are within a blockage circle. This is shown on the output in Figure 11.

Figure 11 contains the complete output for the sample problem in Section VIIIB.

VIII. LOAD FORMAT AND SAMPLE PROBLEM

A. Load Format

A few basic rules for writing a load format for the GE 225 Fortran are as follows:

1. Fixed point numbers must have an X before the number and must be immediately followed by a comma (i.e., X31,). "X31," will be misinterpreted.

2. Each value must be separated by a comma (i.e., .21, .2646, ...).

3. Every data card must end with an asterisk (i.e., .28, .29, .30*).
4. Floating point numbers can be written either in decimal form without exponent or with exponent (i.e., .000539 or 5.39E-4).

5. Any one data card cannot contain values listed on more than one read statement (RCD).

6. Data must be set up in the same order as it is listed in the read statement (RCD).

7. All floating point numbers must have decimal points (i.e., the expression 31 is not permitted).

These rules must be followed explicitly.

In the following discussion the input will be separated into blocks. Each block of data must go on one or more cards. The first item in each new block must be the first item on a new card; i.e., no card may contain data from two blocks.

a. Block 1

(1) A fixed point number expressing the number of values in the input table of M vs X/D. This number is of course also the number of values in the θ vs X/D table and the R/D vs X/D table and must not exceed 60.

(2) A fixed point number giving the number of X/D locations at which the user desires to increment his intersection region.

(3) A fixed point number with its value determined from the following.

   (a) X + 1 if oblique shock relations are desired in the solution of the temperature and pressure profiles.

   (b) X + 0 if Rankine-Huginot relations are desired.

   (c) X - 1 if isentropic flow relations are needed.

(4) A floating point number giving one-half the distance between the centerlines of the two motors which are generating the intersection region. This number is the actual distance nondimensionalized by the exit diameter of the motor.

(5) The value of the chamber total pressure; atmospheres ~ floating point.
(6) The value of the chamber total temperature; °R ~ floating point.

(7) The exhaust gas specific heat ratio; floating point.

b. **Block 2**

A floating point number expressing the base pressure to exit pressure ratio for which the method of characteristics run was made.

c. **Block 3**

A set of floating point numbers in ascending order expressing the X/D coordinates of the points at which the M and θ will be given.

d. **Block 4**

A set of floating point numbers expressing the Mach numbers corresponding to the X/D locations given in Block 3.

e. **Block 5**

A set of floating point numbers giving the corresponding θ values; radians.

f. **Block 6**

A set of floating point numbers giving the corresponding R/D's. (Note: Together Blocks 3, 4, 5, and 6 give the columns of a table into which the machine may go to interpolate the value of any parameter (M, θ, R/D) at any given X/D value during later execution stages. As a result, extreme care must be taken to assure that proper correspondence is maintained between the elements in the columns.)

g. **Block 7**

A series of floating point numbers giving the X/D locations at which the plume is desired to be incremented. The number of these points is given in Block 1, part 2.

h. **Block 8 and Block 9**

These two blocks contain the maximum values of M and θ, respectively, which are used to prevent the program from solving for a nonexistent shock angle. Referring to Figure 12, notice that for a Mach number 4.0 and a flow angle θ of 55 degrees no shock angle θ exists. Thus, to prevent the program from attempting to solve for shock angles
in cases like this, a table of Mach numbers and the maximum corresponding angles is input. Figure 12 shows such a table for Figure 12. This table must have ten values.

i. Block 10

(1) Exit diameter of the rocket engines used on the base being considered. A floating point number ~ feet.

(2) The percent water vapor in the exhaust plumes. Floating point ~ dimensionless.

(3) A selected X/D location equal to the last X/D location in Block 7 plus .1. Floating point ~ nondimensionalized.

j. Block 11

(1) The distance in the plane of the point being analyzed from the point of view to the axial line of the intersecting region. This value is always given a negative sign. A floating point number ~ nondimensionalized (see Figure 10).

(2) The angle formed by the semi-major axis, BDIS(J), of the elliptical plane for the intersection region and a corresponding projection of part of the beam into the same elliptical plane. A floating point number ~ radians (see Figure 14).

(3) The perpendicular distance between the plane of the point being analyzed and the nozzle exit plane. A floating point number ~ nondimensionalized (see Figure 10).

k. Block 12

(1) The number of blockage circles being considered. Must not exceed 40. A fixed point number ~ dimensionless.

(2) The x-coordinate of the center of the intersection region being considered. A floating point number (including sign) ~ feet.

(3) The y-coordinate of the center of the intersection region being considered. A floating point signed number ~ feet.
1. **Block 13**

A series of numbers expressing the x-coordinates of the blockage circle centers. Floating point signed numbers \(~\) feet.

m. **Block 14**

A series of numbers expressing the y-coordinate of the blockage circle centers. Floating point signed numbers \(~\) feet.

n. **Block 15**

A series of numbers expressing the z-coordinate of the blockage circle centers. Floating point signed number \(~\) feet.

o. **Block 16**

A series of numbers expressing the radius of each of the blockage circles. A floating point number \(~\) feet.

p. **Block 17**

(1) A value, either +1.0 or 0.0 depending upon whether a new intersection region is to be analyzed next or just a new point. This number is a command to the machine.

\[+1.0\text{ if a new point is to be analyzed or if this is the last piece of data.}\]

\[0.0\text{ if a new intersection region is to be considered.}\]

(2) This value is +1.0 if the intersection region being analyzed has an x-coordinate greater than the x-coordinate of the point being considered. If the intersection region has an x-coordinate less than the point being considered, then the value in this position is the x-coordinate. A floating point negative (all x-coordinates are negative according to the defined coordinate system in Section IIID) number \(~\) feet. As clarification, a case where the x-coordinate must be input in this manner is illustrated in Figure 15.
B. Sample Problem

As a sample problem an analysis of the radiation incident on a point on the thrust structure of the S-II stage from two of the eight I.R.'s will be made. Figure 16 shows the base with some of the necessary geometrical input. Figure 17 shows the same base drawn in the necessary coordinate system to input the coordinates of the intersection regions and blockage regions. Other necessary input data concerning the engine and other features of the base are:

Chamber Total Pressure: 43.004899 atmospheres.
Chamber Total Temperature: 5760 °R
Exit Diameter: 6.6666 feet
Distance between Engine Centerlines: 12.37 feet
Specific Heat Ratio of Exhaust Gases: 1.23
Percent Water Vapor in Exhaust Gases: 63% = .63.

Now a method of characteristics analysis of a J-2 engine free plume expansion into quiescent air must be obtained. From this we develop the following table.

**TABLE I**

(Note that the axial distance and radius have already been divided through by the exit diameter, 6.6666 ft.)

<table>
<thead>
<tr>
<th>A X/D</th>
<th>B M</th>
<th>C Theta - θ</th>
<th>D Free Plume Expansion Radius - R/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>.55300</td>
<td>5.55</td>
<td>.59932</td>
<td>.932</td>
</tr>
<tr>
<td>.60577</td>
<td>5.678</td>
<td>.61166</td>
<td>.971</td>
</tr>
<tr>
<td>.68793</td>
<td>6.121</td>
<td>.69823</td>
<td>1.022</td>
</tr>
<tr>
<td>.75513</td>
<td>6.027</td>
<td>.65766</td>
<td>1.062</td>
</tr>
<tr>
<td>.82400</td>
<td>6.01</td>
<td>.63600</td>
<td>1.103</td>
</tr>
<tr>
<td>.87308</td>
<td>5.89</td>
<td>.60129</td>
<td>1.131</td>
</tr>
<tr>
<td>.91254</td>
<td>5.874</td>
<td>.58471</td>
<td>1.456</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
<td>----</td>
<td>------------------------</td>
</tr>
<tr>
<td>X/D</td>
<td>M</td>
<td>Theta - θ</td>
<td>Free Plume Expansion Radius (R/D)</td>
</tr>
<tr>
<td>.96408</td>
<td>5.840</td>
<td>.56404</td>
<td>1.182</td>
</tr>
<tr>
<td>1.05794</td>
<td>5.80</td>
<td>.53164</td>
<td>1.242</td>
</tr>
<tr>
<td>1.09118</td>
<td>5.78</td>
<td>.51986</td>
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<tr>
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<td>5.723</td>
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<td>1.323</td>
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<tr>
<td>1.27162</td>
<td>5.715</td>
<td>.46950</td>
<td>1.355</td>
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<tr>
<td>1.33487</td>
<td>5.710</td>
<td>.45436</td>
<td>1.388</td>
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<tr>
<td>1.43185</td>
<td>5.69</td>
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<tr>
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<td>5.69</td>
<td>.41471</td>
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<tr>
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<td>5.69</td>
<td>.38945</td>
<td>1.561</td>
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<td>5.70</td>
<td>.37429</td>
<td>1.609</td>
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<td>5.705</td>
<td>.36038</td>
<td>1.660</td>
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<td>5.705</td>
<td>.34923</td>
<td>1.710</td>
</tr>
<tr>
<td>2.10755</td>
<td>5.715</td>
<td>.33942</td>
<td>1.743</td>
</tr>
<tr>
<td>2.20516</td>
<td>5.718</td>
<td>.33107</td>
<td>1.783</td>
</tr>
<tr>
<td>2.31099</td>
<td>5.718</td>
<td>.32418</td>
<td>1.825</td>
</tr>
<tr>
<td>2.39077</td>
<td>5.716</td>
<td>.32047</td>
<td>1.851</td>
</tr>
<tr>
<td>2.40765</td>
<td>5.701</td>
<td>.32655</td>
<td>1.862</td>
</tr>
<tr>
<td>2.51795</td>
<td>5.705</td>
<td>.32794</td>
<td>1.902</td>
</tr>
<tr>
<td>2.6743</td>
<td>5.717</td>
<td>.31689</td>
<td>1.955</td>
</tr>
<tr>
<td>2.82225</td>
<td>5.72</td>
<td>.30810</td>
<td>2.005</td>
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<tr>
<td>2.96077</td>
<td>5.722</td>
<td>.30217</td>
<td>2.050</td>
</tr>
<tr>
<td>3.08055</td>
<td>5.725</td>
<td>.29927</td>
<td>2.086</td>
</tr>
<tr>
<td>3.19207</td>
<td>5.719</td>
<td>.29961</td>
<td>2.117</td>
</tr>
<tr>
<td>3.35044</td>
<td>5.685</td>
<td>.31006</td>
<td>2.165</td>
</tr>
</tbody>
</table>
It has been decided to break up the plume into increments at the following X/D locations:

\[ .56, .60, .7, .8, .9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, \text{and} \ 3.3. \]

In addition, the graph shown in Figure 12 has been made and the values shown in Figure 13 have been chosen as the maximum Thetas and Mach numbers. With this data, the first part of the program may be loaded.

1. **Block 1**
   a. There are 32 figures in each column of Table I. The value loaded here is X32.
   
   b. According to the X/D locations shown above, the region is to be incremented at 25 X/D's. Value loaded is X25.
   
   c. Oblique shock theory is desired. X + 1.
   
   d. One half the distance between rocket engine centerlines divided by the exit diameter is .92773.
   
   e. Value is 43.004899.
   
   f. Value is 5760.0.
   
   g. Value is 1.23.

2. **Block 2**
   The pressure ratio for this run was .0185.

3. **Block 3**
   Load here column A of Table I in order going down, taking as many cards as needed.

4. **Block 4**
   Load here column B.
5. **Block 5**
   Load here column C.

6. **Block 6**
   Load here column D.

7. **Block 7**
   Load here the 25 values of $X/D$ locations listed on page 28.

8. **Block 8**
   Load here column A of Figure 13.

9. **Block 9**
   Load here column B of Figure 13.

10. **Block 10**
    a. The exit diameter of the engine is 6.666 feet.
    b. Value is .63.
    c. Since the last value in column A of the input table is 3.35044, the value loaded here is 3.45044.

11. **Block 11**
    a. Referring to Figure 16 and remembering to nondimensionalize by division by the exit diameter, the value loaded here is - .81404.
    b. From the same figure, the value (in radians) loaded is 1.57079.
    c. Value here is 2.09559.

Now another table is made. Referring to Figure 17, locate the blockage regions (which are the heat shield and the five motors) and prepare the following table.
TABLE II

<table>
<thead>
<tr>
<th>Blockage</th>
<th>x-coordinate</th>
<th>y-coordinate</th>
<th>z-coordinate</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>Heat Shield</td>
<td>0.0</td>
<td>0.0</td>
<td>-3.4</td>
<td>10.25</td>
</tr>
<tr>
<td>Engine I</td>
<td>-6.19</td>
<td>6.19</td>
<td>0.0</td>
<td>3.33</td>
</tr>
<tr>
<td>Engine II</td>
<td>6.19</td>
<td>6.19</td>
<td>0.0</td>
<td>3.33</td>
</tr>
<tr>
<td>Engine III</td>
<td>6.19</td>
<td>-6.19</td>
<td>0.0</td>
<td>3.33</td>
</tr>
<tr>
<td>Engine IV</td>
<td>-6.19</td>
<td>-6.19</td>
<td>0.0</td>
<td>3.33</td>
</tr>
<tr>
<td>Engine V</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Also it is noted that the intersection region is located at X = -6.19, y = 0.0. Now this new data is loaded.

12. **Block 12**
   
a. There are six blockage regions so X6 is loaded here.
b. -6.19 is the value as noted above.
c. 0.0 is loaded here.

13. **Block 13**
   
Load here column A of Table II.

14. **Block 14**
   
Load here column B of Table II.

15. **Block 15**
   
Load here column C of Table II.

16. **Block 16**
   
Load here column D of Table II.

17. **Block 17**
   
a. Since a new intersection region is to be analyzed the first tendency would be to load 0.0 in this place.
However, it is apparent that actually the new intersection region is formed by two motors separated by the same distance as before and formed by engines with the same characteristics as before. Thus, in effect the same intersection region is being analyzed but from a different point. 1.0 is input here. This tells the machine that it need not calculate new temperature and pressure profiles and that it may start execution with Chain II instead of Chain I.

b. 1.0 is input here because clearly the intersection region is at a larger x-coordinate than is the point.

The input for the first intersection region analysis is now complete. Directly behind it the data for the second intersection region is loaded. Of course, since the machine already has in memory the necessary intersection region data, Blocks 1 through 9 need not be input again. Starting with Block 10 and proceeding in the same manner as before the following is loaded:

18. Block 10
   a. 6.6667
   b. .63
   c. 3.35044.

19. Block 11
   a. -1.97363
   b. .4895
   c. 2.09559.

20. Block 12
   a. X6
   b. 0.0
   c. +6.19.
21. Block 13
Column A, Table II.

22. Block 14
Column B, Table II.

23. Block 15
Column C, Table II.

24. Block 16
Column D, Table II.

25. Block 17
a. Load here now 1.0. The machine will continue to execute, will try to read data for Block 10 and finding none, will stop.

b. 1.0.

Figure 18 presents the input for this problem as it appears on the input cards.

IX. CONCLUSIONS AND RECOMMENDATIONS

1. This program may be used to estimate radiant heat flux rates to various missile bases from intersection regions produced by H2-O2 engines.

2. Possible inaccuracies can exist in the calculated intersection region properties, the emissivity data, or in the application of equation (15).

3. Other inaccuracies arise because of the fact that second and higher order intersections are ignored, radiation from plume to plume is ignored, and blockage by intervening gas masses is ignored.

4. Further work is needed in all the areas mentioned in 2 and 3 above. In particular, an accurate method to calculate the region properties and an accurate method of accounting for second order regions is essential before accurate estimates can be made.
5. Measurements, preferably inflight, must be made before these analytical approaches can be completely trusted. Instrumentation must be developed that will measure these low transient heat flux rates.
FIGURE 1. INTERPOLATED MACH NUMBER (M) AND FLOW DIRECTION ANGLE (θ) ALONG A LINE OF CONSTANT RADIUS WITHIN THE EXHAUST PLUME
FIGURE 2. GEOMETRY OF THE CROSS SECTION OF THE INTERSECTION REGION
FIGURE 3. MODEL OF SEMI-MINOR AXIS PROFILE OF THE INTERSECTION REGION
FIGURE 4. GEOMETRY FOR DETERMINING THE SEMI-MINOR AXIS PROFILE OF THE INTERSECTION REGION CROSS SECTION
FIGURE 5. SKETCH OF TYPICAL INCREMENT RADIATING TO A POINT ON THE BASE
FIGURE 6. RIGHT-HANDED CARTESIAN COORDINATE SYSTEM USED TO LOCATE POINTS ON BASE
Line of Sight. HYP is located directly below this line in a plane parallel to xy plane.

Plane of Obstruction

\( (x_1, y_1) \)

\( (x_0, y_0, z_0) \)

\( P (x', 0, z') \)

FIGURE 7. PLANE OF OBSTRUCTION GEOMETRY
FIGURE 8. RADIATION PROGRAM PRINTOUT
49    FM(J) = A(1,2)  
      T(J) = A(1,3)  
      RD(J) = A(1,4)  

52    IF (1,6 - GAMMA) 61, 63, 63  
61    PRINT 220, GAMMA  
     GO TO 71  
63    DO 70 K = 1, 10  
84    IF (TABLE(K) = FM(J)) 70, 7001, 7001  
70    CONTINUE  
    PRINT 221  
    GO TO 72  
7001  IF (TABLE(K) - T(J)) 71, 70, 76  
71    PRINT 222  
     IF (J - 1) 78, 78, 72  
78    T(J) = A(1,3)  
79    MCON = 0  
80    DELTA = 1.1344  
     GO TO 145  
84    AA = GAMMA + 1.  
     AC = FM(J)/Q  
     AAA = GAMMA + 1.  
     BB = FM(J) / Q  
     CC = 1. / (GAMMA * FM(J) ** Q)  
     DELI = ASINF(1. * FM(J))  
     DLMIL = CC * AA * AC ** 2 - 1. * (AA + 1. + AA * BB + AA * AC ** 1. ** 2.5)  
     DELM = ASINF(DLMIL ** 5)  
     DUDE = DELM - DELI  
     GO TO 100  
99    DDEL = T(J) / THETA * DDEL  
100   DELTA = DELI + DDEL  
     THET = TANF(DELTA) ! (AA + 1. / (FM(J) * SINF(DELTA)) ** Q - 1. * 1. - 1.)  
     THETA = ATANF(1. / THET)  
     IF (ABS(THETA - T(J)) - 0001) 195, 195, 120  
120   GO TO 99  
145   IF (MCON) 146, 157, 174  
146   IF (J - 1) 147, 147, 148  
147   PRINT 207  
148   CP = Z * SINF(T(J)) ** Q  
     ORP = (CP * (GAMMA / Q) ** FM ** 2. ** 1.)  
     OPC = (GAMMA - 1.) / Q  
     ODP = 1. + OPC ** FM ** 1.  
     OPC = ODP ** (GAMMA / (GAMMA - 1.))  
     SAV = ODP  
84    P(J) = PO * SAV  
84    TEMP(J) = T(J) * SAV ** ((GAMMA - 1.) / GAMA)  
84    PRINT 208 * XO(J) * FM(J), T(J), P(J), TEMP(J)  
     GO TO 190  
157   IF (J - 1) 158, 158, 129  
158   PRINT 210  
159   CP = Z * SINF(T(J)) ** Q  
     OPC = (CP * (GAMMA / Q) ** FM ** 2. ** 1.)  
     OPC = (GAMMA - 1.) / Q  
     ODP = 1. + OPC ** FM ** 1.  

FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
START
CHAIN
COMMON L*GAMMA*, XD(60), FR(60), T(60), RU(60)
COMM0N P(60), TEMP(60), DD(60), AUS(60)
COMM0N FG(60), ANGLE(60), FLE(60), PHN=0.16, EMIC(60), DIA=0.1, FX1
COMM0N FK*, SXD, RHAT(60)
DIMENSION RC(60), UHIS(60), RAIN(60), EMIC(60), DFD(60), ARF(60)
DIMENSION XDU(60), FNRC(60)
RC=A, DIA=FX1, XD(J+1)
RC=FK*, PHN=FX1
UG=2
VG=SGFPHI**Q
VGG=CSFPHI**Q
DO 2959 J=1,L
V=ADIS(J)*HDIS(J)
IF (V=ADIS(J)**Q
VL=HDIS(J)**Q
RC(J)=VC*(SGRTF/1, (VE*VG+VF*VGG))
TDIS(J)=VC*(SGRTF/1, (VE*VGG+VF*VG))
2959 CONTINUE
PRINT 252
PRINT 426
FOMAT(131X)HCNAM (C)/*
PRINT 263
FOMAT(30X)HFNAM
DO 267 J=1,L
PRINT 267, J
FOMAT(35X)HJN=, I(J)
PRINT 269, J
FOMAT(30X)HJD(J)=E13+5//
IF (HJN(J)=RC(J)) <89, 371, 75
271 FLE(J)=50, 0
ANGLE(J)=1, 57079637
GO TO 297
275 UA=ABS(FK)*-RC(J)
UB=KD(J)+UX
ANG(J)=ATAN(UA/UB)
XDF(J)=KD(J)+TANF(ANGLE(J))*RC(J)
DO 281 IF(J+1)
IF (XDF(J)=KD(J)) 291, 291, 291
281 CONTINUE
FLE(J)=(RC(J)+RCL(J))/COSF(ANGLE(J))
GO TO 294
285 FLE(J)=25,
UB=KD(J)+UX
UC=RC(J)*AABS(FK)
ANGLE(J)=ATAN(UA/UC)
GO TO 297
291 IF (XDF(J)=KD(J)) 293, 291
FLE(J)=(RC(J)+FNRC(J))/COSF(ANGLE(J))
294 IF (FLE(J)=50,) 297, 297, 297
295 FLE(J)=50,
297 XD=SGF(ANGLE(J))*FLE(J)
XD=XDF(J)+X01
DO 301 IF(J+1)
IF (XD(J)=XDS) 301, 314, 314
301 CONTINUE
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
FIGURE 8. RADIATION PROGRAM PRINTOUT (Continued)
52

FIGURE 8. RADIATION PROGRAM PRINTOUT (Concluded)
FIGURE 9. FLOW DIAGRAM
FIGURE 9. FLOW DIAGRAM (Continued)
FIGURE 9. FLOW DIAGRAM (Continued)
FIGURE 9. FLOW DIAGRAM (Continued)
FIGURE 9. FLOW DIAGRAM (Continued)
FIGURE 9. FLOW DIAGRAM (Continued)
FIGURE 9. FLOW DIAGRAM (Concluded)
FIGURE 9. FLOW DIAGRAM (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Continued)
FIGURE 11. RADIATION PROGRAM, SAMPLE PRINT OUT (Concluded)
FIGURE 12. SHOCK ANGLE, DELTA, VERSUS FLOW DIRECTION, T(J), FOR SELECTED MACH NUMBERS, FM(J), USING OBLIQUE SHOCK THEORY
<table>
<thead>
<tr>
<th>Mach Number (FM(J))</th>
<th>Theta (Radians)</th>
<th>Theta (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>.02827</td>
<td>1.62</td>
</tr>
<tr>
<td>1.4</td>
<td>.18325</td>
<td>10.5</td>
</tr>
<tr>
<td>1.6</td>
<td>.28796</td>
<td>16.5</td>
</tr>
<tr>
<td>1.8</td>
<td>.38045</td>
<td>21.8</td>
</tr>
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<td>2.0</td>
<td>.45724</td>
<td>26.2</td>
</tr>
<tr>
<td>2.4</td>
<td>.57417</td>
<td>32.9</td>
</tr>
<tr>
<td>3.0</td>
<td>.68935</td>
<td>39.5</td>
</tr>
<tr>
<td>4.0</td>
<td>.79232</td>
<td>45.4</td>
</tr>
<tr>
<td>5.0</td>
<td>.84293</td>
<td>48.3</td>
</tr>
<tr>
<td>10.0</td>
<td>.91972</td>
<td>52.7</td>
</tr>
</tbody>
</table>

NOTE: Values at 1.4, 1.8, 5.0, and 10.0 are interpolated from Figure 12. Notice that because of the purpose of the table our values for the maximum Theta at any given Mach number are on the low side of the actual maximum.

FIGURE 13. MAXIMUM VALUES OF THETA FOR SELECTED MACH NUMBERS
NOTE: This is a typical intersection region (I.R.). The region minor axis is on a line joining the rocket engine centers and the major axis is perpendicular to it as shown.

NOTE: PHI must never exceed 1.57079 radians. It is always the smallest angle between FK and BDIS.

FIGURE 14. GEOMETRY OF INTERSECTION REGION LOCATION
NOTE: $-X_2 > -X_1$ but $-X_2 < 0$ and $-X_2 < X_3$. Thus $-X_2$ must be input to analyze I.R.(1) but not to analyze I.R.(2), I.R.(3), or I.R.(4).

I.R.(3) located at $(X_3, 0)$

I.R.(2) located at $(0, Y_1)$

I.R.(4) located at $(0, -Y_3)$

Rocket Engine,

Point to be analyzed located at $(-X_2, 0)$

I.R.(1) located at $(-X_1, 0)$

FIGURE 15. BASE GEOMETRY FOR SAMPLE PROBLEM ~ CASE IN WHICH THE X-COORDINATE MUST BE INPUT
The heat shield and engines are not pictured here to avoid confusion.

FIGURE 16. BASE GEOMETRY FOR SAMPLE PROBLEM
FIGURE 18. CHAIN INPUT DATA FOR INTERSECTION REGIONS
FIGURE 18. CHAIN INPUT DATA FOR INTERSECTION REGIONS (Concluded)
FIGURE 19. GEOMETRY OF SEGMENTED INTERSECTION REGION USED TO DETERMINE FORM FACTOR (SEE APPENDIX A)
APPENDIX A

With reference to Figure 19, the form factor $F$ from the region to area $dA_1$ is given by Reference 2 to be

$$F_{dA_1-A_2} = \oint_c \frac{(y_2 - y_1) \, dx_2 - (x_2 - x_1) \, dy_2}{2\pi r^2}. \tag{33}$$

Since $A_2$ is assumed parallel to the xz plane then $dy_2 = 0$ and

$$F_{dA_1-A_2} = \oint_c \frac{(y_2 - y_1) \, dx_2}{2\pi r^2}. \tag{34}$$

By choosing $dA_1$ at the origin

$$y_1 = x_1 = z_1 = 0. \tag{35}$$

Then,

$$F_{dA_1-A_2} = \frac{K}{2\pi} \oint_c \frac{dx_2}{r^2}. \tag{36}$$

where

$$K = y_2. \tag{37}$$

Now the magnitude of the radius vector, $r$, is defined by

$$r^2 = x_2^2 + y_2^2 + z_2^2 = x_2^2 + K^2 + z_2^2 \tag{38}$$
but
\[ z_2 = m x_2 + b. \]  

Therefore,
\[ r^2 = (1 + m^2) x_2^2 + (2mb)x_2 + (K^2 + b^2) \]  

and
\[ F_{dA_1-A_2} = K \frac{2\pi}{c} \oint \frac{dx_2}{(1 + m^2) x_2^2 + (2mb)x_2 + (K^2 + b^2)}. \]  

Integrating and proceeding around the trapezoid in the direction indicated yields
\[ I_2 = \frac{K}{2\pi} \left( \frac{1}{\sqrt{K^2 + (b_1 + b_2)^2}} \right) \tan^{-1} \left( x \sqrt{\frac{1}{K^2 + (b_1 + b_2)^2}} \right) \]  

and
\[ I_4 = \frac{K}{2\pi} \left( \frac{1}{\sqrt{K^2 + (b_1 + b_2 + b_3)^2}} \right) \tan^{-1} \left( x \sqrt{\frac{1}{K^2 + (b_1 + b_2 + b_3)^2}} \right). \]  

and
\[ F_{dA_1-A_2} = I_2 + I_4. \]
REFERENCES


A FORTRAN PROGRAM TO CALCULATE THE THERMAL RADIATION TO THE BASE OF A MULTI-ENGINE SPACE VEHICLE

By E. R. Heatherly, M. J. Dash, G. R. Davidson and C. A. Rafferty

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This document has also been reviewed and approved for technical accuracy.

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