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CONSTANT AMPLITUDE, VARIABLE PHASE FILTERS

by Lonnie Joe Rogers and David S. Hepler

Goddard Space Flight Center

Greenbelt, Md.

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SUMMARY

A lossless, constant amplitude, variable phase filter is derived from a general bridge-T network. Certain balance and load conditions are necessary to maintain a constant voltage while changing the phase. By using a voltage variable capacitor, the phase of a sinusoidal voltage can be controlled by the network. The phase response, as a function of voltage, is shown for several specific networks. Examples of methods for altering the phase response of a particular network are demonstrated. An application of the network as a phase modulator for a radio frequency transmitter is illustrated.

author

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INTRODUCTION

With the increased use of phase modulation in recent years as a means of transmitting information on a carrier signal, and the extensive use of transistors as signal processors in satellites, needs for various special circuits have arisen. One such circuit is the constant amplitude, variable phase filter, a network that can produce voltage controlled phase changes in the carrier signal without changing its amplitude. This has many advantages over conventional phase modulation techniques.

Constant-amplitude phase modulation permits class C transistor power amplifiers to operate at maximum efficiency. In narrow-band phase-lock reception, short term oscillator stability is a necessity. The variable phase filter is an asset to oscillator stability, resulting in simplified stabilization circuitry.

The network described here provides large phase deviations with either S-shaped or linear response curves. It is based upon a design developed for an early satellite transmitter (Reference 1). Synthesis techniques will be used to find certain boundary conditions, and then some of the simpler forms will be analyzed.

VARIABLE PHASE FILTER FROM A BALANCED LATTICE

Constant Amplitude Network

The constant amplitude, variable phase filter is expected to have the following characteristics: (1) constant input impedance, (2) control of the phase of the sinusoidal voltage, (3) constant amplitude independent of phase change, and (4) zero loss. The lossless nature is not a characteristic by definition but better fits the applications discussed here.

The network presently has the form of a lossless two-port circuit terminated in its characteristic impedance, a pure resitance (R) as shown in Figure 1; however, it may be a balanced lattice, a bridged-T, or a more complicated four-terminal form. It is desired that the network present a constant resistance while one or more internal impedance elements are changed to control the phase. If the input resistance remains a constant R_0 and the network is loaded with that resistance, then the requirement for constant amplitude is satisfied.

For illustrative purposes only, assume the network exists in a balanced lattice form. This form is chosen because of the available information in textbooks on conditions and requirements for constant resistance (Reference 2, pp. 477-479, and Reference 3, pp. 172-182). The necessary and sufficient conditions that the balanced lattice be a constant resistance is that $\sqrt{Z_a Z_b}$ equal R_0, where Z_a and Z_b are the lattice impedances as shown in Figure 2. This result can be verified by computing the image input impedance, Z_{I_1} or Z_{I_M} which is equal to the square root of the product of the short circuit input impedance and the open circuit input impedance. The quantities Z_{11} and $1/Y_{11}$ are the respective open circuit and short circuit input impedances Z_{oc} and Z_{sc}, and

$$Z_{I_1} = \sqrt{Z_{oc}\, Z_{sc}} = \sqrt{\frac{Z_{11}}{Y_{11}}} \quad . \tag{1}$$

From Figure 2, Z_{11} and Y_{11} can be found:

$$Z_{11} = \frac{1}{2\left(Z_a + Z_b\right)}$$

and

$$Y_{11} = \frac{1}{\dfrac{2Z_aZ_b}{Z_a + Z_b}} = \frac{Z_a + Z_b}{2Z_aZ_b} \quad . \tag{2}$$

Then the image input impedance, Z_{I_1}, becomes:

$$Z_{I_1} = \sqrt{\frac{Z_{11}}{Y_{11}}} = \sqrt{\frac{1/2\left(Z_a + Z_b\right)}{\dfrac{Z_a + Z_b}{2Z_aZ_b}}} = \sqrt{Z_aZ_b} \quad . \tag{3}$$

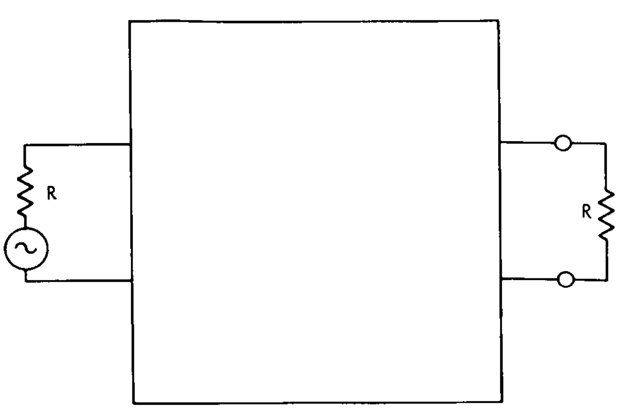

Figure 1—Zero loss network.

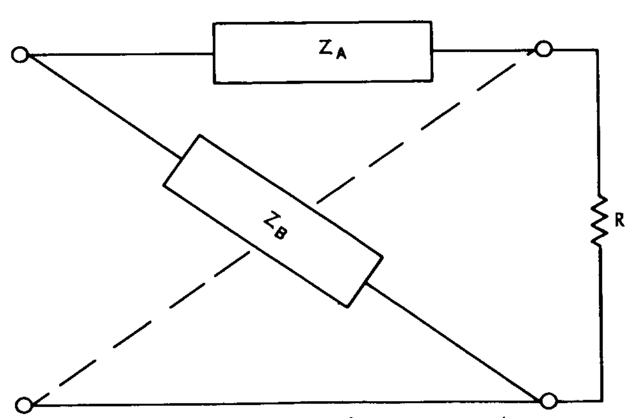

Figure 2—Balanced lattice network.

If Z_{I_1} is equal to the load resistance R_L , then the network has a constant resistance or a constant amplitude output. Then the basic design criterion for the lattice network is:

$$\left.\begin{aligned} R_L &= \sqrt{Z_a Z_b} \ , \\ \text{so that} \qquad & \\ R_L^{\,2} &= Z_a Z_b = R_0^{\,2} \ . \end{aligned}\right\} \tag{4}$$

Equation 4 shows that however Z_a and Z_b are varied, their product must be a real number. Obviously, Z_a and Z_b may be resistors but this would not meet the "lossless" criterion. Equation 4 insures that the network will have a constant resistance, but does not allow a change in output phase. The term impedance (Z) will be used for generality throughout the discussion, but to preserve the lossless nature of the network the impedance will be reactive when a choice is available.

To continue the illustration, the problem is to make a variable phase filter from the balanced lattice structure. Assume for the present that the only variable element is a variable capacitance with a reactance X_c ; then a variable phase filter can be derived by using classical network theory. Let Z_a be a capacitive reactance $-jX_c$ paralleled with an arbitrary inductor jX_L as shown in Figure 3. Impedance Z_b is derived from Equation 4. Thus

$$Z_a = \frac{(-jX_c)\,(jX_L)}{-jX_c + jX_L} = \frac{X_c X_L}{j\,(X_L - X_c)} \ , \tag{5}$$

and

$$Z_b = \frac{R_0^{\,2}}{Z_a} \ . \tag{4}$$

From this

$$Z_b = \frac{R_0^{\,2}}{\dfrac{X_c X_L}{j\,(X_L - X_c)}} = \frac{R_0^{\,2}\ j\,(X_L - X_c)}{X_c X_L} = \frac{jR_0^{\,2}}{X_c} - \frac{jR_0^{\,2}}{X_L} \ .$$

Adding and subtracting jX_L from Z_b , we have

$$Z_b = jX_L + \frac{-jR_0^{\,2}}{X_L} + -jX_L + \frac{jR_0^{\,2}}{X_c} = \frac{jR_0^{\,2}}{X_c} + jX_L + \frac{-j\,(R_0^{\,2} + X_L^{\,2})}{X_L} \ . \tag{6}$$

Three Terminal Form

It is desirable to reduce the lattice network to a three-terminal form. This will be done on a step by step basis, but no attempt will be made to justify the changes since they are shown in some textbooks (Reference 2, pp. 174-178).

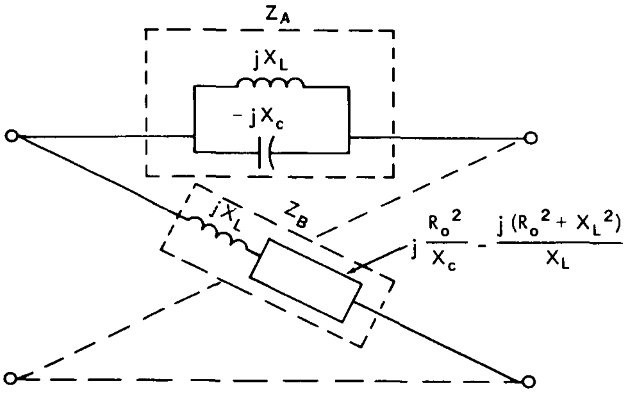

Figure 3—Network meeting "constant amplitude" and "lossless" conditions.

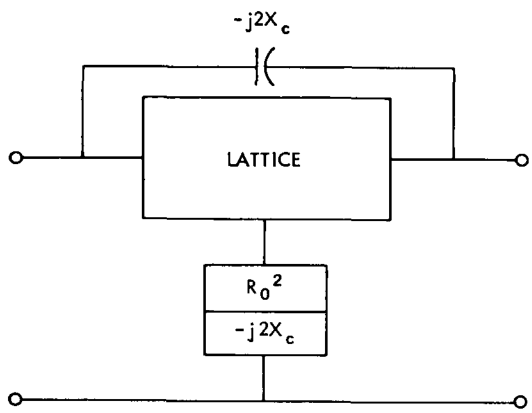

Figure 4—Network reduced to three terminal form.

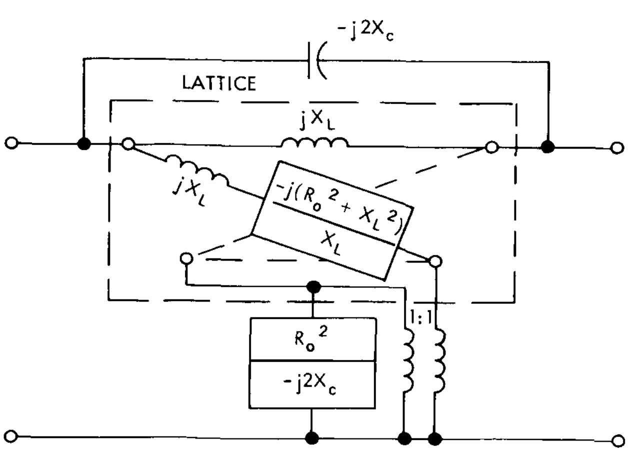

Figure 5—Overall network.

The paralleled element $-jX_c$ of Z_a can be brought outside the lattice to an unbalanced three terminal network, and the series element jR_0^2/X_c can be put into the center leg of the external three terminal network (Figure 4). Figure 5 shows the overall network and also reveals the problem of connecting the two lower terminals to the common center leg. This has been encountered in similar balanced lattice reductions and can be solved by using an ideal transformer (Reference 2). Since the lattice will be completely reduced, its use will not be necessary in the final form.

The common series element jX_L in the new lattice impedances Z_a' and Z_b' may be removed, as shown in Figure 6. Now the remaining lattice has a impedance in parallel with itself, and its effective value is one-half its original value. Removing this impedance and redrawing the network shows its final form (Figure 7).

The upper center leg impedance $-j(R_0^2 + X_L^2)/2X_L$ will be a constant after the values of R_0 and X_L have been selected. The impedance $jR_0^2/2X_c$ is realized by a capacitor with a reactance $-j2X_c$, connected by a quarter-wave transmission line having a characteristic impedance R_0 (Figure 8). The line may be either real or artificial and will be the primary factor limiting the bandwidth of the network. The reactance $-jX_c$, assumed to be the variable element, will consist of two identical variable capacitors placed at the specified points in the network.

Since the elements are lossless and the equivalent lattice meets the "constant resistance" criterion, the magnitude of the voltage transfer ratio (E_2/E_1) will be unity when the network is terminated in its characteristic impedance. The phase function, θ, will be a function of the chosen constants and of X_c.

So far, what has been done was for illustrative purposes and showed that a general constant amplitude, variable phase filter is realizable. The phase response as a function of X_c has not been determined.

Because of the previous assumptions (arbitrary inductive reactance jX_L and variable capacitive reactance $-jX_c$) and the limited flexibility of the design procedure, no further analysis will be attempted on this particular network. Emphasis will be placed on designing a general network that has a constant amplitude and variable phase but is more flexible in design.

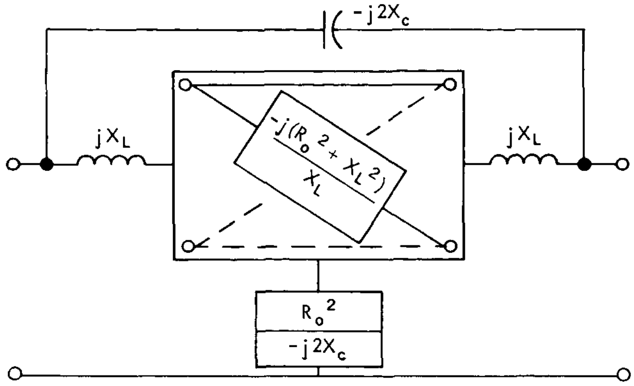

Figure 6—Network with jX_L removed from lattice.

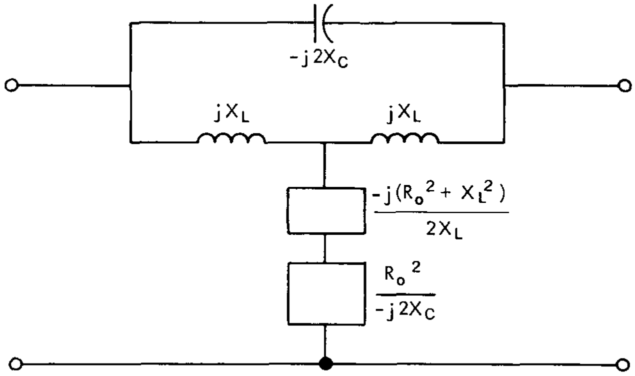

Figure 7—Final form of constant amplitude variable phase filter.

THREE-TERMINAL VARIABLE PHASE FILTER

Matrix Equation

It has been shown that the desired filter exists in a three-terminal form; now an attempt will be made to synthesize such a network. Again the two-port approach will be used with the help of the matrix equations. The Z_{ij} and Y_{ij} parameters given below are illustrated in Figure 9:

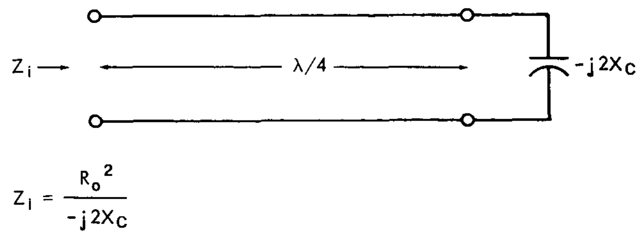

Figure 8—Quarter-wave transmission line with characteristic impedance R_0.

$$E_1 = Z_{11}I_1 + Z_{12}I_2 , \qquad (7)$$

$$E_2 = Z_{11}I_1 + Z_{22}I_2 , \qquad (8)$$

$$I_1 = Y_{11}E_1 + Y_{12}E_2 , \qquad (9)$$

$$I_2 = Y_{21}E_1 + Y_{22}E_2 . \qquad (10)$$

Since the network is passive and reciprocal, Z_{12} is equal to Z_{21} and Y_{12} is equal to Y_{21}. Another simplification that will greatly reduce the algebra without limiting the usefulness is making the network symmetrical as well as three-terminal; that is, $Z_{11} = Z_{22}$ and $Y_{11} = Y_{22}$. The three-terminal equations in simplified form are

$$E_1 = Z_{11}I_1 + Z_{12}I_2 \,, \tag{11}$$

$$E_2 = Z_{12}I_1 + Z_{11}I_2 \,, \tag{12}$$

$$I_1 = Y_{11}E_1 + Y_{12}E_2 \,, \tag{13}$$

$$I_2 = Y_{12}E_1 + Y_{11}E_2 \,. \tag{14}$$

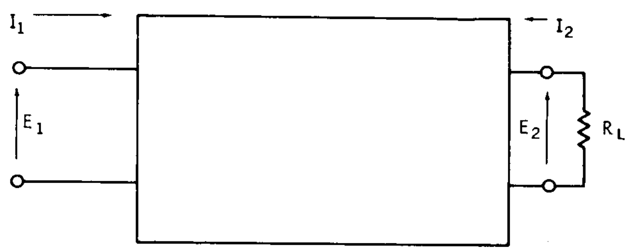

Figure 9–General two-port network.

To find the voltage (or current) transfer ratio E_2/E_1, assume that the two-port is loaded in a pure resistance, R. Later the effects of the value of R will be discussed. Notice that $E_2/I_2 = -R$ because of the convention of sign (Figure 9).

$$\frac{E_2}{I_2} = -R = \frac{-1}{G} \,. \tag{15}$$

Solving Equation 11 and 12 with Equation 15 results in

$$\frac{I_2}{I_1} = \frac{Y_{12}}{|Y|R + Y_{11}} = \frac{-Z_{12}}{R + Z_{11}} \,. \tag{16}$$

By requiring that the network's input impedance be equal to its load resistance $(E_1/I_1) = R$, the following relations can be derived:

$$\frac{E_2}{E_1} = \frac{G - Y_{11}}{Y_{12}} = \frac{-Y_{12}}{G + Y_{11}} \,, \tag{17}$$

$$\frac{E_2}{E_1} = \frac{R - Z_{11}}{-Z_{12}} = \frac{+Z_{12}}{R + Z_{11}} \,, \tag{18}$$

$$\frac{I_2}{I_1} = \frac{-E_2}{E_1} \,. \tag{19}$$

The requirement that made the network's input impedance be equal to its load resistance R, simplified the immittance determinants to

$$R^2 = Z_{11}^2 - Z_{12}^2 = [Z] \ , \tag{20}$$

and

$$G^2 = Y_{11}^2 - Y_{12}^2 = [Y] \ . \tag{21}$$

These equations are useful for checking and interrelating Equations 15 to 17 which were previously derived. They also note that $Z_{11} + Z_{12}$ and $Z_{11} - Z_{12}$ are conjugate impedances having a product that is a pure resistance.

Little has been said about the form of the network. From the previous section, it is known that the three terminal network can be a bridged-T and that form will be assumed. The presently defined form of the bridged-T is shown in Figure 10 with arm impedances (Z) made equal for symmetry. Although they will be chosen as reactive to keep the transfer function lossless, impedances Z_1 and Z_2 are still arbitrary.

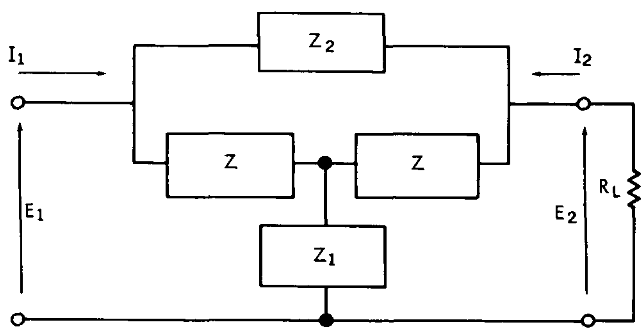

Figure 10—Three terminal bridged-T network.

To find the voltage transfer function, it will be necessary to find Z_{11}, Z_{12} or Y_{11}, Y_{12}. The algebra is somewhat tedious but not new (Reference 4, p. 69); and the results are as follows:

$$Z_{11} = Z_{22} = \frac{2ZZ_1 + Z_2Z_1 + Z^2 + ZZ_2}{2Z + Z_2} \ , \tag{22}$$

$$Z_{12} = Z_{21} = \frac{Z^2 + 2ZZ_1 + Z_1Z_2}{2Z + Z_2} \ , \tag{23}$$

$$Y_{12} = Y_{21} = \frac{Z^2 + 2ZZ_1 + Z_1Z_2}{Z_2\left(Z^2 + 2ZZ_1\right)} \tag{24}$$

$$Y_{11} = Y_{22} = \frac{ZZ_2 + Z_1Z_2 + Z^2 + 2ZZ_1}{Z_2\left(Z^2 + 2ZZ_1\right)} \ . \tag{25}$$

The respective matrices for the transfer functions are:

$$[Y] = \frac{2Z + Z_2}{ZZ_2\left(Z + 2Z_1\right)} \ , \tag{26}$$

and

$$[Z] = \frac{ZZ_2\left(Z + 2Z_1\right)}{2Z + Z_2} \ . \tag{27}$$

Before computing the voltage (or current) transfer ratio, a closer look into the boundary conditions will be helpful. One condition is Equation 20. Inserting Z_{11} and Z_{12} into Equation 20 and simplifying, results in

$$Z_1 = \frac{R^2}{2Z} - \frac{Z}{2} + \frac{R^2}{Z_2} \quad . \tag{28}$$

The implications of this equation may not be obvious until it is compared with Figure 7. The first two terms will be a reactive constant for any given conditions, and its third term is realizable with a quarter-wave transmission line. Impedance Z_1 is composed of a constant reactance plus a varying function of Z_2 .

There is physical significance attached to the reappearing quarter-wave transmission line. A varying capacitance on the end of a quarter-wave transmission line appears to be a varying inductance on its open circuit end. Solving Equation 20 for Z_2 will give a different result that also has significance:

$$Z_2 = \frac{2ZR^2}{2Z_1Z - R^2 + Z^2} = \frac{1}{\frac{Z_1}{R^2} + \frac{Z^2 - R^2}{2ZR^2}} = \frac{1}{\frac{1}{\frac{R^2}{Z_1}} + \frac{1}{\frac{2ZR^2}{Z^2 - R^2}}} \quad . \tag{29}$$

Equation 29 appears to be a constant impedance in parallel with a quarter-wave transmission line which is terminated in Z_1 . Noting the position of Z_2 and Z_1 in the network (Figure 10), it can be concluded that placing an impedance in series with Z_1 is equivalent to placing an element in parallel with Z_2. This result can be important in eliminating unnecessary components when constructing the network.

Equation 28 offers another advantage when analyzing a network; in that the network response equation can be reduced to contain only one variable (Z_1 or Z_2), although the actual network has both. This could be expected, since Equation 28 is a boundary condition for constant resistance, giving a finite relationship between Z_1 and Z_2 that must be maintained.

Equation 28 will be preferred over Equation 29 throughout this paper due to its simpler separation of constants and variables. Impedance Z_2 will be used to determine the phase response.

Voltage Transfer Ratio

The voltage transfer ratio may be found from either Equation 17 or 18 (repeated below) by replacing the Z and Y parameters with their derived equivalents. The phase response can then be determined from the voltage transfer ratio:

$$\frac{E_2}{E_1} = \frac{R - Z_{11}}{-Z_{12}} = \frac{Z_{21}}{R + Z_{11}} \quad , \tag{18}$$

$$\frac{E_2}{E_1} = \frac{G - Y_{11}}{Y_{12}} = \frac{-Y_{12}}{G + Y_{11}}, \tag{17}$$

$$\frac{E_2}{E_1} = \frac{-R(2Z + Z_2) + ZZ_2}{Z^2 + 2ZZ_1 + Z_1Z_2} + 1 = \frac{1}{\dfrac{R(2Z + Z_2) + ZZ_2}{2ZZ_1 + Z^2 + Z_1Z_2} + 1}, \tag{30}$$

$$\frac{E_2}{E_1} = \frac{GZZ_2(2Z_1 + Z) - ZZ_2}{Z^2 + 2ZZ_1 + Z_1Z_2} - 1 = \frac{-1}{\dfrac{GZ_2Z(2Z_1 + Z) + ZZ_2}{2ZZ_1 + Z^2 + Z_1Z_2} + 1}. \tag{31}$$

Equations 30 and 31 are in their simplest form until more is known about the network. The network under study is now shown in Figure 11. As mentioned earlier, the impedances are reactive in order to preserve the lossless nature of the network. Each impedance may contain a number of elements rather than a simple one. The cluster of elements, however, must be reducable to a definable value at any operating point.

For simplicity the arm elements Z will be chosen to be a single reactive element X. Impedances Z_1 and Z_2 will be in general a number of reactive elements that form X_1 and X_2. The reduced network is shown in Figure 12.

Using X, X_2, and R_0 as the elements to determine the phase response was justified in Equation 28, and it remains to determine the voltage transfer ratio.

The remaining variables are X_2, X, and R_0 (frequency will be assumed constant for any one design) and one of the variables must be eliminated in order to determine the phase response. One method that will maintain generality is to define X_2 and X in terms of the characteristic impedance R_0.

The phase response of the network can be determined without assigning a value to R_0. Reactances X_1, X_2, and X will have value that are normalized to the magnitude of the characteristic impedance, R_0. That is, if the arm impedance Z is equal to jR_0 and R_0 is equal to 100, then Z is $j100$. If R_0 is equal to 1,000, then Z will be $j\,1{,}000$. Although this normalization may seem more complicated, the results will be more versatile.

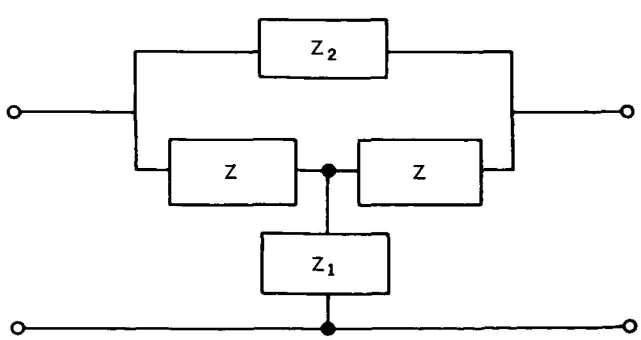

Figure 11—General bridged-T network.

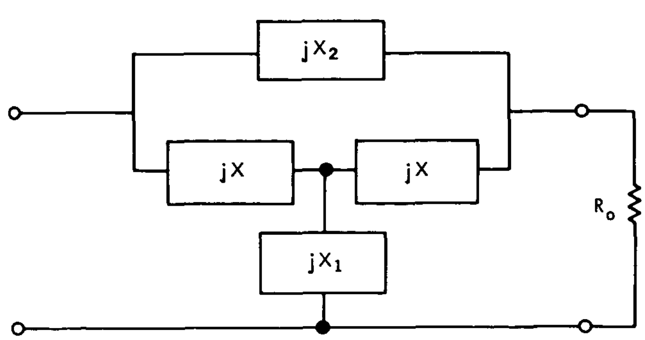

Figure 12—Lossless bridged-T network.

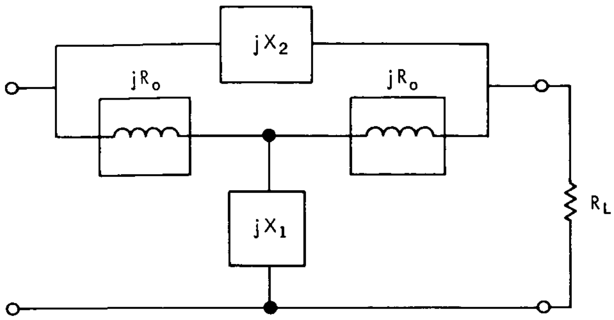

Figure 13—Network with arm impedances equal to jR_0.

To continue the analysis, a value will be assigned to the arm impedances. A large number of values would be necessary for a complete analysis, but only six values $j\,\frac{1}{2}\,R_0$, jR_0, $j2R_0$, $-j\,\frac{1}{2}\,R_0$, $-jR_0$, and $-j2R_0$ will be used for an indicative result. The case of X equal to jR_0 will be developed on a step by step basis, while only the results will be shown for the others. The network is shown in Figure 13.

From Equation 28, Z_1 can be expressed as shown:

$$Z_1 = \frac{R^2 - Z^2}{2Z} + \frac{R^2}{Z_2} = jX_1 = \frac{R^2 + R^2}{j2R} + \frac{R^2}{jX_2}, \tag{28}$$

and

$$jX_1 = -jR + \frac{-jR^2}{X_2} = -j\left(R + \frac{R^2}{X_2}\right).$$

Then

$$\frac{E_2}{E_1} = \frac{-R(2Z + Z_2) + ZZ_2 + Z^2 + 2ZZ_1 + Z_1Z_2}{Z^2 + 2ZZ_1 + Z_1Z_2}, \tag{32}$$

and

$$\frac{E_2}{E_1} = \frac{2R^2 + 2RX_2 - jX_2(X_2 + 2R)}{2RX_2 + 2R^2 + X_2{}^2}, \tag{33}$$

and finally

$$\theta = \tan^{-1}\frac{-X_2(X_2 + 2R)}{2R(R + X_2)}. \tag{34}$$

Equation 33 and 34 describe the phase and amplitude response. It will now be shown that the magnitude of E_2/E_1 equals one. Note that E_2/E_1 has the complex form of $(a + jb)/c$. The magnitude of a complex number $x + jy$ is $\sqrt{x^2 + y^2}$; then it follows that the magnitude of E_2/E_1 is $\sqrt{(a^2 + b^2)/c^2}$. Since, in this case, a, b, and c will be polynomials; c will remain under the square root; and the proof that the magnitude of E_2/E_1 equals one will be completed when c^2 is term-by-term equal to $a^2 + b^2$:

$$\left|\frac{E_2}{E_1}\right| = \sqrt{\frac{[2R^2 + 2RX_2]^2 + [X_2(X_2 + 2R)]^2}{[2RX_2 + 2R^3 + X_2{}^2]^2}}, \tag{35}$$

$$\left|\frac{E_2}{E_1}\right| = \sqrt{\frac{X_2^4 + 4RX_2^3 + 8R^2X_2^2 + 8R^3X_2 + 4R^4}{X_2^4 + 4RX_2^3 + 8R^2X_2^2 + 8R^3X_2 + 4R^4}} = 1 \tag{36}$$

Equation 36 applies only when the network is loaded with its characteristic impedance R_0. The condition of any load R_L will be considered later. The magnitude function will be computed for the network $Z = jR_0$, and will be unity when Z is equal to any reactive value, inductive or capacitive.

Consideration should be given to other values of Z besides jR_0 before proceeding any further in evaluating the phase response of the network. As mentioned, the values of Z to be used as indicative results are $j\,\frac{1}{2}\,R_0$, jR_0, $j2R_0$, $-j\,\frac{1}{2}\,R_0$, $-jR_0$, and $-j2R_0$. Using Equations 27 and 29, an expression for the phase angle, θ, as a function of R and X_2 can be extracted. The circuit for X_2 is shown in Figure 14.

$$\text{For } Z = j\,\frac{R_0}{2}, \quad \theta = \tan^{-1}\frac{-4X_2(R + X_2)}{3X_2^2 + 8RX_2 + 4R^2}; \tag{37}$$

$$\text{for } Z = jR_0, \quad \theta = \tan^{-1}\frac{-X_2(X_2 + 2R)}{2R(R + X_2)}; \tag{38}$$

$$\text{for } Z = j2R_0, \quad \theta = \tan^{-1}\frac{-4X_2(4R + X_2)}{-3X_2^2 + 8RX_2 + 16R^2}; \tag{39}$$

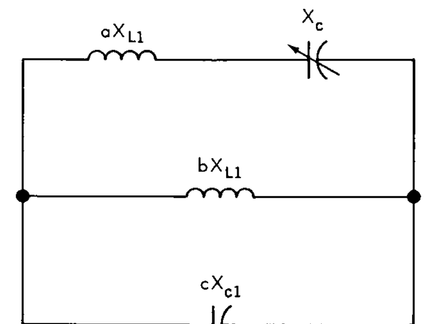

Figure 14—Reactance X_2.

$$\text{for } Z = -j\,\frac{R_0}{2}, \quad \theta = \tan^{-1}\frac{4X_2(X_2 - R)}{3X_2^2 - 8RX_2 + 4R^2}; \tag{40}$$

$$\text{for } Z = -jR_0, \quad \theta = \tan^{-1}\frac{X_2(X_2 - 2R)}{2R(R - X_2)}; \tag{41}$$

$$\text{for } Z = -j2R_0, \quad \theta = \tan^{-1}\frac{+4X_2(X_2 - 4R)}{-3X_2^2 - 8RX_2 + 16R^2}. \tag{42}$$

If X_2 is expressed by a normalized number as the magnitude of R, such as jKR where $\infty \leq K \leq -\infty$, curves can be drawn of (θ) as a function of the magnitude of X_2. Curves are shown for Equation 37 through 42, which provide useful information for the network designer (Figure 15). A point of interest is that $X_2 = 0$ when $\theta = 0$. This could have been anticipated, noting that X_2 is the bridge element and $X_2 = 0$ is equivalent to a short circuit across the network.

Reactance X_2 can be either a variable element or a combination of fixed and variable elements. For applications discussed in this paper, X_2 will consist of one variable capacitor and no more than three fixed elements, as shown in Figure 14. Magnitude constants a, b, and c are arbitrary and may be zero.

Variable Capacitance Diode

An electronic means of varying a circuit capacitance is now available in a special class of diodes known as voltage variable capacitance diodes (varactors). In these P-N junction diodes, the capacity across the junction varies inversely as a function of the applied voltage. Over most of the useful range of the diode, the capacity varies inversely with the square root of the applied voltage; that is, C equals $K/\sqrt{V}$ where V is the applied voltage and K is a constant depending upon the particular diode. Three practical numbers for K that are commercially available are 20×10^{-12}, 44×10^{-12}, and 94×10^{-12} farads. Most manufacturers use four volts as the standard for measurement of the low voltage diodes for determining K. The diode constant K can be replaced by a new constant K_1, by normalizing the standard to one volt. This will be done using the new practical values as 10, 22, and 47 picofarads, and the normalized voltage will be designated as E_N. Thus C equals $K_1/\sqrt{E_N}$.

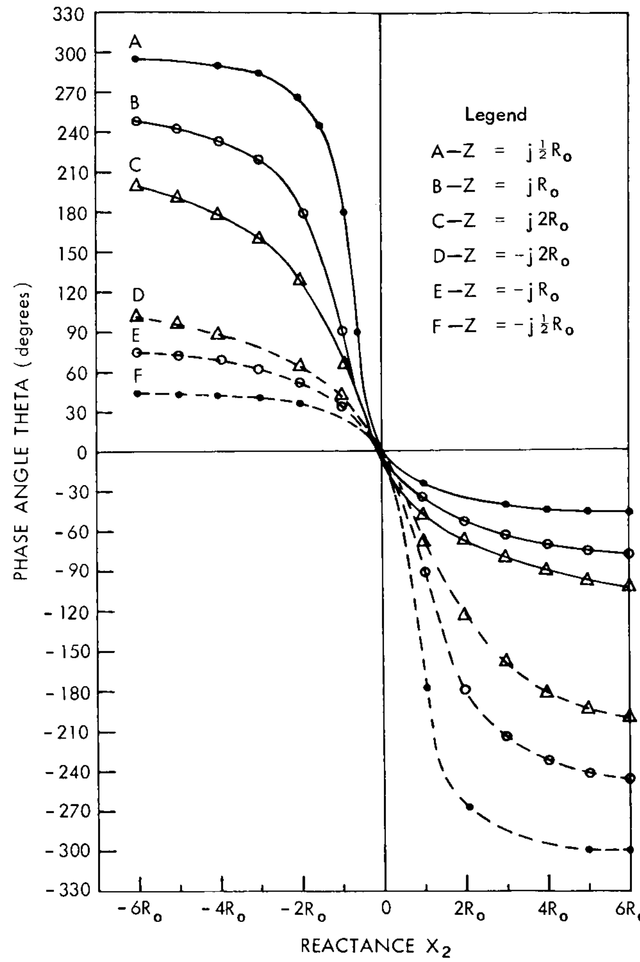

Figure 15—Phase shift as a function of X_2.

The capacitor's reactance ($X_c = 1/\omega C$) is: $X_c = (1/\omega K_1)/\sqrt{E_N} = \sqrt{E_N}/\omega K_1$. For any one frequency and particular diode, ωK_1 is a constant that can be replaced by M, where $M = 1/\omega K_1$. Reactance X_c is now $M\sqrt{E_N}$. For frequencies between 20 and 150 Mc, values of $M = 100$ are easily available and simplify calculations. A value of $M = 100$ may also be attainable at audio and microwave frequencies making the discussion directly applicable there. Due to the present availability of diodes, however, minor modifications may be necessary at audio and microwave frequencies. All phase response calculations will use $M = 100$, and effects of other values can be determined by moving along the normalized voltage scale since $X_c = 100\sqrt{E_N}$.

To illustrate the usable range of the formulas stated, a graph of C versus voltage is shown along with the formula $C = K/\sqrt{E_N}$ (Figure 16). As expected, the formula is a useful approximation for a definite range of values. All calculations using the formula will be reasonably accurate.

Phase and Voltage Characteristics

Analysis of the constant amplitude, variable phase network will be complete with an appropriate reactance X_2. Reactance X_2 can be a complicated combination of elements; however, the analysis will also be complicated. The general reactance X_2 used in this paper is shown in Figure 17.

Multiplier constants a, b, and c are magnitude determining values. Note that b and c will never occur together in the same X_2. For any one frequency the two reactances X_{L2} and X_{C2} can be combined into an equivalent reactance that is either inductive or capacitive. It can be seen now that the number of phase versus voltage curves that can be drawn is large. For this reason, only the ones that portray a significant shape are shown in this paper, although a complete catalog has been made.

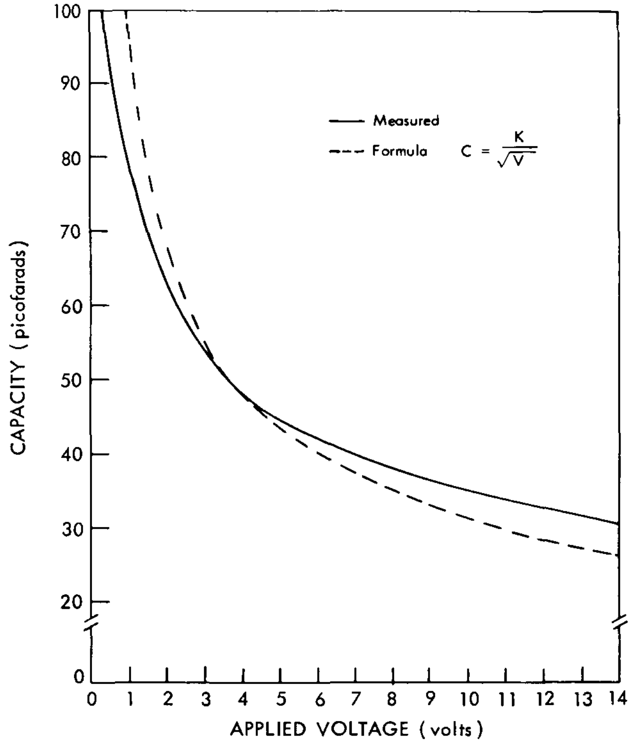

Figure 16—Capacity versus voltage curve.

Two shapes of the phase response are of particular interest. The response that rises rapidly from zero to approach 360° (Figure 18) is interesting but has no application in this project. The linear phase versus voltage response (Figure 19) is the most desirable response and has wide usage in radio frequency phase modulators and phase control elements.

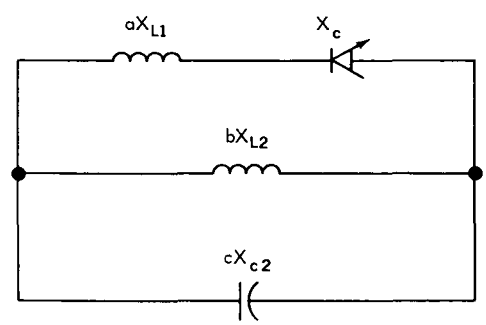

Figure 17—Voltage controlled reactance X_2.

The S-shaped phase response (Figure 20) has a particular application as a telemetry transmitter modulator where the signals are all pulses and square waves. If the bias or center of operation is chosen in the center of the peak-to-peak deviation and the modulating voltage exceeds a prescribed minimum value, then the phase deviation of the transmitted signal can be made almost independent of the peak-to-peak value of the modulating square wave voltage.

Other curves are shown with their respective constants. By careful examination, the reader may be able to get a feeling for "shaping" some of the response curves by changing the proper constants. Computation of these curves was aided by a digital computer.

When plotting these curves, sometimes it is difficult to establish a reference or the beginning of a curve. A method of spot-checking values is desirable and available. When $X_2 = 0$, the phase shift through the network is zero (Figure 15). This is because X_2 is the network bridge element and a value of zero is a short-circuit bypass across the network. Other points easy to check are available when the network will reduce to a simple Pi or Tee configuration, giving 90 or 270 degrees of phase shift (artifical quarter-wave transmission line).

Sample Design for a Modulator Circuit

In designing a phase modulator circuit for a transmitter, there are some considerations that should be noted. The first, and perhaps the most important, is choosing the characteristic impedance. An examination of Figure 21 will show that the modulator is also a power transferring network. The basic formula $P = Vp^2/2R$ describes a relationship between impedance, power, and peak voltage.

A problem which occurs with the varactor is that both the controlled and control signals are present at the varactor. In a modulator circuit, for example, both the carrier and the modulation signals are present at the varactor. Since the varactor is a back-biased diode, forward rectification of the controlled signal will occur when the controlled signal voltage exceeds the control voltage more than the diode offset voltage, which is usually about 0.7 volt at 25°C. The result of the rectification will be an unpredicted operation which none of the derived formulas will apply. It will be necessary for proper operation that the bias-plus-control voltage exceed the peak controlled signal voltage.

For the modulator circuit, assume the modulator power level is at least 10 milliwatts. If the characteristic impedance is 100 ohms, the peak radio frequency voltage is 1.4 volts, as follows:

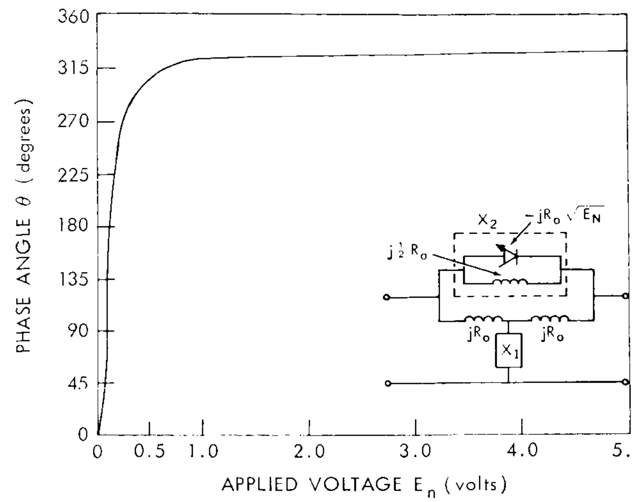

Figure 18—Exponential rise phase response.

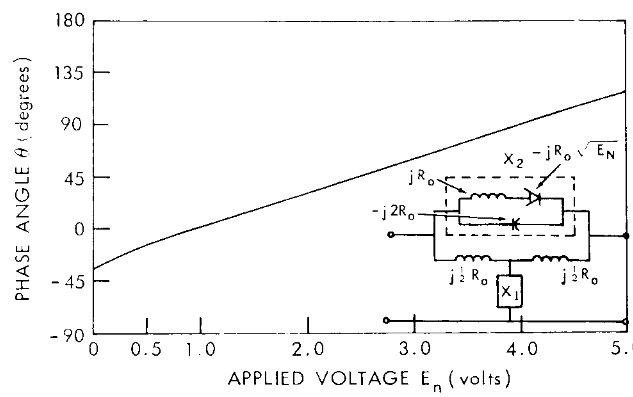

Figure 19—Linear phase response.

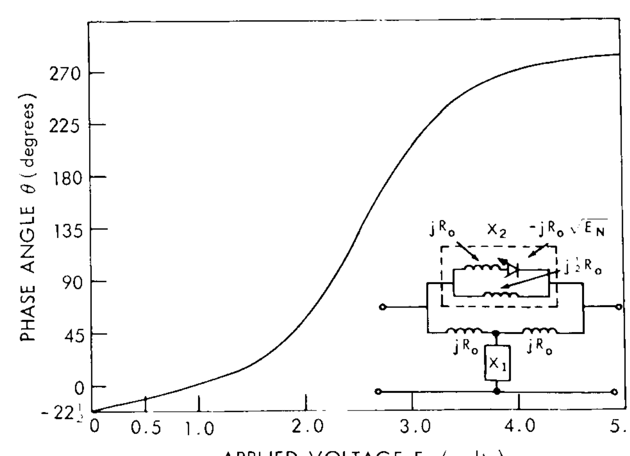

Figure 20—S-shaped phase response.

$$V_p^2 = 2RP \; ;$$

$$V_p = \sqrt{2RP} = \sqrt{2(100)(.01)} = 1.4 \; .$$

Using the diode offset voltage as a safety factor, the minimum control voltage is 1.4 volts. Table 1 shows the peak radio frequency voltage for other impedances and a power of 10 milliwatts, but it will increase as a square root function for higher characteristic impedances. The characteristic impedance must be low (50, 100, 150 ohms) in order to maintain a low value of V_p when transferring power.

Table 1
Peak Voltage — Resistance

Resistance (ohms)	Power (milliwatts)	V_p (volts)
50	10	1.0
100	10	1.4
200	10	2.0
1000	10	4.46

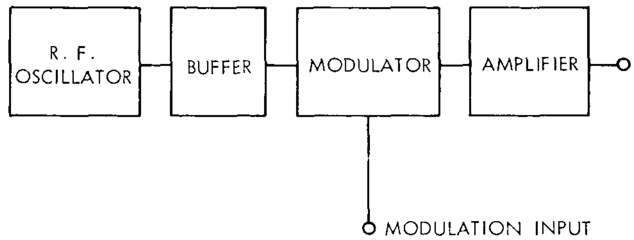

Figure 21—Typical transmitter elements.

At characteristic impedances below 50 ohms, lead inductances and stray capacitances become very troublesome at frequencies above 100 Mc. The 100 ohm characteristic impedance is a reasonable compromise and is also close to the input impedance of a common base transistor amplifier operating at 10 milliwatts input at VHF frequencies. Effects of a mismatched impedance are mentioned in the next section.

Generation of carrier frequency harmonics, a desired effect in other varactor circuits such as frequency multipliers, is an unwanted effect but can easily be eliminated with a tuned filter. Usually, the modulator is followed by a sufficient number of amplifiers or multipliers so that the generated harmonics can be neglected.

From the graph of a particular response the expected performance can be determined. A bias point must be chosen to prevent the minimum modulating signals from becoming equal to V_p. For a power of 10 milliwatts and $R = 100$ ohms, V_p is 1.4 volts. A typical marked graph showing these features is illustrated in Figure 22.

A Specific Example

Examination of a particular type of circuit may clarify some of the mentioned points. A typical circuit will be examined using easily attainable, practical values. The following design criteria are given:

$$R_0 = 100 \; .$$

$$Z = jR_0 = j\,100 \; .$$

$$jX_2 = jR_0 \left(1 - \sqrt{E_N}\right) = j\,100 \left(1 - \sqrt{E_N}\right) \; .$$

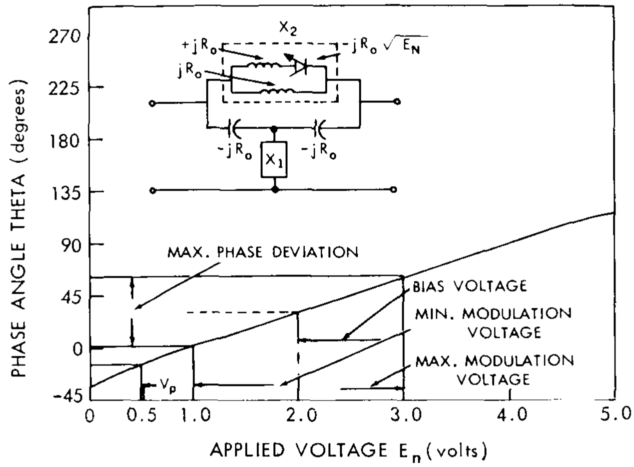

Figure 22—Expected performance curve.

From Equation 28:

$$Z_1 = \frac{R^2 - Z^2}{2Z} + \frac{R^2}{Z_2} = jX_1 \ ,$$

$$jX_1 = \frac{100^2 + 100^2}{2j100} + \frac{100^2}{jX_2} \ , \tag{43}$$

$$jX_1 = -j100 + \frac{100^2}{jX_2} \ .$$

The described network is now complete and is shown in Figure 23, with all impedances realized by actual components. When using an artificial quarter-wave transmission line, a choice is available. While two capacitors and one inductor may be used or two inductors and one capacitor, two capacitors and one inductor are used here.

A close examination of Figure 23 illustrates that components can be reduced by combining certain impedances. This has been achieved in Figure 24, but that network is not ready for use. Any applied control voltage across the center varactor would immediately be shorted to ground or a common circuit point. Additional modifications shown in Figure 25 make the network more useful. Input and output coupling capacitors have been added to isolate the phase control voltage. Adding radio frequency, self resonant coils (RF chokes) separates the carrier voltage from the control voltage and supplies a control voltage common return on the upper diode. An isolating capacitor has been added to the center leg variable capacitor. If the RF carrier frequency and the control voltage frequency approach the same value, isolation and interference troubles will increase.

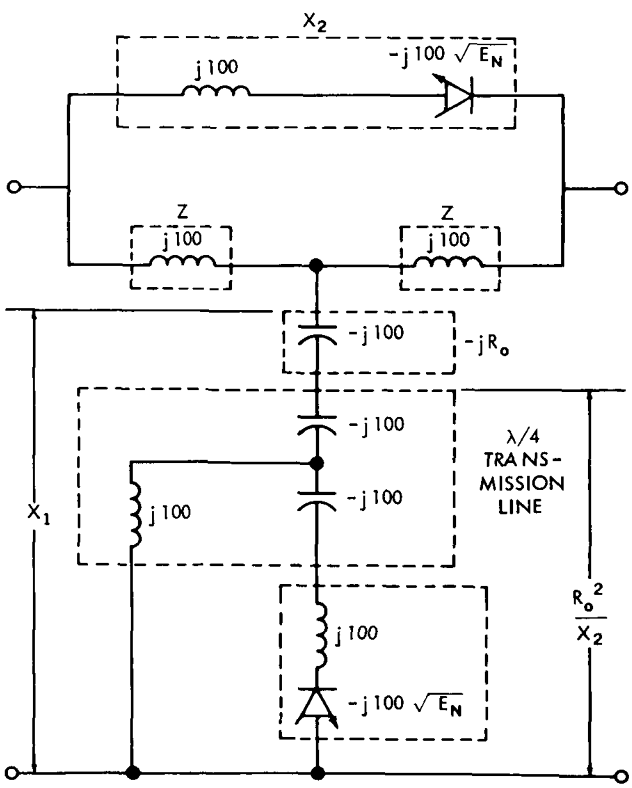

Figure 23–Filter with all elements realized.

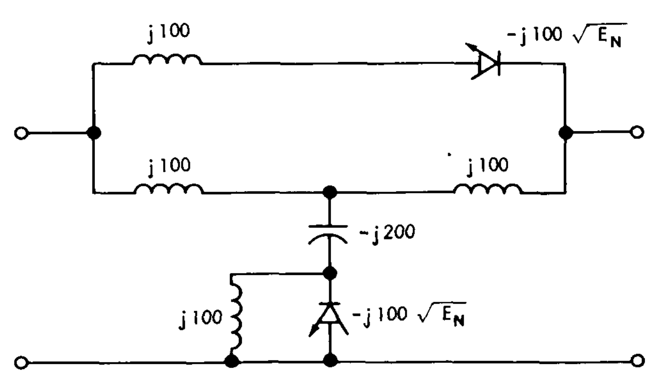

Figure 24–Simplified network.

Another useful variation of this circuit can be derived from transformer theory. Consider the imperfect center-tapped transformer and one of its equivalent circuits with inductance L_1 from center tap to each side (Figure 26a). The equivalent circuit (Figure 26b) has an arm inductance of $L_1 + M$ and a center leg inductance of $-M$. The equivalent circuit is valid as long as no attempt is made to connect to the inaccessible center point.

This imperfect transformer can replace the inductors Z of Figure 23 if $\omega(L_1 + M) = j100$ or, in a general circuit, if $\omega(L_1 + M) = jX_0$. The center leg capacitor ($-j200$ of Figure 24) will have to be changed to the equivalent value of $-j200$ in series with $-j\omega M$. This change is usually small, and sometimes this capacitor is a variable trimmer to account for other small circuit variations. The modified and simplified network is illustrated in Figure 27.

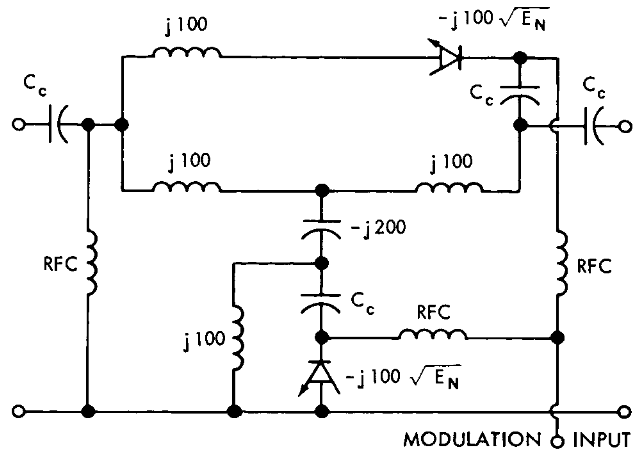

Figure 25–Filter with practical additions.

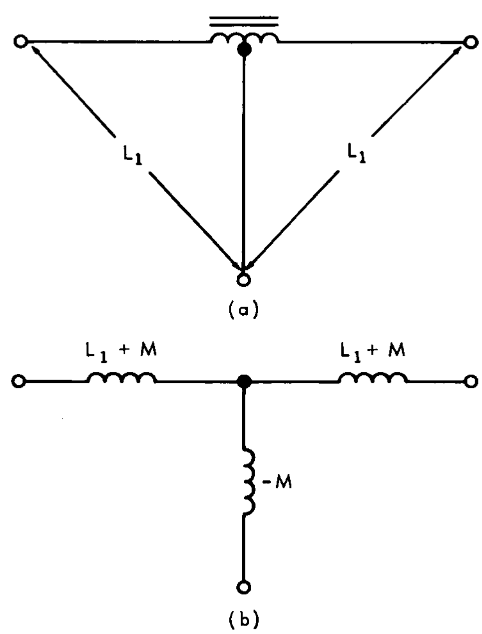

Figure 26–Center-tapped transformer and its equivalent circuit.

It remains to determine how the engineering model can compare with the predicted performance. Several models have been constructed, some of which are pictured at the end of this report. Figure 28 compares the measured performance of one filter with its predicted performance. As expected, the largest deviation occurs at high and low voltages where the capacitance approximation has the largest error. Near $E_N = 0$ the response is more linear than is predicted by the derived equations. This can be explained by the fact that the varactor capacity does not go to infinity as the formula indicates, but reaches a finite value. The result is a lesser phase change than predicted; and for some filters the response is linear to zero voltage.

Effects of Impedance Mismatch

When the designed modulator was added to transmitter circuitry, a noticeable change in its characteristics was observed, since the filter termination is a resistive load other than its characteristic impedance R_0.

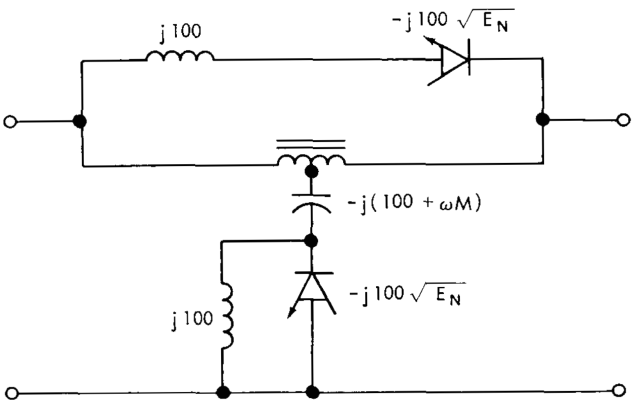

Figure 27—Transformer type variable phase filter.

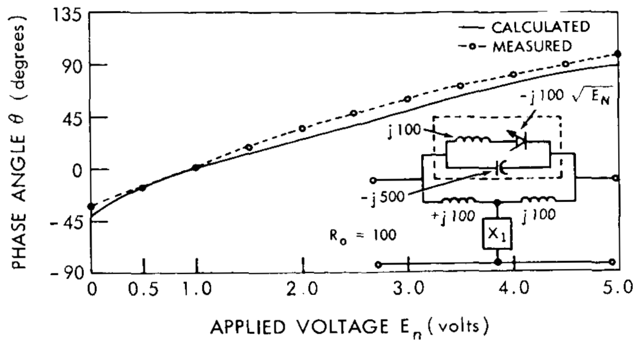

Figure 28—Performance comparison curve.

The original equations can be used to determine the change, however, they become much more complicated. The network will appear as shown in Figure 13, where R_L is not necessarily equal to R_0. To normalize the load resistance, let $R_L = R_0 + \Delta$ (Δ may be positive or negative), and for simplicity let $Z = jR_0$. Then

$$\frac{E_2}{E_1} = \frac{\left(R_L{}^2 - R_0{}^2\right) X_2{}^2 + 4R_L{}^2 R_0 X_2 + 4R_0 R_L{}^2 - j\, 2R_0 R_L X_2 \left(X_2 + 2R_0\right)}{\left(R_L{}^2 + R_0{}^2\right) X_2{}^2 + 4R_L{}^2 R_0 X_2 + 4R_0{}^2 R_L{}^2} . \tag{44}$$

A first approximation will be used and all terms with Δ^2 will be dropped. Let $R_0 = 100$ and $X_2 = 100\ (1 - \sqrt{E_N})$. Then

$$R_L = R_0 + \Delta , \tag{45}$$

and

$$R_L{}^2 = R_0{}^2 + 2R_0\Delta + \Delta^2 \approx R_0{}^2 + 2R_0\Delta . \tag{46}$$

The phase difference θ is

$$\theta = \tan^{-1} \frac{-2R_0 R_L X_2 \left(X_2 + 2R_0\right)}{\left(R_L{}^2 - R_0\right) X_2{}^2 + 4R_L{}^2 R_0 X_2 + 4R_0{}^2 R_2{}^2} , \tag{47}$$

and

$$\theta = \tan^{-1} \frac{(100 - \Delta)\left[- E_N + 4\sqrt{E_N} - 3\right]}{\Delta E_N - 2\sqrt{E_N}(100 + 3\Delta) + 400 + 7\Delta} \tag{48}$$

Finally,

$$\left|\frac{E_2}{E_1}\right| = \sqrt{\frac{\left[(R_0 + 2\Delta)X_2{}^4 + 2R_0\left(2R_0 + 5\Delta\right)X_2{}^3 + 4R_0{}^2\left(2R_0 + 7\Delta\right)X_2{}^2 + 8R_0{}^3\left(R_0 + 4\Delta\right)X_2 + 4R_0{}^4\left(R_0 + 4\Delta\right)\right]}{\left[(R_0 + \Delta)\,X_2{}^4 + 4R_0\left(R_0 + 3\Delta\right)X_2{}^2 + 4R_0{}^2\left(2R_0 + 7\Delta\right)X_2{}^2 + 8R_0{}^3\left(R_0 + 4\Delta\right)X_2 + 4R_0{}^4\left(R_0 + 4\Delta\right)\right]}}\,, \tag{49}$$

and

$$\left|\frac{E_2}{E_1}\right| = \sqrt{\frac{\left[(100 + 2\Delta)X_2{}^4 + 200(200 + 5\Delta)X_2{}^3 + 4(10)^4(200 + 7\Delta)X_2{}^2 + 8(10)^6(100 + 4\Delta) + 4(10)^8(100 + 4\Delta)\right]}{\left[(100 + \Delta)X_2{}^4 + 400(100 + 3\Delta)X_2{}^3 + 4(10)^4(200 + 7\Delta)X_2{}^2 + 8(10)^6(100 + 4\Delta)X_2 + 4(10)^8(100 + 4\Delta)\right]}}\,. \tag{50}$$

A graph of Equation 48 (Figure 29) is given to illustrate the effect of a load resistance unequal to it characteristic impedance. Since R_0 was 100 ohms, the values of Δ are normalized to percent. If $\Delta = +10\%$, for example, R_L is $R_0 + .10\ R_0$, representing a 10% increase above R_0. These curves, therefore, are applicable to any value of R_0, and Δ can be taken as a percent of R_0.

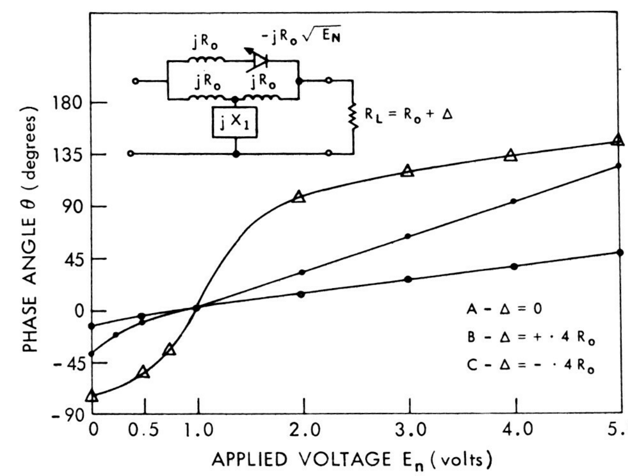

Figure 29—Effects on phase response due to impedance mismatch.

CONCLUSION

This analysis has shown that it is possible to derive filters with constant amplitude, variable phase characteristics. The derived filters may be used at any frequency at which the required component values are realizable.

The utility of the variable phase filter analysis was illustrated by the design and development of several radio frequency phase modulators. The many possible applications of the variable phase filter easily justify the effort of analysis and design.

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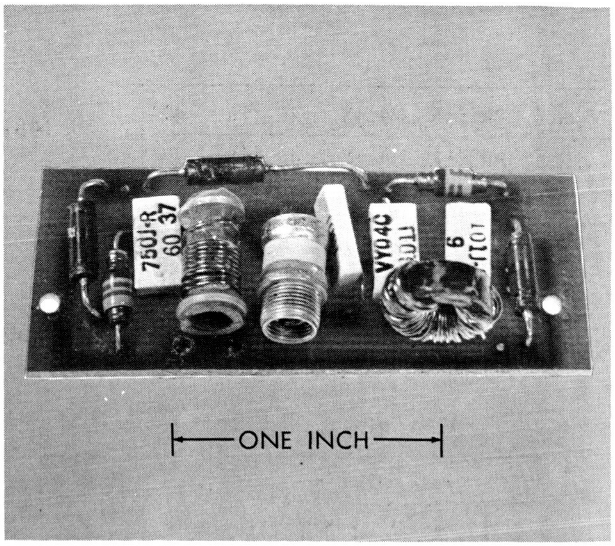

Figure 30—The 17 Mc phase modulator.

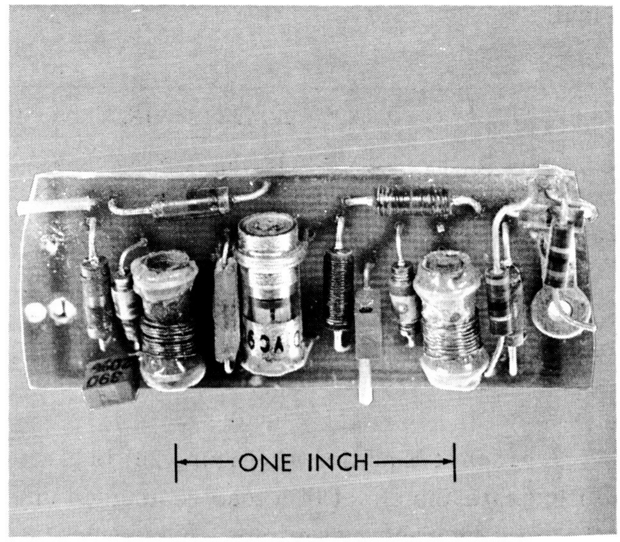

Figure 31—The 22 Mc phase modulator.

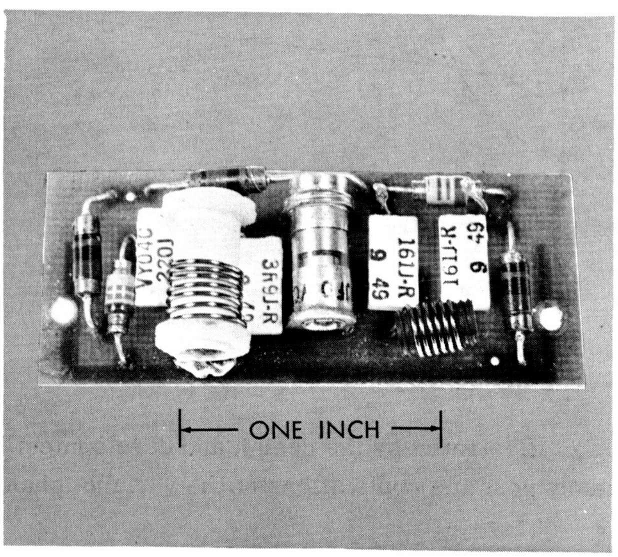

Figure 32—The 68 Mc phase modulator.

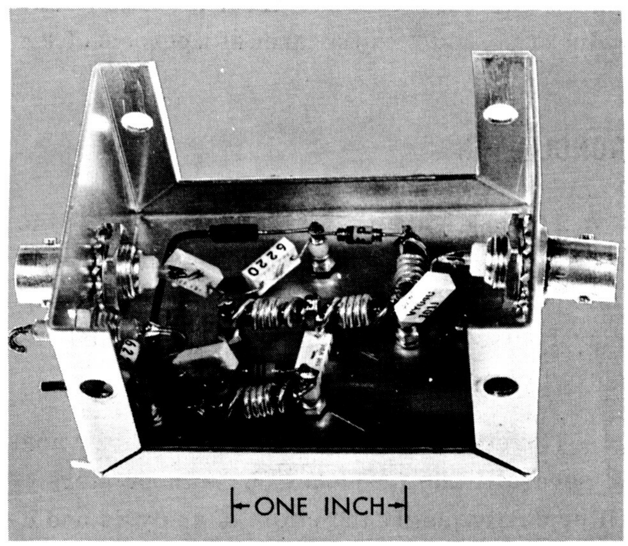

Figure 33—The 148 Mc phase modulator.

Appendix A

Partial List of Symbols Used

Symbol	Meaning
C_c	Coupling capacity
E_1	Input voltage
E_2	Output voltage
E_N	Normalized voltage
R	Resistance
R_L	Load resistance
R_0	Characteristic resistance
V	Applied voltage
V_P	Peak voltage
X	Reactive impedance
X_c	Capacitive reactance
X_L	Inductive reactance
Z	Impedance
θ	Phase shift
λ	Wave length

NASA-Langley, 1965