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THE SOUND FIELD FOR SINGULARITIES IN MOTION

by

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THE SOUND FIELD FOR SINGULARITIES IN MOTION

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SUMMARY

A general expression for the sound field of a point force in arbitrary motion is found. The expression reduces to the classical results for both uniform rectilinear motion and uniform circular motion of a point force. The solution shows that a fluctuating force in motion will have two components in its radiative sound field; the first an essentially dipole term due to the time rate of change of the force, and the second an essentially quadrupole term due to the acceleration of the system in which the force is acting. This second effect has not been explicitly recognized in previous work. Applications include the vortex noise in rotating systems and helicopter main rotor noise.

An expression for the sound field of a point source in arbitrary motion is also determined. This expression does not reduce to the classical result for uniform motion, and it is concluded that the classical result does not take proper account of the momentum output associated with the source. The present result also shows that a constant velocity mass source moving in an accelerative manner will radiate sound. A practical example is the tip jet rotor.

The sound field for a point acoustic stress in arbitrary motion is determined and the accelerative effects are shown to produce higher order poles than the quadrupole normally associated with the acoustic stress tensor. Some preliminary interpretation of this result is presented. In all three cases the effect of a volume distribution of acoustic sources may be found by integration, and in all cases the effect of the acceleration terms becomes greater as speed is increased.

1. Introduction

For over a century it has been known that motion has an important effect on sound generation. The well known Doppler effect on frequency was first shown in 1842, and more recently the prediction of the effects of source convection formed an important part of the work of Lighthill (1952) "On Sound Generated Aerodynamically". In this paper, and others on the subject, attention has been focussed on source convection at constant velocity. For example, the sound field for a point force in uniform rectilinear motion is given by Lighthill (1962) as

$$\rho - \rho_0 = \left[\frac{x_i - y_i}{4 \pi a_0^3 r^2 (1 - M_r)^2} \frac{\partial F_i}{\partial t} \right] \tag{1}$$

Here x_i, y_i ($i = 1, 2, 3$) are the cartesian coordinates of observer and source respectively, and M_r is the component of convection Mach number in the direction $\underset{\sim}{r}$ of the observer. Equation (1) implies that if the force were constant no sound would be radiated. This is indeed true for the uniform motion of a constant force in a straight line but it is known that convection of constant forces can give rise to radiated sound. The sound generation by propeller thrust forces is a case in point.

It appears that the assumptions made in the derivation of equation (1) have been sufficiently restrictive to remove some of the terms giving rise to radiated sound. Physical intuition suggests the possible importance of the acceleration effects which have been removed from equation (1) by restriction to the case of constant velocity. For example, it might be expected that centrifugal accelerations would play a part in the noise generation by propeller thrust forces. Further, the well-known effect of shock wave generation by an accelerating piston calls attention to the possibility of a parallel acoustic case.

The object of this paper is to derive the equations of the sound fields for singularities in motion under less restrictive assumptions, and to investigate some of the additional effects that appear. The sound fields for extended distributions of acoustic sources may be readily obtained by integration, but the evaluation of these integrals will depend critically on the retarded time differences occurring within each source distribution. However, when the extent of a source is small compared with a typical wavelength of the sound generated, then the retarded time differences are small and the source may again be reduced, for calculation purposes, to an acoustic point singularity.

In the work which follows, the sound field for a point force in arbitrary motion is first found and the effects of acceleration discussed. The result is used to calculate the sound radiation from a propeller and also for more general rotary cases. A dimensional analysis is used to obtain a preliminary estimate of the relative magnitudes of the sound generated during uniform and accelerated motions, and it is shown that acceleration effects will generally become more important at higher velocities. Finally the results for both simple point sources and for point acoustic stresses in motion are found. The effect of arbitrary motion on the sound radiation of a simple source is readily understood, but only a preliminary interpretation of the result for the acoustic stress is given.

2. The General Equation for Sound Generation

Lighthill (1952) showed how the exact equations for sound generation could be obtained directly from the exact equations of aerodynamics. Following Lighthill, the equation for conservation of mass is written, in tensor notation, as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = Q \tag{2}$$

where Q is the rate of introduction of mass per unit volume. The equation for conservation of momentum is

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}(\rho v_i v_j) + \frac{\partial}{\partial x_j} p_{ij} = F_i \tag{3}$$

where F_i is the external force per unit volume acting on the fluid. Rewriting equation (3) gives

$$\frac{\partial}{\partial t}(\rho v_i) + a_o^2 \frac{\partial \rho}{\partial x_i} = - \frac{\partial}{\partial x_j}(T_{ij}) + F_i \tag{4}$$

where

$$T_{ij} = \rho v_i v_j + p_{ij} - a_o^2 \rho \delta_{ij} \tag{5}$$

$$\delta_{ij} = 1, \ i = j; \ = 0, i \neq j$$

T_{ij} may be regarded as an "acoustic stress".

Eliminating ρv_i between (2) and (4) by differentiation and subtraction gives finally

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} - \frac{\partial F_i}{\partial x_i} + \frac{\partial Q}{\partial t} \tag{6}$$

The left-hand side of equation (6) is recognizable as the equation for sound propagation in a uniform acoustic medium at rest. The terms on the right are volume distributions representing the various possible sources of sound present in the fluid.

The solution to (6) (for an unbounded fluid) is well known. If the right-hand side of (6) is written as $g\ (\underset{\sim}{y})$ the solution to (6) is

$$\rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \left[\frac{g}{r}\right] d\,v\,(\underset{\sim}{y}) \tag{7}$$

The square brackets imply evaluation at the retarded time $t' = t - r/a_o$ where t is the time of observation and r is the distance from source to observer. $\underset{\sim}{y}$ is a dummy variable of integration referring to the source position.

3. The Sound Field for a Point Force in Motion

The acoustic effects of a volume distribution of forces are found by putting

$$g = - \frac{\partial F_i}{\partial y_i}$$

as suggested by equation (6). Using (7) the sound field may be written down as

$$\rho - \rho_o = - \frac{1}{4\pi a_o^2} \int_V \left[\frac{1}{r} \; \frac{\partial F_i}{\partial y_i} \right] d\,v \qquad (8)$$

The reduction of the volume distribution of forces in equation (8) to the particular case of a point force may be accomplished by an appropriate introduction of three-dimensional Dirac δ functions. It is tempting to introduce the δ function as a multiple inside the square brackets at equation (8), but in fact it is necessary to introduce the δ function within the differential replacing F_i by $F_i\delta$. The new F_i may be regarded as a function of time only. Introduction of the δ function outside the differential would be a means of representing the effects of a concentrated force gradient and, although leading to a valid mathematical result, this case is not of practical interest. The sound field from a point force is thus given directly from (8) as

$$\rho - \rho_o = - \frac{1}{4\pi a_o^2} \int_V \left[\frac{1}{r} \; \frac{\partial}{\partial y_i} (F_i\delta) \right] d\,v \qquad (9)$$

In equation (9) $\delta = \delta(\underset{\sim}{y} - \underset{\sim}{y}_o)$ so that $\underset{\sim}{y}_o$ is the position of existence of the δ function. In order to study the effect of a moving point force it is sufficient to let $\underset{\sim}{y}_o$ be a function of the time t.

The evaluation of the integral in equation (9) depends critically on a proper treatment of the square brackets, which require evaluation of their contents at the retarded time $t' = t - r/a_o$. The square brackets may be treated as the retarded time operator, and laws established for their effects.

Consider any function $f(\underset{\sim}{y}, t)$. The chain rule shows

$$\frac{\partial}{\partial y_i}\left[f\right] = \left[\frac{\partial f}{\partial y_i} + \frac{(x_i - y_i)}{a_o r}\frac{\partial f}{\partial t}\right] \qquad (10)$$

Note also

$$\frac{\partial}{\partial t}\,[f] = \left[\frac{\partial f}{\partial t}\right]$$

Now for the moving singularity

$$\frac{\partial \delta}{\partial t} = -\frac{\partial \delta}{\partial y_i}\frac{\partial y_{oi}}{\partial t} \qquad (11)$$

where y_{oi} is the component of $\underset{\sim}{y}_o$ in the i direction.

Thus from (10) and (11), with some rearrangement

$$\left[\frac{\partial \delta}{\partial t}\right] = \left[-\frac{a_o M_i}{1 - M_r}\right] \frac{\partial}{\partial y_i} [\delta] \tag{12}$$

Here M_r is the component of the instantaneous convection Mach number in the direction $\mathbf{r}$ of the observer. In the terminology of Lighthill (1952) $M_r = M_c \cos\theta$. Also

$$M_r = \frac{(x_i - y_i)\, M_i}{r} \tag{13}$$

where $M_i = \dfrac{1}{a_o} \dfrac{\partial y_{oi}}{\partial t}$ is the component of the instantaneous convection Mach number $\mathbf{M}$ in the i direction.

Now using (10) and (12) in (9)

$$\rho - \rho_o = -\frac{1}{4\pi a_o^2} \int_V \left\{ \left[\frac{1}{r}\right] \frac{\partial}{\partial y_i} \left[F_i \delta\right] - \left[\frac{x_i - y_i}{a_o r^2} \frac{\partial F_i}{\partial t}\right] [\delta] \right.$$

$$\left. + \left[\frac{(x_i - y_i)}{r^2} \frac{F_i M_j}{1 - M_r}\right] \frac{\partial}{\partial y_j} [\delta] \right\} dv \tag{14}$$

The first and third terms in the integral may be rearranged to eliminate the differentiation of $[\delta]$ by applying a form of Green's identity viz:

$$\int_S U V_i n_i \, ds = \int_V \left\{ \frac{\partial U}{\partial x_i} V_i + U \frac{\partial V_i}{\partial x_i} \right\} dv$$

Since, in each case, the surface integral vanishes if the contour does not pass through a singularity, (14) may be rewritten as

$$\rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \left\{ [F_i] \frac{\partial}{\partial y_i} \left[\frac{1}{r}\right] + \left[\frac{(x_i - y_i)}{a_o r^2} \frac{\partial F_i}{\partial t} \right] \right.$$

$$\left. + \frac{\partial}{\partial y_j} \left[\frac{(x_i - y_i) F_i M_j}{r^2 (1 - M_r)} \right] \right\} [\delta] \, dv \qquad (15)$$

Now

$$[\delta] = \delta \{ \underset{\sim}{y} - \underset{\sim}{y}_o (t - r/a_o) \}$$

To evaluate the integral of $[\delta]$ the variable is changed from $\underset{\sim}{y}$ to $\underset{\sim}{\eta}$ where

$$\underset{\sim}{\eta} = \underset{\sim}{y} - \underset{\sim}{y}_o (t - r/a_o)$$

This will introduce the Jacobian of the transformation which may readily be calculated to give

$$dv\,(\underset{\sim}{\eta}) = (1-M_r)\,dv\,(\underset{\sim}{y})$$

Equation (15) may now be integrated directly, yielding

$$\rho - \rho_o = \frac{1}{4\pi a_o^2 \left[1 - M_r\right]} \left\{ \left[F_i\right] \frac{\partial}{\partial y_i}\left[\frac{1}{r}\right] + \left[\frac{(x_i - y_i)}{a_o r^2} \frac{\partial F_i}{\partial t}\right] + \frac{\partial}{\partial y_\gamma}\left[\frac{(x_i - y_i)\; F_i M_\gamma}{r^2 (1-M_r)}\right] \right\} \tag{16}$$

where all quantities in (16) are evaluated on the path of the singularity. Using (10) on the last term of (16), only the radiative terms are retained to give

$$\rho - \rho_o = \frac{1}{4\pi a_o^2 \left[1-M_r\right]} \left[\frac{(x_i - y_i)}{a_o r^2} \frac{\partial F_i}{\partial t} + \frac{(x_i - y_i)(x_\gamma - y_\gamma)}{a_o r^3} \frac{\partial}{\partial t} \left\{\frac{M_\gamma F_i}{1 - M_r}\right\}\right]$$

and performing the differentiation on the last term gives the far field sound radiation from a point force in arbitrary motion as

$$\rho - \rho_o = \left[\frac{(x_i - y_i)}{4\pi \, (1-M_r)^2 \, a_o^3 \, r^2} \left\{ \frac{\partial F_i}{\partial t} + \frac{F_i}{1-M_r} \frac{\partial M_r}{\partial t} \right\} \right] \qquad (17)$$

<u>This equation is the basic result of the paper.</u>

In equation (17) $\frac{\partial M_r}{\partial t}$ is the component of acceleration in the $\underset{\sim}{r}$ direction

$$\frac{\partial M_r}{\partial t} = \frac{(x_i - y_i)}{r} \frac{\partial M_i}{\partial t}$$

Examination of equation (17) shows first of all that there are two terms. For constant convection velocity the second vanishes and the expression reduces to the accepted result for a point force convected at uniform speed as given in equation (1). But if the motion is accelerative, then there is a second term present which will give rise to a radiated sound field even when the applied force is constant.

The near field terms may also be calculated, and are found to be

$$\rho - \rho_o = \left[\frac{1}{4\pi \, a_o^2 \, (1-M_r)^2 r^2} \left\{ \frac{F_i \, (x_i - y_i)}{r} \frac{(1 - M^2)}{(1 - M_r)} - F_i \, M_i \right\} \right] \qquad (18)$$

The near field terms given by equation (18) show no dependence on the acceleration, and are in fact identical to the case of convection at constant velocity.

4. <u>Dimensional Analysis of the Sound Generated</u>

In the light of equation (17) it is now possible to make some general comments on the effects of acceleration on the noise generation of a moving force, and of the importance of the various parameters in the overall sound production. By putting $F \propto \rho_o U^2 \ell^2$ i.e. a typical pressure multiplied by a typical area, then equation (17) shows

$$\rho - \rho_o \propto \frac{1}{r a_o^3} \left\{ \frac{\rho_o U^2 \ell^2 n}{(1 - M_r)^2} + \frac{\rho_o U^2 \ell^2}{(1 - M_r)^3} \frac{\partial M_r}{\partial t} \right\}$$

where n is a typical frequency. A typical value of $\frac{\partial M_r}{\partial t}$ will depend on the system under consideration, but in many cases $\frac{\partial M_r}{\partial t} \propto \frac{Un}{a_o}$ Invoking constancy of Strouhal number, and putting $n \propto U/\ell$, yields finally

$$\frac{\rho - \rho_o}{\rho_o} \propto \frac{\ell}{r} \left(\frac{U^3}{(1-M_r)^2 a_o^3} + \frac{U^4}{(1-M_r)^3 a_o^4} \right) \tag{19}$$

The first term in equation (17) gave the conventional dipole result for uniformly moving forces. Similarly, the first term in equation (19) shows dipole dependence on velocity and convection Mach number. But the second term in equation (19), which corresponds to the acceleration effects is dependent on $U^4 (1-M_r)^{-3}$ This dependence is typical of the uniformly convected quadrupoles investigated by Lighthill (1952) and leads to the suggestion that it may be useful to consider the sound output from a generally moving force as composed of two parts: A dipole contribution from the fluctuating part of the force, and a quadrupole contribution due to the acceleration effects, which may be expected to be particularly important at high velocities. Additional support for this viewpoint comes from a further examination of equation (17). The first term has a two lobe space distribution, whereas the second term has a four lobe distribution, so that the order of spherical harmonic associated with each term is again that appropriate to the dipole and quadrupole respectively.

The distinction between the various orders of acoustic radiation is often drawn from the form of the acoustic forcing function on the right-hand side of equation (6). Monopole, dipole and quadrupole types of radiation are associated with zero, one and two space derivatives respectively. In the present case the quadrupole-like radiation patterns have arisen from an acoustic forcing function with only one space derivative, so that classification as dipole or quadrupole radiation depends on the choice of definition. However, the sound radiation due to the acceleration effects of moving forces seems to be most consistently classified as a quadrupole field. It should be noted that this classification is based only on the far field sound radiation. The near field radiation in the present case is identical to that usually associated with a dipole.

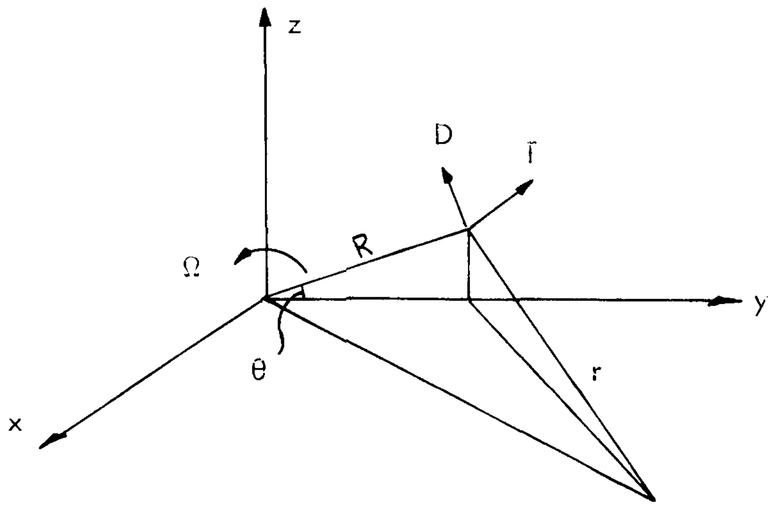
5. Application to a Static Propeller

Equation (17) may be rewritten to give the sound pressure as

$$p = \left[\frac{(x_i - y_i)}{4\pi a_o r^2 (1-M_r)^2} \frac{\partial F_i}{\partial t} + \frac{F_i}{1-M_r} \frac{\partial M_r}{\partial t} \right] \tag{20}$$

Equation (20) refers to the effect of an isolated point force. In order to apply it to the case of a propeller, the distributed loading over the propeller blade is reduced to an equivalent point force as is common in propeller noise theory. The sound radiation from a static propeller may then be considered via the case of circular motion of a point force acting at some effective radius from the hub. In principle, the effective radius is that at which the point drag force gives rise to the total torque on the blade, but in practice some care is required in choosing a different optimum effective radius for the acoustic calculations.

The distributed force on the blade is condensed to a single thrust T parallel to the propeller axis, and drag D perpendicular to and in the plane of the propeller acting at distance R from the hub.



Set up cartesian coordinates centered on the propeller axis with x, y, z corresponding to $i = 1, 2, 3$, x along the propeller axis and the observer in the xy plane. Suppose the propeller is rotating with angular velocity Ω so that its current position is given by $\theta = \Omega t$ with $\theta = 0$ along the y axis. The force on the air is equal and opposite to that on the blade and is thus in a thrust in the negative x direction an a drag in the direction of rotation, as shown.

If M is the rotational Mach number, $M = \Omega R/a_o$, then $\underset{\sim}{r} = (x_i - y_i) = (x, y - R\cos\theta, -R\sin\theta)$, $\underset{\sim}{M} = (0, -M\sin\theta, M\cos\theta)$ and $F_i = (-T, -D\sin\theta, D\cos\theta)$. Substituting into equation (20) gives, after some cancellation,

$$p = \left[\left\{\frac{MTx}{r} - D\right\}\frac{\Omega(y\cos\theta - R)}{4\pi(1 + My\sin\theta/r)^3 a_o r^2}\right] \tag{21}$$

A formula equaivalent to (21) was found by Van de Vooren and Zandbergen (1963).

However, the Fourier coefficients of (21) can be evaluated.

Now $r^2 = |\underset{\sim}{r}|^2 = x^2 + y^2 + R^2 - 2y\,R\cos\theta$

So that in the far field where $r >> R$

$$r \approx r_o - \frac{yR}{r}\cos\theta$$

with $r_o^2 = x^2 + y^2 + R^2$. Equation (21) is evaluated at the retarded time $t' = t - r/a_o$ and t only appears in equation (21) through θ.

Now

$$[\theta] = \Omega\left(t - \frac{r_o}{a_o}\right) + \frac{My}{r}\cos\theta$$

If r is now regarded as constant, the problem reduces to that of the calculation of the Fourier coefficients in τ of $\cos\theta\,(1 + \alpha\sin\theta)^{-3}$

where $\theta = \tau + \alpha\cos\theta$, with $\tau = \Omega\left(t - \frac{r_o}{a_o}\right)$ and $\alpha = \frac{My}{r}$

Put $$\pi a_n = \int_{\alpha}^{2\pi+\alpha} \frac{\cos\theta\,\cos n\tau}{(1 + \alpha\sin\theta)^3}\,d\tau$$

The limits of the integral may be over any interval of 2π.

Changing the variable by $\tau = \theta - \alpha\cos\theta$ yields

$$\pi a_n = \int_0^{2\pi} \frac{\cos\theta \cos n(\theta - \alpha\cos\theta)}{(1 + \alpha \sin\theta)^2} d\theta \tag{22}$$

But

$$\int \frac{\cos\theta}{(1+\alpha \sin\theta)^2} d\theta = - \frac{1}{\alpha(1+\alpha\sin\theta)}$$

So that (22) may be integrated by parts to give

$$\pi a_n = - \frac{1}{\alpha} \int_o^{2\pi} \frac{n(1+\alpha\sin\theta)}{1+\alpha\sin\theta} \sin n(\theta - \alpha\cos\theta)\, d\theta = 0$$

Applying the same technique to

$$\pi b_n = \int_\alpha^{2\pi+\alpha} \frac{\cos\theta \sin n\tau}{(1+\alpha\sin\theta)^3} d\tau$$

gives

$$\pi b_n = \frac{n}{\alpha} \int_o^{2\pi} \cos n(\theta - \alpha\cos\theta)\, d\theta$$

which is the well-known form for Bessel functions. Hence

$$b_n = \frac{2n}{\alpha} J_n(n\alpha) \tag{23}$$

where J_n is a Bessel function of the first kind and nth order. So that in the present case the sound pressure field for the circular motion of a point force can be written as a Fourier series whose nth coefficient is

$$|a_n| = \left\{ \frac{MTx}{r} - D \right\} \frac{\Omega y}{4\pi a_o r^2} \left\{ \frac{2n}{M\, y/r} \quad J_n \left(\frac{n\, M\, y}{r} \right) \right\}$$

If B equally spaced blades are present, harmonics which are not multiples of the number of blades will cancel giving

$$|a_m| = \frac{m\, B\, \Omega}{2\pi a_o r} \left\{ \frac{Tx}{r} - \frac{D}{M} \right\} J_{mB} \left(\frac{m\, B\, M\, y}{r} \right) \tag{24}$$

The quadrupole effect of acceleration may be observed in either equation (21) or (24). The noise radiation due to the drag D is dipole in nature, while the noise due to the thrust T has the velocity and directional dependence of a quadrupole.

Equation (24) is identical to the expression found by Gutin (1936) for the same case except for the trivial difference (to the order of approximation considered) that instead of r , the retarded distance from the force, he has r_1, the distance from the propeller center. Gutin's result has been substantiated by experiment. The reduction of equation (17) to this classical result provides a satisfying test of the present approach, particulary when the two expressions initially appear so different. However this agreement is not as surprising as it might first appear since the physical basis for Gutlin's theory is identical to that of the present work, i.e. the moving isolated force is replaced by a string of impulses acting along its path at the appropriate times.

In principle, the extension of equation (17) to cover the distributions of forces acting on the blade is a straightforward integration over the desired area of action. The use of equation (17) or its particular case (21) will generally be convenient in computational methods, particularly if the higher harmonics of the sound pressure are required.

6. Vortex Noise from Rotating Systems and Helicopter Noise

Equation (17) represents the far field pressure for a quite arbitrary motion. In practice, however, most systems will be moving in some more regular manner. Two cases in which the noise has to be calculated for fluctuating forces on a system with acceleration are the vortex noise from rotating systems and helicopter noise. Subjectively, the vortex or random part of the noise from a rotating system seems considerably higher than would be expected if an equivalent cascade of blades were moving linearly. Equation (17) shows part of the reason why this is so. The vortex noise is caused by the passage over the blades of the random pressure patterns associated with the turbulent boundary layer. Equation (17) shows that these random pressures have two effects. Firstly, they will radiate noise by reason of their rate of change. Secondly, the instantaneous pressure pattern over the blades will radiate noise by virtue of the centrifugal acceleration of the pattern. Calculations of vortex noise to date have only considered the noise due to the first effect, but it can be seen that noise from the second effect may also be important, particularly at high rotational speeds. However, a proper formulation of the vortex noise problem will be difficult because of the complex nature of the turbulent pressure distribution over the blades.

A similar problem arises when calculating the noise from helicopters. A helicopter has a high level of vortex noise, but in addition, the spectrum of the main rotor rotational noise is typified by the presence of a large number of the higher harmonics of the blade passage frequency. These harmonics are not found in conventional propeller noise theory. It is hoped that they can be predicted by an application of equation (17), which will enable the large cyclic changes in blade loading to be included in the calculation.

The complete equation for the noise generation by an arbitrary point force in a convected rotating system may be written down using equation (17). The only changes required from the case of the propeller are that the thrust and drag forces become functions of time, and the convection Mach number is $\underset{\sim}{M} = (M_{o1}, M_{o2} - M \sin\theta, M_{o3} - M\cos\theta)$ where $\underset{\sim}{M}_o$ is the constant convection Mach number of the centre of the rotating system. If M_{or} is the component of $\underset{\sim}{M}_o$ in direction $\underset{\sim}{r}$ then the generalized form of equation (21) is

$$p = \left[\frac{\{MTx - rD(1 - M_{or})\}\,\Omega\,(y\cos\theta - R)}{4\pi a_o \{r(1 - M_{or}) + My\sin\theta\}^3} - \frac{y\frac{\partial D}{\partial t}\sin\theta + \frac{x\partial T}{\partial t}}{4\pi a_o\{r(1 - M_{or}) + My\sin\theta\}^2}\right] \tag{25}$$

Equation (25) may be used directly for calculation of helicopter main rotor noise, and will also serve as a basis for calculations of vortex noise. In Equation (25) $M = \Omega r/a_o$ as before.

7. Simple Sources in Motion

The usual result for simple sources in motion has been established by replacing $\frac{\partial Q}{\partial t}$ in equation (6) by q. The effect of the moving point source q is given, using (7) as

$$\rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \left[\frac{q}{r} \; \delta \left(\underset{\sim}{y} - \underset{\sim}{y}_o \right) \right] dv$$

The evaluation of the integral is accomplished by change of variable, as in the case of the moving force, giving

$$\rho - \rho_o = \frac{q}{4\pi a_o^2 \; r\,(1-M_r)} \tag{26}$$

This is the result quoted by Lighthill (1962) for the simple source moving at constant velocity, although the present derivation is valid for an arbitrarily moving q. This result is mathematically satisfactory and is equivalent to the scalar part of the result obtained for the field of a moving electron. In the case of the moving electron q does have a physical meaning, i.e. the charge. But in the acoustic case q represents a double time rate of introduction of mass, and the physical interpretation is less direct.

A possible practical case is a <u>single</u> time rate of introduction of mass as in an arbitrarily moving jet of air. The mathematical formulation of the problem requires the introduction of the source term g in equation (7) as $\frac{\partial}{\partial t}(Q\delta)$ rather than as the expression $\delta \frac{\partial Q}{\partial t}$ used above. This will give the solution as

$$\rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \left[\frac{1}{r} \frac{\partial}{\partial t} (Q\delta) \right] dv \tag{27}$$

Expanding and using (12)

$$\rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \left[\frac{1}{r} \frac{\partial Q}{\partial t} \right] [\delta] - \left[\frac{Q}{r} \frac{a_o M_i}{1 - M_r} \right] \frac{\partial}{\partial y_i} [\delta] \quad dv$$

Using Green's identity and then evaluating the integral of $[\delta]$ as before

$$\rho - \rho_o = \frac{1}{4\pi a_o^2 [1 - M_r]} \left\{ \left[\frac{1}{r} \frac{\partial Q}{\partial t} \right] + \frac{\partial}{\partial y_i} \left[\frac{Q}{r} \frac{a_o M_i}{1 - M_r} \right] \right\}$$

Now using (10), the radiative terms are

$$\rho - \rho_o = \left[\frac{1}{4\pi a_o^2 (1 - M_r)^2 r} \left\{ \frac{\partial Q}{\partial t} + \frac{Q \frac{\partial M_r}{\partial t}}{1 - M_r} \right\} \right] \tag{28}$$

and the near field terms are

$$\rho - \rho_o = \left[\frac{Q\,(M_r - M^2)}{4\pi\, a_o^2\, (1 - M_r)^3\, r^2} \right] \tag{29}$$

For the case of convection at constant velocity, equation (28) reduces to a result differing by a factor of $(1 - M_r)$ from equation (26). This result is surprising, but its meaning may be clarified by rewriting equation (28) as

$$\rho - \rho_o = \left[\frac{\frac{\partial Q}{\partial t}}{4\pi\, a_o^2\, (1 - M_r)\, r} + \frac{(x_i - y_i)}{4\pi\, a_o^2\, (1 - M_r)^2\, r^2} \right.$$

$$\left. \times \left\{ \frac{\partial Q M_i}{\partial t} + \frac{Q M_i}{1 - M_r} \quad \frac{\partial M_r}{\partial t} \right\} \right] \tag{30}$$

The first part of equation (30) is identical with equation (26) and the second part is the same as equation (17) with F_i put equal to Qv_i, which is the rate of introduction of momentum from the moving system. Thus, the sound field given by (28) is the sum of both mass and momentum terms. The near field terms are also the result of momentum input, and equation (29) may be found by putting $F_i = Qv_i$ in equation (18).

Clearly the momentum terms cannot be ignored. Even for such an idealized case as a convected pulsating sphere the momentum is not instantaneously balanced. One may imagine that, during the convection of a pulsating sphere, the mass output occurs at a different location from the mass input and thus gives rise to a net momentum fluctuation. The sound field is thus given by equation (28) rather than the generally accepted result of equation (26). It should be noted that the mathematical validity of equation (26) is not in question. The error lies in the incorrect modelling of the boundary conditions as being solely a double time rate of introduction of mass.

A dimensional analysis for this case gives little additional information, but it is of interest to note from equation (30) that the radiation from an arbitrarily moving "simple source" gives rise to three distinct orders of sound radiation: the pure monopole effect of the second rate of introduction of mass, the dipole effect of the convected momentum, and the quadrupole effect of acceleration. An additional result from equation (28) is to show that the introduction of mass at a constant rate in an accelerated system will give rise to radiated noise. This result may be applied to tip jet rotors or similar devices, and if required, the sound field may be calculated following the methods of Section 5.

8. Acoustic Stresses in Motion

The methods used to calculate the effect of a point force in motion may be extended to the case of an acoustic stress in motion. The mathematical formulation of the problem is straightforward, although the algebra involved is somewhat tedious.

Clearly, the solution for the general case of an acoustic stress in motion is given, using equations (6) and (7), by

$$\rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \left[\frac{1}{r} \frac{\partial^2}{\partial y_i \partial y_j} \left\{ T_{ij}\, \delta \right\} \right] dv \tag{31}$$

As before, the reduction of this integral depends on taking proper account of the retarded time differences within it. The first requirement is to establish formulae showing the effect of the square bracket retarded time operator under double differentiation. Differentiating equation (10) gives, after some rearrangement,

$$\frac{\partial^2 [f]}{\partial y_i \partial y_j} = \left[\frac{\partial^2 f}{\partial y_i \partial y_j} \right] + \frac{(x_j - y_j)}{a_o r} \frac{\partial}{\partial y_i} \left[\frac{\partial f}{\partial t} \right] + \frac{\partial}{\partial y_j} \left[\frac{(x_i - y_i)}{a_o r} \frac{\partial f}{\partial t} \right]$$

$$- \left[\frac{(x_i - y_i)(x_j - y_j)}{a_o^2 r^2} \frac{\partial^2 f}{\partial t^2} \right] \tag{32}$$

Differentiation of equation (12) with respect to t gives, again after rearrangement,

$$\left[\frac{\partial^2 \delta}{\partial t^2}\right] = \left[\frac{a_o M_i}{1 - M_r}\right] \frac{\partial}{\partial y_i} \left\{ \left[\frac{a_o M_j}{1 - M_r}\right] \frac{\partial [\delta]}{\partial y_j} \right\}$$

$$- \frac{\partial [\delta]}{\partial y_i} \frac{\partial}{\partial t} \left[\frac{a_o M_i}{1 - M_r}\right] \qquad (33)$$

The only radiative term in equation (32) is the last, so that expanding (31) and using equation (10), (12), (32) and (33) gives the terms contributing to the far field sound radiation as

$$\rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \left[\frac{(x_i - y_i)(x_j - y_j)}{a_o^2 r^3}\right] \left\{ [\delta] \frac{\partial^2 T_{ij}}{\partial t^2} + \frac{2\partial T_{ij}}{\partial t} \left[\frac{-a_o M_k}{1-M_r}\right] \frac{\partial [\delta]}{\partial y_k} \right.$$

$$\left. + \left[T_{ij}\right] \left\{ \left[\frac{a_o M_k}{1-M_r}\right] \frac{\partial}{\partial y_k} \left(\left[\frac{a_o M_l}{1-M_r}\right] \frac{\partial [\delta]}{\partial y_l} \right) - \frac{\partial [\delta]}{\partial y_k} \frac{\partial}{\partial t} \left[\frac{a_o M_k}{1 - M_r}\right] \right\} \right\} dv \qquad (34)$$

Now using Green's Identity the appropriate number of times

$$\rho - \rho_o = \frac{1}{4\pi a_o^2}\int_V [\delta] \Bigg\{ \left[\frac{(x_i - y_i)(x_j - y_j)}{a_o^2 r^3} \frac{\partial^2 T_{ij}}{\partial t^2} \right] + \frac{2\partial}{\partial y_k}\left[\frac{(x_i - y_i)(x_j - y_j)}{a_o^2 r^3} \left\{ \frac{a_o M_k}{1 - M_r} \right\} \frac{\partial T_{ij}}{\partial t} \right]$$

$$+ \frac{\partial}{\partial y_\ell}\left\{ \left[\frac{a_o M_\ell}{1 - M_r} \right] \frac{\partial}{\partial y_k}\left[\frac{(x_i - y_i)(x_j - y_j)}{a_o^2 r^3} \left\{ \frac{a_o M_k}{1 - M_r} \right\} T_{ij} \right] \right\}$$

$$+ \frac{\partial}{\partial y_k}\left\{ \left[\frac{(x_i - y_i)(x_j - y_j)}{a_o^2 r^3} \; T_{ij} \; \frac{\partial}{\partial t}\left(\frac{a_o M_k}{1 - M_r} \right) \right] \right\} \Bigg\} dv \qquad (35)$$

The integral of $[\delta]$ may be evaluated as in the case of the moving force. Then, using (10) and (32) in (35) and retaining only the radiative terms, yields, after much algebra, the required solution. The far field sound of a generally moving point acoustic stress is thus

$$\rho - \rho_o = \Bigg[\frac{(x_i - y_i)(x_j - y_j)}{4\pi a_o^4 r^3 (1 - M_r)^3} \Bigg\{ \frac{\partial^2 T_{ij}}{\partial t^2} + \frac{\partial T_{ij}}{\partial t} \; \frac{3 \frac{\partial M_r}{\partial t}}{1 - M_r} + \frac{T_{ij}}{1 - M_r} \; \frac{\partial^2 M_r}{\partial t^2}$$

$$+ \frac{3 \; T_{ij}}{(1 - M_r)^2} \left(\frac{\partial M_r}{\partial t} \right)^2 \Bigg\} \Bigg] \qquad (36)$$

The physical interpretation of equation (36) is not immediately clear. The result should show a simple relationship to the result for the field of a generally moving point force given by equation (17). In order to see this relationship equation (17) is rewritten as,

$$\rho - \rho_o = \left[\frac{(x_i - y_i)}{(1-M_r)\, a_o r} \; \frac{\partial}{\partial t} \left(\frac{F_i}{4\pi\, a_o^2\, r\, (1 - M_r)} \right) \right] \tag{37}$$

In a letter to the writer Professor Lighthill showed how this form of (17) is far from being a casual relationship, and can be found directly by an alternative method of proof. His method is based on relating the solution for a given order of acoustic forcing function to a result for the next lower order. Equation (37) gives the result for a first order function and it will be observed that the term under differentiation in Equation (37) is the solution for a zeroth order function (Equation 26). Professor Lighthill's method also gives the result for the sound field of an acoustic stress in arbitrary motion as

$$\rho - \rho_o = \left[\frac{(x_i - y_i)}{(1 - M_r)\, a_o r} \; \frac{\partial}{\partial t} \left\{ \frac{(x_j - y_j)}{(1-M_r)\, a_o r} \; \frac{\partial}{\partial t} \left(\frac{T_{ij}}{4\pi\, a_o^2\, r\, (1 - M_r)} \right) \right\} \right] \tag{38}$$

An acoustic stress is a second order forcing function and it may again be observed that the result for a first order function (Equation 37) is under differentiation in Equation (38).

The relationship of equation (36) to equation (17) is apparent when they are written in their alternative forms of equations (38) and (37) respectively. The equivalence of equations (36) and (38) may be verified by differentiation. As would be expected, equation (36) reduces to the result obtained by Lighthill (1952) for the case of convection at constant velocity. For a more general motion the first term of equation (36) is quadrupole, the second two terms are octupole, and the last term is the next order higher pole, for which a consistent terminology might have the name "sexdecupole". The definition of these orders is based on their velocity, convection Mach number, and directional dependence as in the discussion of the orders for the case of the moving force. Each term is the result of only two space derivatives in the acoustic forcing function.

Equation (36) indicates that accelerated stresses would produce additional noise over the case of uniform convection. This effect may be of practical importance, for instance, in the calculation of the noise radiation from deflected jets or rockets, and the turbulent sound generation from tip jet rotors.

However, no firm conclusions as to the practical significance of equation (36) can be drawn at this stage. The velocity dependence of the radiated sound intensity from the three orders of pole is on the eighth, tenth and twelfth power respectively. To the writer's knowledge, no experimental investigation has yet found overall powers higher than the eighth, and this fact tends to discount any major significance of the acceleration effects. However, it is possible that other parameters are coming into play. Lighthill (1954) has shown how even higher powers of the velocity would be expected from the convection Mach number dependence of the sound radiation, but that this effect is conveniently balanced by the reduction in turbulence intensity with increase in velocity. It is possible that some similar balance could occur in the present case. In addition, this velocity dependence has been derived by putting a typical acceleration proportional to U^2/ℓ , as in Section 4. This proportionality may not always occur.

Equation (36) may be particularly suitable for calculations in certain types of noise fields. The equations for the sound radiation from a propeller may be found using many different approaches, but the method presented in Section 3 is particularly convenient since the force is constant relative to the moving system. In the same way it may be possible, theoretically or experimentally, to find paths such that T_{ij} is substantially constant along them. The calculation of the noise field using equation (36) would then be straightforward. The strength of the acoustic stress distribution is unaffected by the system used to calculate its sound radiation. The strength continues to be essentially dependent on $\rho v_i v_j$ as shown by Lighthill (1952), and these velocities are still measured relative to the undisturbed atmosphere.

The calculation of the radiation from an extended distribution represents a further problem. It may be noted that the concentrated acoustic stress considered in equation (36) is a representation of a turbulent eddy under Lighthill's (1954) hypothesis that the turbulence may be considered, for sound radiation purposes, as a collection of independent point quadrupoles, one in each "correlation volume". But considerably more analysis will be required before the full implications of equation (36) become apparent.

Conclusions

It has been shown that the radiative pressure field for a point force in arbitrary motion is given by

$$p = \left[\frac{(x_i - y_i)}{4\pi a_o r^2 (1 - M_r)^2} \left\{ \frac{\partial F_i}{\partial t} + \frac{F_i}{1 - M_r} \frac{\partial M_r}{\partial t} \right\} \right]$$

This expression reduces to the previously obtained result for the case of uniform rectilinear motion as quoted by Lighthill (1962), and the complete equation yields the same result for circular motion of a point force as that obtained by Gutin (1936). The expression shows that a fluctuating force in motion will have two components in its radiative sound field. The first is the conventional dipole term due to the time rate of change of the force, and the second is essentially a quadrupole term due to the acceleration of the system in which it is acting. This second term gives rise to radiated sound even when the force is constant. The vortex noise in rotating systems and helicopter main rotor noise provide two examples where the above equation may usefully be applied.

The arguments that lead to the above expression may be applied to the point source in arbitrary motion, and give the radiated sound field for this case as

$$p = \left[\frac{1}{4\pi\, r\, (1 - M_r)^2} \left\{ \frac{\partial Q}{\partial t} + \frac{Q}{1 - M_r} \; \frac{\partial M_r}{\partial t} \right\} \right]$$

This result differs from the generally accepted result by a factor of $(1 - M_r)$ even for the case of convection at constant velocity. This disagreement has been shown to be the result of the neglect of the momentum terms in the derivation of the classical result. The classical result is mathematically valid, but should not be directly applied to a real physical situation. The above result also shows how the sound radiation from a "simple source" in arbitrary motion will in fact consist of pure monopole, dipole and quadrupole terms. The quadrupole term will still exist even for a non-fluctuating source in accelerated motion, such as a tip jet rotor.

The expression for the sound field generated by a point acoustic stress in arbitrary motion has also been derived. It is

$$p = \left[\frac{(x_i - y_i)(x_j - y_j)}{4\pi\, a_o^2\, r^3\, (1 - M_r)^3} \left\{ \frac{\partial^2 T_{ij}}{\partial t^2} + \frac{\partial T_{ij}}{\partial t} \; \frac{3 \frac{\partial M_r}{\partial t}}{1 - M_r} + \frac{T_{ij}}{1 - M_r} \; \frac{\partial^2 M_r}{\partial t^2} \right. \right.$$

$$\left. \left. + \frac{3\, T_{ij}}{(1 - M_r)^2} \left(\frac{\partial M_r}{\partial t} \right)^2 \right\} \right]$$

The relationship of this result to the case of the convected point source and point force has been shown. The result reduces to that of Lighthill (1952) for uniform rectilinear motion and, as before, the effect of the acceleration terms is to produce higher order poles in the radiated sound field. It is possible that the result will have both theoretical and practical applications, for example, to deflected (and thus accelerated) rocket exhaust flows. In all three cases the effect of an extended distribution of acoustic sources can be determined by an integration over the region of action (taking appropriate account of retarded time), and in all cases the effect of the acceleration terms becomes greater as velocity is increased.

Note: This work was initiated while the author was at the Institute of Sound and Vibration, Southampton, England.

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