TURBULENCE IN THE UPPER ATMOSPHERE

by C. G. Justus and H. D. Edwards

Georgia Tech Project A652-001

Contract No. NsG 304-63

FACILITY FORM 602

N65 19753 (ACCESSION NUMBER) — (THRU)

56 (PAGES) — 1 (CODE)

CR 57258 (NASA CR OR TMX OR AD NUMBER) — 13 (CATEGORY)

Prepared for
National Aeronautics and Space Administration
Washington 25, D. C.

GPO PRICE $ _____

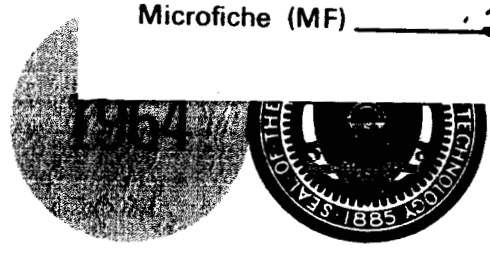
OTS PRICE(S) $ _____

UNPUBLISHED PRELIMINARY DATA

Hard copy (HC) 3.00

Microfiche (MF) .50

July 1964



Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia

REPORTS CONTROL No. 6

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*The studies reported here were also supported by the Air Force Cambridge Research Laboratories under Contract Af19(628)3305.

SCIENTISTS

The following scientists contributed to the technical work

1. Dr. H. D. Edwards - - - - - - - - - - - - - - - Project Director
2. Dr. W. M. Schofield - - - - - - - - - - - - - - Research Physicist
3. R. N. Fuller- - - - - - - - - - - - - - - - - - Research Assistant
4. C. G. Justus- - - - - - - - - - - - - - - - - - Graduate Research Assistant
5. A. J. Lineberger- - - - - - - - - - - - - - - - Graduate Research Assistant
6. W. B. Moseley - - - - - - - - - - - - - - - - - Graduate Research Assistant
7. J. M. Reynolds- - - - - - - - - - - - - - - - - Graduate Research Assistant
8. J. R. Williams- - - - - - - - - - - - - - - - - Graduate Research Assistant
9. A. Woodrum- - - - - - - - - - - - - - - - - - - Graduate Research Assistant
10. H. W. Morgan- - - - - - - - - - - - - - - - - - Student Assistant
11. J. E. Westmoreland- - - - - - - - - - - - - - - Student Assistant
12. B. H. Zellner - - - - - - - - - - - - - - - - - Student Assistant

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TURBULENCE IN THE UPPER ATMOSPHERE

C. G. Justus and H. D. Edwards
Georgia Institute of Technology
Atlanta, Georgia

ABSTRACT

19753

By direct investigation of fluctuating turbulent velocities determined from chemical releases, data have been obtained which indicate that turbulence observed up to altitudes near 105 km is ambient turbulence, produced by wind shears. This turbulence is isotropic only for scales $\approx$ 1 km or less. The smallest eddies are on the order of 20 m in size with characteristic velocities and times of about 2 m/sec and 10 sec. The largest eddies observed are about 6 km in size with characteristic velocities and times of about 15 m/sec and 400 sec. It is thought that tidal waves and gravity wave components may acccunt for the major portion of all velocity fluctuations with size scales greater than 6 km.

The turbopause, or upper height limit of turbulence of the scales observed, usually occurs within $\pm$ 5 km of the 105 km level. This cutoff altitude appears to be the result of the rate of energy supplied by the wind shears becoming too small to maintain the turbulence in the presence of a dissipation rate which is increasing rapidly with altitude.

Growth measurements on diffusing globules produced in the chemical release clouds indicates that the Batchelor form of turbulent diffusion is applicable in this height range.

AUTHOR →

INTRODUCTION

Edwards et al (1963) and other groups have reported that chemical release clouds usually become globular in appearance below some altitude, usually around 105-110. It is presumed that these globules are associated with turbulence but there remains some question (Nawrocki and Papa (1963) and Cote (1962)) as to whether this turbulence is naturally occurring ambient turbulence or is in some way produced by either the ejection mechanism or a reaction of the ejection vehicle with the ambient.

In the following report an attempt is made to examine the turbulence quantitatively, comparing results whenever possible with the predictions of existing turbulence theories. It is hoped that these results will shed some light on this problem of the exact cause of the turbulence.

In an attempt to unify the thinking with regard to turbulence and turbulent eddies Stewart (1959) proposed the following definitions:

Turbulence - "A fluid is said to be turbulent if each component of the velocity is distributed irregularly and aperiodically in time and space, if the flow is characterized by a transfer of energy from larger to smaller scales of motion, and if the mean separation of neighboring fluid particles tends to increase with time."

Eddy - An eddy is "a volume of fluid moving more or less coherently with respect to the mean flow."

These definitions exclude from the realm of turbulence such two dimensional phenomena as vortex sheets, whirlpools, convection cells, and internal waves.

Turbulent motion is in many ways analogous to the random molecular motions responsible for the phenomena of viscosity, diffusion, and conductivity in

gases. There are, however, several differences between these two types of motion. First, a molecule is under the influence of a very small number of other molecules in the gas and each molecule moves about somewhat freely, whereas a fluid element in turbulent motion cannot move independently of the general motion of the other fluid elements. Turbulent motion is less random, or more ordered, than molecular motions. Secondly, turbulent motion requires a continuous source of energy to maintain it. If the air is thermally unstable, that is, cool air over warm air, the potential energy of the unstable arrangement can supply the turbulence. In a stably stratified region of the atmosphere, such as that above 85 km, wind shears provide the only source of energy for maintaining turbulence.

According to standard turbulence theories (See Townsend (1956).) the eddies in a turbulent field have a spectrum of sizes ranging from the largest eddies, which are being supplied with energy from the source, through the intermediate sized energy containing eddies on down to the smallest eddies, which lose their energy by the effects of viscosity. Each eddy size interacts extensively only with other eddies of neighboring size so that eddies containing energy lose energy to only slightly smaller eddies which in turn lose their energy to still slightly smaller eddies, and so on until the smallest eddy size is reached.* The effect of viscosity is to remove energy from only one size of eddies and not to redistribute it among other eddy sizes, although the viscous stresses can convert energy into heat or accelerate neighboring particles.

---

* "Big whirls have little whirls that feed on their velocity
And little whirls have lesser whirls, and so on to viscosity."
- L. F. Richardson

Previous investigators have employed two primary means of investigating the ionospheric altitude region near 100 km with regard to the existence of turbulence. These methods are (1) direct investigation of the turbulent velocity fluctuations and (2) investigation of the diffusion characteristics of materials released into the atmosphere.

Up until the present time the first method has been used only in connection with radio echo observations of meteor trails such as those by Greenhow and Neufeld (1959a, 1959b, 1960). Here the instantaneous horizontal wind is U + u where U is the mean horizontal wind determined, for example, by averaging all the wind determinations over a one hour period, and u is considered to be the instantaneous turbulent wind fluctuation.

Accurate methods have now been developed for determining winds by triangulation and tracking of artificial chemical clouds released into the atmosphere by rockets. (See Albritton, et al (1962); Justus, et al (1964a, 1964b).) Using these methods it is possible to measure instantaneous wind velocities over a range of altitudes. Averaging of the wind data allows the determination of the north-south and east-west components of the mean winds, U (z) and V (z), as functions of altitude, z. The vertical component of mean wind W (z) is considered to be zero. The instantaneous velocity components determined at a height z are then U (z) + u (z), V (z) + v (z), and w (z), where u, v, and w are considered to be the instantaneous components of the turbulent velocity fluctuations. Note that w is not necessarily zero. The equivalent notation (u_1, u_2, u_3) is sometimes used instead of (u, v, w) for the components of the turbulent velocity.

SCALES OF THE MOTION

Vertical Autocorrelation Scale of the Mean Winds

Since the mean winds U (z) and V (z) are not static or uniform there can also be scales associated with the mean wind field. Liller and Whipple (1954) performed a vertical autocorrelation analysis on the mean wind profile determined from visual meteor trails. The vertical autocorrelation coefficient G_U (δ z) is given by

$$G_U(\delta z) = \frac{\Sigma [U(z)\, U(z + \delta z)]}{\{\Sigma [U(\delta)]^2\, \Sigma [U(z + \delta z)]^2\}^{\frac{1}{2}}} \qquad (1)$$

for the U component of the mean wind profile. A correlation coefficient for the V component is given by substitution of V for U in (1). The sums in (1) run over all pairs of data points separated by a vertical distance δ z, or the sums may be replaced by integration if U is considered as a continuous function. Physically, this analysis determines the degree of correlation between the wind profile and the wind profile displaced by an amount δ z in the vertical direction. The values of the coefficient G should start at + 1 for δ z = 0, go to 0 for $\delta z = \frac{1}{4}\lambda$, and on to - 1 for $\delta z = \frac{1}{2}\lambda$, where λ is the vertical "wave length" of the mean wind component profile.

Liller and Whipple obtained an average value of 5.2 km for L_v, the distance δ z for which G = 0. This value is applicable for the height region somewhere between 82 and 113 km. Using the Liller and Whipple wind data Hines (1960) subtracted a constant shear wind from the wind profile and multiplied the residual wind by a height varying scaling factor to compensate for the increase in amplitude of the wind with altitude. After this alteration Hines obtained a value of 4.0 km for L_{v2}, the modified zero autocorrelation distance.

Wind data obtained from about 30 chemical releases launched from Eglin AFB, Florida during 1962 and 1963 has been put to vertical autocorrelation analysis. The data was broken up into three height intervals 70-90 km, 90-115 km, and 115-170 km. The representative mean altitude for each height range was 81 km, 104 km and 130 km. Three types of zero autocorrelation lengths were calculated: L_v for unmodified data, L_{v1} for residual winds after subtraction of a representative constant shear wind, and L_{v2} for the residual winds multiplied by a height scaling factor.

Figure 1 shows a typical set of results for G_U and G_V. They were obtained from a release occurring at 05:15 CST on 16 October 1962 and covering an altitude range of 92-106 km. The autocorrelation curve G_V (δ z) crosses zero at L_v = 8.6 km, the corresponding value for G_U (δ z) is 7.5 km.

Figure 2 shows the average results for L_v, L_{v1}, and L_{v2} in each of the three height ranges. The Liller and Whipple value agrees well with the L_v data and the Hines value shows good agreement with the L_{v2} data. All three L scales show an increase in magnitude with increasing altitude, the rate of increase being about the same in all three cases.

Zimmerman (1964) has suggested that the vertical scale of the mean winds is connected with the scale height, H. A graph of H versus altitude is also included in Figure 2 for comparison. It appears that there is reasonable correspondence between the values for L_v and H in the height range from 90-105 km but the L_v values fall below the values for H in the height regions above and below this.

Motion Spectrum Scale of the Mean Winds

Another method for obtaining information about the scale of the mean

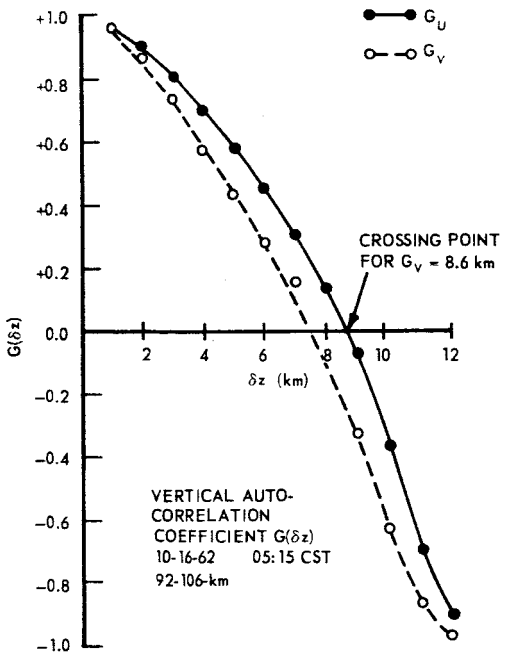

Figure 1. Typical Vertical Autocorrelation Curve.

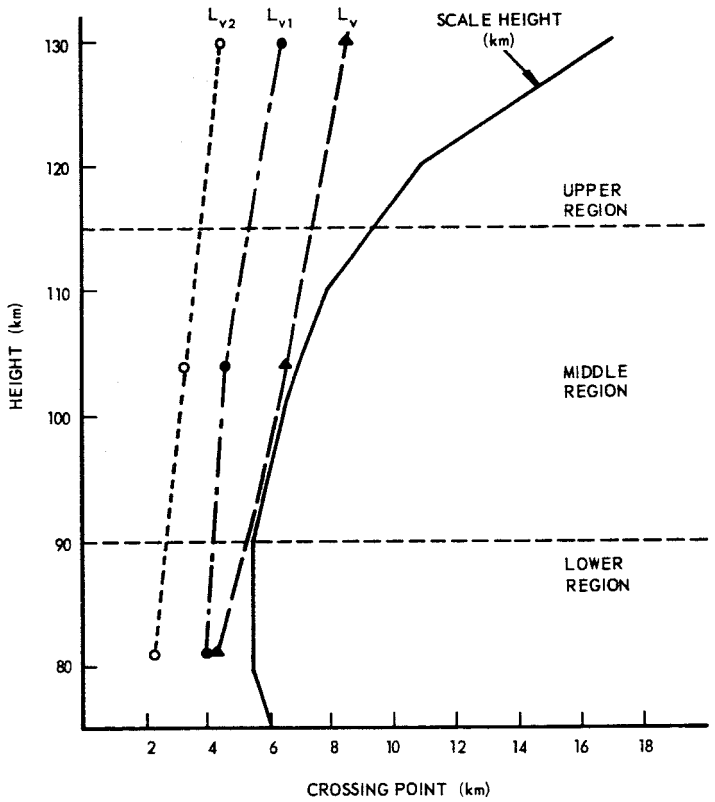

Figure 2. Vertical Autocorrelation Scales Versus Altitude.

winds is the evaluation of the point $\delta z = L_s$ at which the motion spectrum function is a maximum. The motion spectrum function, $F(\delta z)$, is obtained by averaging the square velocity differences, that is

$$F(\delta z) = \langle [U(z) - U(z + \delta z)]^2 \rangle \qquad (2)$$

and similarly for the V component, where the average is taken over all data points separated by a height difference δz.

Blamont and de Jager (1961, 1962) report results of a motion spectrum analysis of four sodium trails showing that for small values of δz, $F(\delta z) \sim (\delta z)^n$ where $n = 1.4 \pm 0.2$. They also report that $F(\delta z)$ graphs show maxima which correspond with the vertical correlation length. Evaluation of the point $\delta z = L_s$ at which these maxima occur thus provides another method of estimating the vertical scale of the mean winds.

The chemical release wind data used in the vertical autocorrelation analysis was also analyzed in a similar fashion for motion spectrum features, the data being divided into three height ranges, roughly, 70 to 93 km, 93 to 112 km and 112 to 143 km. Figure 3 shows the results of a typical motion spectrum analysis. These data are from a release occurring 3 December 1962 at 18:50 CST and are valid for the height region 112-143 km. It is seen that for small δz the exponent n for the spectrum function power law is 1.71. A maximum in $F(\delta z)$ occurs at $\delta z = 12$ km, thus L_s is 12 km for this graph.

Figure 4 shows the averaged results of the evaluation of the maximum point L_s for each of the three altitude regions. The L_s values, like the L_v vertical autocorrelation scales, agree with the scale height, especially in the lower and middle height regions, L_v being somewhat less than the scale height in the upper region.

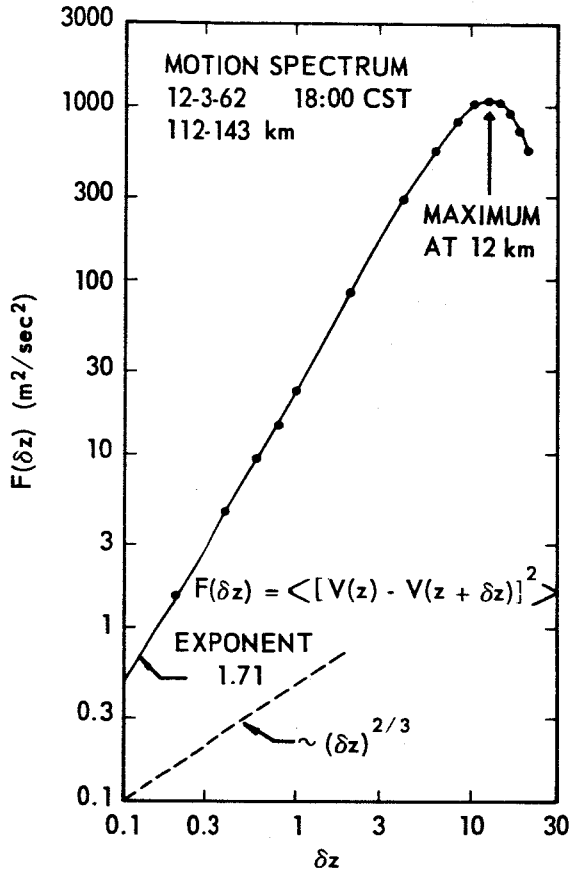

Figure 3. Typical Motion Spectrum Curve.

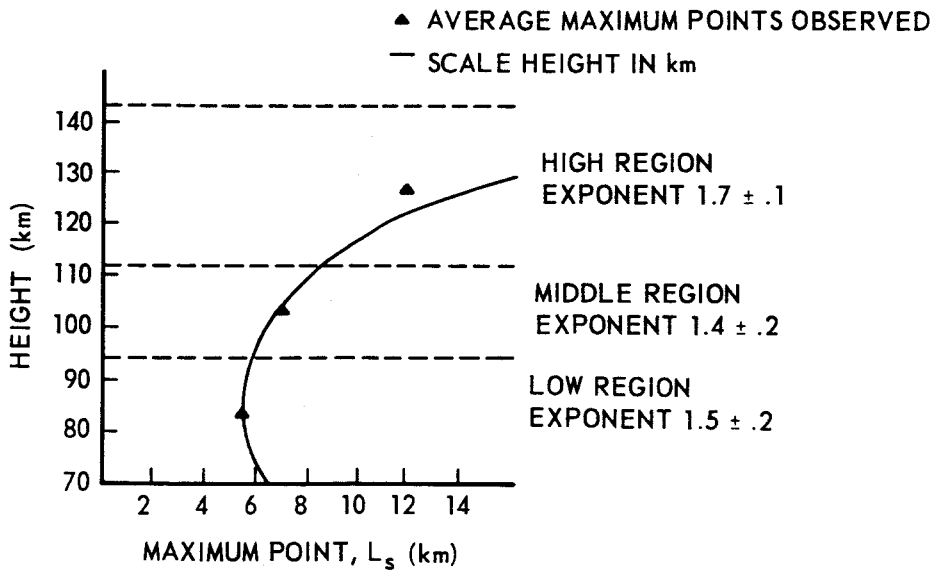

Figure 4. Motion Spectrum Scale Versus Altitude.

Figure 4 also tabulates the average exponents observed for F (δ z). The exponent is almost constant over the altitude range examined, increasing only slightly with increasing altitude.

Zimmerman (1962) has pointed out that the motion spectrum analysis by Blamont and de Jager (1961), yielding an exponent of 1.4 $\pm$ .2 is in agreement with the shear turbulence theory of Tchen (1954) which predicts an exponent of 4/3. Zimmerman also shows that the data of Blamont and de Jager support Tchen's theory with regard to the energy spectrum. He calculates the average turbulent energy per unit mass E, given by

$$E = \left| < U^2 (z) > - < U^2 (z + \delta z) > \right| \qquad (3)$$

and shows that $E \sim \delta z^{2/3}$ for small δz.

Roper and Elford (1963) report that the motion spectrum function F (δ z) is proportional to $\delta z^{4/3}$ if only height separation is considered, but that F (δ r) is proportional to $\delta r^{2/3}$ if F is considered as a function of the total separation δ r.

Analysis of chemical release wind data is presently underway to calculate F (δ r) and E. Preliminary results tend to confirm the conclusions of Zimmerman, and Roper and Elford.

Motion Spectrum of the Turbulent Winds

Based on work by Kolmogoroff (1941) it was shown by Batchelor (1947) that the turbulent motion spectrum function

$$f (\underline{\delta r}) = < [u (\underline{r}) - u (\underline{r} + \underline{\delta r})]^2 > \qquad (4a)$$

obeys the relationship

$$f(\underline{\delta r}) = C_1 (\varepsilon_t \delta r)^{2/3} \left(1 + \frac{\delta r_1}{3 \delta r^2} \right) \qquad (4b)$$

where C_1 is an absolute constant, δr is the magnitude of $\underline{\delta r}$ and δr_1 is the component of $\underline{\delta r}$ in the direction of the turbulent wind component u. See Sutton (1953) for a summary of the work of Kolmogoroff and Batchelor on this topic.

For most purposes (4 b) may be written as

$$f(\delta r) \sim (\delta r)^{2/3} \qquad (5)$$

which is known as Kolmogoroff's Law.

Turbulent winds determined from 13 chemical releases launched from Eglin AFB, Florida from 1959 to 1963 have been analyzed to determine the applicability of (5) to the turbulence of the upper atmosphere. The results, shown in Figure 5, indicate that Kolmogoroff's Law is not followed for $\delta r \gtrsim 1$ km.

The figure indicates that the law may be followed for $\delta r < 1$ km but the data points are too uncertain to be conclusive. Kolmogoroff's Law is based on the assumption of isotropic turbulence and it is shown later that the turbulence may be isotropic only for eddy scales of 1 km or less.

The Mixing Length Scale

An analogy has been made between molecular and turbulent motions by introducing the concept of mixing length. According to the mixing length idea of turbulent motion, eddies transport momentum from one level of the flow to another and the transport of momentum from the level z to the level

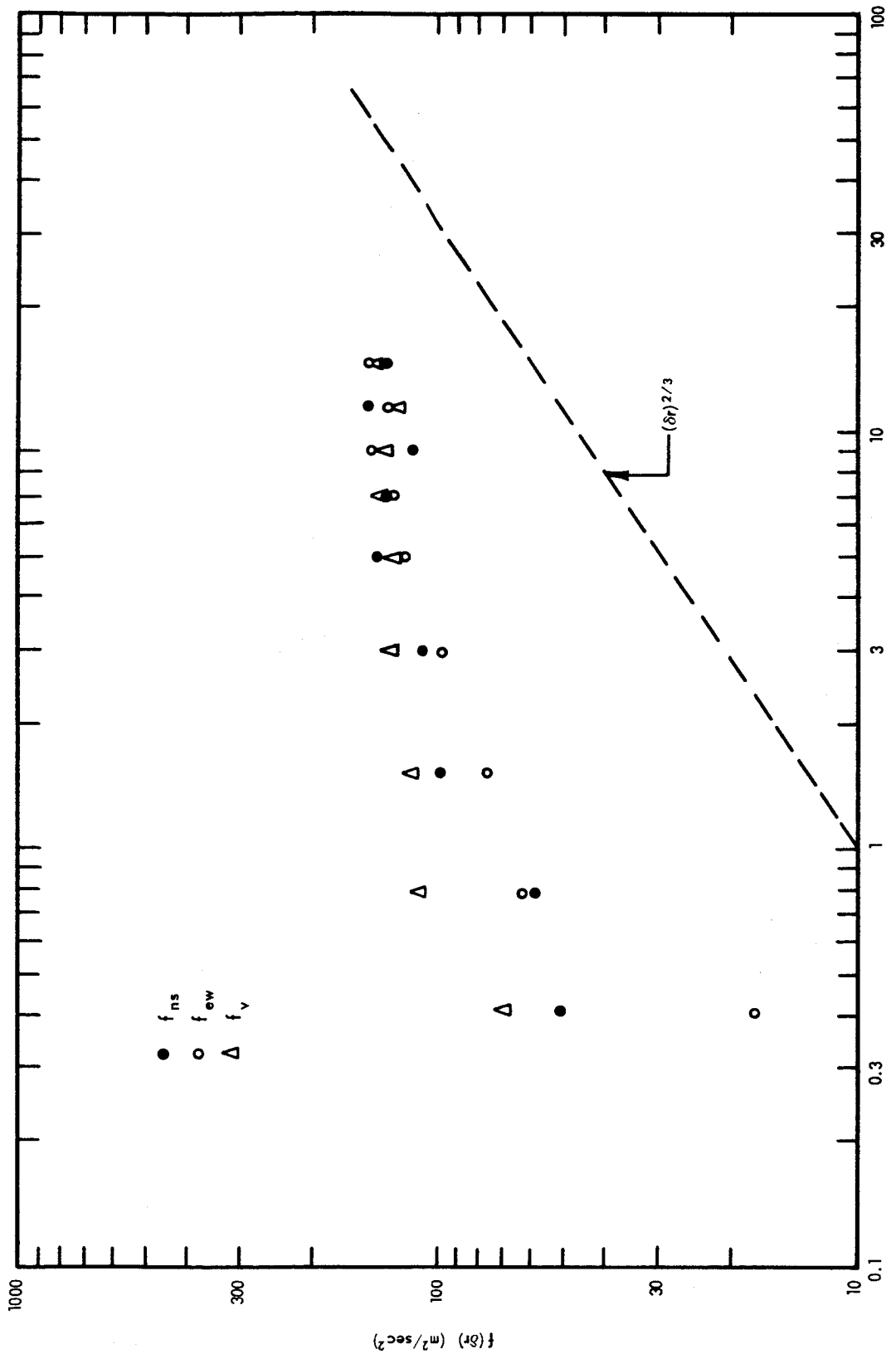

Figure 5. Motion Spectrum of Turbulent Winds.

$z + L_m$ produces a fluctuation, u, in the mean velocity, U, given by

$$u = U(z + L_m) - U(z) \approx L_m \frac{\partial U}{\partial z} \tag{6}$$

here L_m is the mixing length. Thus the magnitude of the mixing length can be found approximately by

$$L_m \approx \frac{u}{(\frac{\partial U}{\partial z})} \tag{7}$$

Figure 6 shows the averaged results obtained from several chemical releases. The mixing length scale is seen to remain fairly constant from 92 to 108 km at a value close to 0.8 km and then increase rapidly above 108 km to about 3.3 km at 112 km altitude.

Scale of the Smallest Eddies

Standard theories of homogeneous turbulence provide a method of evaluating the scale of the smallest, energy dissipating eddies. The size of these eddies, l_e, should be given by

$$l_e = (\frac{\eta^3}{\varepsilon})^{\frac{1}{4}} \tag{8}$$

where η is the kinematic viscosity of the atmosphere, and ε is either the rate, ε_s, at which the wind shears supply energy to the turbulence or the rate, ε_t, at which the turbulence is dissipating energy as heat. For isotropic turbulence ε_s is presumed to be approximately equal to ε_t. The kinematic viscosity can be evaluated up to a height of 90 km from values given in the U. S. Standard Atmosphere (1962) and above this altitude from the formula used for generating the Standard Atmosphere tables

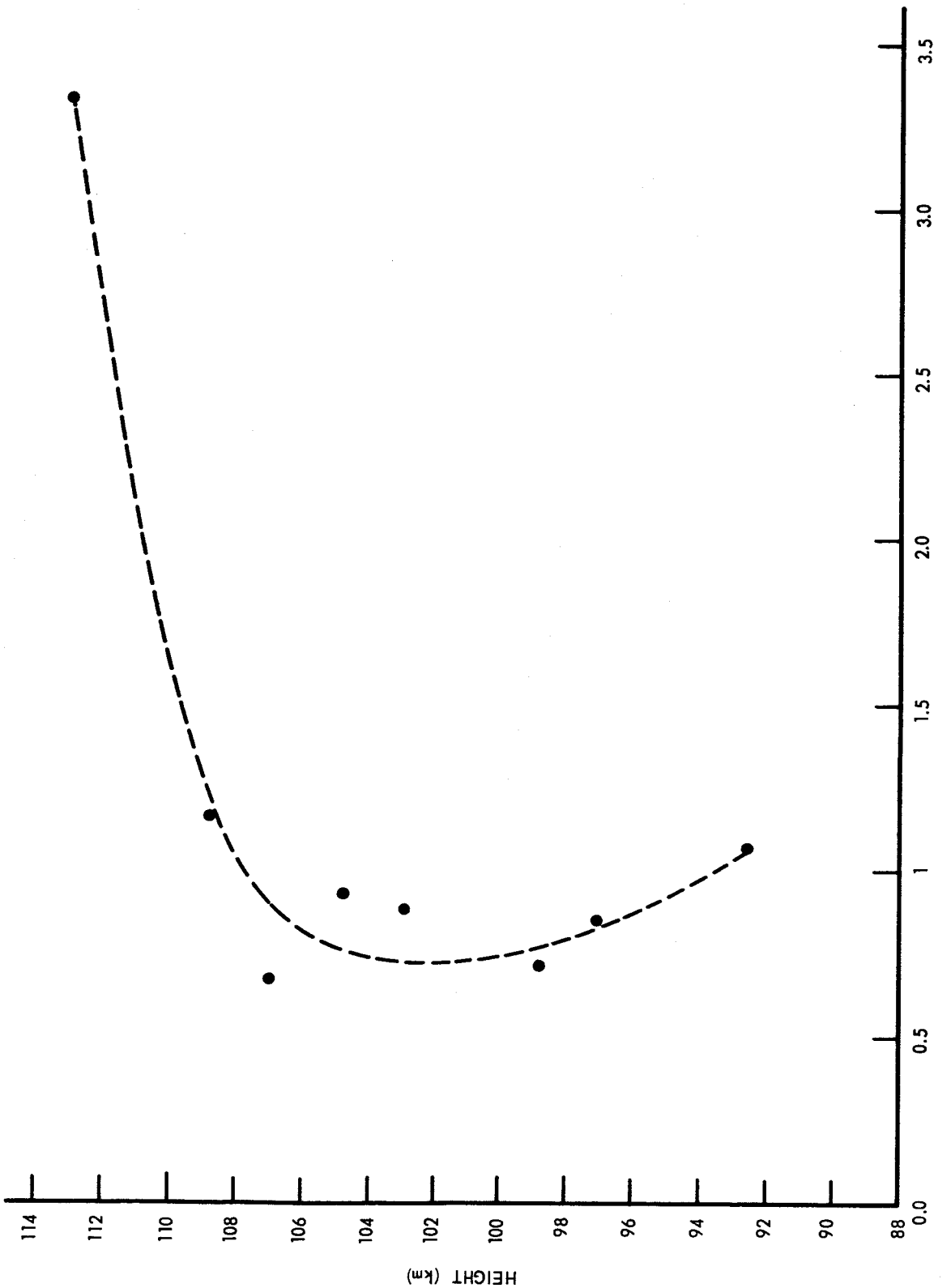

Figure 6. Mixing Length Versus Altitude.

$$\eta = \frac{\beta T^{3/2}}{\rho (T + S)} \qquad (m^2/sec) \qquad (9)$$

where $\beta = 1.458 \times 10^{-6}$, $S = 110.4\ ^{o}K$, and T and ρ are the atmospheric temperature and density in ^{o}K and kg/m^3.

Actual evaluations of ε_t and ε_s show that these quantities are not the same throughout the upper atmosphere. (See later sections of this report for evaluation of these parameters.) Tables 1 and 2 show values of l_e calculated from ε_s and ε_t for three altitudes. Both sets of values show that l_e is of the order of 10 m and that l_e is increasing in magnatude with increasing altitude.

TABLE I

VALUES OF l_e USING ε_s

Height (km)	η (m^2/sec)	ε_s (m^2/sec^3)	l_e (m)
92	5.7	0.30	5
100	28	0.37	16
108	120	0.46	44

TABLE 2

VALUES OF l_e USING ε_t

Height (km)	η (m^2/sec)	ε_t (m^2/sec^3)	l_e (m)
92	5.7	0.012	11
100	28	0.14	20
108	120	1.1	35

Correlation Scales of the Turbulent Velocities

The general double-velocity correlation coefficient $g_{ij}\ (\underline{x};\ \underline{r})$ is given by

$$g_{ij}\ (x;\ r) = \frac{< u_i\ (\underline{x})\ u_j\ (\underline{x} + \underline{r}) >}{[\ < u_i^{\ 2}\ (\underline{x}) \times u_j^{\ 2}\ (x + r) > \ ]^{\frac{1}{2}}} \quad i,j = 1,2,3 \qquad (10)$$

where now $\underline{x}$ is the position vector $(x_1,\ x_2,\ x_3,)$, $\underline{r}$ is the vector $(r_1,\ r_2,\ r_3)$ and $u_1,\ u_2,\ u_3$ are the components of the turbulent velocity previously denoted by u, v, and w, and the mean values are taken with respect to time. As seen from the defining formula, the correlation coefficient $g_{ij}\ (\underline{x};\ \underline{r})$ serves as a measure of the degree of correlation between the turbulent velocity fluctuations $u_i\ (\underline{x})$ and $u_j\ (\underline{x} + \underline{r})$, and therefore $g_{ij}\ (\underline{x};\ \underline{r}) = 1$ for $\underline{r} = 0$ and $g_{ij}\ (\underline{x};\ \underline{r})$ approaches 0 as $|\underline{r}|$ approaches the size of the largest eddies.

Of special interest are the particular set of one-dimensional correlation coefficients $g_{ii}\ (\underline{x};\ r,\ 0,\ 0)$, $g_{ii}\ (\underline{x};\ 0,\ r,\ 0)$ and $g_{ii}\ (\underline{x};\ 0,\ 0,\ r)$ or, if the correlation coefficients are presumed to be independent of the position in the turbulent field, the coefficients may be denoted by $g_{ii}\ (r,\ 0,\ 0)$, $g_{ii}\ (0,\ r,\ 0)$ and $g_{ii}\ (0,\ 0,\ r)$. If the correlation coefficient is also isotropic the coefficient may be characterized by the single function g (r) given by

$$g\ (r) = g_{11}\ (r,\ 0,\ 0) = g_{22}\ (0,\ r,\ 0) = g_{33}\ (0,\ 0,\ r)$$

An integral scale of the turbulent motion, L_i is defined to be

$$L_i = \int_0^\infty g\ (r)\ dr \qquad (11)$$

The scale of the largest eddies may be defined as the point $r = L_o$ at which $g(r)$ is first zero, or, more loosely, the point L_o at which an extrapolation of $g(r)$ is zero should $g(r)$ remain positive.

It can be shown (Taylor (1938)) that to second order in r, $g(r)$ is given by

$$g(r) = \frac{\left\langle \left(\frac{\partial u}{\partial r} \right)^2 \right\rangle}{2 \langle u^2 \rangle} r^2 = 1 - \frac{r^2}{L_p^2} \tag{12}$$

where u is the turbulent velocity component in the appropriate direction to correspond to r, and

$$L_p = \left[\frac{2 \langle u^2 \rangle}{\left\langle \left(\frac{\partial u}{\partial r} \right)^2 \right\rangle} \right]^{\frac{1}{2}} \tag{13}$$

is the dissipation length parameter. L_p is considerably larger in magnitude than the scale of the smallest, energy dissipating eddies. L_p is a length corresponding to eddies which contain a negligible portion of the total energy and are responsible for a negligible part of the total dissipation of energy as heat.

Townsend (1956) gives a relation showing that the scale of the dissipating eddies, here denoted by l_c, is given by

$$l_c^2 = \frac{L_p^3}{10 \sqrt{15}\, L_o} \tag{14}$$

As an approximation to $g_{ii}(r, 0, 0)$, $g_{ii}(0, r, 0)$ and $g_{ii}(0, 0, r)$ for $i = 1, 2$ (the horizontal components), Greenhow and Neufeld (1959b) calculated

$$g\,(r,\,z) = \frac{\Sigma\,[\,u'\,(\underline{x})\,u'\,(\underline{x}+\underline{r})\,]}{[\,\Sigma\,u'^{2}\,(\underline{x})\,\Sigma\,u'^{2}\,(\underline{x}+\underline{r})\,]^{\frac{1}{2}}} \tag{15}$$

where, here, u' is the horizontal component of the turbulent velocity as defined in the introduction for the meteor type data, $\underline{r}$ is the vector with magnitude r and vertical component z, and the summations extend over all positions $\underline{x}$ at which wind determinations were made.

Greenhow and Neufeld plot their determination of g (r, z) versus both r and z and conclude from these graphs that the vertical scale of the largest eddies is 6 km. They report that consideration of echo pairs of the same height separation but different horizontal separations leads to a horizontal rate of decay in the correlation which indicates a horizontal scale of the order of 100 to 200 km. Using 150 km for their value of L_o and 2.4 km as their value of L_p, they deduce from (14) a size $l_c \approx 50$ m for the smaller eddies.

Wind data from 13 chemical releases launched from Eglin AFB, Florida between 1959 and 1963 have been used to calculate correlation coefficients analogous to the Greenhow and Neufeld approximation by computing g (r, z) by (15), where now u' stands for the horizontal component of the turbulent velocity $(u^2 + v^2)^{\frac{1}{2}}$ with a + or - sign attached according to whether the magnitude of the instantaneous wind is greater or less than the magnitude of the mean wind.

A horizontal correlation coefficient g (r, h) can be defined by

$$g\,(r,\,h) = \frac{\Sigma\,[\,u'\,(\underline{x})\,u'\,(\underline{x}+\underline{r}')\,]}{[\,\Sigma\,u'^{2}\,(\underline{x})\,\Sigma\,u'^{2}\,(\underline{x}+\underline{r}')\,]} \tag{16}$$

where $\underline{r}'$ is a vector with magnitude r and horizontal component h, and the summation extends over all positions $\underline{x}$ for which wind data were obtained such that z, the vertical component of $\underline{r}'$, is less than 1 km. Thus g (r, h) is an

approximation to g_{ii} (r, 0, 0) or g_{ii} (0, r, 0) and should yield information concerning the horizontal scale of the eddies.

Figure 7 and 8 show the averaged results of g (r, z) versus z and g (r, h) versus h. Both coefficients tend to zero at about 6 km, confirming the 6 km vertical scale deduced by Greenhow and Neufeld but indicating an equal 6 km horizontal scale instead of their 100-200 km value.

The horizontal scale of the mean winds, that is the horizontal distance over which the mean wind profile maintains some degree of correlation, is certainly greater than 100 km. Simultaneous wind data obtained from separate chemical release clouds some 100 km apart indicate that this scale may be of the order of 1000 km, since the wind profiles still have a positive correlation of about 0.9 for this 100 km separation. It could be that the horizontal scale that Greenhow and Neufeld reported was more nearly the scale of the mean winds than the scale of the turbulent winds. This discrepancy might result from their method of defining the turbulent components, which differs from the definition used with regard to the chemical release wind determinations.

Correlation coefficients similar to those in Figure 7 and 8 but calculated by using w, the vertical component of the turbulent velocity, in (15) and (16) instead of the horizontal component do not show a similar fall off in the correlation. Further investigation of this is planned and it is also planned that as a next approximation to the actual one-dimensional correlation coefficients the quantities g (r, h) and g (r, z) can be evaluated using complete set of components u, v, and w in (15) and (16) instead of only horizontal and vertical components u' and w.

If the data from Figure 7 is used to estimate the integral scale of turbulence L_i one obtains a value $L_i \approx 1$ km. The dissipation length para-

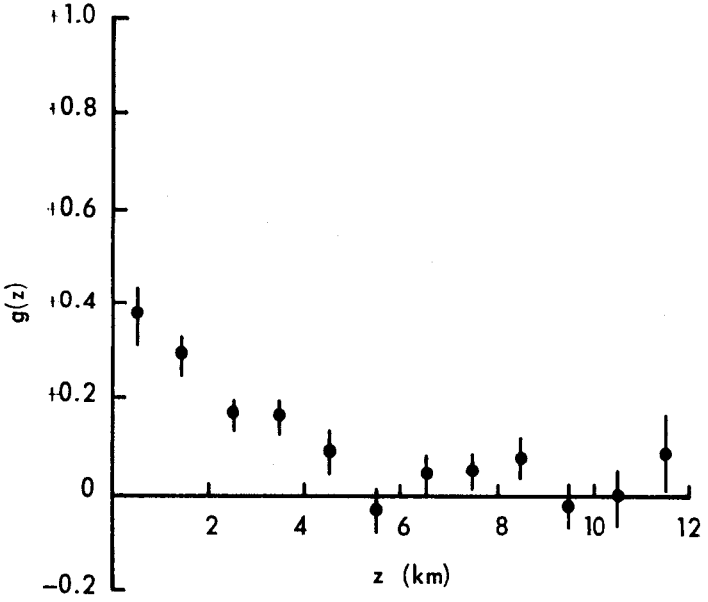

Figure 7. Spatial Correlation Coefficient for Vertical Separation.

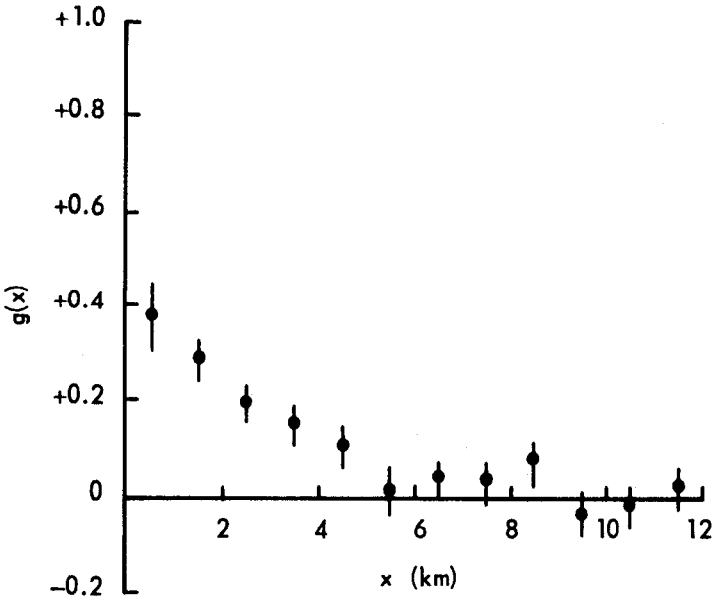

Figure 8. Spatial Correlation Coefficient for Horizontal Separation.

meter L_p is the point where a parabola $g(h) = 1 - z^2/L_p 2$ passes through zero. If the data point $z = 0.5$ km, $g = 0.27$ is used from Figure 7 then

$$L_p = \left(\frac{z^2}{1 - g(z)}\right)^{\frac{1}{2}} = 0.6 \text{ km} \qquad (17)$$

This value disagrees somewhat with the Greenhow and Neufeld value of 2.4 km because the data points of their correlation curve approach the value $g = 1$ more closely for small z. The fact that the data points seen in Figures 7 and 8 do not seem to approach $g = 1$ for z and h approaching zero may be due to the fact that an extimated r.m.s. error of about 3 - 5 m/sec is present in all of the wind determinations from the chemical releases whereas the r.m.s. value of the magnitude of the turbulent winds is about 16 m/sec. Since the errors constitute such a substantial percentage of the fluctuations being observed there may be some degree of destruction of correlation because of the errors. However it is interesting to note that Figures 7 and 8 show the same quantative shape as correlation coefficients calculated by Townsend (1956) for a field of turbulence in which there are only two distinct eddy sizes.

If the values $L_p = 0.6$ km and $L_o = 6$ km are used in (14) the resultant value for l_c is 18 m. This value fits into the range of values shown in Tables 1 and 2 for the size of the smallest eddies. From the fact that a scale L_o of about 6 km results from both Figure 7 and Figure 8 data it would seem that the turbulence is almost isotropic, however, Dougherty (1961) has shown that the turbulence should be isotropic only in the range from l_c to l_i. The magnitude of l_c, the scale of the smallest eddies has been seen to be about 20 m, and l_i is given by

$$l_i = \frac{\varepsilon_t^{\frac{1}{2}}}{\omega_g^{3/2}} \qquad (18)$$

where ε_t, again is the rate of turbulent energy dissipation and ω_g is the Brunt Vaisala frequency given by

$$\omega_g^2 = \frac{g}{T}\left(\frac{\partial T}{\partial z} + \frac{g}{C_p}\right) \tag{19}$$

where T is temperature, g is the acceleration of gravity, z is the vertical coordinate and C_p is the specific heat at constant pressure. The result (18) is based on Bolgiano's theory (1959, 1960) which hypothesizes that the isotropic inertial subrange exists only for scales between l_c and l_i, and that an anisotropic bouyancy subrange exists for scales between l_i and l_b. Data from which N can be computed are available in the U. S. Standard Atmosphere (1962). Using these values ω_g is about 2.4×10^{-2} sec^{-1} for 90 km < z < 115 km. Using this value, Table 3 lists values for l_i for several altitudes, as determined by values of ε_t from data presented later in this paper.

TABLE 3

Height (km)	ε_t (m^2/sec^3)	l_i (km)
90	.0065	0.022
100	0.13	0.097
110	2.8	0.45
115	13	0.97

Thus, if the values of Table 3 are correct, the turbulence can be isotropic only for the smaller size eddies and indeed there is essentially no isotropic subrange below about 90 km. However, many globules in the height region 92-108

km remain approximately spherical up to sizes of 1km or greater, but some of the globules larger than 1 km do become stretched into elliptical shapes. Therefore a more realistic value for the scale of isotropy may be about 1 km throughout the entire height region 90-110 km.

Dissipation Length Parameter from Turbulent Winds

Actual data on the turbulent wind components u, v and w allow the calculation of the dissipation length parameter, now called L_t, by the relation (13)

$$L_t = \left[\frac{2 < u^2 >}{< \left(\frac{\partial u}{\partial x} \right)^2 >} \right]^{\frac{1}{2}} \qquad (20)$$

and similar relations obtained by replacing u by v or w and x by y or z. Here the partial derivatives have to be estimated by ratios of finite differences Δu and Δx.

Wind data obtained from 13 chemical releases shows an average value for L_t of 0.65 km, in very good agreement with the value of L_p, the dissipation length parameter determined from the correlation coefficient data.

Globule Size Scales and the Globule Cut-off Altitude

From observations on sodium trail releases near 100 km Blamont and de Jager (1961, 1962) report globules ranging in size from 70 meters to 1 km, with an average size of 0.5 km. Globule size data has been obtained from 13 chemical releases (primarily Cesium Nitrate-Aluminum) launched from Eglin AFB Florida between 1959 and 1963. Globules smaller than about 200 meters could not be measured from the photographs that were taken, but data on the growth

of 209 separate globules with diameters greater than 200 meters was obtained. Five of the releases had sufficient numbers of globules so that a size spectrum could be determined, that is, the number of globules of a given size could be plotted against globule diameters.

All five of these size spectrum plots have a peak at 0.7 km, and one has an additional peak at 1.2 km. The peak at 1.2 km occurs because of larger globules appearing in the height region 104-108 km. A large number of the individual growth curves of the globules level off at a diameter of about 0.7 km (or at 1.2 km for the higher ones) and then continue to increase (note Figure 9). Thus the reason for the peaks in the spectrum plots is that the growing globules tend to spend more time at this leveling off diameter than at other sizes. This leveling off scale for the globules, L_d, illustrated in Figure 10, increases from 0.4 km at $97\frac{1}{2}$ km altitude to 0.75 km at 99 km and remains constant until at 106 km it begins to increase again and reaches 1.6 km at a height of 112 km. The mechanism that produces this leveling off in the growth curve is not known at the present.

The globular appearance of the chemical releases ceases at some altitude, H_t, called the turbopause altitude, between 100 and 115 km. Both point releases (i.e. chemicals released explosively at one or more points) and trail releases which covered an altitude range including H_t exhibited globules on that part of the cloud below H_t and were smooth above H_t. Blamont (1960) reported H_t to be at 102 km, and Manring (1962) at 102 ± 4 km. A total of 56 observations on 19 chemical releases showed H_t to lie between the limits 96 and 115 km with an average of 106 ± 4 rms.

Two of the releases produced one trail as the rocket ascended and another trail as the rocket descended. The up trails of these two releases showed

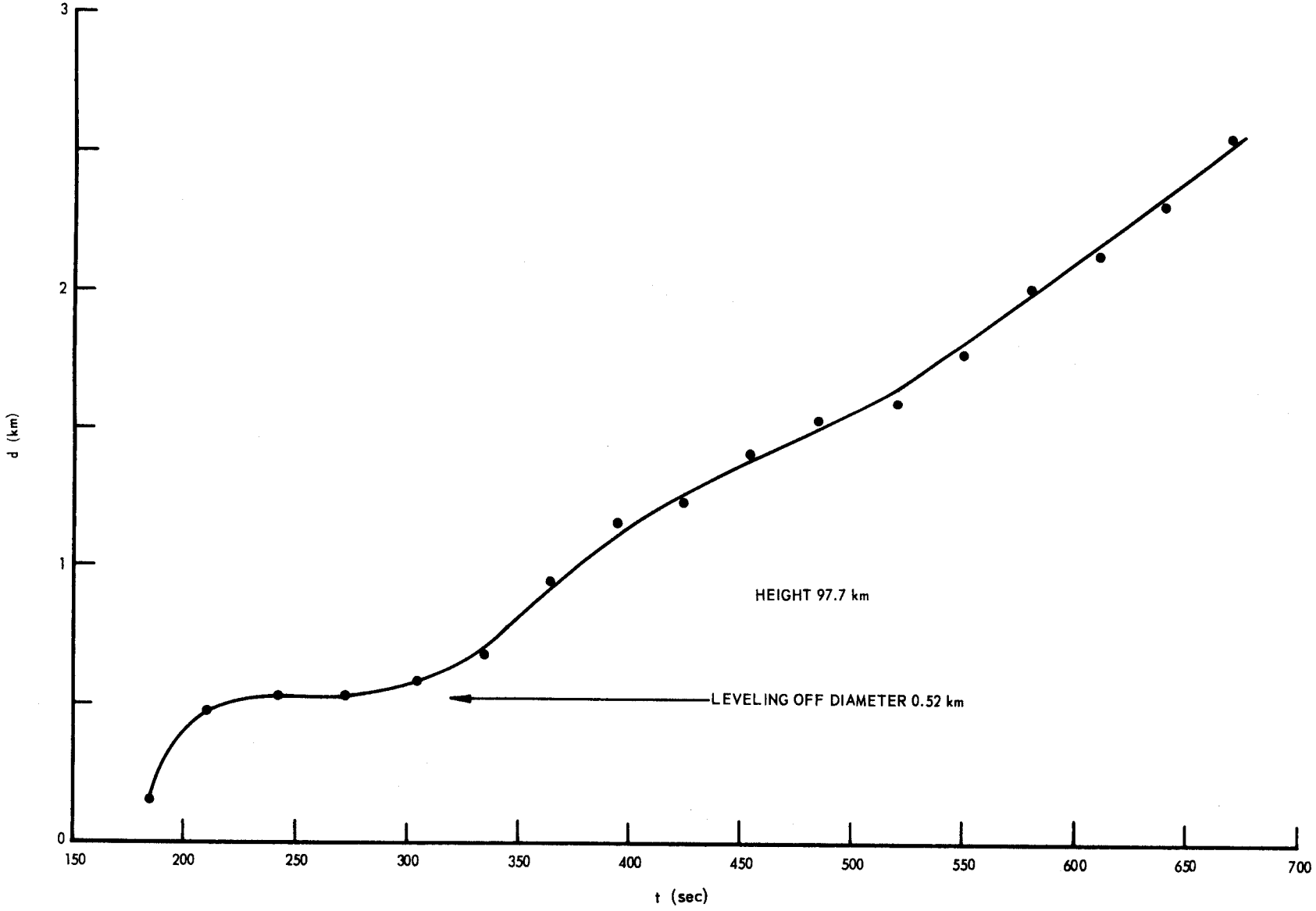

Figure 9. Globule Growth Curve - Diameter Versus Time.

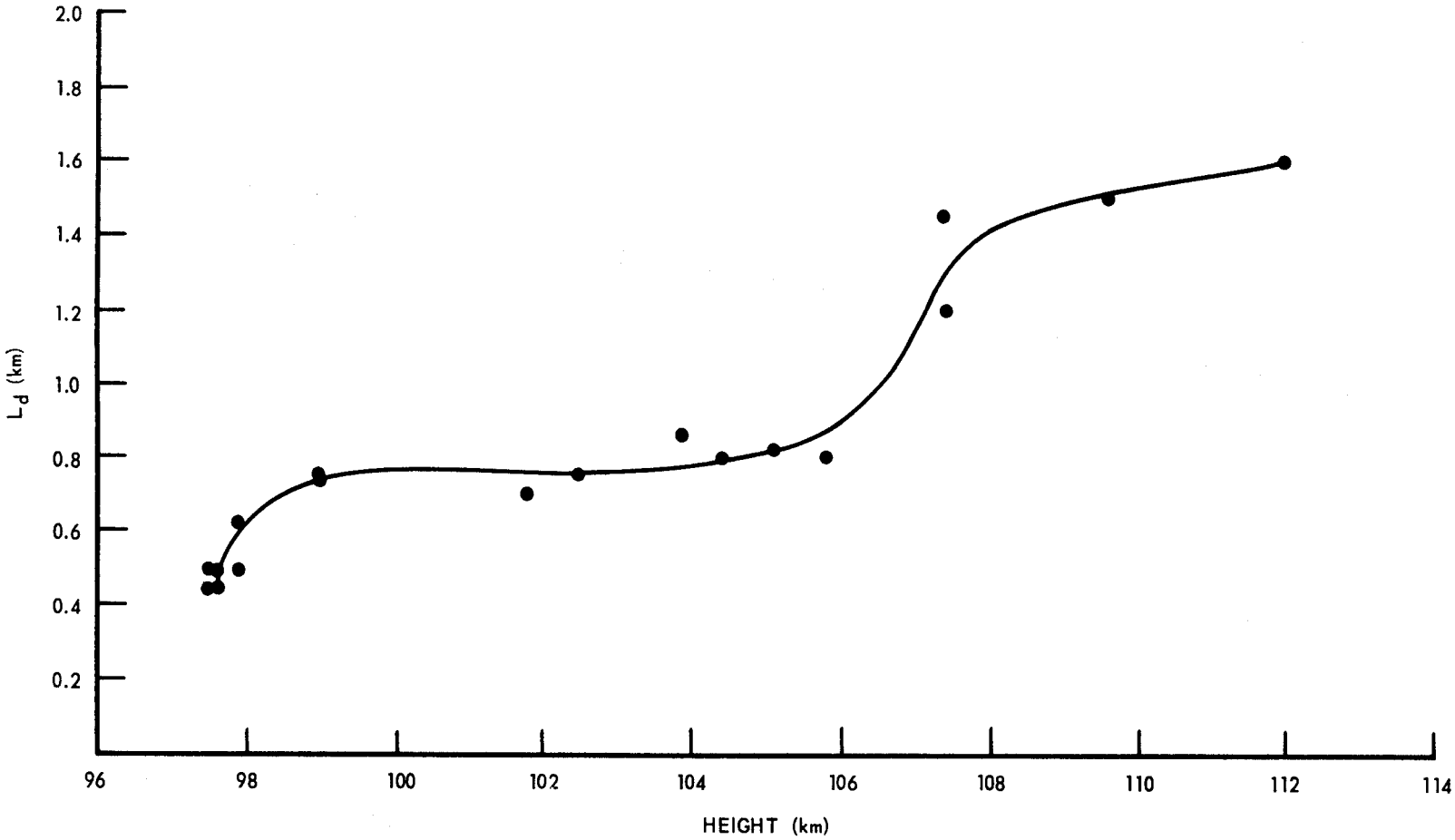

Figure 10. Leveling-off Diameter Versus Altitude.

H_t to be 106.5 and 107.0 km, and the down trails indicated H_t values of 104.3 and 105.2 km. In both cases H_t on the down trail showed a shift of 2 km downward with respect to the up trail. This may not be significant because solid particles which followed the trajectory and reentered the atmosphere made the down trails at H_t more difficult to observe and so the down trail values are much less accurate. It is interesting to note, however, that there appears to be about a 2 km downward shift of the wind pattern of the down trail with respect to the up trail, and this apparent shift in H_t may be associated with the similar shift in the wind profile.

Time Scales of the Motion

In addition to size scales of the motion one can speak of time scales of the motion. The time scale of a particular size scale of eddy would be the time over which that size eddy maintains its identity, or the time over which the turbulent velocity fluctuations maintain some degree of correlation. The size scale, L, the time scale, t_1, and the characteristic velocity fluctuation, v, of the eddy should be related by

$$L = v\, t_1 \tag{21}$$

according to Batchelor (1953).

The time correlation coefficient of, say, the u component of turbulent velocity would be

$$g(\delta t) = \frac{\Sigma [\, u(t)\, u(t + \delta t)\,]}{[\, \Sigma u^2(t)\, \Sigma u^2(t + \delta t)\,]^{\frac{1}{2}}} \tag{22}$$

where the summations extend over all observations separated by a time interval δt.

Greenhow and Neufeld (1959b, 1960) report results of an analysis using (22) to calculate time correlation of the turbulent winds. In this analysis the winds were averaged over twenty minute intervals, a harmonic analysis was made of these winds to determine prevailing, diurnal and semi-diurnal components, and the turbulent winds were defined as the residual from the 20 minute average winds after subtracting off the harmonic functions. The first zero of the time correlation coefficient is at $\delta t \approx 100$ min according to the Greenhow and Neufeld data. This value, they point out, verifies (21) if one uses L = 150 km and v = 25 m/sec which are appropriate figures for the large eddies according to their data.

Again, however, the definition of turbulent winds employed by Greenhow and Neufeld may mean that their value for t_1 applies not to the turbulent winds but more closely to the mean winds. The data of Greenhow and Neufeld (1956, 1960) as well as other meteor investigations indicate that the semi-diurnal component is the one of largest magnitude. This is confirmed by observations by Rosenberg and Edwards (1964) of winds throughout the night determined by a series of chemical release trails. The semi-diurnal component, having a period of 12 hours, would have a zero on the time correlation curve for δt = 3 hrs = 180 min. Actual calculation of the time correlation coefficient from the chemical release wind data shows that the first zero on the correlation curve of the mean winds is at about 200 min.

If we assume (21) is valid and use L = 6 km and v = 15 m/sec as indicated by the chemical release data then $t_1 \approx 400$ sec. Investigation is underway to determine if motions of this time scale are detectable.

The time scale, τ, of the smallest eddies can be determined by an equation

complementary to (8)

$$\tau = (\eta/\varepsilon)^{\frac{1}{2}} \tag{23}$$

where, as in (8), η is the kinematic viscosity and ε is either ε_t, the energy dissipated as heat, or ε_s the energy supplied by the shears. Tables 4 and 5 show several values of τ deduced from both ε_s and ε_t for the height range 92-108 km. Both tables show τ to be on the order of 10 sec over this height interval. The values given for the velocity fluctuations, v_e, of the small eddies are calculated from (21) using the appropriate size scales from Tables 1 and 2.

TABLE 4

VALUES OF τ FROM ε_s

Height (km)	η (m^2/sec)	ε_s (m^2/sec^3)	τ (sec)	v_e (m/sec)
92	5.7	0.30	4	1
100	28	0.37	9	2
108	120	0.46	16	3

TABLE 5

VALUES OF τ FROM ε_t

Height (km)	η (m^2/sec)	ε_t (m^2/sec^3)	τ (sec)	v (m/sec)
92	5.7	0.012	22	0.5
100	28	0.14	14	1
108	120	1.1	10	3

SUMMARY

Table 6 and 7 give a summary of the various size scales of the motion and tabulate their magnitudes or range of values in the indicated height interval. If no height interval is indicated the applicable interval is probably 90-110 km.

TABLE 6

SIZE SCALES OF THE MEAN MOTION

Scale Description	Symbol Used	Magnitude (km)	Height Range (km)
Horizontal Scale		150 - 1000	≈ 90 - 140
Vertical Autocorrelation Scale	L_v	4.3 - 8.6	81 - 130
"	L_{v1}	4.1 - 6.5	"
"	L_{v2}	2.4 - 4.5	"
Motion Spectrum Scale	L_s	5.5 - 12.0	83 - 127

TABLE 7

SIZE SCALES OF THE TURBULENT MOTION

Scale Description	Symbol Used	Magnitude (km)	Height Range (km)
Vertical and Horizontal Scale of Largest Eddies from Correlation Coefficient	L_o	6	
Integral Scale of Correlation Curve	L_i	1	
Mixing Length Scale	L_m	0.7 - 3.3	92 - 113
Dissipation Length			
(a) from correlation coefficient	L_p	0.6	
(b) from turbulent winds	L_t	0.65	93 - 109
Globule Size Spectrum Peak		0.7 - 1.2	
Globule Leveling Off Diameter	L_d	0.8 - 1.5	
Isotropic Subrange			
(a) from (18)	l_i	0.022 - 0.97	90 - 115
(b) from globule shapes		≈ 1	90 - 110
Scale of Smallest Eddies			
(a) from energy dissipation rate	l_e	0.011 - 0.035	92 - 108
(b) from energy supplied by shears	l_e	0.005 - 0.044	92 - 108
(c) from correlation coefficient	l_c	0.018	

The motion has been separated here into mean winds and turbulent fluctuations. However if no restricting conditions were imposed on the flow field, motions considered here as mean winds could (after substracting off any tidal or other non-random components) be considered as the turbulent component for higher order size and time scale eddies.

If a fluid element is displaced a small amount from its equilibrium position, it experiences a restoring force which produces a harmonic motion with a natural frequency ω_g, the Brunt-Väisälä frequency, given by (19). Layzer (1961) states that if the lifetime t_l of a turbulent eddy is greater than $2\pi/\omega_g$, then it is possible to follow a typical fluid element throughout a complete gravitational oscillation. But if a fluid element retains its identity for such a length of time the motion is not true turbulence. Again using the value $\omega_g = 2.4 \times 10^{-2}$ sec^{-1} we see that motions with time scales great than about 260 seconds would not be true turbulence.

This value of ω_g may be in error since it is based on standard atmosphere conditions and an extrapolation of C_p from the values for air at low pressure laboratory conditions. Table 8 shows the values of $\partial T/\partial z$ and g/C_p which are the two least accurately known terms in ω_g. It is seen that in the range 90-115 km the g/C_p term contributes more heavily to ω_g than the $\partial T/\partial z$ term. Thus a large uncertainty in g/C_p would represent a correspondingly large uncertainty in ω_g. Consequently it may be that values of $t_l \approx 400$ sec, previously deduced from the chemical release winds, would represent true turbulent motion. But it would be difficult to reconcile a turbulent motion with a time scale of 100 minutes or more with the above argument.

TABLE 8

$\frac{\partial T}{\partial z}$ AND g/C_p VERSUS HEIGHT

Z (km)	$\frac{\partial T}{\partial z}$ (^oK/km)	$\frac{g}{C_p}$ (^oK/km)
90	0.00	9.54
95	2.94	9.53
100	3.85	9.51
105	4.70	9.49
110	7.00	9.47
115	9.25	9.43

It seems quite likely that the features of the mean winds can be explained by a combination of tidal waves and other ordered motion such as the gravity waves proposed by Hines (1959, 1960). If this is the case then true turbulence will be confined to the size and time scales discussed here as being associated with the turbulent motion. Further work must be done, however, to clarify the exact nature of the motions here described as the mean winds, and to resolve these motions into tidal wave, gravity wave, and other ordered motions which may contribute to the total mean winds.

ENERGY BALANCE OF THE MOTION

If the flow is statistically steady the energy balance equation is

$$\varepsilon_s = \varepsilon_g + \varepsilon_t \qquad (24)$$

where ε_s is the rate per unit mass of atmosphere at which the turbulence extracts energy from the wind shears of the mean flow, ε_g is the rate per unit mass of atmosphere at which the turbulence does work against gravity, and ε_t is the rate per unit mass of atmosphere at which turbulent energy is dissipated as heat by viscous effects. The rate of supply ε_s is given by

$$\varepsilon_s = \sum_1 \sum_j u_i \, u_j \frac{\partial U_i}{\partial x_j} \tag{25}$$

where the U's are components of the mean velocity, the u's are components of the turbulent velocity and the x's are coordinates, x_3 being the vertical coordinate, and x_1 and x_2 appropriate horizontal coordinates. An order of magnitude relations for ε_s is

$$\varepsilon_s \approx \frac{v^3}{L} \tag{26}$$

where v and L are the characteristic turbulent velocity and scale of the energy containing eddies.

The mixing length theory provides an approximate relation for ε_g

$$\varepsilon_g = \overline{u_3 \, L_m} \, \omega_g^2 \approx \frac{\overline{u_1 \, u_3} \, \omega_g^2}{\left(\frac{\partial U_1}{\partial x_3} \right)} \tag{27}$$

where L_m is the mixing length given by (7) and ω_g is the Brunt-Väisälä frequency given by (19).

According to Lamb (1945) the general expression for ε_t is

$$\varepsilon_t = \eta \left[2 \sum_i \overline{\left(\frac{\partial u_i}{\partial x_i} \right)^2} + \sum_{cyc} \overline{\left(\frac{\partial u_1}{\partial u_3} + \frac{\partial u_3}{\partial u_1} \right)^2} \right] - \frac{2\eta}{3} \left(\overline{\sum_i \frac{\partial u_i}{\partial x_i}} \right)^2 \tag{28}$$

where η is the kinematic viscosity and $\sum_{cyc}$ indicates summation over a full cyclic permutation of indices. If the atmosphere is assumed incompressible then $\sum_i \frac{\partial u_i}{\partial x_i} = 0$, from the continuity equation, so the last term in (28) would disappear. If, in addition, the turbulence is assumed to be isotropic then it can be shown (Taylor (1935)) that

$$\varepsilon_t = 6\,\eta\left[\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} + \overline{\left(\frac{\partial u_i}{\partial x_2}\right)^2} + \overline{\frac{\partial u_2}{\partial x_1}\,\frac{\partial u_1}{\partial x_2}}\right] \tag{29}$$

which can be reduced to

$$\varepsilon_t = \frac{15\,\eta}{2}\,\overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} \tag{30}$$

since the terms in (29) are not all independent for an isotropic field and incompressible flow.

The turbulent wind components of wind data obtained from chemical release clouds may be used in (25) and in (27) through (30) to evaluate ε_s and ε_g and give three separate evaluations of ε_t. For this procedure averages such as $\overline{u_i\,u_j}$ are obtained by averaging over all wind determinations in a finite height interval, usually about 2 km. The derivatives are approximated by ratios of finite differences $\Delta u_i/\Delta x_j$. The results of these evaluations of the energy terms, shown in Figure 12, will be discussed later in this report.

The manner in which a cloud of material diffuses after injection into the turbulence offers another method for evaluating ε_t. Cote (1962) summarizes the theoretical results of several authors, Lin (1960), Tchen (1961), Bolgiano (1959) and Roberts (1960) for the dispersion law that should be followed by the diffusing material. The various dispersion relations are

$$< r^2 > \sim (t - t_o)^2 \tag{31}$$

$$< r^2 > \sim \varepsilon_t (t - t_o)^3 \tag{32}$$

$$< r^2 > \sim \varepsilon_t 2 (t - t_o)^4 \tag{33}$$

$$< r^2 > \sim Z (t - t_o)^5 \tag{34}$$

$$< r^2 > \sim \varepsilon_t 4 (t - t_o)^6 \tag{35}$$

where $< r^2 >$ is the mean square separation between particles of the injected material, t_o is some appropriate reference time, presumably the time of injection, and Z in (34) is a function of the atmospheric potential density fluctuation and the vertical turbulent velocity fluctuations. The relations (31), (32), (33), and (35) result from the use of different forms of the theoretical energy spectrum function, which gives a description of the distribution of the energy within the energy containing eddies. Formula (34) is based on Bolgiano's theory, mentioned in a previous section.

The relation (32) was first deduced by Batchelor (1950) from Kolmogoroff's similarity hypothesis. It is this form of dispersion which presumably holds if Kolmogoroff's principle is valid. The complete relation for describing the diffusion of a cloud injected into the turbulent field would be

$$d^2 = \frac{16}{3} \varepsilon_t (t - t_o)^3 \tag{36}$$

where d is the diameter of the cloud, or trail of material.

Figure 11 shows a plot on log-log scale of the diameter squared of a globule versus $t - t_o$. This is the growth curve for the same globule shown in Figure 9. It is seen that the points at early times when the globule is going through the leveling off phase do not follow a power law, but above the leveling off size the growth follows the relation (36) closer than any other

of the diffusion formulas, the actual exponent being 2.62 in this case.

Each of the formulas (31), (32), (33) and (35) is of the form

$$< r^2 > \sim \varepsilon_t^{\alpha-2} (t - t_o)^{\alpha}. \tag{37}$$

If the 16/3 factor of (36) is assumed, a least squares fit of globule growth data to the formula

$$d^2 = 16/3 \, \varepsilon_t^{\alpha-2} (t - t_o)^{\alpha} \tag{38}$$

can be made by assuming some reasonable value of t_o. This procedure allows solution for both α and ε_t. The average value thus obtained for α was 3.0 $\pm$ 0.4 rms. The resultant ε_t data are shown in Figure 12.

Data were available primarily in the region from 92-108 km for using in (25), (27) through (30), and (37). A small amount of data were also available in the range 108-112 km. Functions of the arbitrary form $\varepsilon = \exp(a + bz)$ were fit through the data obtained by each of these procedures. The resultant curves, straight lines on a semi-log plot, are shown in Figure 12. The curves from each of the three turbulent wind methods for evaluating ε_t disagree among themselves at some altitudes by about a factor of five, and the results from (29) and (30), which should yield identical values of ε_t for isotropic incompressible flow, disagree at some altitudes by about a factor of 2. The curve for ε_t obtained from globule growth rates is in rather poor agreement with the other ε_t data, especially at low altitudes. There was considerable scatter, however, in the data points for ε_t from globule growth, some of the individual data points actually falling in the region of the other three ε_t curves. All of the four ε_t curves show ε_t increasing rapidly with

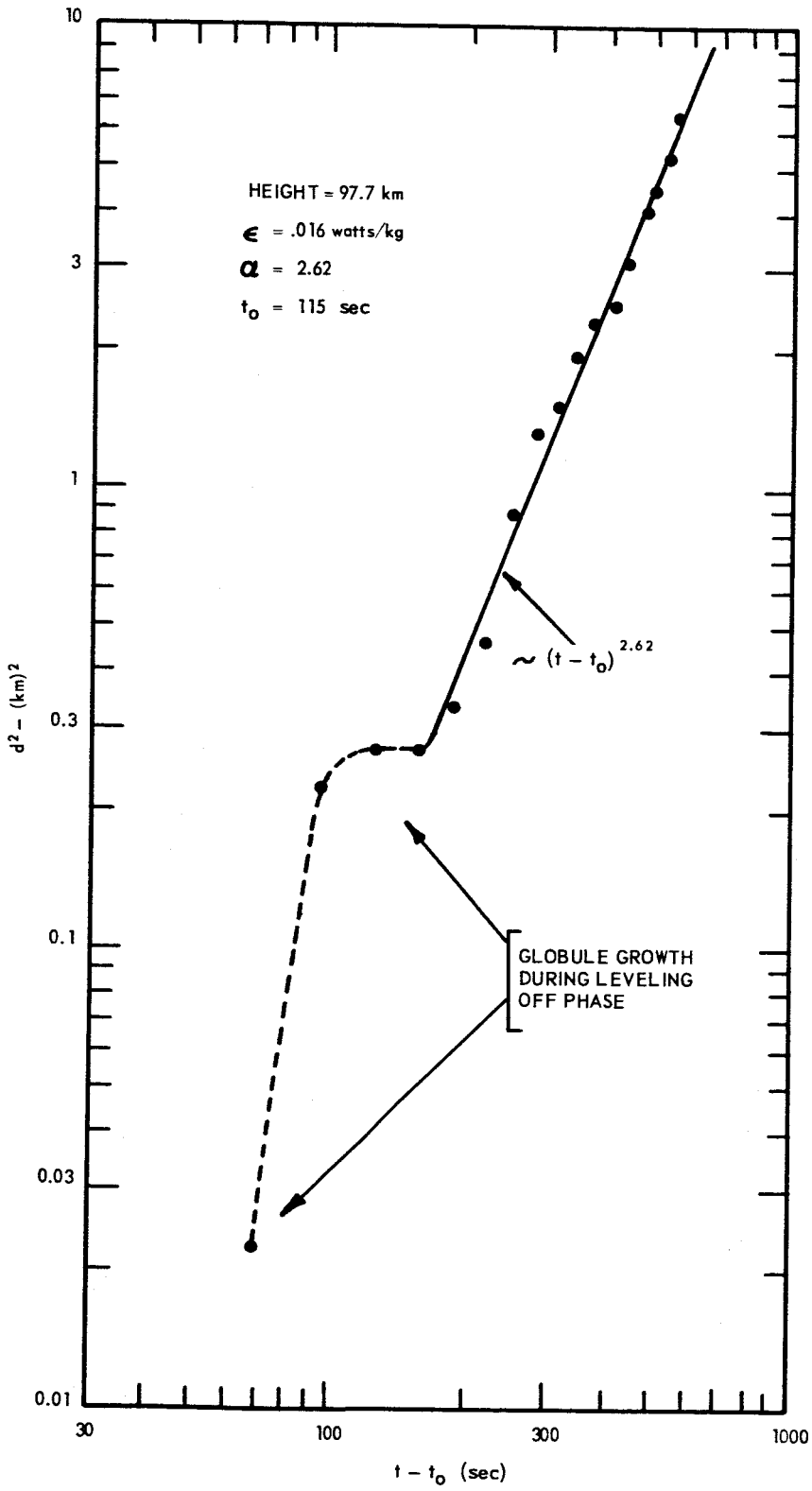

Figure 11. Globule Growth on Log-Log Scale - Diameter Squared Versus Time.

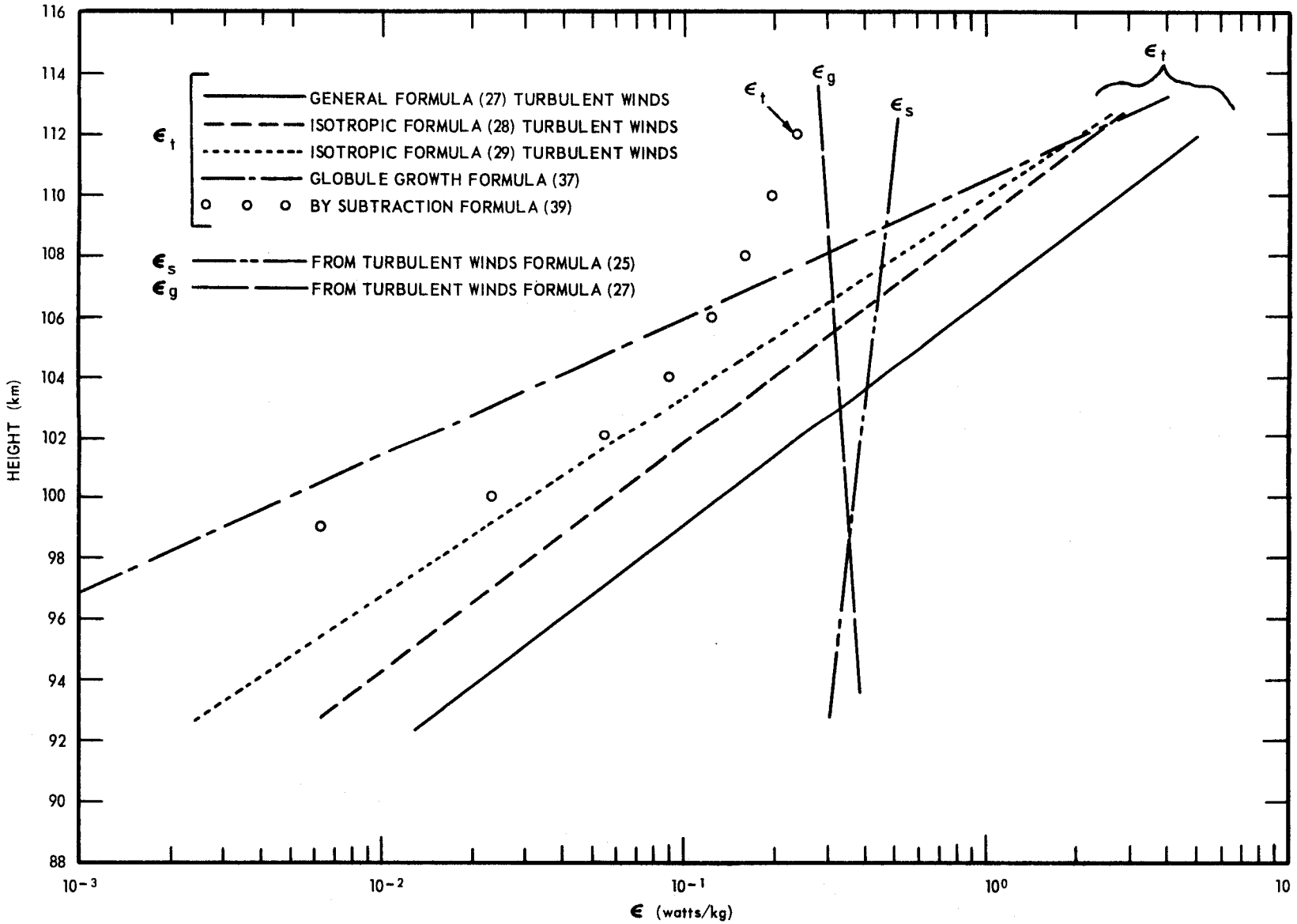

Figure 12. Variations of Energy Balance Terms with Altitude.

altitude, changing by two orders of magnitude or more in 15 km.

The curves for ε_s and ε_g show comparatively little variation with altitude. The actual curve for ε_g is greater in magnitude than ε_s from $98\frac{1}{2}$ km down, which is physically unreasonable, but it should be realized that each of the curves ε_s and ε_g should have an uncertainty of about a factor of two at each altitude. This $\varepsilon_g > \varepsilon_s$ anomaly can be rectified well within these limits of accuracy. It should be noted that an additional estimate of ε_t obtained by subtraction of the ε_g curve from the ε_s curve by

$$\varepsilon_t = \varepsilon_s - \varepsilon_g \tag{39}$$

agrees reasonably well with the other values for ε_t in the height range from 100 to 108 km.

The point at which the ε_t curve intersects the ε_s curve should represent the absolute upper limit for the existence of turbulence, since at this intersection point the rate of energy supply would be equal to the rate of energy being dissipated by viscous effects alone. The heights of these intersection points for the various ε_t curves range from 104 to 109 km, with 107 km representing an approximate mean. This 107 km altitude agrees with the observed average value of 106 km for the turbopause altitude.

If the values $v = 15$ m/sec and $L = 6$ km are used in (26) as the appropriate values of rms turbulent velocity and scale, as determined from the chemical release wind data, one obtains the value $\varepsilon_s \approx 0.6$ watts/kg. Similarly if one uses in (27) mean values of $\overline{u_1 u_3} = .11$ m^2/sec^2, $\omega_g^2 = 6 \times 10^{-4}$ sec^{-2} and $\partial U_1/\partial x_3 = 20$ m/sec km, which are approximate average values for these quantities throughout the height range 90 - 110 km, one obtains $\varepsilon_g \approx 0.3$ watts/kg. Both

of these estimated values agree fairly well with the curves for ε_s and ε_g in Figure 12.

Figure 13 shows a summary by Lettau (1961) of values obtained for ε_t from diffusion and wind profile observations in the altitude range from 1 cm to 40 km. The figure also illustrates the ε_t variation versus altitude as determined from the chemical release diffusion and wind data in the approximate height range 90 to 110 km. It appears from Figure 13 that ε_t continues to decrease with decreasing altitude below 90 km, diminishing by an additional 3 or more orders of magnitude from 90 to 30 km.

CRITERIA FOR THE ONSET OF TURBULENCE

Reynolds Criterion

Working on flow experiments in long, straight pipes, Reynolds showed in 1883 that the motion became turbulent when the Reynolds number exceeded a critical value (≈ 2000). The Reynolds number is defined as

$$R_e = \frac{ud}{\eta} \tag{40}$$

where u is the flow velocity, d is the pipe diameter and η is the kinematic viscosity.

In the free atmosphere it is stability that primarily determines whether or not turbulence is present. If temperature decreases with altitude so that the region is gravitationally unstable the velocity gradient will almost never be so small that turbulence will be inhibited by a low Reynolds number. When the region is gravitationally stable the flow will be laminar in weak velocity gradients, but if the velocity gradient is sufficiently large the region will be turbulent in spite of the gravitational stability.

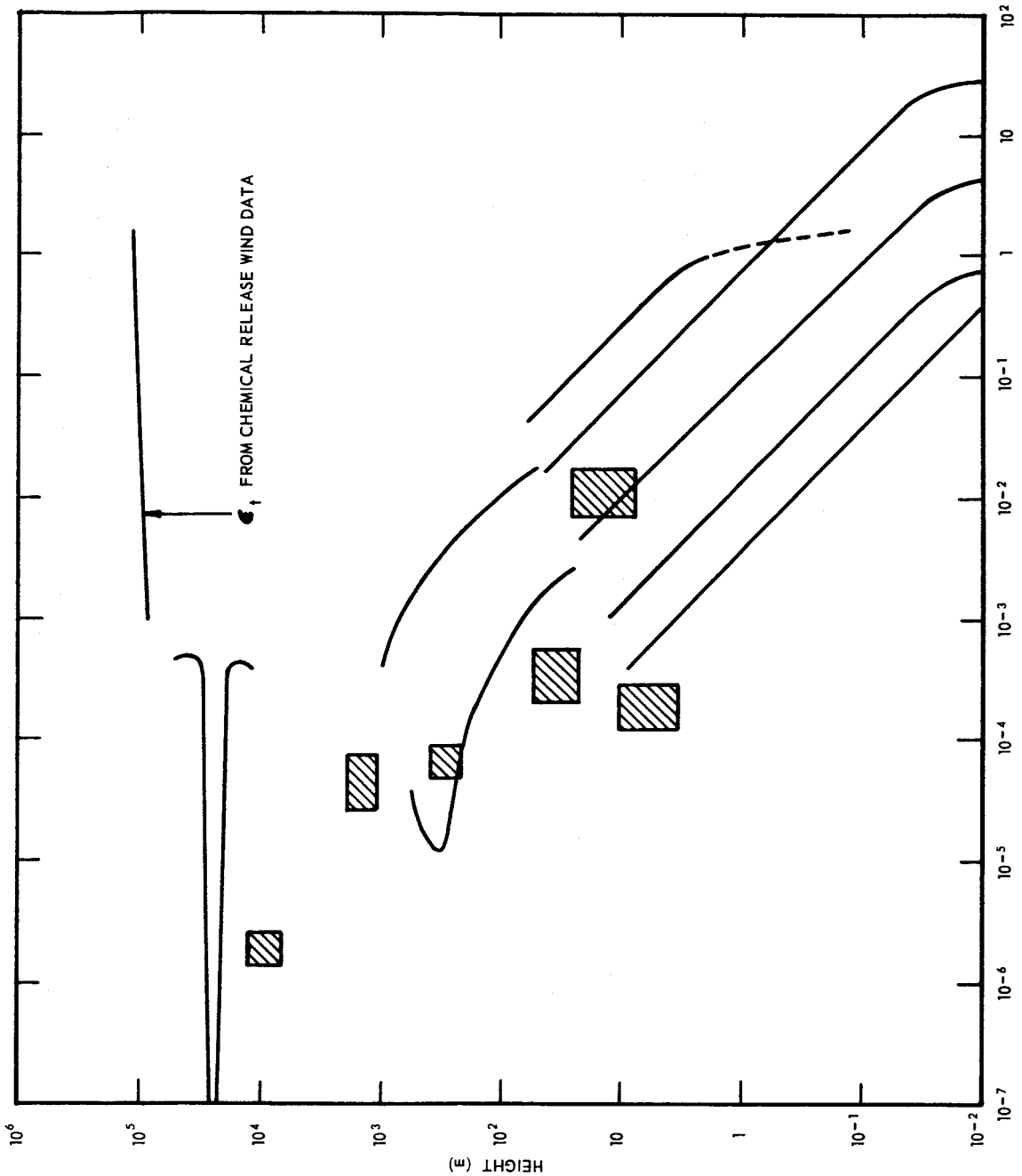

Figure 13. Viscous Energy Dissipation Rate from Altitudes of 1 cm to 110 km.

Consequently the Reynolds number has little bearing on the existence of turbulence in the height region above 85 km and we must examine other criteria for the onset of turbulence.

Richardson's Criterion

Richardson's theory (1920) is based on the assumption that if $\varepsilon_t > 0$ turbulence exists. From (24) it is seen that this condition is equivalent to $\varepsilon_s - \varepsilon_g > 0$ or $\varepsilon_g/\varepsilon_s < 1$. Using the approximation

$$\varepsilon_s = | \overline{uw} | \frac{\partial U}{\partial z} \tag{41}$$

Richardson's forms for ε_s and ε_g are

$$\varepsilon_s = K_E \omega_s^2 \tag{42}$$

$$\varepsilon_g = K_C \omega_g^2 \tag{43}$$

where $\omega_s = \partial U/\partial z$. From the equations (27) and (41) it is seen that $K_C = \overline{wL_m}$, and $K_E = \overline{uw}/(\partial U/\partial z)$ or, using the mixing length theory, K_E is defined by

$$K_E = L_m^2 \left(\frac{\partial U}{\partial z}\right)^2. \tag{44}$$

Richardson assumed $K_E = K_C$ so that the condition $\varepsilon_g/\varepsilon_s < 1$ is equivalent to the Richardson number, R_i, being less than 1, where the Richardson number is defined by

$$R_i = \frac{\omega_g^2}{\omega_s^2} \tag{45}$$

Therefore the Richardson criterion may be outlined as

$R_i < 1$ turbulent

$R_i > 1$ laminar. (46)

Townsend's Criterion

Townsend (1957) developed a more elegant criterion for turbulence based on an analogy between turbulent motion and Brownian motion. For this theory the quantity ω_t defined by

$$\varepsilon_t = (\overline{u^2} + \overline{v^2} + \overline{w^2})\,\omega_t \tag{47}$$

is important. Using the Brownian motion analogy, Townsend arrives at the result

$$\overline{wL_m} = \frac{\overline{w^2}}{2\omega_t} . \tag{48}$$

Using this in (43), ε_g may be written

$$\varepsilon_g = \tfrac{1}{2} k_g \overline{w^2}\,\frac{\omega_g^2}{\omega_t}$$

where $k_g = 1$ if the Brownian motion analogy holds exactly, so k_g should be a constant close to unity.

Equations (47) and (42) are rewritten as

$$\varepsilon_t = \frac{3}{2} k_t \overline{w^2}\,\omega_t \tag{49}$$

and

$$\varepsilon_s = \frac{2}{5} k_s \overline{w^2}\,\omega_s \tag{50}$$

where the coefficients k_t and k_s are of order unity.

Defining the Richardson flux number, R_f, as

$$R_f = \frac{\varepsilon_g}{\varepsilon_s} \tag{51}$$

Townsend derives the result

$$(1 - R_f)R_f = \frac{75}{16} \frac{k_g k_t}{k_s^2} R_i \tag{52}$$

The left hand side of (52) is a minimum at $R_f = \frac{1}{2}$ and at this point R_i is the critical value R_{ic} given by

$$R_{ic} = \frac{4}{75} \frac{k_s^2}{k_g k_t} \approx 0.05. \tag{53}$$

The flow will be turbulent for all $R_i < R_{ic}$.

Layzer's Criterion

Layzer (1961) extended the ideas of Townsend by imposing the additional restriction $\omega_t > \omega_g$. This is the same condition $t_l < 2\pi/\omega_g$ discussed in the section on time scales of the motion. For this to be true it is necessary that the condition

$$R_f < \frac{k_g}{k_g + 3k_g} \tag{54}$$

be true. Combining (52) and (54), Layzer derives the condition

$$R_i < R_{ic} = \frac{4}{5} \left(\frac{k_s}{k_g + 3k_g} \right)^2 \approx 0.05 \tag{55}$$

where, again, the flow will be turbulent for all $R_i < R_{ic}$.

Observations on Chemical Release Wind Data

Townsend's and Layzer's interia are designed to eliminate quantities such as $\overline{uw}$ and $\overline{u^2}$ which are considered unknowns. The mathematical steps of these

theories can be retraced making only the assumption

$$wL_m = \frac{k_j \overline{w^2}}{\omega_t} \tag{56}$$

where k_j is of order unity. Starting with equations (27), (41), and (47), if all the quantities involving turbulent velocity fluctuations are retained then the modified Townsend and Layzer criteria are

$$R_i < \frac{8(\overline{uw})^2}{(\overline{u^2} + \overline{v^2} + \overline{w^2})k_j \overline{w^2}} \qquad \text{(Townsend)} \tag{57}$$

$$R_i < \frac{4(\overline{uw})^2}{[\,\overline{u^2} + \overline{v^2} + \overline{w^2}(1 + 2k_j)\,]^2} \qquad \text{(Layzer)} \tag{58}$$

Each of these critical values is on the order of 0.01 for the wind data obtained from the chemical releases.

The average Richardson number in the height range 90-110 km is on the order of unity, however, so that there is little chance of satisfying Townsend's or Layzer's criteria in this altitude region. The percentage occurrance of Richardson number less than one have been calculated for the chemical release wind data and the results are shown in Figure 14. It is seen that the maximum occurrance is around 105 km, R_i being less than 1 about 35 percent of the time there. At 125 km the occurrance of $R_i < 1$ has fallen virtually to zero and at 90 km R_i is less than 1 only about 10 percent of the time.

CONCLUSIONS

From the data presented here it seems that the turbulence observed up to altitudes near 105 km is ambient turbulence produced by wind shears, and is

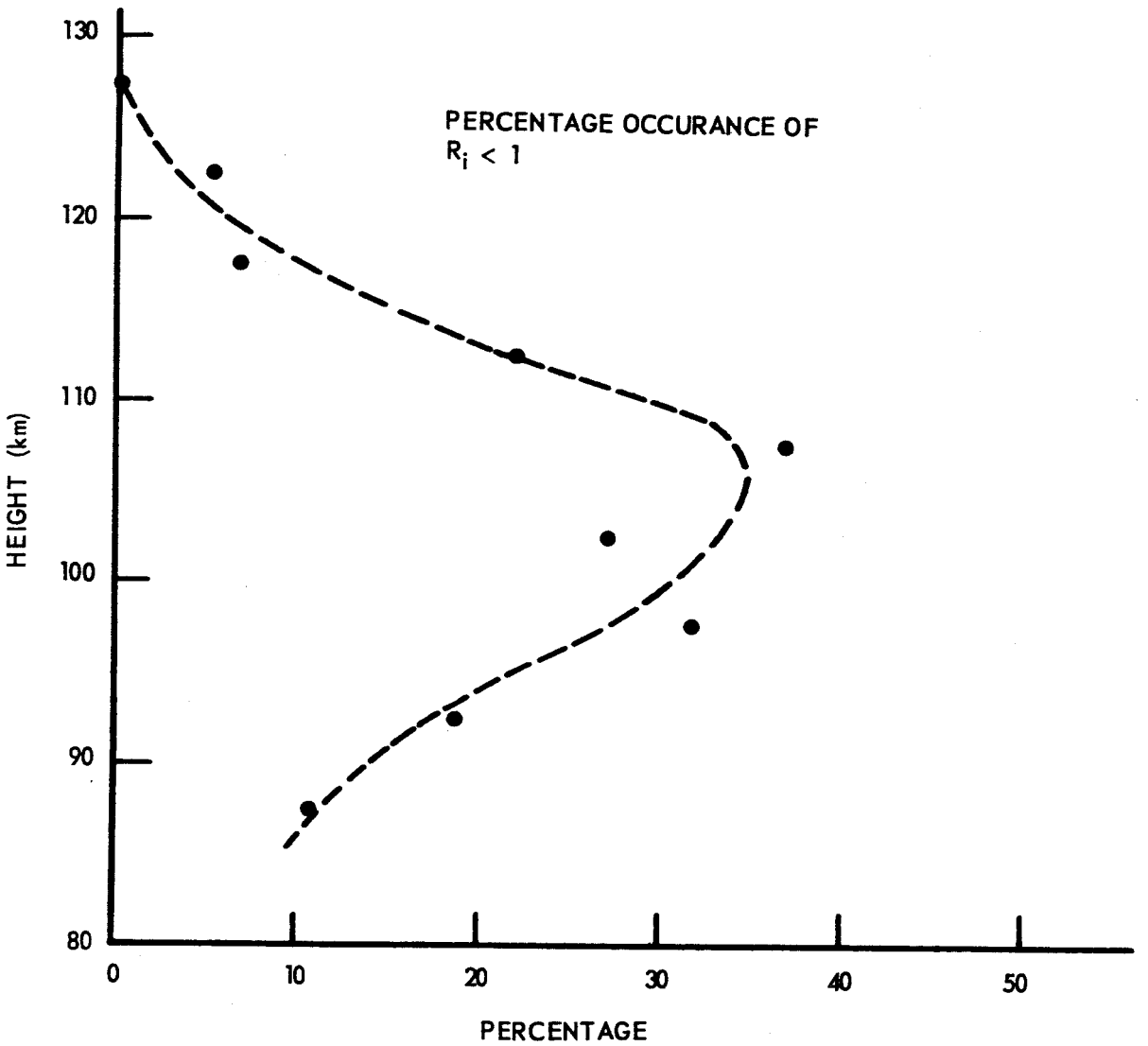

Figure 14. Percentage Occurance of $R_i < 1$ Versus Altitude.

isotropic only for scales $\approx$ 1 km or less. The smallest eddies are on the order of 20 m in size with characteristic velocities and times of about 2 m/sec and 10 sec. The largest eddies are about 6 km in size with characteristic velocities and times of about 15 m/sec and 400 sec. The winds which over short time intervals are considered to be the mean winds may have turbulent components of larger horizontal scale than 6 km, and longer time scale than 400 sec, but probably tidal waves and gravitational waves account for most of the fluctuations of this scale.

There are four parameters of the turbulence that depend on the factor ω_g. They are: 1) the scale of isotropy, l_i, in (18); 2) the time scale, t_g, of gravitational oscillations given by $t_g = 2\pi/\omega_g$; 3) the gravitational term in the energy balance, ε_g, in (27); 4) the Richardson number, R_i, in (45). The conclusions of the previous paragraph would be strengthened if l_i and t_g were larger and if R_i and ε_g were smaller than the calculated values. All of these conditions would require ω_g to be smaller than the calculated value. It may be that equation (19) for ω_g, which is based on standard thermodynamic laws, is not applicable to the near plasma conditions of the 100 km height region, or if (19) is valid, the presumed values of C_p may not be applicable at this altitude.

The upper height limit of turbulence of size scale less than 6 km usually occurs within $\pm$ 5 km of the 105 km level. From the data presented here it appears that this cutoff altitude, or turbopause, is caused by the rate of energy supplied by wind shears becoming too small to maintain the turbulence in the presence of dissipation rates which are increasing rapidly with altitude.

The Batchelor form of turbulent diffusion (32) has been shown as most corresponding to observed growth rates of individual globules. The actual exponent observed for the diffusion power law was 3.0 ± 0.4 rms. Cote (1962) has measured diffusion rates of chemical release trails. He reported that, without further study, it could not be shown conclusively whether or not the diffusion power law observed for the trails was different from the $d^2 \sim t^2$ relation expected for diffusion of a turbulent wake or turbulent jet. Wake or jet type diffusion would presumably be produced by rocket generated turbulence.

No turbulent diffusion measurements on the expansion of complete trails have been made yet by this group but work is underway on such measurements. If an exponent of 3 were observed for diffusion of trails as well as of individual globules, this would definitely rule out the possibility that the observed turbulence is in any way produced by the rocket or the ejection mechanism.

Further investigations of the spatial and time correlations of the turbulent wind components are planned. In particular, more accurate approximations to the one dimensional correlation coefficients g_{ii} will be evaluated, and a search for motion fluctuations of the expected time scale of 400 sec will be carried out.

Preliminary results indicate that Tchen's theory (1954) is applicable to the motion and energy spectrum of the mean winds for vertical separations, but that a Kolmogoroff motion spectrum may be followed if total spatial separation is considered in the evaluation of the spectrum function.

In addition, it is hoped that future work can establish what portions of the mean wind features are attributable to tidal wave, gravitational wave, and possibly large scale turbulent components.

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