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THE MOTION OF GASEOUS STREAMS IN THE BINARY SYSTEM

by

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ABSTRACT

We have studied several problems in connection with the gaseous flow in the close binary system. First, the statistical property of the Jacobian constants of colliding particles in the system is examined and a conversation law for their mean value established. Then the velocity-independent nature of the rate of change in angular momentum of a particle moving in the binary system is pointed out. These properties prompt us to derive for the gaseous flow a set of differential equations that provides a point of view lying in the middle between the orbital approach (which neglects both pressure and collision) and the hydrodynamic approach (which includes both) because our equations take into account the collision but not the pressure. The equations have been solved under the same approximation as Prendergast (1960) has assumed and have been found to yield a similar result as was obtained by him from the hydrodynamic equations.

Because of our emphasis on the Jacobian constant and angular momentum in the treatment of gaseous flow we have called attention to the fact that some combinations of these two physical quantities are incompatible in a certain region of space which we have called the forbidden zone.

Finally, the formation, the evolution and the significance of rotating rings observed in many Algol-type eclipsing binaries are discussed in an effort to understand the mode of ejection of matter from the secondary surface.

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I. INTRODUCTION

The problem of gaseous streams observed in some binary systems has been studied theoretically either as individual bodies moving in orbits independently of each other (Kopal 1959, Gould 1959) or as an aerodynamical flow (Prendergast 1960). Both approaches encounter difficulties, though of entirely different nature. In the present paper, we shall call the attention to a few general properties of the motion of gaseous particles in the binary systems, which lead us to a theory that somewhat reconciles these two fundamentally different approaches and thereby makes the flow problem easier to comprehend.

II. STATISTICAL PROPERTY OF THE JACOBIAN CONSTANTS DURING A COLLISION OF PARTICLES

One of the differences between the two approaches mentioned in Section I concerns the collision of particles. While the neglect of this important process makes the orbital approach unrealistic, some results obtained in celestial mechanics of the motion of an infinitesimal body in a gravitational field of two revolving components has its physical significance. This is because of the statistical property of the Jacobian constants that we will discuss in this Section.

Let us assume that the two stars are revolving around each other in circular orbits. This is generally true for close binaries (Struve 1950). Thus, a motion of a particle in such a system is identical to what is treated in the restricted three-body problem in celestial mechanics (e.g., Moulton 1914; Brouwer and Clemence 1961). Following the standard treatment of the problem we shall choose as the unit of length, the separation between the two components, as the unit of mass, the total mass of the two components, and as the unit of time, the period of the orbital motion of the two components divided by 2π. In such a unit system the gravitational constant is one. Let us now denote by μ the mass of the secondary component. Thus, the mass of the primary will be $1-\mu$. If, furthermore, a rotating coordinate system xyz is so chosen that its origin is at the center of mass of the two components, its x-axis coincides at all times with the line joining the two components and its z-axis is perpendicular to the orbital plane of the stars, then the coordinates of the primary will be $x_1 = -\mu$, $y_1 = o$, $z_1 = o$ and those of the secondary will be $x_2 = 1 - \mu$, $y_2 = o$, $z_2 = o$ in consistency with the adopted unit system.

The equations of motion of the third infinitesimal body in the restricted three-body problem admits an integral, frequently known as the Jacobian integral, as follows:

$$C = 2U - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \qquad (1)$$

where

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \qquad (2)$$

Here r_1 and r_2 are respectively the distances of the infinitesimal body from the primary $(1-\mu)$ and the secondary (μ) component, while (x,y,z) the three vector components of $\vec{r}$, are the coordinates of the infinitesimal body. The dot represents as usual the time derivative. The integration constant C, is known as the Jacobian constant.

We can transform equation (1) into a stationary system. Let us now consider a collision of n particles of mass, m_i (i = 1, 2 ...n). Since we have particles of atomic sizes, the coordinates may be regarded as the same for all colliding particles at the instant of collision. Moreover, the total kinetic energy of the colliding particles conserves during an elastic collision. It follows from these considerations as well as the definition of C by equation (1) that

$$\sum_{i=1}^{n} m_i C_i' = \sum_{i=1}^{n} m_i C_i \qquad (3)$$

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where C_i and C_i' are respectively the Jacobian constant of the i-th particle before and after the collision. Thus, if we define an average C, such that

$$\langle C \rangle \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} m_i C_i \,, \qquad (4)$$

$\langle C \rangle$ will be an invariant under the physical processes of elastic collisions. However, it may be noted that the dispersion of C's from their average value will in general change after each collision.

For inelastic collisions an equation connecting various C_i and C_i' can always be obtained from the energy consideration if we know the detailed process of the collision. We shall assume in the present paper that the collisions that take place among particles in the binary system are statistically elastic, i.e., endoergic collisions balancing exoergic ones.

As a result of the constancy of $\langle C \rangle$ during collision, the problem of gaseous flow is considerably simplified because we have now a macroscopic quantity, $\langle C \rangle$, to deal with instead of following the courses of numerous particles in the system. Thus, the gaseous particles must maintain a constant value of $\langle C \rangle$ in their stream motion. Indeed, Prendergast (1960) has shown that C is a constant along a stream line. This situation resembles the introduction of the concept of temperature and pressure which simplifies our study of the chaotic motion of molecules in gases in free space. Therefore, whatever is the nature of ejection that occurs on the stellar surface, the mean value of C's of ejected particles and their dispersion serve as two of the most characteristic indices of the mode of ejection as regards the course of their subsequent motion.

It should be noted, however, that although the gaseous particles maintain a constant $\langle C \rangle$, the mean flow does not follow the orbit derived from the equations of motion of the three-body problem. It is physically obvious that all those loops, cusps, sudden reversal in the direction of motion, and other erratic behavior found in the orbits of the three-body problems must be completely erased by collisions.

III. THE RATE OF CHANGE OF ANGULAR MOMENTUM

The angular momentum (the z-component) per unit mass of the third body with respect to the center of mass of the binary system will be denoted by h. It is given by:

$$h = x^2 + y^2 + x\frac{dy}{dt} - y\frac{dx}{dt} \qquad (5)$$

and varies with time because the third body is continuously interacting with the two revolving component stars.

Although h varies with time, there are two points which make it physically significant. First, the total angular momentum is conserved among the colliding particles if the collision takes place rapidly. Secondly, it can be easily shown from the equations of motions in the restricted three-body problem that

$$\frac{dh}{dt} = \mu(1-\mu)y\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right). \qquad (6)$$

The significance of equation (6) derives from the fact that $^{dh}/dt$ is a function of coordinates of the third body only, being independent of its velocity. Moreover, it is anti-symmetric with respect to the x-axis and to the line bisecting the separation segment between the two finite bodies. Thus, it vanishes on these two lines. Here we see physically why five Lagrangian points all lie on either of these two lines. It follows that any steady flow in a closed curve must cross either one or both of these lines so that h will recover to its original value after the completion of the circuitous flow. Indeed, this is the case of rotating gaseous rings frequently observed around the primary component in many an Algol type binary system (Joy 1942, 1947; also Sahade 1960).

In the three dimensional case, $^{dh}/dt$ vanishes in the XZ plane and in the plane bisecting the line joining the two components. Thus, if we divide space into four regions by these two planes the sign of $^{dh}/dt$ is positive in the two regions and negative in the other two.

IV. GASEOUS FLOW DERIVED FROM THE C AND h CONSIDERATIONS

In the two dimensional case, a knowledge of C and h at every point defines completely the flow pattern. Therefore, a velocity vector field of gaseous motion in a binary system can be defined by two scalar fields of h and C. Since the average values of C and h do not change by collision, we may write

$$\frac{\partial C}{\partial t} + \vec{u} \cdot \nabla C = 0 \qquad (7)$$

from the constancy of C and

$$\frac{\partial h}{\partial t} + \vec{u} \cdot \nabla h = \mu(1-\mu)\left(\frac{1}{R_1^3} - \frac{1}{R_2^3}\right) \qquad (8)$$

from equation (6) when we follow the stream lines. In writing these equations where $\vec{u}$ denotes velocity at point (x, y) we have made an additional assumption that C and h are continuous over the plane. In this way, we have derived two flow equations from the results of celestial mechanics.

By imposing the continuity condition of C and h over space we are able to take advantage of the result derived from celestial mechanics but at the same time to discard as meaningless the seemingly erratic and infinitely varied forms of orbits that one may actually obtain by a straight integration of the equations of motion in the three-body problem. Thus, equations (7) and (8) are proposed here not as a result of mathematical formality but they correspond to a physical process that involves collisions under the conservation law of C and h.

Since the pressure is not included in equations (7) and (8), the present formulation of the flow problem lies between the orbital approach (which neglects both pressure and collision) and the bona fide hydrodynamic equations of flow (which include both.) Thus, the present treatment is mathematically equivalent to the hydrodynamic approach when the pressure is neglected. In presenting the problem in this manner we gain a better physical insight because both C and h are physical quantities.

Actually an inclusion of pressure would make the problem very difficult. Indeed, Prendergast (1960) who started directly from hydrodynamic equations also neglected the pressure term when he came to the stage of solving the equations. Thus, the equations (7) and (8) should be equivalent to what Prendergast has used, although the basic approach is different.

What we will show in the following is that we can obtain a similar solution as obtained previously by Prendergast. Also by following the present derivation we can see very clearly the conditions under which the solution will be valid.

Following Prendergast, we shall consider the two-dimensional case, neglect the velocity component at right angles to the zero-velocity curves and choose a right-handed orthogonal curvilinear coordinate system (ξ, η, z) where ξ is the label of the zero-velocity curves, namely

$$U = \xi, \tag{9}$$

U is given by equation (2) and η is an angular measure along the zero-velocity curve. If we now denote Q_ξ and Q_η the metric coefficients corresponding to this coordinate system, we can express equations (7) and (8) in this new system as follows:

$$\frac{u_\xi}{Q_\xi}\left[1-\left(u_\xi\frac{\partial u_\xi}{\partial \xi}+u_\eta\frac{\partial u_\eta}{\partial \xi}\right)\right]-\frac{u_\eta}{Q_\eta}\left(u_\xi\frac{\partial u_\xi}{\partial \eta}+u_\eta\frac{\partial u_\eta}{\partial \eta}\right)=0 \tag{10}$$

and

$$\begin{aligned}&\frac{u_\xi}{Q_\xi}\left(\frac{\partial r^2}{\partial \xi}+r_\xi\frac{\partial u_\eta}{\partial \xi}+\frac{\partial r_\xi}{\partial \xi}u_\eta-r_\eta\frac{\partial u_\xi}{\partial \xi}-\frac{\partial r_\eta}{\partial \xi}u_\xi\right)\\&+\frac{u_\eta}{Q_\eta}\left(\frac{\partial r^2}{\partial \eta}+r_\xi\frac{\partial u_\eta}{\partial \eta}+\frac{\partial r_\xi}{\partial \eta}u_\eta-r_\eta\frac{\partial u_\xi}{\partial \eta}-\frac{\partial r_\eta}{\partial \eta}u_\xi\right)\\&=\mu(1-\mu)y\left(\frac{1}{r_1^3}-\frac{1}{r_2^3}\right)\end{aligned} \tag{11}$$

if we assume a steady state of flow. Here the subscripts ξ and η denote respectively the components of the vector in the ξ and η direction.

If we neglect u_ξ, we obtain

$$\frac{\partial u_\eta}{\partial \eta} = 0 \tag{12}$$

from equation (10) and

$$\frac{u_\eta}{Q_\eta}\frac{\partial r^2}{\partial \eta} + \frac{u_\eta}{Q_\eta}\left(r_\xi \frac{\partial u_\eta}{\partial \eta} + \frac{\partial r_\xi}{\partial \eta} u_\eta\right) = \mu(1-\mu)\, y\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right) \tag{13}$$

from equation (11). Combining equations (12) and (13) we obtain a second degree algebraic equation for u_η

$$\frac{u_\eta}{Q_\eta}\frac{\partial r^2}{\partial \eta} + \frac{u_\eta^2}{Q_\eta}\frac{\partial r_\xi}{\partial \eta} = \mu(1-\mu)\, y\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right) \tag{14}$$

By neglecting u_ξ in one of the two hydrodynamic equations of flow and u_ξ^2 in the other, Prendergast has also obtained a second degree algebraic equation for u_η. While the coefficients in the equation here (which involves $\partial r^2/\partial\eta$ & $\partial r_\xi/\partial \eta$) are completely different from those in Prendergast's equation, (which involves $\partial Q_\eta/\partial \xi$), both results are valid under the same approximation. Now it appears from equation (10) that in order to neglect u_ξ, $\partial u_\eta/\partial\eta$ must be small. This is the condition for the validity of our solution.

Physically we have started by assuming that the flow follows the zero-velocity curves, but once we have found u_η, according to equation (14) we immediately see that u_ξ does not vanish because $\partial u_\eta/\partial\eta$ does not. So the flow cannot exactly follow the zero-velocity curves. Whether we can find a converging field of velocities for this steady state by an iterating process, we do not know. Neither has Prendergast commented on this possibility. ~~Intuitively it appears that it cannot be done because of the basic conflict between equations (12) and (14). It should be noted that this conflict arises not only in the flow forced to follow zero-velocity curves but in any other flow whose stream lines may be given by $f(x, y, \xi) = 0$.~~

However, near the two finite bodies, the variations of velocity with η is small, so the flow pattern obtained here represents a good approximation. This explains why gaseous rings are frequently observed around the primary component of many an Algol-type eclipsing binary.

In order to complete the analogy of the present calculation with Prendergast's, we may write equation (10) by neglecting u_ξ^2 and higher order terms, as follows:

$$u_\xi = \left(\frac{Q_\xi}{Q_\eta}\right) u_\eta^2 \frac{\partial u_\eta}{\partial \eta} \left(1 - u_\eta \frac{\partial u_\eta}{\partial \xi}\right)^{-1}$$

We can now examine the asymptotic behavior of u_η for small values of r_1 (or similarly of r_2). In the immediate neighborhood of the $1-\mu$ component, we may take the position of this component as the origin and use the polar coordinate system (r_1, φ) where φ is the angle that the radius vector $\vec{r}_1$ makes with the x-axis, being counted, as usual, positive in the counter-clockwise direction from the positive x-axis. The zero-velocity curves in the immediate neighborhood can be approximated by circles. Thus, we have

$$\xi = U \rightarrow \frac{1-\mu}{r_1}, \quad Q_\eta = r_1 \tag{15}$$

and

$$\eta = -\varphi \tag{16}$$

since (ξ, η, z) is a right-handed coordinate system.

It can be easily shown that equation (14) reduces to

$$u_\eta^2 - 2r_1 u_\eta - \frac{1-\mu}{r_1} = 0, \tag{17}$$

under the approximation given by equations (15) and (16). We take the negative sign before the square root in the solution of this quadratic equation, because as Prendergast has pointed out, the velocity should vanish if the force vanishes. Retaining the dominant terms, we find the solution for small r_1

$$u_\eta = r_1 - \left(\frac{1-\mu}{r_1}\right)^{1/2} . \tag{18}$$

This asymptotic expression represents the Keplerian velocity in the neighborhood of the $1-\mu$ component in the rotating coordinate system.

In order to study the asymptotic behavior at large distances from both the components, we use the polar coordinates (r, θ) and expand all quantities in terms of $1/r$, θ being now the angle between the radius vector $\vec{r}$ and the x-axis. It can be shown that the angle that the normal to the zero-velocity curve makes with the radius vector $\vec{r}$ decreases as $1/r^5$. Consequently, we may take the radius vector as the normal to the curve as a first approximation for zero-velocity curves at large distances. Similarly, we can show that the radius of curvature may be set equal to r if we neglect terms of $1/r^5$ and higher orders. With these approximations we can easily derive from equation (14)

$$u_\eta = -r, \tag{19}$$

if we remember that in this asymptotic case

$$\eta = \theta . \tag{20}$$

According to equation (19) the gas remains at rest in the stationary frame of reference. Thus, the asymptotic behavior in both cases is, as it should be, identical to what Prendergast has obtained.

The numerical evaluation of u_η is not simple but it can be done. We shall illustrate it by computing values of u_η at those two points in the flow around the $1\text{-}\mu$ component where a stream line intersects with the x-axis. At these two points we can take advantage of the simplification arising from the symmetry of the zero-velocity curves with respect to the x-axis.

Let point A(x_0,0) be the one of these points on the right side of the 1-μ component and point B(x_0,0) the one on the other side. Thus, $x_0 > -\mu$ at A and $x_0 < -\mu$ at B. The relation between x_0, r_1 and r_2 at both A and B can be easily obtained.

Since Q_η denotes the radius of curvature of the zero-velocity curve at A or B then we have in the immediate neighborhood of A or B

$$r^2 = Q_\eta^2 + (x_0 \mp Q_\eta)^2 + 2Q_\eta (x_0 \mp Q_\eta) \cos\varphi \tag{21}$$

and

$$r_\xi = -Q_\eta - (x_0 \mp Q_\eta) \cos\varphi , \tag{22}$$

where φ represents the angle which the normal to the zero-velocity curve at any point near A or B makes with the positive x-axis. When two signs appear together, the upper one corresponds to point A and its neighborhood and the lower one points to B and its neighborhood. Again $\eta = -\varphi$ as in the first case of asymptotic expansions. With the aid of equations (21) and (22) we may reduce equation (14) to

$$u_\eta^2 - 2Q_\eta u_\eta + k Q_\eta^2 = 0 \tag{23}$$

where

$$k = \frac{\mu(1-\mu)}{x_0 \mp Q_\eta}\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right) \tag{24}$$

for point A and B respectively according to whether we take the upper or lower sign in equation (24). The solution of equation (23) is

$$u_\eta = Q_\eta \left[1 - (1-k)^{1/2}\right], \tag{25}$$

the minus sign before the parenthesis has been chosen in order to agree with the asymptotic behavior found previously in equation (18). The radius of curvature Q_η can be derived from equation (9). We shall omit its long expression here.

We have computed u_η according to equation (25) for several cases of x_0 with $\mu=0.2$. It appears evident from the results of computations that velocities thus obtained are very near to those found in the periodic orbits which we may obtain either by the numerical process of successive approximation (Huang and Wade 1963) or by the series solution (Huang 1964). In the second and third column of Table 1 we have given a few velocities of a particle as it crosses the x-axis obtained by equation (25) and from the periodic solutions respectively. Needless to say, the disagreement in sign between $\dot{y}$ and u_η in one half of the cases in Table 1 arises purely from the difference in the coordinate system. We should compare only the magnitudes between the second and third column. As would be expected, the agreement between two kinds of computations becomes better and better as we approach more and more to the star. Thus, the gaseous rings found observationally in many binary systems may be regarded equivalently either as a hydrodynamic flow or as motions of particles in a continuous series of periodic orbits that exist around the component.

V. THE FORBIDDEN ZONE

Since our approach to the problem of gaseous flow in the binary system emphasizes the two physical quantities C and h, it is interesting to point out that at some points in space, a certain combination of values for these two quantities is incompatible. In other words, with a given value of C and h, sometimes the particle cannot go into a certain region of space which we shall call the forbidden zone. It can be easily seen as follows. We may express equations (1) and (5) simply as

$$\dot{x}^2+\dot{y}^2+\dot{z}^2=A, \tag{26}$$

and

$$x\dot{y}-y\dot{x}=B, \tag{27}$$

where A and B are functions of x, y, z, C and h and can be easily found from equations (1) and (5). Now in the (x, y, z) velocity space, equation (26) represents a sphere with the center at the origin and with a radius equal to $A^{1/2}$, while equation (27) represents a plane. It is then obvious that for any given combination of A, B, x, y, z (or equivalently C, h, x, y, z) the two surfaces may or may not intersect with each other. If they do not, no real velocity components (x, y, z) will satisfy both equations. This means that the particle with the given values of C and h cannot reach the point (x, y, z). In other words, the point (x, y, z) lies in the forbidden zone associated with the given C and h values.

The firbodden zone can be easily calculated from the condition that the distance of the origin from the plane given by equation (27) in the velocity place is greater than the radius, $A^{1/2}$ of equation (26). Explicitly, the forbidden zone is given by

$$B^2 > A(x^2 + y^2) \qquad (28)$$

We shall illustrate the forbidden zone only in the x, y plane. It can obtained by plotting the curve defined by the following equation

$$f(r,\theta) \equiv r^2 + r\left[r^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - C\right] - h = 0, \qquad (29)$$

where θ which enters into the expression for r_1 and r_2 denotes the angle between the radius vector $\vec{r}$ of the third body and the positive x-axis.

Although h is a physical quantity, it is not a constant of motion in the restricted three-body problem. Consequently, the forbidden zone is not as important as the zero-velocity curves in depicting the motion of particles. However, combined with the property of $^{dh}/_{dt}$ discussed in the previous section, the forbidden zone may serve some useful purpose of excluding certain modes of gaseous flow in the binary system.

Figure 1 illustrates three forbidden zones in the x, y plane for C = 3.5 and for three values of h. The areas that include the origin are forbidden to particles having the assumed C and h values. Because of the symmetry with respect to the x-axis, only one half of the zone is drawn in each case. However, the signs of $^{dh}/_{dt}$ are marked in the figure in all four quadrants at the corners.

VI. SOME REMARKS CONCERNING THE ROTATING GASEOUS RINGS OBSERVED IN THE BINARY SYSTEM

1. The Chance of Ring Formation

Let us denote C_1 and C_2 as the value of C that corresponds respectively to the innermost and outermost contact surface (Kuiper, 1941). The latter will be hereafter called, for the sake of brevity, the S_1 and S_2 surface. Both C_1 and C_2 have been computed by Kopal (e.g. 1959) and Kuiper and Johnson (1956).

According to the result obtained in the restricted three-body problem, those ejected particles whose C values are greater than C_1 cannot penetrate the S_1 surface, and those whose C values are greater than C_2 cannot penetrate the S_2 surface. It follows that those particles whose C values are less than C_2 could escape from the system and those whose C values are larger than C_1 will remain inside the S_1 surface.

The quantity,

$$\Delta C = C_1 - C_2 , \qquad (30)$$

measures the closeness of the two critical zero-velocity surfaces S_1 and S_2 and may have an important effect on the flow of matter ejected by the secondary component into the primary lobe of the S_1 surface. We do not mean that only those particles with C values between C_1 and C_2 will penetrate into this volume, since any particle with $C < C_1$ can move into it. But the amount of accumulation of matter inside this volume at any given time perhaps increases with the increase of ΔC. It follows from this reasoning that formation of gaseous rings around the primary $(1-\mu)$ component favors large values of μ, as we can easily see, for example, from Kuiper and Johnson's Table, that ΔC increases with μ.

On the other hand, we have pointed out (Huang and Struve, 1956) that from the consideration of available space for their ring formation around the primary component, gaseous rings have a better chance to exist in binaries of small μ. From the two arguments we may conclude that perhaps formation of gaseous rings has its highest chance in binaries with μ neither near the maximum end of 0.5 nor near the minimum end of approaching zero. Observationally gaseous rings have been found in binaries with μ around 0.2. While this result agrees with the prediction from the previous simple arguments, it may also be caused by the effect of observational selection, since it is extremely difficult to measure μ when it is much less than 0.1.

In passing, it may be noted that following the argrument of available space we have predicted a few eclipsing binaries in which gaseous ring may be expected but not yet observationally detected (Huang and Struve, 1956). Among these predicted stars is β Per. (Algol) whose emission feature was later discovered by Struve and Sahade, 1957). While they have concluded that the emission feature does not indicate a ring structure, it nevertheless reveals an accumulation of gases in the system. And the accumulation of gases is a necessary condition for the ring formation.

2. Ring Formation, Mass Dissipation and Ejection Velocities

It is evident from observations that the gaseous particles flowing in the binary system come from the component stars themselves (e.g. Wood 1950; Kopal 1959; Sahade 1960). In fact, it is usually the less massive component that is losing mass. Accordingly, we will assume the injection of particles into the system by the μ component.

Let $\vec{V}$ be the velocity of ejection with respect to that point of the stellar surface from which the particle is ejected. If the secondary component is rotating axially as a rigid body with an angular velocity $\vec{\omega}$ with respect to a stationary frame of reference, any particle that is attached to the surface rotates with a velocity $\vec{\omega} \times \vec{R_2}$ where $\vec{R_2}$ is the radius vector of a point on the secondary surface from its center. Since the center of the secondary revolves with a unit angular velocity $\vec{k}$ in the z-direction in its orbit, the ejection velocity in the xyz coordinate system is given by

$$\frac{d\vec{r}}{dt} = \vec{V} + (\vec{\omega} - \vec{k}) \times \vec{R_2}. \qquad (31)$$

In order to compute the C values of ejected particles, we can take advantage of the fact that the surface of the secondary coincides with the secondary lobe of the S_1 surface. Thus, it follows from equation (1) that

$$C = C_1 - \left(\frac{d\vec{r}}{dt}\right)^2 \qquad (32)$$

If axial rotation and orbital revolution of the secondary (μ) component are synchronized,

$$C = C_1 - V^2.$$

While the particles ejected from the secondary component has values always less than C_1 according to equation (32), the C values corresponding to those periodic orbits close to the $1-\mu$ component are greater than C_1. It becomes evident that before the ejected particles accumulate to form gaseous rings close to the primary component,

they must have collided one another many times such that C's of some particles have been increased to the necessary values to make the ring formation possible. Because of the conservation law given by equation (3) we may expect that C's of other particles must have been reduced as a result of collisions. Since particles of small C correspond to high velocities, they will easily escape from the system. It can, therefore, be concluded that the formation of gaseous rings of small radii around the primary component must be accompanied by dissipation of mass from the system.

If V should be very large, it would be doubtful whether the velocities of an appreciable amount of particles can be reduced by collisions to make ring formation possible. Therefore, we would suggest that the ejection velocities from the secondary are in general, small if gaseous rings are observed around the primary component. That is why we have classified ejection leading to the ring formation as a slow mode (Huang 1963).

3. Evolution and Physical Significance of Rotating Gaseous Rings

The gaseous rings around the primary component formed by the material from the secondary component cannot be permanent. Because of the tidal friction, the rapidly rotating ring will gradually lose the angular momentum to its original source of orbital motion. As the angular momentum of rotating particles decreases, they fall into the primary. Therefore, without other disturbances, the rotating rings represent only an intermediate step in the transfer of mass from the secondary to the primary component. If the ring will be dissipated easily, its presence can only indicate an active secondary at the epoch of observation.

We know solar surface activities because they provide us with a disk to observe. In the case of stars, little can be learned about their surface conditions because they appear to us as point sources, although eclipsing binaries have revealed some of the secrets of the stellar surface. Now from the observable behavior of gaseous rings we can derive, according to the present idea of ring formation, the mode of ejection of mass from stellar surface. Thus, if the ring can maintain its existence only when matter is continually supplied to it by the secondary component, its fluctuations in intensity or even its disappearance and re-emergence, which have been actually observed (Wyse 1934; Joy 1947; McNamara 1951) can only reflect the manner in which matter from the secondary is ejected.

Rings may disappear when the secondary ceases to eject matter. In this case, their disappearance would be gradual. Rings may also disappear when the secondary suddenly ejects a large number of particles of high velocities. The latter simply sweeps all rotating particles off their orbit. In this case, the disappearance of rings would most likely occur suddenly. Perhaps the fluctuation of light intensity of gaseous rings and sometimes their total disappearance actually observed are due to the second cause.

In any case, from what has been observed of the gaseous rings the ejection of matter from the stellar surface does not resemble a continuous steady process such as the evaporation from a liquid surface. If there is ever a steady background flow out of the secondary, it is superimposed by intermittent bursts like the prominence activities on the solar surface. Thus, by observing the variation in intensity and structure of the emission lines that are produced by the rotating rings we will be able to learn something about the manner of how a component star loses its mass when its evolutionary stage of expansion brings it to touch the S_1 surface. Since such an empirical knowledge is unlikely to be found elsewhere, the importance of observing gaseous emission in binary systems cannot be exaggerated.

There remains the question whether a rotating ring or disk can be formed around the secondary (less massive) component when the primary component is losing mass. It is obvious that the ring would be less stable around the secondary than around the primary because of a larger perturbation in the first case. Also the available volume for ring formation is smaller in the first than the second case. But there is no a priori reason to believe that rings cannot be formed around the secondary component. However, observationally we have never found a gaseous ring around the secondary component.

Actually we have found few binaries whose more massive component has filled the primary lobe of the S_1 surface while those of less massive components remain small compared with the secondary lobe of the S_1 surface. Therefore, the impending question is not why we have not found any gaseous ring around the less massive component, but rather why systems whose more massive component injects particles into the secondary lobe should be so rare. Presumably, some selection effect plays a role here, but it is unlikely that this is the sole cause.

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LEGEND

Figure 1 - The Forbidden Zone - Particles with given C and h at any instant cannot be found in a certain region of space, called the forbidden zone. Three forbidden zones in the xy plane are illustrated here for three pairs of (C, h). They are (3.5, 1), (3.5, 0.6) and (3.5, 0.3). Because of symmetry with respect to the x-axis, only one half of each zone is shown here. The forbidden zone corresponding to (3.5, 1) lies between two curves, while that corresponding to each of the other two pairs of (C, h) lies inside a single closed curve. The sign of $^{dh}/_{dt}$ is marked at each corner of the four quanrants formed by x = o and y = 0.5.

TABLE 1

A Comparison of Velocities at Points on the x-axis Obtained from Equation (14) and from Periodic Solutions

x_0	u_η From Eq.(14)	$\dot{y}$ From Periodic Solutions
.054472	-1.492	+1.525
-.450000	-1.529	-1.556
.001841	-1.775	+1.791
-.400000	-1.794	-1.809
-.049380	-2.148	+2.154
-.350000	-2.156	-2.164
-.099858	-2.724	+2.726
-.300000	-2.727	-2.730
-.149986	-3.949	+3.949
-.250000	-3.950	-3.950