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ABSTRACT

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The Kramers-Unsöld theory of continuum intensity for electron-ion recombination is formulated quantum mechanically. The valid conditions for the theory in a non-equilibrium plasma are deduced by using Griem's criterion for partial local thermodynamic equilibrium. The results indicate that the conditions are much less restricted than those required for the Saha equation to be valid.

AUTHOR

Kramers (Ref. 1) and Unsöld (Ref. 2) have derived the expression for the intensity of electron-ion recombination continuum radiation from a plasma by calculating the coefficient of absorption of a photon interacting with atoms and using the radiational equilibrium argument to get the intensity of radiation. In such an approach the number density of the atoms N_a will appear in the expression for the radiation and the electron density N_e is introduced by eliminating N_a through the use of the Saha equation. Because of the nature of the mathematical derivation one is apt to tie the physical restraints associated with the Saha equation to the theory for the continuum radiation (Ref. 3). The conditions required for the Saha equation to be valid, stated in Griem's paper (Ref. 4), are that the plasma be in complete local thermodynamic equilibrium. These conditions usually cannot be expected in most laboratory plasmas. However, the electron-ion recombination continuum intensity theory can be formulated by using the free-bound transition probability of an electron, integrating over all possible electron velocities, and summing over only the energy states contributing to the continuum radiation at a given frequency. The assumption made in such a calculation is that the energy of the free electrons has a Maxwellian distribution, and there are no excited electrons in the ions. The problem is then reduced to that of a hydrogenlike plasma.

The transition probability for the transition from states i to f per unit time for the recombination process, according to first-order perturbation theory, can be expressed as (Ref. 5, 6)

$$A_{if} = \frac{2\pi}{\hbar}|r_{if}|^2 \rho_f$$

where r_{if} is the position matrix element for dipole radiation and ρ_f is the density of final states. After summing over the final-state orbital and azimuthal quantum numbers and the two directions of polarization for the emitting photon, and averaging over the initial electron spins, the transition probability for a transition to a state of principal quantum number n with the emission of a photon of energy $h\nu$ is given by (Ref. 7)

$$A_{if} = \frac{KI^2 wgV}{\nu n^3 (\frac{1}{2}m_e w^2)} d\nu\, \delta\left(h\nu - \tfrac{1}{2}m_e w^2 - \frac{1}{n^2}\right) \quad (1)$$

where I is the ground-state energy, w the velocity of free electrons, ν the frequency of the photon emitted, n the principal quantum number, V the volume of the system, and δ the δ-function. The Gaunt factor g is approximately unity and is set equal to unity in the present discussion. The constant K in Eq. (1) stands for $2^4 he^2z^2/3^{5/2}m_e^2c^3$, where h is Planck's constant, c the velocity of light in vacuum, e and m_e the electronic charge and electronic mass, respectively, and z the effective charge seen by the bound electron ($z = 1$ for a hydrogen atom).

The intensity of radiation $\varepsilon_{\nu n}$ per unit volume per unit frequency at frequency ν due to a transition of free electrons to a particular state of quantum number n is obtained by multiplying Eq. (1) by $h\nu$, the number density of ions n_i, the number of electrons per unit volume with velocity w, and integrating over all possible values of $w\,(0 \rightarrow \infty)$ in a Maxwellian electron gas. This yields

$$\varepsilon_{\nu n} = \frac{8\pi K I^2 N_e^2}{m_e^2}\left(\frac{m_e}{2\pi kT_e}\right)^{3/2}\frac{1}{n^3}\exp\left(\frac{\dfrac{I}{n^2} - h\nu}{kT_e}\right) \tag{2}$$

The intensity of the continuum radiation ε_ν at frequency ν for all possible transitions is given by summing Eq. (2) over all n such that $h\nu \geqq I/n^2$:

$$\varepsilon_\nu = \frac{8\pi K I^2 n_e^2}{m_e^2}\left(\frac{m_e}{2\pi kT_e}\right)^{3/2} e^{-h\nu/kT_e}\int_{n_m}^{\infty} e^{I/n^2kT_e}\,dn$$

$$= C\frac{n_e^2}{T_e^{1/2}}\left(1 - e^{-h\nu/kT_e}\right) \tag{3}$$

where C stands for $2IK/(2\pi m_e k)^{1/2}$, and n_m is equal to $(I/h\nu)^{1/2}$. In obtaining Eq. (3), the summation over n is replaced by integration, for at reasonably large n the energy level is close enough to justify such a replacement. The last factor in Eq. (3) gives the reduction of intensity by induced emission.

The inequality

$$h\nu \geqq \frac{I}{n^2} \tag{4}$$

allows one to calculate the minimum value of quantum number n_m above which the transition to a quantum state will have a contribution of continuum intensity at frequency ν. The energy levels with quantum number smaller than n_m will contribute no continuum intensity at the particular frequency.

The valid conditions for Eq. (3) are that the energy of the free electrons has a Maxwellian distribution and all the electrons in the ions are in their ground states. To verify Eq. (3), an experimental measurement in the decaying plasma is performed. A column of argon plasma is produced by discharging a capacitor bank of 16 μf capacitance energized with a 7500-V dc source through a discharge tube. After a period of about 50 μsec after the cessation of the discharge, the free-electron energy distribution is thought to relax from the Druyvesteyn distribution (Ref. 8) to the Maxwellian one (in the order of 5 μsec) and the excited electrons in the ions to deplete to the ground states (in the order of 10^{-8} sec). The further excitation of the electrons in the ions, after cessation of the discharge, due to electron-ion collisions and photoexcitation is unlikely to happen because the electron temperature is of the order of 1.5 eV at the moment the measurement is initiated, which is small in comparison with the first excitation energy of argon (13.4 eV), and the plasma is optically thin (330 μHg initial pressure). The electron temperature of the decaying plasma as a function of time is measured by the spectral-line intensity ratio techniques, and the electron density as a function of time is measured by the ion current probe and microwave techniques. The relative intensity of continuum radiation at 5530Å is obtained by using a spectroscope and photomultiplier. The results of the measurements are shown in Fig. 1. The electron density obtained from the

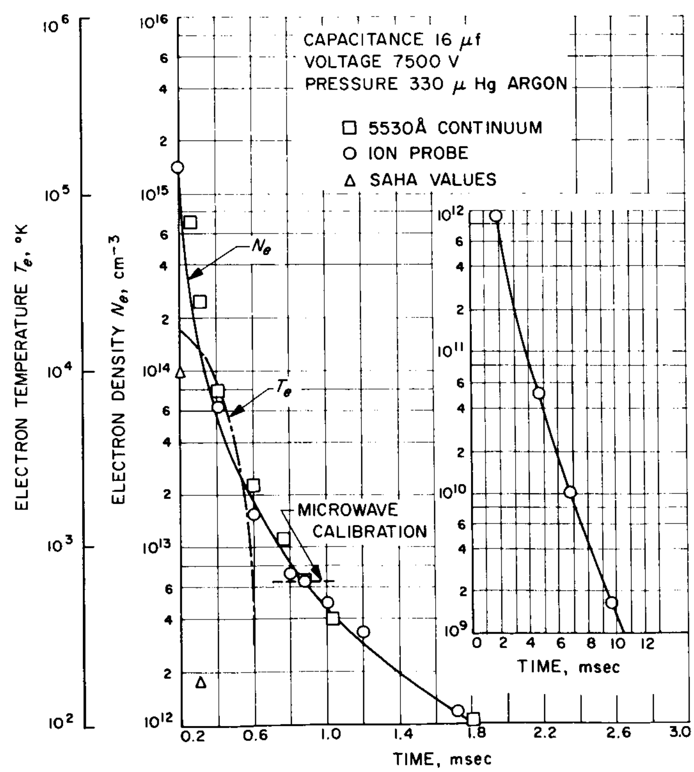

Fig. 1. Electron temperature and electron density as a function of time

continuum intensity is calculated by using Eq. (3) (continuum due to free-free transition is negligible in the temperature range of the present experiment). The constant C in Eq. (3) is evaluated from the known electron density (microwave data), known electron temperature (spectral-line ratio data) and relative continuum intensity at one particular instant of the discharge. This point is at the microwave cutoff point. The percentage difference of the electron density obtained from the continuum intensity, and from the microwave and ion probe data, is less than 30%, covering the electron density range from 10^{12} to 10^{15} cm^{-3}; while the values of electron density calculated from the Saha equation differ by two orders of magnitude from the experimental data, as also indicated in Fig. 1. This fact reveals that the plasma is not in a complete local thermodynamic equilibrium, but that the two valid conditions mentioned above do exist for the continuum radiation measured.

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