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July 1963

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* Supported by the Air Force Office of Scientific Research under Contract No. AF 49(636)-886.

✝ Supported by the National Aeronautics and Space Administration under Contract No. NASw-698.

The Chapman-Ferraro theory (Chapman, 1960; Ferraro, 1960) of magnetic storms is based on the assumption that a cool, highly conductive, non-magnetic plasma of density N_o streams from the sun with speed U_o and that the thinnest transition layer with the earth's field is formed at some $B_o = B(r_o)$. For equal mass streaming particles, the layer thickness is of order

$$\delta = c/\omega_p = 2U_o/\omega_c(r_o) \ (\omega_p{}^2 = 4\pi Ne^2/M, \ \ \omega_c = eB/Mc) \ , \qquad (1)$$

$$(M_+ N_+ + M_- N_-) U^2 + B^2/8\pi = \text{const.} = B_o{}^2/8\pi(r < r_o) = 2N_o MU_o{}^2(r > r_o + \delta) \ , \qquad (2)$$

and B_o is twice the undisturbed geomagnetic field, B_G, at the boundary, $r = r_o$. More sophisticated analyses (see Dungey, 1958; Rosenbluth, 1957) which include electric fields ($M_+ - M_-$ effects) lead to a layer thickness on the order of $\delta = c/\omega_p^- \approx U_o/(\omega_c^+ \omega_c^-)^{\frac{1}{2}}$, with electrons and protons exchanging energies within the sheath, and regaining the original speeds as they are reflected back to the $B = 0$ region (Fig. 1); this sheath has an electrostatic instability (Piddington, 1960) but we argue below that the maximum broadening is on the order of an ion gyroradius. Grad (1961) also investigated the thermal broadening of an equal mass sheath and he found that simple monotonic velocity distributions never produce broadening much greater than an order of magnitude. In recent years considerable emphasis has therefore been placed upon the problem of generalizing the above pressure balance relation in order to construct a three dimensional cavity (Beard, 1960; Spreiter and Briggs, 1962, etc).

Nevertheless, space probe measurements near the interface suggest that the simple thin sheath picture is not a good one. The Pioneer probes (Sonett, 1963) measured a broad disordered field in the region $r \simeq 8 - 13R_e$

along the earth-sun line, and Explorer X (Heppner, Ness, Scearce and Skillman, 1963) found a similar region beyond the night side of the cavity. In all cases, the field fluctuates greatly, but the mean field approximates the geomagnetic field in magnitude and direction. In fact, Explorer X seemed to traverse a Chapman-Ferraro boundary six times during a single outward passage. Recently, the sunlit region was extensively mapped by Explorer XII. It was found that the characteristics of the disordered region are highly variable, with occasional sharp but continuous transitions to a fluctuating region (i.e., no geomagnetic field doubling and cancellation) as on the 30 Sept., 1961 outbound pass; even when a definite "discontinuity" is seen (13 Sept., 1961 inbound pass) the field in the disordered region ranges between 75 and 20 gammas (Cahill and Amazeen, 1963). Geomagnetically trapped high energy particles only exist in the relatively stable region, but kilovolt energy electrons are seen between r_o and r_1, the inner and outer boundaries of the disordered quasi-geomagnetic field (Freeman, Van Allen and Cahill, 1963).

Since the idealized Chapmann-Ferraro sheath should be on the order of 1 - 100 km, some new explanation for the long range disordering ($r_1 - r_o \simeq 20{,}000 - 30{,}000$ km) must be sought. One theory which has been discussed recently (Axford, 1962; Kellogg, 1962) involves the formation of a shock layer at the standoff distance $r \simeq r_1$. The interplanetary field, B_I, is used to define a solar wind Alfvén speed, $V_A = (B_I/4\pi NM)^{\frac{1}{2}}$, and it is observed that the earth moves at supersonic speeds through the magneto-hydrodynamic (mhd) fluid. Hydrodynamic analogies suggest standing shock wave formation and the outer boundary may be placed at $r_1 \simeq 12 - 14R_e$ (Spreiter and Jones, 1963). Furthermore, two-dimensional mhd models

(Auer, Hurwitz and Kilb, 1962) have been invoked to account for the fluctuations below r_1.

We feel that several objections to the shock wave theory exist: (1) In the mhd theories the streaming plasma is required to carry a transverse field component B_I, $\vec{U} \times \vec{B}_I \neq 0$. However, although the Parker model (Parker, 1958) of the solar field leads to a large transverse component $B_\phi = (\Omega r \sin \theta / U) B_r$ in an inertial frame, the inertial observer also sees a large electric field, $\vec{E}_I = - \vec{U} \times \vec{B}_I / c$, associated with solar rotation. In the solar frame of reference $\vec{E}_I' = 0$, $\vec{U}' \times \vec{B}_I' = 0$ and thus the relevant speed is U_E, the component of the earth's orbital velocity normal to $\vec{B}_I$. This is related to U, by $U_E = \Omega_1 r / (1 + \Omega_o^2 r^2 / U^2)^{\frac{1}{2}}$, where Ω_o and Ω_1 correspond to 24.7 and 27 day rotation periods respectively. For $U \simeq (300 - 400)$ km/sec, we find $U_E \simeq (228 - 272)$ km/sec, while $V_A \simeq 242 \,|\vec{B}_I(\gamma)/20|$ km/sec for N = 5; since thermal effects may increase the wave speed (Kellogg, 1962) many cases which seem "supersonic" can, in fact, involve "subsonic" streaming. (2) Even if Parker's model is not correct in the sense that it underestimates the transverse component of B_I, the strong and rapid fluctuations in $\vec{B}_I$ (Coleman, Davis, Smith and Sonett, 1962) and N suggest that a definite Alfven speed cannot be well defined over a region of 20,000 to 30,000 km. (3) The apparent qualitative "memory" of the magnitude and direction of B_G for $r_o < r < r_1$ is not characteristic of the disordered region behind a collisionless-shock. (4) Experimental studies of plasma shock waves (Kantrovitz, Patrick and Petschek, 1960) indicate that the disordered interaction region extends only a distance on the order of several ion gyro-radii behind the shock; although these shocks may not be of the collisionless variety, no extremely broad disordered regions have been found experimentally.

The presence of high energy electrons in the disordered region led Freeman, Van Allen and Cahill (1963) to conjecture that a Chapman-Ferraro sheath might somehow be broadened to include the entire region $r_o < r < r_1$. In this note we wish to outline a qualitative model of the formation of such a sheath and to compare the predictions with experiment. The main features of the model can best be described by studying the temporal development from an initial minimum sheath formed at t = 0. We assume that no standing shock is formed and that: (1) The very low energy whistler-medium density distribution is valid out to the magnetospheric boundary. (2) The fluctuations in B_I and sheath instabilities induce "fast" diffusion of this plasma past the interface; the trapped plasma then allows the geomagnetic field to reappear past r_o, to form a broad oscillatory region in which field and plasma coexist. (3) The solar wind incident on this region generates an instability as the electrons locally acquire proton energies. This leads to fluctuations which enhance the diffusion and ultimately generate the hot electrons in the final equilibrium distribution. We essentially revive the ring current theory suggested in connection with the Pioneer data (Smith, Coleman, Judge and Sonett, 1960), but we propose a definite source for the current, and we discuss its inherent instability.

Let us consider a minimum Chapman-Ferraro sheath formed at t = 0, as in Fig. 1. This configuration is already unstable (Piddington, 1960) if $U_- = (M_+/M_-)^{\frac{1}{2}} U_o > (2kT_o^-/M_-)^{\frac{1}{2}}$, (See Fig. 1) so that one would expect the two-stream plasma instability (electrons flowing through electrons) to destroy the initial configuration. However, it seems plausible to anticipate that this instability will merely heat the electrons to a temperature corresponding to approximately one-half the directed energy of the solar plasma, and the effect would be to broaden the sheath to, say, $\delta' \simeq U_o/\omega_c^+ \simeq 43c/\omega_p^-$, by decoupling the ions and electrons. The presence of some initial thermal distribution in the incident solar wind should not have any drastic influence on this minimum layer, according to Grad's (1961) analysis. While the instability and heating associated with $M_+ - M_-$ effects will undoubtedly modify these thermal effects, there is at present no indication that the true equilibrium state of an initial Chapman-Ferraro sheath is broadened by the aforementioned mechanisms to the extent required to explain the observations in the region 8Re - 13Re. One therefore is constrained to start with a Chapman-Ferraro sheath broadened to perhaps $\delta' \simeq 100$ km, and to seek some other, more effective broadening agent. As stated above, we postulate the existence of a trapped plasma which forms exterior to the initial sheath as this sheath relaxes, and describe a plausible source for such a distribution of trapped particles in the broad disordered region.

Let us consider the thermal plasma within the cavity and show how this can provide trapped particles in the exterior. Whistler analysis (Scarf and Liemohn, 1963) suggests $N_-(r) \simeq 1.4 \times 10^3 \, (Re/r)^3$, $2.5 < r/Re < 5$, and this may be extrapolated to the boundary of the magnetosphere [whistlers propagating to such high altitudes have recently been detected (Carpenter, 1963)] to yield $N_-(8Re) \simeq 27 \text{ cm}^{-3}$, for instance. This plasma is usually neglected beyond r_o, but it is important to examine the diffusion of the whistler medium into the sheath and beyond. The known fluctuations in B_I;

together with the inherent instability of the Chapman-Ferraro sheath indicate that one should utilize the fast diffusion expression (Spitzer, 1956; also see Spitzer, 1960),

$$V_{\perp} = \frac{-c}{16N_{-}eB} \nabla p = - \frac{4 \times 10^{18} \nabla p}{N_{-}B} \left(\frac{cm}{sec}\right) \quad . \tag{3}$$

This isothermal plasma ($T = T_o$) will therefore diffuse quite freely across the magnetic field into the plasma free region. If $B = \frac{1}{2}(B_o + B_I) \simeq 50 \, \gamma$, $(dp/dr) \simeq NkT/\delta'$ and $\delta' \simeq 100$ km, then the initial diffusion speed at the exit plane is

$$V_{\perp}(r_o + \delta) \simeq 100(T_o/10^3 \, {}^oK) \text{ cm/sec} \quad , \tag{4}$$

and the initial containment time is thus

$$t_D(\text{sec}) \simeq 10^3(T_o = 10^5 \, {}^oK) - 10^5(T_o = 10^3 \, {}^oK) \quad . \tag{5}$$

Clearly, the thermal plasma within the cavity rapidly populates the region initially free of trapped particles, reducing the pressure gradient. Since this thermal plasma has a finite conductivity it is also clear that the geomagnetic field will diffuse outward as well, and that it will ultimately reappear in the plasma filled region beyond r_o. Of course, the latter process is rather slow, and it is of interest to see if a more rapid field producing mechanism can be found for $r > r_o$.

In this connection, we note that the particles which have diffused across r_o are trapped in the sense that their orbits do not include the asymptotes $r \rightarrow \infty$, (another source of trapped thermal plasma beyond r_o is the charge exchange reaction between solar wind protons and H, He, etc., atoms in ballistic orbits; we do not analyze this source at this time because the rate of depletion during solar storms is uncertain). Furthermore, these particles interact electromagnetically with the incident wind and with the local magnetic field. Grad (1961) has emphasized that the self-consistent equilibrium solutions for a plasma-field interface are extremely sensitive to the presence of trapped orbits, and that an arbitrary magnetic field profile can be reproduced in a self-consistent calculation by placing trapped particles on magnetic lines in the appropriate manner. For instance, in a two-dimensional model for which $P_y = MV_y + eA_y(x)$ $(B_z(x) = dA_y/dx)$ is a constant of motion of each particle, Grad shows how to construct a distribution function $f(P_y) \rightarrow f[A(x)]$ which is consistent with any given $B(x)$. In essence, the trapped particles can cluster into groups (in configuration and velocity space) which produce currents, and modify the field; the clustering depends on the trapped particle orbits, on the interaction with free particles, and with the self-consistent field, but the presence of trapped particles generally precludes the formation of a minimum profile with a smooth monotonic transition between incident plasma and applied field.

It is impossible at present to discuss the dynamics of this approach to equilibrium. However, we now show that a smooth but oscillatory ring-current broadened field cannot represent the final equilibrium state of this system because of electrostatic restoring forces associated with finite $M_+ - M_-$ effects in the incident wind.

Consider the sub-group of electron-proton pairs impinging on an oscillatory quasi-geomagnetic field with asymptotic speed $V \rightarrow V_o$,

$$\int \vec{v}\, f_{FREE}(\vec{v}, \vec{r})\, d^3v \rightarrow \vec{U}_o, \; r \rightarrow \infty \quad . \tag{6}$$

Beyond $r_1 \simeq 12$ - 14Re, the interplanetary field fluctuations effectively mask the dipole-like field. If the particles sense a varying but ordered field within this boundary, they begin to execute orbits such as those shown in Fig 1. Some electrons in this group attain maximum speeds of $v_- = (M_+/M_-)^{\frac{1}{2}} v_o$ in the direction $-v_o \times B$, and we have a local two-stream instability associated with the electrons flowing through an ambient plasma. This initially generates electrostatic instabilities which propagate along $-\underset{\sim}{v}_o \times \underset{\sim}{B}$ with $\omega \simeq kv_- \simeq \omega_p$ (ambient). However, this electron instability must be quenched by local electron heating with kT_- tending to $\frac{1}{2}(M_-v_-^2/2) = \frac{1}{2}(M_+v_o^2/2)$. As the heating progresses, new instabilities are triggered. Ion waves which propagate with speeds on the order of $(kT_-/M_+)^{\frac{1}{2}}$ are usually heavily damped, but if $T_- >> T_+$, the Landau damping becomes negligible. In this case, a small current of particles with drift speeds on the order of $v_o \simeq (kT_-/M_+)^{\frac{1}{2}}$ can overcome the residual damping and ion-wave instabilities arise (Bernstein and Kulsrud, 1960). All wavelengths greater than the electron Debye length, λ_{D-}, are generated, but the shortest waves travel primarily along the magnetic field ($k_\perp U_o << \omega_c^+$).

Let us consider some mean numerical values. For U_o = 600 km/sec we find U_- = 2.6 x 10^4 km/sec, and kT_- tends to about 900 ev. If $N \simeq 10/cm^3$, the threshold wavelength, λ_{D-}, is near 0.1 km, which is small compared to any gyroradii in a 20 - 100 γ magnetic field. The local fluctuations in $\underset{\sim}{E}$ and N generate local magnetic fluctuations, and a given value of B occurs at many different points. Thus, we envisage the formation of a broad disordered region containing hot electrons and fluctuating $\underset{\sim}{E}$ and $\underset{\sim}{B}$ fields. The maximum penetration into the field is determined by the highest energy pairs in the wind, and we therefore anticipate that the condition $B^2/8\pi = NMU_o^2$ defines a radius somewhat beyond r_o.

An important aspect of the presence of sharp fluctuations in $B(r_o < r < r_1)$ is that this configuration tends to perpetuate itself, once it has been formed. If B does fluctuate sharply, then there can be no well defined position for the specular reflection of any given incident electron-proton pair. Furthermore, the diffusion of the whistler-medium is greatly enhanced by the fluctuations, and thus the trapped particles should diffuse out until they are stopped by the interplanetary field. Finally, incoming particles may be trapped by being scattered from the disordered fields. Of course, if the wind becomes intense, we might expect compression of this disordered region, with formation of a transient Chapman-Ferraro sheath. This is depicted in Fig. 2.

Let us now summarize the physical conjectures upon which the model is based: a) A broad fluctuating region with hot electrons is a possible self-consistent configuration in the presence of an appropriate trapped particle distribution. b) The inner boundary of the sheath which varies with latitude, longitude, time and solar activity, is determined by specular reflection of the high energy tail in the solar wind (non-singular thermal distributions have plasma penetrating only a finite distance into a non-uniform field). The outer boundary is determined by the maximum extent of the trapped particle distribution; diffusion effects should carry this distribution outward so that r_1 is determined by the magnitude and noise level of the interplanetary field. c) Some electrons attain drift speeds on the order of $(M_+/M_-)^{\frac{1}{2}} U_o$, leading to a local double stream instability. The consequence of this instability is the thermalization of this drift energy, yielding hot electrons. The high temperature achieved by these electrons acts to limit the growth of this instability but an unstable equilibrium configuration results.

Of course, a quantitative verification of this model requires construction of an appropriate self-consistent equilibrium distribution which will yield $j_-(r)$, $B(r)$, etc., and bear a sensible relation to trapped particle production and depletion rates. To the extent that the particles can be described by isotropic pressure tensors, the local constant of motion is

$$\sum(p + NMU^2) + \frac{(B^2 - E^2/c^2)}{8\pi} \quad , \tag{7}$$

where the sum includes protons, electrons, trapped and free particles. This problem is now being examined in detail.

Acknowledgement: We are very grateful to B. D. Fried for several extremely stimulating and helpful suggestions.

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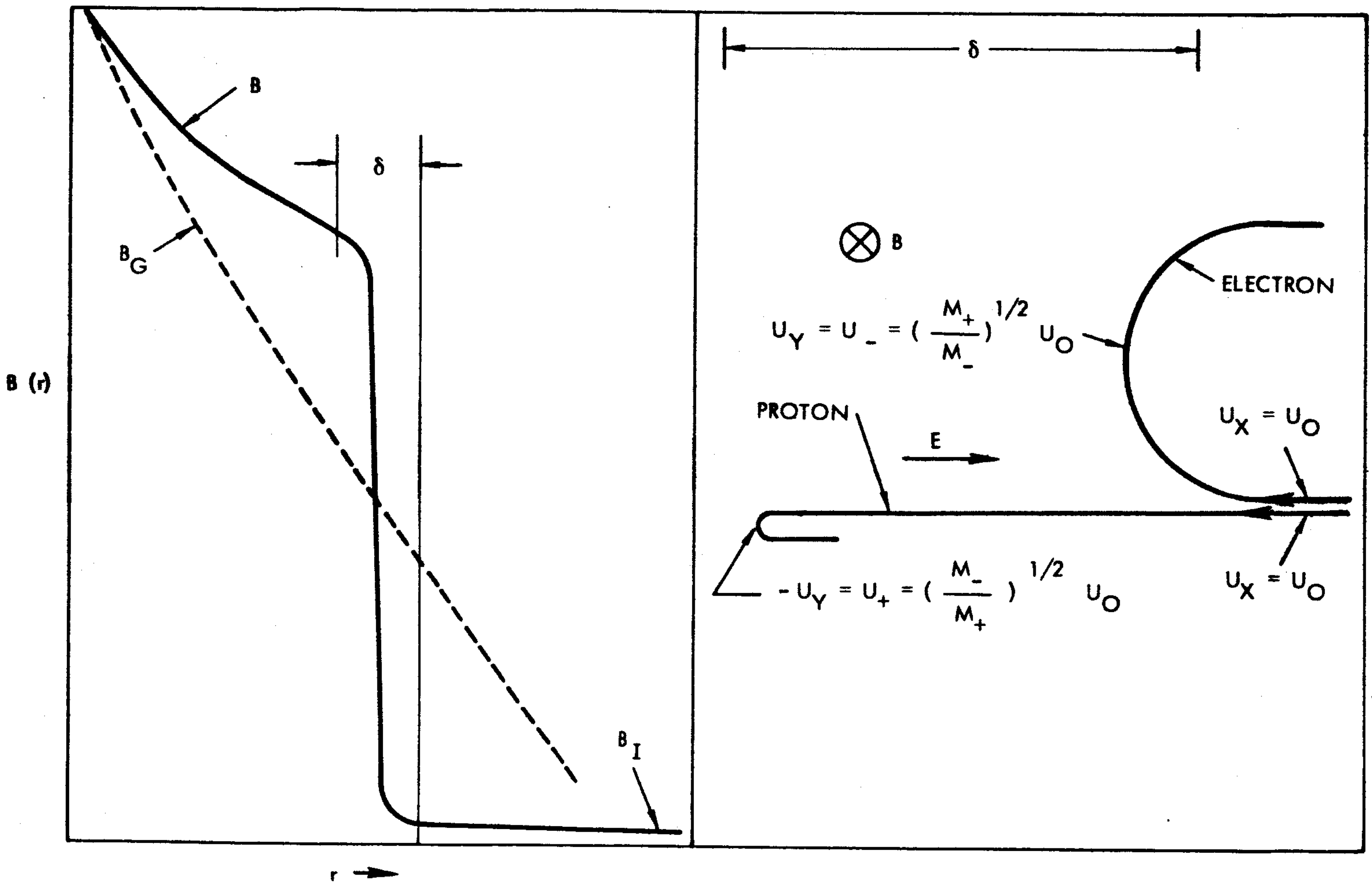

Figure 1. The idealized Chapman-Ferraro thin sheath. A current $J_y = -eN\left[\left(\frac{M_+}{M_-}\right)^{\frac{1}{2}} + \left(\frac{M_-}{M_+}\right)^{\frac{1}{2}}\right]\frac{U_o}{C}$ flow transverse to B_G. This shields B_G beyond $r_o + \delta$, and produces field doubling immediately below r_o. In fact, the electrostatic forces which produce the local equipartition of streaming energy also induce instabilities in this system.

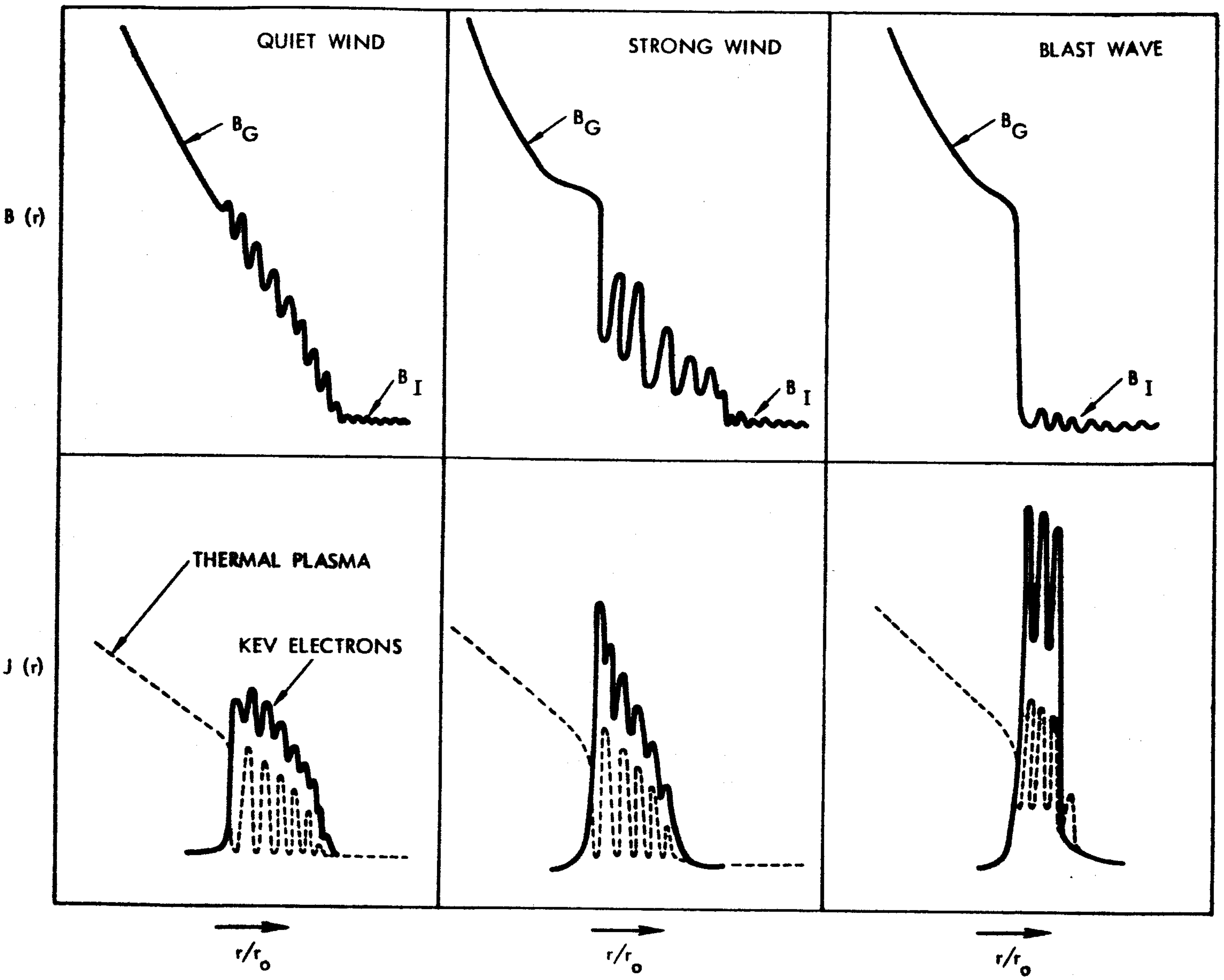

Figure 2. The anticipated self-consistent magnetic profile and current system as a function of distance and solar activity. The thermal plasma is the whistler medium which has diffused past r_o. The kilovolt electrons are solar wind particles which receive proton energies upon being accelerated by fluctuating electrostatic fields associated with charge separation.