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FINITE AND INFINITE H-PLANE BIFURCATION OF WAVEGUIDE WITH ANISOTROPIC PLASMA MEDIUM

S. W. Lee, Y. T. Lo and R. Mittra

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ABSTRACT

In this paper, the problem of H-plane bifurcation in a parallel plate waveguide filled with homogenous, antisotropic, temperate plasma is considered. The static magnetic field is assumed to be along the edge of the septum. By using the Wiener-Hopf technique, the complete field solutions are obtained for both semi-infinite and finite bifurcations.

author

1. INTRODUCTION

The classical problem of an infinite bifurcation in a parallel plate waveguide filled with an isotropic medium has been considered by many authors. Marcuvitz[1] solved the problem by using the Wiener-Hopf technique, Hurd and Gruenberg[2] by the (mode matching) function-theoretic approach, and Mittra[3] by inverting the infinite matrix. Later, Mittra[4] also extended his method to the case of a finite bifurcation.

In this paper the same problem is considered except for the medium which is assumed to be an anisotropic plasma. To make this problem mathematically tractable, the plasma under consideration is assumed to have the commonly accepted model with the following properties: (1) the plasma is an electron gas with stationary ions which provide a neutralizing background; (2) the plasma is cold, homogeneous and without interaction between particles, (3) all nonlinear effects are negligible. The purpose of the present paper is to consider the problems of both infinite and finite bifurcations in a parallel plate waveguide filled with a plasma having properties described above, and to present solutions based on the Wiener-Hopf technique.

The solution will be restricted to the case when the dc magnetic field is oriented parallel to the edge of the septum.

2. FORMULATION OF PROBLEM

The geometry of the problem to be considered is shown in Figure 1, which shows a parallel plate waveguide of spacing a containing an infinitely thin and perfectly conducting septum extending from $z = 0$ to $z = \ell$ and located in the plane $x = b$. Let the medium in the guide be a plasma and let the dc magnetic field be oriented along y-direction. Then it is characterized by the relative dielectric tensor

$$\underset{\approx}{K} = \epsilon_1 \hat{x}\hat{x} - i\epsilon_2 \hat{x}\hat{z} + \epsilon_3 \hat{y}\hat{y} + i\epsilon_2 \hat{z}\hat{x} + \epsilon_1 \hat{z}\hat{z} \qquad (2\text{-}1)$$

where

$$\epsilon_1 = 1 - (\omega_p/\omega)^2 \left[1 - (\omega_c/\omega)^2\right]^{-1}, \; \epsilon_2 = (\omega_p/\omega)^2 \left[\omega/\omega_c - \omega_c/\omega\right]^{-1}$$

$$\epsilon_3 = 1 - (\omega_p/\omega)^2, \; \omega_c = - e B_o/m, \; \omega_p{}^2 = Ne^2/m\epsilon_o,$$

e = charge of an electron, m = mass of an electron,

N = average number density of electrons.

The time dependance $e^{-i\omega t}$ is suppressed throughout this paper. Assume that there is no variation along y-axis. Then from the Maxwell equations,

$$\nabla \times \underset{\sim}{E} = i\,\omega\,\mu_o\,\underset{\sim}{H} \qquad \nabla \times \underset{\sim}{H} = -\,i\,\omega\,\epsilon_o\,\underset{\approx}{K} \cdot \underset{\sim}{E}, \qquad (2\text{-}2)$$

it is readily shown that the wave equation for E_y and H_y are uncoupled because the wave number in the direction along $\underset{\sim}{B}_o$ is zero. Consequently, we can consider separately the solution with $H_y = 0$ and $E_y = 0$.

For the case $H_y = o$, the nonvanishing field components are E_y, H_x and H_z; hence, the field is transverse-electric with respect to z, or TE mode. The wave equation for E_y is given by

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \epsilon_3\, k_o{}^2\right] E_y(x,z) = 0$$

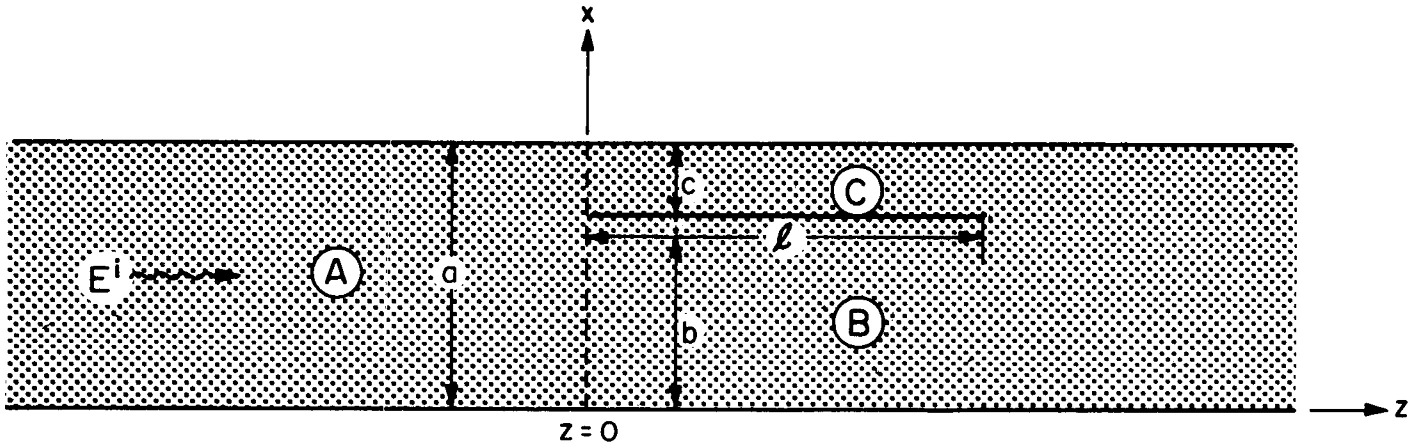

Figure 1. Geometry of the H-plane bifurcation parallel-plate waveguide with anisotropic plasma medium. The static magnetic field is along y-direction.

where $k_o^2 = \omega \mu_o \epsilon_o$ is the free space wave number. The boundary condition on the conducting plates is the usual one, namely $E_y = 0$. It is clear that the problem becomes identical to the one in which the guide is filled with an isotropic medium having a relative dielectric constant ϵ_3. This is to be expected since in this case the electron motion due to rf field is parallel to $\underset{\sim}{B}_o$; consequently, the anistropic affects are not introduced.

We shall, therefore, concentrate on the alternative case, that of $E_y = 0$, for which the nonvanishing field components are E_x, E_z and H_y. Thus, the fields are transverse-magnetic with respect to z, i.e., TM. The wave equation for H_y, which may be derived straightforwardly, reads

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right] H_y(x,z) = 0 \tag{2-3}$$

where $k^2 = k_o^2 \, \epsilon/\epsilon_1$ and $\epsilon = \epsilon_1^2 - \epsilon_2^2$. It is clear that the TM-mode can propagate only when k^2 is positive and real. In other words, the passbands are given by

$$-\omega_c/2 + (\omega_p^2 + \omega_c^2/4)^{1/2} < \omega < (\omega_p^2 + \omega_c^2)^{1/2} \quad \text{lower band} \tag{2-4}$$

$$\omega_c/2 + (\omega_p^2 + \omega_c^2/4)^{1/2} < \omega < \infty \quad \text{upper band} \tag{2-5}$$

We shall limit our following discussion to above frequency ranges. The electric field components of the TM-mode are given by

$$E_x(x,z) = \left[h_1 \frac{\partial}{\partial x} - h_2 \frac{\partial}{\partial z}\right] H_y(x,z) \tag{2-6}$$

$$E_z(x,z) = \left[h_1 \frac{\partial}{\partial z} + h_2 \frac{\partial}{\partial x}\right] H_y(x,z) \tag{2-7}$$

where $h_1 = -\epsilon_2/\omega\epsilon_o\epsilon$ and $h_2 = i\epsilon_1/\omega\epsilon_o\epsilon$. Now for simplicity assume that only the dominant TM-mode propagates in the large guide A (expept for the degenerate TEM-mode), and the incident wave is

$$E_z^i = \text{Sin}\,\frac{\pi}{a}x \;\; e^{-\gamma_{1a} z} \tag{2-8}$$

where $\gamma_{1a} = [(\pi/a)^2 - k^2]^{1/2}$ under the assumption is a negative imaginary number. Let the Fourier transform of H_y be

$$\Phi(x,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{-\infty}^{\infty} H_y(x,z)\, e^{i\alpha z}\, dz \tag{2-9}$$

where $\alpha = \sigma + i\tau$. Then the Fourier transform of Equation (2-3) with respect to z becomes

$$\left[\frac{\partial^2}{\partial x^2} - \gamma^2\right] \Phi(x,\alpha) = 0 \tag{2-10}$$

where $\gamma = (\alpha^2 - k^2)^{1/2}$. We shall consider the branch of γ such that $\gamma \longrightarrow \sigma$ as $\alpha \longrightarrow +\infty$. It is clear that the above equation is defined only in the strip $|\tau| < \mathrm{Re}\ \gamma_{1a}$. For analytical convenience, let us suppose that $k = k_1 + ik_2$ where k_1 and k_2 are positive real numbers. The introduction of a finite imaginary part in k may be interpreted in terms of finite losses in the medium and eventually k_2 may be set equal to zero. It can be shown that γ always has a positive real part when α lies in the strip $|\tau| < k_2$. Let $\mathrm{Re}\ \gamma_{1a} > k_2$; hence, Equation (2-10) holds in the strip $|\tau| < k_2$, and its solution is

$$\Phi(x,\alpha) = \begin{cases} A_1(\alpha) \cosh \gamma(x-a) + B_1(\alpha) \sinh \gamma(x-a), & x > b \\ A_2(\alpha) \cosh \gamma x + B_2(\alpha) \sinh \gamma x, & x < b \end{cases}$$

Applying the conditions:

(1) $E_z(x,z) = 0$ at $x = 0$, a for all z,

(2) $E_z(x = b + 0, z) = E_z(x = b - 0, z)$ for all z,

we have after some manipulations,

$$\Phi(x,\alpha) = \begin{cases} A_1(\alpha)\left[\cosh \gamma(x-a) + (ih_1\alpha/h_2\gamma) \sinh \gamma(x-a)\right], & x > b \\ -A_1(\alpha)[\sinh \gamma c/\sinh \gamma b][\cosh \gamma x + (ih_1\alpha/h_2\gamma) \sinh \gamma x], & x < b \end{cases} \tag{2-11}$$

which gives the Fourier transform of $H_y(x,z)$. Let $\Psi(x,\alpha)$ be the Fourier transform of $E_z(x,z)$. Then using Equation (2-7) one has

$$\Psi(x,\alpha) = \begin{cases} -A_1(\alpha)[h_1^2 + h_2^2/h_2\gamma]\ L(\alpha)\ \sinh \gamma(x-a) & x > b \\ A_1(\alpha)[\sinh \gamma c/\sinh \gamma b][h_1^2 + h_2^2/h_2\gamma]\ L(\alpha)\ \sinh \gamma x & x < b \end{cases} \qquad (2\text{-}12)$$

where $L(\alpha) = [h_2^2 k^2/(h_1^2 + h_2^2)] - \alpha^2$. The problem now is to solve for $A_1(\alpha)$, which must be determined from the condition that the tangential electric field is zero on the septum. This will be discussed in the following two sections.

3. INFINITE SEPTUM CASE

In this section we let $\ell = \infty$, i.e., consider the case of a semi-infinite bifurcation. Following the standard notation used in the Wiener-Hopf technique,[5] we write

$$\Phi_+(x,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \int_0^{\infty} H_y(x,z)\, e^{i\alpha z}\, dz$$

$$\Phi_-(x,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{-\infty}^{0} H_y(x,z)\, e^{i\alpha z}\, dz$$

Let similar definitions be introduced for $\Psi_+(x,\alpha)$ and $\Psi_-(x,\alpha)$ as well. Then setting x = b, Equation (2-11) and (2-12) may be rewritten as

$$\Phi_+(b+0,\alpha) + \Phi_-(b+0,\alpha) = A_1(\alpha)\left[\cosh \gamma c - (ih_1\alpha/h_2\gamma) \sinh \gamma c\right] \tag{3-1}$$

$$\Phi_+(b-0,\alpha)+\Phi_-(b-0,\alpha) = - A_1(\alpha)[\sinh \gamma c/\sinh \gamma b][\cosh \gamma b+(ih_1\alpha/h_2\gamma)\sinh \gamma b] \tag{3-2}$$

$$\Psi_+(b,\alpha) + \Psi_-(b,\alpha) = A_1(\alpha)\ [h_1^{\,2}+ h_2^{\,2}/h_2\gamma]\ L(\alpha)\ \sinh \gamma c \tag{3-3}$$

Since H_y, and hence Φ, is continuous at x = b and z < 0, taking the difference of Equations (3-1) and (3-2) obtains

$$I_+(\alpha) = \Phi_+(b+0,\alpha) - \Phi_+(b-0,\alpha) = A_1(\alpha)\cdot \sinh \gamma a/\sinh \gamma b \tag{3-4}$$

Now making use of the boundary condition that the total tangenial electric field vanishes on the septum x = b, z > 0, we have

$$\Psi_+(b,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \sin \frac{\pi b}{a}\ \frac{1}{(i\alpha - \gamma_{1a})} \tag{3-5}$$

Substitution of Equation (3-4) and (3-5) into (3-3) gives

$$\left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\,\frac{1}{(i\alpha-\gamma_{1a})} + \Psi_-(\alpha) = \frac{bc}{a}\,\frac{h_1^2+h_2^2}{h_2}\,\frac{L(\alpha)}{H(\alpha)}\,I_+(\alpha) \qquad (3\text{-}6)$$

where $H(\alpha) = [bc\ \gamma \sinh \gamma a/a \sinh \gamma b \sinh \gamma c]$. In the above equation, which is a Wiener-Hopf type, the unknown $\Psi_-(\alpha)$ and $I_+(\alpha)$ are analytic in the lower half plane $\tau < k_2$ and upper half plane $\tau > -k_2$, respectively. Following the standard technique, we decomposite $L(\alpha)$ and $H(\alpha)$ into $L_+(\alpha)$, $L_-(\alpha)$, $H_+(\alpha)$ and $H_-(\alpha)$ analytic in the respective upper and lower half plane. After some manipulations, the above equation becomes

$$\left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\,\frac{1}{(i\alpha-\gamma_{1a})}\left[\frac{H_-(\alpha)}{L_-(\alpha)} - \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}\right] + \frac{\Psi_-(\alpha)\,H_-(\alpha)}{L_-(\alpha)}$$

$$= \frac{bc}{a}\,\frac{h_1^2+h_2^2}{h_2}\,\frac{L_+(\alpha)\,I_+(\alpha)}{H_+(\alpha)} - \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\,\frac{1}{(i\alpha-\gamma_{1a})}\,\frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \qquad (3\text{-}7)$$

where:

$$L(\alpha) = L_+(\alpha)\,L_-(\alpha), \quad H(\alpha) = H_+(\alpha)\,H_-(\alpha)$$

$$L_+(\alpha) = L_-(-\alpha) = -\left[\frac{h_2 k}{(h_1^2+h_2^2)^{1/2}} + \alpha\right] = [\alpha_o - \alpha]$$

$$H_+(\alpha) = H_-(-\alpha) = \exp[-\Gamma(\alpha)]\prod_{n=1}^{\infty}\frac{(a/n\pi)[\gamma_{na}-i\alpha]}{(b/n\pi)[\gamma_{nb}-i\alpha]\,(c/n\pi)[\gamma_{nc}-i\alpha]}$$

$$\Gamma(\alpha) = (i\alpha/\pi)(a \ln a - b \ln b - c \ln c)$$

$$\gamma_{na} = [(n\pi/a)^2 - k^2]^{1/2}, \quad \gamma_{nb} = [(n\pi/b)^2 - k^2]^{1/2}, \quad \gamma_{nc} = [(n\pi/c)^2 - k^2]^{1/2}$$

The left hand side of this equation is analytic in the lower half plane $\tau < k_2$; the right hand side in the upper half plane $\tau > k_2$. Since these two regions overlap, by analytic continuation both sides must be equal to an entire function $P(\alpha)$. Since $I_+(\alpha)$ is the induced current on the septum, it behaves as $\alpha^{-3/2}$ as $\alpha \longrightarrow \infty$ in the upper half plane. Then by Liouville's theorm,[5] it can be shown that $P(\alpha)$ is identically zero. Thus, one obtains the value of $I_+(\alpha)$:

$$I_+(\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{a}{bc} \frac{h_2}{h_1^2 + h_2^2} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{H_+(\alpha)}{L_+(\alpha)} \frac{1}{i(\alpha + i\gamma_{1a})} \tag{3-8}$$

and hence $A_1(\alpha)$ via Equation (3-4). Substituting $A_1(\alpha)$ into Equation (2-11) and taking the inverse Fourier transform, one immediately obtains

$$H_y(x,z) = \left\{ \frac{1}{2\pi i} \frac{h_2}{h_1^2 + h_2^2} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \sin\frac{\pi b}{a} \right\}$$

$$\times \begin{cases} \displaystyle\int_P \frac{1}{L_+(\alpha)H_-(\alpha)} \frac{1}{(\alpha + i\gamma_{1a})} \left[\frac{\gamma \cosh \gamma(x-a)}{\sinh \gamma c} + \frac{ih_1\alpha}{h_2} \frac{\sinh \gamma(x-a)}{\sinh \gamma c} \right] e^{-i\alpha z} dz, & x > b \\ \displaystyle\int_P \frac{-1}{L_+(\alpha)H_-(\alpha)} \frac{1}{(\alpha + i\gamma_{1a})} \left[\frac{\gamma \cosh \gamma x}{\sinh \gamma b} - \frac{ih_1\alpha}{h_2} \frac{\sinh \gamma x}{\sinh \gamma b} \right] e^{-i\alpha z} dz, & x < b \end{cases}$$

where the integration path P is shown in Figure 2, with $P = P_1$ for $z > 0$, and $P = P_2$ for $z < 0$. The only singularities in the integrands are simple poles, and the residue contributions of these poles are easily obtained. Evaluating the integral, one has, after some algebra, for $z > 0$ and $x > b$,

$$H_y(x,z) = \sin\frac{\pi b}{a} \frac{h_2}{h_1^2 + h_2^2} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{H_-(\alpha_o)} \frac{1}{\alpha_o + i\gamma_{1a}} \frac{\gamma_o}{\sinh \gamma_o c} e^{\gamma_o(x-a)} e^{-i\alpha_o z}$$

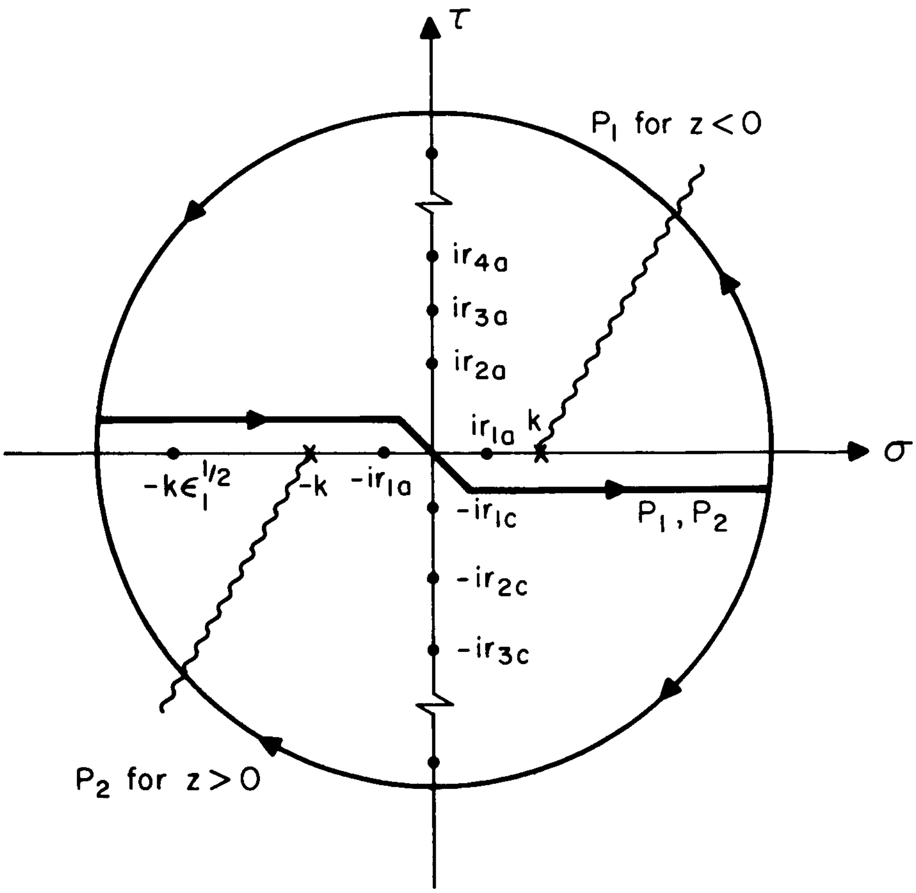

Figure 2. Contour of integration for $H_y(x,z)$ in the case of infinite bifurcation.

$$+ \frac{h_2}{h_1^2+h_2^2} \frac{1}{L(-i\gamma_{1a})} \left[\frac{\pi}{a} \cos \frac{\pi}{a}(x-a) + \frac{h_1\gamma_{1a}}{h_2} \sin \frac{\pi}{a}(x-a) \right] e^{-\gamma_{1a}z} - \sin \frac{\pi b}{a} \frac{h_2}{h_1^2+h_2^2} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{L_+(-i\gamma_{nc})H_+(-i\gamma_{nc})} \frac{1}{\gamma_{nc} - \gamma_{1a}} \left[\frac{n\pi}{c} \cos \frac{n\pi}{c} x + \frac{h_1\gamma_{nc}}{h_2} \sin \frac{n\pi}{c} (x-a) \right] e^{-\gamma_{nc}z}$$

$$E_x(x,y) = \sin \frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{H_-(\alpha_o)} \frac{i\alpha_o}{\alpha_o+i\gamma_{1a}} \frac{\gamma_o}{\sinh \gamma_o c} e^{\gamma_o(x-a)} e^{-i\alpha_o z} \qquad (3\text{-}9)$$

$$+ \frac{1}{L(-i\gamma_{1a})} \left[\gamma_{1a} \frac{\pi}{a} \cos \frac{\pi}{a}(x-a) + \frac{h_1\alpha_o^2}{h_2} \sin \frac{\pi}{a}(x-a) \right] e^{-\gamma_{1a}z} - \sin \frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{L_+(-i\gamma_{nc})H_+(-i\gamma_{nc})} \frac{1}{\gamma_{nb}-\gamma_{1a}} \left[\gamma_{nc} \frac{n\pi}{c} \cos \frac{n\pi}{c}(x-a) + \frac{h_1\alpha_o^2}{h_2} \sin \frac{n\pi}{c}(x-a) \right] e^{-\gamma_{nc}z}$$

$$E_z(x,z) = \sin \frac{\pi}{a}(x-a) e^{-\gamma_{1a}z} + \sin \frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}$$

$$\sum_{n=1}^{\infty} \frac{L_-(-i\gamma_{nc})}{H_-(-i\gamma_{nc})} \frac{(-1)^n}{\gamma_{1a} - \gamma_{nc}} \sin \frac{n\pi}{c} (a-x) e^{-\gamma_{nc}z}$$

For $z > 0$ and $x < b$, $H_y(x,z)$, $E_x(x,z)$ and $E_z(x,z)$ have the same expressions as given above except that $(x-a)$, $n\pi/c$, γ_{nc}, and c are replaced by x, $n\pi/b$, γ_{nb} and b, respectively, and the sign of all terms are changed. For $z < 0$, the evaluation of the integral for H_y and so forth gives

$$H_y(x,z) = \sum_{n=1}^{\infty} \sin\frac{\pi b}{a} \frac{h_2}{h_1^2+h_2^2} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{L_+(i\gamma_{na})} \frac{1}{H_-'(i\gamma_{na})} \frac{1}{(i\gamma_{na}+i\gamma_{1a})} \frac{(-1)}{\sin\frac{n\pi}{a}b}$$

$$\left[\frac{n\pi}{a}\cos\frac{n\pi}{a}x - \frac{h_1\gamma_{na}}{h_2}\sin\frac{n\pi}{a}x\right] e^{\gamma_{na}z}$$

$$E_x(x,z) = \sum_{n=1}^{\infty} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{L_+(i\gamma_{na})} \frac{1}{H_-'(i\gamma_{na})} \frac{1}{(i\gamma_{na}+i\gamma_{1a})} \frac{1}{\sin\frac{n\pi}{a}b} \tag{3-10}$$

$$\left[\gamma_{na}\frac{n\pi}{a}\cos\frac{n\pi}{a}x - \frac{h_1\alpha_o^2}{h_2}\sin\frac{n\pi}{a}x\right] e^{\gamma_{na}z}$$

$$E_z(x,z) = \sum_{n=1}^{\infty} \frac{H_-(-i\gamma_{1a})}{H_-'(i\gamma_{na})} \frac{L_-(i\gamma_{na})}{L_-(-i\gamma_{1a})} \frac{1}{(i\gamma_{na}+i\gamma_{1a})} \sin\frac{n\pi}{a}x\, e^{\gamma_{na}z}$$

where $\alpha_o = -h_2k/h_1^2 + h_2^2 = -k_o\epsilon_1^{1/2}$, $\gamma_o = (\alpha_o^2 - k^2)^{1/2} = k_o\epsilon_1^{-1/2}\epsilon_2$, and $H_-'(i\gamma_{na})$ is the derivative of $H_-(i\gamma_{na})$ with respect to $(i\gamma_{na})$.

Equation (3-9) and (3-10) comprise the complete and exact solution for the case of semi-infinite bifurcation. It may be noted that the modes with the variation of $e^{\gamma_o x}\, e^{-i\alpha_o z}$ type are TEM because the corresponding $E_z(x,z)$ is identically zero. However, it is different from the ordinary TEM mode of an isotropic guide in the following ways: (a) rather than being independent of the transverse dimension, it varies exponentially along x; (b) in the scattered field given by Equation (3-9), it is not continous at x = b, and (c) whether it propagates or not solely depends on the properties of the plasma medium. Explicitly, it is propagating if $\alpha_o = -k_o\epsilon_1^{\frac{1}{2}}$, is a negative real number, or $\omega > (\omega_p^2 + \omega_c^2)^{\frac{1}{2}}$. Because of the identity $\omega_c/2 + (\omega_p^2 + \omega_c^2/4)^{\frac{1}{2}} \geq (\omega_p^2 + \omega_c^2)^{\frac{1}{2}}$, we conclude that for the frequency in the upper band defined by Equation (2-5), the TEM mode propagates. Conversely, for frequency in the lower band defined by Equation (2-4), the TEM mode attenuates.

Since the TEM mode has zero $E_z(x,z)$ component, it is seen that the incident field in the above analysis is not uniquely specified by merely giving E_z^i, as in Equation (2-8). However, this does not affect the uniqueness of our solution as far as the scattered field is concerned, because an incident TEM mode does not give rise to any scattered field since it propagates freely in the guide without being disturbed by the septum.

4. FINITE SEPTUM CASE

Now consider the case of a finite septum. First, the following notations are introduced for the partial Fourier transform of $H_y(x,z)$:

$$\Phi_-(x,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{-\infty}^{0} H_y(x,z)\, e^{i\alpha z}\, dz$$

$$\Phi_1(x,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{0}^{\ell} H_y(x,z)\, e^{i\alpha z}\, dz \tag{4-1}$$

$$\Phi_+(x,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{\ell}^{\infty} H_y(x,z)\, e^{i\alpha(z-\ell)}\, dz$$

It is clear that $\Phi_-(x,\alpha)$ and $\Phi_+(x,\alpha)$ are analytic in the lower and upper half plane, respectively. The function $\Phi_1(x,\alpha)$ is an entire function of α, and in particular, analytic in the upper half plane including the point at infinity. However, the function $e^{i\alpha\ell}\,\Phi_1(x,\alpha)$ is analytic in the lower half plane as may be seen from the representation:

$$e^{i\alpha\ell}\,\Phi_1(x,\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{0}^{\ell} H_y(x,z)\, e^{i\alpha(z-\ell)}\, dz$$

$$= \left(\frac{1}{2\pi}\right)^{1/2} \int_{-\ell}^{0} H_y(x,u+\ell)\, e^{i\alpha u}\, du.$$

Rewriting Equation (2-11) in the above notations and evaluating it at $x = b$, we have

$$e^{i\alpha\ell}\,\Phi_+(b+0,\alpha) + \Phi_1(b+0,\alpha) + \Phi_-(b+0,\alpha) = A_1\left[\cosh \gamma c - \frac{ih_1\alpha}{h_2\gamma}\sinh \gamma c\right]$$

$$e^{i\alpha\ell}\,\Phi_+(b-0,\alpha) + \Phi_1(b-0,\alpha) + \Phi_-(b-0,\alpha) = -A_1 \frac{\sinh \gamma c}{\sinh \gamma b}\left[\cosh \gamma b + \frac{ih_1\alpha}{h_2\gamma}\sinh \gamma b\right]$$

Since $H_y(x,z)$, and hence $\Phi(x,\alpha)$, is continuous for $x = b$ and $-\infty < z < 0$, and $x = b$ and $\ell < z < +\infty$, the difference of the above two equations gives

$$I_+(\alpha) = \Phi_1(b+0,\alpha) - \Phi_1(b-0,\alpha) = A_1 \sinh \gamma a/\sinh \gamma b. \tag{4-2}$$

Let us use the same notations for $\Psi(x,z)$, the Fourier transform of $E_z(x,z)$, as in Equation (4-1). Then letting $x = b$, Equation (2-12) becomes

$$e^{i\alpha\ell}\,\Psi_+(\alpha) + \Psi_1(\alpha) + \Psi_-(\alpha) = A_1[h_1^2 + h_2^2/h_2\gamma]\, L(\alpha) \sinh \gamma c \tag{4-3}$$

Now, applying the boundary condition that the total tangential electric field vanishes on the septum, we have

$$E_z(b,z) + (\sin \pi b/a)\, e^{-\gamma_{1a} z} = 0 \quad \text{for} \quad 0 < z < \ell$$

or

$$\Psi_1(\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\left[\frac{1}{i(\alpha+\gamma_{1a})} - \frac{e^{i(\alpha+i\gamma_{1a})\ell}}{i(\alpha+i\gamma_{1a})}\right] \tag{4-4}$$

Substitution of Equations (4-2) and (4-4) into (4-3) results in

$$e^{i\alpha\ell}\,\Psi_+(\alpha) + \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\left[\frac{1}{i(\alpha+i\gamma_{1a})} - \frac{e^{i(\alpha+i\gamma_{1a})\ell}}{i(\alpha+i\gamma_{1a})\ell}\right] + \Psi_-(\alpha)$$
$$= \frac{bc}{a}\,\frac{h_1^2 + h_2^2}{h_2}\,\frac{L(\alpha)}{H(\alpha)}\, I_+(\alpha) \tag{4-5}$$

Now, multiplying the above equation by $e^{i\alpha\ell} H_+(\alpha)/L_+(\alpha)$, we have

$$\frac{\Psi_+(\alpha)H_+(\alpha)}{L_+(\alpha)} - \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{e^{-\gamma_{1a}\ell}}{i(\alpha+i\gamma_{1a})} \frac{H_+(\alpha)}{L_+(\alpha)} + E_+(\alpha) + F_+(\alpha)$$
$$= \frac{bc}{a} \frac{h_1^2+h_2^2}{h_2} \frac{L_-(\alpha)}{H_-(\alpha)} [e^{-i\alpha\ell} I_+(\alpha)] - E_-(\alpha) - F_-(\alpha) \tag{4-6}$$

where the following notations have been used:

$$\left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{e^{-i\alpha\ell}}{i(\alpha+i\gamma_{1a})} \frac{H_+(\alpha)}{L_+(\alpha)} = E_+(\alpha) + E_-(\alpha) \tag{4-7}$$

$$\frac{\Psi_-(\alpha)H_+(\alpha)}{L_+(\alpha)} e^{-i\alpha\ell} = F_+(\alpha) + F_-(\alpha) \tag{4-8}$$

Similarly, multiplying Equation (4-5) by $H_-(\alpha)/L_-(\alpha)$, we have, after some re-arrangements,

$$N_-(\alpha)+\left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{1}{i(\alpha+i\gamma_{1a})} \left[\frac{H_-(\alpha)}{L_-(\alpha)} - \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}\right] + M_-(\alpha) + \frac{\Psi_-(\alpha)H_-(\alpha)}{L_-(\alpha)}$$
$$= - N_+(\alpha) - \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \sin\frac{\pi b}{a} \frac{1}{i(\alpha+i\gamma_{1a})} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} - M_+(\alpha) + \frac{bc}{a} \frac{h_1^2+h_2^2}{h_2} \frac{L_+(\alpha)}{H_+(\alpha)} I_+(\alpha) \tag{4-9}$$

where the following notations have been used:

$$\frac{\Psi_+(\alpha)H_-(\alpha)}{L_-(\alpha)} e^{i\alpha\ell} = N_+(\alpha) + N_-(\alpha) \tag{4-10}$$

$$-\left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{e^{i(\alpha+i\gamma_{1a})\ell}}{i(\alpha+i\gamma_{1a})} \frac{H_-(\alpha)}{L_-(\alpha)} = M_+(\alpha) + M_-(\alpha) \tag{4-11}$$

Notice that the right hand side of Equation (4-6) and the left hand side of Equation (4-9) are analytic in the upper half plane $\tau > -k_2$; while the left hand side of Equation (4-6) and the right hand side of Equation (4-9) are analytic in the lower half plane $\tau < k_2$. Hence, by analytic continuation, Equation (4-6) and (4-9) are equal to some integral functions in α-plane. By examining the asymptotic behavior of the functions in these two equations as $\alpha \longrightarrow \infty$, it can be shown that the integral functions in both equations are identically zero. At this point, it should be noted also that the decompositions in Equation (4-7), (4-8), (4-10) and (4-11) are very difficult or impossible because of the presence of some unknown functions. Nevertheless, the explicit expressions for these decompositions are not needed at this moment, and they are introduced simply for the convenience in notations. Remembering the decomposition theorem,[5] the functions $F_+(\alpha)$ and $F_-(\alpha)$, for example, are given by

$$F_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id} \frac{\Psi_-(\zeta)H_+(\zeta)e^{-i\zeta\ell}}{(\zeta-\alpha)L_+(\zeta)}\, d\zeta$$

$$\tau < d < k_2 \qquad (4\text{-}12)$$

$$F_-(\alpha) = \frac{-1}{2\pi i}\int_{-\infty-id}^{\infty-id} \frac{\Psi_-(\zeta)H_+(\zeta)e^{-i\zeta\ell}}{(\zeta-\alpha)L_+(\zeta)}\, d\zeta$$

There exist similiar expressions for $E_+(\alpha)$ and $E_-(\alpha)$, $N_+(\alpha)$ and $N_-(\alpha)$, and $M_+(\alpha)$ and $M_-(\alpha)$. For the following analysis, it will be convenient to introduce some more notations, viz.,

$$J_+(\alpha) = \Psi_+(\alpha) - \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\, \frac{e^{-\gamma_{1a}\ell}}{i(\alpha+i\gamma_{1a})}$$

$$(4\text{-}13)$$

$$J_{(-)}(\alpha) = \Psi_-(\alpha) + \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\, \frac{1}{i(\alpha+i\gamma_{1a})}$$

where $J_+(\alpha)$ is analytic in the upper half plane $\tau > -k_2$; and $J_{(-)}(\alpha)$ is analytic in the lower half plane $\tau < k_2$ except for a pole at $\alpha = -i\gamma_{1a}$, and the parenthesis in the subscript of $J_{(-)}(\alpha)$ simply denotes the existence of this pole. Returning to Equation (4-6) and (4-9) and equating their left hand sides to zero, we obtain after introducing the notations in Equation (4-12) and (4-13),

$$\frac{J_+(\alpha)H_+(\alpha)}{L_+(\alpha)} + \frac{1}{2\pi i}\int_{-\infty-id}^{\infty-id} \frac{J_{(-)}(\zeta)H_+(\zeta)}{L_+(\zeta)} \frac{e^{-i\zeta\ell}}{\zeta-\alpha} d\zeta = 0 \tag{4-14}$$

$$\frac{J_{(-)}(\alpha)H_-(\alpha)}{L_-(\alpha)} + \frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id} \frac{J_+(\zeta)H_+(\zeta)}{L_+(\zeta)} \frac{e^{i\zeta\ell}}{\zeta-\alpha} d\zeta$$

$$- \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha+i\gamma_{1a})} = 0 \tag{4-15}$$

The unknowns in the above equations are $J_+(\alpha)$ and $J_{(-)}(\alpha)$. In order to decouple these two equations, we replace α by $(-\alpha)$ in Equation (4-14) and ζ by $(-\zeta)$ in (4-15). Since $H_+(-\alpha) = H_-(\alpha)$ and $L_+(-\alpha) = L_-(\alpha)$, there results

$$\frac{J_+(-\alpha)H_-(\alpha)}{L_-(\alpha)} + \frac{1}{2\pi i}\int_{-\infty-id}^{\infty-id} \frac{J_{(-)}(\zeta)H_+(\zeta)}{L_+(\zeta)} \frac{e^{-i\zeta\ell}}{\zeta+\alpha} d\zeta = 0$$

$$\frac{J_{(-)}(\alpha)H_-(\alpha)}{L_-(\alpha)} + \frac{1}{2\pi i}\int_{-\infty-id}^{\infty-id} \frac{J_+(-\zeta)H_+(\zeta)}{L_+(\zeta)} \frac{e^{-i\zeta\ell}}{\zeta+\alpha} d\zeta$$

$$- \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha+i\gamma_{1a})} = 0$$

The sum and difference of the above two equations yield

$$\frac{S_{(-)}(\alpha)H_-(\alpha)}{L_-(\alpha)} + \frac{1}{2\pi i}\int_{-\infty-id}^{\infty+id} \frac{S_{(-)}(\zeta)H_+(\zeta)}{L_+(\zeta)} \frac{e^{-i\zeta\ell}}{\zeta+\alpha}\, d\zeta$$

$$- \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha+i\gamma_{1a})} = 0 \qquad (4\text{-}16)$$

$$\frac{D_{(-)}(\alpha)H_-(\alpha)}{L_-(\alpha)} - \frac{1}{2\pi i}\int_{-\infty-id}^{\infty-id} \frac{D_{(-)}(\zeta)H_+(\zeta)}{L_+(\zeta)} \frac{e^{-i\zeta\ell}}{\zeta+\alpha}\, d\zeta$$

$$- \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha+i\gamma_{1a})} = 0 \qquad (4\text{-}17)$$

where $S_{(-)}(\alpha) = J_{(-)}(\alpha) + J_+(-\alpha)$, $D_{(-)}(\alpha) = J_{(-)}(\alpha) - J_+(-\alpha)$.

The parentheses in the subscripts of $S_{(-)}(\alpha)$ and $D_{(-)}(\alpha)$ are also used to denote the existence of the pole in the lower half plane at $\alpha = -i\gamma_{1a}$. There is only one unknown appearing in each of these equations; therefore, the solution of $S_{(-)}(\alpha)$ and $D_{(-)}(\alpha)$ could be obtained if the integrals were known.

First, consider the integral in Equation (4-16). Writing out the functions $H_+(\zeta)$ and $L_+(\zeta)$ in an explicit manner, we have

$$G = \frac{1}{2\pi i}\int_{-\infty-id}^{\infty+id} \frac{S_{(-)}(\zeta)H_+(\zeta)}{L_+(\zeta)} \frac{e^{-i\zeta\ell}}{\zeta+\alpha}\, d\zeta$$

$$= \frac{1}{2\pi i}\int_P \frac{S_{(-)}(\zeta) \prod_{n=1}^{\infty} (a/n\pi)(\gamma_{na} - i\zeta) \; e^{-\Gamma(\zeta)}}{(a_o-\zeta) \prod_{n=1}^{\infty} (b/n\pi)(\gamma_{nb} - i\zeta)(c/n\pi)(\gamma_{nc} - i\zeta)} \frac{e^{-i\zeta\ell}}{\zeta+\alpha}\, d\zeta$$

Because of the factor $e^{-i\zeta\ell}$, we can close the path of integration by an infinite semicircle in the lower half plane denoted by P as shown in Figure 3. The following observations may be made in connection with the integral for G.

(a) The only pole of $S_{(-)}(\zeta)$ in the lower half plane is at $\zeta = -i\gamma_{1a}$, however, this is cancelled by a zero of $H_+(\zeta)$.

(b) The pole at $\zeta = -\alpha$ is not inside the contour P since we can choose (-d) arbitrarily close to $(-k_2)$.

(c) The pole contributions at $\zeta = -i\gamma_{nb}$ and $\zeta = -i\gamma_{nc}$ will contain the decaying factors $e^{-\gamma_{nb}\ell}$ and $e^{-\gamma_{nc}\ell}$, respectively, where, under previous assumption, γ_{nb} and γ_{nc} are real and positive. Thus, for sufficiently large ℓ, the contributions due to these poles are negligible.

(d) The pole at $\zeta = \alpha_o$ will contribute only when α_o is a real number, i.e., for frequencies in the upper band; otherwise, its contribution becomes negligible for large ℓ due to the same reason as in (c).

Therefore, for large ℓ, and the frequency in the upper band, one obtains the following approximation for G,

$$G \approx [S_{(-)}(\alpha_o)H_+(\alpha_o)/L_-(\alpha)]\ e^{-i\alpha_o\ell}$$

Substitution of this approximation into Equation (4-16) results in

$$\frac{S_{(-)}(\alpha)H_-(\alpha)}{L_-(\alpha)} + \frac{S_{(-)}(\alpha_o)H_+(\alpha_o)}{L_-(\alpha)} e^{-i\alpha_o\ell} - \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\, \frac{H_-(-\gamma_{1a})}{L_-(-\gamma_{1a})}\, \frac{1}{i(\alpha+i\gamma_{1a})} = 0 \quad (4\text{-}18)$$

To evaluate $S_{(-)}(\alpha_o)$, one simply sets $\alpha = \alpha_o$ in the above equation and obtains

$$S_{(-)}(\alpha_o) = \left[\frac{H_-(\alpha_o)+H_+(\alpha_o)e^{-i\alpha_o\ell}}{L_-(\alpha_o)}\right]^{-1} \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\, \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}\, \frac{1}{i(\alpha_o+i\gamma_{1a})} \quad (4\text{-}19)$$

Then by putting $S_{(-)}(\alpha_o)$ back into Equation (4-18), one obtains an expression

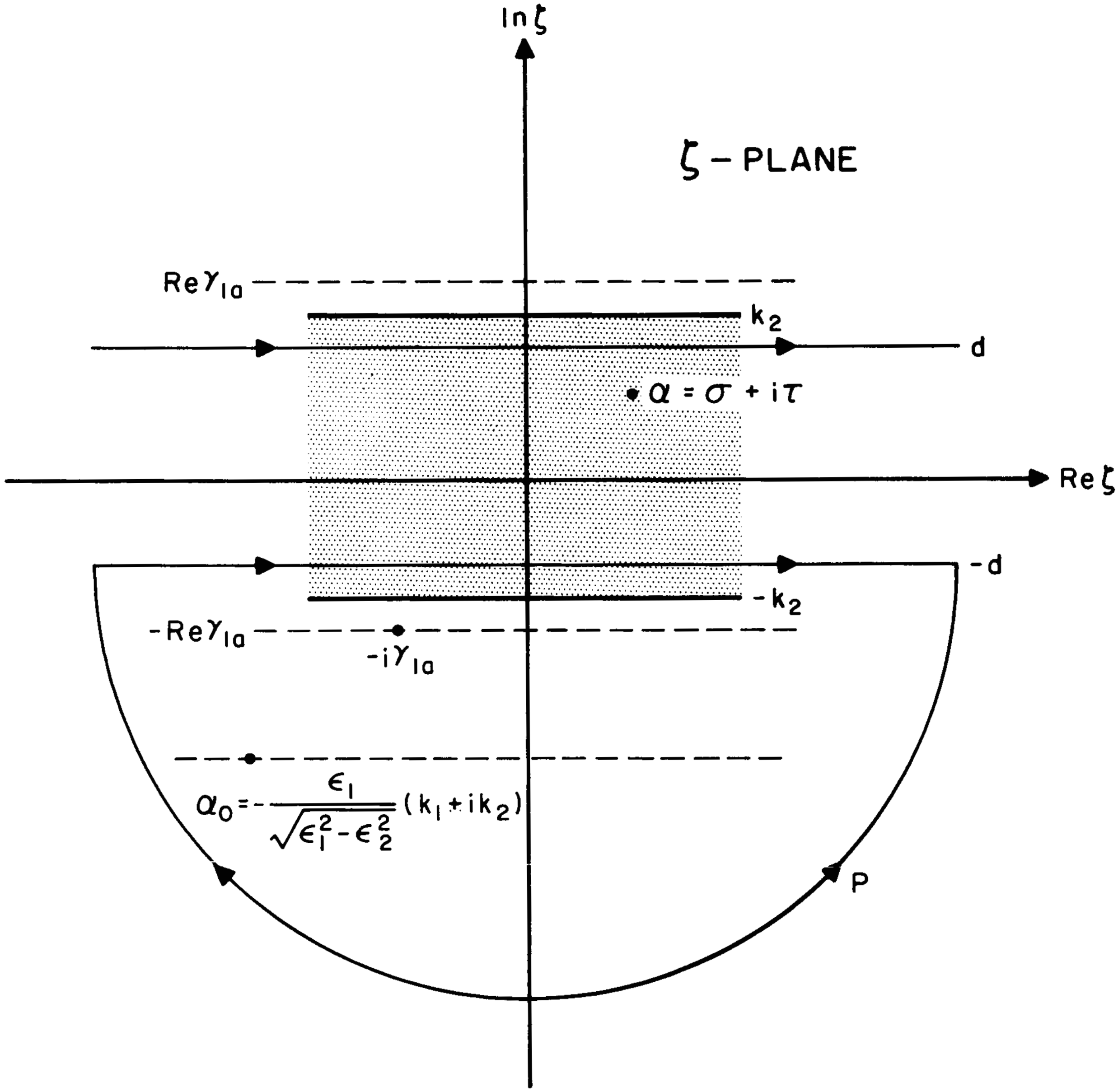

Figure 3. Contour of integration for G in Equation (4-16).

for $S_{(-)}(\alpha)$ or $[\Psi_-(\alpha) + \Psi_-(-\alpha)]$, namely,

$$\Psi_-(\alpha)+\Psi_+(-\alpha) = \left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha+i\gamma_{1a})} \frac{L_-(\alpha)}{H_-(\alpha)} - \frac{S_{(-)}(\alpha_o)H_+(\alpha_o)}{H_-(\alpha)} e^{+i\alpha_o\ell}$$

$$-\left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a}\left[\frac{1}{i(\alpha+i\gamma_{1a})} - \frac{e^{-\gamma_{1a}\ell}}{i(-\alpha+i\gamma_{1a})}\right] \qquad (4\text{-}20)$$

Similiarly, by starting with Equations (4-17) and following the same procedure, one obtains

$$\Psi_-(\alpha)-\Psi_+(-\alpha) = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha+i\gamma_{1a})} \frac{L_-(\alpha)}{H_-(\alpha)} + \frac{D_{(-)}(\alpha_o)H_+(\alpha_o)}{H_-(\alpha)} e^{-i\alpha_o\ell}$$

$$-\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \sin\frac{\pi b}{a}\left[\frac{1}{i(\alpha+i\gamma_{1a})} + \frac{e^{-\gamma_{1a}\ell}}{i(-\alpha+i\gamma_{1a})}\right] \qquad (4\text{-}21)$$

where $D_{(-)}(\alpha_o)$ is given by

$$D_{(-)}(\alpha_o) = \left[\frac{H_-(\alpha_o)-H_+(\alpha_o)e^{-i\alpha_o\ell}}{L_-(\alpha_o)}\right]^{-1} \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha_o+i\gamma_{1a})} \qquad (4\text{-}22)$$

From Equation (4-20) and (4-21), one readily obtains the solution for $\Psi_-(\alpha)$ and $\Psi_+(-\alpha)$:

$$\Psi_-(\alpha) = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{i(\alpha+i\gamma_{1a})} \frac{L_-(\alpha)}{H_-(\alpha)} + \frac{H_+(\alpha_o)e^{-i\alpha_o\ell}}{2H_-(\alpha)} [D_{(-)}(\alpha_o)-S_{(-)}(\alpha_o)]$$

$$-\left(\frac{1}{2\pi}\right)^{1/2} \sin\frac{\pi b}{a} \frac{1}{i(\alpha+i\gamma_{1a})}$$

$$\Psi_+(\alpha) = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \sin\frac{\pi b}{a} \frac{e^{-\gamma_{1a}\ell}}{i(\alpha+i\gamma_{1a})} - \frac{H_+(\alpha_o)e^{i\alpha_o\ell}}{H_+(\alpha)} [D_{(-)}(\alpha_o) + S_{(-)}(\alpha_o)]$$

where the sign of α in the expression $\Psi_+(-\alpha)$ has been changed in order to have the solution for $\Psi_+(\alpha)$. Substituting the above results into Equation (4-3), one can solve for $A_1(\alpha)$, and hence $\Phi(x,\alpha)$, via Equation (2-11). It suffices to consider only the region $x > b$, where it is found that

$$\Phi(x,\alpha) = \frac{h_2}{h_1^2+h_2^2} \Bigg\{ \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{(i\alpha-\gamma_{1a})} \frac{1}{L_+(\alpha)H_-(\alpha)}$$

$$+ \frac{H_+(\alpha_o)e^{-i\alpha_o\ell}}{2L(\alpha)H_-(\alpha)} [D_{(-)}(\alpha_o) - S_{(-)}(\alpha_o)] \tag{4-23}$$

$$- \frac{H_+(\alpha_o)e^{i(\alpha-\alpha_o)\ell}}{2L(\alpha)H_+(\alpha)} [D_{(-)}(\alpha_o) + S_{(-)}(\alpha_o)] \Bigg\} \left\{ \frac{\gamma\cosh(x-a)}{\sinh\gamma c} + \frac{ih_1\alpha}{h_2} \frac{\sinh\gamma(x-a)}{\sinh\gamma c} \right\}$$

The scattered magnetic field $H_y(x,z)$ is obtained by taking the inverse Fourier transform of $\Phi(x,\alpha)$, namely for $x > b$,

$$H_y(x,z) = \frac{h_2}{h_1^2+h_2^2} \frac{1}{2\pi i} \int_{P_1} \left\{ \frac{\gamma\cosh(x-a)}{\sinh\gamma c} + \frac{ih_1\alpha}{h_2} \frac{\sinh\gamma(x-a)}{\sinh\gamma c} \right\}$$

$$\Bigg\{ \sin\frac{\pi b}{a} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{(\alpha+i\gamma_{1a})} \frac{1}{L_+(\alpha)H_-(\alpha)}$$

$$+ \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{iH_+(\alpha_o)e^{-i\alpha_o\ell}}{L(\alpha)H_-(\alpha)} [D_{(-)}(\alpha_o) - S_{(-)}(\alpha_o)] \Bigg\} e^{-i\alpha z}\, d\alpha$$

$$- \frac{h_2}{h_1^2+h_2^2} \frac{1}{2\pi i} \int_{P_2} \left\{ \frac{\gamma \cosh(x-a)}{\sinh \gamma c} + \frac{ih_1 a}{h_2} \frac{\sinh\gamma(x-a)}{\sinh \gamma c} \right\} \left\{ \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{iH_+(a_o)e^{-ia_o \ell}}{L(a)H_+(a)} \right\}$$

$$\left\{ D_{(-)}(a_o) + S_{(-)}(a_o) \right\} e^{-ia(z-\ell)} \, da \qquad (4\text{-}24)$$

where P_1 and P_2 are the paths of integration determined by the convergence of respective integrals as shown in Figure 4. Since only simple poles are involved, the evaluation of the above integrals is straightforward. However, the complete field expressions become quite complicated; therefore, only the expression for propagating modes are given below:

For $x > b$ and $z < 0$,

$$H_y(x,z) = \frac{h_2}{h_1^2+h_2^2} \left\{ \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{L_+(i\gamma_{1a})H'_-(i\gamma_{1a})} \frac{1}{2i\gamma_{1a}} + \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{iH_+(a_o)e^{-ia_o\ell}}{L(i\gamma_{1a})H'_-(i\gamma_{1a})} \right.$$

$$\left. [D_{(-)}(a_o) - S_{(-)}(a_o)] \right\} \left\{ \frac{\pi}{a} \cos\frac{\pi}{a} x - \frac{h_1\gamma_{1a}}{h_2} \sin \frac{\pi}{a} x \right\} e^{\gamma_{1a} z}$$

For $x > b$ and $0 \leq z \leq \ell$,

$$H_y(x,z) = \frac{h_2}{h_1^2+h_2^2} \frac{1}{L(-i\gamma_{1a})} \left[\frac{\pi}{a} \cos \frac{\pi}{a}(x-a) + \frac{h_1\gamma_{1a}}{h_2} \sin \frac{\pi}{a}(x-a) \right] e^{-\gamma_{1a} z}$$

$$+ \frac{h_2}{h_1^2+h_2^2} \sin \frac{\pi b}{a} \frac{h_2}{h_1^2+h_2^2} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{H_-(a_o)} \frac{1}{a_o+i\gamma_{1a}} \frac{\gamma_o}{\sinh\gamma_o c} e^{\gamma_o (x-a)} e^{-ia_o z}$$

$$+ \frac{ih_2}{h_1^2+h_2^2} \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{H_+(a_o)e^{-ia_o\ell}}{H_-(a_o)L_-(a_o)} [D_{(-)}(a_o) - S_{(-)}(a_o)] \frac{\gamma_o}{\sinh\gamma_o c} e^{\gamma_o(x-a)} e^{-ia_o z}$$

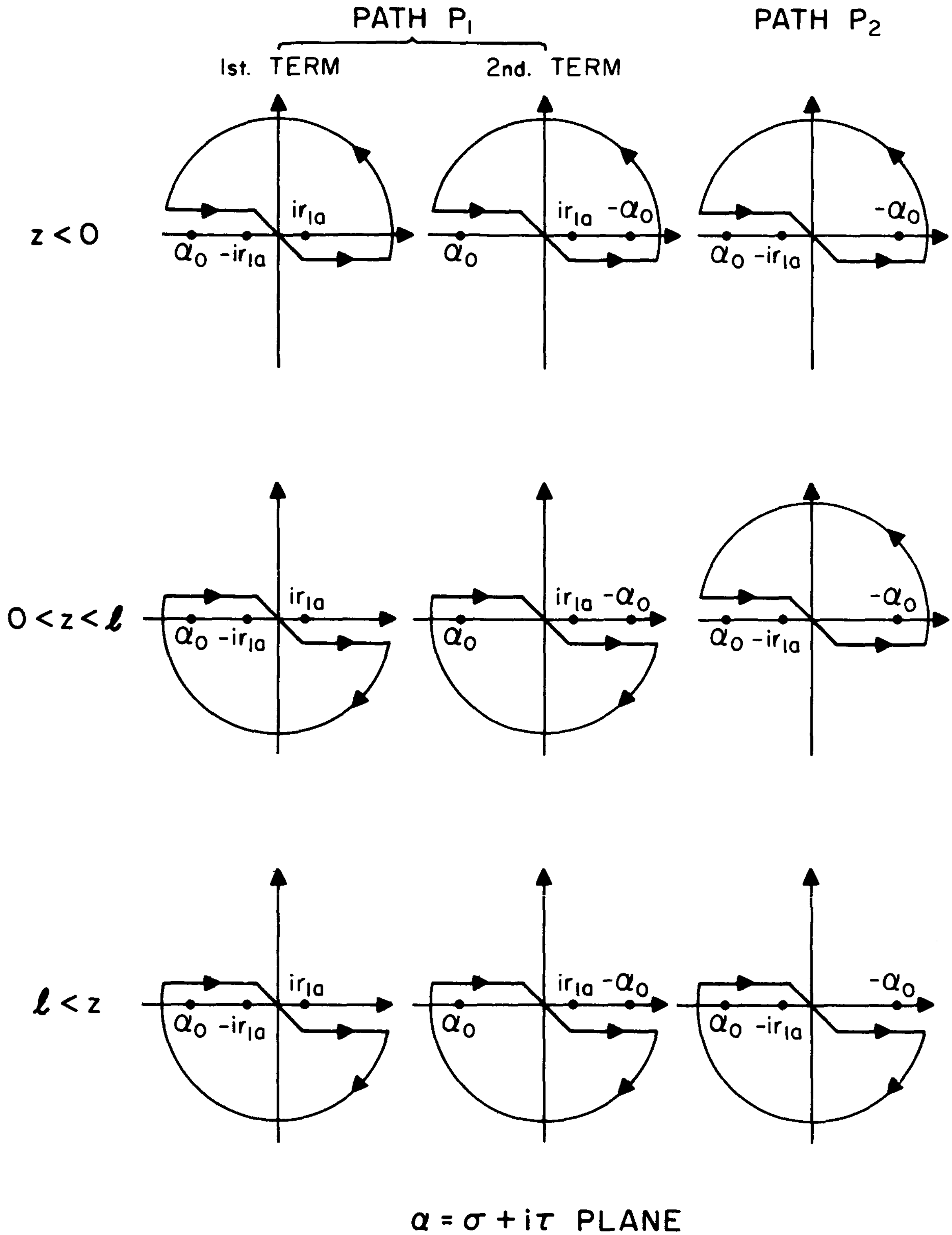

Figure 4. Contour of integration for $H_y(x,z)$ in the case of finite bifurcation.

$$- \frac{ih_2}{h_1^2+h_2^2} \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{H_+(\alpha_o)e^{-iz\alpha_o \ell}}{H_-(\alpha_o)L_-(\alpha_o)} \; [D_{(-)}(\alpha_o) + S_{(-)}(\alpha_o)] \; \frac{\gamma_o}{\sinh \gamma_o c} \; e^{\gamma_o (a-x)} \; e^{i\alpha_o z}$$

For $x \geq b$ and $z > \ell$,

$$H_y(x,z) = \frac{h_2}{h_1^2+h_2^2} \left\{ \frac{1}{L(-i\gamma_{1a})} e^{-\gamma_{1a} z} - \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{iH_+(\alpha_o)e^{-i\alpha_o \ell}}{H_+'(-i\gamma_{1a})L(-i\gamma_{1a})} [D_{(-)}(\alpha_o)+S_{(-)}(\alpha_o)] \right.$$

$$\left. e^{-i\gamma_{1a}(z-\ell)} \right\} \left\{ \frac{\pi}{a} \cos \frac{\pi}{a}(x-a) + \frac{h_1 \gamma_{1a}}{h_2} \sin \frac{\pi}{a}(x-a) \right\}$$

For $x < b$, the magnetic field $H_y(x,z)$ has the same expression as above except that $(x-a)$ is replaced by x, and the sign of the entire expression is changed. Accordingly, the electric fields $E_x(x,z)$ and $E_z(x,z)$ can be detained from Equation (2-6) and (2-7). Thus, the case of finite septum for large ℓ is solved with the approximation described above.

5. CONCLUSION

In this paper the problem of H-plane bifurcation in a parallel plate waveguide filled with anisotropic plasma is solved provided that the dc magnetic field is along the edge of the septum, and there is no field variation in this direction. When the septum is extended to infinity, the exact field solution is obtained. When the septum is of finite length ℓ, an approximate solution is found for large ℓ. In both cases the incident field is assumed to be the dominant TE or TM mode. In the trivial case when the incident field is of TEM mode, the wave travels freely in the guide and the septum causes no modification on the fields. In case of an incident TE mode, the problem becomes identical to that for an isotropic medium in the guide. In case of an incident TM mode, a scattered TEM mode is produced in the bifurcated portion. Although in this paper for simplicity only one propagating mode is assumed in the un-bifurcated portion, a more general case can be solved with no difficulty. In the case of finite bifurcation, where ℓ is not very large, a more accurate approximation can be obtained by retaining more terms in the integral G in Equation (4-16).

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