X-524-64-205

NASA TMX-55129

FACILITY FORM 602

N65 18254
(ACCESSION NUMBER)

20
(PAGES)

TMX-55129
(NASA CR OR TMX OR AD NUMBER)

(THRU)

1
(CODE)

16
(CATEGORY)

GEOS-1 LASER RETROREFLECTOR DESIGN AND PRELIMINARY SIGNAL CALCULATIONS

BY
H. H. PLOTKIN

GPO PRICE $ ________

OTS PRICE(S) $ ________

Hard copy (HC) $1.00

Microfiche (MF) $0.50

JULY 1964

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

GEOS-1 LASER RETROREFLECTOR DESIGN AND PRELIMINARY SIGNAL CALCULATIONS

July 1964

Approved: Robert J. Coates

Robert J. Coates
Chief, Advanced Development Division

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

GEOS-I LASER RETROREFLECTOR DESIGN AND PRELIMINARY SIGNAL CALCULATIONS

Introduction

GEOS will be attitude stabilized along the gradient of the gravitational field, so that the normal to the bottom plane surface will point toward the center of the earth. On this bottom plane will be mounted an array of cube-corner prisms, each with an entrance aperture approximately 1 inch in diameter. These will form a retro-reflecting surface and will enhance the reflected energy received in the vicinity of a ground laser source directed at the satellite. The aims of this paper are (a) to discuss the advisability of tilting the symmetry axes of the cube-corners at various angles to the plane normal in an attempt to equalize the effective reflecting area presented to rays incident over a large range of angles, and (b) to perform preliminary signal calculations which would be useful for the design of ground stations.

Variation of Effective Reflecting Area with Angle of Incidence

As a satellite moves over the laser reflection station during a pass, the angle between the line of sight and the satellite symmetry axis will change, as shown in Fig. 1. The individual cube-corner prisms will be identical to those used on the S-66 Beacon Explorer Satellite in which the corners of the triangular front face have been cut off to form an aperture in the form of a regular hexagon. For those prisms, the effective reflecting area has previously been calculated by P. O. Minott (Monthly Research and Advanced Technological Development Activity Report for April 1963). In addition to the angle of incidence, α , the variation of area depends upon the width across flats, w, the depth of the prism, d, (which can be expressed in terms of w) and its index of refraction, N. When rotated about an axis parallel to one of the sides of the hexagon, the expression for the effective area can be written

$$A = (w \tan 30^{\circ}) \cos\alpha \left[(w - 2d \tan\alpha') + (w - 2d \tan\alpha')^2 \frac{\sin 30^{\circ}}{w} \right]$$

$$\text{where } \alpha' = \arc \sin \left(\frac{\sin\alpha}{N} \right) \qquad (1)$$

A plot of the relative projected area versus angle of incidence is presented in Fig. 2, in which the area at normal incidence has been normalized to unity. The curve resulting from rotation about a corner of the hexagon instead of a side does not vary significantly from this.

In order to maximize the angle of the cone about zenith in which a ground station would find effective laser tracking possible, it has been suggested that the GEOS reflector consist of an array of cube corners with their axes tilted at a variety of angles with respect to the satellite symmetry axis. Thus, if the distribution of tilt angles were chosen properly, it was hoped that the satellite might present a reasonably large effective area even when the zenith distance, θ, was relatively large. Further consideration of the proposal shows that there is little or nothing to be gained by pursuing this approach.

Suppose the zenith distance were such that the line of sight made an angle α_1 with the normal to the bottom surface. Then, if all the cube corners were parallel and pointing down, the relative projected area would be f_1 (see Fig. 3). Now, if a group of prisms were tilted so as to point closer to the line of sight, making an angle α_2 with the laser beam, these reflectors would have a larger effective relative area f_2. At the same time, it is clear that another equal group of prisms would have to be tilted away from the line of sight by the same angle in order to maintain rotational symmetry about the satellite axis. These reflectors would present the smaller relative area f_3. Where the function $f(\alpha)$ is linear, the loss of area in the group tilted away from the laser just offsets the gain experienced by those tilted in the favorable direction. Where the second derivative of $f(\alpha)$ is negative (near zenith), the result is a net loss of reflecting area, and where the second derivative of $f(\alpha)$ is positive, (nearer to the horizon) there is a net gain.

The small improvement in effective area that might be experienced at large zenith distances by distributing the cube corner directions in this manner does not contribute appreciably to the usefulness of the satellite because other parameters which also determine the signal strength will be found to vary in a much more significant way. In particular, the received energy falls off very rapidly as the satellite moves away from zenith because of increasing range and increasing atmospheric attenuation. In addition, any attempt to tilt the cube corners without lifting them out of the plane of the bottom surface would result in shadowing effects and/or degraded geometrical packing efficiency. Our conclusions therefore are to arrange as many reflectors as possible into a mosaic on the available area, with all of them pointing parallel to the gravitational gradient.

Reflected Signal Vs. Zenith Angle

We use the following parameters:

E_t = the energy in the transmitted laser pulse

δ_t = half-angle of the transmitted beam divergence cone. We assume the energy is uniformly distributed over this cone.

$A_o f(\alpha)$ = effective retro-reflecting area of the array, for light incident at an angle α to the symmetry axis. A_o is the maximum projected area and $f(o) = 1$. Fig. 2 is a plot of $f(\alpha)$.

ρ = effective reflectivity

δ_c = half-angle of the reflected beam divergence. Again, we assume that the energy is uniformly distributed in this cone.

$\tau(\theta)$ = one-way transmission through the atmosphere for light at zenith distance θ.

a = slant range from observer to satellite

d = receiving telescope aperture

η = receiving telescope transmission.

The ratio of received signal to transmitted pulse energy is then given by

$$\frac{Es}{E} = \frac{A_o f(\alpha)\left[\tau(\theta)\right]^2 \eta d^2}{4\pi \delta_t^2 \delta_c^2 a^4} = KG(\theta) \tag{2}$$

$$\text{where } K = \frac{\rho A_o \eta d^2}{4\pi \delta_t^2 \delta_c^2} \tag{3}$$

is a constant, and

$$G(\theta) = \frac{f(\alpha)\left[\tau(\theta)\right]^2}{a^4} \tag{4}$$

is a variable over the duration of a pass since it is a function of zenith distance θ. From Fig. 1 and the law of cosines,

$$a = -R\cos\theta + \left[R^2\cos^2\theta + h(2R+h)\right]^{1/2} \tag{5}$$

From the law of sines,

$$\sin\alpha = \frac{R}{R+h}\sin\theta \tag{6}$$

These equations enable us to calculate expected signal levels for a satellite at different positions in the sky.

Variation of Atmospheric Transmission

To estimate the effect of zenith angle on atmospheric transmission, we assume the very rough model shown in Fig. 4. The atmosphere here is a uniformly dense layer of thickness T. A light beam passing once through the entire atmosphere in a vertical direction will be attenuated so that its intensity is reduced to a fraction τ_o of its initial value. At a zenith angle θ, the atmospheric path length is $t(\theta)$ and the one-way transmission is

$$\tau(\theta) = \tau^{t(\theta)/T} \tag{7}$$

$$\text{where } t(\theta) = \left\{-R\cos\theta + \left[R^2\cos^2\theta + T(2R+T)\right]^{1/2}\right\} \tag{8}$$

Note that in the limit of a flat earth ($R \rightarrow \infty$), equation (6) may be written

$$t(\theta) = R\cos\theta\left\{\left[1 + \frac{2T}{R\cos^2\theta} + \frac{T^2}{R^2\cos^2\theta}\right]^{1/2} - 1\right\}$$

$$= R\cos\theta\left\{1 + \frac{T}{R\cos^2\theta} + \frac{T^2}{R^2}\left(\frac{1}{2\cos^2\theta} - \frac{1}{2\cos^4\theta}\right) + \cdots - 1\right\}$$

$$t(\theta) \doteq T\sec\theta \tag{9}$$

and

$$\tau(\theta) \doteq \tau_o^{\sec\theta} \tag{10}$$

This is the usual expression used by astronomers for atmospheric transmission. It is only accurate for zenith angles such that

$$\left|\frac{T^2}{R^2}\left(\frac{1}{2\cos^2\theta} - \frac{1}{2\cos^4\theta}\right)\right| < 0.1\frac{T}{R\cos^2\theta}$$

or

$$\sec^2\theta < 1 + 0.2\frac{R}{T} \tag{11}$$

If we take for the thickness of our model atmosphere $T = 25$ km, and for the radius of the earth $R = 6,310$ km, we find that the condition in eq. (11) is satisfied by

$$\theta < 80^\circ \tag{12}$$

Since we will never operate at zenith angles even as large as this, we may as well use eq. (10) instead of the more complicated expressions (7) and (8).

The value of τ_o is chosen to be 0.7, from several conservative estimates in the literature. Fig. 5 is the resulting variation of atmospheric transmission with zenith angle.

Example of Numerical Signal Calculations

The expected orbit for GEOS-I is one with a perigee of 600 nautical miles and an apogee of about 1200 nautical miles. We choose an altitude of 1500 km (about 800 nautical miles) as a typical example because it results in fairly large off-axis incidence angles α so we can study the drop-off in effective area. The corresponding relation between α and zenith angle θ is shown in Fig. 6. When the effective reflecting area is then plotted as a function of θ instead of α, as required in eq. (4), it appears as in Fig. 7. The last of the variable factors, slant-range, a, is plotted in Fig. 8.

We next turn to a calculation of the constant K in eq. (3). The specification on the cube corner reflectors is that 50% of the light incident on the effective reflecting area shall be retro-reflected within a cone 20 seconds of arc in diameter. We thus have

$$\rho = 0.5$$
$$\delta_c = 10 \overset{\frown}{\sec} = 5 \times 10^{-5} \text{ radians.}$$

The transmitted laser beam will be taken with a divergence equal to that planned for S-66 since it seems to be a good compromise between aiming ability and energy concentration.

$$\delta_t = 5 \times 10^{-4} \text{ radians}$$

The receiving telescope is assumed to have an aperture with a diameter

$$d = 16 \text{ inches} = 0.4064 \text{ meters}$$

and the transmission of the receiving optics is

$$\eta = 0.5$$

The reflector arrays will be fitted into four flat panels which take on the trapezoidal shapes shown in Fig. 9. Around the perimeter of each panel will probably be a wasted strip about 1/2 inch wide because of the hexagonal shape of the reflectors. The useable area of these panels adds up to about

$$A_o = 0.178 \text{ sq. meters.}$$

When the appropriate values are inserted into eq. (3) we obtain

$$K = 0.9358 \text{ km}^4$$

The ratio of received to transmitted energy from eq. (2) is plotted in Fig. 10. The scale on the right has been converted into received photons at the sensor, per joule of energy in the transmitted ruby laser pulse.

Using a photomultiplier detector we have found that 10^4 photons results in a good signal, even against a bright sky, with proper filtering. This would allow us, from Fig. 10, to get range information from GEOS down to a zenith angle of at least 53°, with much better signals near zenith. For other geometrical arrangements, other detectors and lasers, similar calculations may be performed. However, it is already clear from the foregoing that the laser reflection panels on GEOS will be very valuable for range and photographic tracking with geodetic precision.

ACKNOWLEDGMENT

Much of this paper developed from valuable discussions with William M. Meyers of General Electric Missile and Space Division.

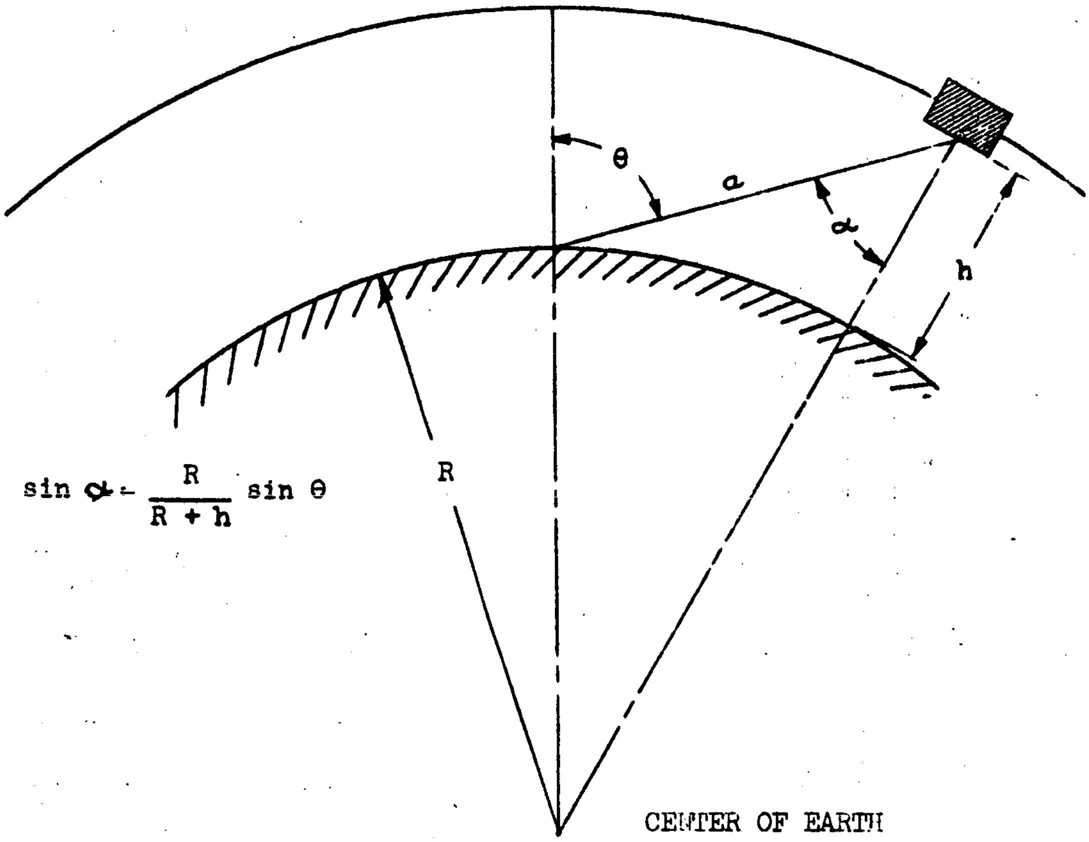


FIGURE 1

GEOMETRY OF A SATELLITE PASS

$f(\alpha)$ VS α

RELATIVE EFFECTIVE AREA OF A SINGLE FUZED QUARTZ CUBE-CORNER AS A FUNCTION OF THE ANGLE BETWEEN INCIDENT RAY AND SYMMETRY AXIS

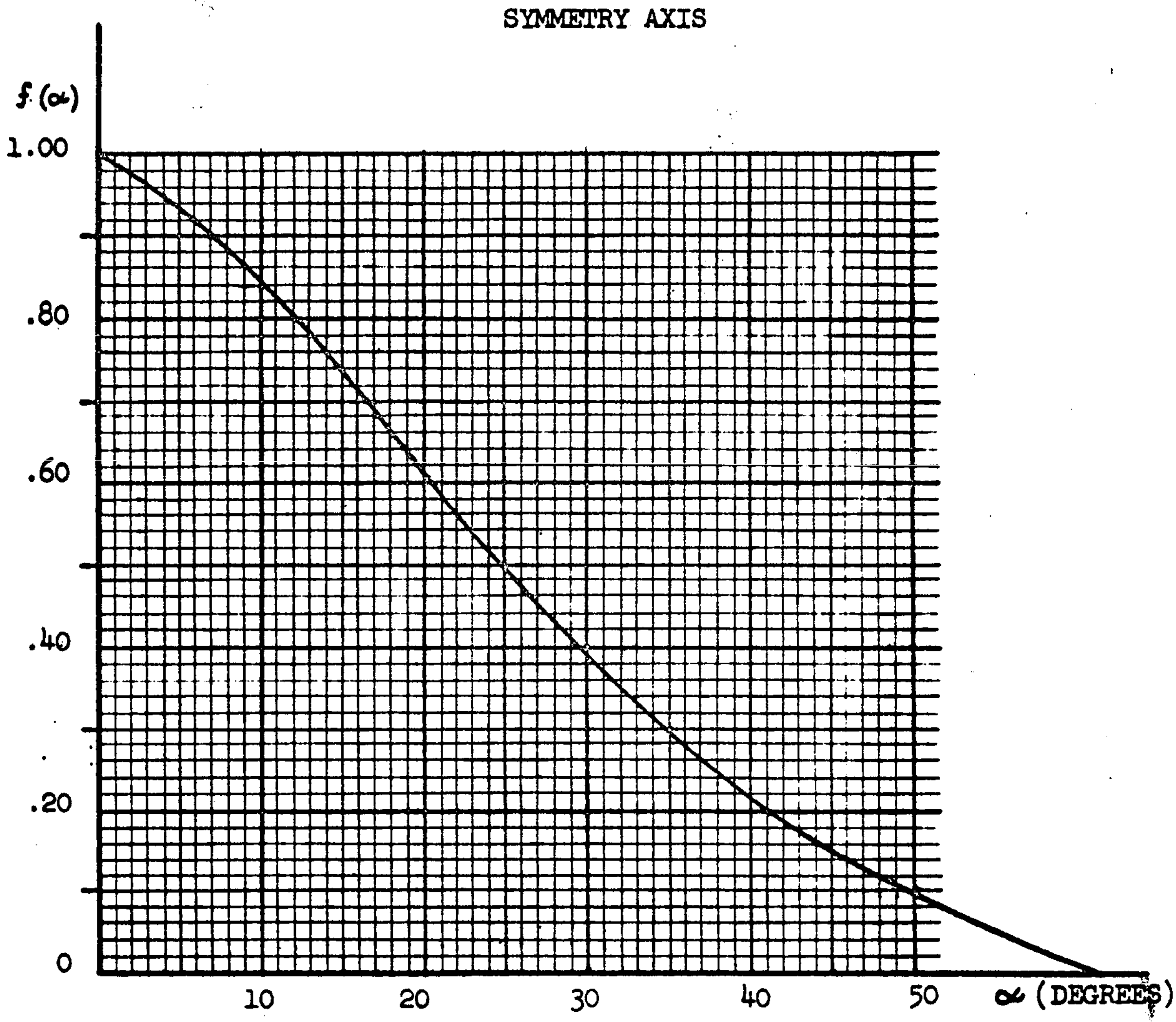


FIGURE 2

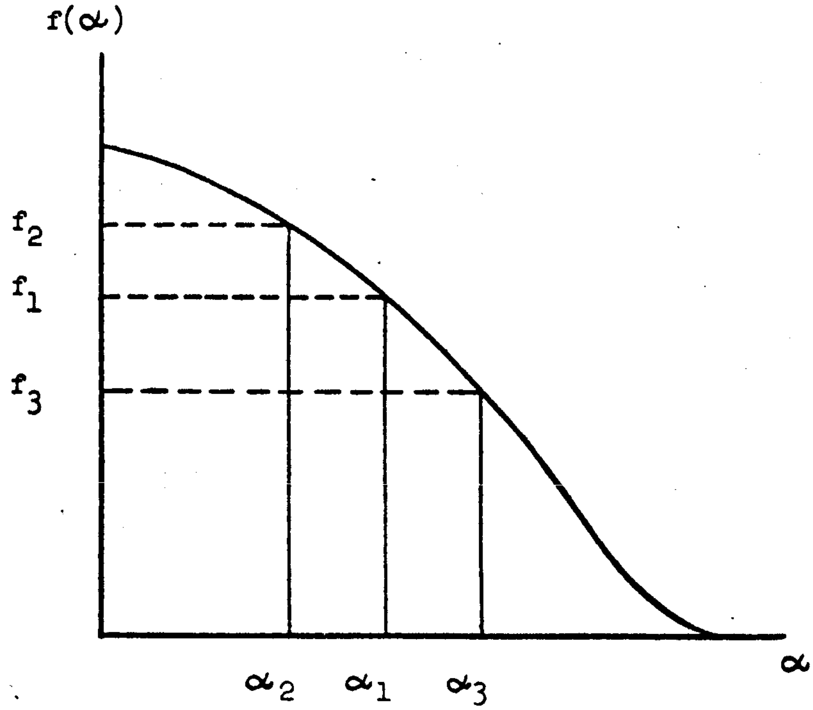


FIGURE 3

THE EFFECT OF TILTING GROUPS OF CUBE-CORNER PRISMS

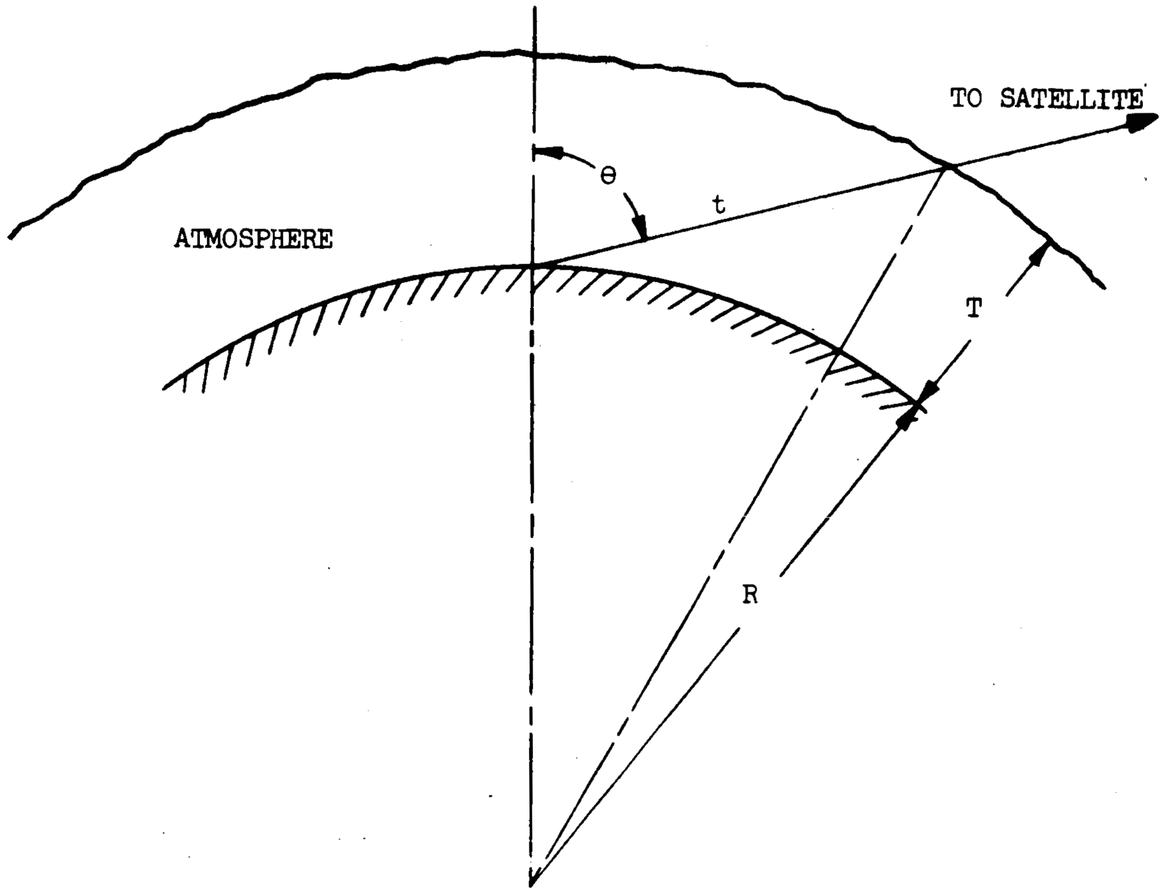


FIGURE 4

ATMOSPHERIC ATTENUATION GEOMETRY

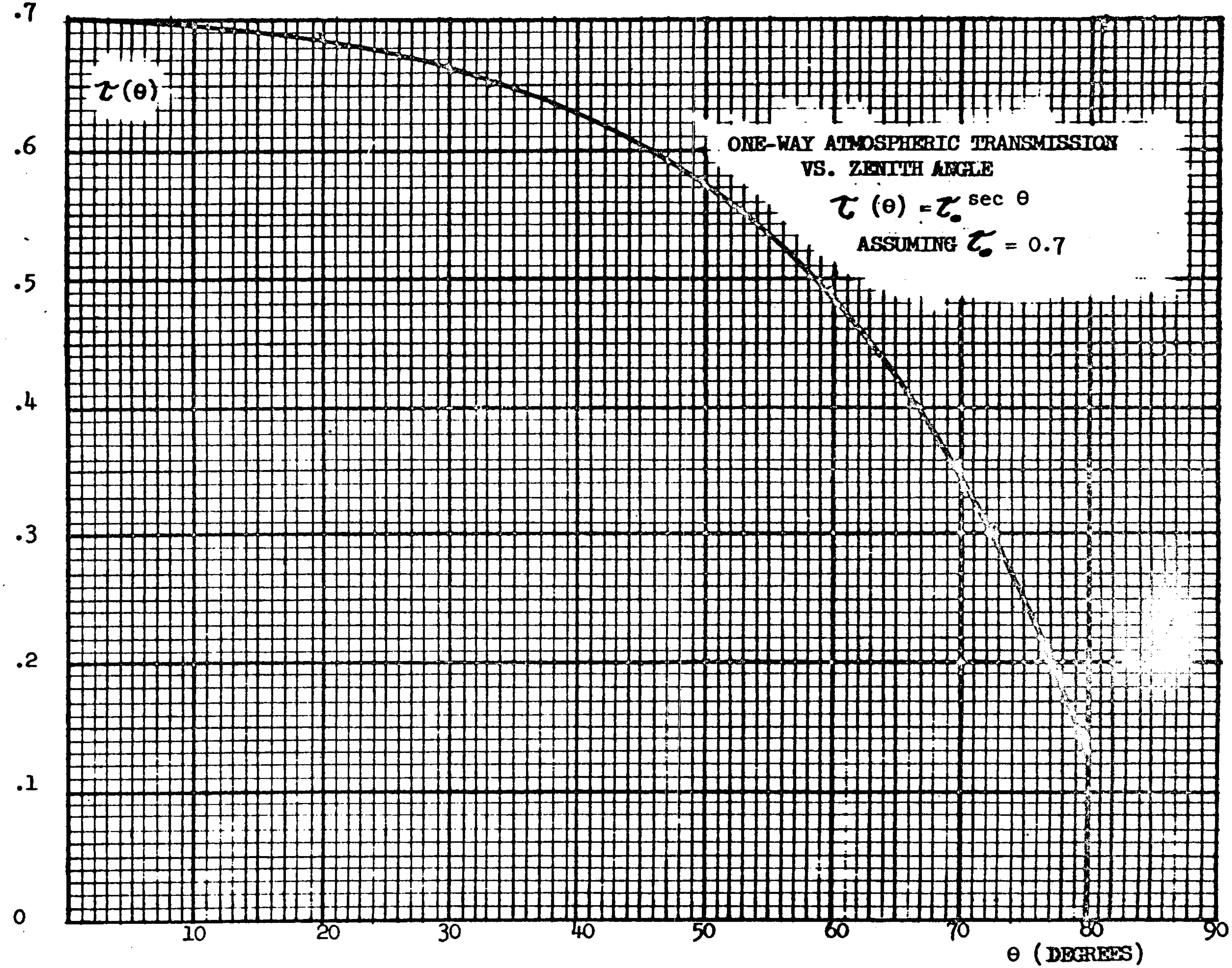

τ(θ)
ONE-WAY ATMOSPHERIC TRANSMISSION
VS. ZENITH ANGLE
$\tau(\theta) = \tau_0^{\sec\theta}$
ASSUMING τ_0 = 0.7
.7
.6
.5
.4
.3
.2
.1
0
10
20
30
40
50
60
70
80
90
θ (DEGREES)

FIGURE 5

α VS. θ FOR GEOS AT 1500 KM ALTITUDE

$$\alpha = \text{ARC SIN}\left[\frac{R}{R+h} \quad \text{SIN}\ \theta\right]$$

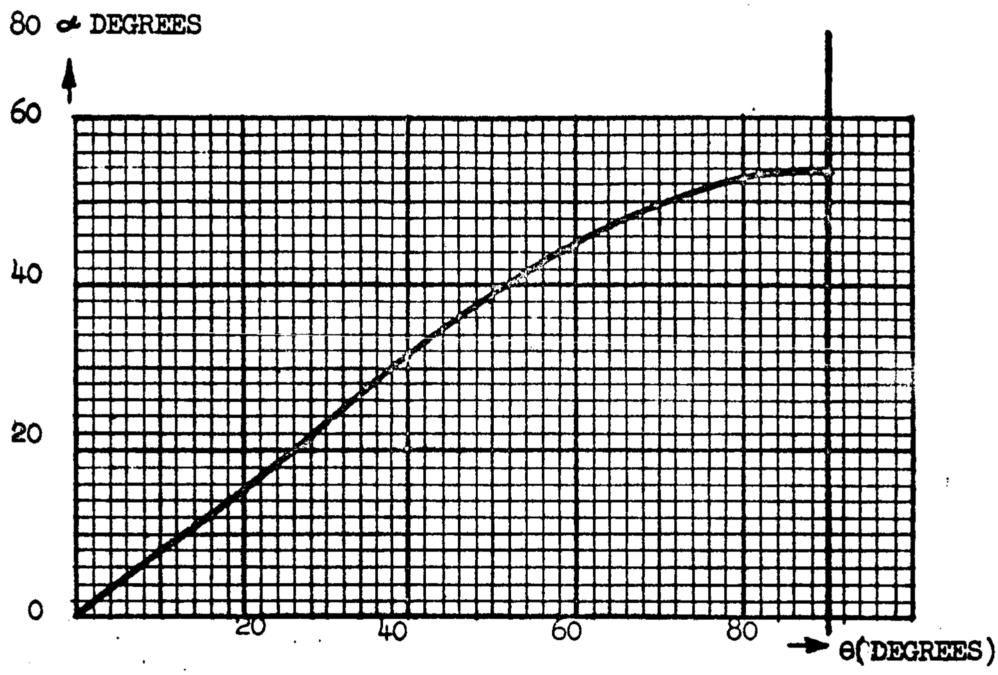
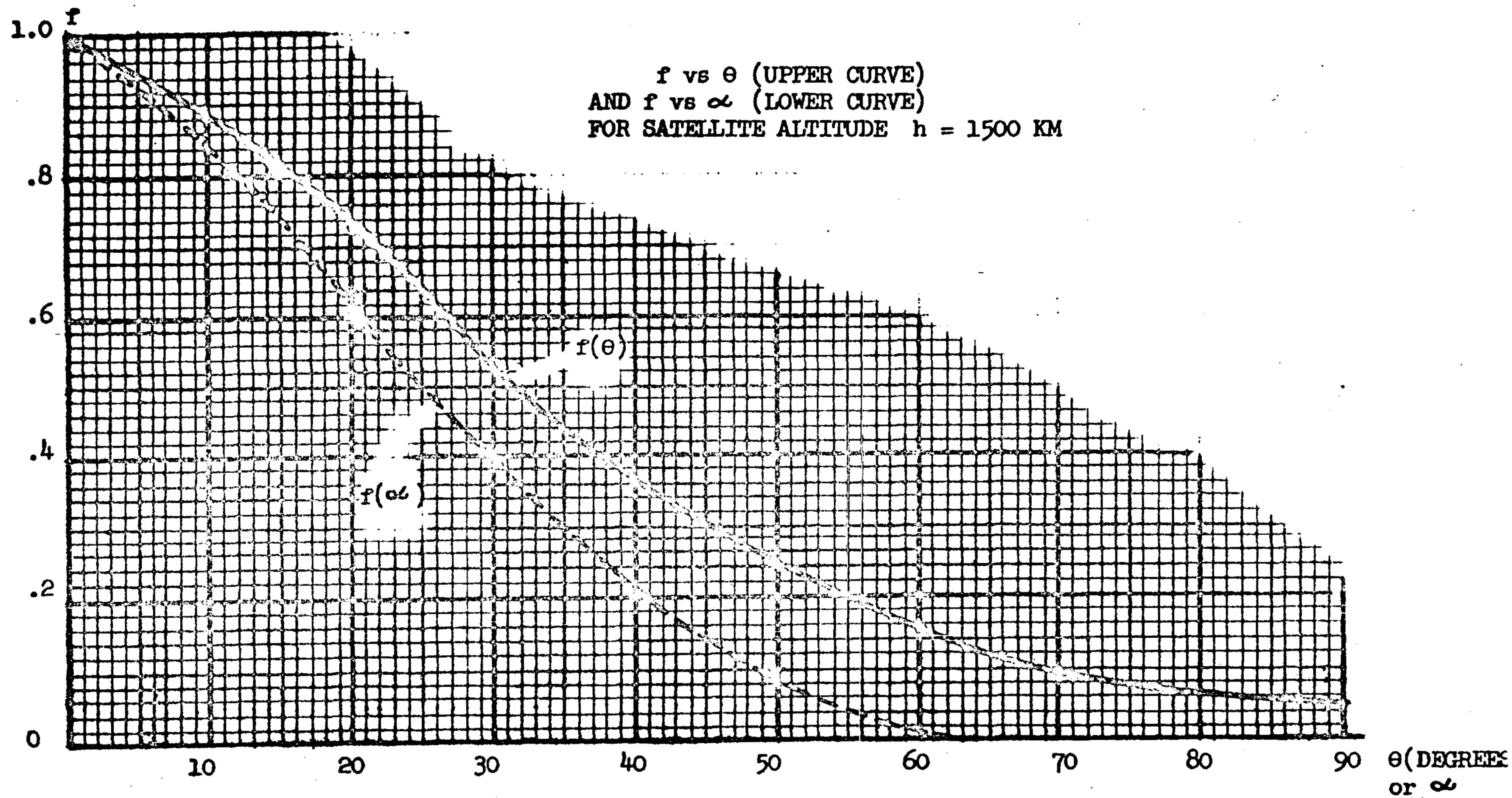


FIGURE 6



f vs θ (UPPER CURVE)
AND f vs α (LOWER CURVE)
FOR SATELLITE ALTITUDE h = 1500 KM
f
1.0
.8
.6
.4
.2
0
f(θ)
f(α)
10
20
30
40
50
60
70
80
90
θ(DEGREES
or α

FIGURE 7

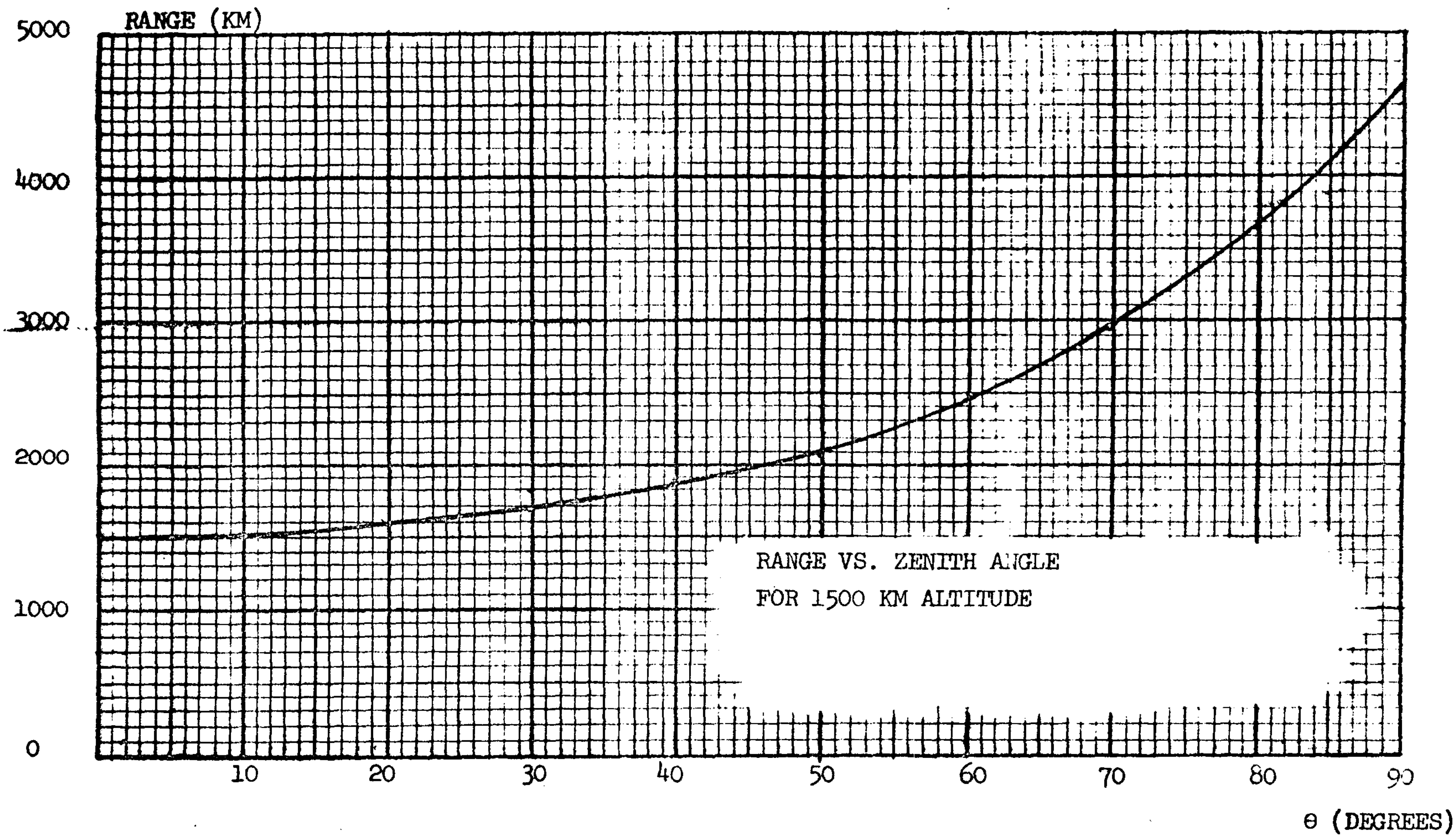

RANGE (KM)
5000
4000
3000
2000
1000
0
10
20
30
40
50
60
70
80
90
θ (DEGREES)
RANGE VS. ZENITH ANGLE
FOR 1500 KM ALTITUDE

FIGURE 8

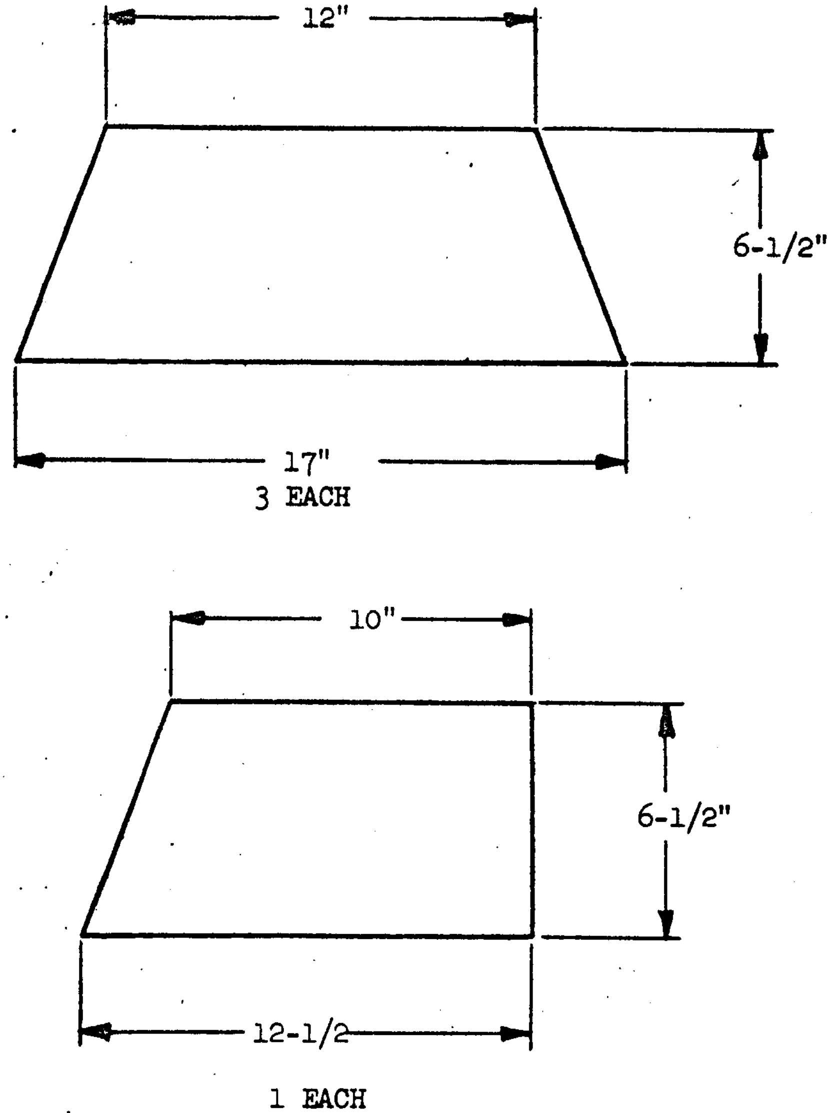


FIGURE 9

APPROXIMATE GEOMETRY OF REFLECTOR PANELS

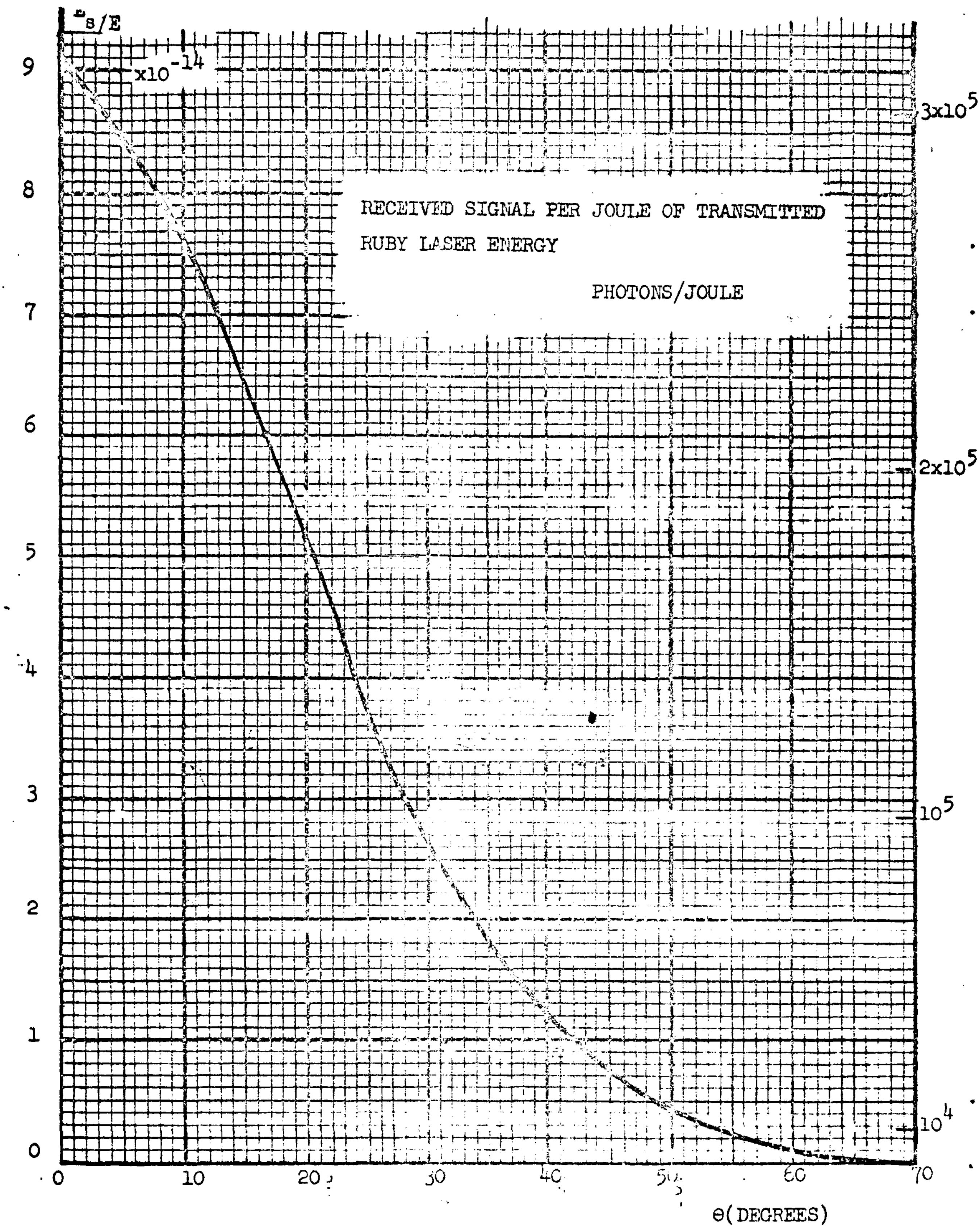

E_s/E
$\times 10^{-14}$
RECEIVED SIGNAL PER JOULE OF TRANSMITTED RUBY LASER ENERGY
PHOTONS/JOULE
9
8
7
6
5
4
3
2
1
0
3×10^5
2×10^5
10^5
10^4
0
10
20
30
40
50
60
70
θ(DEGREES)

FIGURE 10