TECHNICAL MEMORANDUM NO. 41

WIDE BAND PHASE DISTORTION EQUALIZATION

C. Capetanopoulos

July 1964

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
CONTRACT NAS 5-3508

LABORATORY FOR ELECTROSCIENCE RESEARCH

DEPARTMENT OF ELECTRICAL ENGINEERING

SCHOOL OF ENGINEERING AND SCIENCE

NEW YORK UNIVERSITY New York 53, New York

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ABSTRACT

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This report gives the design criteria for wide band phase realization. The design of passive lattice phase equalizers is introduced. These are all-pass passive networks that can correct the phase response of a particular system without affecting its amplitude response; they have the advantage of requiring no power and have been proven to give reliable performance in many systems where phase equalization is required.

The main part of this report consists of the design of a particular lattice equalizer whose phase vs. frequency characteristic has the form of an -S- curve. This particular phase characteristic can be used for phase correction in a wide variety of systems. One particularly effective method for almost any phase correction required is to cascade a number of these lattice networks designed for a so-called "staggered" arrangement.

AUTHOR

TABLE OF CONTENTS

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INTRODUCTION

The departure of the phase vs. frequency curve from a straight line for a tape recorder accounts for the distortion in a signal that is due to the different time delays of the different frequency components of the signal.

Since the time delay τ_d is defined as $\tau_d = \frac{d\phi}{d\omega}$, it is evident that unless $\phi = k\omega + c$ there will be time delay distortion.

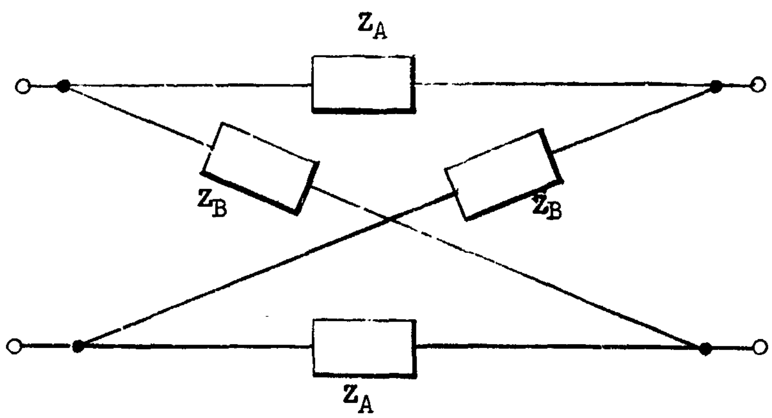
It is experimentally observed that the time delay is minimum for a certain band of frequencies and increases for frequencies outside this band. Thus it is typical to represent the time delay vs. frequency curve by a parabola centered at a frequency ω_1. Then the phase vs. frequency curve has the form of $y = AX^3 + B$ with its point of inflection displaced to the right by ω_1 and upwards so that a tangent through this point will cross the ordinate at the zero or $2n\pi$ point, as shown in Figure 1.

To compensate for the distortion of the signal arising from such a phase characteristic, a network should be designed which would have complementary phase vs. frequency characteristics (i.e., and -S-curve) that is, it would introduce little delay at low and high frequencies and considerable delay in the center band of frequencies.

A typical phase vs. frequency curve is shown in Figure 1, the minimum delay is assumed to occur at 100 kc. and the phase variation is over 4π radians to assure appreciable time delay.

DESIGN PROCEDURE

The basic network of the phase distortion equalizer will be a lattice network. This network has the advantage over all other passive two-ports of offering the designer a greater versatility in his specifications. Consider the following lattice network:



The design equations for the symmetrical lattice are the following:

The open and short circuited impedances are

$$Z_{OC} = \frac{1}{2}(Z_A+Z_B) \quad ; \quad Z_{SC} = \frac{2Z_AZ_B}{Z_A + Z_B}$$

The characteristic (or image) impedance is

$$Z_O = \sqrt{Z_AZ_B}$$

The propagation constant is given by

$$e^{\gamma} = e^{\alpha}e^{j\beta} = \frac{1 + \sqrt{Z_A/Z_B}}{1 - \sqrt{Z_A/Z_B}} \quad ; \quad \Phi = 2 \tan^{-1} \sqrt{- \frac{Z_A}{Z_B}}$$

It is desired that the lattice network be an all-pass structure so that the amplitude characteristic will not need any further compensation.

In order that the symmetrical lattice be an all-pass structure it is necessary that Z_A and Z_B be pure reactances of opposite sign at all frequencies. Under these circumstances

$$e^{\alpha}e^{j\beta} = \frac{1 + jX}{1 - jX} \quad \text{and thus} \quad \alpha = 0$$

Then the characteristic impedance is purely real

$$Z_0 = R_0$$

The simplest all-pass lattice has the form

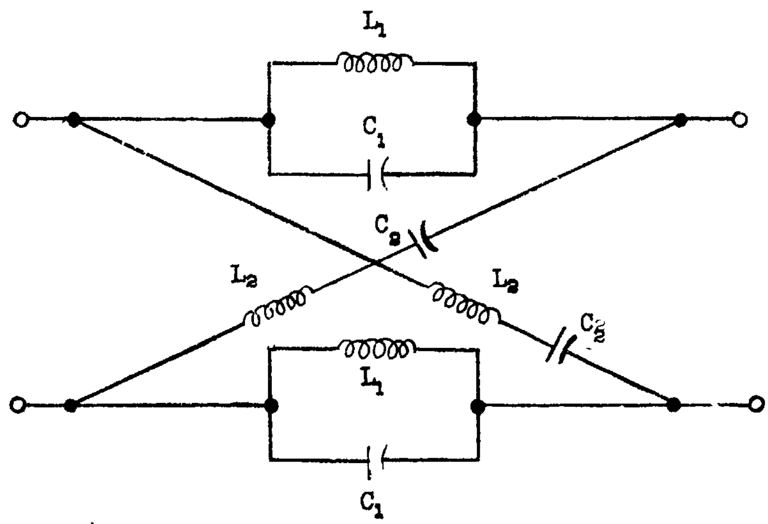
$$Z_A = L \qquad\qquad Z_B = C$$

For this network it can be shown that the phase angle varies with frequency as

$$\phi = 2 \tan^{-1} k\omega$$

The slope of the phase vs. frequency curve which represents the time delay $\left(\tau_d = \frac{d\phi}{d\omega}\right)$ is a monotonically decreasing function of the frequency, thus such a network cannot improve the low frequency phase response.

Consider the following symmetrical lattice network.



$$Z_A = \frac{\left(j\omega L_1\right)\left(\frac{1}{j\omega C_1}\right)}{j\omega L_1 + \frac{1}{j\omega C_1}} = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1}$$

$$Z_B = j\omega L_2 + \frac{1}{j\omega C_2} = \frac{1 - \omega^2 L_2 C_2}{j\omega C_2}$$

$$Z_0 = \sqrt{\frac{j\omega L_1}{1 - \omega^2 L_1 C_1} \cdot \frac{1 - \omega^2 L_2 C_2}{j\omega C_2}} = \sqrt{\frac{L_1}{C_2} \cdot \frac{1 - \omega^2 L_2 C_2}{1 - \omega^2 L_2 C_2}}$$

$$\frac{Z_A}{Z_B} = \frac{\dfrac{j\omega L_1}{1 - \omega^2 L_1 C_1}}{\dfrac{1 - \omega^2 L_2 C_2}{j\omega C_2}} = - \frac{\omega^2 L_1 C_2}{[1 - \omega^2 L_1 C_1][1 - \omega^2 L_2 C_2]}$$

In order that the lattice structure be all-pass it is necessary that the ratio Z_A/Z_B be negative for all ω. This is possible if the denominator of the expression is a perfect square. Thus

$$L_1 C_1 = L_2 C_2 = b \qquad \text{Condition for All-Pass}$$

Then the phase angle $\phi = 2 \tan^{-1} \sqrt{-\frac{Z_A}{Z_B}}$ becomes

$$\phi = 2 \tan^{-1} \left[\frac{\omega \sqrt{L_1 C_2}}{1 - \omega^2 L_1 C_1}\right]$$

If we let $a = \sqrt{L_1 C_2}$

$$\phi = 2 \tan^{-1} \left[\frac{a\omega}{1 - b\omega^2}\right]$$

In order to improve the low frequency phase response it is desired that the phase vs. frequency response of the equalizing network have a region where it is concave upwards, this in turn implies that the curve must have a point of inflection.

So a relationship between a and b must be obtained in order that the curve have a point of inflection.

The slope of the phase curve is given in

$$\frac{d\phi}{d\omega} = 2 \frac{1}{1 + \frac{a^2\omega^2}{(1-b\omega^2)^2}} \left\{\frac{(1-b\omega^2)a + a\omega(2b\omega)}{(1-b\omega^2)}\right\} = 2 \left[\frac{a + ab\omega^2}{(1-b\omega^2)^2 + a^2\omega^2}\right]$$

The point of inflection is given by the condition

$$\frac{d^2\phi}{d\omega^2} = b^3\omega_1^4 + b^2\omega_1^2 + a^2 - 3b = 0 \qquad (1)$$

The solution

$$\omega_1^2 = \frac{-b \pm \sqrt{1 - 4b(a^2-3b)}}{2b^2}$$

In order that ω be a real frequency it is necessary that

$$-b + \sqrt{1 - 4b(a^2-3b)} > 0$$

If $b \gg 1$ the condition reduces to

$$a^2 < \frac{11}{4} b$$

Under this condition there will be one and only one point of inflection in the phase curve. Equation (1) may be solved for b for two different values of a. Thus:

$$b = \frac{1}{\omega_1^2} \quad (\text{for } a^2 = b)$$

$$b = \frac{2.1}{\omega_1^2} \quad (\text{for } a^2 \ll b)$$

The design equations for the network are as follows:

(a) $$L_1 C_1 = L_2 C_2 = \frac{1}{\omega_1^2}$$

[where ω_1 is the desired frequency at which the point of inflection occurs]

(b) $$Z_O = R_O = \sqrt{\frac{L_1}{C_2}}$$

(c) $$L_1 C_2 < \frac{11}{4\omega_1^2}$$

It can be seen that the designer has two degrees of freedom. He can arbitrarily specify the characteristic resistance and the frequency at which the point of inflection will occur. Values of the different elements must be chosen such that condition c is satisfied. The phase angle is given by

$$\phi = 2 \tan^{-1} \frac{\omega \sqrt{L_1 C_2}}{1 - \frac{\omega^2}{\omega_1^2}}$$

TYPICAL DESIGN

Let it be required to design a compensating network having the characteristic of Figure 2 (II), with a characteristic resistance of 100 Ω and a center (inflection) frequency of 100 kc., and a phase variation over 4π radians.

Two identical stages will be used to obtain the phase variation of 4π radians. The conditions to be satisfied are:

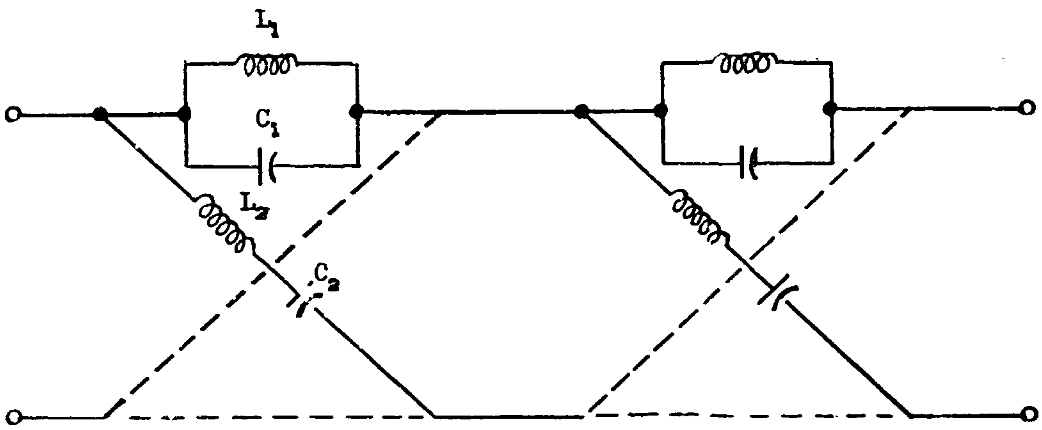
$$L_1 C_2 = \frac{1}{10(2\pi)^2 \times 10^{10}} \qquad \frac{L_1}{C_2} = 10^4$$

$$L_1 C_1 = \frac{2}{(2\pi)^2 \times 10^{10}} \qquad L_2 C_2 = \frac{2}{(2\pi)^2 \times 10^{10}}$$

The elements are

$$L_1 = \frac{1}{\sqrt{10}\,(2\pi)} \times 10^{-3} \text{ Hy} \quad ; \quad C_2 = \frac{1}{\sqrt{10}\,(2\pi)} \times 10^{-7} \text{ Fd} \quad ;$$

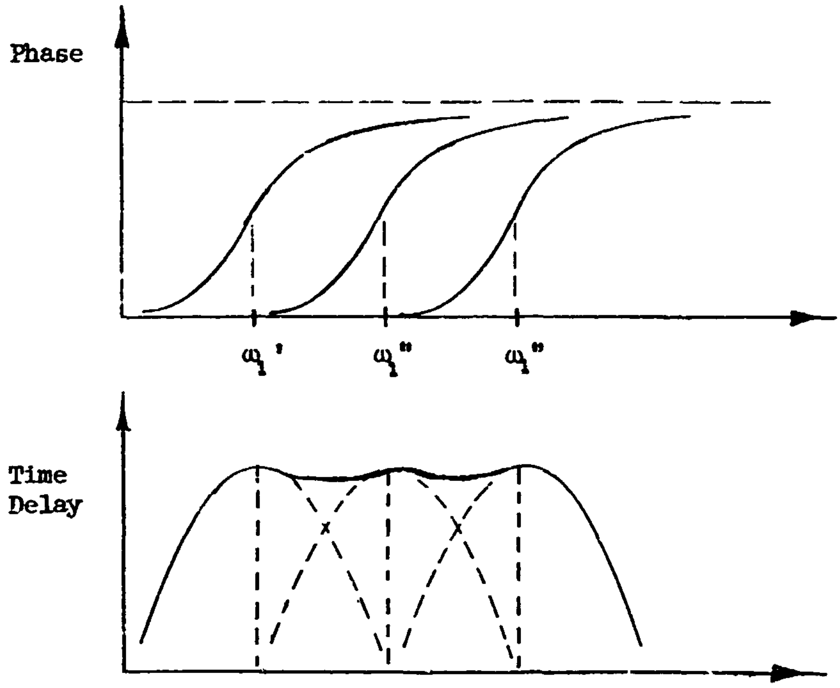
$$C_1 = \frac{2\sqrt{10}}{2\pi} \times 10^{-7} \text{ Fd} \quad ; \quad L_2 = \frac{\sqrt{10}}{2\pi} \times 10^{-3} \text{ Hy}$$



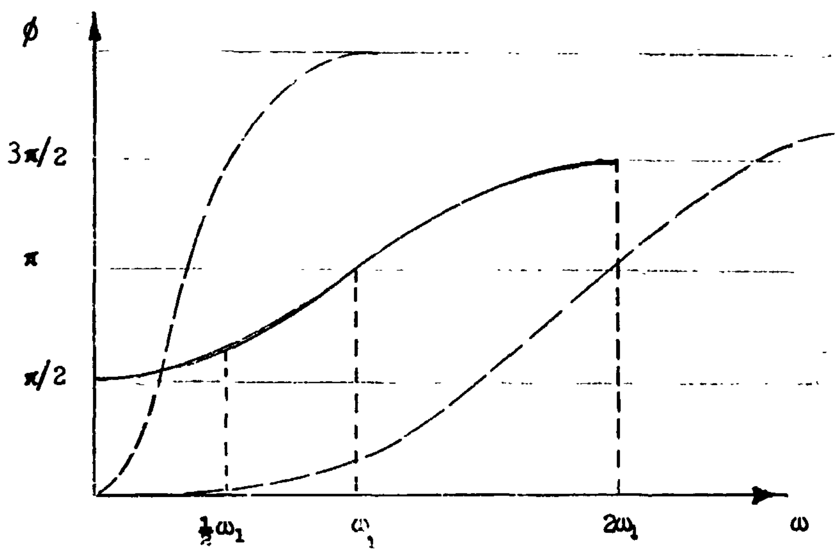
CONCLUSIONS

In the previous example a single phase equalizing network was designed to compensate the distortion arising from a parabolic time dealy vs. frequency curve.

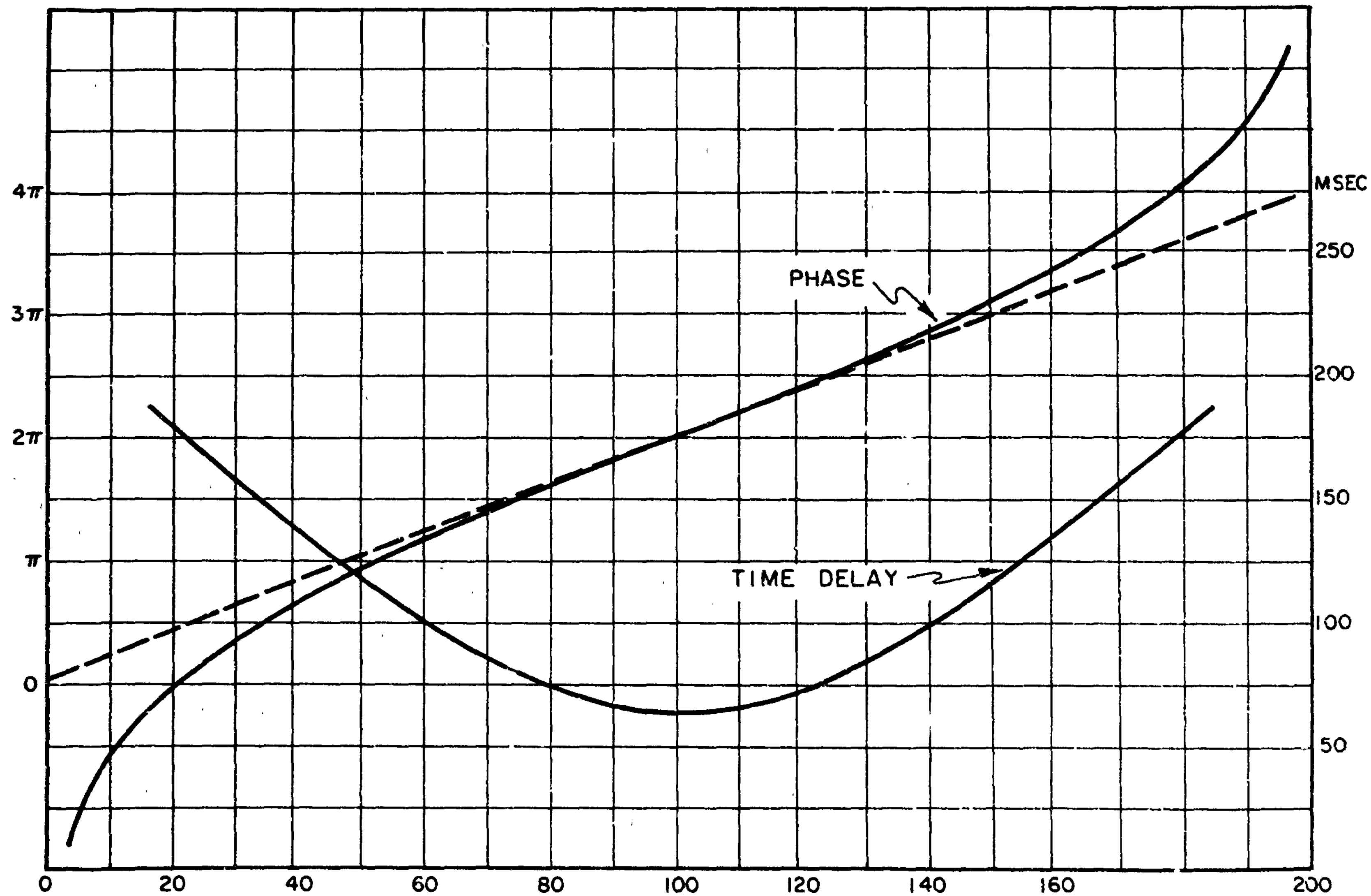
However, it is often common that the time delay vs. frequency curve instead of being a parabola is essentially flat over a range of frequencies. Under these conditions a single network as the one described will not be satisfactory. A considerable improvement should result if 3 networks having phase characteristics of Figure 2 (III) with identical characteristic impedances but center (inflection) frequencies properly chosen to give a considerable and approximately constant time delay over the center band of frequencies as shown. If it is desired to have a continuous variation over the shape of the frequency curve, this can be done by varying the inductance L_1, but the other elements must be properly chosen and mechanically linked to this inductance.



The lattice network designed can be used as a basic structure to equalize a wide variety of time delay vs. frequency curves. Consider for example that the phase vs. frequency curve for a particular tape recorder has the form shown below.

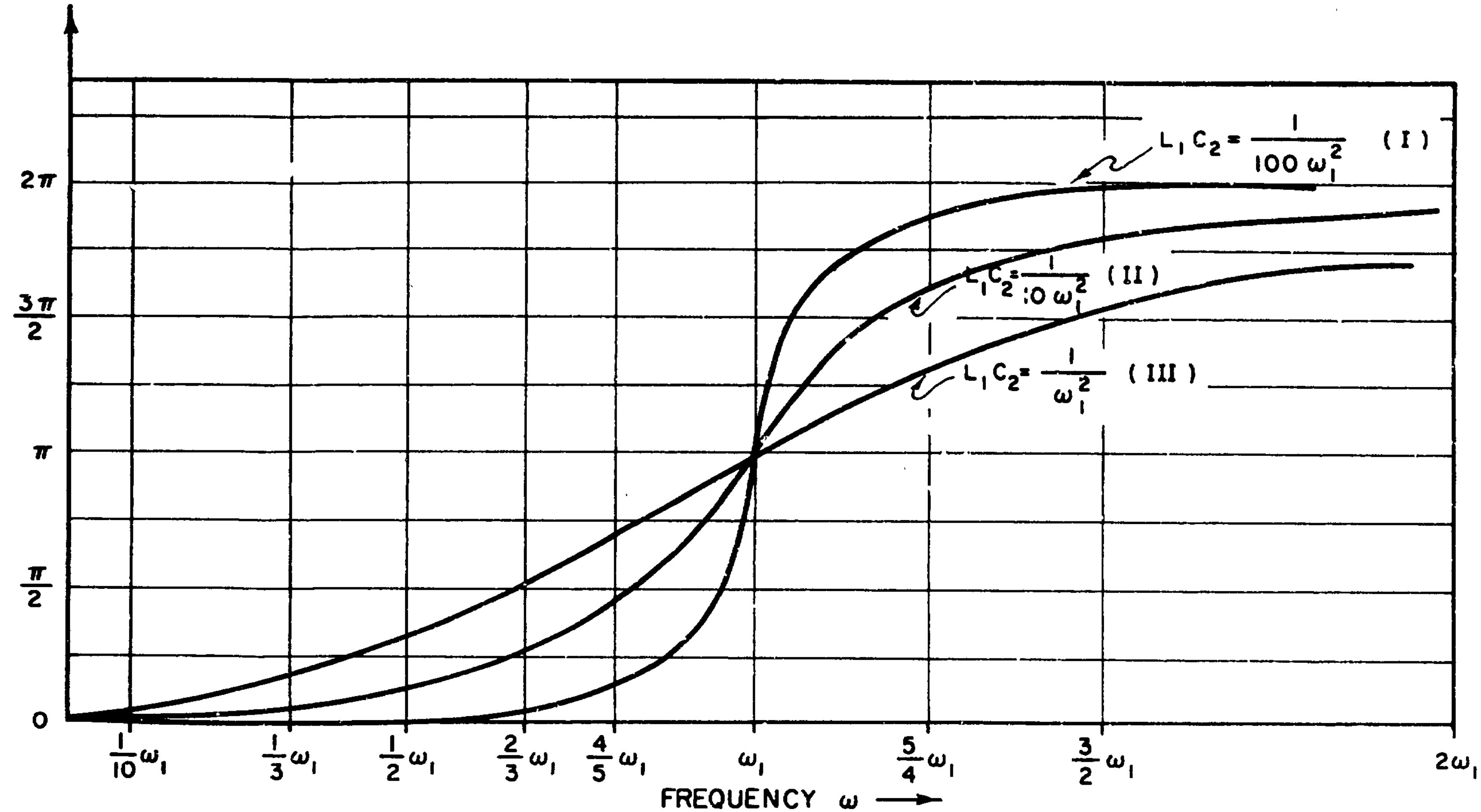


This phase variation might be explained in terms of simple RC network equivalents representing the record and playback systems. This phase curve indicates that there is a considerable time delay over the center of the frequency range and very little delay outside the band. Phase equalization can be provided by constructing two lattice networks in cascade having identical image impedances, with their inflection frequencies chosen such as to provide a considerable time delay over the low and high frequency band. The inflection frequency for the first lattice should be chosen to be a little less than half the inflection frequency of the phase curve. For the second lattice its inflection frequency should be chosen to be approximately twice that of the phase curve. The slopes could then be adjusted for best equalization.



4π
3π
2π
π
0
MSEC
250
200
150
100
50
PHASE
TIME DELAY
0
20
40
60
80
100
120
140
160
200
FREQ. IN KC

FIG. 1



F I G. 2

PHASE VS. FREQENCY FOR THE LATTICE NETWORK