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Construction of Nonlinear Programming Test Problems*

by

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In order to test a nonlinear programming algorithm it is very useful to be able to construct test problems with known optimum solutions. The purpose of this note is to describe a simple procedure for constructing such test problems. We will describe the procedure for a concave maximization problem subject to concave constraints.

The concave maximization problem is

$$\max_x \{ \varphi(x) \mid h_i(x) \geq 0, i = 1, 2, \dots, k \},$$

where $x \in E^m$, and $\varphi(x)$ and $h_i(x)$ are real valued concave functions of x . The procedure will be described for $\varphi(x) = \theta(x) + c'x$, and $h_i(x) = q_i(x) + b_i$, $i = 1, \dots, k$, where $\theta(x)$ and $q_i(x)$, $i = 1, \dots, k$ are any selected differentiable concave functions of x , c is a vector $\in E^m$ and the b_i are scalars.

Step I

Choose any $x^0 \in E^m$ as a desired optimum point, and any set of $u_i^0 \geq 0$, $i = 1, \dots, k$, as the corresponding optimum dual solution. That is, we first specify the primal and dual solution to the problem.

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Step II

Choose b_i , $i = 1, \dots, k$, so that $h_i(x^0) = 0$ for $u_i > 0$ and $h_i(x^0) \geq 0$ for $u_i = 0$. Note that $u_i > 0$ means that the i^{th} constraint is active.

Step III

Let

$$c = -\nabla\theta(x^0) - \sum_{i=1}^k u_i^0 \nabla q_i(x^0)$$

This choice satisfies the Kuhn-Tucker condition $\nabla\varphi(x^0) + \sum_{i=1}^k u_i^0 \nabla h_i(x^0) = 0$, and therefore ensures that x^0 is an optimum solution to the concave programming problem.

We will illustrate this procedure by applying it to the quadratic problem where $\theta(x) = x'Q_0x$, $q_i(x) = x'Q_i x + a_i'x$, and the Q_i , $i = 0, 1, \dots, k$, are negative semi-definite matrices.

Example (quadratic problem with four variables and three constraints)

Let

$$Q_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad Q_1 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix},$$
$$Q_3 = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad a_1 = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad a_3 = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}.$$

Step I

$$\text{Let } x^0 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ -1 \end{pmatrix} \quad \text{and } u^0 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

Step II

$$h_1(x^0) = x^{0'} Q_1 x^0 + a_1' x^0 + b_1 = -8 + b_1$$

$$\text{Since } u_1 > 0, \quad b_1 = 8.$$

$$h_2(x^0) = x^{0'} Q_2 x^0 + a_2' x^0 + b_2 = -9 + b_2$$

$$\text{Since } u_2 = 0, \quad \text{we choose } b_2 = 10 \text{ so that } h_2(x^0) = 1 > 0.$$

$$h_3(x^0) = x^{0'} Q_3 x^0 + a_3' x^0 + b_3 = -5 + b_3$$

$$\text{Since } u_3 > 0, \quad b_3 = 5.$$

Step III

$$c = \begin{pmatrix} 2x_1^0 \\ 2x_2^0 \\ 4x_3^0 \\ 2x_4^0 \end{pmatrix} + \begin{pmatrix} 2x_1^0 + 1 \\ 2x_2^0 - 1 \\ 2x_3^0 + 1 \\ 2x_4^0 - 1 \end{pmatrix} + 2 \begin{pmatrix} 4x_1^0 + 2 \\ 2x_2^0 - 1 \\ 2x_3^0 \\ 0 \quad -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 21 \\ -7 \end{pmatrix}$$

The constructed problem is:

$$\begin{aligned} \text{minimize } & \varphi = -x_1^2 - x_2^2 - 2x_3^2 - x_4^2 + 5x_1 + 5x_2 + 21x_3 - 7x_4 \\ \text{subject to } & -x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_1 + x_2 - x_3 + x_4 + 8 \geq 0 \\ & -x_1^2 - 2x_2^2 - x_3^2 - 2x_4^2 + x_1 + x_4 + 10 \geq 0 \\ & -2x_1^2 - x_2^2 - x_3^2 - 2x_1 + x_2 + x_4 + 5 \geq 0 \end{aligned}$$

and has as its optimum function value $\varphi(x^0) = 44$.