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School of Electrical Engineering
Electronic Systems Research Laboratory
Purdue University
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STUDY OF LUNAR REFLECTIVE COMPONENTS OF SOLAR RADIO EMISSION

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by

G. R. Cooper

C. D. McGillem

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Introduction and Mathematical Analysis

Methods of measuring the radio frequency scattering properties of the lunar surface are limited to the backscattering case because of the requirement that the transmitter and receiver be in essentially the same direction relative to the scattering point on the lunar surface. As a consequence of this limitation, it is not possible to obtain a satisfactory estimate of the scattering law of the surface in which varying angles of incidence and scattering are taken into account.

As a method of overcoming this limitation, it is proposed to utilize solar radio frequency emission as the energy source and to measure the scattering of this radiation by the lunar surface. Because of the variations in the relative positions of the sun, moon, and earth which occur as the moon goes through its phases, a wide range of scattering angles can be obtained. Also, a wide frequency range is available as a result of the broad spectrum of solar radiation. By measuring the reflection coefficient of the lunar surface as a function of angle of incidence, frequency, and polarization, it may be possible to obtain much more satisfactory estimates of dielectric constant, conductivity, and roughness of the lunar surface.

Before discussing the details of the detection method we are proposing, it might be worthwhile to examine the magnitude of the power involved here.

In order to obtain some insight into the problem, consider first a case where the moon is represented as a collection of stationary scatterers. If $x(t)$ is the signal from the sun and a_k is the net scattering coefficient of the scatterers whose signals are received with delay τ_k relative to the direct signal from the sun, then the received signal is given by

$$y(t) = \sum_k a_k \, x(t-\tau_k)$$

the crosscorrelation of $y(t)$ and $x(t)$ is found to be:

$$R_{yx}(\tau_o) = \sum_k a_k \, R_{xx}(\tau_o - \tau_k).$$

The crosscorrelation function is therefore a sum of terms, each of which is a reproduction of the correlation function of $x(t)$ located at a particular delay τ_k and having an amplitude determined by a particular scattering coefficient, a_k. The coefficients a_k can be related to the power reflecting properties of the scatterer and for the case of wideband noise -- such that $R_{xx}(\tau)$ is zero except when $\tau = 0$ -- it is found that the mean square value of the received signal is related to the scattering coefficients by:

$$\overline{y^2(t)} = R_{xx}(0) \sum_k a_k^{\,2}$$

Since $R_{xx}(0)$ is the mean square value of the transmitted signal, i.e., the transmitted powers, it follows that $a_k^{\,2}$ is the net power reflection coefficient corresponding to delay τ_k and can be related to the range, antenna gain, surface reflection coefficients and other parameters.

When motion of the scatterers is taken into account, the problem becomes considerably more complicated. For this case it is found that the crosscorrelation of the transmitted (i.e., solar) signal and reflected (i.e., lunar) signal leads to the following expression.

$$R_{yx}(\tau_o, t) = \sum_k b_k \, R_{xx}[\tau_o - \tau_k + (\beta_k - 1)t]$$

$$= b_1 \, R_{xx} \, [\tau_o - \tau_1 + (\beta_1 - 1)t] + b_2 \, R_{xx} \, [\tau_o - \tau_2 + (\beta_2 - 1)t] + \ldots$$

$$+ b_N \, R_{xx} \, [\tau_o - \tau_N + (\beta_N - 1)t]$$

where:

$$\beta_k = 1 - \frac{v_k}{c}$$

v_k = velocity of kth scatterer

c = velocity of light

$b_k = a_k \sqrt{\beta_k}$

In a practical correlator the time correlation function will be measured and this is related to the time average of the ensemble correlation function. It is seen that the ensemble correlation function given above is a function of time and so its time average will depend on the form of $R_{xx}(\tau)$. If $R_{xx}(\tau)$ is oscillatory, as in the case of bandlimited noise, then the time average will be zero and the correlator output would be zero. By correlating with a modified function, $z = x(\beta_m t - \tau)$, a more useful result is obtained. What this amounts to is using a correlator with a continuously varying delay given by $\tau = \tau_o + (1 - \beta_m)t$. Computing the correlation function for this case gives:

$$R_{yz}(\tau_o) = E\left\{\sum_k b_k\, x(\beta_k t - \tau_k)\, x(\beta_m t - \tau_o)\right\}$$

$$= \sum_k b_k\, R_{xx}\left[(\beta_k - \beta_m)t - (\tau_k - \tau_o)\right]$$

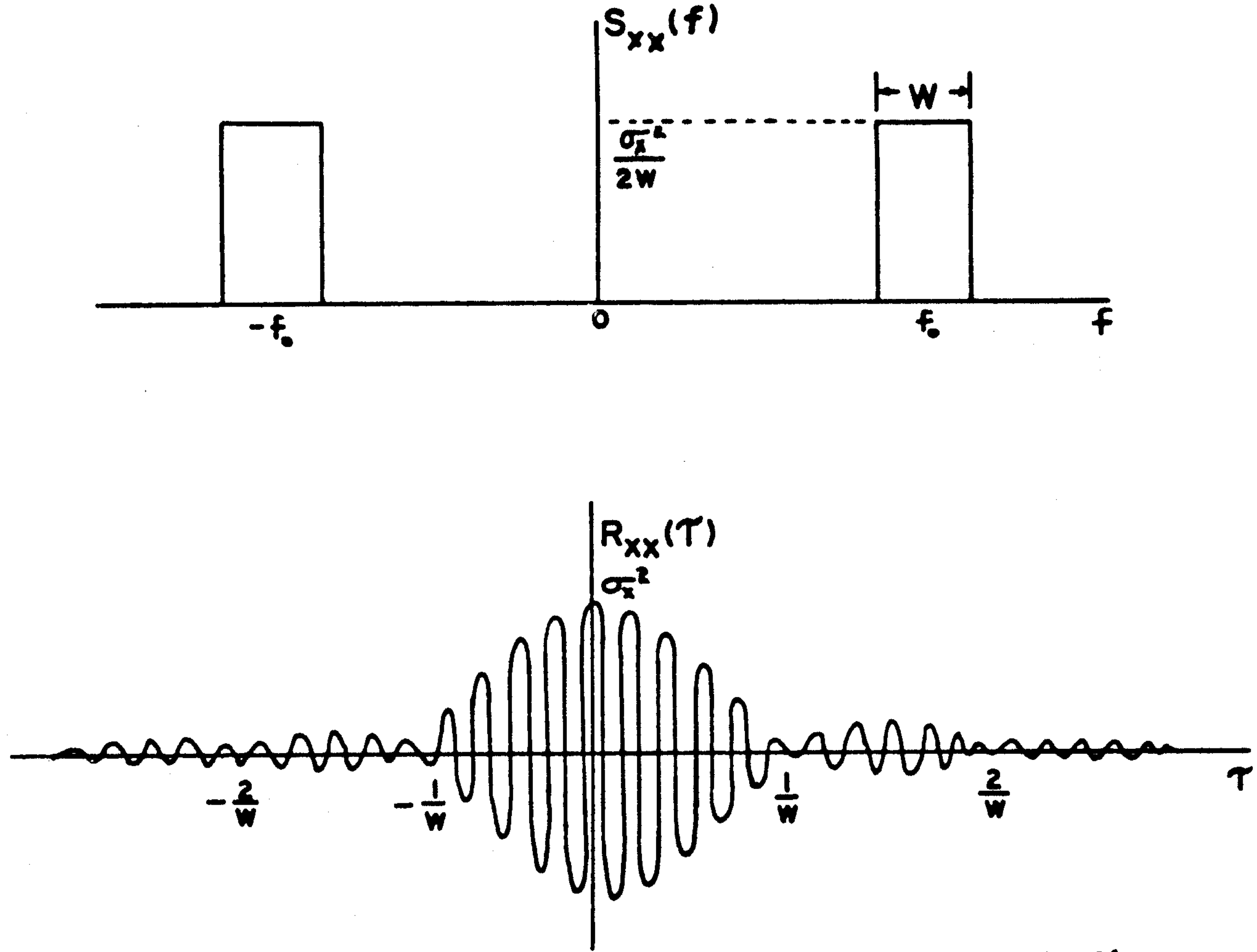
This correlation function will have a time average of zero for all $\beta_k \neq \beta_m$. However, by setting $\beta_m = \beta_k$ the time average becomes:

$$\widetilde{R}_{yz}(\tau_o) = b_m\, R_{xx}(\tau_m - \tau_o)$$

In a practical measurement $x(t)$ can be represented as bandlimited white noise in which the bandwidth, w cps, is determined by the receiving system. If the mean square value of the noise is σ_x^2, the spectral density will be $G_{xx}(f) = \sigma_x^2/2w$. If the receiver bandwidth has a center frequency of f_o, the corresponding autocorrelation function will be given by:

$$R_{xx}(\tau) = \sigma_x^2 \left[\frac{\sin \pi w\tau}{\pi w\tau}\right] \cos 2\pi f_o \tau$$

Fig. 1 shows a pictorial representation of the spectral density and the autocorrelation function of $x(t)$. For most cases of interest, the bandwidth, w, will be small compared to the center frequency, f_o. Accordingly, there will be a number of oscillations of cos $(2\pi f_o \tau)$ in the interval before the first zero of $R_{xx}(\tau)$ which occurs at $\tau = 1/w$.



$$R_{xx}(\tau) = \sigma_x^2 \frac{\mathrm{SIN}(\pi W \tau)}{\pi W \tau} \mathrm{COS}(2\pi f_0 \tau)$$

FIG. 1 SPECTRAL DENSITY AND CORRELATION FUNCTION OF BANDLIMITED NOISE

Substituting the expression for $R_{xx}(\tau)$ into the equation for $\tilde{R}_{yz}(\tau_o)$ gives:

$$\tilde{R}_{yz}(\tau_o) = b_m \sigma_x^2 \left[\frac{\sin \pi w(\tau_m - \tau_o)}{\pi w(\tau_m - \tau_o)} \right] \cos 2\pi f_o(\tau_m - \tau_o)$$

If $(\tau_m - \tau_o)$ is small compared to the reciprocal of the bandwidth -- i.e., if the correlator delay is set to correspond closely to some particular locus of constant time delay on the lunar surface -- then the $(\sin x)/x$ factor in the above equation can be approximated as unity, and the crosscorrelation function becomes:

$$\tilde{R}_{yz}(\tau_o) = b_m \sigma_x^2 \cos 2\pi f_o(\tau_m - \tau_o)$$

A similar approximation cannot be made for the cosine function, since in a practical case f_o might be 10^{10} cps and, accordingly, $(\tau_m - \tau_o)$ would have to be much smaller than 10^{-10} in order for the function $\cos 2\pi f_o(\tau_m - \tau_o)$ to be near unity. If there were an uncertainity in $(\tau_m - \tau_o)$ in the order of one part in 10^{10}, then the $\cos 2\pi f_o(\tau_m - \tau_o)$ might have any value from +1 to -1, and no correction would be possible. This difficulty can be overcome by employing a correlator which measures the envelope, $R_o(\tau_m - \tau_o)$, of the correlation function rather than the total correlation function. In this way the necessity for the precise adjustment of $\tau_m = \tau_o$ will be avoided and the correlator output will be:

$$b_m R_o(0) = b_m R_{xx}(0) = b_m \sigma_x^2.$$

<u>Correlator Design</u>

The correlator design that has evolved is an extension of the polarity coincidence type which has been used successfully by other experimenters. The extension is in several directions: a narrow sampling gate permits correlation of wideband signals using a low sampling frequency; an additional channel is included in the correlator to permit measurement of the envelope of the correlation function; a vernier differential delay between samplers is provided to allow the delay time to be controlled to a fraction of the repetition period and to be varied as a function of time.

The basic correlator block diagram is shown in Fig. 2. The operation is as follows. The input signals from the sun and moon are received in separate receivers with separate antennas. Each signal is heterodyned from a bandpass signal at the receiver center frequency down to a low pass video signal. Separate local oscillators are used with a frequency displacement corresponding to the desired delay rate; and in the case of the signal from the moon, a dual mixer is used which provides two signal channels in quadrature with each other. The signals are then passed through a wideband clipper that preserves the zero crossing information, and are periodically sampled with a very narrow pulse to determine whether the signals are positive or negative at the sampling instant. Clock frequencies for the two samplers are different by an amount corresponding to the desired delay rate. The result of each sampling operation is coded as a binary digit by the pulse generator which produces a positive pulse for a positive sample and no pulse for a negative sample. The pulse train from the sun channel is then delayed an amount τ_o and correlated with each of the two lunar channels in the polarity coincidence circuitry and counters. The normalized crosscorrelation coefficient of the clipped signals in each channel is the net change in the counter for that channel divided by the total number of counts. Combining the two correlation coefficients in this fashion gives the amplitude of the envelope of the correlation function. A detailed derivation of the mathematical relationships for the correlator has been carried out.

One of the major problems in the correlator is that of obtaining the large initial delay required to bring the direct solar signal into time synchronization with the lunar reflected signal. The magnitude of this delay is shown in Fig. 3. It was primarily for the purpose of solving this problem that a sampled system with digital processing was selected. After the incoming analog signals are clipped, sampled, and converted to binary digits,

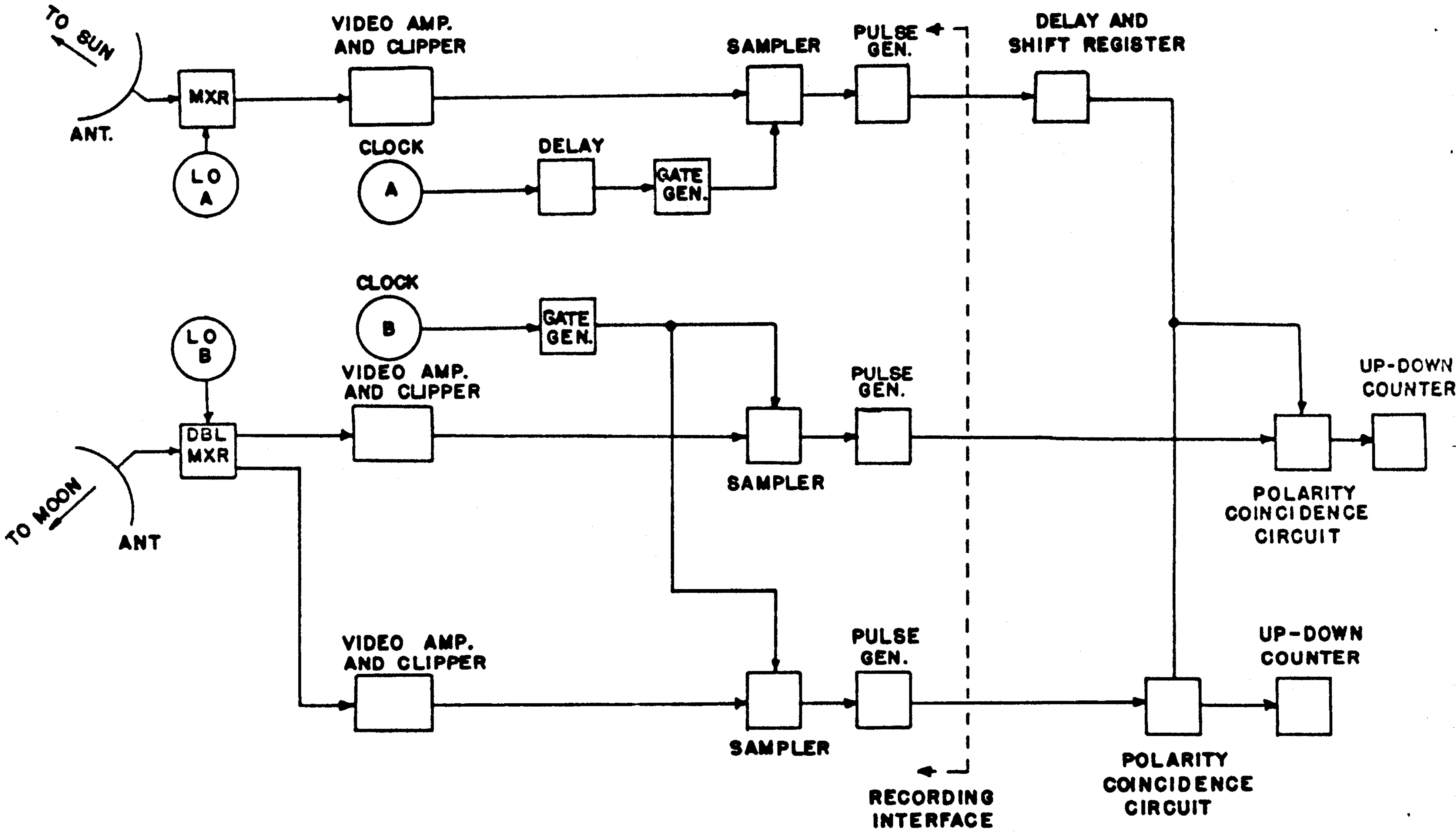


FIG. 2 BLOCK DIAGRAM OF DIGITAL CORRELATOR

FIG. 3 INITIAL DELAY OF LUNAR REFLECTED SIGNAL

the signals in the sun channel must be delayed before the correlation function can be computed. This delay can be accomplished by delay lines, shift registers, magnetic core buffers, or other straightforward methods for the digital signals. It is virtually impossible to delay the original analog signals due to the long delay times (0.5 to 2 sec.) and the large bandwidths (100 mc). In practice, it is planned to record the signals at the output of the pulse generators on magnetic tape and carry out the remainder of the processing subsequent to the actual data acquisition. Conventional digital circuitry is used in the correlator in order to simplify any interface problems with the digital delay method employed.

Performance of the correlator can be analyzed by estimating the variance in the output as a function of the input signal to noise ratio. The correlator output is an unbiased estimate of the envelope of the normalized crosscorrelation function of the input signals. This, in turn, gives an unbiased estimate of the scattering parameter b_n as follows:

$$b_n^* = \frac{\pi}{2} \frac{\sigma_N}{\sigma_x} \rho_{yxc}^*(\tau)$$

where:

b_n^* = estimate of b_n

σ_n = rms value of noise in lunar channel

σ_x = rms value of solar signal

ρ_{yxc}^* = estimate of normalized crosscorrelation function.

The variance of this estimate is found to be

$$\mathrm{Var}\,[b_m^*] = \left(\frac{\pi}{2}\right)^2 \left(\frac{\sigma_n}{\sigma_x}\right)^2 \frac{1}{N}$$

where N is the number of samples used in making the estimate. Fig. 4 shows a plot of signal-to-noise ratio improvement versus number of samples. For typical signal levels and receiver noise figures, it is expected that N will be in the order of 10^6 to give good estimates of the scattering parameters.

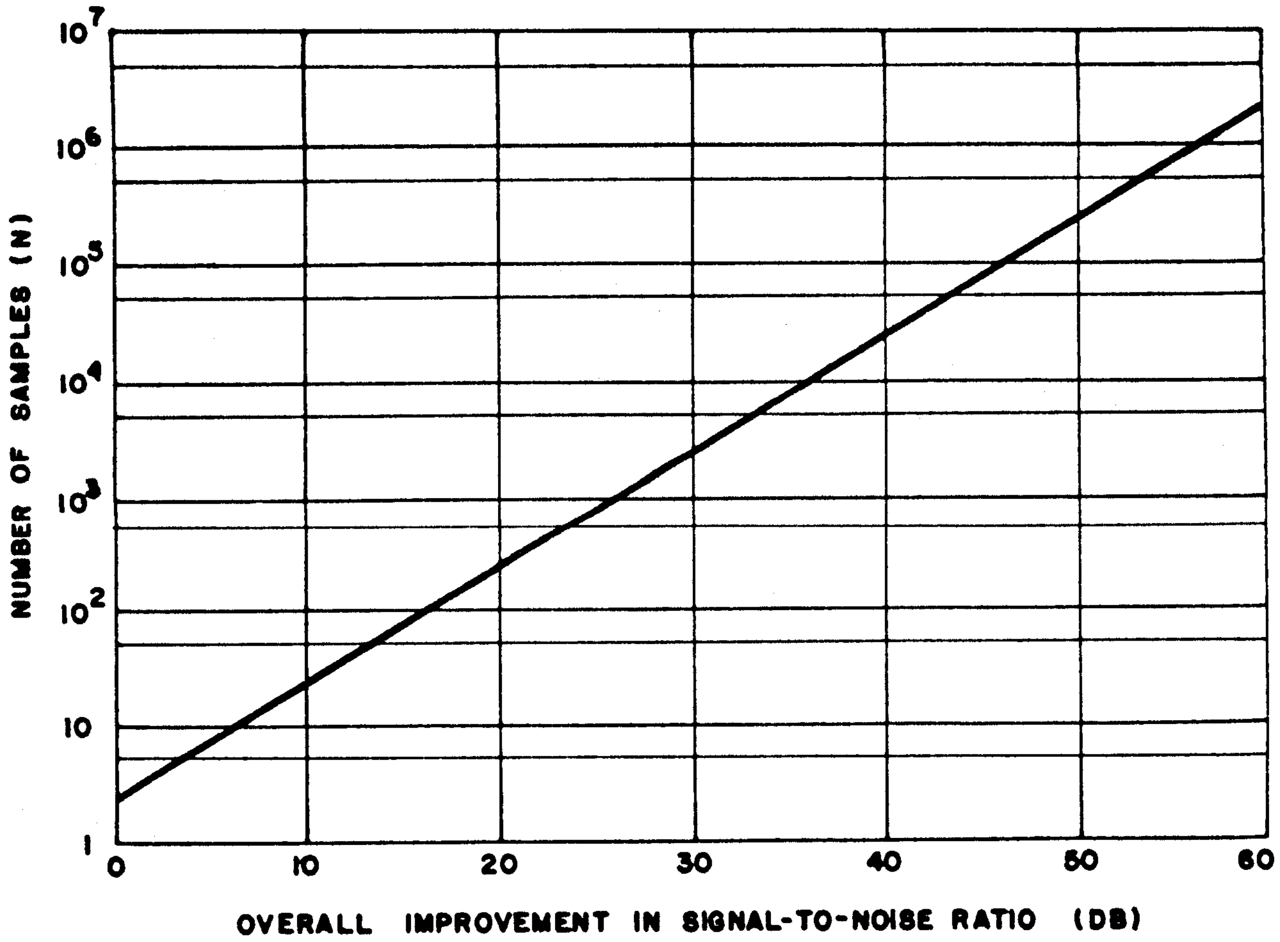


FIG. 4 NUMBER OF SAMPLES VERSUS SIGNAL-TO-NOISE RATIO IMPROVEMENT

Interpretation of Measurements

The correlator output, when multiplied by the rms values of the lunar channel signal and the sun channel signal, gives a direct measure of the signal reflected with the delay and delay rate set in the correlator. The expression for the signal is $b_k \sigma_x^2$ where b_k^2 is, for all practical purposes, the fraction of the power in the solar signal reflected with a delay τ_k and delay rate $(1 - \beta_k)$. This value of b_k^2 can be related to more familiar measures of scattering by defining a radar scattering cross-section, σ_k, and equating the received power as expressed in terms of a_k^2 and σ_k. If we let w_s be the power per unit area of the solar signal as received on earth, then the power per unit area at the moon would be essentially the same. The signal scattered from the moon and received at earth with an antenna of gain G would be

$$w_s^2 \left(\frac{\sigma_k}{4\pi D_e^2} \right) \left(\frac{G \lambda^2}{4\pi} \right)$$

where: λ = wavelength

D_e = distance from moon to earth

This power can be equated to the power expressed in terms of b_k as:

$$b_k^2 w_s^2 = w_s^2 \frac{\sigma_k}{4\pi D_e^2} \frac{G \lambda^2}{4\pi}$$

$$b_k^2 = \frac{\sigma_k G\lambda^2}{(4\pi D_e)^2}$$

The measured variation of σ_k with angle of incidence and angle of scattering will allow precise determination of the scattering law of the lunar surface. Measurements at various polarizations and frequencies will provide further data upon which to compute the dielectric constant and other parameters of the lunar surface.

Measurement of the scattering coefficient for regions away from the specular reflecting point will provide data on the roughness of the surface and should permit more definite conclusions to be drawn than are possible with the backscatter data of active radars.

The resolution of details on the lunar surface is directly related to the capability of the correlator to separate various delays and delay rates. The resolution of delays is inversely proportional to the bandwidth of the correlator and, for a bandwidth of 30 mc, permits a theoretical resolution away from the lunar equator of less than 10 meters. Near the equator the resolution is substantially less. The resolution of delay rates is proportional to the integration time and can be increased by increasing this time. As the integration time is increased, however, the stability requirements on the local oscillator and clock frequencies also increase.

Fig. 5 shows the grid produced by intersection of loci of constant delay and loci of constant delay rate. The variation in delay rate over the lunar surface is due primarily to rotation of the moon and is superimposed on the large delay rate common to all points on the surface resulting from orbital motion and rotation of the earth. The delay rate resolution and antenna beamwidth will determine how much of a particular locus will contribute to the net signal obtained after integration. The specularly reflected signal occurs at the minimum delay and accordingly corresponds to an essentially constant delay locus. Therefore this signal can be resolved independently of the delay rate resolution. It is this component that is expected to give the largest signal and which will provide the most accurate data on the effects of angle of incidence, polarization, etc.

Present Status and Future Work

The various components of the correlator have been assembled and tested with the exception of local oscillators and tape recorder. It is planned that future work will include programming the IBM 7094 computer to handle the data delay and computation of the correlation functions for the early experiments. The correlator hardware will be assembled and tested by the summer of 1965, and it is hoped to obtain experimental data during the summer and fall. Theoretical studies are continuing to investigate the nature of scattering from rough surfaces and the effects of a distributed source on the measured correlation function. It is planned that a detailed summary report will be issued to cover the technical aspects of this program in more detail.