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An Evaluation of the Main

Geomagnetic Field 1940 - 1962

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December 1964

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Figure Captions

Fig. 1 Location of Magnetic Survey Observations available 1955-1962. Only one point plotted in each 0.5° x 0.5° latitude - longitude block.

Fig. 2 Location of data selected 1940-1962 for field analysis. Shaded areas designate those for which some data are available for the years 1900-1939.

Fig. 3 (a-d) Contours of the geomagnetic field in gauss (F, H, Z) or degrees (I), synthesized from the coefficients in Table 4 for epoch 1965.0.

(a) F (Total Intensity). All centers are 'Highs' except the South American 'Low' of 0.238 Γ.

(b) H (Horizontal Intensity). The + signs are the dip pole positions (north: 75.6° N, 101°W; south: 66.3° S, 141° E). The two centers near the equator are 'High', the center near the southern tip of Africa is a 'Low'.

(c) I (Inclination)

(d) Z (Vertical Intensity)

Fig. 4 (a-b) Contours of the secular change of the geomagnetic field in gamma/year synthesized from the coefficients given in Table 4.

(a) $\dot{F}$ (Total Intensity) secular change

(b) $\dot{Z}$ (Vertical Intensity) secular change

Fig. 5 Magnetic survey tracks selected for comparison with computed fields. Shown are data positions from the U. S. aircraft project MAGNET, the U. S. ships <u>Rehoboth</u> and <u>Vema</u>, the Japanese ship <u>Soya</u>, and the Russian ship <u>Zarya</u>.

Fig. 6 (a-d) Plots of measured minus computed values of the magnetic field components taken over the tracks illustrated in Fig. 5. The ordinate scale is marked in 400γ increments. Model fields used are A from Table 4 and LME as referenced in Table 7. (no separate captions)

Fig. 7 (a-c) Comparison between observatory annual means (dots) and field components computed using coefficients A (solid lines) and LME (dashed lines).

(a) Sodankyla, Finland: 67° 22.0' N, 26° 39.0' E (1914-1944); 67° 22.1' N, 26° 37.8' E (1946-1961).

(b) San Juan, Puerto Rico: 18° 22.9' N, 66° 7.1' W (1926-1963)

(c) Alibag, India: 18° 38.3' N, 72° 52.3' E (1904-1961).

Abstract

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A new determination is made of the geomagnetic field and its secular change using a set of 21695 selected survey and observatory annual mean data available for the interval 1940-1963. The field is given by a series of 63 spherical harmonics (g_n^m and h_n^m to n=m=7) and 35 time derivatives ($\dot{g}_n^m$ and $\dot{h}_n^m$ to n=m=5). Although it is inferred from these results that a better fit should be possible with existing data if more coefficients are used, the improvement is sufficiently substantial to recommend that it be used as a replacement for the previously derived coefficients (Jensen and Cain, 1962). One refinement in the present determination includes the use of the oblate earth in place of the spherical approximation. It is illustrated that this modification is necessary to obtain meaningful extrapolations above the earth's surface.

The standard errors and confidence levels are given for the coefficients to show that almost all are well determined. The rms deviations of this fit from several selections of survey data are used in comparison with similar results from other coefficient sets to show that this set of coefficients is considerably more accurate than any published previously and is comparable to a recent determination by Leaton et al (1964).

Introduction: The history of the determination of the main geomagnetic field goes back over a century with excellent bibliographies to be found in such works as those by Chapman and Bartels (1940a), Vestine (1947), Mauersberger (1952), Fanselau (1956), and Kautzleben (1963). Some of the more recent publications on the subject include those by Vestine (1960), Euler (1963), Fougere (1963a), Heppner (1963), Leaton (1963), Vestine et al (1963a,b).

In this long history the techniques of evaluation of the main field have generally included first determining magnetic charts from the data and then performing a spherical harmonic analysis on equi-spaced grid points from the charts. The basic evaluation of the data has thus occurred in the map-making phase of the processes, rather than in the spherical harmonic analysis itself. These maps have generally been hand-contoured from a collection of magnetic survey data that have been extrapolated to the epoch of the map by attempting to interpolate the secular change in the field using the set of data from magnetic observatories and repeat stations. Although the data analysis is made very difficult by the poor distribution over the earth of the total set of survey and secular change data, the accuracy of the final results has also been limited by the errors introduced in producing the charts.

It was with the idea of instead determining a fit directly to the survey data themselves that Jensen and Cain (1962) made a computation of the main field using a selection from 74000 H and F data available for the period 1940-1960. Although the resulting set of spherical harmonics was an improvement over the previously available coefficients (cf. Leonard, 1963) for the field near its epoch of 1960, there were still serious discrepancies between the computed field and measurements made near the earth's surface.

Using similar techniques of curve fitting, a set of spherical harmonics was also computed (Cain, et al, 1962) from the Vanguard 3 magnetic data. As pointed out by Cain and Hendricks (1964) the mere fitting of magnetic field data covering a given volume with a set of spherical harmonic coefficients does not necessarily imply that the main field is correctly evaluated elsewhere.

Assuming that one wishes to compute a set of spherical harmonic coefficients representing the main field directly from the total set of available data there are several approaches that can be taken. The analysis can either include the determination of the coefficients of secular change along with the spatial terms, or instead, the data can be adjusted to a particular epoch by a separate analysis of secular change and then the spatial analysis made on the adjusted data. The only data of sufficient quality to attempt a determination of secular change are the annual mean values from the set of about 100 magnetic observatories. However, since these observatories are mainly confined to the major land masses and are predominently in the northern hemisphere, a determination of secular change from them alone cannot be representative of secular change over the whole earth. The quality of repeat station data is sufficiently low, particularly in remote areas, that it is unlikely that their inclusion would aid substantially in improving this determination. For this reason the approach followed here has been to perform the analysis on uncorrected data making the simultaneous determination of the time derivatives of the spherical harmonic coefficients along with the spatial terms themselves. In this way both the observatory and survey data contribute to the determination of secular change.

A second basic decision to be made in the determination of the main field is whether to use X, Y, Z as derived from the original observations of such components as D, H, I and F or to use the observations themselves. This is a difficult choice since the use of X, Y, Z data requires only the solution to linear equations whereas the use of angular data D and I or combined component data H and F requires that a non-linear analysis must be performed. However, with the present distribution of data where the observations in some of the remote regions are only of F it is clear that it is not always feasible to compute the vectors X, Y and Z. Also, the only presently available satellite data is F (Vanguard 3) which should be used since they are the only data which have an appreciable distribution in altitude. A further problem with the use of X, Y and Z data is that not only are the errors more difficult to estimate, but also they can be systematically enhanced over different parts of the globe. For example, Z may be obtained from observations of the horizontal intensity H and the inclination I, from $Z=H \tan I$. The inequality $|\delta Z| \leq |\delta H \tan I| + |H \sec^2 I \delta I|$ shows that the errors in the computed Z are commensurate with those of

I in low latitudes and reflect those of both H and I in middle latitudes. However, writing the second term as $|(F\delta I)/\cos I|$ it is apparent that the errors in Z reach large proportions in high latitudes as $\cos I \to 0$.

Thus the decision was made here to use only observed components in the analysis and also to attempt weighting them according to their estimated accuracy. This choice implies that if all three components are measured, they are treated as three separate observations and included independently into the fit. The weighted sums of squares of the quantities (measured minus computed) ΔX, ΔY, ΔZ, ΔH, and ΔF are minimized. The angular quantities ΔD and ΔI are taken into account by converting to force units using the additional weighting factors H and F (computed) respectively.

A third decision in the analysis of the main field is whether to add the refinement of including the earth's oblateness. As we will illustrate subsequently there are compelling reasons to include the earth's shape if one expects to realistically extrapolate the result above the surface. We thus chose to make this deviation from past practice even though the resulting coefficients will not be directly comparable with those determined previously. There will be a slight additional complication in their use since one must take the earth's shape into account for synthesis of the field even at the earth's surface.

2. Method

We here follow the usual spherical harmonic potential expansion (cf. Chapman and Bartels, 1940 p. 639)

$$V = a \sum_{n=1}^{\infty} \{ (\tfrac{r}{a})^n T_n^e + (\tfrac{a}{r})^{n+1} T_n^i \}$$

where $T_n = \sum_{m=0}^{n} (g_n^m \cos m\varphi + h_n^m \sin m\varphi) P_n^m (\theta)$,

r, θ, φ = spherical coordinates corresponding to geocentric radius, colatitude, and longitude respectively, g_n^m, h_n^m = Gauss (Schmidt normalized) coefficients ($g_1^o < 0$), and $P_n^m (\theta)$ = Schmidt's quasi-normalized polynomials. The field can be derived by a straightforward evaluation of $\overline{F} = - \nabla V$. Although the T_n^e terms are included here in the expression for V, they were neglected in the determination of the coefficients of the internal field. This omission was deliberate for a number of practical and theoretical reasons. First, it is now estimated that the true value of such external terms could only contribute at the level of a few tens of gammas at most from either the ring current (Akasofu, Cain and Chapman, 1962; Hoffman and Bracken, 1964) or the cavity field (Mead, 1964, Midgley, 1964) and would thus be very difficult to determine in the anomaly field 'noise' in the data. Secondly, as will be shown subsequently, the computer core size was already exceeded by the number of significant coefficients required to fit the internal field.

Now let one component of the field (X, Y, Z, D, I, H or F) be represented by the functional form:

$$C = C(g, r, \theta, \varphi, t)$$

where g is now the set of all coefficients (e.g. g_n^m, h_n^m, $\dot{g}_n^m$...) and r, θ, φ and t are the coordinates of the observation point. If there is given a set of observations C_i we then attempt to find a set of g's to make the sum of squares of the difference between the observed C_i and computed C (at the coordinates of the observation) a minimum. The quantity to be minimized χ^2 is thus:

$$\chi^2 = \sum_i [C_i - C(g, r_i, \theta_i, \varphi_i, t_i]^2$$

where the summation on i is taken over all component observations. Following the usual least squares method we differentiate in turn by each of the g's and solve the resulting expressions simultaneously

$$\frac{\partial\chi^2}{\partial g_k} = 0 = -2\sum_i [C_i - C] \left(\frac{\partial C}{\partial g_k}\right) \qquad k = 1, 2, \ldots n$$

However, if the function C is obtained from the spherical harmonic expansion of the geomagnetic potential function, the above set of equations is non-linear in the g's, and direct methods of solution are unknown if not impossible. A method of successive approximation was formulated to avoid this problem.

Assuming that some approximate values of the g's are available, the expression for C can be expanded into a Taylor's series to first order:

$$C = C(g, r, \theta, \varphi, t) \simeq C_o (g, r, \theta, \varphi\ t) + \sum_{k=1}^{n} \left(\frac{\partial C}{\partial g_k}\right)_o \delta g_k$$

where C_o and $\left(\frac{\partial C}{\partial g_k}\right)_o$ are the function C and its derivative with respect to g_k evaluated using the approximate values of the g's, and δg_k is the correction to be applied to the g_k parameter. If the least squares procedure is now followed, the resulting set of equations is linear in the δg_k's and can be readily solved.

Before writing the normal equations we first modify the expression for χ^2 to include the weights for different types of observations and different components as follows:

$$\chi^2 = \sum_i [C_i - C(g, r_i, \theta_i, \varphi_i, t_i)]^2 \quad w_i$$

where the weights are applied to each component observation C_i as $w=1/\sigma$ and σ is the estimated accuracy (in gammas) for that class of observation. For the components D and I these estimated accuracies (in radians) are multiplied by (the computed) H and F respectively.

On substituting for C,

$$\chi^2 = \sum_i [C_i - C_o - \sum_{k=1}^{n} \left(\frac{\partial C}{\partial g_k}\right) \delta g_k]^2 \quad w_i$$

The least squares procedure is to minimize with respect to δg_k. Differentiating with respect to δg_k and setting the partial derivatives equal to zero gives

$$\frac{\partial \chi^2}{\partial(\delta g_k)} = 0 = -2 \sum_i [C_i - C_o - \sum_{j=1}^{n} \left(\frac{\partial C}{\partial g_j}\right)_o \delta g_j] \left(\frac{\partial C}{\partial g_k}\right)_o w_i$$

for all k.

The resulting kth normal equation is then

$$\sum_{j=1}^{n} \delta g_j \sum_i w_i \left(\frac{\partial C}{\partial g_j}\right)_o \left(\frac{\partial C}{\partial g_k}\right)_o = \sum_i w_i \; (C_i - C_o) \quad \left(\frac{\partial C}{\partial g_k}\right)_o$$

If we now let

$$D_{jk} = \sum_i w_i \left(\frac{\partial C}{\partial g_j}\right)_o \left(\frac{\partial C}{\partial g_k}\right)_o$$

and

$$W_k = \sum_i w_i \; (C_i - C_o)$$

we may write the normal equations as

$$\sum_{j=1}^{n} \delta g_j \; D_{jk} = W_k$$

from which

$$\delta g_k = \sum_{j=1}^{n} \; D^{-1}_{jk} \; W_k$$

where D^{-1} are the elements of the matrix inverse of D.

Of course such a computation is not meaningful unless some measure of the error of the results is evaluated. If we assume that the errors in the measurements C_i are normally distributed, we can draw some conclusions about the coefficients found.

We know that $\sigma^2 = \{\sum_i (\Delta C_i)^2 \; w_i\} / \sum_i \; w_i$

is a minimum variance estimator of the true variance. Secondly,

$$t_\alpha = (g_i - g_i^o) \, \sigma_{g_i}$$

has a "t" distribution where $\sigma_{g_i} = \sigma \sqrt{D^{-1}_{ii}}$ and g_i is one of the coefficients g_n^m, h_n^m, $\dot{g}_n^m$, $\dot{h}_n^m$ ----. Thus if we wish to test the hypothesis that $g_i \neq 0$, we must set $g_i^o = 0$ and compare g_i/σ_{gi} with the value of the "t" distribution for any given confidence level. For the present case where the number of degrees of freedom, $N_{\text{data points}} - N_{\text{coefficients}}$ is large the "t" distribution is given to adequate accuracy by its asymptotic form, the normal distribution, so that $t_{95} = 1.96$ and $t_{50} = 0.67$ are the 95% and 50% confidence levels respectively. If $g_i/\sigma_{gi} \geq t_{95}$, then with 95% confidence $g_i \neq 0$. Each g_i can thus be said to lie within the range $\pm\, 1.96\sigma_{g_i}$ with 95% confidence and the narrower range $\pm\, 0.67\sigma_{gi}$

with 50% confidence. Although these estimates are statistically valid they must be viewed with some additional information to decide whether a particular spherical harmonic makes a meaningful contribution to the potential expansion. Since the pair of g_n^m, h_n^m, may be considered as separate components of a vector, we have chosen to estimate the sigma of the magnitude $R_n^m = \sqrt{g_n^{m2} + h_n^{m2}}$ as the rms σ of the individual σ's for g and h. In addition, the set of R/σ_R is considered as a whole for a given degree n before assessing its contribution to the expansion. A further comparison is made between the σ's and the comparative values of g_k using different selections of data. A truncation of the potential expansion is thus made only on n based on these two sources of information.

The formulation up to the derivation of the g_k is basically the same as that used by Jensen and Cain (1962). The refinements added in the present work include the addition of the weights w_i and the extension of the procedure to include components other than H and F. Also, in the prior analysis there were some initial difficulties with the computation and inversion of D_{jk} that were erroneously thought to be due to the size of the matrix. In this previous calculation instead of determining the g_k's for all values of k (i.e. for g_1^0, g_1^1, h_1^1, g_2^0----g_n^m, h_n^m, where n = m = maximum degree and order of the spherical harmonic expansion), an iterative procedure was used whereby sets of only 16 or less δg_k's were computed at once holding the rest constant. That is, corrections were made first on g_1^0, g_1^1, h_1^1,---g_3^3, h_3^3 holding all other g's and h's constant. Another pass through the data was then made correcting only on the n=4 (g_4^0, g_4^1,--h_4^1,h_4^2--h_4^4) terms. Each degree harmonic was corrected in turn up to the maximum and then the process repeated until the corrections became small. Although improvements in existing sets of coefficients were in fact deduced there was sufficient correlation between the various harmonics to cause the resultant solutions to never quite stabilize. For the present calculation, however we adopted the procedure of solving for all the values of k simultaneously. This technique was found to have good results if enough data points were used and if the matrix of values entering the normal equations was formed and solved in double precision (17 digit) arithmetic. The number of data necessary appears to depend jointly on the distribution and type of component and the accuracy of the initial estimates for g_k. F data are troublesome if the original estimate is bad or if the data are poorly distributed. Component data do not need to be well distributed. The δg_k become small ($\sim 10^{-6} g_k$) after only two passes through the data whereas using the prior procedure the g_k continued to change slowly and systematically for the correlated coefficients even after several iterations.

A further refinement in the present computation scheme over that used by Jensen and Cain (1962) is the allowance for using a spheroidal earth instead of assuming sphericity. This is done by entering into the normal equations for each datum the geocentric coordinates r, θ instead of the geodetic colatitude and an r derived from the mean earth radius. Most previous works have used $\theta = 90^\circ - \lambda$ and $r = 6371.2 + h$ where

λ is the geodetic latitude and $\underline{h}$ the altitude of the observation above the earth's surface. We instead now use the relations

$$\cot\theta = \frac{\{h\sqrt{A^2\cos^2\lambda+B^2\sin^2\lambda}+B^2\}}{h\sqrt{A^2\cos^2\lambda+B^2\sin\lambda}+A^2}\tan\lambda \qquad \text{and}$$

$$r^2 = h^2 + 2h\sqrt{A^2\cos^2\lambda+B^2\sin^2\lambda} + \frac{A^4\cos^2\lambda+B^4\sin^2\lambda}{A^2\cos^2\lambda+B^2\sin^2\lambda}$$

for determining θ, r given h and λ, where A and B are respectively the earth's equatorial and polar radius. The value used in the potential expansion V for $\underline{a}$ was 6371.2 Km to correspond to the earth's mean radius. Thus in recomputing the field from the derived set of coefficients a truly geocentric $\underline{r}$ and θ must be used. The correction for $\underline{r}$ is the more significant of these whereas the correction for the latitude is quite small. To be exactly consistent with the directions of the measured data on the spheroidal earth (e.g. X and Z) a small rotation also needs to be made from the geocentric directions derived from $\overline{F} = -\nabla V$. That is, if the geocentric components of F are given by

$$F_r = \frac{-\partial V}{\partial r},\quad F_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta},\quad \text{and}\quad F_\varphi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi},$$

then the components X, Y and Z are given by

$$X = -F_\theta\cos\delta - F_r\sin\delta$$

$$Y = F_\varphi$$

$$Z = F_\theta\sin\delta - F_r\cos\delta$$

where δ is the angle between geodetic and geocentric latitude and can be computed from expressions such as $\sin\delta = \sin\lambda\sin\theta - \cos\lambda\cos\theta$. Since

δ is only of the order of 0.2° the correction is not of much consequence for most evaluations of the field at the earth's surface.

Both Schmidt (Chapman and Bartels, 1940, pg. 641) and Jones and Melotte (1953) contended that the use of a spheroidal earth in these calculations was not a meaningful improvement. However, the accuracy of these past workers was only of the order of one percent. Further they did not need to extrapolate the field to any significant distance above the earth's surface. Making the approximations $r = 6371.2 + h$ and $\theta = 90^{\circ} - \lambda$ during the analysis for g_n^m and h_n^m meant that in the synthesis of the field components one also used the same relations. This is mathematically equivalent to mapping the field from the spheroid onto the mean sphere in finding the coefficients and then mapping from the sphere back to the spheroid for their evaluation. The inherent errors in this process can best be illustrated by the following numerical example.

A set of spherical harmonic coefficients was taken to represent the exact potential function, $V(r, \theta, \varphi)$. The form of this potential is not important other than that it used 48 coefficients and did represent the actual geomagnetic field to about 2%. Using this potential, a set of "observed" data was calculated to simulate exactly what an observer would measure if he were to occupy each point with his magnetometers, levels, and surveying instruments on the spheroidal earth. The "observed" F, H, X, Y, and Z were computed over a 10° mesh in longitude and latitude, and at altitudes of 0, 10, 1000, 10,000 km. Using the set of Z's for zero altitude, a set of coefficients was calculated assuming a spherical earth exactly as has been done by many investigations in the past. In order to make this fitted field agree with the "observed" field to less than one gamma error, it was necessary to compute g_n^m's and h_n^m's up to n = 16, m = 16. The difficult areas to fit were near the poles.

Now using this set of coefficients, "computed" values of the field were calculated for the same points as the "observed" set, now assuming a spherical earth. These two sets of field points were compared and the largest errors are given in the table:

Maximum errors (gamma)

Altitude (km.)	Z	X	Y	H	F
0	0	154	25	153	149
10	0	152	25	151	148
1000	49	87	13	86	65
10,000	10	7	1	7	10

When the absolute magnitudes of the field are taken into account the result is that the maximum error at all altitudes is of the order of 0.5%. This percentage error would be the approximate inconsistency that would constitute a 'noise' level if the shape of the earth were ignored in a field determination.

3. Selection of Data

Another decision necessary in computations such as this is whether to perform some reduction and smoothing of the data prior to analysis or instead to analyze the raw data. As previously dicussed most of the prior determinations of the main field have been done using compiled charts. This technique represents the ultimate degree of data reduction which utilizes little of the potential form of the field and raises many questions which are more properly left to the data analysis. For example, map data have frequently been generated for the earth's surface by merely using a dipole relation to "reduce" airborne observations. Such procedures obviously destroy what little r dependent data are available for aiding in the determination of the potential terms.

There are, however, valid reasons for considering some reduction of the survey data prior to analysis. The main theoretical incentive is that in making a spherical harmonic analysis determining a non-infinite set of coefficients from an irregularly distributed set of data there is a tendency for the neglected higher order harmonics to influence the values of the lower order terms. This 'aliasing' (Blackman and Tukey, 1958) might be reduced by spatially smoothing the data so that only those wavelengths are left which correspond to the degrees of harmonics n determined. Thus in any analysis of a given set of data the values of the lower order terms change somewhat as more coefficients are added to the series unless such smoothing is done. However, the influence of the high order irregularities can be minimized, particularly for the lowest degree terms, by carrying the determination to a sufficiently high degree. This procedure is practicable so long as the neglected harmonics are of sufficiently low amplitude. Fortunately, as shown by Alldredge (1963) the amplitudes do in fact appear to decrease at least for degrees of n continuing beyond 10.

There are, of course, practical reasons for wishing to reduce the set of data to be analyzed, in that the computation time would be impractically large if a numerical fit were made to all available survey data in one computation. However, the nature of the data themselves make the decisions as to the type of smoothing very complex. For this reason we have here chosen to instead make a selection from the currently available data with simple criteria based on the result desired. Since the original intent of the study was to develop the most accurate reference field for the current epoch for use in conjunction with the analysis of data from satellites to be launched during the IQSY, the latest set of survey observations that give an adequate coverage of the earth was selected.

The set of all data available from the Geomagnetism Division of the U. S. Coast and Geodetic Survey for the period 1900-1962 comprised 215,757 observations of one or more components or a total of about 450,000 component observations (Hendricks and Cain, 1963). The percentage distribution of these data by decade is given in Table 1 and by altitude in Table 2. The main bulk of the latest data are from airborne observations (Serson, 1957, 1960; USNOO, 1963; Behrendt and Wold, 1963). The only satellite data included in this set are from Vanguard 3 (Cain, et al 1962). A plot of the positions of all data taken since 1955 is given in Figure 1. Only one observation per 0.5° by 0.5° block of longitude and latitude is represented on this diagram. As i. bvious

Table 1

Distribution of magnetic survey data (1900-1962) by altitude.

Altitude (Km)	Percent
0 (surface)	55
0.1-2 (airborne)	4
2 - 4 (airborne)	32
4 - 8 (airborne)	8
510-3750 (Vanguard 3)	1

Table 2

Distribution of magnetic survey data by Decade

Years	Percent
1900 - 1909	7
1910 - 1919	12
1920 - 1929	7
1930 - 1939	6
1940 - 1949	25
1950 - 1959	27
1960 - 1962	15

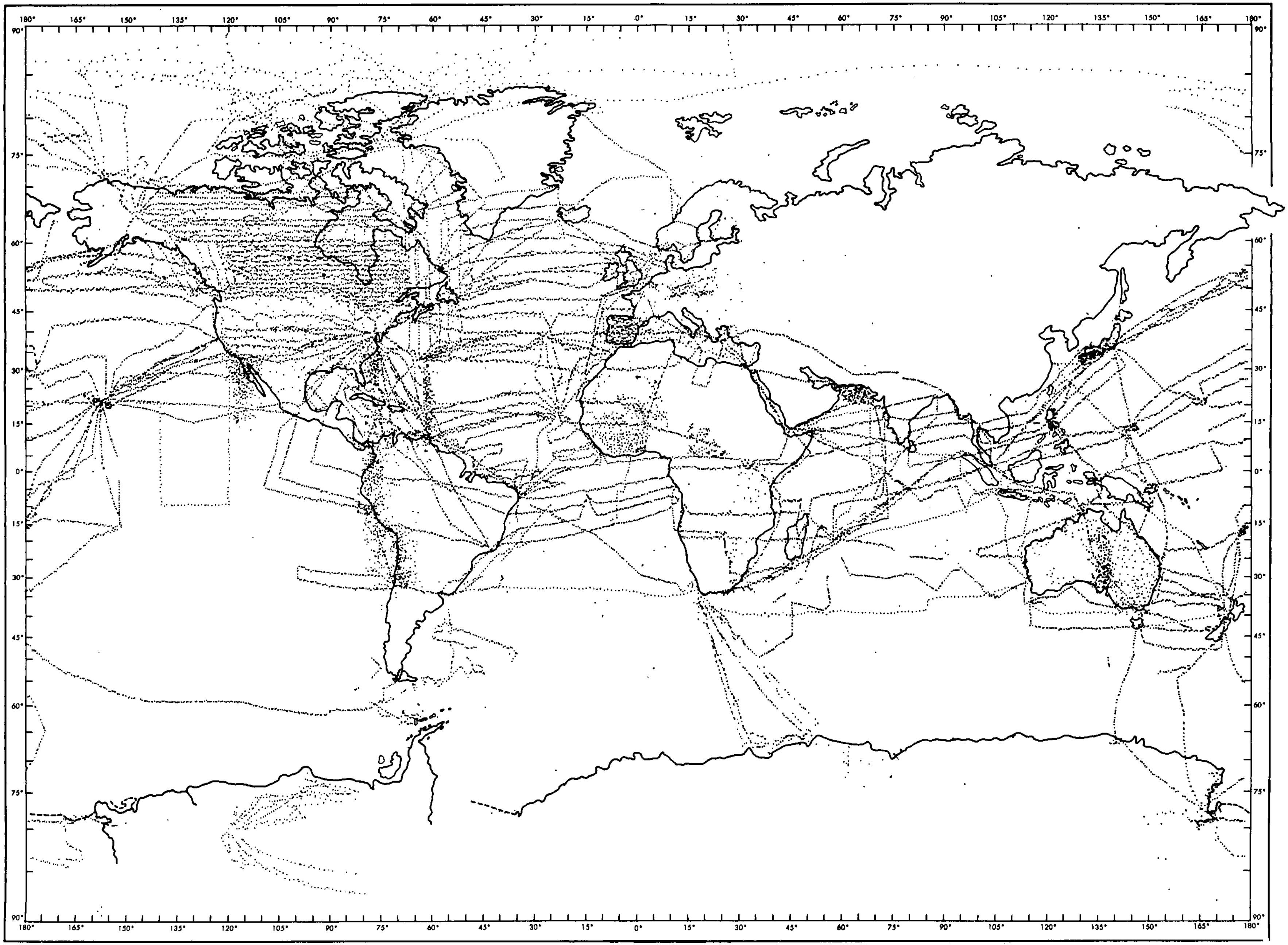

180° 165° 150° 135° 120° 105° 90° 75° 60° 45° 30° 15° 0° 15° 30° 45° 60° 75° 90° 105° 120° 135° 150° 165° 180°
90° 75° 60° 45° 30° 15° 0° 15° 30° 45° 60° 75° 90°

Figure 1

from this Figure any determination of the earth's field which would be representative of that over the whole earth would require data to fill in the large gaps over the southern oceans and Asia.

In order to provide a reasonably uniform distribution of data and to reduce the number to that practical for analysis, an equal-area grid was constructed based on the area of a $2^o \times 2^o$ latitude-longitude block at the equator. The earth was thus divided into 180 lunes of longitude and segments of latitude within each lune so the area would be the same as that for the lowest latitude blocks bounded by the equator and the $\pm 2^o$ parallels. There were thus 58 segments in latitude from pole-to-pole with the longest of almost 15^o near the poles. The latest survey or annual mean observations were then selected from all data for each such block. It was surprising to find only 8112 of the total 10440 such blocks filled by data after 1900! A time-area study of the resulting selected data was then made and it was found that a relatively small quantity would be lost by deleting all data prior to 1940. The main reason for this fact is that the latest observed data available for the Asian continent is given for epoch 1940.0. The only other substantial area that would be lost by restricting the data to post 1940 observations was the South Pacific. However, these South Pacific data are now almost worthless since they were taken over forty years ago and there are no magnetic observatories in adjacent areas to aid in relating them to present data.

One danger in making a fit to the resulting data set was that it would be difficult to distinguish secular changes from the ordinary spatial variations. In order to connect the different areas in time a selection of observatory annual means was added from the period 1940-1962. To avoid distributing these too densely in areas of high observatory density the number of annual means was limited to one per year per 10^{o} by 10^{o} block of latitude and longitude.

Figure 2 is a plot of the positions of the data actually used in this computation. The shaded areas indicate the largest of those for which data were available only prior to 1940 and were hence discarded.

In preparing these data for analysis an effort was made to select from each data source only the observed components. Since each component is fitted separately the use of derived data would have placed too high a weight on a given observation. In instances where no choice was obvious from the information at hand a maximum of three components was selected.

The set of weights w_i previously defined was then assigned to various data classes according to the estimated accuracies of the observations. This assignment of weights as well as the selection of the observed components was made with the cooperation of the Geomagnetism Division of the U. S. Coast and Geodetic Survey. These estimates cannot be considered more than qualitative since they are complicated by many factors. For example, in the previous formulation there is no allowance for errors in the position (or time) for a given observation. Thus the

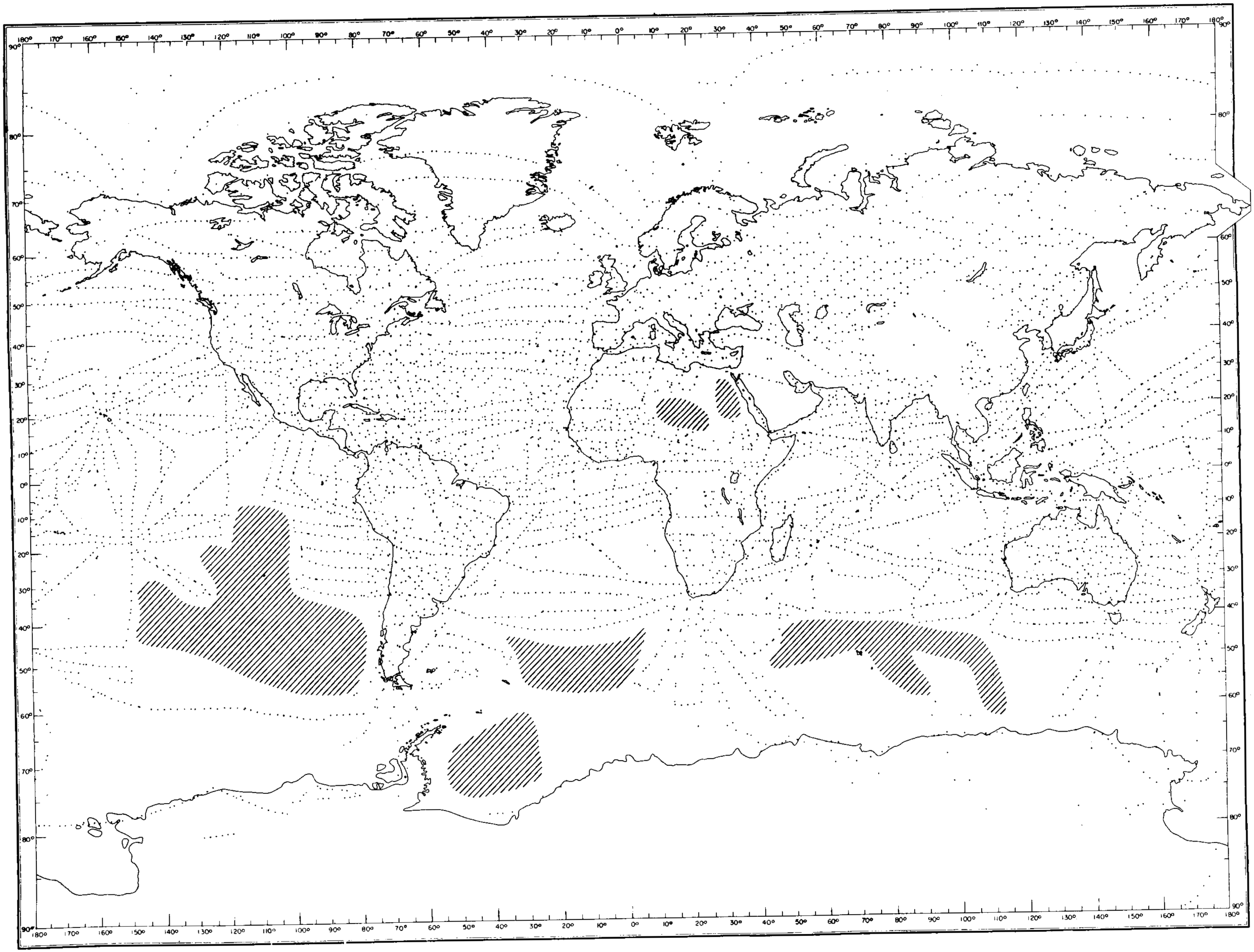


Fig. 2

weights must include some allowance for positional errors. This factor is most clearly shown by the Vanguard 3 data where it is noted by Cain et al (1962) that whereas the errors in the actual observation were only of the order of a gamma, the several kilometer error in position provided an inherent 'noise' level of the order of 10γ to the data. Positional errors become even more important near the earth's surface where the spatial gradients are larger. The gradients that are significant are of course not those of the crustal anomalies but those of the derived field. The weight assignments also depend on the smoothing involved in the observations. This smoothing is either done spatially by averaging observations over a track segment of the observing vehicle (e.g. Zarya $\simeq$ 300 Km; Project Magnet $\simeq$ 8 Km for D and I but < .5 Km for F) or temporally by determining the mean field at one location over the period of several days (e.g. repeat stations) or up to a year (magnetic observatories). Table 3 gives the estimated accuracies used in assigning weights. Since the weights for D and I were 'converted' to force units they should be multiplied by factors of the order of 500 (H or F ~ 30000γ) for comparison with the other accuracies. Further studies will need to be made to ascertain whether these assignments provide a meaningful improvement over unweighted fits.

4. Results

Prior to the determination of the final set of coefficients to best fit the selected data, preliminary calculations were made to obtain a qualitative picture of the meaning of the results and to decide on the

Table 3

Estimated accuracies σ for various classes of data used in computing the spherical harmonic coefficients A. Weights are $1/\sigma$ for H,Z,F, $180/\sigma\pi H$ for D and $180/\sigma\pi F$ for I. Dashes appear if elements were not observed for this Data Class.

Data Class	D (degrees)	I (degrees)	H (gamma)	Z (gamma)	F (gamma)
Observatory annual means	.0033	.006	5	15	15
Repeat Stations	.033	.083	5	—	2
Land Survey	.1	.1	30	50	—
Shipboard	.083	.083	25	—	—
Ship-towed Magnetometer:					
Proton	—	—	—	—	10
Fluxgate	—	—	—	—	40
Airborne*	.3	.1	60	60	30
Satellite (Vanguard 3)	—	—	—	—	10

* Observed Elements are:

Project Magnet DIF

Canadian Airborne DHZ

Univ. of Wisconsin F

maximum number of coefficients that would be statistically valid. Expanding the coefficients in time about a mean time (~1957) for the selected data we may write for g_n^m

$$g_n^m(t) = g_n^m(\bar{t}) + (t-\bar{t})\, \dot{g}_n^m(\bar{t}) + (t-\bar{t})^2\, \ddot{g}_n^m(\bar{t})$$

and a similar expression for $h_n^m(t)$. The parameters varied in these preliminary calculations were the maximum degree n_m for the spatial terms, $\dot{n}_m$ and $\ddot{n}_m$ for the first and second time derivatives respectively, and a parameter k denoting the selection of each kth observation. The results that were derived from these fits were the coefficients, their first and second time derivatives and the corresponding standard errors σ_n^m, $\dot{\sigma}_n^m$ and $\ddot{\sigma}_n^m$ of the amplitudes $R_n^m = \sqrt{{g_n^m}^2 + {h_n^m}^2}$, ---- as previously defined. One interesting result from using various non-multiple values of k was that for different subsets of the selected data the variations in individual parameters did in fact match those estimated by their standard errors. This fact gave some confidence that the standard error estimates are realistic. The only flaw in this reasoning lies in the large gaps in the time - space distributions of the data which allows only small variations in the distributions by the choice of various values of k. The real test of the validity of the standard error estimates can only be made when at least the spatial coverage of data is made more uniform.

The immediate practical problem that arose in the computation of the results was the limitation in the maximum number of coefficients that could be used. Including the various check columns, the number of computer core locations required to store the matrix of constants for the normal equations was given by $N(N+5)$ where $N = n_m\ (n_m + 2) + \dot{n}_m\ (\dot{n}_m + 2) + \ddot{n}_m\ (\ddot{n}_m+2)$. Thus the storage varied as the fourth power of the maximum degree of the coefficients. One alleviating factor that was soon apparent was that none of the amplitudes of the second time derivatives exceeded the 1.96 $\ddot{\sigma}$ (95% confidence) level and very few attained 0.67 $\ddot{\sigma}$ (50% confidence). This fact resulted in eliminating their computation by setting $\ddot{n}_m=0$ for the remainder of the tests. The implication here is only that the curvature in the secular change of the spatial terms is not significant for the selected data. It is well known that at a given location on the earth there is a measurable deviation from linearity over such spans as the 23 years covered by the data. It is likely that it would be necessary to take a much larger and more uniformly distributed set of data over a longer period before a meaningful determination could be made of the second derivatives.

The final fit to the selected data was thus made with $\underline{k}=1$ (all data), $\ddot{n}_m=0$, $\dot{n}_m=5$ and $n_m=7$. Since some of the data were anomalous or in error, no data were accepted which differed from computed values by more than 2000γ, a value of the order of five times the standard deviation. Of the total number of 21779 component observations this rejection criterion only eliminated 84 observations. The resulting best set of coefficients fit to the remaining 21695 observations is given in Table 4. The spatial terms are rounded to the nearest gamma and the first time derivatives to the nearest tenth γ/year. Although the average time of

Table 4

n	m	g	$\dot{g}$	h	$\dot{h}$
1	0	-30426	18.9		
1	1	- 2174	7.3	5761	-1.9
2	0	- 1548	-24.8		
2	1	3000	-0.8	-1949	-14.0
2	2	1574	0.8	201	-17.7
3	0	1323	-0.4		
3	1	-2009	-10.5	-442	1.9
3	2	1275	3.4	233	4.0
3	3	877	-1.9	-118	-9.0
4	0	957	0.8		
4	1	797	5.4	149	-0.9
4	2	527	-1.9	-266	-1.7
4	3	-400	-0.2	-4	3.2
4	4	273	0.8	-262	-5.5
5	0	-241	3.5		
5	1	353	-0.7	0	1.8
5	2	231	2.5	124	2.9
5	3	-33	0.6	-104	-0.8
5	4	-147	0.0	-98	-0.4
5	5	-79	1.6	75	-0.2
6	0	58			
6	1	71		6	
6	2	20		86	
6	3	-241		58	
6	4	-19		-18	
6	5	-0		-25	
6	6	-100		5	
7	0	90			
7	1	-47		-51	
7	2	-2		-22	
7	3	-26		6	
7	4	-11		-38	
7	5	26		44	
7	6	6		-4	
7	7	6		-26	

Best fit to selected data in gamma for the spatial terms and γ/year for first derivatives. Epoch is 1960.0.

the data was 1957.2, the epoch was arbitrarily chosen as 1960.0. The values of the amplitudes of the coefficients R and $\dot{R}$, and the corresponding standard errors σ and $\dot{\sigma}$ are given in Table 5 along with their ratios R/σ and $\dot{R}/\dot{\sigma}$. As can be seen from this table only one of the R values and four of the $\dot{R}$'s fall below the 95% confidence (1.96σ) level. From the previous experience in testing the fitting process with lower values of n_m and $\dot{n}_m$ it is believed that significant coefficients could have been obtained for at least one higher degree for both the spatial terms and their derivatives if allowed by the computer core size.

The weighted rms deviations between the field and the data by component are given in Table 6. As in most of these fits it is found that the residual to the Z component data is about 50% higher than for the other components. The overall distribution of these deviations differs slightly from gaussian in that there are many data beyond 2σ with over 1% beyond four standard deviations. This non-gaussian component is of course mainly due to the anomaly 'noise' in the data. However, a gaussian curve with a σ of about 200γ fits the error distribution fairly well up to about 400γ.

In order to further investigate how the coefficients fit the selected data, weighted averages were taken over 10° blocks in latitude and longitude and the resulting signed means printed. The resulting numerical 'maps' were qualitatively inspected to determine the areas where the computed field was predominantly above or below the measured. It was possible to pick out areas of the order of 50° whose deviations

TABLE 5

N	M	R	σ	R/σ	$\dot{R}$	$\dot{\sigma}$	$\dot{R}/\dot{\sigma}$
2	1	30426	9	3437	18.9	1.4	13.3
2	2	6158	9	656	7.6	1.3	5.7
3	1	1548	8	203	24.8	1.2	21.4
3	2	3577	8	476	14.1	1.1	12.4
3	3	1587	8	203	17.7	1.1	16.9
4	1	1323	6	204	.4	1.0	.4
4	2	2057	6	322	10.7	.9	11.5
4	3	1296	6	206	5.2	.9	6.0
4	4	885	7	120	9.2	.9	10.7
5	1	957	6	169	0.8	.9	.9
5	2	811	5	149	5.4	.8	6.6
5	3	591	6	103	2.5	.8	3.1
5	4	400	6	63	3.2	.7	4.8
5	5	378	6	63	5.5	.8	6.8
6	1	241	5	49	3.5	.6	5.5
6	2	353	5	69	1.9	.7	2.7
6	3	262	5	53	3.8	.6	5.9
6	4	109	5	22	1.0	.6	1.6
6	5	177	5	33	.4	.6	.6
6	6	109	6	19	1.6	.8	2.1
7	1	58	4	14			
7	2	71	5	15			
7	3	88	4	20			

TABLE 5 (Cont.)

N	M	R	σ	R/σ	$\dot{R}$	$\dot{\sigma}$	$\dot{R}/\dot{\sigma}$
7	4	247	5	49			
7	5	26	5	5			
7	6	25	5	5			
7	7	101	5	20			
8	1	99	3	26			
8	2	70	4	19			
8	3	23	4	6			
8	4	27	4	7			
8	5	39	4	10			
8	6	51	4	13			
8	7	7	4	2			
8	8	27	5	5			

Amplitudes of the main geomagnetic field R and of their first time derivatives $\dot{R}$ in γ and γ/year respectively. Standard errors σ and $\dot{\sigma}$ and the ratio R/σ and $\dot{R}/\dot{\sigma}$ are also given. The ratios should exceed 1.96 for a 95% confidence and 0.67 for a 50% confidence.

TABLE 6

RMS deviations by component between computed field and selected data with weights based on Table 3.

ELEMENT	SIGMA (γ)	OBSERVATIONS
D	202	5998
I	234	5087
H	211	2958
F	223	5436
Z	344	2216
ALL	214	21695

were systematically a few tens up to a few hundreds of gammas. Thus from evidence such as this and the other factors mentioned it is concluded that there are significant spherical harmonics in the selected data of degree beyond seven.

Illustration of results:

Since the accuracies of most computed fields are now within a percent of the measured values, world maps synthesized from the harmonic coefficients all appear very similar. Nevertheless, it is useful to attempt to illustrate the scale of the irregularities as represented by the harmonic descriptions. Maps of F, H, I, and Z for epoch 1965.0 are given in Fig. 3 as drawn by the techniques previously described by Cain and Neilon (1963). On comparing this Figure with the 1960.0 maps given in the earlier paper one notes only slight differences in spite of the epoch being five years later and the fact that the present charts use 63 instead of 48 spatial coefficients. The reasons for this are that the percentage changes in the field during a five year interval are small and that the amplitudes of the additional fifteen coefficients are small compared to those of the lower terms.

Using such a smooth representation it is of course not possible to illustrate the numerous small scale anomalies that are found near the earth's surface. The contours are thus much smoother than are found on the standard world magnetic charts which are normally constructed by other methods. One peculiarity of standard charts, however, is that they attempt to show as much detail as the data allows with the result that there are frequently kinks around anomalies in isolated areas of good data coverage, but smooth contours elsewhere. It is likely

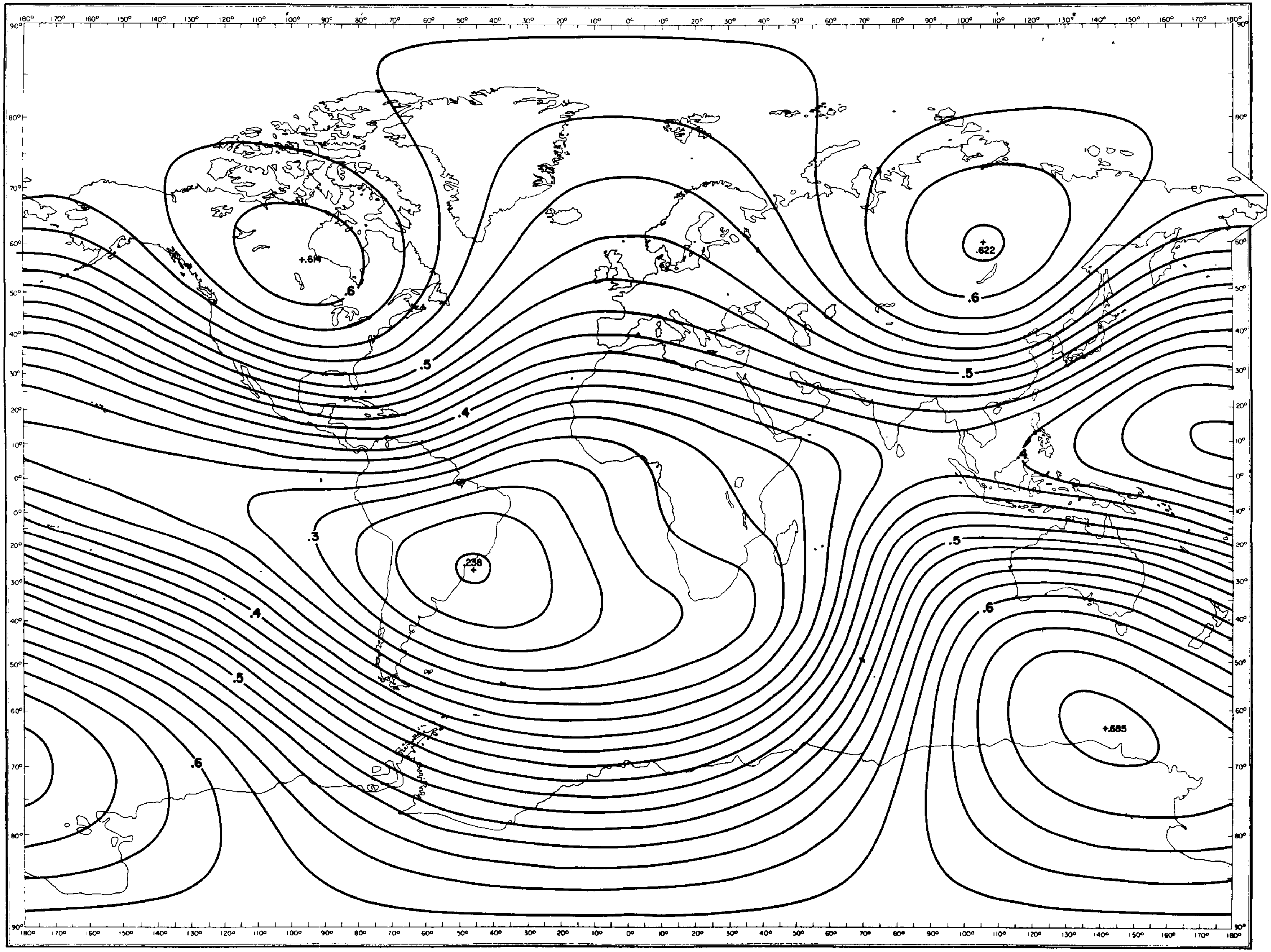


Fig. 3a

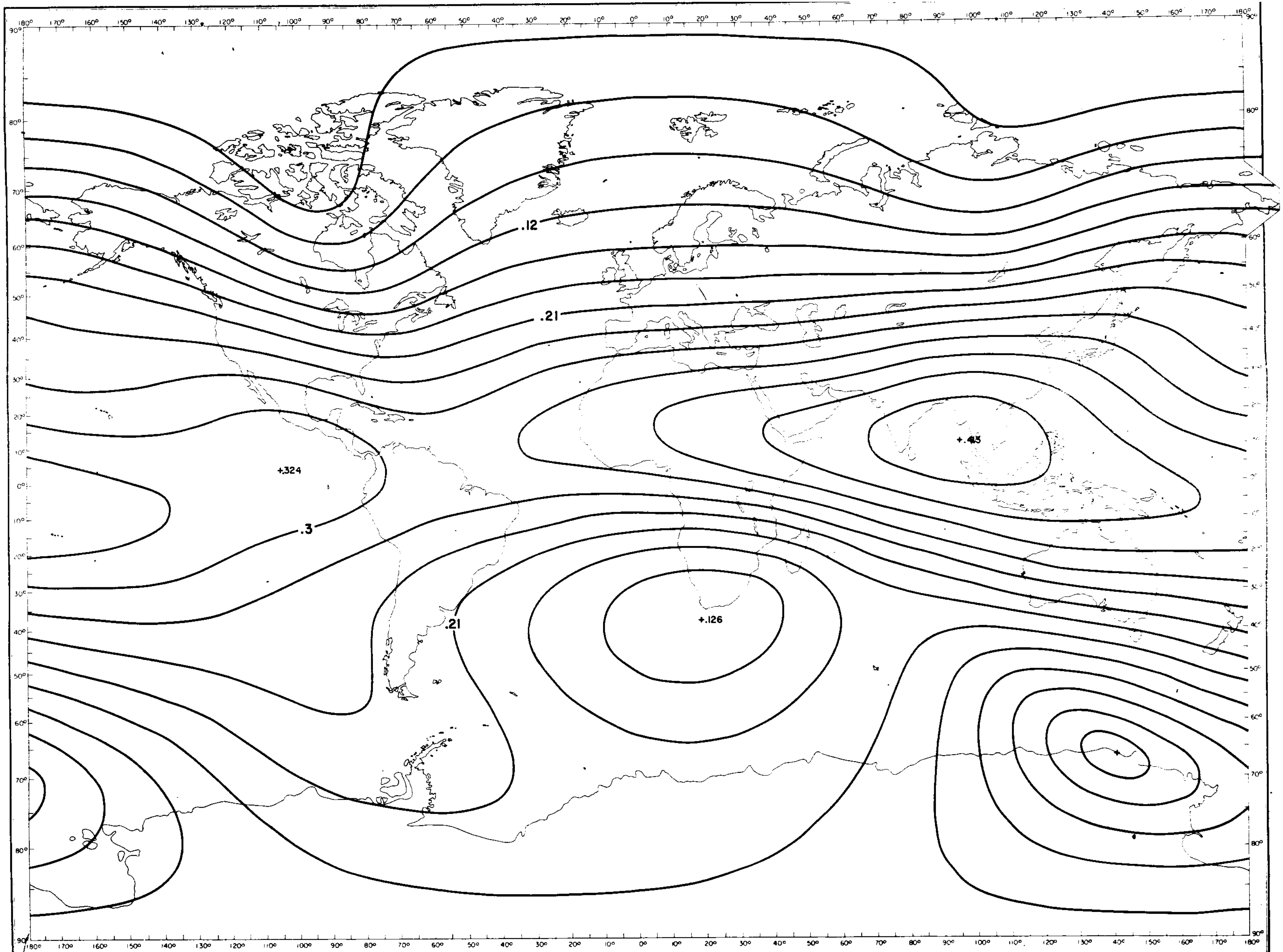


Fig. 3b

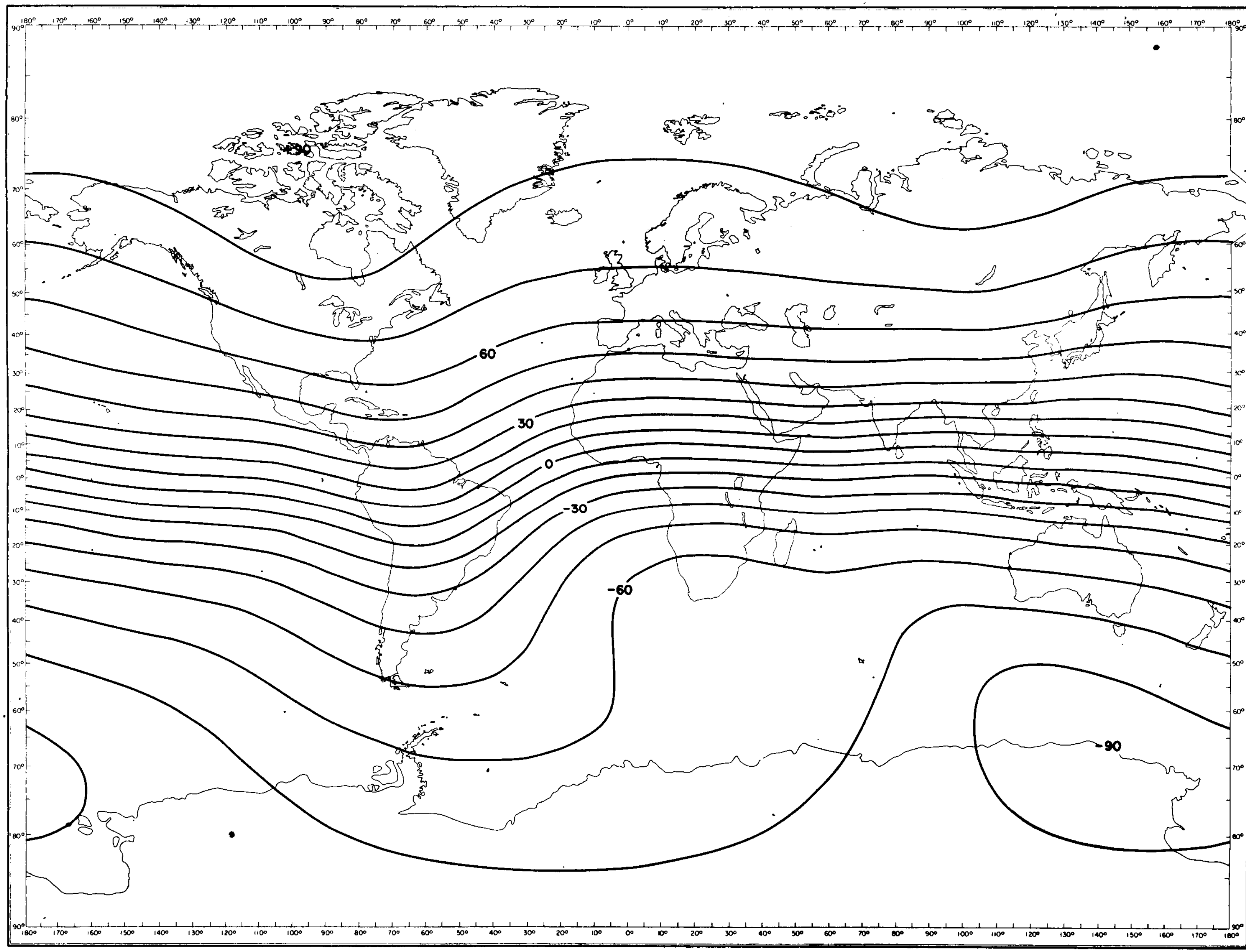


Fig. 3c

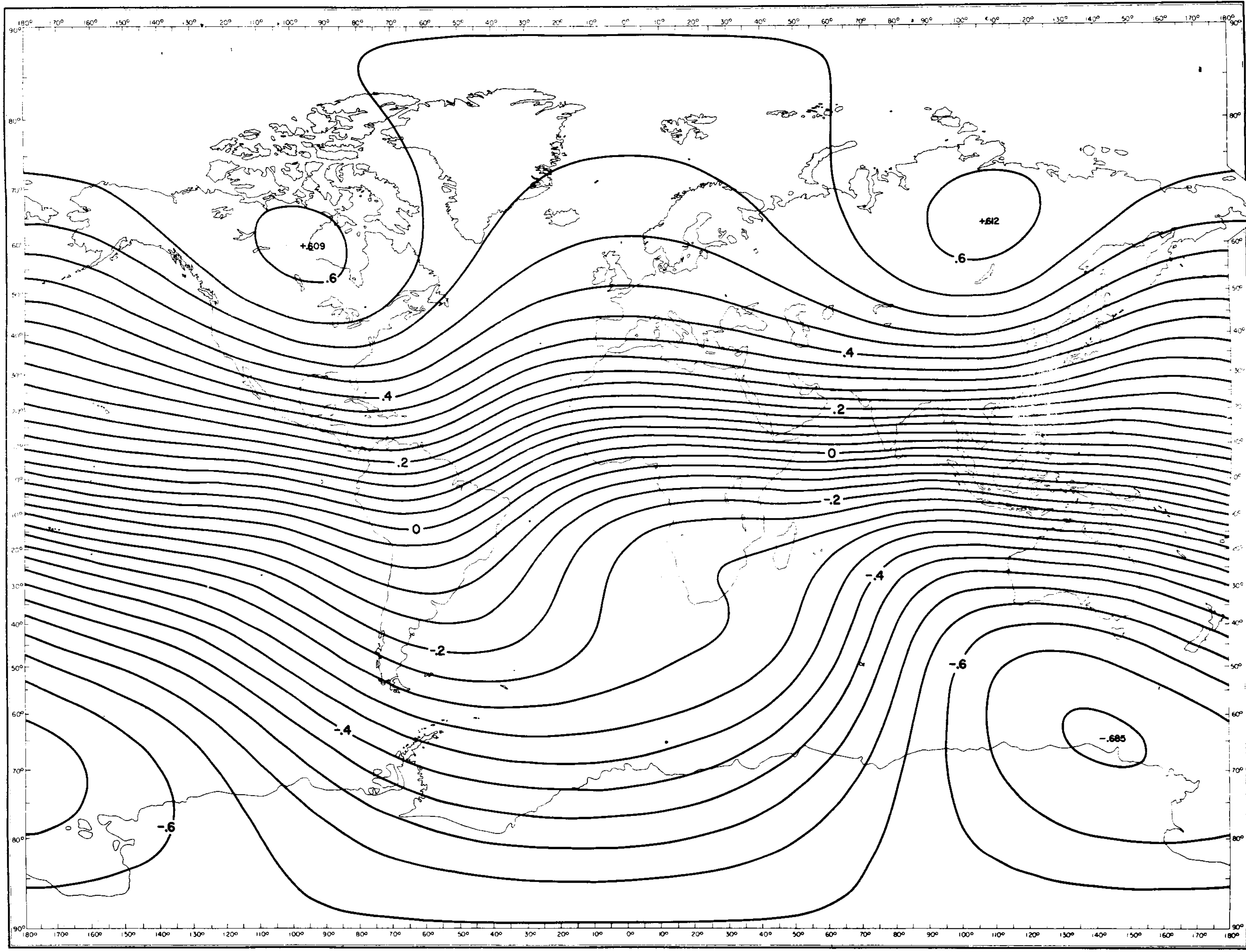


Fig. 3d

such maps as shown in Fig. 3 give a more realistic estimate of the field averaged over the whole of the earth's surface.

A prominent feature of the secular change coefficients given in Table 4 is the fact that the $\dot{g}_1^0$ term does not dominate the series as does the g_1^0. Even though there are fewer secular change terms determinable from the data, the resulting secular change maps are at least as complex as those for the field itself. The secular change in two of the components, F and Z, is drawn in Figure 4. The characteristic feature of the $\dot{F}$ map is that the earth is almost equally divided between positive and negative regions. The gross pattern of positive $\dot{Z}$ is equally simple but with the centers of negative change occurring in low latitudes and the positives centered only in high latitudes and over Asia. Since the estimated confidences in the major secular change terms are far smaller than those for the coefficients of the main field (cf. Table 5) the details of the patterns illustrated are equally less likely to be significant.

5. Comparison With Other Models

In order to evaluate the way in which the presently derived set of coefficients compare with previous models we have made a study of the deviations between the available data and the computed field components at the points of observation. Complete validation can never be made since it is not possible to make comparisons in regions that are void of data. Indeed, as new data are accrued it should always be possible (provided enough parameters are used) to incorporate this information

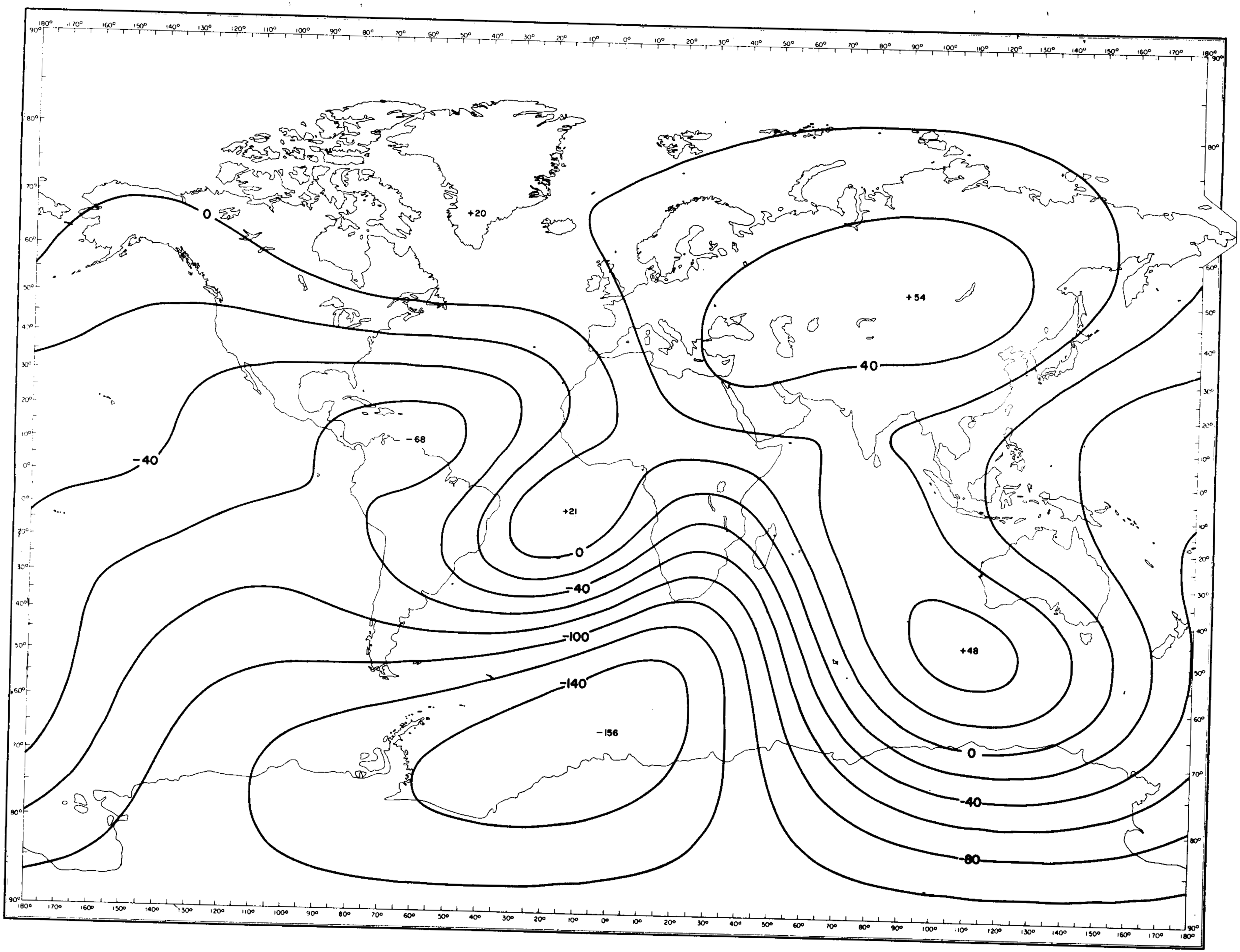


Fig. 4a

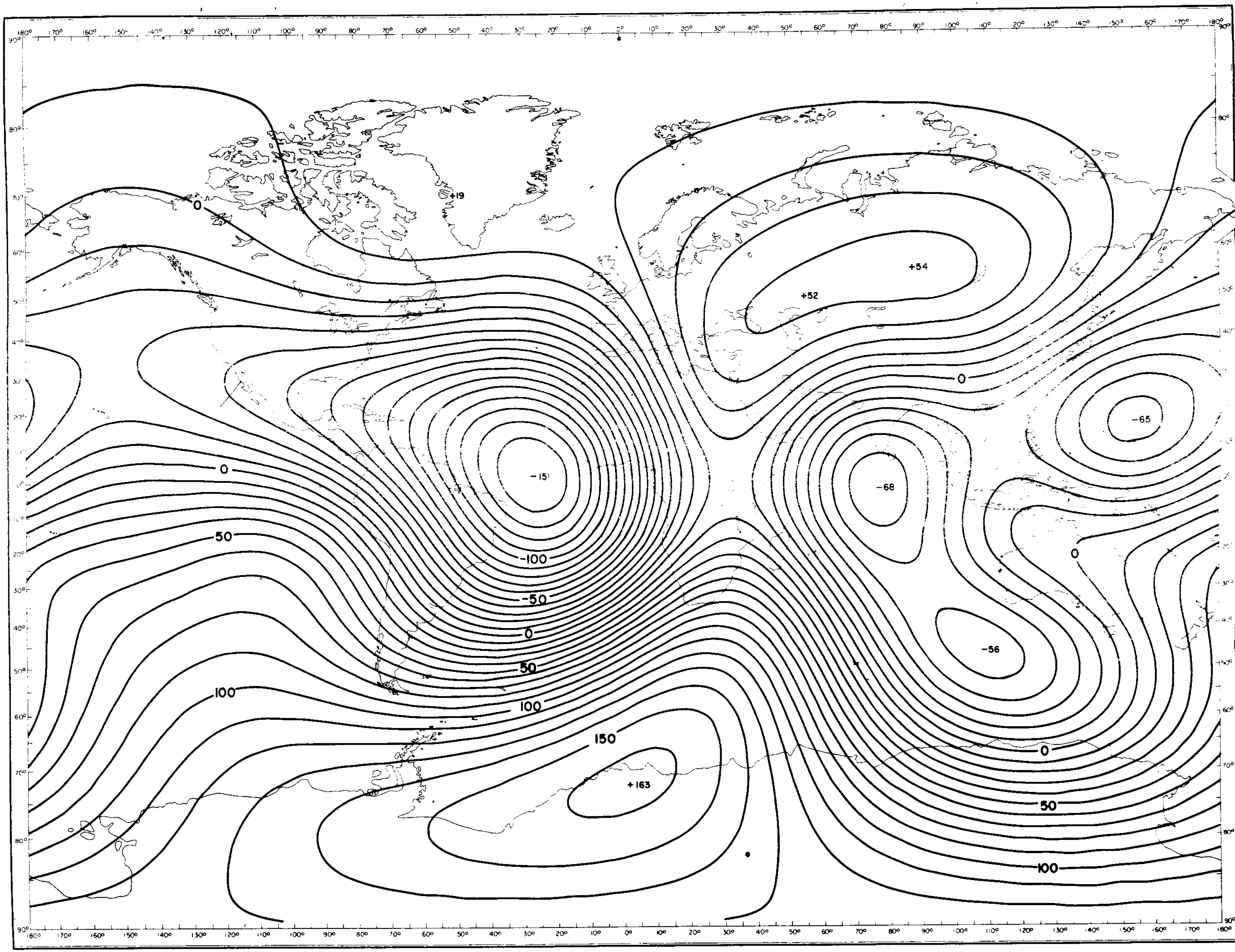


Fig. 4b

into the computer programs and improve the field coefficients accordingly! The point of this study then can only be to make a comparison of the older models to see how they fit the data at various epochs and to see whether substantial improvements can be made in the newer models using the most recently available data. Less extensive but more detailed comparisons have also been done by Heuring (1964) using the set of Vanguard 3 data.

A description of the various models used is given in Table 7. Although this list does not include all published models, it contains most of those widely used in the past with the exception of the 569 coefficients derived by Jensen and Whitaker (1960) from the 1955 U. S. Z charts. This last model was omitted from the comparison since it has been shown in the past to be no substantial improvement over most other models in spite of its much larger number of coefficients.

A statistical comparison was first made between all available survey data for the period 1940-1963 and the various models listed in Table 7. The data set used was a slightly later version of the one described in Tables 1 and 2 so that some data were available for 1963 that were not included in the original selection. The rms deviations between the observed and computed components were derived as described in the fitting of the data except that no weighting factors were used initially. The rms deviations of the angular components ΔD and ΔI were computed separately in angular units but then weighted by H and F respectively when combined with the sums of squares from the ΔF, ΔH, and ΔZ. The data were grouped

TABLE 7

Symbol	Reference*
A	Set of coefficients listed in Table 4
LME	Leaton, Malin and Evans, 1964. (A fit with $n_m = 8$ and $\dot{n}_m = 7$ to a set of X and Y computed data assuming a spherical earth).
F+L	(Finch and Leaton, 1957)
F+L(t)	The (t) added to F+L indicates that the secular change terms of Leaton (1962) were used.
J+C	Jensen and Cain (1962)
N+O	(Nagata and Oguti, 1962) The use of the (2) means that both the internal and external fields were added. If (i) only appears, the evaluation was made using only the internal field.
V	Vestine (1960)
J+M	Jones and Melotte (1953)
USSR	Adam et al (1962) Table 4, Adam et al (1963) secular change from appendix (61 observatories 1954-1959)
F+K	Fauselau and Kautzleben (1956)

*A listing of some of these coefficients can be found in a report by the authors (Cain et al, 1964) according to each year 1940-1963 and by the source of the observations. The source categories were: observatory annual means, surface (land or sea surveys), airborne, and satellite (Vanguard-3 F only). Results were tabulated for each of the component-source-year groups. Summaries were also tabulated by source-year and year alone and also for each source-component or source alone for all years 1940-1963. Using the set of coefficients A, data were first rejected if their deviation exceeded 3000γ in the components, F, H, Z or 10° in the angles I or D. In each succeeding comparison the same rejection criteria were maintained for the remaining data. A summary of the results by year is given in Table 8 for the eleven different models. The last column is labeled "Maximum number of observations" since in order to conserve computer time, only a fraction of the available data were used for each model. The figures in the table represent the rms deviations from the largest data sample in those cases where different sample sizes were used. In making such comparisons using different subsets of the data the results varied up to a few tens of gamma. Thus only those differences exceeding this amount should be considered significant.

The two set of field coefficients, A and LME, appear to have the lowest residuals to the data of all coefficients sets over most of the interval 1940-1963. This improvement is striking even over the F+L(t) coefficients which are the next best set over the whole interval 1940-1964. Particular other sets of coefficients show relatively low residuals near their epoch time but not elsewhere. Noteworthy in this respect are

TABLE 8

The rms deviation (γ) by year between unweighted magnetic survey data and magnetic fields calculated by different sets of spherical harmonic coefficients as listed in Table 7

* designates use of time derivatives

Year	A *	LME *	F+L(t) *	F+L	J+C	N+O(2) *	N+O(i) *	V *	J+M	USSR	F+K	Maximum Number of Observations
1940	300	440	400	420	860	440	450	360	490	860	380	62000
1941	280	320	400	530	620	420	410	390	590	970	470	2000
1942	300	330	360	450	630	350	360	300	490	880	370	3000
1943	260	330	380	490	610	390	380	340	500	1090	390	2000
1944	240	261	260	420	700	310	270	290	450	1010	440	11000
1945	250	240	290	390	550	330	310	270	520	930	540	2000
1946	310	320	340	400	520	380	370	400	540	970	470	2000
1947	240	260	330	400	480	410	380	430	560	960	500	2000
1948	230	230	310	390	470	340	310	370	520	940	500	2000
1949	240	260	310	340	410	390	310	440	510	940	500	2000
1950	340	330	390	370	440	410	400	490	580	1050	560	9000
1951	320	320	440	450	420	460	380	500	630	1230	590	13000
1952	220	220	300	350	440	430	370	420	650	1040	620	2000
1953	230	210	260	280	370	370	280	390	740	1250	660	10000
1954	250	230	310	310	340	400	270	400	850	1220	680	13000
1955	240	220	310	310	340	390	310	400	610	1120	620	11000
1956	250	210	340	380	510	390	340	580	780	1450	1030	12000
1957	260	230	350	410	490	490	400	650	870	1210	1010	10000
1958	340	320	430	460	480	490	490	710	990	1260	950	15000
1959	240	230	390	420	400	520	460	640	890	1170	880	22000
1960	260	240	400	460	440	540	450	660	1040	1390	1000	36000
1961	240	220	420	450	420	510	420	600	1080	1330	770	31000
1962	280	240	460	500	430	500	410	580	1280	2370	1070	8000
1963	250	270	440	520	420	520	480	960	1130	1360	1490	10000
Epoch	1960	1965	1955	1955	1960	1958.5	1958.5	1945	1942	1958	1945	

Vestine's (1960) 1945-epoch coefficients which appear to give consistently low residuals over the years 1942-1945. The N+O(i) set appear to give consistently good results in the interval 1952-1956 and for a few previous years but tend to become worse at and after their epoch of 1958.5. This tendency for field models to better fit the data prior to their epoch is the result of the fact that most authors are attempting to compute a field valid at the time of publication based only on past data. This result indicates that 'correcting' data to a given epoch is not on the whole successful.

The set of coefficients derived by Nagata and Oguti (1962) were used to determine how the addition of external terms change the residuals to the data. As can be seen from the column N+O(2) and N+O(i), the use of internal terms only gives a better fit. The reason for this discrepancy is probably that the external terms represent only the errors in the maps from which the determinations were made and are unrelated to any realistic external field.

The comparison between the Finch and Leaton (1957) coefficients with and without Leaton's (1963) time derivatives is indicative that the use of secular change terms is important even over a few years. The intent of the computation made by Jensen and Cain (1962) was to improve the residuals over those of F+L (no time derivatives). It can be seen that for the years 1959-1963 the J+C residuals are indeed less that the F+L by about 10%. However, when the time derivatives are added (F+L(t)), this improvement is eliminated so that the two sets of coefficients are merely comparable.

Having established that the presently derived coefficients A and LME clearly give results that are much more representative of the field than any of the others it is useful to compare these in more detail to see where the differences may lie. In the yearly comparisons of Table 8 the residuals are smaller for the A field than LME for only nine of the 24 years and except for the 1963 data are all grouped near the beginning of the interval. All of these comparisons were made using the survey data with equal weights whereas the fit resulting in the model A was actually made with weights established by Table 3. Table 8 was constructed in this unweighted way in order to simplify the discussion and to see whether the use of weights made any appreciable difference in the apparent relative merits of the various fields. If the data are instead weighted according to Table 3 and a comparison again made of the residuals against the various computed fields, the A and LME sets again have the lowest residuals by a factor of the order of two from the others. The relation between A and LME however is altered in that the yearly residuals for A nearly all become the smaller. As summarized in Table 9 the difference averages a third for 1940-1955 but only 7% for the data after 1955. It is thus apparent that at this point the relative merits of the two sets of coefficients depend on whether the weights are accepted as realistic.

In both Tables 8 and 9 the data for 1940 are fit much better by A

TABLE 9

A comparison of the rms residuals between observed data and data synthesized using fields A and LME and the weighting factors based on Table 3.

	rms Residuals (gamma)				
Years	A	LME	Years	A	LME
1940	286	433	1952	217	244
1941	237	336	1953	204	207
1942	225	278	1954	245	242
1943	241	323	1955	197	206
1944	240	260	1956	232	223
1945	209	281	1957	222	256
1946	243	277	1958	258	272
1947	201	254	1959	140	154
1948	218	253	1960	249	270
1949	183	217	1961	225	236
1950	290	307	1962	264	258
1951	246	242	1963	239	266
1940-1955	249	322			
1955-1963	210	224			

than by LME. The majority of data for this year is that from the Soviet Union. As there was only observatory data since this time from that area there is possibly a major difference between A and LME over Asia. Since Leaton, Malin and Evans (1964) indicate that they used for their analysis the data taken from the USSR 1950 T(=F) and 1955 D, H and Z charts, one may conclude that there are some large differences between those charts and the total set of digital data on file. One other possibility is that the use of only single time derivatives may not be adequate for representing the data for this region weighted as heavily as it is with data at 1940.

Table 9 indicates fairly low residuals for weighted data for both models for 1959. The reason for this is the inclusion of the Vanguard 3 data which have relatively high weights and match either model to ~50γ rms. The fact that it is possible to fit these satellite data to a precision of 17γ (Cain and Hendricks, 1964) is further indication that higher order harmonics are needed. One possibly significant difference between the A and LME models is that whereas the rms deviations to the Vanguard 3 data are almost the same, the mean errors for A are close to zero whereas the measured F average about 20γ below that predicted by LME at all altitudes. We cannot offer any clear physical or mathematical reason for this discrepancy except to reiterate some of the differences in derivation that might be significant. The most apparent difference of course is that the fit to derive A included the Vanguard 3 data whereas LME did not. Thus recalculating the field in the volume of the

Vanguard 3 measurements becomes an extrapolation for LME but an interpolation using the A coefficients. Secondly, the LME field was based on the spherical earth approximation which we have already shown can give rise to errors at altitude.

Bearing in mind this average difference between A and LME using one sample of low altitude satellite data one may also consider how the field models extrapolate to larger distances. One such sample of data on which such comparisons can be made is the small amount of Explorer X total field data available for March 25, 1961 from 1544 to 1740 U.T. which range from dipole latitudes (cf. Chapman, 1963) -8 to -31° and altitudes from 1.7 to 7 earth radii. As can be seen in Fig. 22 of the paper by Heppner et al (1963) the measured field was below that computed by F+L by some 40γ at the lowest altitude but became equal to and greater than that computed as the altitude was increased. Since the lowest altitude portion of the curve will show the least distortion from the effects of trapped particles and the magnetopause we use it to compare with the field models. The maximum ΔF (measured minus computed) for seven of the field models is given in Table 10. It is interesting to note that the ΔF for all four fields LME, J+C, N+O(i), and F+L(t) is very nearly -30γ whereas the lowest deviation of -18γ occurs for A. Ignoring the altitude uncertainty mentioned by Heppner et al (1963) for the measurement it would appear that the A model gives the best prediction of the main field.

Another independent set of information relating to the way in which various field models fit a low latitude field line is provided by

Table 10

Difference ΔF (measured minus computed) between total field observed by Explorer X at 14°SOUTH, 4°WEST, 5000 Km altitude, and field computed by several models.

Field	ΔF
A	-18
LME	-30
J+C	-30
N+O(i)	-29
F+L	-42
F+L(t)	-30
USSR	-92

Table 11

Distance R (km) between position of observed artificial aurora (Leonard, 1963) and traces to 100 Km altitude using field models indicated.

Model	R(Km)
F+L	56
J+C	46
LME	35
A	26

Leonard's (1963) observations of the artificial aurora conjugate to the Johnson Island bomb tests. In his Figure 3 Leonard shows the position of the observed aurora and the E-layer intersections traced with various field models. Assuming an approximate injection point of 16.50°N, 169.64°W and 400 Km altitude (which appears to match the positions given in his Fig. 3) we tabulate in Table 11 the distance from the observed position given by Leonard (16.6°S, 175.82°W) to the point intersecting 100 Km altitude as traced from the estimated injection point, using four field models. As can be seen in this table, the A trace matches the observed position with the smallest error.

In order to obtain a detailed picture of the way in which the models fit the near surface data we have plotted some of the deviations for a few of the ship and aircraft data. A map showing the tracks illustrated is given in Figure 5. Most of these were chosen to be near the regions void of data and should thus result in the largest deviations. The detailed plots of the differences of the observations from the A and LME models for each of these tracks are given in Figure 6. The differences shown are the measured less computed as observed except for the angular observations ΔD and ΔI which were again converted to force units. There are several useful conclusions that can be gleaned from such plots in regard to the way in which these two model fields fit the observations. The most striking feature is that there are long-period systematic deviations for either field model which indicate that either more or better coefficients need to be derived. The rms deviations given in

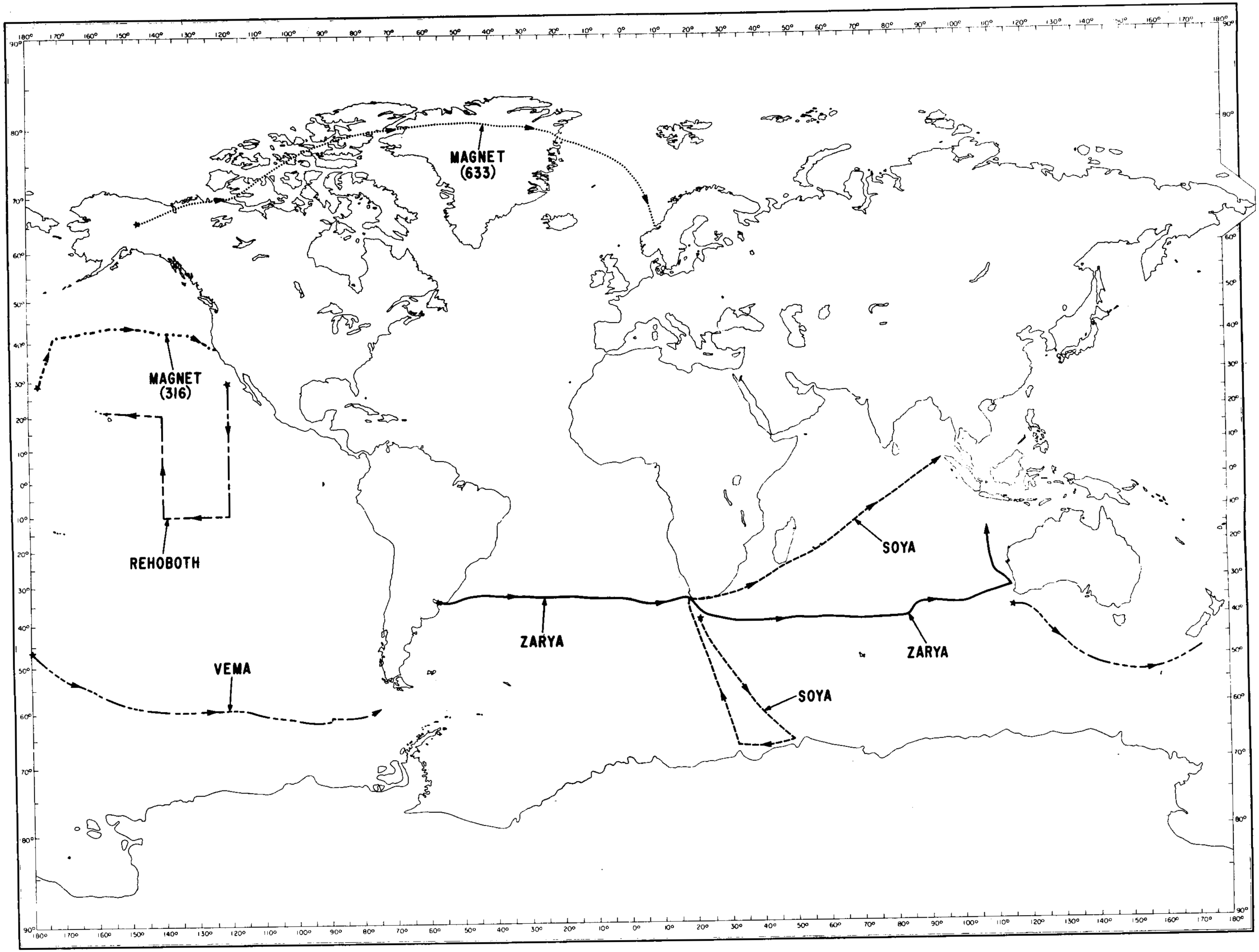


Fig. 5

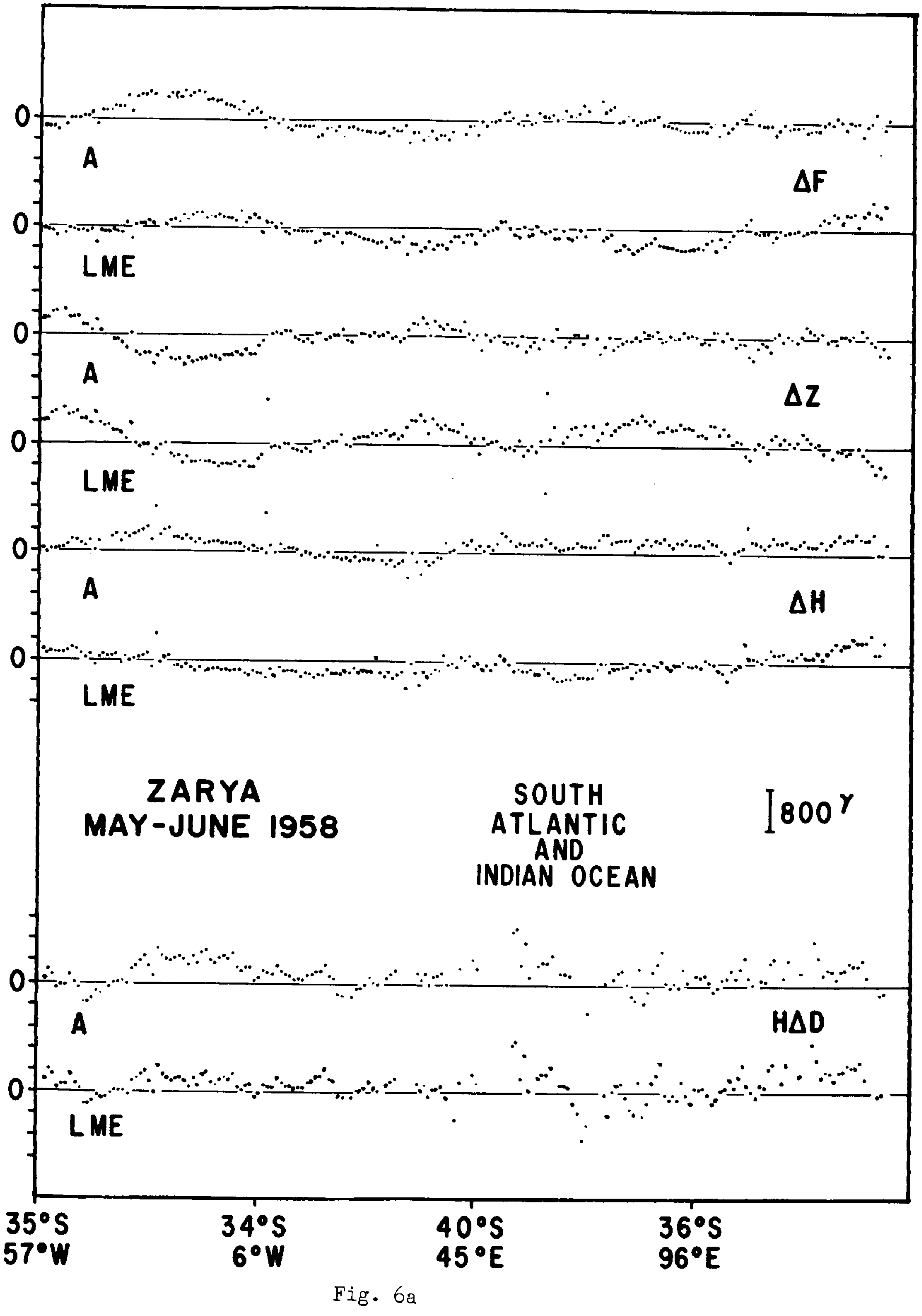

0
A
ΔF
0
LME
0
A
ΔZ
0
LME
0
A
ΔH
0
LME
ZARYA
MAY-JUNE 1958
SOUTH
ATLANTIC
AND
INDIAN OCEAN
800γ
0
A
HΔD
0
LME
35°S
57°W
34°S
6°W
40°S
45°E
36°S
96°E

Fig. 6a

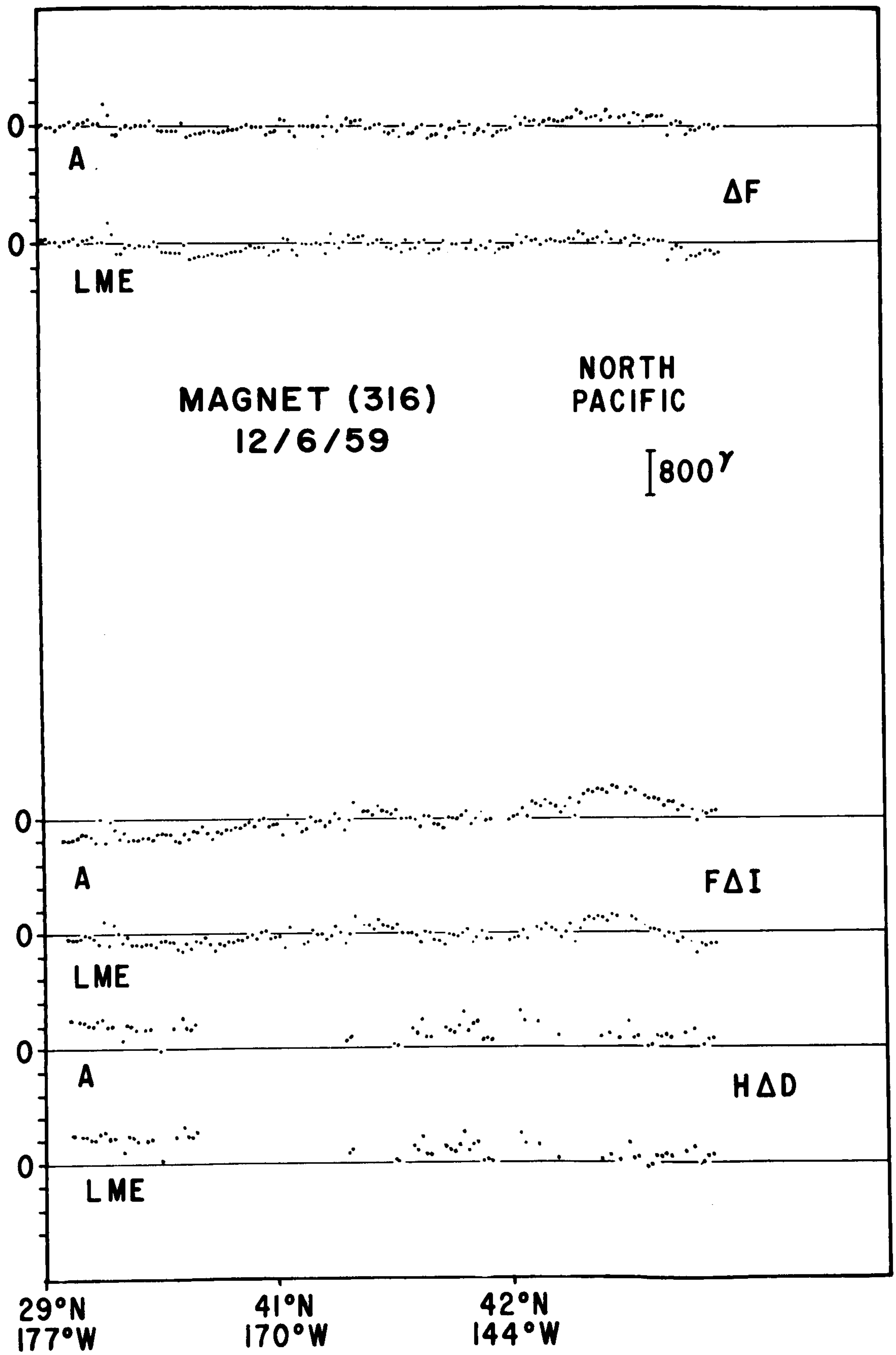
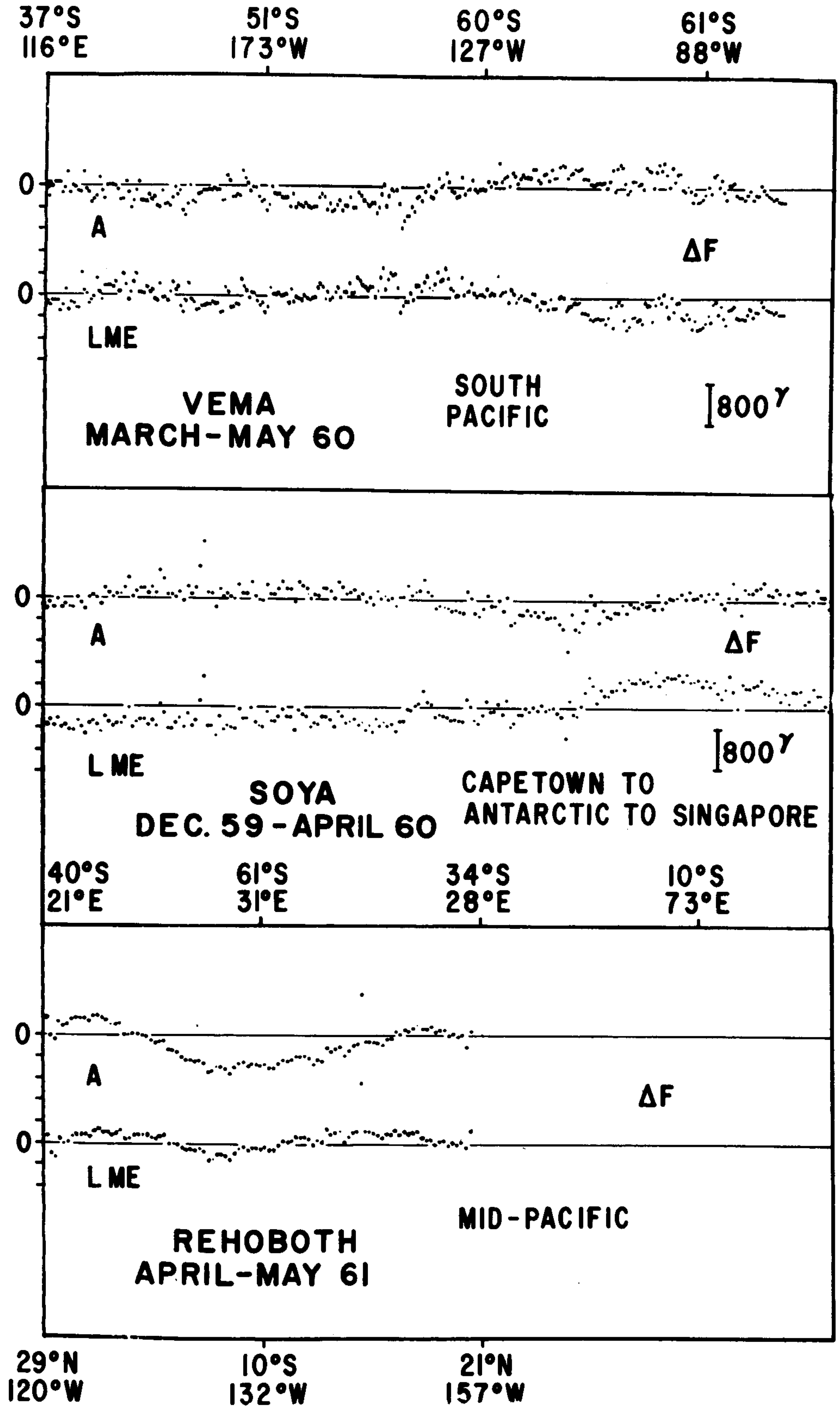

0
A
ΔF
0
LME
MAGNET (316)
12/6/59
NORTH
PACIFIC
800γ
0
A
FΔI
0
LME
0
A
HΔD
0
LME
29°N
177°W
41°N
170°W
42°N
144°W

Fig. 6b



37°S
116°E
51°S
173°W
60°S
127°W
61°S
88°W
0
A
ΔF
0
LME
VEMA
MARCH-MAY 60
SOUTH
PACIFIC
800γ
0
A
ΔF
0
L ME
800γ
SOYA
DEC. 59-APRIL 60
CAPETOWN TO
ANTARCTIC TO SINGAPORE
40°S
21°E
61°S
31°E
34°S
28°E
10°S
73°E
0
A
ΔF
0
L ME
MID-PACIFIC
REHOBOTH
APRIL-MAY 61
29°N
120°W
10°S
132°W
21°N
157°W

Fig. 6c

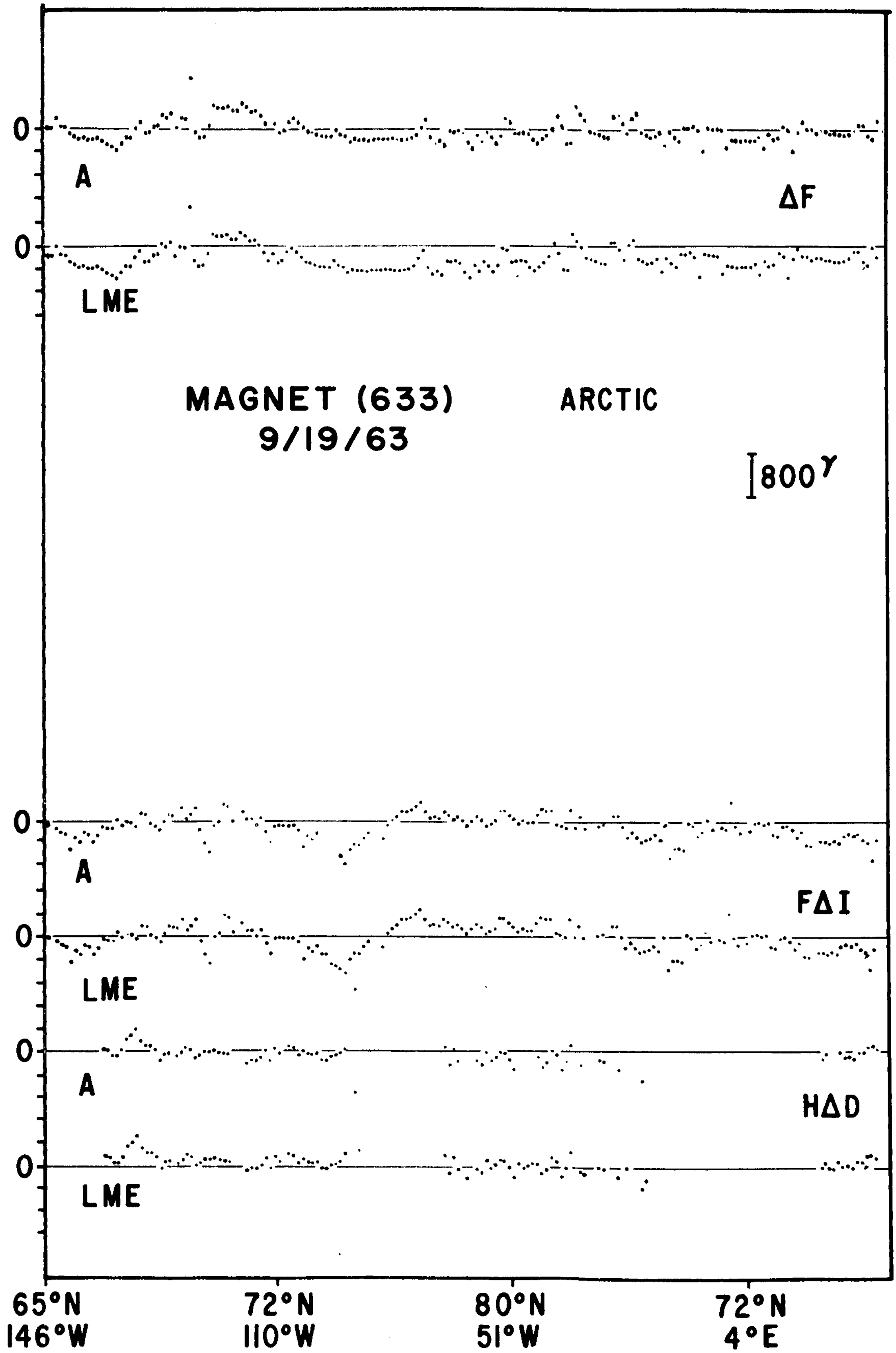
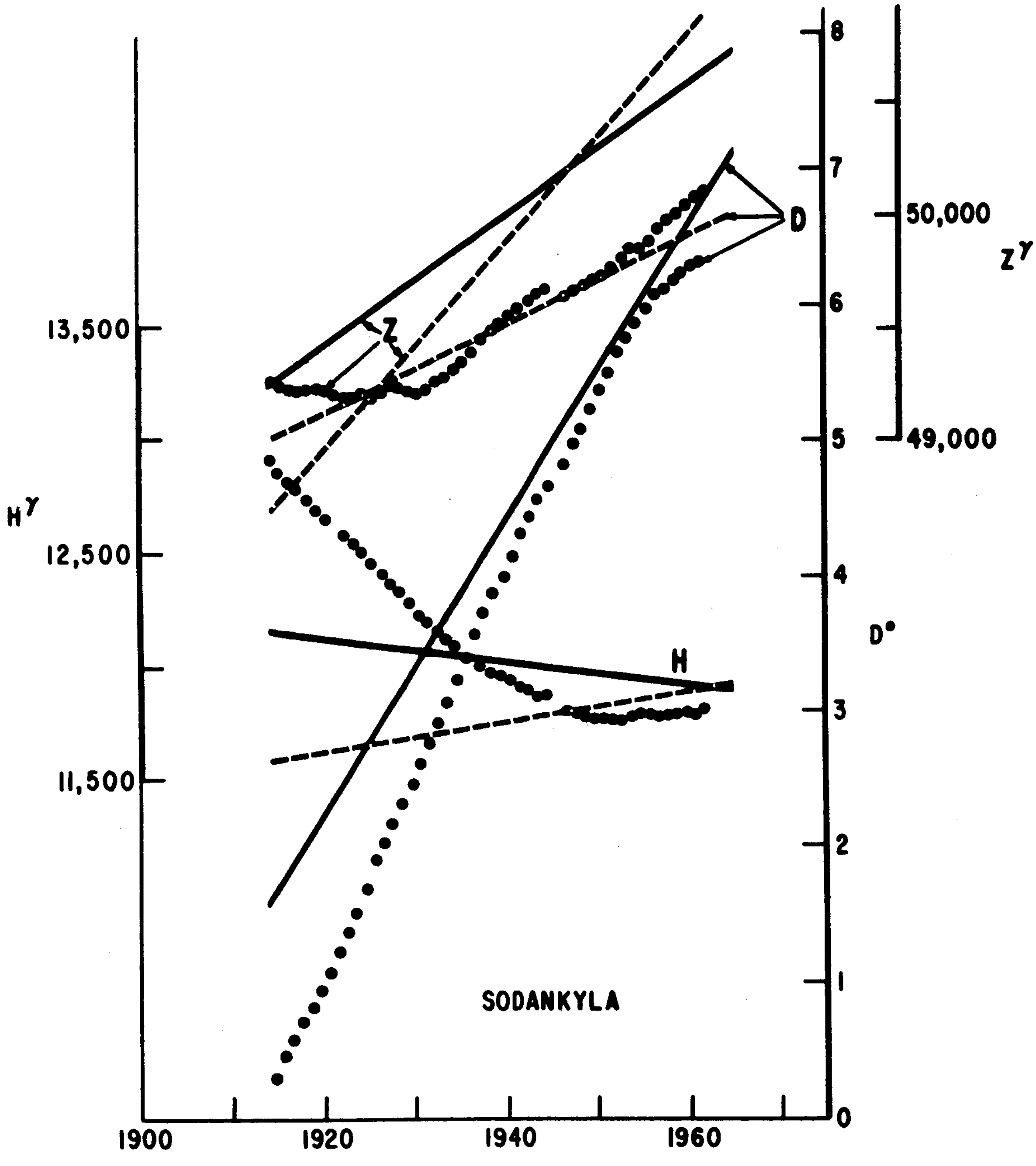

0
A
ΔF
0
LME
MAGNET (633)
ARCTIC
9/19/63
800γ
0
A
FΔI
0
LME
0
A
HΔD
0
LME
65°N
146°W
72°N
110°W
80°N
51°W
72°N
4°E

Fig. 6d

Table 9 thus appear to reflect significant deviations over large distances in addition to the scatter in the observations caused by short-wavelength surface anomalies. Although there are many instances where the data deviations from the two different models are merely different, there are more instances of similarities. For example, for MAGNET (316) the computed D is too low by about a degree for both A and LME. One also finds that the structure of the deviations is often quite similar and that the wavelengths are less than $360/n_m$ so as to again confirm that higher order harmonics are indeed necessary. Such examples can clearly be seen in MAGNET (633) ΔF and FΔI and most of the ZARYA plots.

One can also compare the secular change observed at the magnetic observatories with that predicted by the A and LME models. Sample plots of the annual mean observations at three observatories Alibag, San Juan, and Sodankyla are given in Fig. 7. The dots represent the observations, the solid lines the field components predicted by A and the dashed lines those predicted by LME. The LME model was based on data estimating the secular change at 1965.0 whereas the A model used data selected over the whole interval 1940-1962 but weighted towards more recent times so that the mean date was 1957.2. Thus one would expect the slopes of LME computed values to match those at 1965.0 while the A values should approximate the mean slope over at least the decade of the 1950's. As seen in Fig. 7 this tendency is evident only for the H and D curves at Sodankyla and Alibag. The absolute differences between the annual means at the three observatories and that computed by the two field models are of the same order as one might expect at three surface positions.



13,500
12,500
11,500
H^{γ}
1900
1920
1940
1960
0
1
2
3
4
5
6
7
8
$D°$
49,000
50,000
Z^{γ}
Z
D
H
SODANKYLA

Fig. 7a

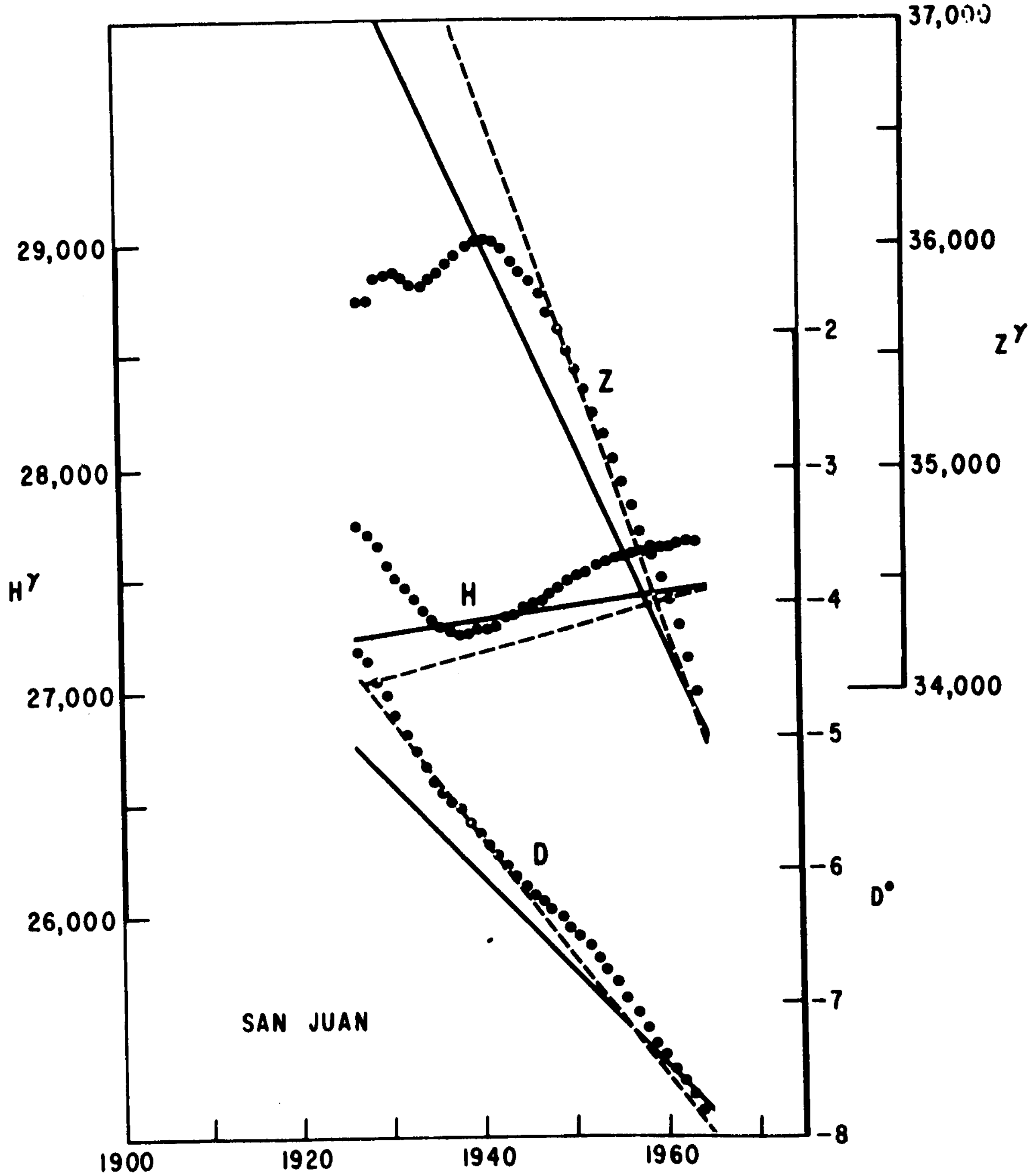
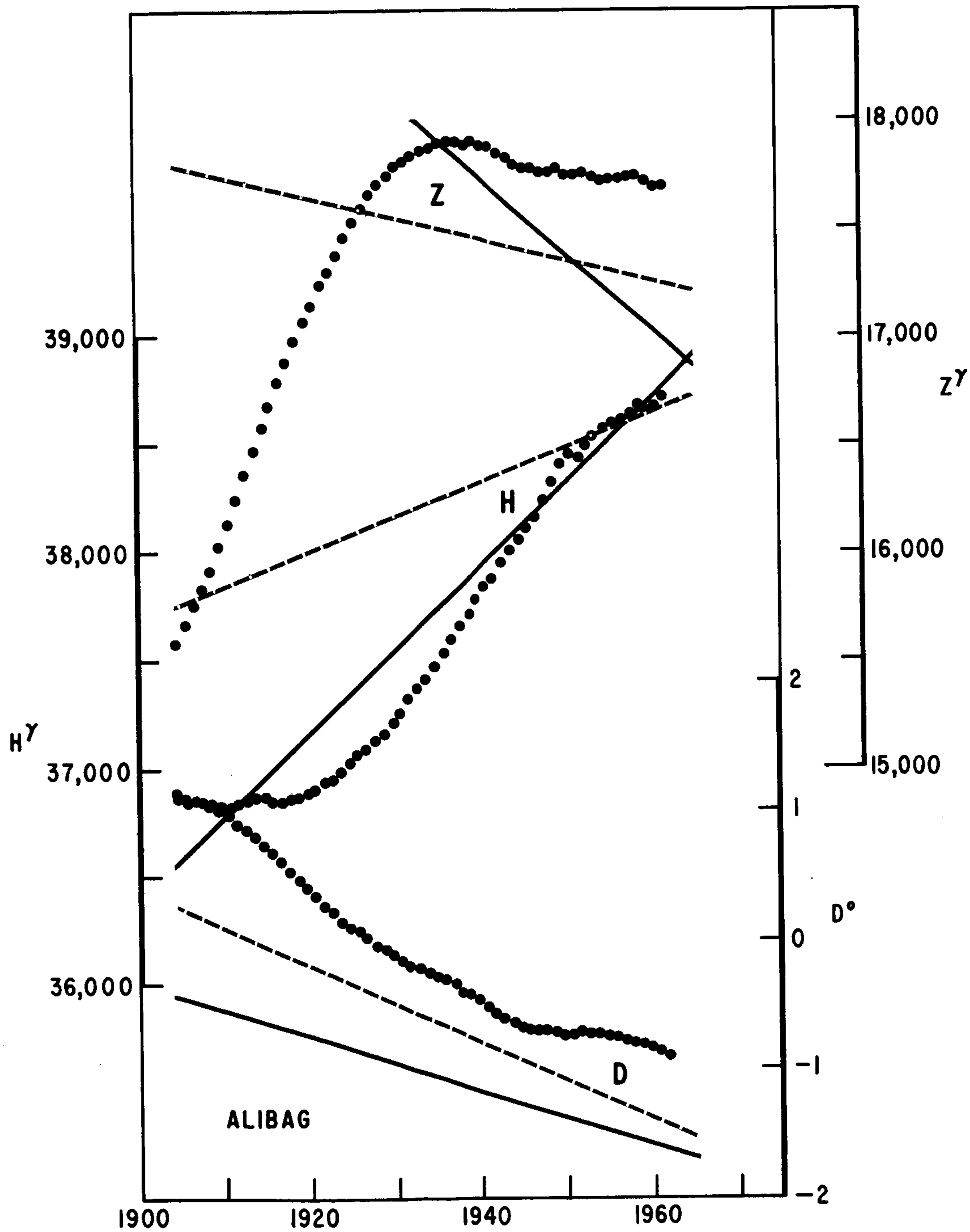

37,000
36,000
35,000
34,000
Z γ
29,000
28,000
27,000
26,000
H γ
Z
H
D
-2
-3
-4
-5
-6
-7
-8
D°
SAN JUAN
1900
1920
1940
1960

Fig. 7b



39,000
38,000
37,000
36,000
H^γ
18,000
17,000
16,000
15,000
Z^γ
2
1
0
-1
-2
D°
Z
H
D
ALIBAG
1900
1920
1940
1960

Fig. 7c

Conclusions

We have presented a new model of the earth's main magnetic field and secular change which is a significant improvement over all past models for the period 1940-1963 with the exception of one recently derived by Leaton, Malin, and Evans (1964). This last model shows a comparable fit to the data but does not extrapolate as well with altitude perhaps partly due to having used the spherical approximation to the earth's surface. The allowance for the oblateness of the earth was illustrated to be a necessary refinement at the levels of accuracy now being reached.

It was apparent in comparing both of the two models against survey data that there are significant higher order harmonics in the field. The deviations from the fits appear to arise at least as much from long wavelength systematic variations as from the small-scale anomaly "noise". Prior conclusions that coefficients of degree exceeding n=6 are not meaningful (Fanselau and Kautzleben, 1956) appear to result from analyzing charts instead of the data themselves. We must conclude that a better field model can be derived by analysis of the raw data than any existing world chart. It appears that better maps could be derived from such analyses as shown here than from more conventional techniques whereby the data are "corrected" to a given epoch and field contours drawn manually for each component.

The present model does not allow for external fields nor did the evaluation make any corrections to the data for magnetic disturbance. A future calculation is planned to see whether any meaningful improvements in the model could be made by the use of external terms and adjustments for the systematic depressions in the field during and following magnetic disturbance (Sugiura, 1964). These further improvements will also include the use of higher degree terms as necessary to match the complexities of the field.

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REFERENCES

Adam, N. V., N. P. Benkova, V. P. Orlov, N. K. Osipov and L. O. Tyurmina, Spherical analysis of the main geomagnetic field for the epochs 1955 and 1958, Geomag. and Aeron (USSR) (in English), 2(5), 785-796 (original Russian 949-962), 1962.

Adam, N. V., N. P. Benkova, V. P. Orlov N. K Osipov, and L. O. Tyurmina, Spherical analysis of the main geomagnetic field and secular variations, Geomag. Aeron, (USSR) (in English) 3(2), 271-285 (Original Russian 336-353) 1963.

Akasofu, S-I, Joseph C. Cain, and Sydney Chapman, The magnetic field of the quiet time proton belt, J. Geophys. Res., 67, 2645-2647, 1962.

Behrendt, John C., Richard J. Wold, Depth of magnetic 'basement' in West Antarctica, J. Geophys. Res., 68, 1145-1153, 1963.

Behrendt, John C., Richard J. Wold, Aero-magnetic survey in West Antarctica 1963, Research Report No. 63-1, University of Wisconsin, 1963.

Blackman, R. B., and J. W. Tukey, The Measurement of Power Spectra, Dover Publications, Inc., New York, 1958.

Cain, Joseph C., Shirley Hendricks, Walter E. Daniels, Duane C. Jensen, Computation of the main geomagnetic field from spherical harmonic expansions, NASA X-611-64-316, 1964.

Cain, Joseph C., I. R. Shapiro, J.D. Stolarik, and J. P. Heppner, Vanguard 3 magnetic field observations, J. Geophys. Res., 67, 5055-5069, 1962.

Chapman, S. and J. Bartels, Geomagnetism, Oxford University Press, London, 1940.

Euler, Harold C., and Peter E. Wasko, An evaluation of various geomagnetic field equations, NASA MTP-AERO - 63-60, 1963.

Fanselau, G., and H. Kautzleben, The analytical representation of the geomagnetic field, part II, together with a presentation of the initial data from a new computation of the potential up to the 15th degree, Jahrbuch 1956 des Adolf-Schmidt obs. fur erdmag. in Niemegk mit Wissenschaftlichen Mitteilungen, 103-120, 1956, (trans. NASA TT F-8118, 1961).

Fanselau, Von Gerhard, Heinz Kautzleben, Otto Locke, Peter Mauersberger, and Kurt Sellien, Die darstellung des geomagnetischen potentials zur epoch 1945.0 durch eine entwicklung nach kugelfunktionem bis zur 15, ordnung, Pure and Appl. Geophys., 57, 5-30, 1964.

Finch, H. F. and B. R. Leaton, The earth's main magnetic field-Epoch 1955.0, Monthly Not. Roy. Astron. Soc., Geophys. Supplement 7(6), 314-317, November, 1957.

Fougere, P. F., Spherical harmonic analysis, 1. A new method and its verification, J. Geophys. Res., 68, 1131-1139, 1963.

Hendricks, S., and J. C. Cain, World magnetic survey data, NASA X-611-63-178, 1963.

Heppner, J. P., The world magnetic survey, Space Sci, Rev., 11, 315-354, 1963.

Heppner, J. P., N. F. Ness, C. S. Scearce, and T. L. Skillman, Explorer 10 magnetic field measurements, J. Geophys. Res., 68, 1-46, 1963.

Heuring, F. T., The analytic description of the geomagnetic field at satellite altitudes, J. Geophys. Res., 69, 4959-4968, 1964.

Hoffman, R. A., and P. A. Bracken, Magnetic effects of the quiet time proton belt, NASA-X-611-64-186, 1964.

Jensen, D. C., and J. C. Cain, An interim geomagnetic field, (Abstract), J. Geophys. Res., 67, 3568-3569, 1962.

Jones, Sir Harold Spencer, F. R. S., and P. J. Melotte, The harmonic analysis of the earth's magnetic field, for epoch 1942, Monthly Not. Roy. Astron. Soc., Geophys. Supplement 6(7), 409-430, June, 1953.

Kautzleben, H., Analytical representation of the geomagnetic field and of the secular variation, Deutsche Akad. der Wiss. zu Berlin, Potsdam Geom. Inst. Treat. No. 32, Akad. Verlag, Berlin, 1963, (trans. NASA TT-F-8700, 1963).

Leaton, B. R., Geomagnetic secular variation for the epoch 1955.0, Roy. Obs. Bull. 57, 1962.

Leaton, B. R., Remarks on paper by P. F. Fougere, 'Spherical Harmonic Analysis', J. Geophys. Res., 68, 5902, 1963.

Leaton, B. R., S.R.C. Malin and Margaret J. Evans, An analytical representation of the estimated geomagnetic field and its secular change for the epoch 1965.0, IAGA Symposium on Magnetism of the Earth's Interior, Pittsburgh November, 1964.

Leonard, Robert S., Selection of a model of the earth's magnetic field, J. Geophys. Res., 68, 6437-6440, 1963.

Mauersberger, P., A discussion of the variation with time of the parameters of the geomagnetic field based on the existing potential analysis, Abhandlungen des Geophysikalisch Instituts Potsdam, No. 5, 9-58, 1952, (trans. NASA TT-F-8443, (1963).

Mead, Gilbert D., Deformation of the geomagnetic field by the solar wind, J. Geophys. Res., 69, 1181-1195, 1964.

Midgley, James E., Perturbation of the geomagnetic field - a spherical harmonic expansion, J. Geophys. Res., 69, 1197-1200, 1964.

Nagata, T., and T. Oguti, Magnetic charts for the epoch of 1958.5 corrected for the Antarctic region and spherical harmonic coefficients of the revised geomagnetic field, J. Geomag. and Geoelectr. XIV, 125-131, 1962.

Schmidt, Adolf, Tables of normalized spherical functions and their derivatives with the logarithms of their values and formulas for expansions in spherical functions, Engelhard-Reyher Verlag, Gotha, 1935, (trans. NASA TT-F-8888, 1964).

Serson, P. H., Airborne magnetic surveys for world charts, (presented) The Symposium of WMS IAGA, Helsinki, July 1960.

Serson, P. H., and W. L. W. Hannoford, A statistical analysis of magnetic profiles, J. Geophys. Res., 62, 1-18, 1957.

Sugiura, Masahisa, Hourly values of equatorial Dst for the IGY, Annals of the International Geophysical Year, 35, part I, 9-45, 1964.

Vestine, E. H., I. Lange, L. Laporte, and W. E. Scott, The geomagnetic field, its description and analysis, Carnegie Institution of Washington, Publication 580, 1947.

Vestine, E. H., The survey of the geomagnetic field in space, Trans. A. G. U., 41, 4-21, 1960.

Vestine, E. H., W. L. Sibley, J. W. Kern, and J. L. Carlstept, Integral and spherical-harmonic analysis of the geomagnetic field for 1955.0, J. Geomag. and Geoelec., XV, 47-89, 1963.

WMS Notes No. 2, World Magnetic Survey Board, October, 1964.

Airborne geomagnetic data 1953-1961, U. S. Naval Oceanographic Office Spec. Pub. 66, 1963.