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The Relationship Between the System of Astronomical Constants and the Radar Determinations of the Astronomical Unit.

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I. Introduction.

The primary goal of this paper is the preparation of material for the I.A.U. Symposium No. 21 on the System of Astronomical Constant with regard to:

i. The author's work on the radar determination of the astronomical unit (Muhleman, 1962, 1963).

ii. The importance of current radar observations on the System of Astronomical Constants.

iii. The history and evolution of that system.

The discussion will be limited to a great extent to that part of the system of astronomical constants that is obviously effected by the current radar observations. Consequently, the discussion pertaining to the geodetic constants of the Earth, for example, will be primarily limited to points of historical interest.

The plan of the paper will be to discuss the classical work of Newcomb and de Sitter from the standpoint of the definitions of the fundamental constants and the theoretical relationships between them. These theoretical relationships will be applied to the radar results, when possible, with a spirit of "let's see what happens". The determination of the velocity of light will be discussed in some detail because of the singular importance of this constant in radar measurements. The main body of the paper will be devoted to a brief but exact discussion of the determination of the astronomical unit with radar and to an extensive error analysis of the technique.

Because of the many theoretical relationships between the constants, a certain group of them have been selected (primarily by Newcomb and de Sitter) as fundamental constants. This discussion will suggest that

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this particular division of the constants may be profitably revised because of the inclusion of radar measurements of distances and velocities to the observational material of dynamical astronomy.

II. The System of Astronomical Constants.

The fundamental constants of the Earth consisting of the elements of its orbit, the mass, constants specifying size, shape, orientation, rotation, inner constitution, and the velocity of light comprise the system of astronomical constants. The group of constants has been called a system because it comprises a model of the Earth and its motions. The interpretation of all astronomical observations depends on this particular model. Furthermore, because of the many theoretical relationships involving several of the fundamental constants, some of the constants are necessarily systematically related. De Sitter (1938) wrote, "An ideal system of fundamental constants would be one in which these theoretical relations were satisfied rigorously, while the adopted value of each individual constant agreed with its observed value, within the limits of uncertainty of the latter." This ideal has not been realized, even today.

The system of astronomical constants has apparently evolved from Newcomb's work reported in Vol. I and II of the Astronomical Papers and his Astronomical Constants (Newcomb, 1895), which is an exhaustive treatment of the subject as well as a compilation of important formulae. These works, particularly the latter, served as the basis for the system of constants adopted by the Paris conference of 1896. Many of the adopted values were integrated into Newcomb's tables of the Sun and the four inner planets. Partly because of the fundamental importance of these tables, astronomers since Newcomb have been reluctant to change the values of the constants even though several important inconsistencies are known to exist in the system. The general view on this point is adequately expressed in the following quotation from the Explanatory Supplement to the Ephemeris, 1961: "The principal reason for retaining the system unchanged is a consequence of the methods necessarily employed in dynamical astronomy. The value of a constant is never measured directly. The method of differential corrections is employed instead. Observations made at various times are compared with an ephemeris. Analysis of the discrepancies between the observations and the ephemeris yields corrections to the values of the constants used in constructing the ephemeris, which being applied give more accurate values of the constants. If, during the period covered by the observations, the value of any constant entering into the calculation of the ephemeris has been altered, then the ephemeris at times before the alteration is inconsistent with the ephemeris at later times, and an analysis that fails to take account of the change is bound to lead to erroneous conclusions." The consequence of changes in the past, have been to cause erroneous conclusions because of the ignorance of the investigator to the changes. Even when the changes are properly considered, the labor of analysis is greatly increased.

III. Relationships Between the Astronomical Constants

De Sitter (1938) attempted to construct a rigorous system of Astronomical Constants based on the observations available up until 1938. In so doing, he set down a series of relationships and ideas which still serve as a guide for a rediscussion of the fundamental constants. The relationships that appear to be important from the standpoint of interpreting the astronomical unit as determined from radar measurements will be written with little development and will be applied to the numerical results later in this paper.

De Sitter selected 8 constants as "fundamental":

1. $\pi_\odot$, the solar parallax
2. μ^{-1} , the Moon: Earth mass ratio
3. c , the velocity of light
4. (C-A)/C, the dynamic compression
5. R_1 , the mean radius of the Earth
6. g_1 , gravity acceleration at mean latitude
7. κ , a small constant relating to the Earth's interior
8. λ_1 , a small constant relating to the Earth's interior

All of the remaining constants are then considered "derived" constants. By the use of R_1 and g_1, the relationships of geodesy are supposedly simplified since R_1 is defined as the radius on an ellipsoidal Earth at a latitude $\phi = \sin^{-1}\sqrt{1/3}$, where the mass of the Earth acts as a point mass at the center of the Earth. The equatorial radius of the Earth, b, is then given by

$$b = R_1(1 + \tfrac{1}{3}\epsilon - \tfrac{4}{9}\epsilon^2 + \tfrac{8}{9}\kappa) \tag{1}$$

where ϵ is the elliptical flattening of the Earth. De Sitter then defined the ratio of centrifugal force to gravitational force as

$$\rho_1 = \frac{\omega^2 R_1^2}{fM_1} \tag{2}$$

where ω is the angular velocity of the Earth, f the gravitational constant, and M_1, the mass of the Earth. Then it can be shown that

$$g_1 = \frac{fM_1}{R_1^2}\left(1 - \frac{2}{3}\rho_1 + \frac{14}{9}\epsilon^2 - \frac{10}{9}\epsilon\rho_1 - \frac{16}{9}\kappa\right). \qquad (3)$$

De Sitter then adopted the values

$$R_1 = 6{,}371{,}260\ (1 + u),\ \text{m}$$
$$g_1 = 979.770\ (1 + v),\ \text{cm/sec}^2$$
$$H = (C - A)/C = 0.003279423\ (1 + w)$$
$$\kappa = 0.000\ 00050 + 10^{-3}\chi$$
$$\lambda_1 = 0.000\ 40 + \psi$$
$$\pi_\odot = 8''.8030\ (1 + x)$$
$$c = 299{,}774\ (1 + y),\ \text{km/sec}$$
$$\mu^{-1} = 81.53\ (1 + z)$$

The astronomical unit in km is defined by the relationship

$$1\ \text{a.u.} = \frac{b}{\pi_\odot \sin 1''} \qquad (4)$$

which from (1) yields

$$1\ \text{a.u.} = \frac{R_1}{\pi_\odot \sin 1''}\left(1 + \frac{1}{5}\epsilon - \frac{4}{9}\epsilon^2 + \frac{8}{9}\kappa\right). \qquad (5)$$

His derived astronomical unit then becomes

$$1\ \text{a.u.} = 149{,}453{,}000\ \text{km}\ [1 - x + 1.0002u - .0002v + .0009w + .0007\chi + .0009\psi].$$

De Sitter followed this method for all of the derived constants.

I will now present a series of definitions and relationships from de Sitter with little comment that will be employed later in this paper.

All of the relationships are sufficiently standard and require no discussion. From Kepler's law, the semi-major axis, a_0, in a.u.'s is defined by

$$n^2 a_0^3 = k^2(1 + m) \qquad (6)$$

where n is the mean motion of the Earth, k is gauss's constant and m is the ratio of the mass of the Earth plus Moon to the mass of the Sun. The constant of abberation, K, is <u>defined</u> by

$$K = \frac{n\ a\ \sec\Phi}{86400\ C}$$

$$K = \frac{n\ b\ \sec\Phi}{84600\ \pi_\odot\ \sin 1''\ e} \qquad (7)$$

and the light time for 1 a.u., τ, is <u>defined</u>

$$\tau = \frac{b}{c\ \pi_\odot \sin 1''}\text{ , seconds} \qquad (8)$$

Now, using equations (1), (3), (4), (5), and (6) we get after considerable manipulation

$$\left(\frac{1 + m}{m}\right)\pi_\odot^3 = \frac{n^2 R_1(1 + \nu_1)^3}{g_1(1+u)(86400)^2 \sin 1''}\left(1 - \nu_3 + \epsilon - \frac{2}{3}\rho_1 + \right.$$

$$\left. + \frac{5}{9}\epsilon^2 - \frac{16}{9}\epsilon\rho_1 + \frac{8}{9}\kappa\right) \qquad (9)$$

where ν_3 is the fraction of the Earth mass that must be added to M_1 to include the mass of the atmosphere. Equation (9) can be considered as the major relationship given by de Sitter since it relates the mass of the Earth-Moon system to the fundamental constants $\pi_\odot$, R_1, u and g_1.

This expression has been employed by investigators to determine the constants from the motion of Eros in particular (see discussion below on Rabe's work).

The parallactic inequality is the term $-P \sin D$ in the Moon's ecliptic longitude. The value of P is given from Brown's lunar theory,

$$P = 49853''.2 \left(\frac{1-\mu}{1+\mu}\right) \frac{\pi_\odot}{\sin \pi_☾} \qquad (10)$$

and finally, for the constant of the lunar inequality, de Sitter introduces

$$L = \frac{\mu}{1+\mu} \frac{\pi_\odot}{\sin \pi_☾} \qquad (11)$$

The last two expressions are important relationships between μ, $\pi_\odot$ and $\pi_☾$ in terms of the observables P and L that have been utilized to compute one of the three constants, given the other two.

A second consistent system of constants has been presented by Clemence (1948). The major contribution from this paper is a precise statement of the proposed introduction of "ephemeris time". As a consequence of this change (inacted in 1950) several of the inconsistencies of the ephemeris were removed. A precise discussion of this point can be found in the Explanatory Supplement to the Ephemeris.

A second conference on the system of constants was held in Paris in 1950 (see Bull. Astronomique, vol. 15). The recommendation of that conference was that no changes should be made of the system of constants but that the concept of ephemeris time should be made official.

The most current revision of the constants has been given by Brouwer and Clemence (1961) where they have primarily employed current observations to the system developed by de Sitter.

IV. Rabe's Work on Eros Observations.

Rabe (1950) utilized the observations of Eros at three Earth passings to compute the solar parallax, the Earth-Moon mass ratio, and several other planetary masses as well as corrections to the elements of the Earth. In his computation, the observations of 1930-31 were most heavily weighted. The actual procedure used was to compute the mass of the Earth from the perturbations by the Earth on Eros. Once the mass of the Earth was obtained, the solar parallax was computed from equation (9) using de Sitter's constants. The results of Rabe that are of interest here are:

1. $\pi_\odot = 8''.79835 \pm 0''.00039$
2. $\mu^{-1} = 81.375 \pm .026$
3. $m = 328,452 \pm 43$
4. $m_\oplus = 332,480$ (from 2 and 3)

It would be possible to revise Rabe's $\pi_\odot$ using slightly different values in de Sitters equation (9) but this would not be profitable (Brouwer, 1963). However, it has apparently gone unnoticed that the corrections to the elements of the Earth resulting from Rabe's computations are very different from a similar set computed by Duncombe (1958) from the observations of Venus. It appears likely that if Duncombe's corrections were employed in a new solution for Rabe's normal equations a significantly different value of the solar parallax might result. The need for such a revision of the Eros results is clear from the strength of the radar results reported below.

V. The Velocity of Light.

The determinations of the velocity of light have a long and interesting history. An excellent survey of the classical determinations has been given by Bergstrand (1956). The adopted value of c as given in the Nautical Ephemeris is a very old determination by Newcomb and is well known to be grossly in error. A precise value of the velocity of light has not been a particular concern to astronomical questions until the present time. The radar determinations of the astronomical unit and the determination of associated constants by radar and radio tracking of artificial space vehicles are intimately concerned with a precise measurement of the velocity of light, however. It will be shown that, even though the modern value of c is known reliably to six figures, the uncertainty in the light-velocity determinations is the major single source of error in the radar measurements.

A recent survey of the important light-velocity determinations since 1946 has been given by DuMond (1959). His results are shown in Table I. The best single determination is apparently the value found by Froome (1958)

$$299{,}792.50 \pm 0.10 \text{ km/sec}$$

which he obtained by a microwave interferometer technique at 74,500 Mc. I have computed the mean value from Table I, weighting the values with the reciprocal-squares of the quoted uncertainties, and found

$$299{,}792.63 \pm 0.08 \text{ km/sec.}$$

This result is in excellent accord with Froome's individual measurement. This is partially due to the large weight assigned to Froome's 1958 determination, of course. The general agreement to a few parts in 10^6 of all of the modern values shown in Table I is reassuring and it appears highly unlikely that a systematic error larger than 0.3 km/sec could exist.

The International Union of Geodesy and Geophysics, on the recommendation of the XII General Assembly of the International Scientific Radio Union, has adopted the value

$$299{,}792.5 \pm 0.4 \text{ km/sec.}$$

This value has been used in the radar determinations of the astronomical unit.

VI. The Determination of the A.U. by Radar at the 1961 Inferior Conjunction of Venus.

Radar observations have been obtained for Venus around the 1961 inferior conjunction by several groups. The resulting value for the astronomical unit are shown in Table II. All the determinations are in agreement. However, Newcomb's tables of the Sun and Venus were employed in all cases, which, if they cause an important error at all, would effect each determination in essentially the same way. A detailed discussion of these effects is presented below.

1. Instrumentation

Details of the computations of Muhleman, et al (1962a) will be described. A complete discussion of Pettingill's result can be found in Pettingill, et al (1962). The observations reported in that paper have been used to compute a slightly revised value of the A.U.. For purposes of reference, the work of Muhleman, et. al. will be referred to as that of the "Goldstone group" since the observations were made at the Goldstone station of the Jet Propulsion Laboratory, California Institute of Technology.

The observations of the Goldstone group were taken with three fundamentally different radar receiving systems. The observations consisted of the doppler-frequency shift on the 2388 Mc/s carrier and measurements of the propagation time to Venus and back to the Earth by modulating the carrier with either a regular square wave or a pseudo-random code.

The frequency reference for the doppler velocity measurements was an Atomichron cessium-resonance line which had a measured stability of 1 or 2 parts in 10^{10} over a period of about five minutes. All other reference frequencies in the receiver were coherently derived from the standard in such a manner that frequency errors introduced into the system were subsequently subtracted out at some other point in the system (closed-loop system). Consequently, the measurements of the doppler frequency shift are probably accurate to better than 1 part in 10^9. This uncertainty is far smaller than that due to the velocity of light.

The systems of modulation employed by the two methods of measuring the propagation time were designed to have a range resolution of about 100 km. The overall accuracies of this system are about on the order of 100 km except for the uncertainty of c, i.e. about 0.0003 seconds for the Earth-Venus distance.

2. The preparation of the ephemeris.

The doppler frequency shift and the propagation time must be computed from the ephemerides with precision for the comparision with observations. The total propagation time is given by:

i. the time for the signal to travel from the position of the transmitting antenna at time 1 to the <u>surface</u> of venus at time 2,

ii. plus the time for the signal to travel from the surface of Venus at time 2 to the position of the receiving antenna at time 3,

The actual epoch for each observation was taken to be time 3 and the arguments for entries into the tables of the Sun and Venus were computed with a simple iteration scheme. The doppler-frequency shift is a function of

i. the velocity of the <u>center</u> <u>of</u> <u>mass</u> of Venus at the instant the wave front strikes the surface of the planet with respect to the position and velocity of the transmitting station at time 1, $\dot{R}_{12}$,

ii. the velocity and position of the receiving station at the instant the reflected wave front reaches the receiving station, with respect to the velocity of the center of mass of Venus at the instant of reflection, time 2, $\dot{R}_{23}$.

The equation for the conversion of the ephemeris velocities, $\dot{R}_{12}$ and $\dot{R}_{23}$, to doppler frequency shift has been derived by Muhleman (1962) to 2nd order in v/c and is

$$(\tilde{\nu} - \nu) = -\nu\left(\frac{\dot{R}_{12}}{c} + \frac{\dot{R}_{23}}{c} - \frac{\dot{R}_{12}\,\dot{R}_{23}}{c^2} - \frac{\dot{R}_{23}^2}{c^2}\right) \qquad (12)$$

where ν is transmitter frequency and $\tilde{\nu}$ is the received frequency at time 3.

The actual values used in the analysis of the radar observations were computed with a tracking program written for the IBM 7090 computer. The coordinates to be smoothed were obtained directly from Newcomb's tables of the Sun and Venus with corrections for known errors. In particular, a correction of -4."78T was applied to the mean anomaly of the Sun after Clemence (1948). An n-body numerical integration, starting with "injection" position and velocity, was compared with the coordinates written on a magnetic tape from the Newcomb tables, and corrections to the injection conditions were derived using a least-squares iterative procedure. Several iterations yielded the best injection values over a 120-day arc for Venus and a 70-day arc for the Earth. These residuals were reduced to a few parts in 10^7 which is consistent with the roundoff in the tabulated data. Velocity data was obtained at each epoch of interest as a consequence of the Runge-Kutta numerical integration procedure. The velocities obtained in this manner are smooth to seven figures and probably accurate to a few parts in 10^6. The ephemerides obtained with the above technique are considered a smooth equivalent to the numerical tables of Newcomb, including only the change in the argument M referred to above. Subsequently in this paper, the ephemerides will be referred to as the "Newcomb ephemerides".

Duncombe (1958) has obtained a set of corrections to Newcomb's elements from the Venus observations over a period from 1795 to 1949. The published corrections are:

for Earth:

$$\Delta e_{\oplus} = -0.''10 \pm 0.''01 + 0.''00\,T,$$

$$\Delta \epsilon = +0.''04 \pm 0.''01 - (0.''29 \pm 0.''03)\,T,$$

$$\Delta L_{\oplus} = -0.''39 \pm 0.''05 + (0.''45 \pm 0.''15)\,T,$$

$$e\,\Delta \pi = -0.''07 \pm 0.''03 - 0.''09\,T$$

for Venus:

$$\Delta \ell_{♀} = +\,''10 \pm {''06} + ({''53} \pm {''18})\ T,$$

$$\Delta e_{♀} = -\,''12 \pm {''03} + {''01}\ T,$$

$$e_{♀}\Delta \pi_{♀} = +\,''01 \pm {''04} + {''04}\ T,$$

$$\Delta i_{♀} = +\,''08 \pm {''03} - {''02}\ T,$$

$$\sin i_{♀} \Delta \Omega_{♀} = +\,''21 \pm {''03} + {''02}\ T$$

The corrections actually used were supplied by Duncombe (1961) and are only slightly different:

for the Earth:

$$\Delta e_{\oplus} = -\ 0''113\ T,$$

$$\Delta \epsilon = +\ 0''045 - 0''29\ T,$$

$\Delta M_{\oplus} = +\ 4''78\ T$ (all ready applied in the "Newcomb Ephem")

for Venus:

same as above.

The Duncombe corrections were incorporated in the program which evaluated the Newcomb theory and a new ephemeris was generated utilizing the same technique as before. This ephemeris has been called the Duncombe ephemeris.

3. Results.

Observations of Venus were taken at a rate of once per ten seconds from continuous periods of from 5 minutes to one hour. This was normally done once each day for the doppler measurements and the two ranging-systems measurements. Each set of observations was used to compute a separate estimate of the A.U.. The estimate of the A.U. was computed with an iterative least-squares procedure which minimized the observations minus the calculated value by computing a correction to the A.U. value used in the previous iteration. The calculations were performed for both the Newcomb ephemeris and the Duncombe ephemeris. The rms residuals for the velocity observations were about ± 0.1 m/sec and about ± 200 km was obtained for the range residuals. Actually, the residuals varied somewhat with the distance to Venus because of the decrease in the radar-echo power with distance.

The computed A.U. estimates from the velocity observations are shown in Figure 1. This figure shows that the estimates of the A.U. rapidly diverge downward as conjunction (Apr. 11) is approached from the east and return from above immediately after conjunction has passed. The effect of the Duncombe corrections was to raise the estimates on March 23 by 1200 km and on Apr. 7 by about 7000 km. Similarily, on Apr. 13 the estimate was lower by 8900 km and on May3, by 400 km. Clearly, the effect is due to the sensitivity of the doppler velocity (range rate) to errors in the ephemerides as the velocity gets small. The primary correction of Duncombe is to advance the longitude of Venus by about 0".55 relative to that of the Earth. This was apparently not enough to completely straighten the curve. Muhleman, et al (1962) have shown that the effect of an error in the longitudes of Venus and the Earth in the determination of the A.U. is approximately (near conjunction)

$$\delta(\text{A.U.}) \simeq A_{\oplus} \cot(\ell_{♀} - \ell_{\oplus})\, \delta(\ell_{♀} - \ell_{\oplus}) \qquad (13)$$

which is very similar to the behavior shown in Figure 1. A more exact analysis of this problem will be given below.

The estimates of the A.U. computed from the range measurements from the system employing the pseudo-random code modulation are shown in Figure 2. These observations are all post-conjunction. A linear trend with date is evident from the figure, the slope of which was decreased by applying the Duncombe corrections. Muhleman, et al, (1962) have shown that the effect on the A.U. determinations from range data due to only an error in the relative planetary longitudes is approximately

$$\delta(\text{A.U.}) \simeq A_{\oplus} \left(\frac{r_{♀} r_{\oplus}}{r^2} \right) \sin (\ell_{♀} - \ell_{\oplus}) \, \delta(\ell_{♀} - \ell_{\oplus}) \quad (14)$$

where $r_{♀}$ and $r_{\oplus}$ are the heliocentric distances to the planets and r is the distance between them. The equation is in good agreement with the effect observed in Figure 2.

The measured radar propagation times to Venus published by Pettingill, et al (1962) were used to compute the estimates of the A.U. shown in Figure 3. The agreement between these estimates and those computed by Pettingill's group is excellent. A trend similar to that predicted by equation (14) is again evident in the estimates.

The reduction of all of the A.U. estimates to a single result is a considerable task. Because of the apparent errors in the ephemerides (after Duncombe's corrections) it is necessary to proceed somewhat arbitrarily. I have used equation (13) to extrapolate the doppler-A.U. estimates to the east and west elongations where errors in longitude would have a minimal effect. However, an error in e"$\Delta\pi$" may be significant at these points. Equation (14) was employed to interpolate the range-A.U. estimates at conjunction. (Clearly, the total effect of the Duncombe corrections is nearly zero at conjunction). The results of this procedure are:

i. doppler near eastern conjunction. 149,598,750 ± 200 km,

ii. doppler near western elongation.. 149,598,000 ± 1000 km,

iii. range at conjunction .... 149,598,500 ± 150 km,

iv. range at conjunction .... 149,598,800 ± 150 km,

where the value in iv. was computed from range observations from the 2nd ranging system which was independent of the 1st system to a large degree. The uncertainties attached to the above values are estimates based primarily on the scattering in the estimates. The systematic errors will be considered below.

The final value of the A.U. is the mean of the four figures above with weights equal to the reciprocal variances.

149,598,640 ± 200 km.

The value computed from Pettingill's observations utilizing equation (14) for interpolation to conjunction is

149,598,100 ± 400 km

where the uncertainty was taken from Pettingill, et al (1962).

VII. The Determination of the A.U. by Radar at the 1962 Inferior Conjunction of Venus.

The observational program on Venus for 1961 was repeated around the 1962 inferior conjunction. The techniques that were employed in the latter observations were somewhat different. In 1961 two antennas separated by 10 km were operated as a transmitter and receiver pair and, consequently yielded continuous runs of data. However, it was necessary to use a single antenna in 1962 as both the transmitter and the receiver. This was accomplished by transmitting for the propagation time from the Earth to Venus and switching to the receiver mode for a similar length of time. This had the effect of essentially halving the observation time. Furthermore, it was decided that a comparison ephemeris should be constructed over an arc much longer than the 100 day arcs utilized in the previous analysis in order to cover both observational periods with one fit. The ephemeris was prepared in essentially the manner described above but ten year arcs were employed as reported by Peabody and Block (1963). The residuals in positions relative to the Newcomb tables (after a correction of M" = + 4".78 T) exhibited oscillations as large as 5×10^{-7} a.u. in the radius vectors and 0".1 in the longitudes and latitudes with the sideral periods. These residuals have had serious effects on the A.U. results. Primarily for this reason the 1962 results reported here are to be considered as preliminary. However, in all cases the values of the A.U. deduced agree to within the accuracy of the analysis to those found in 1961.

The Calculation of the Astronomical Unit.

The A.U. has been obtained by comparing the observations to the values computed from the astronomical tables using a first guess of the A.U. for entry into the tables and then computing a second estimate of the A.U. from the differences by the classical least squares technique. The process is repeated until the rms differences (residuals) obtained in the n-th iteration are not significantly smaller than those obtained in the (n-1)th iteration. Thus the A.U. is found by assuming that the astronomical tables are correct except for one parameter, - the A.U.. In general, a given residual is given by (after a Taylor's expansion to 1st order)

$$(R_0 - R_c)_i = \left(\frac{\partial R_c}{\partial \alpha_1}\right)_i \delta\alpha_1 + \left(\frac{\partial R_c}{\partial \alpha_2}\right)_i \delta\alpha_2 + \ldots + \left(\frac{\partial R_c}{\partial \alpha_m}\right)_i \delta\alpha_n, \qquad (15)$$

where R_0 is the observed range (for example) and R_c is the range computed from the tables with an assumed value of the A.U.. The $\delta\alpha$'s are the (unknown) errors in the significant parameters of the astronomical theory <u>including</u> the A.U.. Thus, the method employed here assumes that all of the $\delta\alpha$'s are zero except δ A.U.. When the set of equations (15) (the normal equations) are solved in a least squares sense the resulting correction for the A.U. in the case where all of the other $\delta\alpha$'s are zero is

$$\delta \text{ A.U.} = - \sum_i \left(\frac{\partial R_c}{\partial \text{ A.U.}} \right)_i (R_M - R_c)_i \Big/ \sum_i \left(\frac{\partial R_c}{\partial \text{ A.U.}} \right)^2 . \tag{16}$$

A similar expression can be written for δ A.U. for the doppler observations. The solution for a general set of $\delta\alpha$'s merely involves an inversion of the matrix of coefficient from equation (15).

A total of 52 doppler runs were made over the period from October 11, 1962 to December 17, 1962. The average number of samples per run was 141 and the <u>average</u> standard deviation of the final residuals for each run was 2.54 cps. The actual standard deviations are a function of signal-to-noise ratio and they vary from about 3.5 cps at the beginning and end of the observational period to about 1.2 cps at the time of conjunction. Clearly, the uncertainty in a given estimate of the A.U. from any single run depends further on the total doppler shift at that time and is widely variable. At the points of greatest interest in the case of the doppler, i.e. the furthest way from conjunction where the doppler shift is the greatest, the following uncertainties in the A.U. have been computed <u>based entirely on the above internal statistics</u> assuming no correlation between samples:

October 21, $\sigma_{A.U.} = 195$ km

December 12 $\sigma_{A.U.} = 209$ km

The resulting estimates of the A.U. using the so-called Newcomb ephemerides are shown in Figure 4 and 5. They are discussed in detail below.

A total of ten estimates of the A.U. have been made from the range data over a period from November 8, 1962 to December 15, 1962. The average number of samples per run was 472 and the average standard deviation was 614 micro-seconds, round-trip propogation time. However, the range residuals are highly correlated. If we assume that the residuals are correlated over, say, 25 points the average run has an uncertainty of 614 time the square-root of 472/25 or 141 microseconds which corresponds to 42.3 km in round-trip range. Adopting this value for the range uncertainty for a measurement at conjunction gives 79 km in the A.U. based on these statistics along. The resulting estimates of the A.U. are shown in Figure 6.

Range and Doppler A.U. Results.

The doppler A.U. results shown in Figure 4 and 5 exhibit exactly the same variation with date as those reported by Muhleman, et. al. (1962) for 1961. It is certain that this variation is due to errors in the orbital elements of the Earth and Venus employed in Newcomb's tables. In particular, small changes in the mean longitudes and/or the perihelia of the Earth and Venus would essentially remove this variation. The set of corrections to all of the elements that has been computed by Duncombe should be applied to the mean orbital elements used in the Newcomb theory because that theory was the provisional theory utilized by Duncombe in obtaining the corrections. However, several difficulties have been pointed out in this procedure. First of all, Duncombe adopted the concept which attempts to utilize a system of time in close accord to Newtonian time, i.e. uniform time associated with the laws of gravitation. However, for practical reasons, the Ephemeris Time is defined as the time argument for the motion of the Sun (or the Earth) in Newcomb tables of the Sun actually measured utilizing the moon. Since Duncombe employed this concept, it appears that the correction for the mean longitude of the Earth, $\Delta L''$, should have come out of the calculations as precisely zero (Duncombe, personal communication, May 1963). Consequently, on the advice of Duncombe, we have assumed this correction to be zero (with tripidations). Furthermore, Clemence (1943) and Morgan (1945) have obtained a secular correction for the perihelion of the Earth amounting to a correction to the mean anomaly of the Earth of $+4\overset{''}{.}78$ T which has already been applied in the reference ephemeris discussed above. On the basis of this, only the correction $\Delta e''$ and $\Delta \epsilon$ were applied to the reference ephemeris of the Earth and all of the above Venus corrections were applied to the Venus ephemeris for the construction of the so-called Duncombe ephemeris.

A Duncombe ephemeris for the 1962 observations has not been computed as yet. Consequently, it was necessary to analytically compute the change in the A.U. estimate resulting from the Duncombe corrections at each point of interest. It turns out that the effect of the corrections is smallest at specific times in the observational period, i.e., at the points furthest from conjunction for the doppler data and the point at conjunction for the range data. Since these points are the least sensitive to the corrections they are probably the most accurate estimates of the A.U., at least for the types of errors that we are considering. The correction procedure follows from equation (15). If we identify δc_1 with the correction to the A.U. we get, upon solving (15) for δc_1:

$$\delta c_1 = \delta \text{A.U.} = \left\{ (R_M - R_0) - \frac{\partial R_0}{\partial c_2} \delta c_2 - \dots - \frac{\partial R_0}{\partial c_M} \delta c_M \right\} \Big/ \left(\frac{\partial R_0}{\partial c_1} \right)_1 \tag{17}$$

but the term $(R_M - R_0)$ has been iterated to zero. Therefore

$$\delta \text{A.U.} = \left\{ - \frac{\partial R_0}{\partial c_2} \delta c_2 - \dots - \frac{\partial R_0}{\partial c_M} \delta c_M \right\} \Big/ \left(\frac{\partial R_0}{\partial \text{A.U.}} \right) \tag{18}$$

where $\delta c_2 = \Delta L''$, $\delta c_3 = \Delta e''$, etc. The partial derivatives in (18) have been computed from analytical expressions with a digital computer program. An expression similar to (18) can be written for the doppler data. The individual terms in δA.U. are shown in Table III for the doppler observations on October 12 and December 12 and the range observation of November 12, 1962. The actual A.U. estimates listed in Table IV were obtained by computing the weighted mean of the estimates near the data of interest. It is clear from the table that the value for December 12 is anomalously low (also evident from Figure 5). A similar effect was observed in the observations one month after conjunction in 1961 but of much smaller magnitude (see below). Figure 5 suggests that the observations in this region may have been faulty but no explanation can be offered to support this conjecture. Some insight can be gained by the following analysis, however.

The true longitude of the Sun, λ, is computed from Newcomb's tables using the equation

$$\lambda = L'' - (f'' - M'') + \text{perturbation terms} \quad (19)$$

where f" and M" are the true anomaly and mean anomaly of the sun, respectively, and the combination (f" - M") is the equation of center. As is well known, f" may be expanded in terms of M"

$$f'' = M'' + (2e'' - \tfrac{1}{4}e^3) \sin M + \frac{5}{4} e^2 \sin 2M + \ldots \quad (20)$$

and, consequently, to 1st order in e"

$$f'' - M'' = 2e'' \sin M''. \quad (21)$$

Then from (4)

$$\lambda = L'' - 2e'' \sin M'' + \text{perturbation terms}. \quad (22)$$

Now the only change that was made to Newcomb's tables was $\Delta M'' = -4\overset{''}{.}78\ T$. From (22) for a change of M" only, we get

$$\Delta\lambda = 2e'' \Delta M'' \cos M''$$

Actually there is a slight change in the perturbation terms due to a change in ΔM but it is negligible. It turns out that cos M" for October 12 is 0.135 whereas for December 12 cos M" = 0.922. Thus any change in M" has about 7 times the effect on the latter date then on the former date. Actually the inclusion of $-4\overset{''}{.}78\ T$ had an effect on the A.U. estimate for October 12 of +13 km and on December 12, +111 km. Clearly, it is possible to raise the A.U. estimate of December 12 by a very large amount without lowering the estimate on October 12 significantly with a correction to M" (or $e''\Delta\pi''$). However, an impossibly large $\Delta M''$ is required to bring the two estimates into complete agreement. We can conclude from this that the ephemeris errors introduced into the

A.U. computations are probably large compared to the accuracy of the fundamental radar observations. These errors include those in the Newcomb tables, Duncombe corrections to this table, and probably the most significant, errors in our numerical representation of the ephemerides.

Weighted Mean Results and Comparison with Previous Radar Results.

We shall adopt the mean of A.U. estimates reported in the final column of Table IV weighted by estimated variances based on the noise in Figure 5 and 6 and estimated ephemeris uncertainties. Adopting

149,598,719 ± 1000 km,	October 12, 1962
149,599,026 ± 1000 km,	November 12, 1962
149,596,452 ± 2000 km,	December 12, 1962

we obtain as our preliminary 1962 result

149,598,757 ± 670 km

The final A.U. results reported by Muhleman (1963) are shown in Table V.

Conclusions.

The preliminary best value of the astronomical unit from the observations of Venus around the 1962 inferior conjunction is

149,598,757 ± 670 km

where most of the uncertainties are due to ephemeris errors. This result is in complete agreement with the 1961 Goldstone radar result of

149,598,640 ± 200 km

as well as with the results from the 1961 Millstone radar observations.

The remaining uncertainties are primarily linked to the uncertainties in the ephemerides of the Earth and Venus and are of such a nature that the radar observations will ultimately yield definitive corrections to the fundamental ephemerides. This ultimate result is difficult to obtain from an analytical standpoint and will evolve slowly. While it is clear that the observations available at this time are of sufficient quality and quantity to accomplish a good measure of this goal, it should be realized that observations distant from conjunction are required to solve for certain of the corrections that are strongly correlated. In particular, radar observations from the Earth of other planets (or asteriods) are highly desirable for the separation of the effects of the Earth's orbit from those of the orbit of Venus.

VIII. Error Analysis.

1. Velocity of light.

The uncertainty in the vacuum velocity of light was shown to be $\pm$ 0.3 km/sec and this appears pessimistic. The effect on the radar values of the A.U. is then approximately $\pm$ 0.3 x 500 sec or

150 km.

2. Dispersion and refraction.

The effects of signal delays and refraction in the Earths atmosphere are completely negligible at the frequency of operation utilized by the Goldstone group (2300 Mc/s) and Pettingill's Millstone group (440 Mc/s). The effect of refraction in the atmosphere of Venus is probably negligible because the echo power primarily passes through the Venusian atmosphere at normal incidence.

The question of possible delays in the Venusian atmosphere is much more complex, however. An exhaustive discussion of the point has been given by Muhleman (1963a and b). Briefly, the effect of any delay in the atmosphere is to make the propagation time <u>longer</u> than that for the vacuum case and hence, cause the determined value of the A.U. to be <u>larger</u>. Furthermore, according to the modern theories of propagation any delaying medium would have an effect increasing with decreasing frequency; thus the value of the A.U. determined from a radar at 440 Mc/s should be larger than that computed from observations at 2300 Mc/s. In fact, I have shown that if the value of the A.U. from the 2300 Mc/s observations is in error by 100 km then the value measured at 440 Mc/sec should be <u>larger</u> by about 7000 km, whereas the value determined above is actually smaller at 440 Mc/s by 540 km than the value at 2300 Mc/s. Thus, it is unlikely that there is any delay effect at all.

3. The radius of Venus.

The uncertainty in the radius of Venus does not effect the value of the A.U. determined from the doppler frequency. The effect on the range measurements is equal to the radius uncertainty. If the uncertainty of the Venusian radius is taken to be 25 km, the effect on the A.U. is about

89 km.

4. The ephemerides.

The only reasonable estimate of the ephemeris errors are the Duncombe corrections themselves. It is difficult to see how the errors in the ephemerides <u>after</u> corrections could be as large as the corrections themselves. Consequently, we can logically take Duncombe's values as upper bounds on the errors but this appears too pessimistic. I will first analyze the range case.

The range between Venus and the Earth, r, is given by

$$r^2 = r_{♀}^2 + r_{⊕}^2 - 2\, r_{♀}\, r_{⊕} \cos \Theta \tag{23}$$

where $r_{♀}$ and $r_{⊕}$ are the solar distances to the planets and Θ is the heliocentric angle between the Earth and Venus given by

$$\cos \Theta = \cos(\ell_{♀} - \Omega_{♀}) \cos (\ell_{⊕} - \Omega_{♀}) + \sin (\ell_{♀} - \Omega_{♀}) \sin(\ell_{⊕} - \Omega_{♀}) \cos i_{♀} \tag{24}$$

Thus, r is a function of the eccentricities and the arguments of the perihelia through equation (23) and the equations of eliptical motion, and r is a function of $\ell_{♀}$, $\ell_{⊕}$, $\Omega_{♀}$ and $i_{♀}$ through equation (24). We will neglect the uncertainty in the obliquity because its effect on r is very small. Thus, we have

$$r = r(\ell_{⊕}, \ell_{♀}, e_{⊕}, e_{♀}, \pi_{⊕}, \pi_{♀}, \Omega_{♀}, i_{♀}) \tag{25}$$

where we assume that

$$r_{⊕} = \frac{a_{⊕}(1 - e_{⊕}^2)}{1 + e_{⊕} \cos(\ell_{⊕} - \pi_{⊕})} \tag{26}$$

The quantities $a_{⊕}$ and $a_{♀}$ will be assumed precisely known in a.u.'s. Then from equation (23)

$$r dr = r_{♀}(\frac{\partial r_{♀}}{\partial e_{♀}} de_{♀} + \ldots) + r_{⊕}(\frac{\partial r_{⊕}}{\partial e_{⊕}} de_{⊕} + \ldots) + \text{etc.} \tag{27}$$

All of the partial derivatives are then computed from equations (24) and (26). Now, the error in the A.U. due to an error dr is

$$\delta(\text{A.U.}) = A_{⊕} \frac{dr}{r} \tag{28}$$

where $A_{⊕}$ is the value of the A.U. in km. We may then write the expression for δ(A.U.) for small errors in the elements utilizing the partials. Since we are interested in the value of δ(A.U.) at the 1961 inferior conjunction of Venus, the general expression will be given with all of the expression evaluated at that epoch. We get

$$\delta(\text{A.U.}) = 9680 \text{ km} \{0.031\, de_{\oplus} + .0047\, d\ell_{\oplus} - .276\, e_{\oplus} d\pi_{\oplus} +$$

$$- .028\, de_{♀} + .0014\, d\ell_{♀} - .198\, e_{♀} d\pi_{♀} - .029\, di_{♀} - .13\, d\Omega_{♀}\} . \quad (29)$$

With the Duncombe corrections inserted for the differentials

$$\delta(\text{A.U.}) = \{- 30 - 5 + 322 + 33 + 6 + 19 + 19 - 47.4\} \text{ km.}$$

$$(\text{A.U.}) = + 317 \text{ km}$$

Thus, we see that if the ephemerides are in error after correction by as much the corrections themselves, the error in the A.U. from the range observations is about 317 km.

The case for the doppler observations is far more complicated. Since the points of interest in this case are toward the east and west elongations it can be shown that the terms involving sin $i_{♀}$ are negligible to first order and a first order analysis can be carried out in two dimensions. Since the analysis has been carried out in the plane of the ecliptic, the effects of the obliquity can also be ignored. Then the range rate (or doppler velocity) is approximately

$$\dot{r} \simeq V_{♀} (\sin \alpha_{♀} - \gamma_{♀} \cos \alpha_{♀}) - V_{\oplus}(\sin \alpha_{\oplus} + \gamma_{\oplus} \cos \alpha_{\oplus}) \quad (30)$$

where $V_{\oplus}$, $V_{♀}$ = orbital speeds of the Earth and Venus,

$\alpha_{♀}$ = the angle between the Sun and the Earth at Venus, similarly for $\alpha_{\oplus}$.

$v_{\oplus}$, $\gamma_{♀}$ = the angles of the Earth and Venus velocity vectors from the perpendicular to the radius vectors in the orbital planes.

From well known equations of celestial mechanics, to first order in the eccentricities

$$V_{\oplus} \simeq n_{\oplus} a_{\oplus} [1 + e_{\oplus} \cos(\ell_{\oplus} - \pi_{\oplus})] , \tag{31}$$

and

$$\gamma_{\oplus} \simeq e_{\oplus} \sin (\ell_{\oplus} - \pi_{\oplus}). \tag{32}$$

Thus, from equations (26), (30), (31), and (32) $\dot{r}$ can be expressed in terms of the elements and the partial derivatives taken. The results are too complex to profitably write down and I shall merely present the resulting expression for the δ(A.U.) with all of the expressions evaluated at the epoch March 23, 1961, the date of observation nearest the eastern elongation and consequently, the point of greatest interest. I get

$$d\dot{r} = 35.05 \text{ km/sec} \{.13 \, de_{♀} - 1.96 \, d\ell_{♀} - 1.18 \, e_{♀} d\pi_{♀}\} +$$

$$- 29.8 \text{ km/sec} \{-1.0 \, de_{\oplus} - 1.34 \, d\ell_{\oplus} - 1.74 \, e_{\oplus} d\pi_{\oplus}\} .$$

Since

$$\delta(\text{A.U.}) = \frac{A_{\oplus}}{\dot{r}} d\dot{r}, \tag{41}$$

I get, inserting the Duncombe corrections,

$$\delta(\text{A.U.}) = - 1350 \text{ km}.$$

This value is, of course, very large and probably equally pessimistic.

If the uncertainties of the corrections are used the largest term is due to the uncertainty in the longitude of Venus and is 620 km. It is not possible to combine the individual terms in a meaningful statistical manner because the correlation coefficient between the terms may even approach unity. However, it appears safe to say that the error in the A.U. from the doppler observations is less than 620 km. If this circumstance is correct, the doppler value of the A.U. has been weighted twice as heavily as it should have been in the final reduction to a single result. Interestingly enough, the effect of this would be to change the final result toward the value from the Millstone data. Similar values for the 1962 cases may be found in Table III.

IX. Radar Measurements of Mercury

Unequivocal radar contact of Mercury has been accomplished by the Goldstone group. The observations have been made by transmitting a pure CW wave with the Venus radar equipment. The echo signal has been detected by computing the power spectral density of the received signal in a digital computer. The signal spectrum was shifted down near D.C. by continuously adjusting the receiver local oscillator to the ephemeris doppler frequency plus an offset of about 100 cps. An example of such a spectrum taken by R. Carpenter of JPL is shown in Figure 7. The ephemeris was prepared in the same way as the Venus ephemeris. The vertical center line in Figure 7 indicates the frequency about which the observed spectrum would be centered if the ephemeris were perfect and the value used for the A.U. = 149,598,640 km were correct. The arrows indicate the amount that the spectrum would be shifted for an error in the A.U. of ± 5000 km for the observing date of May 8, 1963.

Some error in the measurement of the center frequency is to be expected due to errors in positioning the local oscillator on the order of 1 or 2 cps. Known errors of the ephemerides would have a similar effect. Thus unless the spectrum in Figure 7 was positioned fortuitously the observations yield an excellent verification of the radar value of the A.U..

X. The Related Astronomical Constants

The relationships presented at the beginning of this paper may now be utilized to construct a consistent set of some of the constants based on the A.U. result of 149,598,640 ± 250. From equation (4) using b = 6,347,166 km we get for the solar parallax

$$\pi_{\odot} = 8''.794139 \pm 0''.000015.$$

The light-time for unit distance is given by equation (8)

$$\tau = 499.00728 \pm .00067 \text{ sec.}$$

It should be realized that τ is the most fundamental result from the radar work because it is independent of the speed of light. The aberration constant is also independent of c because of the radar value of the A.U.. From equation (7), (4) and (8)

$$K = \frac{n \sec \phi}{86400} \tau$$

$$K = 20''.495622 \pm .000027.$$

The Earth-Moon mass ratio can be obtained from the lunar inequality, equation (11) which can be written

$$L = \frac{\mu}{1+\mu} \frac{a_{☾}}{\text{A.U.}}$$

and the dependence on c is again removed from the radar results if Kepler's radar value of $a_☾$ is corrected to the same value of c. Using $L = 6.4378 \pm .002$ (Brouwer and Clemence (1961) and $a_☾ = 388{,}400.4$, we get

$$\mu^{-1} = 81.32730 \pm 0.025$$

where the uncertainty is due to that of L.

The coefficient of the parallactic inequality is obtained from equation (10) where again c factors out if radar values of $a_☾$ and A.U. are used:

$$P = -124''\!.9876 \pm .001$$

Finally, a consistent value of the mass of the Earth plus moon can be obtained from an expression given by Brouwer (1963)

$$\frac{S + E + M}{E + M} = 0.0055800307 \frac{(A.U.)^3}{(a_☾)^3}$$

where Brouwer has obtained the constant term from modern measurements of the Earth constants. Note again that for radar values of A.U. and $a_☾$ the errors due to c are removed and we get

$$(E + M)^{-1} = 328{,}903.2$$

The values above cannot be considered definitive until the ephemeris errors are removed from the radar values but it is clear that all the above constants except $\pi_\odot$ are free from the error in the radar A.U. introduced by using a specific value of c. Thus, from this standpoint, the major criticism of the radar method, namely the uncertainty of the propagation velocity, is destroyed.

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Table I. Modern velocity of light determinations*.

Author	Date	Method	c, km/sec
Aslakson	1949	Shoran	299 792 ± 3.5
Hansen & Bol	1950	cavity resonance	299 789.3 ± 1.2
Essen	1950	"	299 792.5 ± 1.0
Bergstrand	1951	Geodimeter	299 793.1 ± .32
Froome	1952	microwave interferometer	299 792.6 ± .7
Mackenzie	1953	Geodimeter	299 792.4 ± .4
Froome	1954	microwave interferometer	299 792.7 ± .3
Plyler, et al	1955	infrared spect.	299 792 ± 6
Florman	1955	microwave interferometer	299 795.1 ± 1.9
Bergstrand	1957	Geodimeter	299 792.8 ± .34
"	"	" survey	299 792.85 ± 0.16
Froome	1958	microwave interferometer	299 792.50 ± .10

*DuMond, J. Ann. Phys. 7, 365, 1959

Table II. Astronomical Unit determinations from radar observations of Venus.*

Good radar methods**	A. U., km.	$\pi_\odot$, sec
D. Muhleman, et al.	149,598,640 250	8.7941379 $\pm$ .000015
G. Pettinghill, et al.	149,597,850 400	8.7941849 $\pm$ .000026
D. Muhleman (revision of Pettinghill's value)	149,598,100 400	8.7941705 $\pm$ .000026
Marginal radar methods		
Thomson, et al	149,601,000 5000	8.7940 $\pm$ .0003
Maron, et al.	149,596,000	8.7943
Kotel'nikov	149,599,500 800	8.7941 $\pm$ .00005

* Muhleman (1963) doctorial thesis.

** Good radar methods are those that observed Venus over a sufficiently long arc to remove the major part of the errors from the ephemerides.

Table III. The Effect of the Duncombe Corrections on the A.U.

	Doppler Oct. 12	Doppler Dec. 12	Range Nov. 12
$\Delta L''$	-119 km	+141 km	-4
$\Delta e''$	+75	+132	-191
$e''\Delta\pi''$	-182	-164	+319
$\Delta \ell$	-441	+506	-1
Δe	+97	-169	-40
$e\Delta\pi$	+45	+41	-45
Δp	-74	-30	-67
Δq	-43	-44	-8
Totals	-642	+413	-37

Table IV. A. U. 1962 Results

	* Newcomb Ephem	**Duncombe Ephem
Doppler, October 12	149,599,060 km	149,598,719 km
Range, November 12	149,599,730 km	149,599,374 km
Doppler, December 12	149,596,452 km	149,596,888 km

*"Newcomb ephemerides" means Newcomb's tables with a mean anomaly correction of $\Delta M_{\oplus} = + 4''.78\ T$.

**"Duncombe ephemerides" means here that only Δe" has been applied for the Earth plus all of the Venus corrections.

Table V. 1961 Radar results. (Ref. 3)

1.	Doppler near eastern elongation	149,598,750 ± 200 km
2.	Doppler near western elongation	149,598,000 ± 1000 km
3.	Range at conjunction (closed loop)	149,598,500 ± 150 km
4.	Range at conjunction (radiometer)	149,598,800 ± 150 km
5.	Millstone result (Ref. 12)	149,597,850 ± 400 km
6.	Muhleman's rework of Millstone data (Ref.3)	149,598,100 ± 400 km
7.	Weighted mean of 1, 2, 3, & 4	149,598,640 ± 200 km

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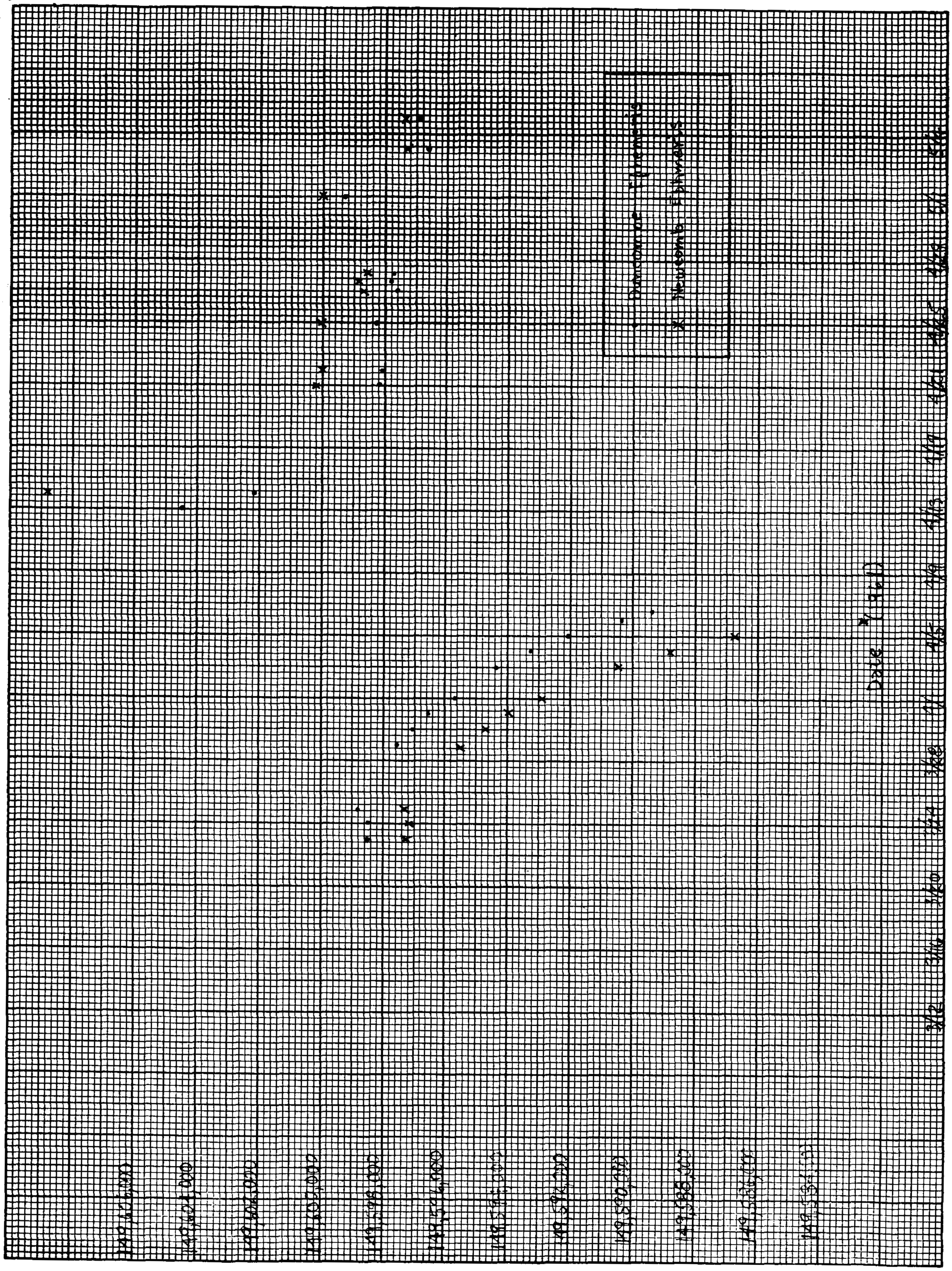

Fig1.The A.U. computed from the Goldstone velocity observations

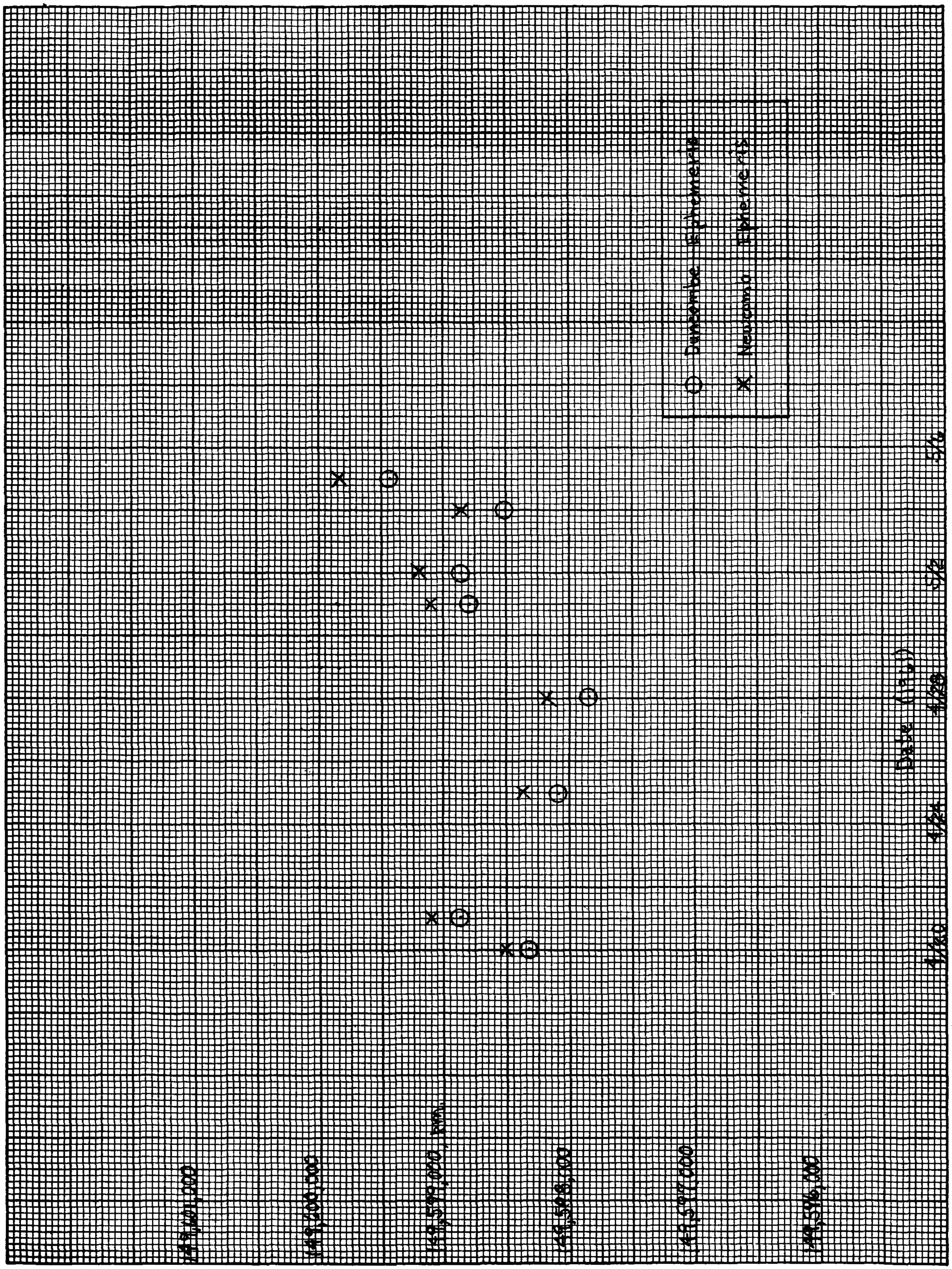

Fig. 2. The A.U. from the Goldstone range observations.

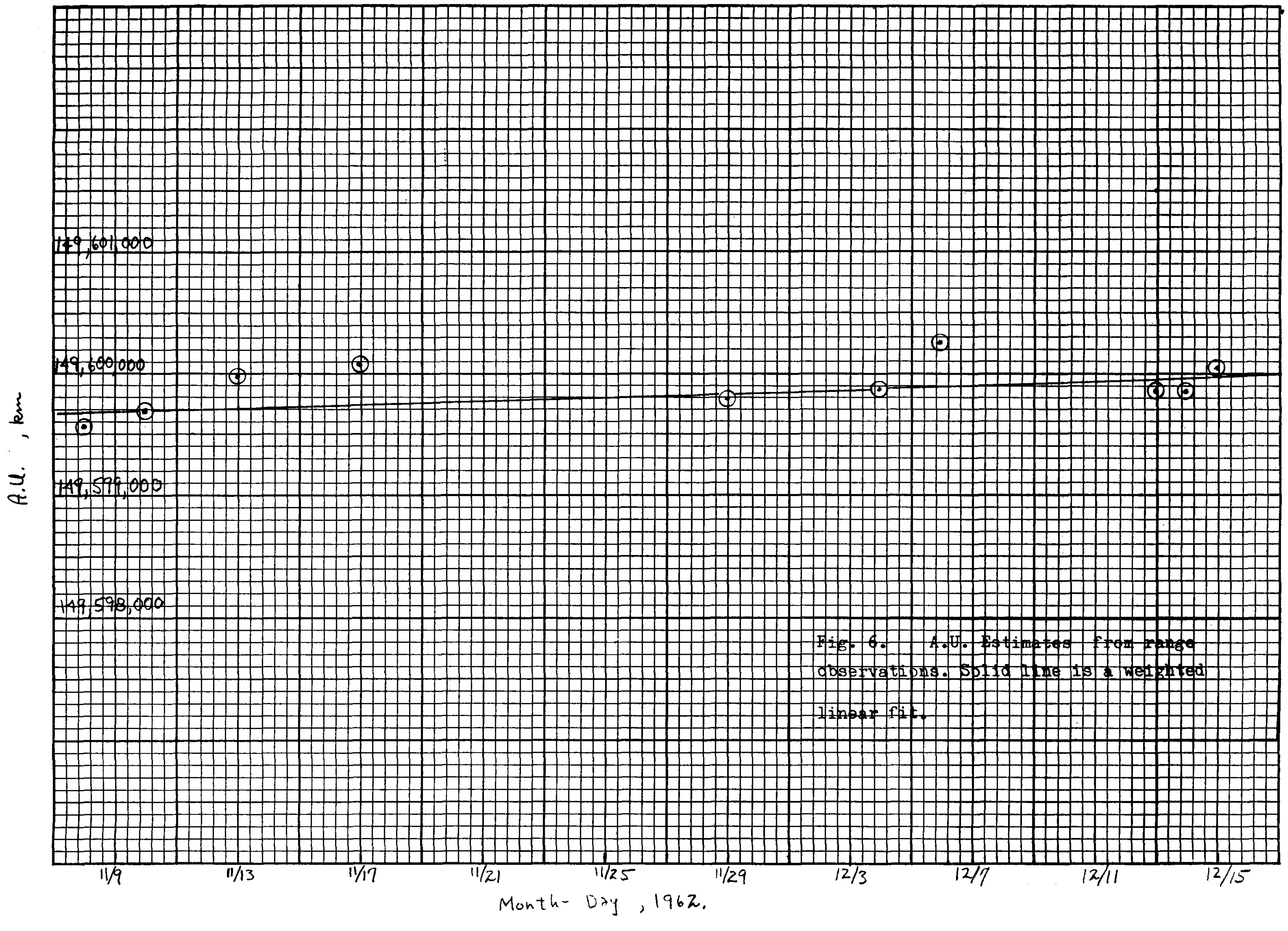

Fig. 6. A.U. Estimates from range observations. Solid line is a weighted linear fit.

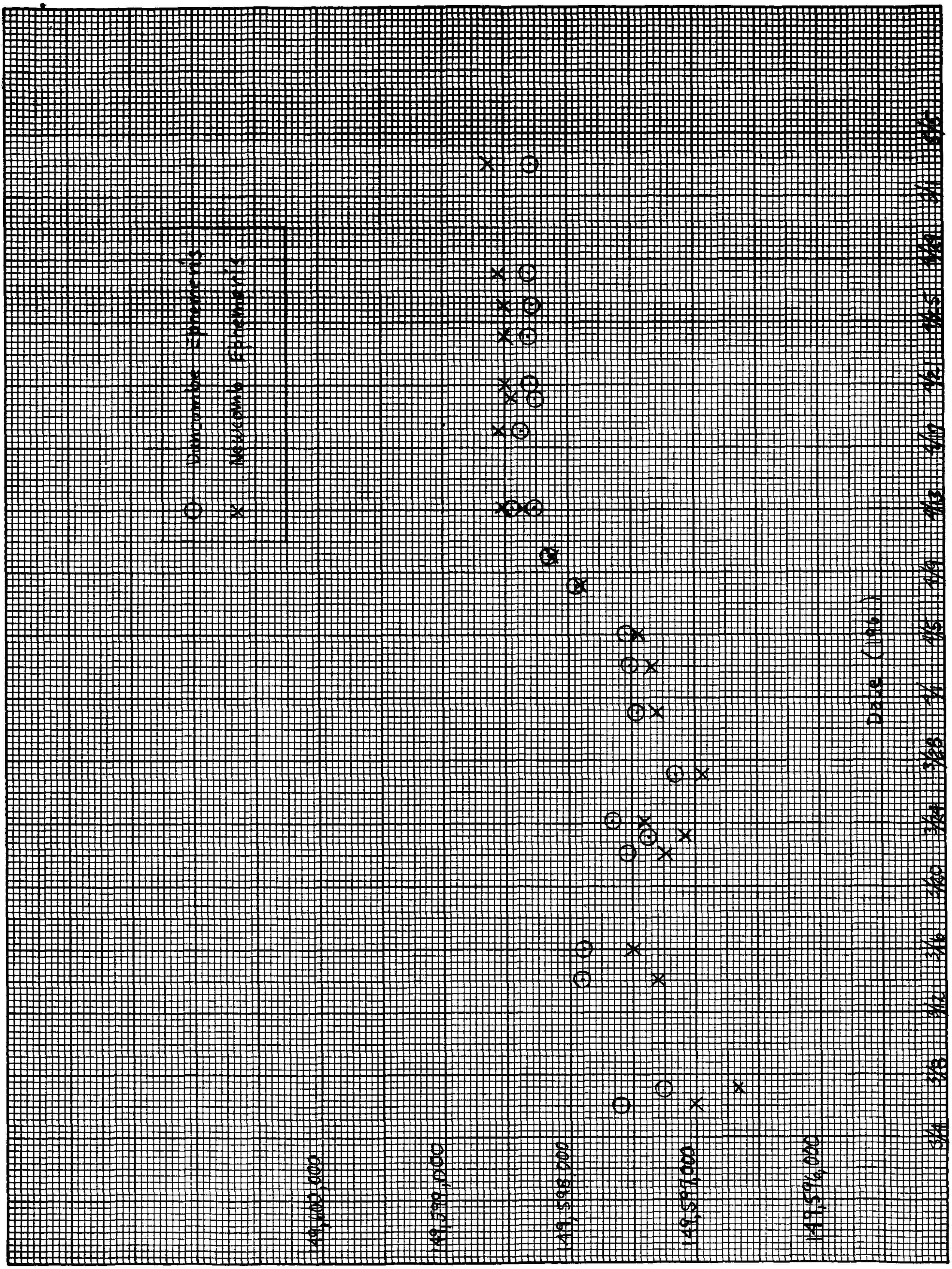

Fig 3. The A.U. computed from the Millstone observations.

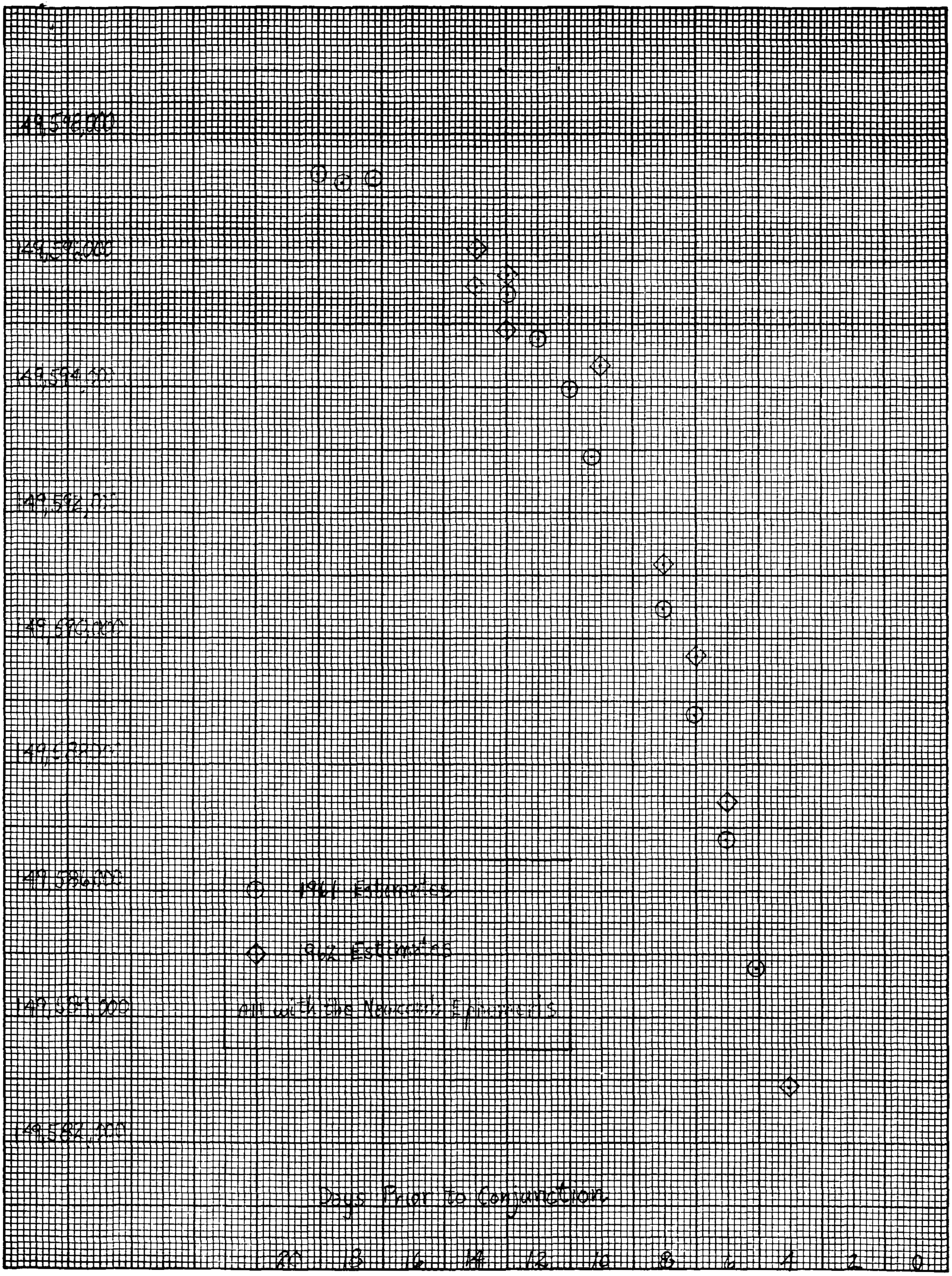

Fig 4. Comparison between the 1961 and 1962 doppler velocity A.U.s

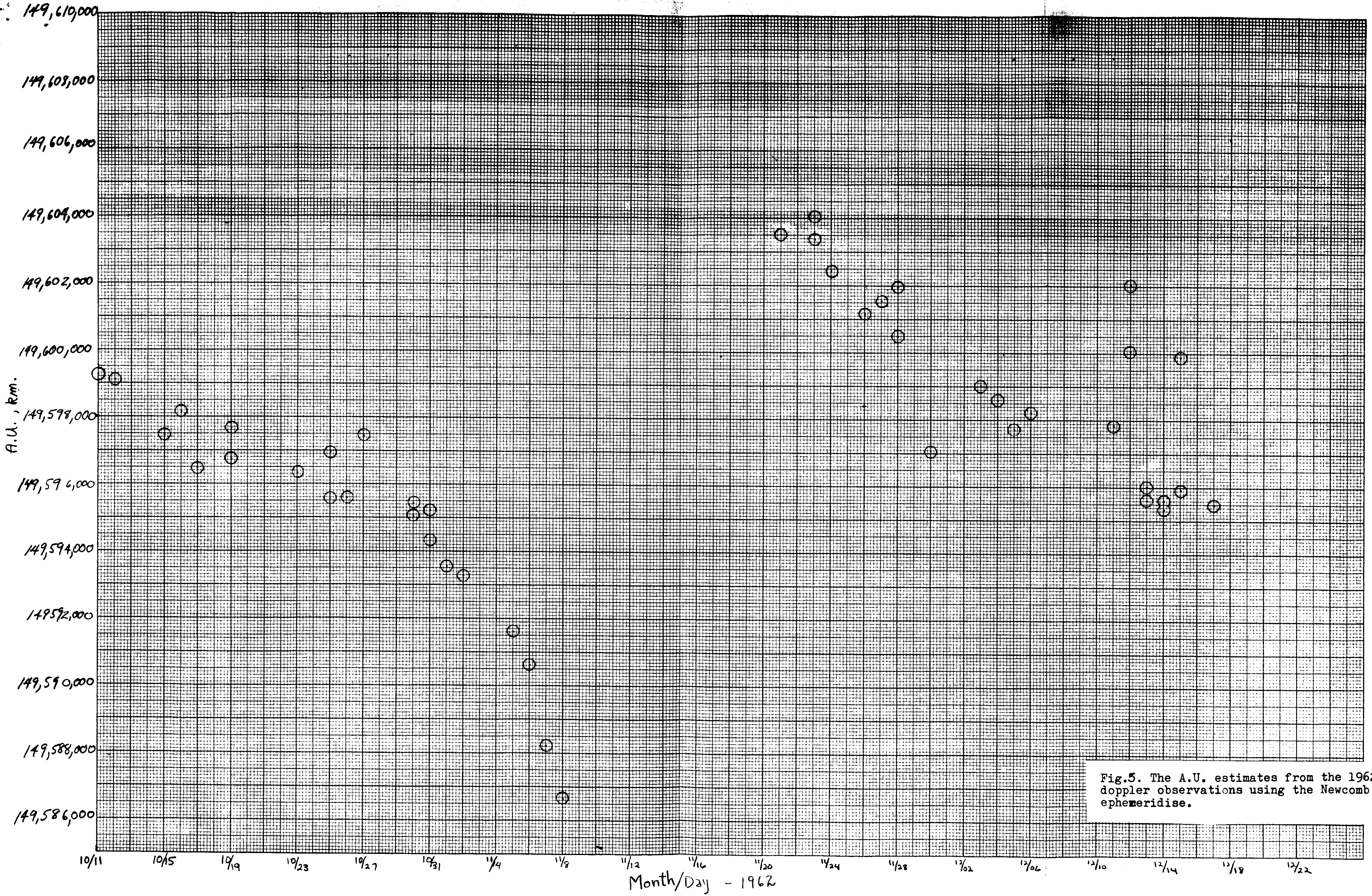

Fig.5. The A.U. estimates from the 1962 doppler observations using the Newcomb ephemeridise.

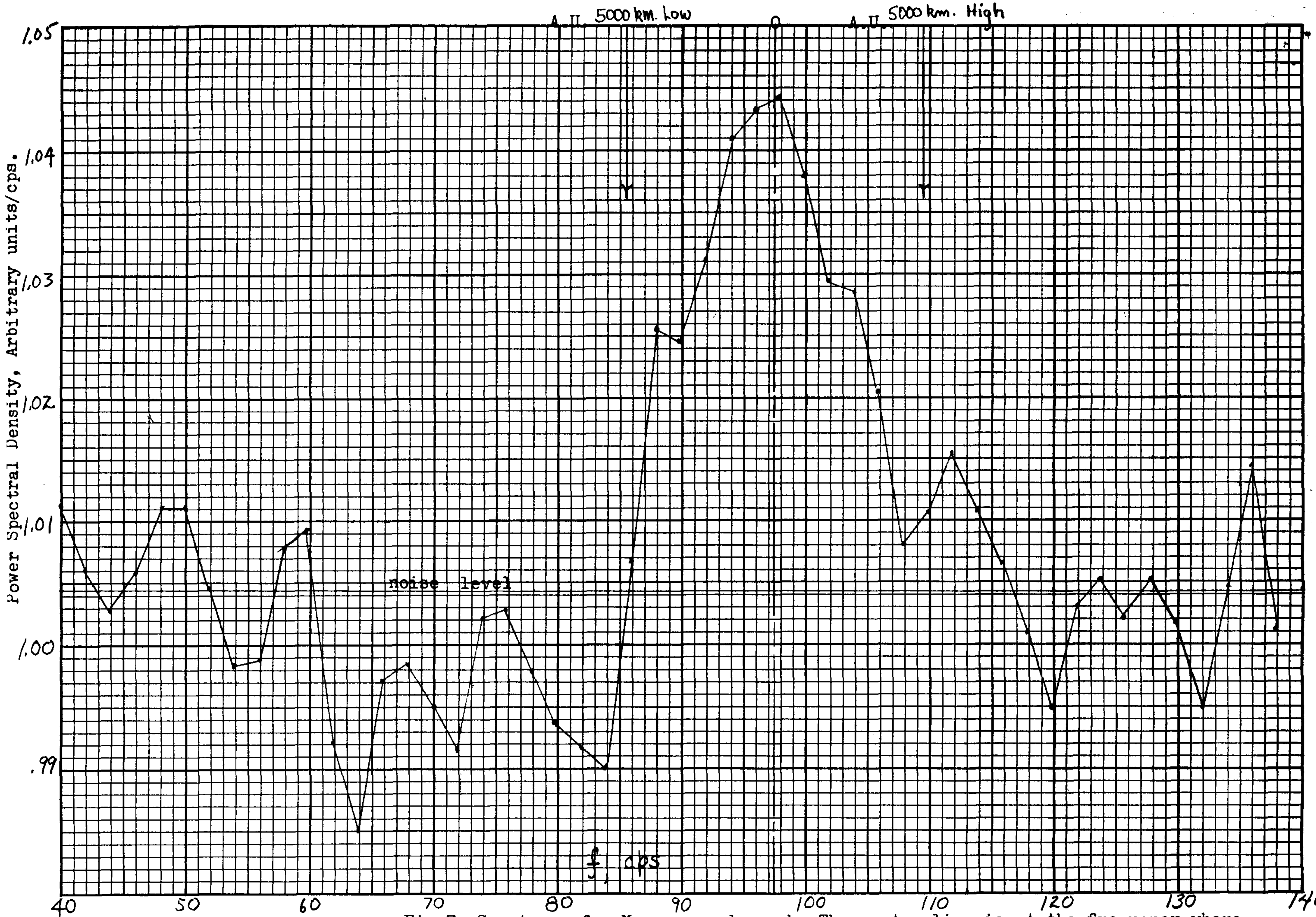

Fig.7. Spectrum of a Mercury radar echo. The center line is at the frequency where the spectrum would fall if A.U. = 149,598,640 km.