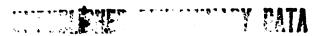
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Construction of Nonlinear Programming Test Problems*

by

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In order to test a nonlinear programming algorithm it is very useful to be able to construct test problems with known optimum solutions. The purpose of this note is to describe a simple procedure for constructing such test problems. We will describe the procedure for a concave maximization problem subject to concave constraints.

The concave maximization problem is

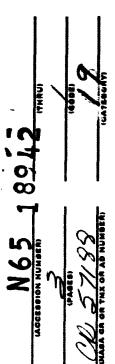
$$\max_{x} \{ \phi(x) | h_{i}(x) \ge 0, i = 1, 2, ..., k \},$$

where $x \in E^{m}$, and $\phi(x)$ and $h_{\underline{i}}(x)$ are real valued concave functions of x. The procedure will be described for $\phi(x) = \theta(x) + c'x$, and $h_{\underline{i}}(x) = q_{\underline{i}}(x) + b_{\underline{i}}$, $i = 1, \ldots, k$, where $\theta(x)$ and $q_{\underline{i}}(x)$, $i = 1, \ldots, k$ are any selected differentiable concave functions of x, c is a vector e E^{m} and the $b_{\underline{i}}$ are scalars.

Ster I

Choose any $x^0 \in \mathbb{R}^m$ as a desired optimum point, and any set of $u^0_1 \geq 0$, $i=1,\ldots,k$, as the corresponding optimum dual solution. That is, we first specify the primal and dual solution to the problem.

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Step II

Choose b_i , i = 1, ..., k, so that $h_i(x^0) = 0$ for $u_i > 0$ and $h_i(x^0) \ge 0$ for $u_i = 0$. Note that $u_i > 0$ means that the $i\frac{th}{t}$ constraint is active.

Step III

Let

$$c = - \nabla \theta(x^{o}) - \sum_{i=1}^{k} u_{i}^{o} \nabla q_{i}(x^{o})$$

This choice satisfies the Kuhn-Tucker condition $\nabla \varphi(\mathbf{x}^0) + \sum_{i=1}^K u_i^0 \nabla h_i(\mathbf{x}^0) = 0$, and therefore ensures that \mathbf{x}^0 is an optimum solution to the concave programming problem.

We will illustrate this procedure by applying it to the quadratic problem where $\theta(x) = x'Q_0x$, $q_i(x) = x'Q_ix + a_i'x$, and the Q_i , i = 0, 1, ..., k, are negative semi-definite matrices.

Example (quadratic problem with four variables and three constraints)

Let

$$\mathbf{Q}_{3} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -2 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}, \qquad
\mathbf{Q}_{1} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}, \qquad
\mathbf{Q}_{2} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -2
\end{pmatrix},$$

$$\mathbf{Q}_{3} = \begin{pmatrix}
-2 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0
\end{pmatrix}, \qquad
\mathbf{Q}_{4} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -2
\end{pmatrix},$$

$$\mathbf{Q}_{5} = \begin{pmatrix}
-2 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0
\end{pmatrix},$$

$$\mathbf{Q}_{1} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -2
\end{pmatrix},$$

$$\mathbf{Q}_{5} = \begin{pmatrix}
-2 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0
\end{pmatrix},$$

$$\mathbf{Q}_{1} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix},$$

$$\mathbf{Q}_{2} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -2
\end{pmatrix},$$

Let
$$x^0 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ -1 \end{pmatrix}$$
 and $u^0 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

Step II

$$h_1(x^0) = x^{0'} Q_1 x^0 + a'_1 x^0 + b_1 = -3 + b_1$$

Since $u_1 > 0$, $b_1 = 8$.
 $h_2(x^0) = x^{0'} Q_2 x^0 + a'_2 x^0 + b_2 = -9 + b_2$
Since $u_2 = 0$, we choose $b_2 = 10$ so that $h_2(x^0) = 1 > 0$.
 $h_3(x^0) = x^{0'} Q_3 x^0 + a'_3 x^0 + b_3 = -5 + b_3$
Since $u_3 > 0$, $b_3 = 5$.

Step III

$$c = \begin{pmatrix} 2x_{1}^{0} \\ 2x_{2}^{0} \\ 4x_{3}^{0} \\ 2x_{1}^{0} \end{pmatrix} + \begin{pmatrix} 2x_{1}^{0} + 1 \\ 2x_{2}^{0} - 1 \\ 2x_{3}^{0} + 1 \\ 2x_{3}^{0} - 1 \end{pmatrix} + 2 \begin{pmatrix} 4x_{1}^{0} + 2 \\ 2x_{2}^{0} - 1 \\ 2x_{2}^{0} \\ 0 - 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 21 \\ -7 \end{pmatrix}$$

The constructed problem is:

and has as its optimum function value $\varphi(x^0) = 44$.