POWER STUDY OF SPIN STABILIZED EGO (S-49)

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MARCH 1965

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INTRODUCTION

The Orbiting Geophysical Observatory (EGO) is currently being operated as a spin stabilized satellite. It is spinning about the $\bar{k}_B$ axis (see Figure 1). Since it was designed to have the solar array continuously normal to the sun's rays, power supply is a critical problem. The capability exists to turn the solar array about its shaft. The purpose of this analysis is to determine the solar array angle which will provide maximum power output. The effects of shadowing by the box and deviation of solar cell short circuit current from a cosine curve is considered.

NOTATION

A   Cross sectional area

$D(i)$ The deviation of the solar cell short circuit current from a cosine curve (see Figure 2).

$f(i)$ Incidence factor defined by equation (7).

$I(t)$ Shadow indicator at time t; 0 in shadow, 1 not in shadow.

I   Solar array current.

i   Angle of incidence.

$\bar{i}, \bar{j}, \bar{k}$ Orthogonal unit vectors.

L   The total number of incremental time steps used in equations (8) and (10).

N   The total number of incremental solar array areas used in equation (5).

1
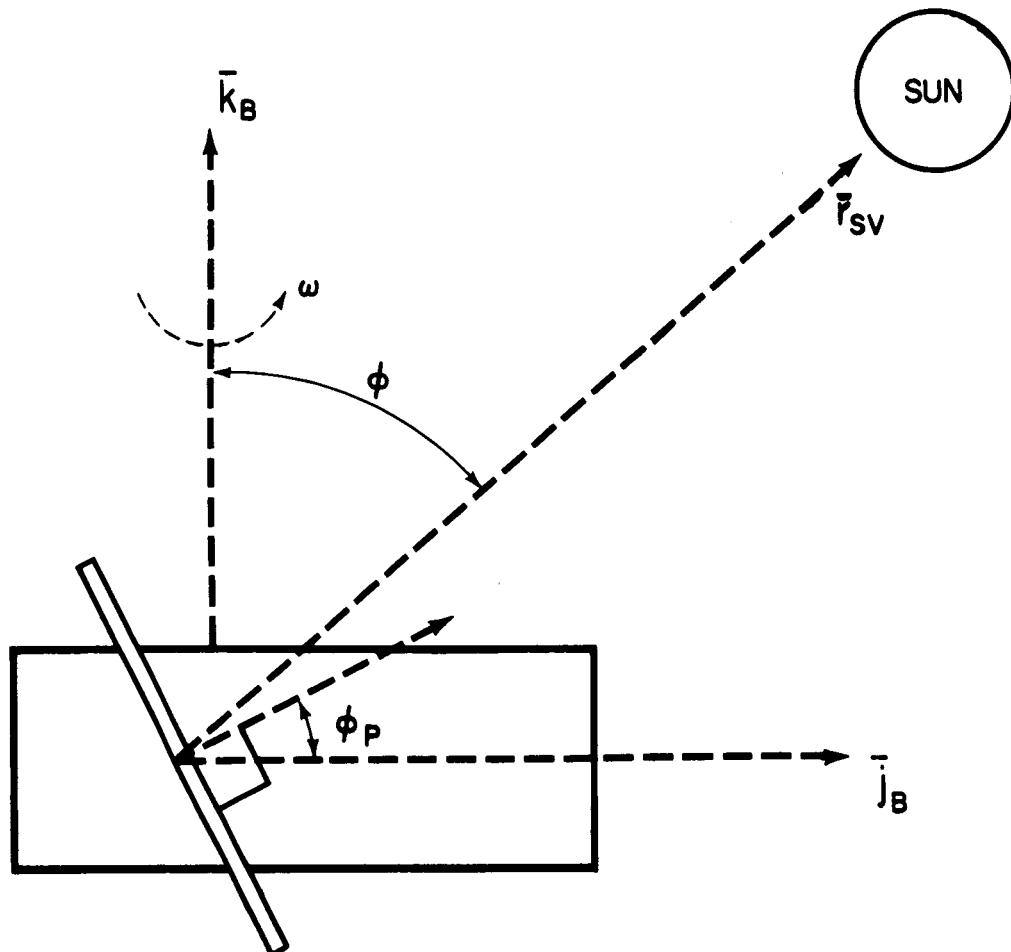
Figure 1

$\bar{n}$ Unit vector normal to solar array surface.

$\dot{q}_i(t)$ The heat flux incident upon the ith surface at time t, which radiates directly from the sun. It is defined by equation (4).

$\bar{q}(t)$ The average energy per unit time per unit area incident upon the solar array at time t. It is defined by equation (5).

$\tilde{q}$ Average energy per unit time per unit area received by the solar array over one revolution of the satellite about its spin axis. See equation (8).

$\bar{r}_{sv}$ The vector which points from the vehicle to the sun. It is given by equation (3).

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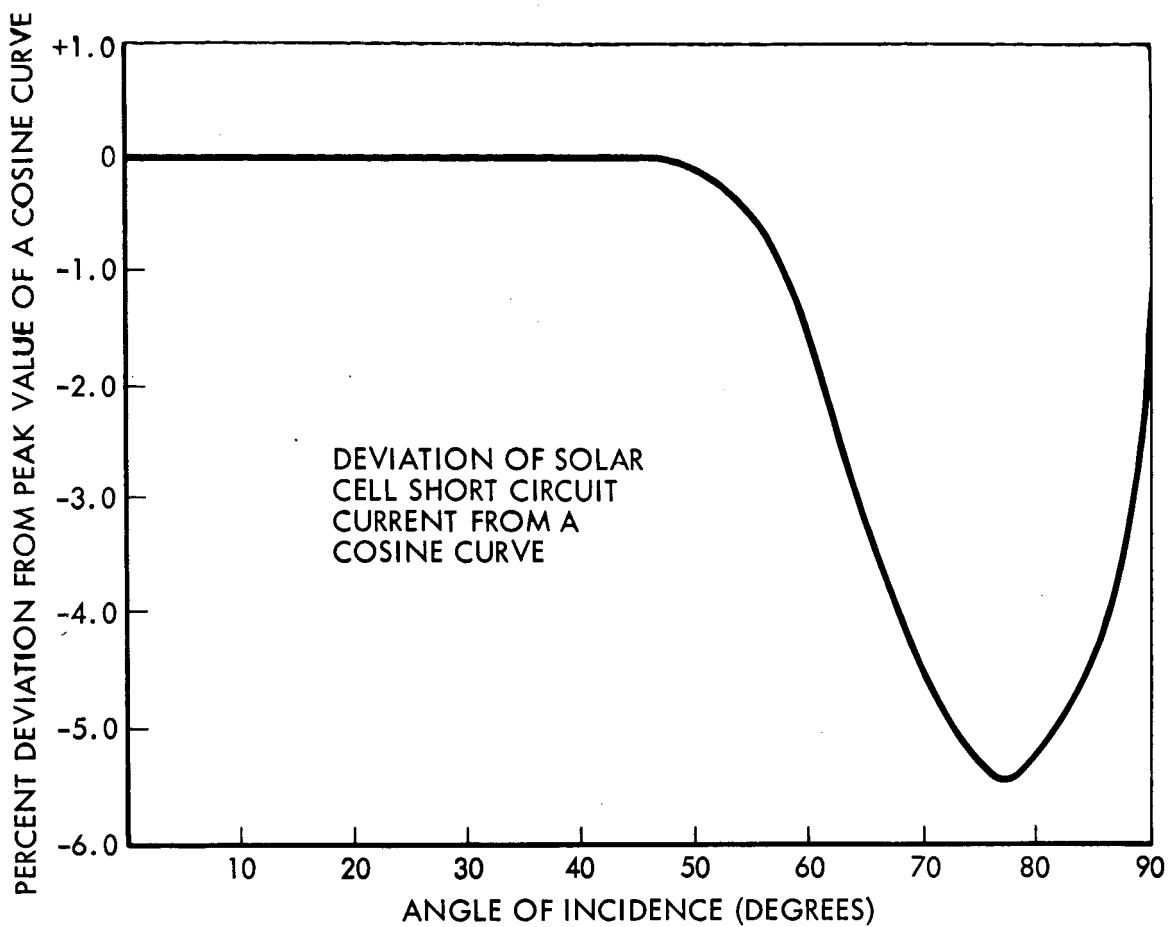
Figure 2

t	Time.
Δt	Integration step; defined by equation (10).
σ	The solar constant.
ϕ_P	The solar array angle; used in equation (2). (Figure 1).
ϕ	The angle between $\bar{r}_{sv}$ and the satellite spin vector $\bar{\omega}$. Used in equation (3).
$\bar{\omega}$	Satellite spin vector.

Subscripts:

    B   Box

    P   Solar array.

ASSUMPTIONS

1. The geometry of S-49 EGO is given by Reference 1.

2. The EGO Satellite is spinning about the $\bar{k}_B$ axis (Figure 1).

3. The solar cells have uniform input-output behavior (all the solar cells behave the same way).

4. The deviation of the output from a cosine curve as the angle of incidence of sunlight is varied is given by Figure 2 (Reference 2).

ANALYSIS

Definition of Axes Systems

The geometry of the OGO satellite is described in a coordinate system attached to the main structure (box coordinates); however, the satellite spin vector $\bar{\omega}$ and the vector from the satellite to the sun are given in an inertial coordinate system. In order to carry out the shadow testing (see Reference 1), the components of all vectors have to be known in a single coordinate system. It is convenient to use the box coordinate system to carry out this testing, since fewer transformations have to be made.

In the following, three coordinate systems are considered. Each coordinate system is defined by a triple $(\bar{i}, \bar{j}, \bar{k})$ of mutually orthogonal unit vectors. The system attached to the box, and solar array are identified by subscripts B and P respectively. Quantities without subscripts refer to the inertial coordinate system ($\bar{i}$ does _not_ point toward Aries and $\bar{j}$ does _not_ point toward the Earth's north pole).

The vectors $\bar{r}_{sv}$ and $\bar{\omega}$ (the position of the sun with respect to the satellite and the satellite spin vector) are given in the inertial coordinate system.

1) **The Inertial To Box Transformation Matrix**

The attitude of the satellite is specified as follows:

a) The satellite is spinning about the $\bar{k}_B$ axis at a rate ω.

b) At some reference time $t = 0$ the coordinate axes $\bar{i}_B$, $\bar{j}_B$, $\bar{k}_B$ are aligned with the inertial axes $\bar{i}$, $\bar{j}$, $\bar{k}$.

c) The solar array is locked at an angle ϕ_P.

The transformations at time t are given by:

$$\begin{vmatrix} x_B \\ y_B \\ z_B \end{vmatrix} = \begin{vmatrix} \cos \omega t_k & \sin \omega t_k & 0 \\ -\sin \omega t_k & \cos \omega t_k & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} \tag{1}$$

$$\begin{vmatrix} x_B \\ y_B \\ z_B \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_P & -\sin \phi_P \\ 0 & \sin \phi_P & \cos \phi_P \end{vmatrix} \begin{vmatrix} x_P \\ y_P \\ z_P \end{vmatrix} \tag{2}$$

2) **The Vehicle-Sun Vector $\bar{r}_{SV}$**

The inertial axes are defined such that the vehicle-sun vector $\bar{r}_{SV}$ lies in the plane of the $\bar{j}$ and $\bar{k}$ vectors. Furthermore, let the angle between the vectors $\bar{r}_{SV}$ and $\bar{k}$ be ϕ.

Then $\bar{r}_{SV}$ is given by

$$\bar{r}_{SV} = \left| \bar{r}_{SV} \right| (\sin \phi \, \bar{j} + \cos \phi \, \bar{k}) \tag{3}$$

MATHEMATICAL MODEL FOR ENERGY INPUT TO AN
ARBITRARILY ORIENTED SURFACE OF OGO

The flux of energy incident upon the ith incremental surface area of the solar array at time t_k, which radiates from the sun is given by:

$$\dot{q}_i(t_k) = I(t_k) \, \sigma \, f(i_i(t_k)) \left[1 - \frac{D(i_i)}{100} \right] \tag{4}$$

The average energy per unit area and per unit time incident upon the solar array at time t_k is given by

$$\overline{\dot{q}(t_k)} = \frac{\left[\sum_{i=1}^{N} \dot{q}_i(t_k) \, A_i \right]}{\sum_{i=1}^{N} A_i} \tag{5}$$

In these equations $I(t_k) = 0$ or 1 depending on whether the experiment is in shadow or sun, σ is the incident radiation on a unit area at a distance of 1 AU from the sun, $D(i_i)$ is the deviation of solar cell short circuit current from a cosine curve (see Figure 2), A_i is the area of the ith incremental surface area, and $\overline{i}_i$ is the angle of incidence given by:

$$\cos i_i = \overline{n}_i \cdot \frac{\overline{r}_{sv}}{|\overline{r}_{sv}|} \tag{6}$$

In this equation, $\overline{n}_i$ is the normal of the ith surface in box coordinates. If $\cos i_i < 0$, the surface faces away from the sun and receives no radiation. This is expressed formally by:

$$f(i_n) = \frac{1}{2} \left(\cos i_n + |\cos i_n| \right) \tag{7}$$

6

The average energy rate per unit area and per unit time over a period is given by:

$$\tilde{\dot{q}} = \frac{1}{L} \left\{ \frac{1}{2} \left[\overline{\dot{q}(t_0)} + \overline{\dot{q}(t_L)} \right] + \sum_{k=1}^{L-1} \overline{\dot{q}(t_k)} \right\} \qquad (8)$$

where

$$t_k = k \Delta t \qquad k = 0, 1, \cdots L \qquad (9)$$

$$\Delta t = \frac{2\pi}{\omega L} \qquad (10)$$

The ratio of average to maximum current generated by the solar array is given by:

$$\frac{I_{av}}{I_{max}} = \frac{\tilde{\dot{q}}}{\sigma} \qquad (11)$$

RESULTS

Figures 3 and 4 present the ratio I_{av}/I_{max} as a function of ϕ with ϕ_p as a parameter. I_{av} is the average current generated by the solar array over one revolution of I_{max} is the current which would be generated if the sun's rays were normal to the array. ϕ is the angle between the satellite's $\bar{k}_B$ axis and the vector which points from the satellite to the sun (Figure 1). ϕ_p is the solar array angle (Figure 1). The solid curves are for a transparent box structure. The dotted curves take into account the blockage of sunlight from the solar arrays by the main box structure. The possible ranges of ϕ and ϕ_p are 0° - 180° and 90° - 270°, respectively. Figure 3 is for $90 \le \phi_p \le 180°$ and Figure 4 is for $180° \le \phi_p \le 270°$. Figure 4 is a mirror image of Figure 3, with the exception that the lower scales are identical.

Figure 3

WITH SHADOW

WITHOUT SHADOW

$|I_{av}/I_{max}|$

ϕ

1.0

0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0   10   20   30   40   50   60   70   80   90   100   110   120   130   140   150   160   170   180

$\phi_p = 90$

$\phi_p = 100$

$\phi_p = 110$

$\phi_p = 120$

$\phi_p = 130$

$\phi_p = 140$

$\phi_p = 150$

$\phi_p = 160$

$\phi_p = 170$

$\phi_p = 180$
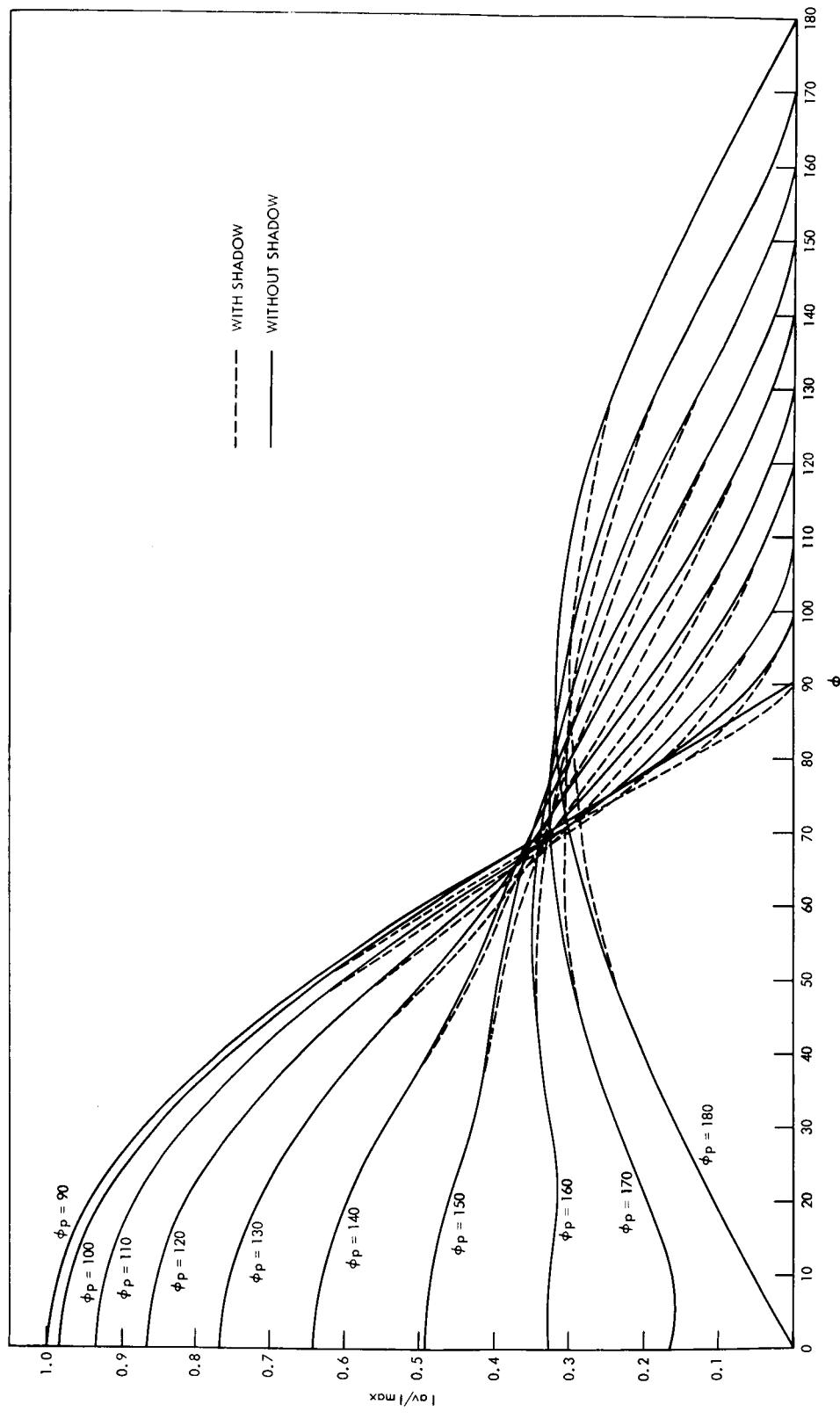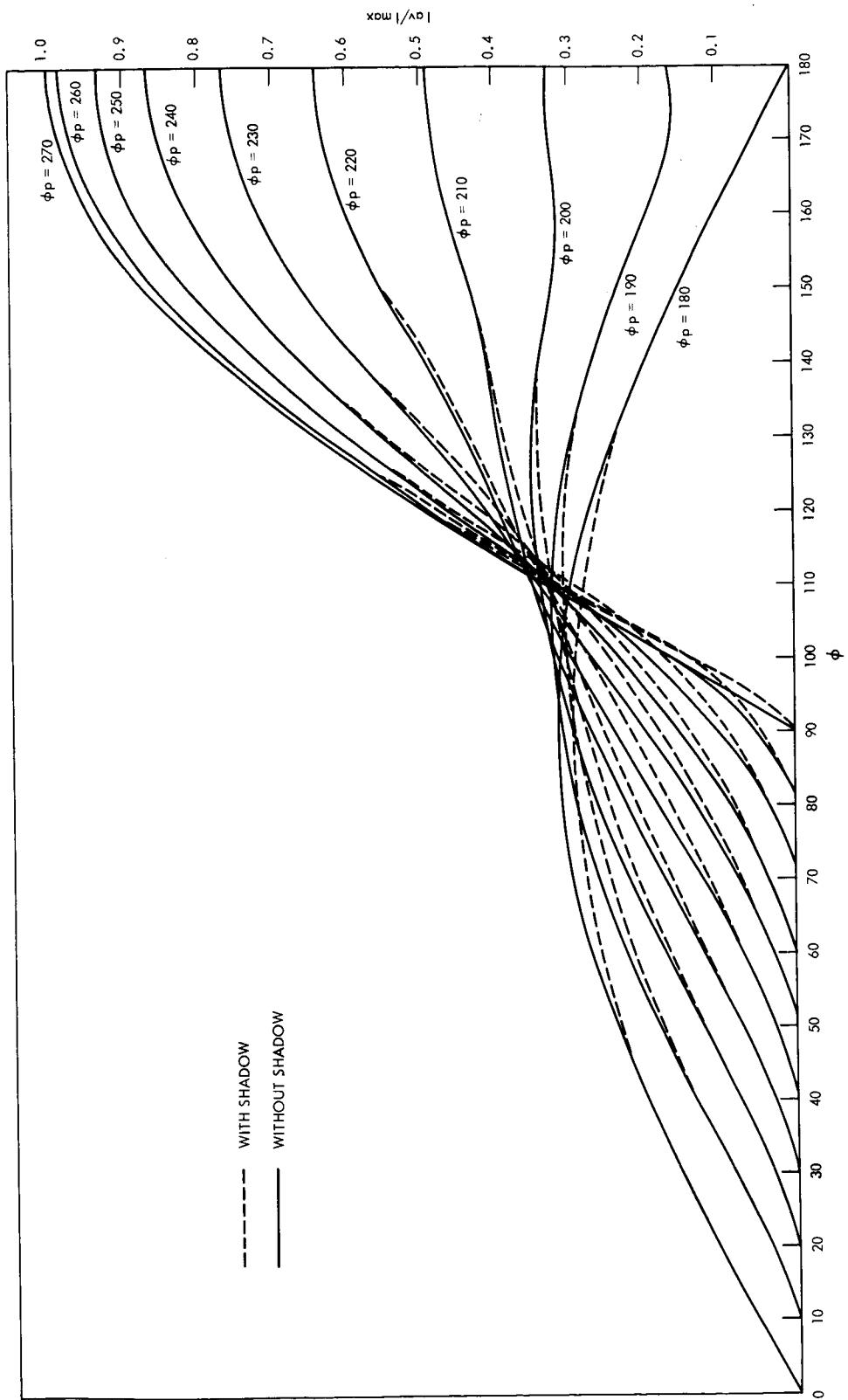
Figure 4

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1. Montgomery et al., Shades of EGO, NASA Report X-640-64-9, Goddard Space Flight Center, Greenbelt, Maryland.

2. Fischell, Robert E., Effects of Passive Attitude Control on Solar Power Systems, Report CF-3077, Johns Hopkins Applied Physics Laboratory, Silver Spring, Maryland.