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TRANSIENT PRESSURE RESPONSE IN A FLUID SYSTEM AS A VIBRATING VALVE CLOSES

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TECHNICAL MEMORANDUM X-53206

TRANSIENT PRESSURE RESPONSE IN A FLUID SYSTEM AS A VIBRATING VALVE CLOSES

By

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ABSTRACT

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Equations are derived that describe the unsteady flow in a pipeline when a vibrating valve closes; a pressure oscillation occurs in addition to water hammer. The results of analytical and experimental investigations show that the pressure oscillation causes an increase in the maximum transient pressure during valve closure that is proportional to the frequency and amplitude of valve vibration.

Because the boundary conditions and natural frequencies may vary, each fluid system must be investigated to determine the relationships between the pressure oscillation and the frequency and amplitude of the valve vibration. The analytical methods reported can be used for such an investigation.

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DEFINITION OF SYMBOLS

Symbol	Definition
a	Velocity of Wave Propagation (in/sec)
A	Acceleration of Control Volume (in/sec^2)
$\overline{A}$	Flow Area of Pipeline (in^2)
A_t	Effective Flow Area of Valve at Time t (in^2)
A_ω	Wetted Area of Control Volume (in^2)
C_1	Constant Dependent Upon Pipeline Mounting
C^+	Characteristic Curve with Positive Slope
C^-	Characteristic Curve with Negative Slope
d	Double Amplitude Displacement of Vibration Exciter (in)
D	Inside Diameter of Pipeline (in)
e	Wall Thickness of Pipeline (in)
E	Modulus of Elasticity of Pipeline Material (lbs/in^2)
f	Darcy - Weisbach Friction Factor
F	Force on Control Volume (lbs)
$\overline{F}$	Vibration Exciter Frequency (cps)
g	Acceleration of Gravity (386.04 in/sec^2)
K	Adiabatic Bulk Modulus of Elasticity of Fluid (lbs/in^2)
ℓ	Length of Pipeline (in)
M	Mass of Control Volume (slugs)

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
P	Piezometric Pressure of Fluid (lbs/in^2)
P_{total}	Total Pressure of Fluid (lbs/in^2)
p	Static Pressure of Fluid (lbs/in^2)
R	Static Pressure Below Valve (lbs/in^2)
t	Time After Valve Closure Begins (sec)
V	Fluid Velocity (in/sec)
$\lvert V \rvert$	Absolute Value of Fluid Velocity (in/sec)
x	Location Along Pipeline (in)
X	Displacement of Vibration Exciter (in)
β	Effective Bulk Modulus of Fluid and Container (lbs/in^2)
ρ	Specific Weight (lbs/in^3)
ϕ	Phase Angle Between Vibration and Start of Valve Closure (radians)
τ_ω	Shearing Stress exerted on stream by pipe wall (psi)
μ	Poisson's Ratio

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By Cecil A. Ponder, Jr.*
George C. Marshall Space Flight Center

SUMMARY

The transient pressure response in a pipeline caused by the closure of a vibrating valve was investigated in order to study the effects of the vibration. The vibrating valve causes a pressure oscillation which increases the maximum transient pressure. Analytical expressions describing the transient flow were derived, and the boundary conditions of a test facility were used in a solution of the equations.

In an experimental investigation, transient pressures were recorded as the valve closed while vibrating at several different frequencies and amplitudes, including the natural frequency of the system. The time of valve closure was varied between .250 and 1.00 second. Two suction line configurations were used. One was a uniform suction line of 4-inch diameter; the other had a 6-inch diameter to 4-inch diameter concentric reducer midway between the reservoir and valve.

The results of the investigation show that the maximum transient pressure is increased only slightly by low level vibration, but as the vibration increases, there is a significant increase in the maximum transient pressure. The acceleration at which this increase in the maximum transient pressure becomes significant depends upon the natural frequency of the system, and the frequency and amplitude of the forced vibrations. With the valve completely closed, a large pressure oscillation was obtained during vibration.

INTRODUCTION

Aerospace engineers have recently begun to study the effects of fluid vibrations on space vehicles. Because high pressures require more rigid ducts and structures, there is particular interest in the magnitude of pressure oscillations produced by the fluid vibrations. The transient pressure that is caused by the closing of a valve has historically been called water hammer. When the closing valve is also

*Chrysler Corporation, Technical Support Contractor

vibrating, the water hammer couples with pressure oscillations, and a higher maximum pressure results. These pressure oscillations, if transmitted throughout the fluid system, create structural problems. Because there was a need for understanding the coupling of fluid vibrations and water hammer, the transient pressure response of the closure of a vibrating valve was investigated (Ref. 1 and 2).

HISTORY OF WATER HAMMER INVESTIGATIONS

There have been numerous analyses of water hammer. A discussion of various methods of reducing water hammer by Michaud (1878) was the first investigation of the phenomena. In 1897, N. E. Joukowsky first connected water hammer with acoustic wave action (Ref. 3 and 4). His analysis was based on instantaneous valve closure and a uniform pipeline through which an inviscid fluid was flowing. He derived the equation:

$$\Delta P = \frac{a\rho}{g} \Delta V \tag{1}$$

Joukowsky conducted an extensive program to verify his equation. However, the effect of compound and branched pipes, or valve closures of finite time was not included in his analysis.

At about the same time Lorenzo Allievi, an Italian engineer, made a thorough study of the water hammer caused by linear valve closures of finite time (Ref. 5). He constructed a chart from which the maximum surge pressure during a linear valve closure could be obtained.

One of the first water hammer studies in the United States was conducted by N. R. Gibson (Ref. 6). He used the method of arithmetic integration to give the same results that Allievi had obtained, yet he had no knowledge of Allievi's studies. E. E. Halmos pointed out the identical results, and in 1925, he published an English translation of Alleivi's works.

Several other investigations of water hammer were made, and in 1927, Ray S. Quick published a comparison of the various theories (Ref. 7). These theories, however, were merely amplifications of the basic ideas of Joukowsky and Allievi.

Nonuniform gate closure was investigated in 1928, by S. Logan Kerr (Ref. 8) in connection with his investigations of valve closures from partial openings. R. W. Angus extended Joukowsky's theory to include a thorough study of water hammer in compound and branched pipe systems (Ref. 9). He relied heavily on graphical solutions.

J. Parmakian published Waterhammer Analysis (Ref. 10) in 1955. It included a derivation of the basic linearized water-hammer equations with an abrupt pressure loss at the end of the pipeline to account for the frictional losses. Graphical methods used to solve water-hammer problems were thoroughly investigated by Louis Bergeron (Ref. 11).

F. M. Wood used Heavisides' operational calculus as an analytical method of predicting the transient pressures in the pipeline (Ref. 12). Although the nonlinear frictional loss term of the water-hammer equations was linearized, he neglected the other nonlinear terms of the wave equations. Wood's analysis gave the results in terms of only the pressure and velocity surges that were produced.

R. George Rich improved Wood's analysis by using the La Place-Mellin transformation so that he worked directly with the total pressure in the pipeline (Ref. 13). He also used the linearized friction loss and ignored the other nonlinear terms of the water-hammer equations. Many authors, including Parmakian, Wood and Rich believed that the solution of the water-hammer equations with the nonlinear terms was impossible (Ref. 14).

However, in 1960, M. Lister published an article on the solution of hyperbolic partial differential equations (Ref. 15). The article showed that the theory of characteristics could be used to solve partial differential equations with two dependent and two independent variables.

V. L. Streeter and C. Lai used this theory of characteristics with an approximation to permit calculation of the transient pressure at specific time intervals (Ref. 16).

The solution used in this investigation is based upon the theory of characteristics as suggested by M. Lister. A grid of characteristics was used, however, to eliminate errors introduced by the method of specific time intervals. The spacing of time was controlled by the number of characteristic grid lines used.

By using the theory of characteristics to solve the wave equations, the transient pressure and fluid velocity are calculated not only at the ends of the pipeline, but also along the length of the pipe. There is also no need for linearizing or neglecting the nonlinear terms. Because the primary purpose of this investigation was to study the effect of a vibrating valve, this additional boundary condition was included in the analysis.

ANALYSIS

Unsteady Flow in a Uniform Pipeline

The solution of the transient pressure is based on a one-dimensional analysis of the transient flow through a uniform pipeline, with boundary conditions given at each end of the pipe, as well as the initial steady state conditions along it. The fluid system investigated is shown in FIG 1. It has a uniform pipeline with a reservoir at one end and a valve at the other end. The basic equations describing the flow through the pipeline are:

a. The momentum equation, Ref. 14, (see Appendix A for derivation)

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{fV}{2D} |V| + \frac{g}{\rho} \frac{\partial P}{\partial x} = 0 \qquad (2)$$

b. The equation of continuity for unsteady flow of a compressible fluid (Ref. 10)

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial V}{\partial x} + V \frac{\partial \rho}{\partial x} = 0 \qquad (3)$$

c. The equation of state for a compressible liquid (Ref. 16)

$$\frac{\partial P}{\partial \rho} = \frac{\beta}{\rho} \qquad (4)$$

Where β is the effective bulk modulus of the liquid in an elastic container defined as (Ref. 10).

$$\beta = \frac{1}{\frac{1}{K} + \frac{C_1 D}{e E}} \qquad (5)$$

where values of C_1, are as follows:

$C_1 = 5/4 - \mu$ for a pipe anchored at the upper end without expansion joints

$C_1 = 1 - \mu^2$. for a pipe anchored against longitudinal movement throughout its length

$C_1 = 1 - \mu/2$ for a pipe with expansion joints

It should be noted that P is the piezometric pressure, and therefore, includes pressure variations caused by elevation; that is, $P = \rho z + p$, where p is the local static pressure and z is the elevation above a datum plane.

Combining the equation of continuity (3) and the equation of state (4)

$$\frac{\partial P}{\partial t} + \beta \frac{\partial V}{\partial x} + V \frac{\partial P}{\partial x} = 0 \tag{6}$$

This equation, along with the momentum equation (2), forms the basic wave equations used in the analytical study of water hammer.

By using the theory of characteristics, a set of characteristic equations is derived in Appendix B. These characteristic equations define two sets of curves as shown in FIG 2. One set of curves has a positive slope and the other set has a negative slope (identified as C^+ and C^-).

The characteristic equations are:

Along C^+

$$(x_o - x_A) = (V_A + a_A)\ (t_o - t_A) \tag{7}$$

$$(V_o - V_A) + \frac{g}{\rho_A a_A} (P_o - P_A) + \frac{f V_A |V_A|}{2D} (t_o - t_A) = 0 \tag{8}$$

Along C^-

$$(x_o - x_B) = (V_B - a_B)\ (t_o - t_B) \tag{9}$$

$$(V_o - V_B) - \frac{g}{\rho_B a_B} (P_o - P_B) + \frac{f V_B |V_B|}{2D} (t_o - t_B) = 0 \tag{10}$$

With the fluid velocity (V), pressure (P), location (x), time (t), and density (ρ) given at points A and B, equations (7), (8), (9), and (10) can be solved simultaneously for the four unknown quantities (P, V, x, t) at point o. It is also necessary to determine the density at point o. The equation of state (4) can be integrated from point A to point o to give the density.

$$\int_A^o dP = \beta \int_A^o \frac{d\rho}{\rho} \tag{11}$$

$$P_o - P_A = \beta \ln \frac{\rho_o}{\rho_A} \tag{12}$$

or

$$\rho_o = \rho_A e^{\left(\frac{P_o - P_A}{\beta}\right)} \tag{13}$$

by using the series expansion (Ref. 17)

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} \tag{14}$$

the density at point o can be approximated with a high degree of accuracy as

$$\rho_o = \rho_A \left(1 + \frac{P_o - P_A}{\beta}\right) \tag{15}$$

It should be noted that $\left(\frac{P_o - P_A}{\beta}\right)$ is of the order of 10^{-6} so that higher order terms may be neglected. As shown in the FIG 2, the quantities at point o are later used to determine the unknown quantities at point S. The number of points for which solutions can be obtained is one less than the number of points for which the conditions initially are known. This problem can be eliminated, however, if the boundary conditions are known.

Boundary Conditions at the Reservoir

It is assumed that the reservoir used is of sufficient capacity to provide a constant total pressure at the intersection of the pipeline and the reservoir. Therefore, returning to FIG 2

$$P_{L_{total}} = P_L + \frac{\rho_L V_L^2}{2g} = \text{CONSTANT} \tag{16}$$

This boundary condition is used in conjunction with the equation for the C^- characteristic to evaluate the unknowns at the left boundary (point L in FIG 2).

$$x_L = 0 \tag{17}$$

From equation (9)

$$(x_L - x_A) = (V_A - a_A)(t_L - t_A) \tag{18}$$

and from equation (10)

$$(V_L - V_A) - \frac{g}{\rho_A a_A}(P_L - P_A) + \frac{f V_A |V_A|}{2D}(t_L - t_A) = 0 \tag{19}$$

With the conditions at A and the total pressure supplied by the reservoir known, the pressure (P_L), velocity (V_L), and time (t_L) can be found.

As before, the density was found by integrating the equation of state from point A to point L

$$\rho_L = \rho_A \left(1 + \frac{P_L - P_A}{\beta}\right) \tag{20}$$

Boundary Conditions at the Valve

With reference to FIG 2, the total velocity of the fluid at the valve (point R) is a function of the pressure drop across the valve, and of the motion of the vibrating valve. The orifice flow equation, with a correction for upstream area, is used to calculate the velocity as a function of the pressure drop across the valve (Ref. 18).

$$V_{R_1} = \frac{A_t}{A}\left[\frac{2g(P_R - R)}{\rho_R\left(1 - \frac{A_t^2}{A^2}\right)}\right]^{1/2} \tag{21}$$

Because the valve was excited sinusoidally by the vibration exciter, the velocity oscillations induced in the water were directly proportional to the velocity of the vibration exciter and to the ratio of the projected area of the vibrating valve to the area of the pipe line. The displacement of the exciter is

$$X = -\frac{d}{2}\cos(2\pi\bar{F}t + \phi) \tag{22}$$

The exciter velocity is found as

$$\frac{dX}{dt} = \pi d\bar{F} \sin(2\pi\bar{F}t + \phi) \tag{23}$$

The ratio of the projected area of the valve to the area of the suction line is $\left(1 - \frac{A_t}{\bar{A}}\right)$. Therefore, the velocity oscillation caused by vibration is

$$V_{R_2} = \left(1 + \frac{A_t}{\bar{A}}\right)\pi d\bar{F} \sin(2\pi\bar{F}t + \phi) \tag{24}$$

The total fluid velocity at the valve face is then given as

$$V_R = V_{R_1} + V_{R_2} = \frac{A_t}{\bar{A}}\left[\frac{2g\,(P_R - R)}{\rho_{R}\left(1 - \frac{A_t^2}{\bar{A}^2}\right)}\right]^{1/2} + \left(1 - \frac{A_t}{\bar{A}}\right)\pi d\bar{F} \sin(2\pi\bar{F}t + \phi) \tag{25}$$

From equations (7) and (8) for the C^+ characteristic

$$t_R = t_M + \frac{x_R - x_M}{(V_M + a_M)} \tag{26}$$

$$V_R - V_M + \frac{g}{\rho_M a_M}(P_R - P_M) + \frac{f V_M |V_M|}{2D}(t_R - t_M) = 0 \tag{27}$$

also at the valve face

$$x_R = \ell \tag{28}$$

The density relationship is again found by integrating the equation of state, this time between x_M and x_R

$$\rho_R = \rho_M\left(1 + \frac{P_R - P_M}{\beta}\right) \tag{29}$$

The simultaneous solution of these equations will then yield P_R, V_R, t_R and ρ_R.

Boundary Conditions for Branched and Compound Lines

Surge pressures in other piping systems can be computed using the characteristic equations (7), (8), (9) and (10) for each uniform section, with the boundary conditions for the case involved. For example, the boundary conditions in FIG 3 require that the flow into an intersection is the same as the flow leaving the intersection, and that the pressure is constant across the intersection. The equations of the boundary conditions would then be:

At junction A

$$V_1 D_1^2 = V_2 D_2^2 \tag{30}$$

$$P_{1A} = P_{2A} \tag{31}$$

At junction B

$$V_2 D_2^2 = V_3 D_3^2 + V_4 D_4^2 \tag{32}$$

$$P_{2B} = P_{3B} = P_{4B} \tag{33}$$

Initial Flow Conditions in the Pipeline

Referring to FIG 2 before calculations can proceed, a set of points must be determined where all of the variables P, V, ρ, x, and t are known. The points used in the analysis are the values when t=0, the steady state flow conditions just as valve closure begins. Because the conditions were steady state ($\frac{\partial}{\partial t} = 0$), the wave equations (2) and (6) reduce to:

$$V \frac{dV}{dx} + \frac{f}{2D} V|V| + \frac{g}{\rho} \frac{dP}{dx} = 0 \tag{34}$$

$$\beta \frac{dV}{dx} + V \frac{dP}{dx} = 0 \tag{35}$$

Combining the two equations to eliminate $\frac{dP}{dx}$ yields

$$\frac{dV}{V} - \frac{g\beta}{\rho V} \frac{dV}{V^2} + \frac{f}{2D} dx = 0 \tag{36}$$

Because the flow is steady and the suction line area is constant, from the requirement of continuity

$$G = \rho V = \text{CONSTANT} \tag{37}$$

$$\therefore \frac{dV}{V} - \frac{g\beta}{G}\frac{dV}{V^2} + \frac{f}{2D}\,dx = 0 \tag{38}$$

Integrating this equation from x to ℓ

$$\int_{V_{o,x}}^{V_{o,\ell}} \frac{dV}{V} - \frac{g\beta}{G}\int_{V_{o,x}}^{V_{o,\ell}} \frac{dV}{V^2} + \frac{f}{2D}\int_{x}^{\ell} dx = 0 \tag{39}$$

The notation $V_{o,x}$ denotes the velocity at time zero, position x.

$$\ln\frac{V_{o,\ell}}{V_{o,x}} + \frac{g\beta}{G}\left(\frac{1}{V_{o,\ell}} - \frac{1}{V_{o,x}}\right) + \frac{f}{2D}\,(\ell - x) = 0 \tag{40}$$

Therefore using the above equation and the known conditions at $x = \ell$, the velocity at any point along the suction line may be calculated.

The pressure at any point along the suction line may be found by integrating equation (35)

$$dP = -\beta\frac{dV}{V} \tag{41}$$

$$\int_{P_{o,x}}^{P_{o,\ell}} dP = -\beta\int_{V_{o,x}}^{V_{o,\ell}} \frac{dV}{V} \tag{42}$$

$$P_{o,\ell} - P_{o,x} = -\beta\,\ln\frac{V_{o,\ell}}{V_{o,x}} \tag{43}$$

And since the flowrate is constant, the density variation along the pipeline may be found

$$G = \rho_{o,x}\,V_{o,x} \tag{44}$$

This then defines the flow under steady state conditions.

Steps of Computations

In solving the characteristic equations, a matrix form was used to keep the variables orderly (FIG 4). Here (a) is a variable analagous to time, and (b) is a variable analagous to the location along the pipeline. Subscripts are used to denote the location of the variables within the matrix. For example $P_{1,2}$ means the pressure at a = 1, b = 2. The steps in computing the unknown quantities are:

a. Divide the suction line into sections. The number (n) of sections chosen will of course determine the degree of accuracy.

b. Set t = 0 for a = 0 and solve the steady state equations (40), (43) and (44) for b = 0 through n. This sets up the initial conditions.

c. The unknown quantities P, V, ρ, and t are then determined for the matrix point (1, 0) using the reservoir boundary conditions (equations 16 and 17) and the C^- characteristic passing through the matrix point (0, 1) (equations 18 and 19). Then density at point (1, 0) is found from equation (20).

d. The unknowns are then determined for a= 1, b=2 through (n-1) by using both characteristic curves and the known values at a=0. For example, the variables are found at matrix point (1, 3) by simultaneous solution of the C^+ characteristic passing through the matrix point (0, 2) and the C^- characteristic (equations 9 and 10) passing through the matrix point (0, 4).

e. The variables are then determined for the matrix point (1, n) using the valve boundary conditions (equations 25 and 28) and the C^+ characteristic (equations 7 and 8) passing through the matrix point (0, n-1). The density at point (1, n) is found from equation (29).

f. Using the results calculated for a = 1, the variables for a = 2 are determined (steps c through e). The calculation then proceeds in this manner as far as desired.

Limitations of Theory

The limitations of the analysis, as applied to the experimental facility, are:

a. The test pipeline contains a bellows that was not considered in the analysis.

b. The analysis assumes an abrupt area change between the pipeline and the reservior so that 100% wave reflection is obtained. The experimental facility did not have an abrupt area change.

c. The analysis assumes only the friction losses in the shear stress between the fluid and the pipeline, omitting the internal viscous dispersion within the fluid.

d. Two phase fluids are not considered so that once vapor pressure is reached, or a bubble exists within the fluid, the equations are invalid.

EXPERIMENTAL INVESTIGATION

Description of Experimental Facility

The experimental program was established to measure the transient pressure response when a vibrating valve is closed. The Experimental Facility is shown in FIG 5. This facility had a uniform pipeline of 4-inch O. D. x .0625-inch wall aluminum tubing attached to the bottom of a 4950 gallon reservoir that could be pressurized to 20 psig. At the lower end of the pipeline, 276 inches below the reservoir, a pneumatically operated 4-inch butterfly valve and a 4-inch diameter bellows were attached. The butterfly valve was bolted to a 22,000 pound force vibration exciter. A 6-inch drain line with a vent was attached to the bottom of the butterfly valve using a bellows to allow the butterfly valve to be vibrated as it was closed.

A second pipeline configuration (see FIG 6) was also used. This pipeline was a 6-inch O. D. x 3/16-inch wall aluminum tube that extended from the reservoir halfway to the butterfly valve. The pipeline then was reduced from 6-inch O. D. to 4-inch O. D; the remaining length of pipeline to the valve was 4-inch O. D. tubing.

Instrumentation

Four pressure transducers were located along the pipeline to collect data. Accelerometers were mounted on the vibration exciter and on the butterfly valve. A position indicator was used to measure the angular position of the butterfly in the valve. These measurements were recorded on an oscillograph using a paper speed of 10 inches/sec.

Also measured during the tests were water temperature, ullage pressure and liquid level in the reservoir, and the frequency and displacement of the vibration exciter.

Control of Test Variables

The most important factors that determine the maximum surge pressure are the time of valve closure, and the variation in fluid velocity while the valve is closing. The time of valve closure was controlled by orifices on the exhaust port of the pneumatic piston that closed the butterfly valve. The variation in fluid velocity while the valve was closing could not be controlled. A turbine flowmeter lacks sufficient response to measure this transient velocity. Because the effective flow area during valve closure could not be directly measured, it was approximated by measuring the effective flow area for various angles of the butterfly valve during steady flow and assuming the effective flow area was the same during unsteady flow.

To determine the effective flow area during steady flow, a turbine flowmeter was installed upstream of the valve. The flowrate through the valve and the pressure drop across it were measured for several valve positions. The orifice flow (Ref. 18), with the correction for upstream area was used to determine the effective flow area for each valve position.

$$Q = A_t \left[\frac{2g\,(\Delta P)}{\rho\left(1 - \frac{A_t^2}{\bar{A}^2}\right)} \right]^{1/2} \tag{45}$$

solving this for A_t

$$A_t = \frac{Q}{\left[\frac{2g\,\Delta P}{\rho} + \frac{Q^2}{\bar{A}^2} \right]^{1/2}} \tag{46}$$

The natural frequency of a fluid system is defined as the frequency at which a maximum pressure oscillation occurs for a given velocity variation of the valve $\left[\left(\frac{\Delta P}{\Delta V}\right) \max\right]$. The natural frequencies of both pipelines, with the valve closed, were determined experimentally by measuring the pressure oscillation at the valve for a given velocity input.

The frequencies of vibration of the valve during closure were then chosen at and between the natural frequencies.

With the valve fully closed, a small velocity oscillation could produce a large pressure oscillation, especially at the natural frequencies of the systems. Therefore, two levels of vibration were used. The first, a low level vibration, was allowed to continue after the valve was closed. However, the second, a high level vibration, was stopped when the valve fully closed.

Description of Tests

Tests were conducted as described below:

With the butterfly valve closed, the reservoir was filled with water and pressurized to 15 psig. For the low level vibration tests, the recorders were started, the vibration exciter was set at the predetermined frequency and displacement, and the butterfly valve was opened. When steady state flow was established, the valve was closed at the predetermined rate. The recorders were stopped after the transient pressures had damped. The procedure was the same for the high level vibration tests except the vibration exciter was turned on after the valve was open, and was turned off as soon as the valve was fully closed.

EXPERIMENTAL RESULTS

Tests of Uniform 4-Inch Pipeline

The natural frequencies of the 4-inch pipeline, between 20 and 100 cps, were determined and are shown in FIG 7. The fundamental natural frequency is at 32 cps, with another resonant point at 96 cps. Frequencies of 32, 48, 64, 80 and 96 cps, were chosen and used as the vibration frequency of the valve during closure.

The steady state effective flow area of the butterfly valve was determined and is shown in FIG 8 as a function of the angular position of the valve. The effective flow area of the fully open valve is seen to be 8.34 in^2, compared to a pipeline flow area of 11.79 in^2. The steady state approximation for a .250 sec closure was then obtained, as shown in FIG 9, by comparing the angular position during closure to the steady state effective flow area of the valve.

The pressure surges created by valve closures with the 4-inch pipeline are shown in Table I. These maximum pressure surges were measured at the pressure transducer located just upstream of the valve.

A typical pressure response, FIG 10, shows the static pressure at the valve for a .265 sec valve closure with (a) no vibration, (b) low level vibration of 32 cps, .010-inch displacement, and (c) high level vibration of 32 cps, .380-inch displacement.

With no vibration, the static pressure increased from the steady flow value of 20.4 psia to a maximum of 117.1 psia. When the valve fully closed, the pressure dropped rapidly and oscillated about the steady state level of 48 psia until it was damped out by the losses of the system. The pressure surge was determined to be 96.7 psi. At the low level vibration of 32 cps, .010 inch displacement, the results were similar. The vibrating valve caused only a small variation in the pressure response during the valve closure. However, the continuation of valve vibration after closure caused a pressure oscillation, the first cycle of which was 25.4 psia to 64.7 psia. After several seconds of vibration, a steady state oscillation as shown in FIG 11 was obtained. Although the velocity input to the valve was sinusoidal, the pressure oscillation measured at the valve was not sinusoidal. This suggests the presence of gas or vapor bubbles in the liquid.

When the valve was closed during high level vibration of 32 cps, .380-inch displacement (FIG 10), a significant change in the maximum surge pressure was obtained. As the valve closed, the vibration caused a pressure oscillation which reached 303.6 psia, giving a surge of 273.2 psi. The minimum pressure during closure was very small (3.5 psia). At the minimum point the curve is flat, probably because of the formation of gas bubbles in the fluid.

The results of a .250 sec valve closure during vibration at 48 cps are shown in FIG 12. The results here are essentially the same as for 32 cps. The effect of vibrating at a frequency other than the natural frequency was twofold; first, it changed the number of pressure oscillations during the closure, and secondly, it required vibration at a higher level to reach the same magnitude of pressure oscillation. This second phenomenon can be seen in FIG 7; as the vibration frequency moves away from the natural frequency, a smaller pressure oscillation is obtained for the same variation of velocity. At a slower rate of valve closure, the pressure response wave lengthened and the maximum pressure surge was reduced (Table I).

Tests of Pipeline Having an Area Reduction

The natural frequencies of the pipeline having an area reduction were determined and are shown in FIG 13. The fundamental frequency is 36.5 cps, with another resonant point at 82 cps. From this figure, frequencies of 30, 36.5, 48, 59, 70, 82, and 96 cps, were chosen and used as the vibration frequency of the valve during closure.

The pressure surges created by valve closures are shown in Table II. A typical pressure response, FIG 14, shows the static pressure at the valve during a .232-sec closure with (a) no vibration, (b) low level vibration at 70 cps, .025-inch displacement, and (c) high level vibration at 70 cps, .080-inch displacement. The results here are similar to those of the uniform 4-inch suction line, except the shape of the pressure response curve was altered slightly.

ANALYTICAL RESULTS

The equations describing the transient flow through a pipeline were programmed for a digital computer. The boundary conditions of a uniform suction line between a reservoir and a valve were used. The equations were then solved using the physical constants of the 4-inch pipeline, and the steady state approximation of the effective flow area during valve closure (FIG 9). The resulting transient pressure at the valve face for a .250 sec closure is shown in FIG 15. Comparing this to the experimental results (FIG 10), a higher pressure is predicted than measured, and the wave shape is also different. A slight modification of the curve of effective flow area, as shown by the dashed line in FIG 9, will yield the correct wave shape and maximum pressure of FIG 16. Since this slight modification gave the correct wave shape, the assumption previously made, that the steady state effective flow area was equal to the transient effective flow area, is only approximate. For comparison, the transient pressure created by an instantaneous closure is predicted [Equation (1)] as 1760 psi.

FIG 17 shows the calculated results under conditions similar to those of FIG 16 except that the valve was vibrating at 32 cps, with a displacement of .030 inches. The vibration only slightly modifies the base curve. However, when the displacement is increased to .380 inches (FIG 18), there is a marked increase in the maximum pressure during the valve closure. These results compare favorably with the experimental results shown in FIG 10. FIG 18 shows that the pressure decreases quite rapidly, and according to the equations, the pressure would continue decreasing to below zero psia. Of course, this could not occur, so computation was stopped. This is a significant point, because if the vapor pressure of the fluid is reached, the fluid will cavitate, similar to the cavitation in ultrasonic cleaning. This is probably what happened in the experimental results shown in FIG 10. When cavitation does occur, the expressions in this analysis do not apply because the assumption of a constant bulk modulus for the fluid is invalid.

The pressure variation at the valve for a sinusoidal vibration of 32 cps with .030 inches displacement with the valve closed is shown in FIG 19. The pressure variation steadily increases, damped only by the losses of the system. Without losses or cavitation, the oscillation would increase undamped for the natural frequency. For other than the natural frequencies, there would be a steady state level even without losses. If the pressure oscillations caused cavitation, large local pressures could be created due to collapse of the bubbles formed.

CONCLUSIONS

Analytical calculations, which are verified by results of the experimental tests, show that a vibrating valve can cause a substantial increase in the maximum transient pressure in a pipeline during valve closure. The magnitude of the transient pressure is directly related to the magnitude of valve vibration, and is also a function of the frequency of forced vibration. If the valve is closed, small valve vibrations at the natural frequency of the fluid system cause large pressure oscillations.

The initial fluid velocity, time of valve closure, and the rate of decrease in flow area are important factors in determining the maximum transient pressure. The magnitude of the transient pressure of a fluid system, with or without vibration, may be calculated by the methods reported if the boundary conditions of the fluid system are known. This method does not neglect the nonlinear terms of the wave equations (including friction), and it provides results along the suction line as well as at each end.

TABLE I

Measured Pressure Surges
Uniform 4-Inch Pipeline

	Test No.	Valve Closure Time (sec)	Vibration Frequency (cps)	Vibration Displacement (in)	Pressure Surge at Valve Face (psi)
FIG 10	2	.268	0	0	97
	14	.270	32	.010	97
	74	.260	32	.380	304
FIG 12	17	.250	48	.038	110
	71	.250	48	.170	165
	30	.260	64	.042	101
	76	.251	64	.095	106
	33	.251	80	.024	102
	67	.265	80	.060	116
	46	.264	96	.010	102
	66	.243	96	.043	154
	80	.504	0	0	52
	102	.449	32	.010	59
	120	.466	32	.380	160
	103	.457	48	.038	69
	117	.462	48	.170	116
	90	.481	64	.042	60

TABLE I (Concluded)

Test No.	Valve Closure Time (sec)	Vibration Frequency (cps)	Vibration Displacement (in)	Pressure Surge at Valve Face (psi)
112	.475	64	.095	70
92	.477	80	.024	64
111	.473	80	.060	122
94	.483	96	.010	64
106	.460	96	.043	110
4	.998	0	0	40
10	1.025	32	.010	56
49	.939	32	.380	148
21	.955	48	.038	62
52	.944	48	.170	75
24	.957	64	.042	44
55	.948	64	.095	156
37	.967	80	.024	51
59	.958	80	.060	165
43	.948	96	.010	54
60	.932	96	.043	113

TABLE II

Measured Pressure Surge
Pipeline with Area Transition

	Test No.	Valve Closure Time (sec)	Vibration Frequency (cps)	Vibration Displacement (in)	Pressure Surge (psi)
	233	.127	0	0	162
	235	.129	30	.030	166
	305	.118	30	.300	192
	238	.130	36.5	.005	162
	307	.118	36.5	.295	193
	242	.130	48	.030	159
	310	.132	48	.150	181
	244	.129	59	.045	155
	313	.135	59	.100	165
	248	.129	70	.025	160
	317	.131	70	.080	171
	251	.130	82	.004	155
	319	.125	82	.058	176
	254	.129	96	.015	158
	322	.124	96	.042	175
FIG 14	230	.238	0	0	93
	210	.224	30	.030	95

TABLE II (Continued)

	Test No.	Valve Closure Time (sec)	Vibration Frequency (cps)	Vibration Displacement (in)	Pressure Surge (psi)
	325	.238	30	.280	120
	212	.222	36.5	.006	86
	328A	.241	36.5	.250	139
	215	.224	48	.030	90
	332	.224	48	.150	106
	218	.223	59	.045	90
	334	.239	59	.100	103
FIG 14	222A	.235	70	.025	102
	337	.222	70	.080	126
	223	.230	82	.004	93
	341	.234	82	.060	150
	228	.228	96	.015	101
	345	.228	96	.042	149
	278	.549	0	0	49
	274	.546	30	.030	55
	348	.561	30	.300	137
	271	.550	36.5	.005	51
	351	.567	36.5	.250	190

TABLE II (Concluded)

Test No.	Valve Closure Time (sec)	Vibration Frequency (cps)	Vibration Displacement (in)	Pressure Surge (psi)
270	.535	48	.030	68
352	.569	48	.150	108
267	.548	59	.045	54
357	.532	59	.100	127
264	.541	70	.025	60
358	.559	70	.080	69
260	.543	82	.004	50
362	.563	82	.060	81
257	.541	96	.015	78
364	.565	96	.042	73
385	.843	30	.300	67
382	.853	36.5	.250	72
380	.839	48	.150	62
376	.821	59	.100	61
375	.820	70	.080	70
371	.826	82	.060	66

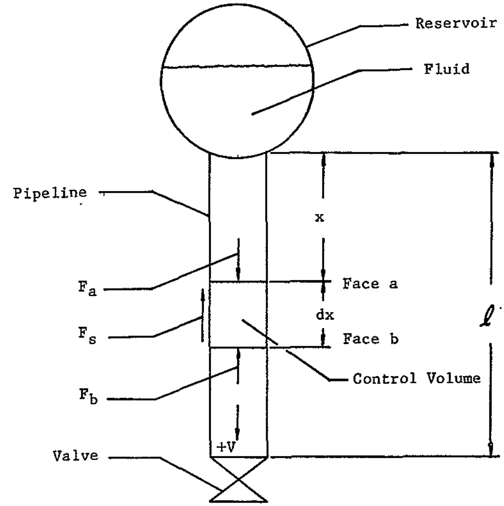

FIGURE 1. SCHEMATIC OF FLUID SYSTEM

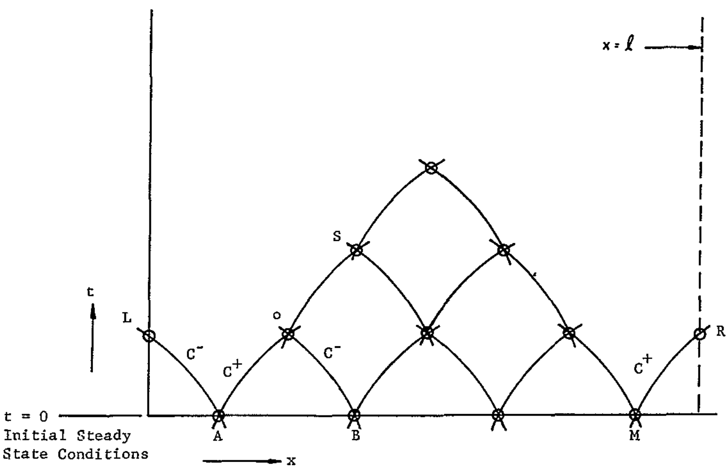

FIGURE 2. CHARACTERISTIC GRID

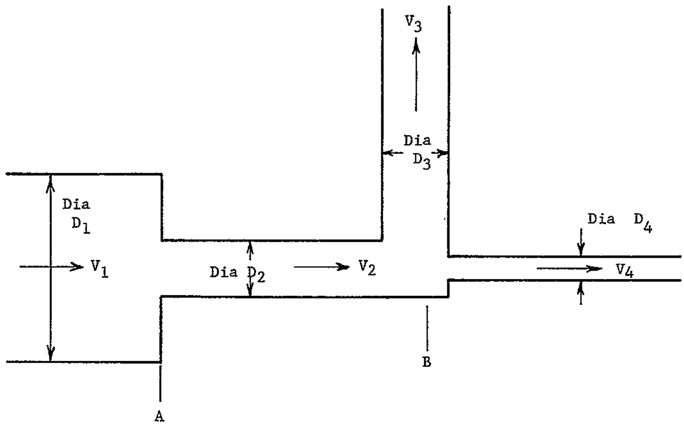

FIGURE 3. SCHEMATIC OF COMPOUND AND BRANCHED FLUID SYSTEMS

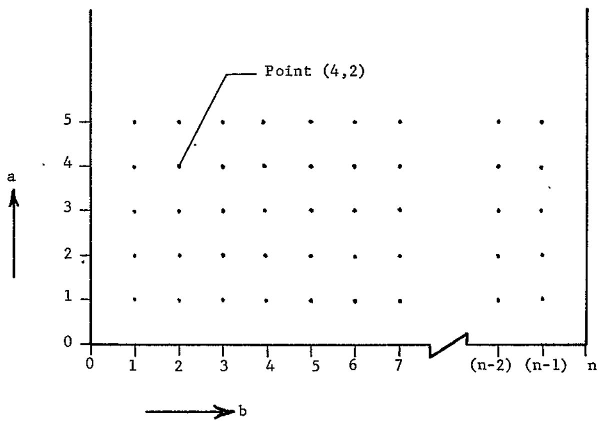

FIGURE 4. MATRIX ARRANGEMENT FOR IDENTIFICATION OF VARIABLES

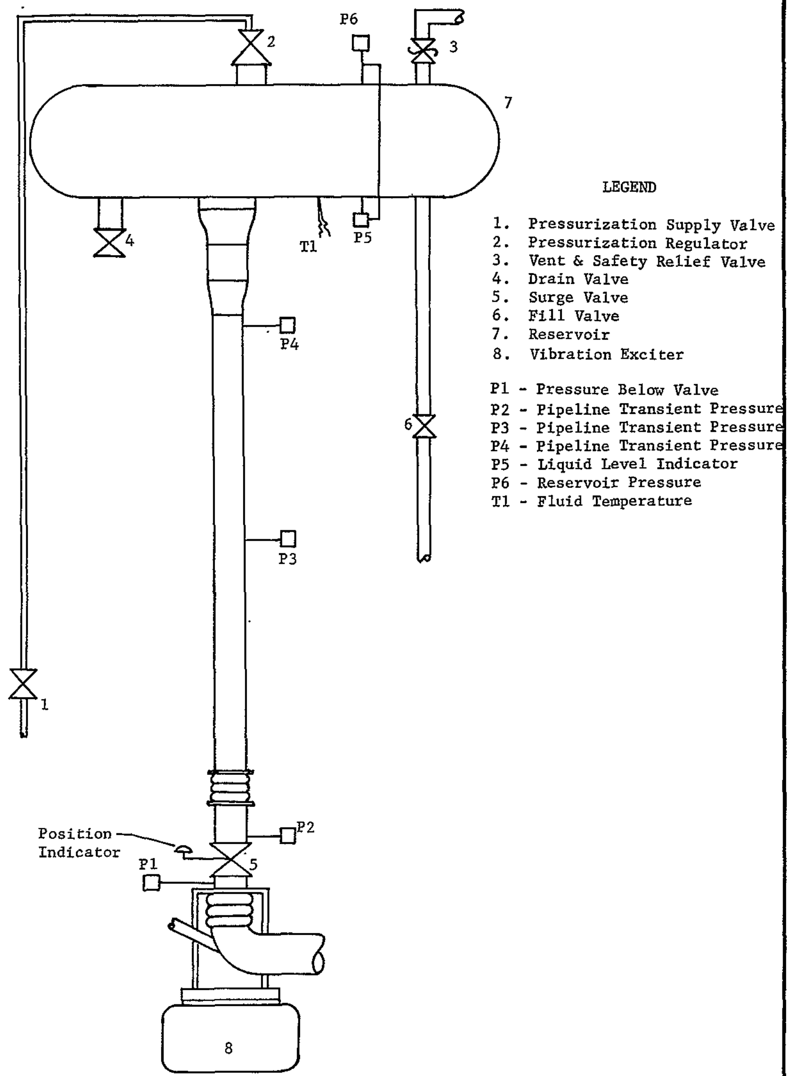

FIGURE 5. SCHEMATIC OF TEST FACILITY
(4-INCH DIAMETER PIPELINE)

R-P&VE-PE
12-1-64

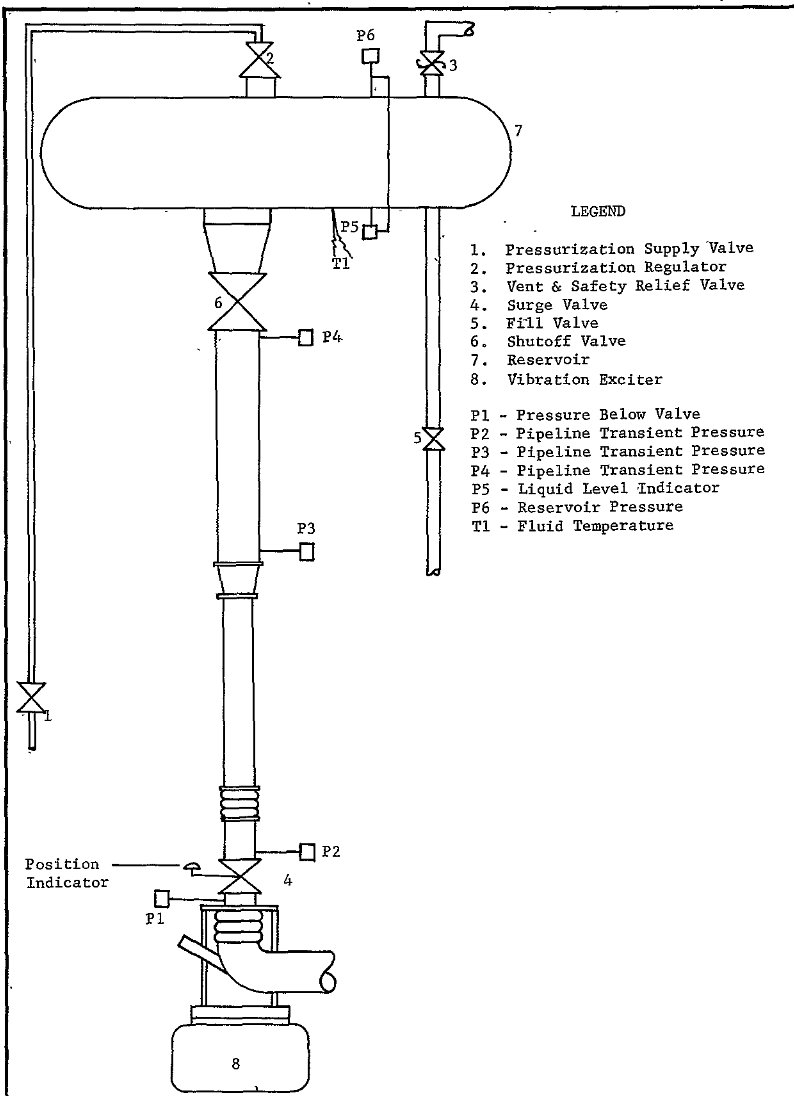

FIGURE 6. SCHEMATIC OF TEST FACILITY
(Pipeline with Area Transition)

R-P&VE-PE
12-1-64

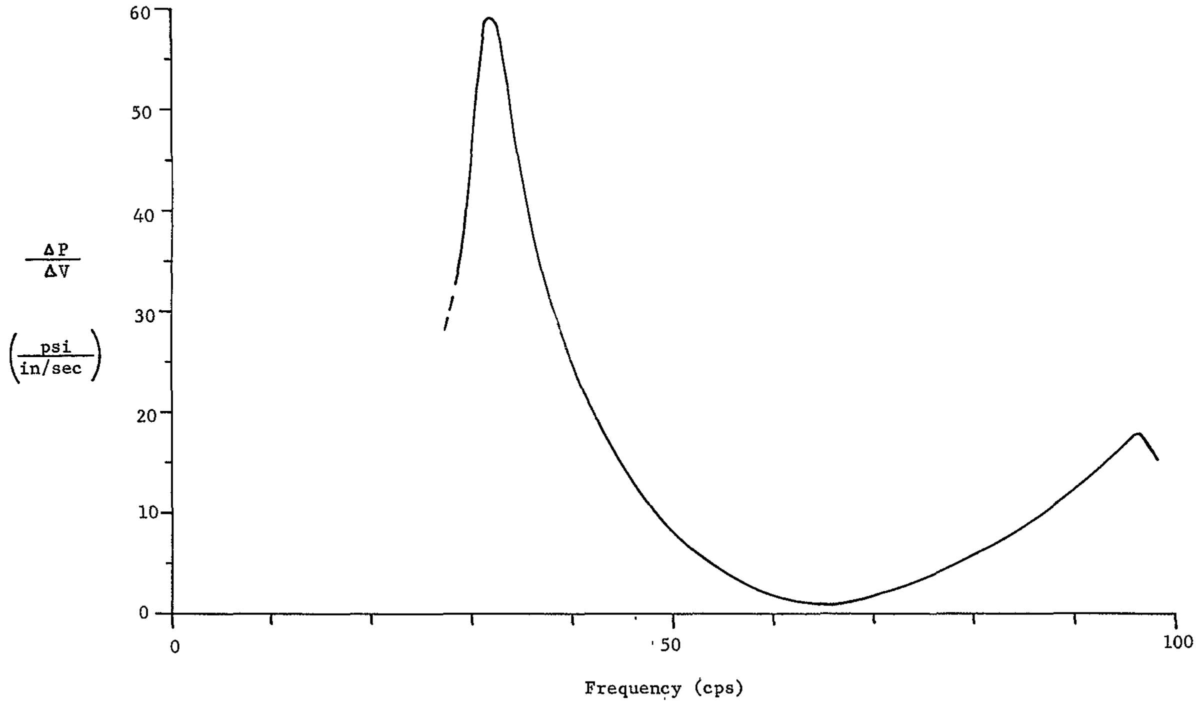

FIGURE 7. NATURAL FREQUENCY OF UNIFORM 4-INCH PIPELINE

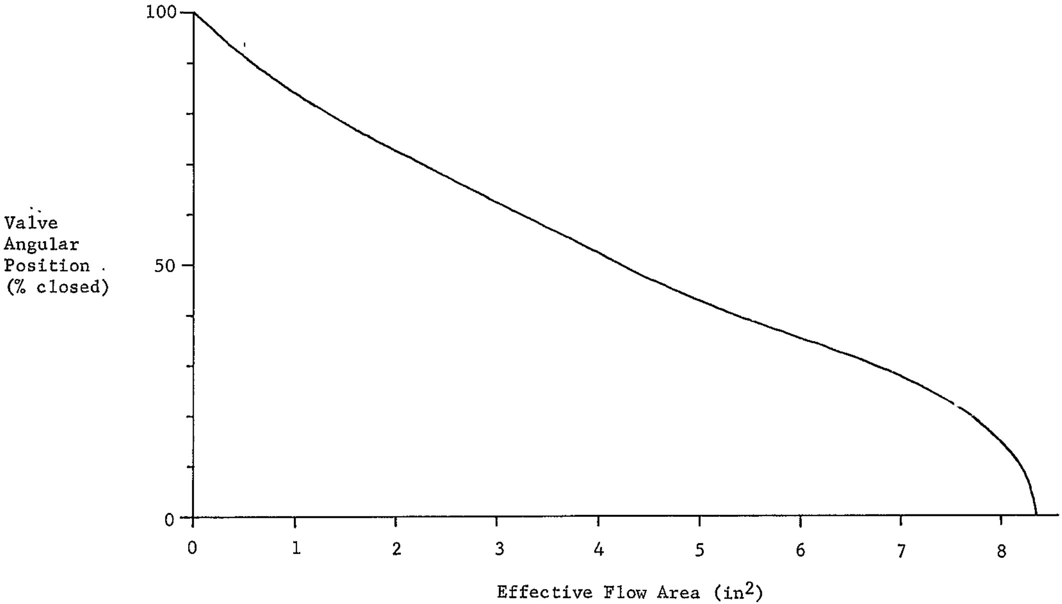

FIGURE 8. STEADY STATE EFFECTIVE FLOW AREA OF BUTTERFLY VALVE

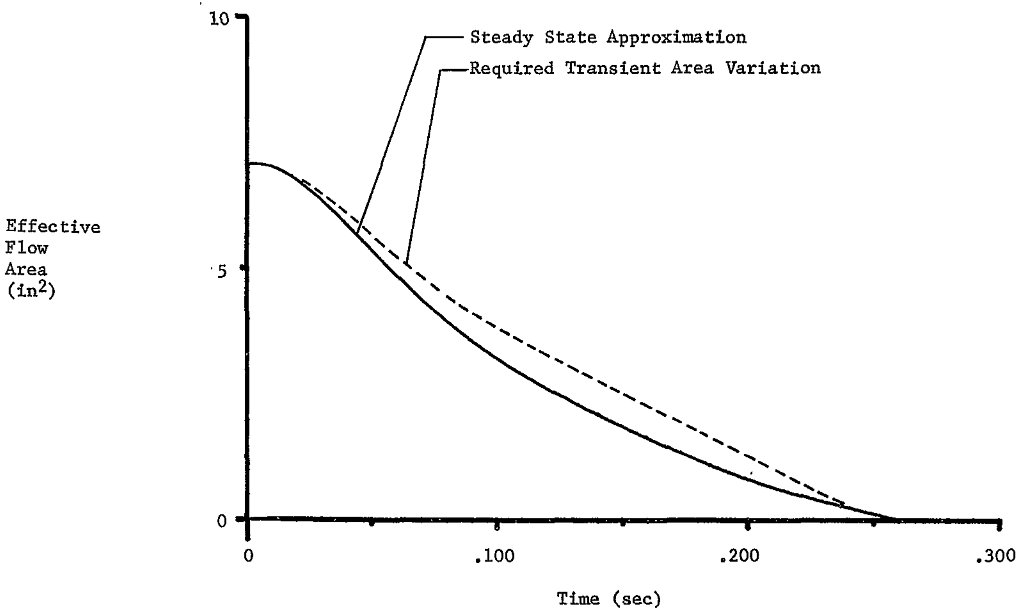

FIGURE 9. EFFECTIVE FLOW AREA DURING VALVE CLOSURE

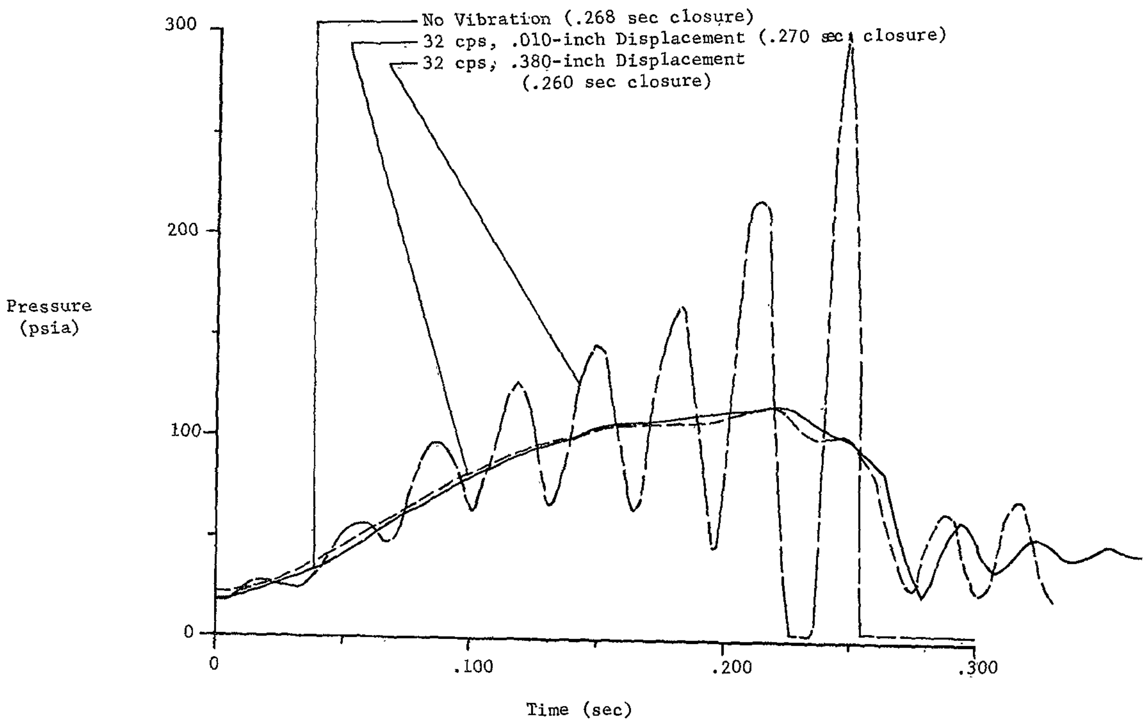

FIGURE 10. MEASURED TRANSIENT PRESSURE, UNIFORM 4-INCH PIPELINE

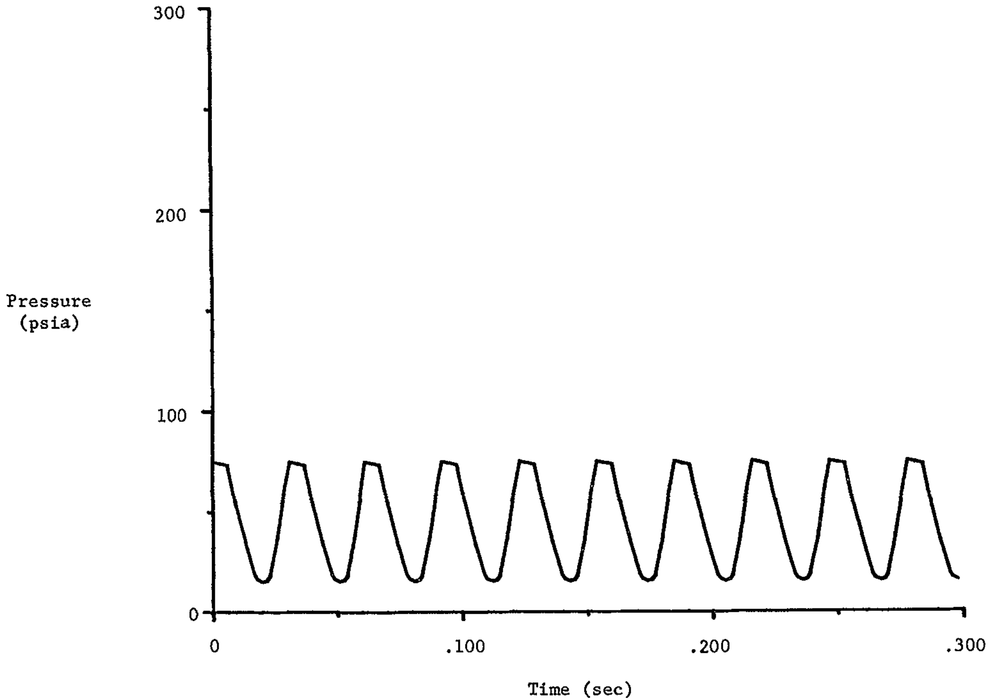

FIGURE 11. STEADY STATE PRESSURE OSCILLATION WITH VALVE CLOSED

(32 CPS, .010-INCH DISPLACEMENT)

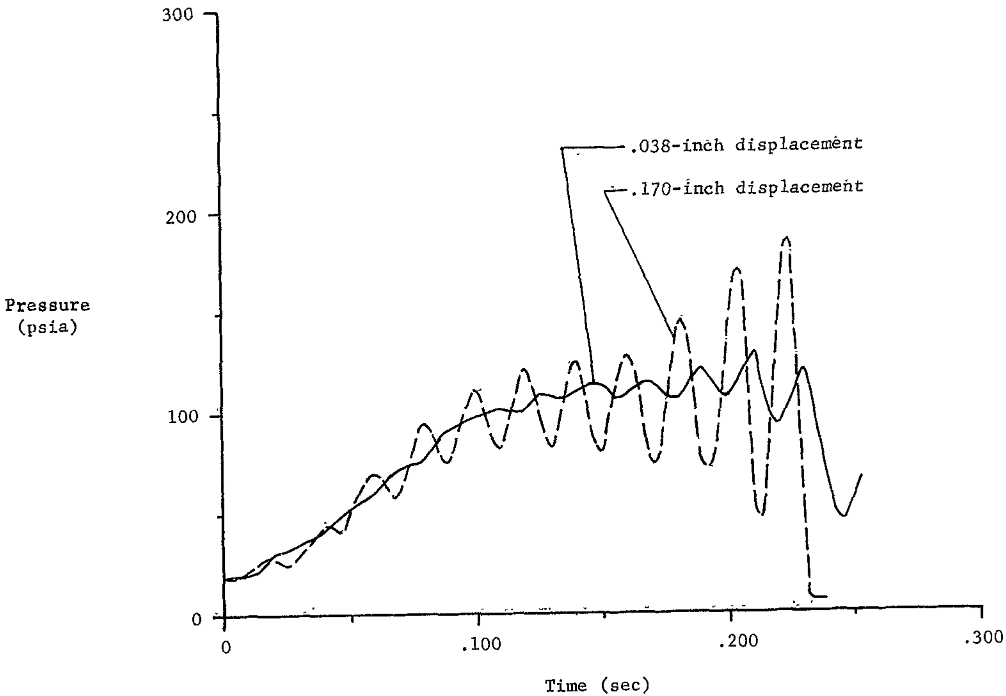

FIGURE 12. MEASURED TRANSIENT PRESSURE, .250 SEC CLOSURE, 48 CPS

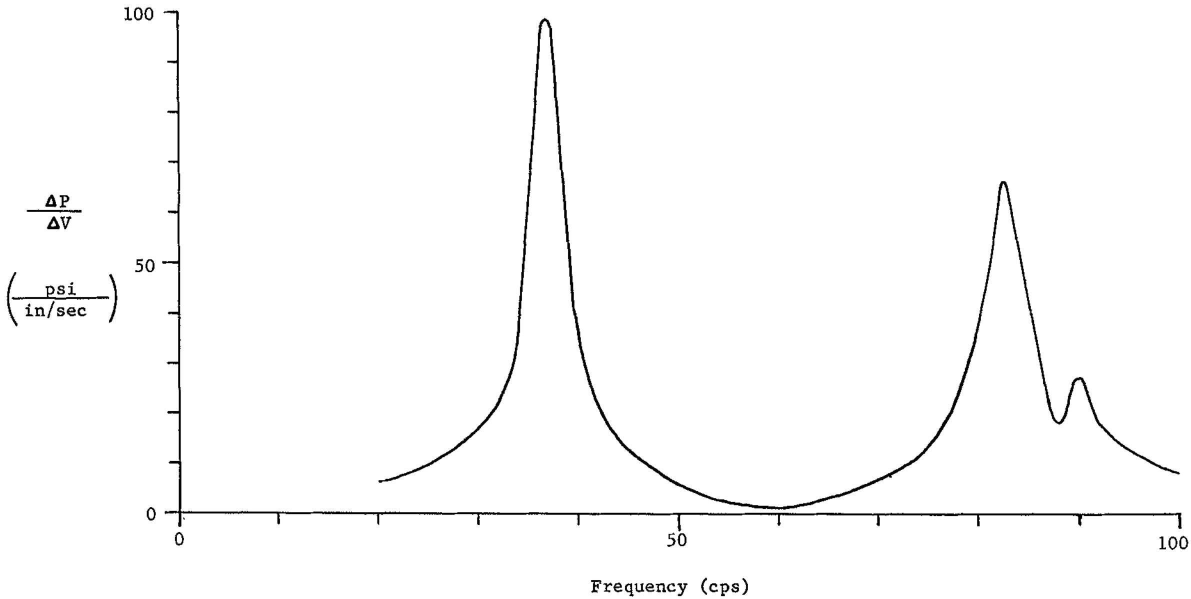

FIGURE 13. NATURAL FREQUENCY OF PIPELINE WITH AREA TRANSITION

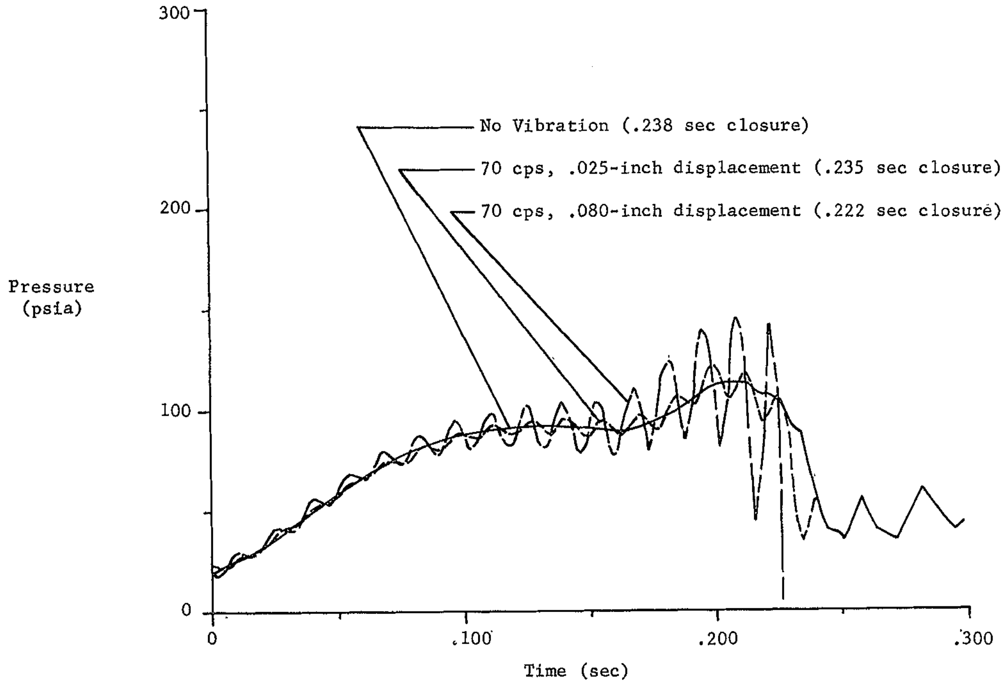

FIGURE 14. MEASURED TRANSIENT PRESSURE, PIPELINE WITH AREA TRANSITION

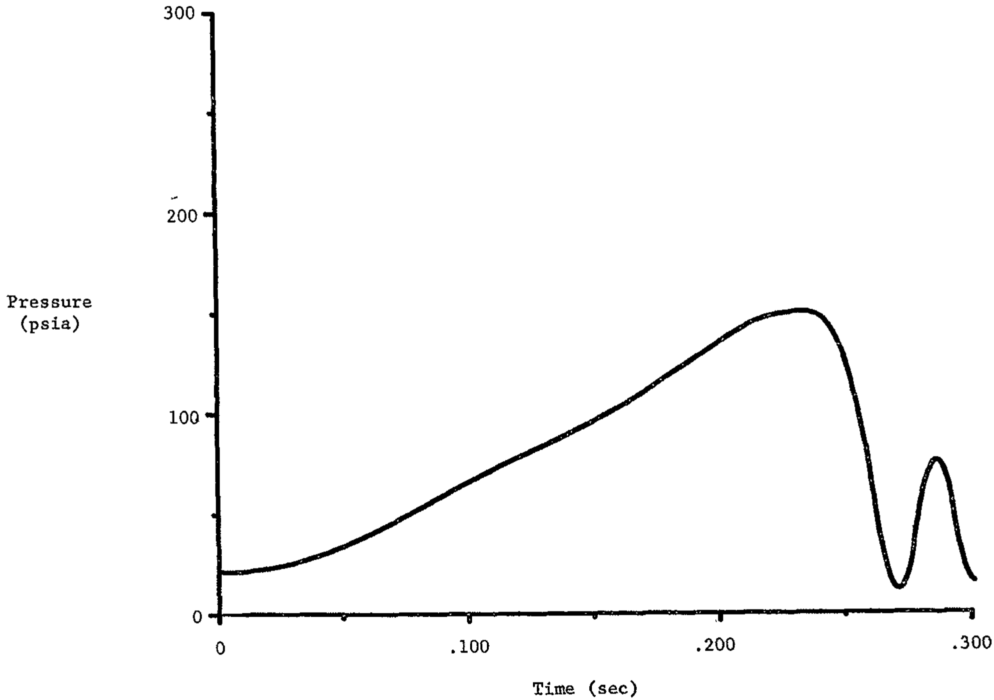

FIGURE 15. CALCULATED PRESSURE, .250 SEC VALVE CLOSURE, STEADY STATE AREA APPROXIMATION

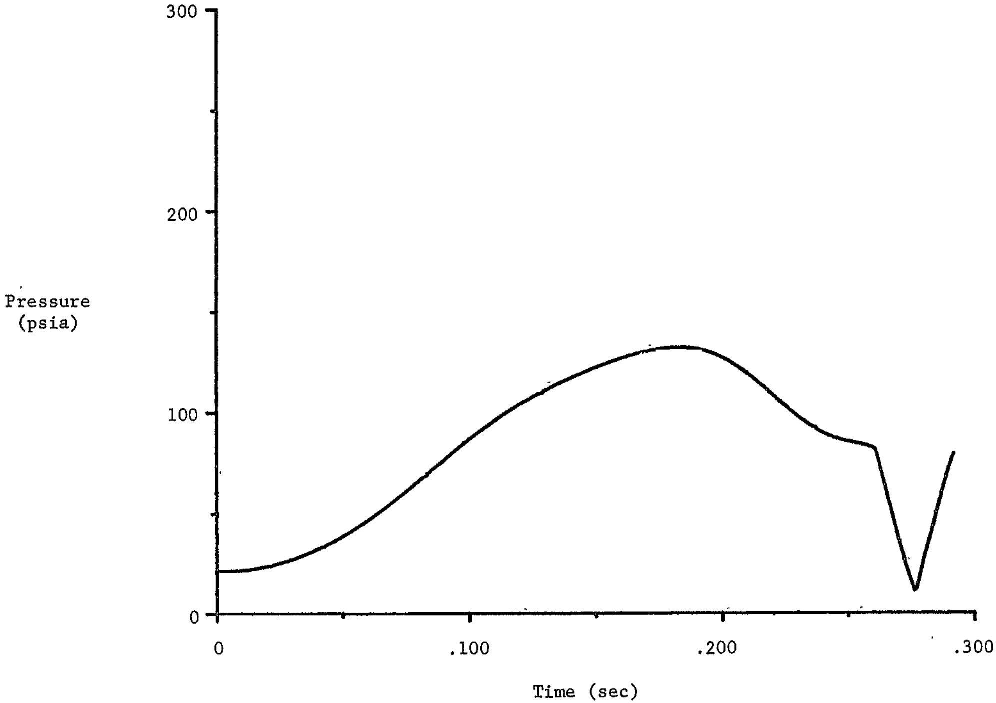

FIGURE 16. CALCULATED PRESSURE, .256 SEC VALVE CLOSURE
WITH REQUIRED TRANSIENT AREA VARIATION

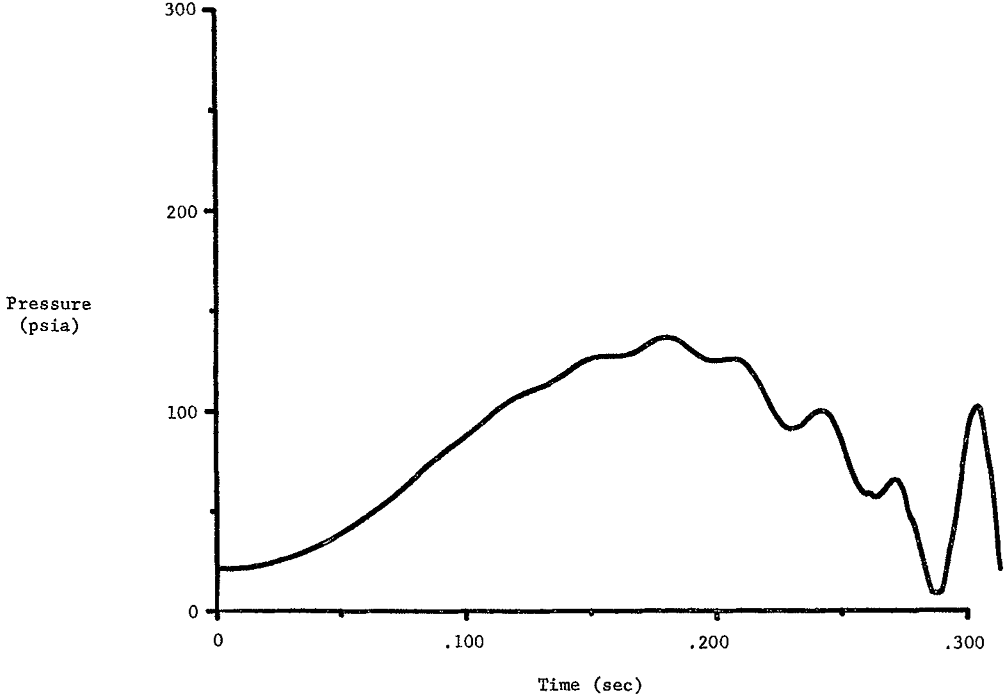

FIGURE 17. CALCULATED PRESSURE, .250 SEC CLOSURE, 32 CPS, .030 INCH DISPLACEMENT

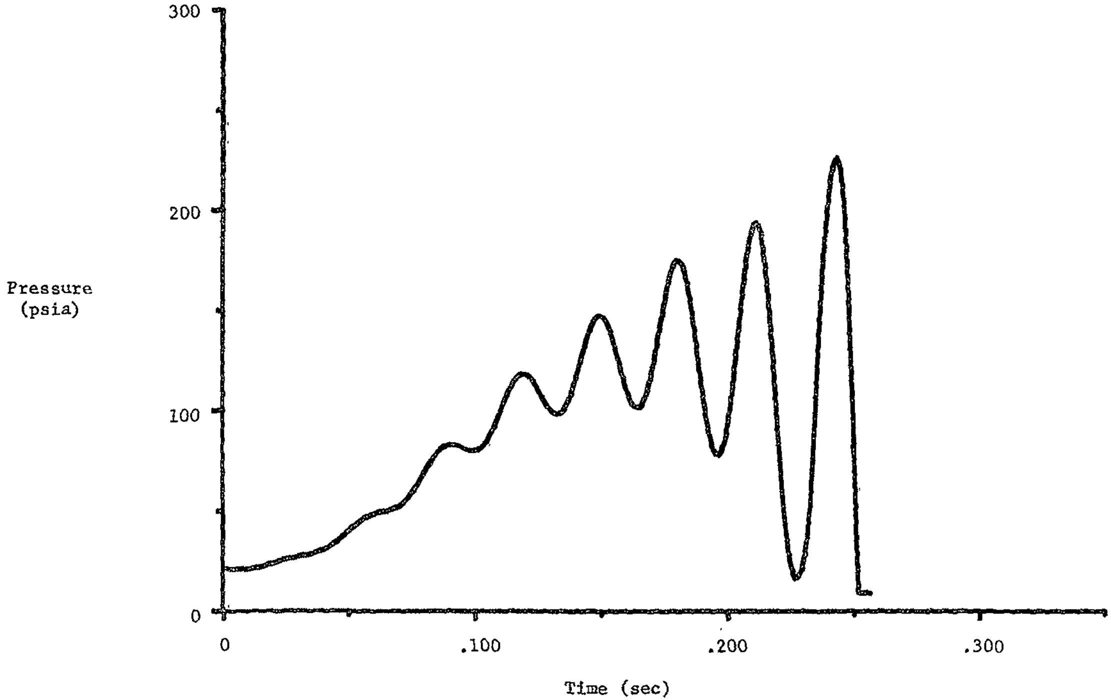

FIGURE 18. CALCULATED PRESSURE, .250 SEC CLOSURE, 32 CPS, .380 INCH

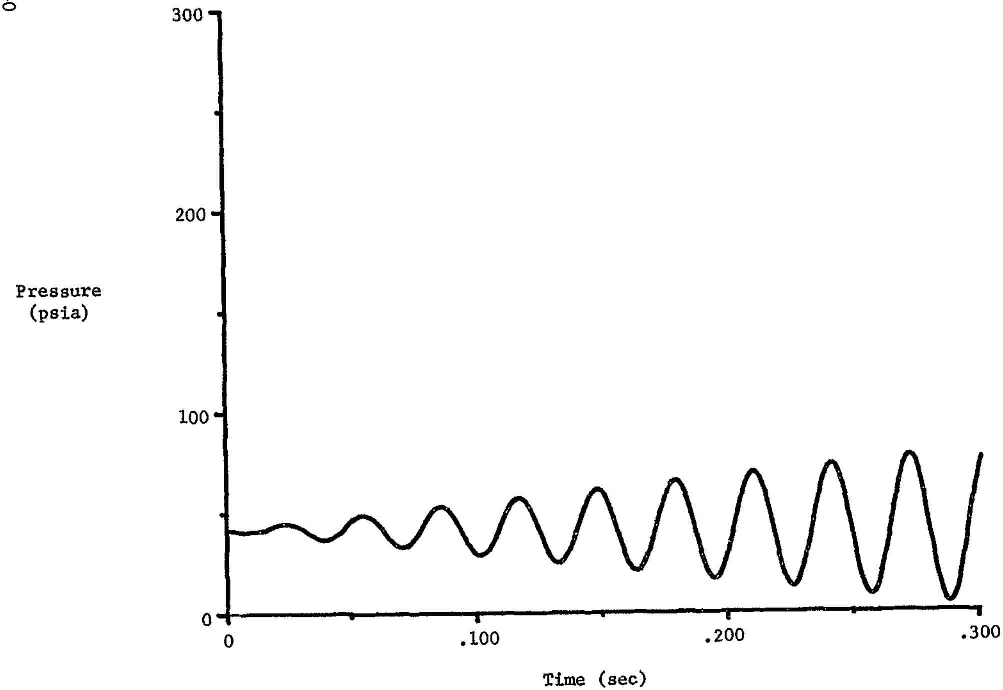

FIGURE 19. CALCULATED PRESSURE OSCILLATION AT NATURAL FREQUENCY (VALVE CLOSED, ZERO DAMPING)

APPENDIX A

DERIVATION OF THE MOMENTUM EQUATION
[Equation (2)]

Consider the control volume of length dx in the uniform pipeline of FIG 1. At face a of the control volume, the pressure (P) acts over the area ($\bar{A}$) exerting a force ($P\bar{A}$) on the control volume.

$$F_a = P\bar{A} \tag{47}$$

(The direction of positive force is the same as the direction of positive velocity)

At face b, the pressure ($P + \frac{\partial P}{\partial x} dx$) acts over the area ($\bar{A}$) exerting a force on the control volume of

$$F_b = -(P + \frac{\partial P}{\partial x} dx)\, \bar{A} \tag{48}$$

A shear force is also exerted on the control volume by the fluid moving past the pipe walls. The direction of the shear force is opposite the direction of the fluid velocity, and its magnitude is $A_\omega t_\omega$. (Where A_ω is the wetted area of the control volume and τ_ω is the shearing stress exerted on the stream by the walls). The wetted area expressed in terms of the pipeline dimensions is

$$A_\omega = \pi D dx \left(\frac{4D}{4D}\right) = \left(\frac{\pi D^2}{4}\right) \frac{4dx}{D} = \frac{4\bar{A}}{D}\, dx \tag{49}$$

The Darcy-Weisbach friction factor is defined as four times the ratio of the wall shearing stress to the dynamic head of the fluid

$$f = \frac{4\,\tau_\omega}{\rho V^2/2g} \tag{50}$$

The magnitude of the shear force in terms of the friction factor is

$$A_\omega \tau_\omega = \frac{4\bar{A}\, dx}{D}\ \frac{f \rho V^2}{2g4} \tag{51}$$

The direction of the shear force is controlled by using the fluid velocity times its absolute magnitude (V|V|) instead of V^2. (V|V|) always has the sign of V, whereas V^2 is always positive. The shear force acting on the control volume then becomes

$$F_s = -\bar{A}\ \frac{dx}{D}\ \frac{f\rho V|V|}{2g} \tag{52}$$

Summing the forces on the control volume

$$\Sigma F = F_a + F_b + F_s = \bar{A}P - \bar{A}\left(P + \frac{\partial P}{\partial x}dx\right) - \bar{A}\ \frac{dx}{D}\ \frac{f\rho V|V|}{2g} \tag{53}$$

The mass of the control volume (which is constant) is

$$M = \frac{\rho}{g}\ \bar{A}dx \tag{54}$$

The acceleration of the control volume is

$$A = \frac{dV}{dt} \tag{55}$$

In unsteady flow, the velocity is a function of both the location along the pipeline (x) and the time (t)

$$V = f\ (x,\ t) \tag{56}$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial t}dt \tag{57}$$

and

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial t} \tag{58}$$

since

$$\frac{dx}{dt} = V \tag{59}$$

$$A = \frac{dV}{dt} = V\ \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} \tag{60}$$

Using Newton's second law

$$\Sigma F = MA \tag{61}$$

$$\bar{A}P - \bar{A}\left(P + \frac{\partial P}{\partial x}dx\right) - \bar{A}\,\frac{f dx}{D}\,\frac{\rho V|V|}{2g} = \frac{\rho}{g}\,\bar{A}dx\left[V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t}\right] \tag{62}$$

Simplifying

$$V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + \frac{g}{\rho}\,\frac{\partial P}{\partial x} + \frac{f}{2D}\,V|V| = 0 \tag{2}$$

APPENDIX B

DERIVATION OF THE CHARACTERISTIC EQUATIONS
[Equations (7), (8), (9), & (10)]

The basic equations used are the wave equations:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{g}{\rho}\frac{\partial P}{\partial x} + \frac{f}{2D}V|V| = 0 \qquad (2)$$

$$\frac{\partial P}{\partial t} + \beta\frac{\partial V}{\partial x} + V\frac{\partial P}{\partial x} = 0 \qquad (6)$$

A linear combination of these equations yields

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{g}{\rho}\frac{\partial P}{\partial x} + \frac{f}{2D}V|V| + \alpha\frac{\partial P}{\partial t} + \alpha\beta\frac{\partial V}{\partial x} + \alpha V\frac{\partial P}{\partial x} = 0 \qquad (63)$$

Rearranging this equation yields

$$\left[\frac{\partial V}{\partial t} + (V + \alpha\beta)\frac{\partial V}{\partial x}\right] + \alpha\left[\frac{\partial P}{\partial t} + \left(V + \frac{g}{\alpha\rho}\right)\frac{\partial P}{\partial x}\right] + \frac{f}{2D}V|V| = 0 \qquad (64)$$

Since P and V are the dependent variables, and x and t are the independent variables.

$$\frac{dP}{dt} = \frac{\partial P}{\partial t} + \frac{dx}{dt}\frac{\partial P}{\partial x} \qquad (65)$$

and

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{dx}{dt}\frac{\partial V}{\partial x} \qquad (66)$$

In equation (64), the total derivations $\frac{dP}{dt}$ and $\frac{dV}{dt}$ can replace the quantities in brackets if

$$\frac{dx}{dt} = V + \alpha\beta = V + \frac{g}{\alpha\rho} \qquad (67)$$

This requires that

$$\alpha = \pm \left(\frac{g}{\beta \rho}\right)^{1/2} \tag{68}$$

Therefore, two equations were obtained, one for the positive α (defining the C^+ characteristic) and another for the negative α (defining the C^- characteristic).

Substituting the positive α into equation (67) yields

$$\frac{dx}{dt} = V + \left(\frac{\beta g}{\rho}\right)^{1/2} \tag{69}$$

Since the velocity of wave propagation is defined as

$$a = \left(\frac{\beta g}{\rho}\right)^{1/2} \tag{70}$$

equation (69) becomes

$$\frac{dx}{dt} = V + a \tag{71}$$

Substituting this into the total differential equations (65) and (66)

$$\frac{dP}{dt} = \frac{\partial P}{\partial t} + (V + a) \frac{\partial P}{\partial x} \tag{72}$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + (V + a) \frac{\partial V}{\partial x} \tag{73}$$

Equation (64) with the positive α substitution becomes

$$\left[\frac{\partial V}{\partial t} + (V + a) \frac{\partial V}{\partial x}\right] + \frac{g}{\rho a} \left[\frac{\partial P}{\partial t} + (V + a) \frac{\partial P}{\partial x}\right] + \frac{f}{2D} V|V| = 0 \tag{74}$$

substituting (72) and (73) into equation (74)

$$\frac{dV}{dt} + \frac{g}{\rho a}\frac{dP}{dt} + \frac{f}{2D} V|V| = 0 \qquad (75)$$

therefore along C^+

$$dx = (V + a)\, dt \qquad (76)$$

$$dV + \frac{g}{\rho a} dP + \frac{f}{2D} V|V| dt = 0 \qquad (77)$$

similarly along C^-

$$dx = (V - a)\, dt \qquad (78)$$

$$dV - \frac{g}{\rho a} dP + \frac{f}{2D} V|V| dt = 0 \qquad (79)$$

These characteristics are shown in FIG 2. If the velocity and pressure are known at any two points on the x-t diagram (points A and B for example) the characteristics can be solved for the conditions at point o. To do this, a linear approximation between A and o and B and o, as shown below, is used.

$$\int_{x_1}^{x_2} f(x)\, dx = f(x_1)(x_2 - x_1) \qquad (80)$$

The characteristics then become

Along C^+

$$(x_o - x_A) = (V_A + a_A)(t_o - t_A) \qquad (7)$$

$$(V_o - V_A) + \frac{g}{\rho_A a_A}(P_o - P_A) + \frac{f}{2D} V_A \left| V_A \right| (t_o - t_A) = 0 \qquad (8)$$

Along C^-

$$(x_o - x_B) = (V_B - a_B)(t_o - t_B) \tag{9}$$

$$(V_o - V_B) - \frac{g}{\rho_B a_B}(P_o - P_B) + \frac{f}{2D} V_B |V_B| (t_o - t_B) = 0 \tag{10}$$

With the conditions given at points A and B, these equations may be solved simultaneously for P_o, V_o, x_o and t_o.

REFERENCES

1. Surge Pressure Study, R-P&VE-PE-64-M-105, March 26, 1964.

2. Experimental Study of Surge Pressures, M-P&VE-PT-308-63, June 28, 1963.

3. Joukowsky, N., "Uber den hydraulischen Stoss in Wasserleitungsrohren," Memoines de l'Academic du Sciences de St. Petersbourg, Vol. 9, 8th Series, 1898.

4. Joukowsky, N., "Waterhammer," translated by Miss O. Simin, Proceedings of the American Water Works Association, Vol. 24, 1904, pp 365-368.

5. Lorenzo, Allievi, Theory of Water-Hammer, translated by Eugene E. Halmos, under auspices of A. S. M. E. and A. S. C. E.

6. Gibson, N. R., "Pressures in Penstocks Caused by the Gradual Closing of Turbine Gates." Transaction of the A. S. C. E., vol. 83, 1920.

7. Quick, Ray S., "Comparison and Limitations of Various Water-Hammer Theories," Mechanical Engineering, Volume 49, No. 5a, Mid-May, 1927.

8. Kerr, S. Logan, "New Aspects of Maximum Pressure Rise in Closed Conduits," Transactions of the A. S. M. E., vol. 51, 1928, HYD-51-3, pp 13-30.

9. Angus, R. W., "Water-Hammer Pressures in Compound and Branched Pipes." Proc. A. S. C. E., 64; 133 (1938).

10. Parmakian, J., Waterhammer Analysis, Prentice Hall Inc., N. Y., 1955.

11. Bergeron, L., Water Hammer in Hydraulics and Wave Surges in Electricity, John Wiley & Sons, New York, 1961.

12. Wood, F. M., "The Application of Heavisides Operational Calculus to the Solution of Problems in Water Hammer." Transactions of the A. S. M. E., vol. 59, 1937, pp 707-713.

REFERENCES (Concluded)

13. Rich, R. George, "Water-Hammer Analysis by the La Place-Mellin Transformation," Transaction of the A. S. M. E., Vol. 67, July 1945, pp 361-376.

14. Streeter, V. L., and Lai, Chintu, "Water-Hammer Analysis Including Fluid Friction," Journal of the Hydraulic Division, Proceedings of the A. S. C. E., vol. 88, No. HY3, May 1962.

15. Lister, M., "The Numerical Solution of Hyperbolic Partial Differential Equations by the Method of Characteristics," Mathematical Methods for Digital Computers, edited by A. Ralston, H. S. Wilf, John Wiley & Sons, N. Y., 1960.

16. Paynter, H. M., "Fluid Transients in Engineering Systems," Handbook of Fluid Mechanics, McGraw-Hill Book Co., New York, 1961, pp 20-1 through 20-47.

17. C. R. C. Standard Mathematical Tables, edited by C. D. Hodgman, Chemical Rubber Publishing Co., Cleveland, Ohio, 1959.

18. Flow of Fluids Through Valves, Fittings and Pipe, by the Engineering Division, Crane Co., Chicago, Technical Paper No. 410, 1957, p 2-14.

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TRANSIENT PRESSURE RESPONSE IN A FLUID SYSTEM AS A VIBRATING VALVE CLOSES

By Cecil A. Ponder, Jr.

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