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by Theodore Katsanis

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Cleveland, Ohio

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SUMMARY

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A method, suitable for computer programing, of analyzing flow on a blade-to-blade surface of a turbomachine is presented. The method is based on an equation for the velocity gradient along an arbitrary quasi-orthogonal between blades and is similar to a method using quasi-orthogonals in a meridional plane. With this method, a streamline analysis can be made for any blade-to-blade stream surface. This surface, if desired, may be assumed to be a surface of revolution generated by a meridional streamline obtained from a meridional streamline analysis. On this stream surface a two-dimensional solution for the velocity and pressure distributions is obtained. With several such blade-to-blade solutions, the velocity distribution throughout the rotor passage can be calculated. Simplifying assumptions for upstream and downstream conditions are made for the purpose of readily obtaining a reasonable approximation near the inlet and outlet.

The method was applied to a radial inflow turbine with splitter vanes. These results are given as a numerical example, and the blade surface velocities obtained are compared with those obtained by a simple linear velocity method. The comparison shows a large discrepancy between the linear velocity method and the method of this report near the outlet, but most of the rest of the blade is in reasonable agreement. The Fortran computer program is included.

Author

INTRODUCTION

Quasi-three-dimensional methods have been developed for analyzing flow through mixed-flow turbomachines. One method is based on a meridional plane solution, with blade surface velocities calculated by some approximate formula. In reference 1 a method is given for obtaining a meridional plane solution using streamlines and quasi-orthogonals to establish a grid for the solution. A quasi-orthogonal is defined to be any curve that intersects every streamline between the flow boundaries exactly once, as does an orthogonal to the streamlines. A quasi-orthogonal, however, is not necessarily orthogonal to any streamline. The usefulness of quasi-orthogonals lies in the fact that they can be predetermined in some convenient manner, and they remain fixed regardless of any change of streamlines. Using this technique it was possible to develop a computer program that would calculate the velocity and pressure distributions without any intermediate graphical procedures or hand calculations even for turbomachines with wide passages and a change in direction from radial to axial within the rotor blade.

From the meridional solution it is possible to obtain blade surface velocities by several methods. One method is by means of an approximate formula based on the assumption of linear velocity between blades and absolute irrotational flow (ref. 2). This gives good results within the blades away from the inlet or outlet. Another method is the use of potential flow equations that may be solved by relaxation methods (ref. 3). The relaxation method gives good solutions but is a rather tedious computation, even using a computer. It was realized that the basic idea used in reference 1 to obtain a meridional solution could be applied to obtain a blade-to-blade solution. By extending the solution upstream and downstream of the rotor, a reasonable solution resulted throughout the rotor, and the solution was obtained with a reasonable effort.

This report presents the analysis method for a blade-to-blade solution using quasi-orthogonals and discusses the numerical techniques required for obtaining solutions using a digital computer. The computer program developed for the blade-to-blade analysis during this study is included. As a numerical example of the application of the analysis method, a radial-inlet mixed-flow gas turbine with splitter blades and of high specific speed is analyzed.

METHOD OF ANALYSIS

The blade-to-blade analysis is based on the assumption of a stream surface extending from the pressure surface of a complete blade to the suction surface of the next complete blade, with the possibility of having a splitter blade between the complete blades. Although not necessary, it will be assumed that the stream surface is a surface of revolution. The shape of the surface of revolution may be determined by a hub-to-shroud streamline analysis, as indicated in reference 1. A separate blade-to-blade solution could be obtained for each streamline in the meridional solution. In many cases three blade-to-blade solutions would be adequate, one at the hub, one at the 50-percent meridional streamline, and one at the shroud. In the analysis presented herein, a correction for a loss in relative total pressure is included in the continuity equation to account for blade losses.

To obtain a reasonably good solution to the equations near the inlet or outlet of the rotor blade, it is necessary to extend the solution beyond the blade itself. The location of the streamline that passes through the stagnation point on the leading or trailing edge of the blade is not known. This location, however, can be approximated by various methods to obtain a better solution near the inlet or outlet. The basic method for a streamline analysis on a blade-to-blade surface for any type of turbomachine rotor or stator is given in the next section. This is followed by the Radial Flow Analysis which is applicable only to radial turbines.

General Flow Distribution Analysis

Analytical equations. - An equation for the directional derivative of the relative velocity based on an assumption of nonviscous flow is derived in reference 1. This differential equation is used to calculate the flow distribution by numerically integrating the equation along an assumed quasi-orthogonal. The directional derivative of the velocity as given by equations (B13) and (B14) of reference 1 is

$$\frac{dW}{dq} = a\frac{dr}{dq} + b\frac{dz}{dq} + c\frac{d\theta}{dq} + \frac{1}{W}\left(\frac{dh_i'}{dq} - \omega\frac{d\lambda}{dq}\right) \tag{1}$$

where

$$\left.\begin{aligned} a &= \frac{W\cos^2\beta\cos\alpha}{r_c} - \frac{W\sin^2\beta}{r} + \sin\alpha\cos\beta\frac{dW_m}{dm} - 2\omega\sin\beta \\ b &= -\frac{W\cos^2\beta\sin\alpha}{r_c} + \cos\alpha\cos\beta\frac{dW_m}{dm} \\ c &= W\sin\alpha\sin\beta\cos\beta + r\cos\beta\left(\frac{dW_\theta}{dm} + 2\omega\sin\alpha\right) \end{aligned}\right\} \tag{2}$$

The coordinate system and notation are shown in figure 1. (All symbols are defined in appendix A.)

These equations give the velocity gradient along an arbitrary quasi-orthogonal on a stream surface where q is the distance along the quasi-orthogonal and where W is considered as a function of q alone. Note that q may be considered as a function of θ, hence W may be considered a function of θ alone, with the understanding that W is to

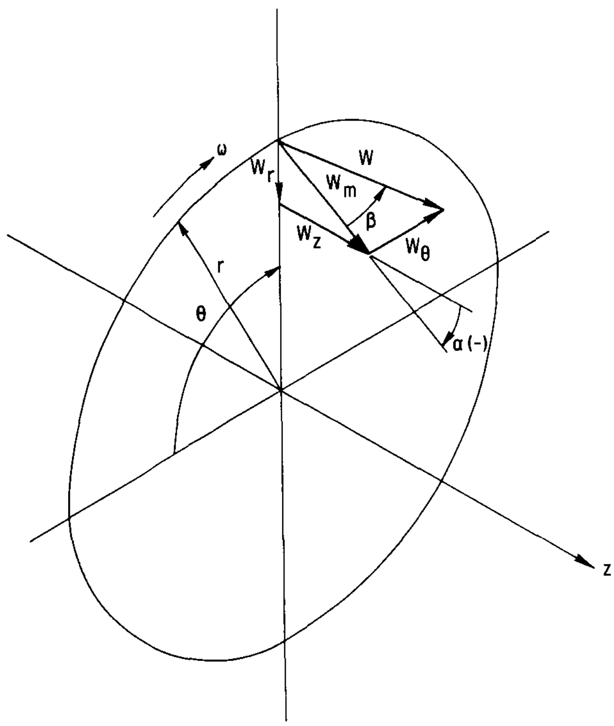


Figure 1. - Coordinate system and velocity components.

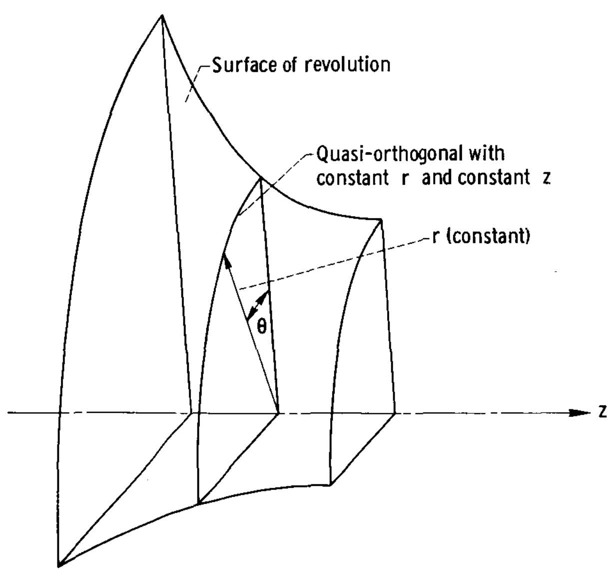


Figure 2. - Quasi-orthogonal on surface of revolution.

be on the quasi-orthogonal. Using the fact that

$$\frac{dW}{d\theta} = \frac{dW}{dq}\frac{dq}{d\theta}$$

results in the following relation being obtained from equation (1)

$$\frac{dW}{d\theta} = a\frac{dr}{d\theta} + b\frac{dz}{d\theta} + c \tag{3}$$

along the quasi-orthogonal since $dh_i'/d\theta = d\lambda/d\theta = 0$. Note that equation (3) is valid even if the stream surface is not a surface of revolution.

In the examples given in this report, the blade-to-blade stream surface is assumed to be a surface of revolution, so that the quasi-orthogonal can be chosen a circular arc as shown in figure 2 with

$$\frac{dr}{d\theta} = \frac{dz}{d\theta} = 0$$

An initial value must be determined for the integration of equation (3). This value is determined by the fact that the weight flow between blades is known. The weight flow across a fixed line from blade to blade can be computed by

$$w = N\int_{\theta_s}^{\theta_p} \rho W_m \, \Delta n \, r \, d\theta \tag{4}$$

where Δn is the normal distance between two closely spaced streamlines in the meridional plane and can be determined from a meridional plane streamline analysis, as indicated in reference 1. The weight flow w is that portion of the total weight flow which is calculated to flow between the streamlines that are used to determine Δn. The density is calculated by the following equations, which are from equations (3) to (5) in reference 1

$$\rho = \left(\frac{T}{T_i'}\right)^{1/(\gamma-1)} \rho_i' - \left[\left(\frac{T}{T_i'}\right)\left(\frac{T_i'}{T''}\right)\right]^{1/(\gamma-1)} \frac{\Delta p''}{RT_i'}\left(\frac{T_i'}{T''}\right) \tag{5}$$

where

$$\frac{T}{T_i'} = 1 - \frac{W^2 + 2\omega\lambda - \omega^2 r^2}{2c_p T_i'} \tag{6}$$

and

$$\frac{T''}{T_i'} = 1 - \frac{2\omega\lambda - \omega^2 r^2}{2c_p T_i'} \tag{7}$$

Equations (5) to (7) and $W_m = W \cos \beta$ give the numerical data for equation (4), which can then be numerically integrated.

Numerical techniques and procedure. - The parameters α, β, r_c, dW_m/dm, and dW_θ/dm must be evaluated for use in equations (2) and (3). For this, a streamline geometry must be established. First, straight lines from blade to blade are established. These lines are the quasi-orthogonals. Since the assumed stream surface is a surface of revolution, the quasi-orthogonals can be chosen so that r and z are constant. For an initial approximation, each quasi-orthogonal is divided into a number of equal spaces. The parameters can now be evaluated in essentially the same manner as in reference 1, using spline fit curves for approximating first and second derivatives (see appendix C of ref. 1). The angle β is given by

$$\tan \beta = r \frac{d\theta}{dm} \tag{8}$$

where $d\theta/dm$ can be calculated from a given streamline pattern by the use of a spline fit curve.

After the parameters in equation (3) have been evaluated, the equation can be integrated by a Runge-Kutta method such as indicated by equation (13) of reference 1. Continuity is used to determine the initial value of W in the numerical integration of equation (3). After equation (3) has been integrated, the resulting flow distribution can be used to obtain an improved streamline pattern. The details of this procedure is given in reference 1 in the section Numerical Techniques and Procedures.

Radial Turbine Analysis

As an illustration of the use of the general flow distribution analysis, the method will be applied to a radial inflow gas turbine with splitter blades. The specific considerations for the analysis of this type of turbine are given herein.

Upstream analysis of radial turbine rotor. - In this section, equations are developed that are useful in extending the solution upstream for a rotor with radial inflow. At the inlet to the rotor of a radial inlet turbine, the relative flow direction usually is not parallel to the mean blade surface but is at a negative angle to the blade (i.e., W_θ is opposite to the direction of rotation). This means that inlet conditions cannot be uniform from blade to blade, since the flow direction must be parallel to the blade at the blade surface. For the purposes of the analysis, the solution was extended upstream one station where uniform inlet conditions were assumed. In most cases inlet conditions are specified at the inlet to the rotor. These conditions will be taken to be average from blade to blade. From this, the conditions at the upstream station can be calculated.

From the assumption of free vortex flow, $d(rV_\theta)/dt = 0$. This means that $rV_\theta = r_iV_{\theta,i} = \lambda$ where the subscript i is used to designate a reference point such as the tip of the rotor. Also, by continuity $\rho rW_m \Delta n = \rho_i r_i W_{m,i} \Delta n_i$, so that

$$\frac{W_{m,i}}{W_m} = \frac{r\rho \Delta n}{r_i\rho_i \Delta n_i}$$

These relations and the fact that $\tan \beta = W_\theta/W_m$ are used to calculate β as follows

$$\tan \beta = \tan \beta_i \frac{W_{m,i}}{W_{\theta,i}} \frac{W_\theta}{W_m}$$

$$= \tan \beta_i \frac{\omega r - V_\theta}{\omega r_i - V_{\theta,i}} \frac{r\rho \Delta n}{r_i\rho_i \Delta n_i}$$

This gives

$$\tan \beta = \tan \beta_i \frac{\omega r^2 - \lambda}{\omega r_i^2 - \lambda} \frac{\rho \Delta n}{\rho_i \Delta n_i} \tag{9}$$

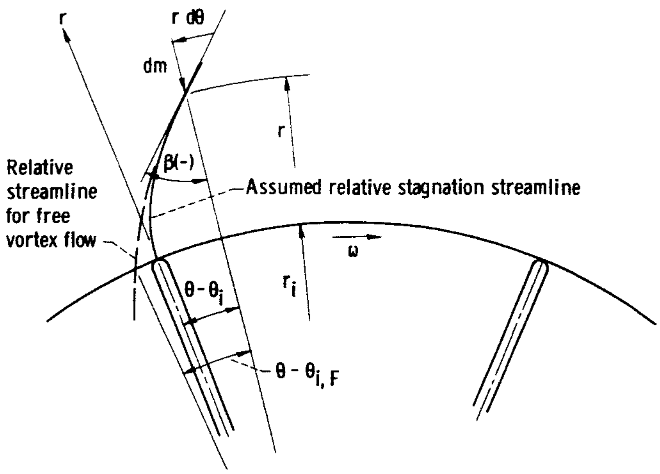


Figure 3. - Relation between stagnation and free vortex relative streamline.

For inward flow $dr/dm = -1$, since dm is positive in the direction of flow (inward), but dr is positive in the outward direction. Since $\tan \beta = r\, d\theta/dm$, this gives $d\theta/dr = -(\tan \beta)/r$.

For the equation of the relative streamline, integrate

$$\frac{d\theta}{dr} = -\frac{\tan \beta}{r} = \frac{\rho\, \Delta n \tan \beta_i}{\rho_i\, \Delta n_i\left(\omega r_i^2 - \lambda\right)}\left(\frac{\lambda}{r} - \omega r\right)$$

This expression can be integrated by parts if $u = \rho\, \Delta n$ and $v = \lambda \log r - (\omega r^2/2)$ in the formula $\int u\, dv = uv - \int v\, du$. This gives

$$\theta - \theta_{i,F} = \frac{\tan \beta_i}{\left(\omega r_i^2 - \lambda\right)\rho_i\, \Delta n_i}\left[\rho\, \Delta n\left(\lambda \log r - \frac{\omega r^2}{2}\right) - \rho_i\, \Delta n_i\left(\lambda \log r_i - \frac{\omega r_i^2}{2}\right)\right.$$

$$\left. - \int_{\rho_i\, \Delta n_i}^{\rho\, \Delta n}\left(\lambda \log r - \frac{\omega r^2}{2}\right) du\right]$$

for free vortex flow.

If the last integral is approximated by the trapezoid rule, the following equation is obtained

$$\theta - \theta_{i,F} = \frac{\tan \beta_i}{\left(\omega r_i^2 - \lambda\right)} \frac{\left(\rho\, \Delta n + \rho_i\, \Delta n_i\right)}{2\rho_i\, \Delta n_i}\left[\lambda \log \frac{r}{r_i} + \omega \frac{\left(r_i^2 - r^2\right)}{2}\right]$$

From figure 3 it can be seen that $\theta - \theta_i$ can be expected to be slightly less than $\theta - \theta_{i,F}$, therefore it is assumed that $\theta - \theta_i = M(\theta - \theta_{i,F})$, which gives

$$\theta - \theta_i = M \frac{\tan \beta_i}{\left(\omega r_i^2 - \lambda\right)} \frac{\left(\rho\, \Delta n + \rho_i\, \Delta n_i\right)}{2\rho_i\, \Delta n_i}\left[\lambda \log \frac{r}{r_i} + \omega \frac{\left(r_i^2 - r^2\right)}{2}\right] \qquad (10)$$

where M is expected to be slightly less than 1.

For purposes of computation, a value of M is assumed, and after a blade-to-blade solution has been computed, the average value of $V_{\theta,i}$ between the blades is checked. If this average is not sufficiently close to the specified value of $V_{\theta,i}$, a different value of M can be tried. It was found that the precise value of M was not critical in its effect on the solution.

To completely specify a spline fit curve passing through a set of points, it is necessary to specify a condition at each end of the curve. In using the spline fit for approximating a streamline in reference 1, it was assumed that the streamline was straightening out at the end, that is, the second derivative at the end point was specified to be one-half the second derivative at the next point. For the blade-to-blade solution at the inlet, however, this is not a reasonable assumption. The coordinates used to specify the streamline are θ and m. For a radial inlet, $d\theta/dm = (\tan \beta)/r$, where $\tan \beta$ is given by equation (9). Thus, the derivative can be specified at the first point. This gives a better approximation to the streamlines.

A similar situation occurs in calculating dW_θ/dm by means of a spline fit curve. In this case,

$$\frac{dW_\theta}{dm} = -\frac{dV_\theta}{dr} + \omega = \frac{r_i V_{\theta,i}}{r^2} + \omega$$

for free vortex flow. Hence, at the first upstream station,

$$\frac{dW_\theta}{dm} = \frac{\lambda}{r^2} + \omega \tag{11}$$

where r is the radius at the upstream station.

For the streamlines that lead to the blade surfaces, there is a sharp curvature just before the blade followed by almost no curvature on the blade itself. To specify this streamline more accurately, additional points on the blade surface were specified between the leading edge and the next station. Uniform inlet conditions are assumed at the upstream station. For the downstream end of the streamline it is still reasonable to assume the streamline is becoming straight.

Downstream analysis for rotor with axial outlet. - It is assumed that the streamline downstream from the stagnation point on a rotor with axial flow at the outlet has a constant angle β. This downstream angle of β can be determined from the blade angle at the outlet by correcting this angle for blade blockage. In the solution of the problem, this assumed streamline downstream from the blade is treated as if it were an extension of the blade. If the assumed streamline is close to the actual streamline, the velocity will

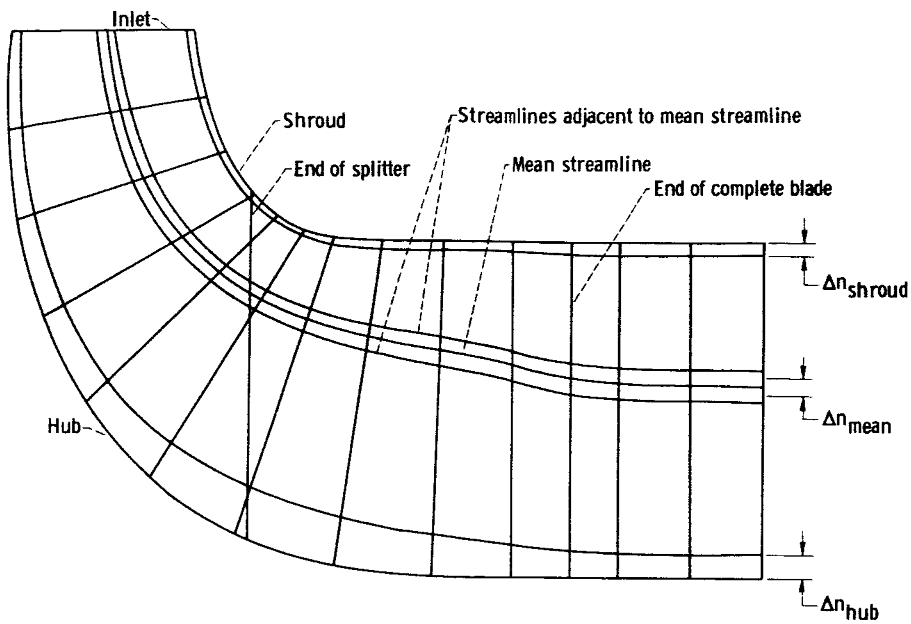


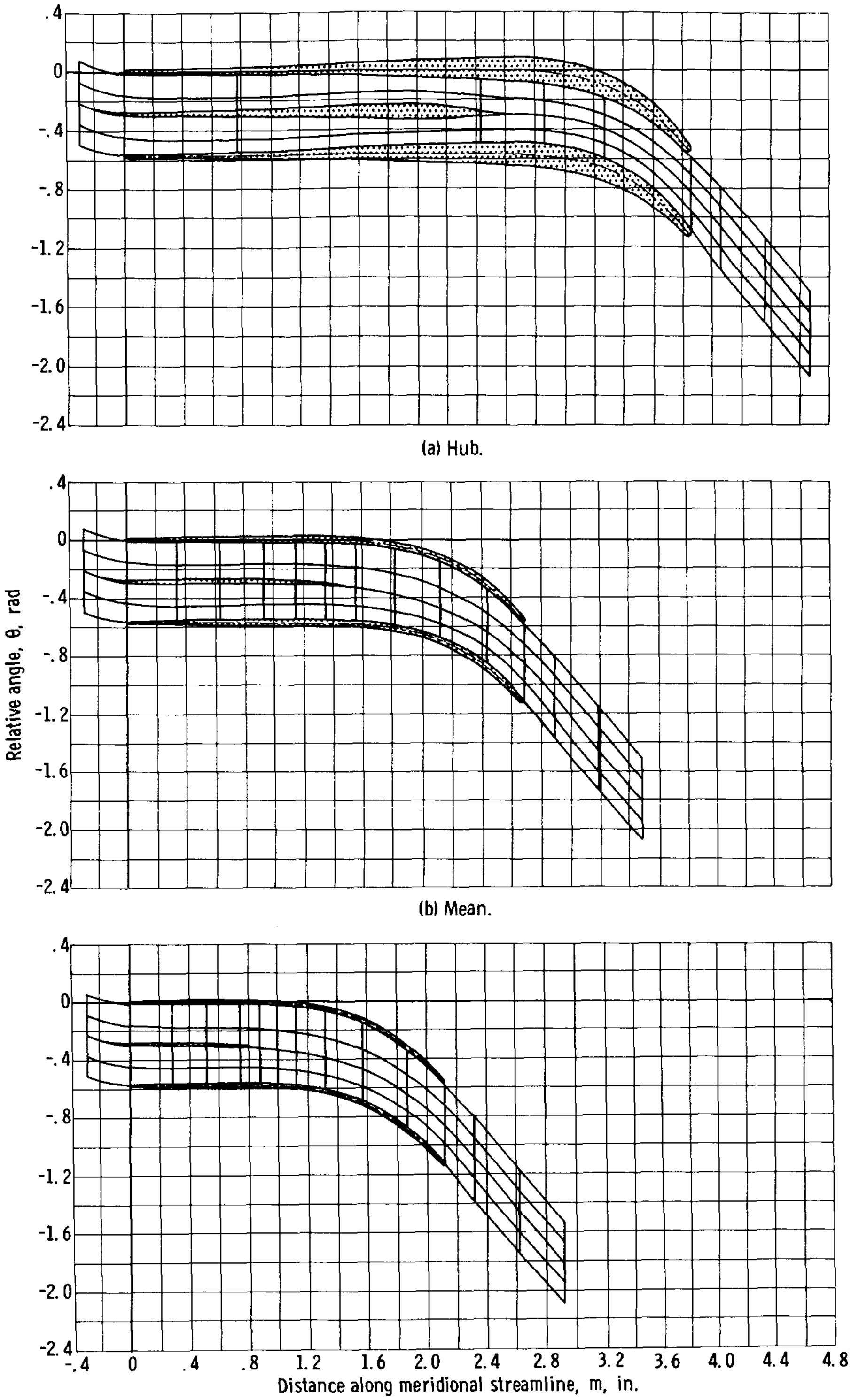
Figure 4. - Hub-shroud profile with streamlines used for blade-to-blade analysis.

have little variation downstream of the rotor. If the variation is too great, it can be reduced by adjustment of the assumed streamline, however, it has been found that a small variation in the assumed streamline has very little effect on the solution within the rotor.

Analysis of rotor with splitter blades. - The method of analysis presented herein has been applied to a turbine rotor with splitter blades. It can be reasonably assumed that the weight flows through the passages on either side of the splitter blade are equal. The 50 percent streamline from blade to blade, therefore, meets the stagnation point on the splitter blade. A detailed analysis of the streamline shape in the neighborhood of the leading and trailing edges would be difficult and has not been attempted. Instead, the streamlines representing the suction and pressure surfaces of a blade are assumed to join one station beyond the end of the blade. This does not give detailed information on what is happening in the immediate neighborhood of the leading or trailing edge, but has little effect on the solution a short distance away. In the solution of the equations, the weight flow on either side of the blades is assumed to be 50 percent of the total weight between complete blades. This is used to evaluate the initial value for the integration of equation (3) on either side of the splitter blade independently. Downstream of the splitter, equation (3) is integrated across the entire width of the passage.

NUMERICAL EXAMPLE

The method outlined has been applied to the analysis of a small radial flow turbine. The calculations were made on a digital computer. The hub-shroud profile is shown in figure 4. The blade has radial elements, except near the trailing edge of the splitter



(c) Shroud.

Figure 5. - Blade-to-blade streamline pattern.

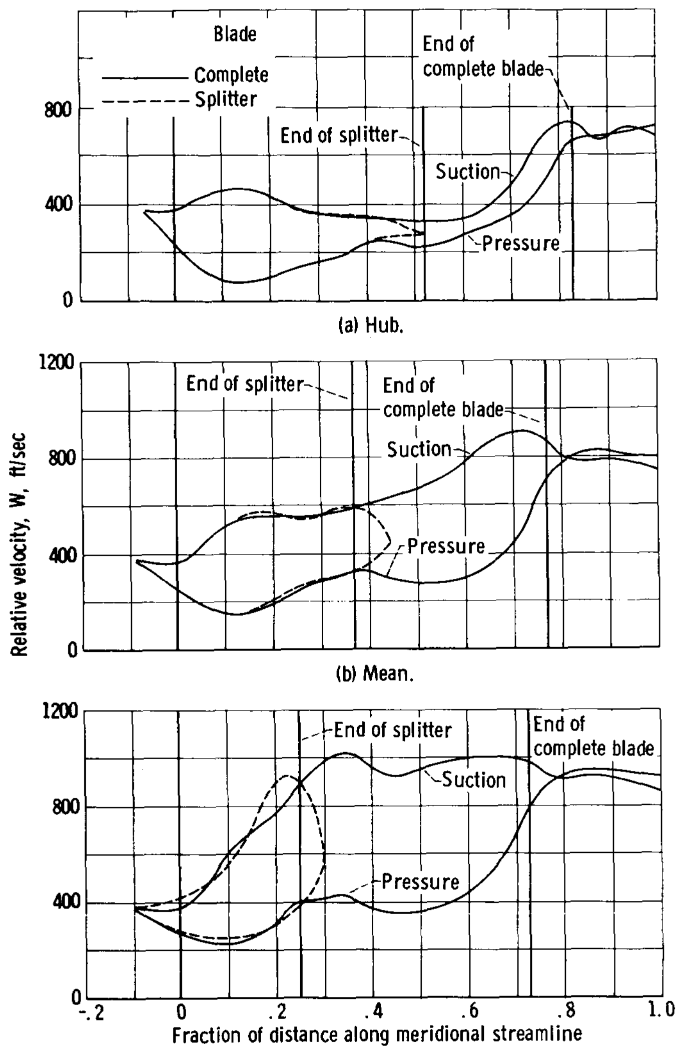
blade, where the taper is not the same on both sides of the blade. The pertinent data for the case analyzed is given in the following list:

Total number of blades	22
Number of splitter blades	11
Inlet radius, r_i, in.	3.01
Rotational speed, rpm	38 500
Fluid	Argon
Weight flow, lb/sec	0.611
Inlet total temperature, T_i', ^{o}R	1950
Absolute tangential velocity at inlet, $V_{\theta,i}$, ft/sec	883
Inlet relative flow angle, β_i, deg	-25.1
Inlet total pressure, p_i', lb/sq in. abs	12.97
Loss of relative total pressure between inlet and outlet (assumed to vary linearly along streamline), $\Delta p''$, lb/sq in.	0.25

The normal blade thickness was given by means of tabulated values on a grid. Blade thickness at any given point was obtained by linear interpolation. It was assumed that h_i' and λ were constant across the inlet.

The first step in the solution was to obtain a meridional plane solution as described in reference 1. This solution was extended downstream by assuming a mean stream surface downstream of the blades. Twenty meridional streamlines were calculated. The meridional streamline pattern obtained at the hub, mean, and shroud is shown in figure 4. This gives the coordinates for a blade-to-blade surface at the mean meridional streamline, as well as the streamline spacing on the quasi-orthogonals at hub, mean, and shroud. With this, the normal streamline spacing and the blade coordinates can be calculated at each of the three surfaces. The blades are extended by streamlines upstream and downstream as explained in the sections on upstream and downstream analysis of a rotor. A computer program has been written for a combined meridional and blade-to-blade analysis, in which all the input data for the blade-to-blade analysis are computed by the program. The combined program is available to qualified persons if a written request is addressed to the author at the Lewis Research Center. The part of the Fortran computer program used for the blade-to-blade analysis for the numerical example is given in appendix B. The solution obtained by the computer program gives blade-to-blade streamlines in θ- and m-coordinates. These streamlines have been plotted in figure 5.

Figure 6 shows the blade loading on main and splitter blades at hub, mean, and shroud. The velocities are increasing over most of the blade; however, there is indicated a large negative velocity gradient on the pressure surface at the inlet near the hub.



(a) Hub.

(b) Mean.

(c) Shroud.

Figure 6. - Blade velocity distribution for numerical example.

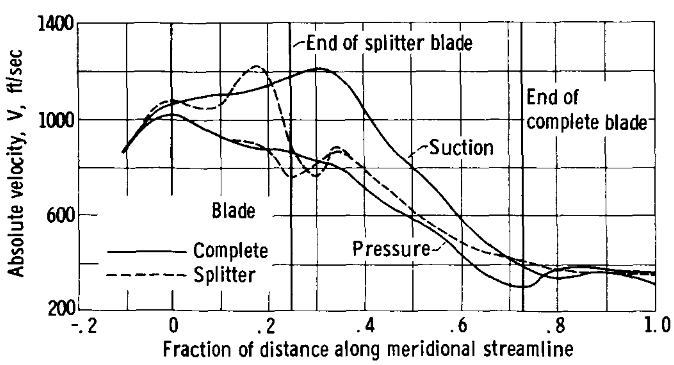
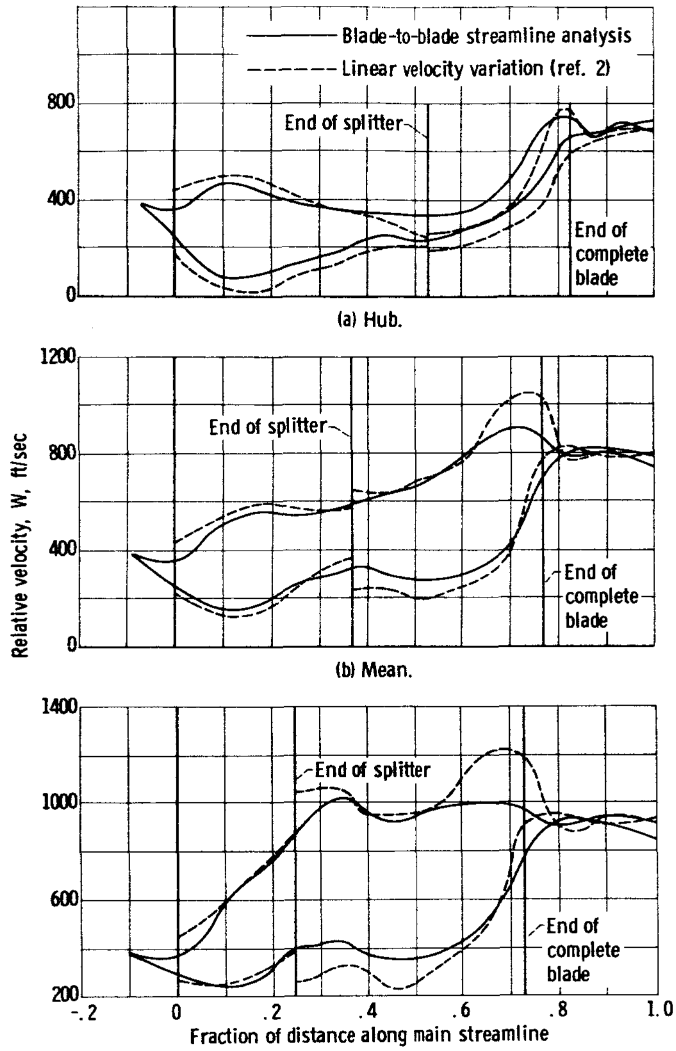


Figure 7. - Shroud absolute velocity distribution for numerical example.

It is even more severe at the end of the splitter blade near the shroud. This indicates the splitter should be extended further from the aerodynamic point of view; however, the splitter is already longer than needed at the hub, and stress considerations may not permit making the splitter extend beyond the point where it ends on the hub. Figure 7 shows the absolute velocity distribution on the stationary shroud. Here, there are large negative velocity gradients. Of course, a negative velocity gradient cannot be eliminated entirely with the outlet absolute velocity lower than the inlet absolute velocity although it can be minimized by careful design. On the pressure surface, the nearly linear distribution is about the best possible for avoiding flow separation, under these circumstances. However, along the shroud there is an increase, and then a greater decrease in a shorter distance. It is difficult to avoid something like this if there is to be any loading of the blades at the shroud.

This type of analysis could be very useful as a design tool, since it points up the location of possible flow separation. Modifications can be made in the geometry to improve on the velocity distribution, until a good design is evolved.

It is interesting to compare the blade surface velocities obtained here with those obtained by the method of reference 2, which is based on the assumption of linear velocity variation



(a) Hub.

(b) Mean.

(c) Shroud.

Figure 8. - Comparison of blade surface velocities with those obtained by linear velocity method of reference 2.

between blades. The comparison is shown in figure 8. The linear velocity method generally shows slightly greater blade loading over most of the blade. Near the outlet and just past the splitter at the shroud there is a rather large discrepancy between the linear velocity method and the blade-to-blade streamline analysis.

CONCLUDING REMARKS

A method for determining the flow distribution in a blade-to-blade surface of a turbomachine is presented. This method is based on a streamline analysis and can be used in conjunction with a meridional streamline analysis to obtain a quasi-three-dimensional solution. The method was applied to a radial inflow turbine with splitter blades. The results are given as a numerical example, and the Fortran computer program is included. The blade-surface velocities obtained are compared with those obtained by the linear velocity method of reference 2. The comparison shows a large discrepancy between the linear velocity method and the method of this report near the outlet but that most of the rest of the blade is in reasonable agreement.

The basic ideas of the methods of streamline analysis on both blade-to-blade surfaces and meridional planes are applicable to any type of turbomachine stator or rotor for either compressible or incompressible flow. The method appears to give reasonable results for the velocity distribution and for pressures throughout the passage.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 17, 1965.

APPENDIX A

SYMBOLS

a parameter, eq. (2)

b parameter, eq. (2)

c parameter, eq. (2)

c_p specific heat at constant pressure, (ft)(lb)/(slug)(^{o}R)

g acceleration due to gravity, ft/sec^2

h enthalpy, (ft)(lb)/slug

M coefficient, eq. (10)

m distance along meridional streamline, ft

N number of complete blades

Δn distance between adjacent streamlines in meridional plane, ft

p pressure, lb/sq ft

$\Delta p''$ loss in relative total pressure between inlet and any point, lb/sq ft

q distance along arbitrary three-dimensional curve, ft

R gas constant, (ft)(lb)/(slug)(^{o}R)

r radius from axis of rotation, ft

r_c radius of curvature of meridional streamline, ft

T temperature, ^{o}R

V absolute fluid velocity, ft/sec

W relative fluid velocity, ft/sec

w weight flow between adjacent blade-to-blade stream surfaces

z axial coordinate

α angle between meridional streamline and z-axis, rad

β angle between relative velocity vector and meridional plane, rad

γ ratio of specific heats

θ relative angular coordinate, rad

λ prerotation; $r_i V_{\theta,i}$, sq ft/sec

ρ mass density, slugs/cu ft

ω rotational speed, rad/sec

Subscripts:

F free vortex flow

i inlet

m component in direction of meridional streamline, ft

p pressure surface of blade

r radial component

s suction surface of blade

θ tangential component

Superscripts:

$'$ absolute stagnation condition

$''$ relative stagnation condition

APPENDIX B

FORTRAN PROGRAMS

Description of Main Program

The Fortran program listed here is for the blade-to-blade analysis of a radial inlet turbine with the same number of splitter blades as complete blades. This program was used to obtain data for the numerical example. It is written in Fortran IV and was run on the IBM 7094 digital computer. A meridional plane solution is required as input for this program. Reference 1 contains a program for the meridional plane analysis. The blade-to-blade program listed here follows the procedure given in the section Numerical techniques and procedure.

The list of program variables given later in this section indicates the equation which is used to calculate a variable or the one in which it is used. In the program, the number of the streamline is indicated by K and the number of the quasi-orthogonal by I. Number 1 is the station upstream of the inlet or the suction blade surfaces. The program is written so that all linear measurements are in inches, angles are in degrees, and pressure is in pounds per square inch for both input and output. Units are changed to feet and radians for computation in the program. All other quantities are in the units specified in appendix A.

It will be noted that a complete listing of input data cards is printed out. In the sample program, for example, the listing gives all the data used as input for the program. All input cards precede the comment card END OF INPUT STATEMENTS.

Program Variables and Definitions

A	temporary storage
AB(J)	temporary storage
AC(J)	temporary storage
AD(J)	temporary storage
AE(J)	temporary storage
AL(I)	α
ALM	λ (input variable)
AR	R (input variable)

B	temporary storage
BA(K)	total weight flow between suction surface of blade and K^{th} streamline
BCDP	integer (input variable), 1 will give THETA and WA as output on cards in binary form after final iteration; 0 will cause this to be omitted
BETA(I, K)	β
BETIN	β_i (input variable)
C	temporary storage
CAL(I)	$\cos \alpha$
CBETA(I, K)	$\cos \beta$
CI	stagnation speed of sound at inlet
CORFAC	percentage of calculated streamline correction to be used for next iteration
CP	c_p
CURV(I, K)	curvature of streamlines in m, θ plane
DELTA	calculated streamline correction
DENSTY	ρg
DN(I)	Δn (input variable), eq. (4)
DTDM(I, K)	$d\theta/dm$, eq. (8)
DTDMI	$d\theta/dm$ at upstream station (input variable)
DWDM(I, K)	dW_θ/dm, eq. (2)
DWDM1	dW_θ/dm at upstream station, eq. (11)
DWDT	$dW/d\theta$, eq. (3)
ERROR	maximum calculated streamline calculation for present iteration
ERROR 1	ERROR from previous iteration
EXPON	$1/(\gamma - 1)$, eq. (5)
GAM	γ
HT	$\Delta\theta$
I	subscript to indicate number of quasi-orthogonal, 1 is upstream of inlet
IM2	I - 2
IND	code number used by subroutine CONTIN

IND 2	code number used by subroutine CONTAL
IP2	I + 2
ITER	number of iterations to be performed after ERROR is less than TOLER or with an increase in ERROR
ITNO	iteration number
J	subscript
K	subscript to indicate number of streamline, 1 at suction surface and KMX at pressure surface
KA	subscript
KHMX	KMX/2
KHP1	(KMX/2) + 1
KHP2	(KMX/2) + 2
KMX	number of streamlines
KMXM1	KMX-1
LAST	KHMX-1
MX	number of quasi-orthogonals
MXBL	number of quasi-orthogonal at end of complete blade (input variable)
MXP2	MX + 2
MXSP	number of quasi-orthogonal at end of splitter blade (input variable)
MXSP1	MXSP + 1
NPRT	data are listed for every $(NPRT)^{th}$ streamline (input variable)
NULL	dummy variable, not used
PLOSS	$\Delta p''$ at outlet, eq. (5)
PRINT	logical variable, when PRINT = TRUE output is listed
PRS(K)	p
R(I)	r (input variable)
RC(I)	temporary storage
RHO	$\rho_i' g$ (input variable)
ROOT	$\sqrt{2}$
RUNO	integer, run number

RR	integer, 1 indicates velocities and streamlines from the previous run will be used as the initial estimate for the present case and 0 indicates that uniform velocity and equally spaced streamlines will be used as the initial estimate for the present case
SA(I, K)	$\sin \alpha \sin \beta \cos \beta$, eq. (2)
SAL(I, K)	$\sin \alpha$
SB(I, K)	$r \cos \beta \left[(dW_\theta/dm) + 2\omega \sin \alpha\right]$, eq. (2)
SBETA(I, K)	$\sin \beta$
SM(I)	distance from inlet along meridional streamline
SMAL(I)	distance from inlet along meridional streamline with additional points just inside rotor inlet to make spline fit curve approximate blade more closely
SRW	integer (input variable) that will cause subroutines to write out data for certain values, used in debugging; 0 indicates that none of the subroutines will write out extra data
T-CURV	curvature of streamlines in θ- and m-coordinates
TEMP	T_i' (input variable)
THAL(I, J)	θ-coordinates of blades with additional points just inside rotor inlet
THETA(I, K)	θ-coordinates of blades and bounding streamlines (input variable)
T1P	T/T_i'
TOLER	(input variable); if maximum calculated streamline correction is less than TOLER, iterations are considered to have converged and desired output is printed
TPP1P	T''/T_i'
TYPE	integer (input variable), 1 indicates THETA and WA are given as input on binary cards and 0 indicates these quantities will be calculated by program
V	V
VTHETA	V_θ
W	ω (input variable)
WA(I, K)	W
WAS	W^* (see eq. (13) in ref. 1)

WASS	W** (see eq. (13) in ref. 1)
WM	W_m
WT	w (input variable)
WTFL(K)	calculated weight flow between assumed blade-to-blade stream surfaces and between suction blade surface and K^{th} streamline, eq. (4)
WTH	w/2
WTHETA(I, K)	W_θ
WTOLER	allowable tolerance on weight flow in satisfying continuity between blades (input variable)
XN	N (input variable)
Z(I)	z (input variable)

Fortran Program Listing

```
      DIMENSION Z(21),R(21),DN(21),SM(21),BA(22),AB(22),AC(22),AL(21),
     1RC(21),CAL(21),SAL(21),PRS(22),WTFL(22),WTHETA(21,22),DWDM(21,22),
     2THETA(21,22),WA(21,22),BETA(21,22),SBETA(21,22),CBETA(21,22),
     3SA(21,22),SB(21,22),CURV(21,22),DTDM(21,22)
      DIMENSION Z1(21,21),R1(21,21),     THH1(21),THHKH(21),THHKP(21),
     1   THHKMX(21),THM1(21),THMKH(21),THMKP(21),THMKMX(21),THS1(21),
     2 THSKH(21),THSKP(21),THSKMX(21)
      DIMENSION THAL(23,4),SMAL(23),AD(22),AE(22),DENSTY(22),BETAI(3)
      INTEGER RUNO,TYPE,BCDP,SRW,RR,HUB,SHROUD
      LOGICAL PRINT
      RUNO=0
    5 READ  (5,1010)MX,KMX,MXSP,RR,W,WT,XN,GAM,AR
      ITNO = 1
      RUNO=RUNO+1
      WRITE (6,1020) RUNO
      WRITE (6,1010)MX,KMX,MXSP,RR,W,WT,XN,GAM,AR
      READ (5,1010)TYPE,BCDP,SRW,MXBL,TEMP,ALM,RHO,TOLER,PLOSS
      WRITE(6,1010)TYPE,BCDP,SRW,MXBL,TEMP,ALM,RHO,TOLER,PLOSS
      PLOSS=PLOSS*144.
      READ (5,1010) NULL,NPRT,ITER,NULL,BETIN,WTOLER,CORFAC
      WRITE(6,1010) NULL,NPRT,ITER,NULL,BETIN,WTOLER,CORFAC
      BETIN =  BETIN/57.29577
      KHMX = KMX/2
      KHP1 = KHMX+1
      KHP2 = KHMX+2
      READ (5,1030) (THETA(I,1),I=1,MX)
      WRITE(6,1030) (THETA(I,1),I=1,MX)
      READ (5,1030)(THETA(I,KMX),I=1,MX)
      WRITE(6,1030)(THETA(I,KMX),I=1,MX)
      READ (5,1030) (THETA(I,KHMX),I=1,MXSP)
```

```
      WRITE(6,1030) (THETA(I,KHMX),I=1,MXSP)
      READ (5,1030) (THETA(I,KHP1),I=1,MXSP)
      WRITE(6,1030) (THETA(I,KHP1),I=1,MXSP)
      IF(RR.EQ.1) GO TO 90
      READ (5,1030)(Z(I),I=1,MX)
      WRITE(6,1030)(Z(I),I=1,MX)
      READ (5,1030)(R(I),I=1,MX)
      WRITE(6,1030)(R(I),I=1,MX)
      READ (5,1030)(DN(I),I=1,MX)
      WRITE(6,1030)(DN(I),I=1,MX)
      DO 9  I=1,MX
      Z(I)=Z(I)/12.
      R(I)=R(I)/12.
    9 DN(I)=DN(I)/12.
   21 IF(TYPE.EQ.1 ) GO TO 30
      WA(1,1)=WT/RHO/DN(1)/R(1)/XN/(THETA(1,KMX)-THETA(1,1))
      DO 23 I=1,MX
      IF((I.EQ.1).OR.(I.GT.MXSP)) THETA(I,KHMX) = (THETA(I,KMX)+THETA
     1  (I,1))/2.
      IF((I.EQ.1).OR.(I.GT.MXSP)) THETA(I,KHP1) = THETA(I,KHMX)
      DO 22 K=1,KHMX
      THETA(I,K) = FLOAT(K-1)/FLOAT(KHMX-1)*(THETA(I,KHMX)-THETA(I,1))
     1  + THETA(I,1)
   22 WA(I,K)=WA(1,1)
      DO 23 K=KHP1,KMX
      THETA(I,K) = FLOAT(K-KHP1)/FLOAT(KMX-KHP1)*(THETA(I,KMX)-
     1    THETA(I,KHP1))+THETA(I,KHP1)
   23 WA(I,K) = WA(1,1)
      READ (5,1010) NEXT
   24 IF(NEXT.EQ.0) GO TO 45
      READ (5,1021)I1,K1,THETA(I1,K1),I2,K2,THETA(I2,K2),I3,K3,THETA(I3,
     1    K3),I4,K4,THETA(I4,K4),I5,K5,THETA(I5,K5),NEXT
      WRITE(6,1021)I1,K1,THETA(I1,K1),I2,K2,THETA(I2,K2),I3,K3,THETA(I3,
     1    K3),I4,K4,THETA(I4,K4),I5,K5,THETA(I5,K5),NEXT
      GO TO 24
C
C       END OF INPUT STATEMENTS
C
   30 CALL BCREAD (THETA(1,1),THETA(21,21))
      CALL BCREAD (WA    (1,1),    WA(21,21))
      WRITE (6,1040)
   45 CONTINUE
      CP=AR*GAM/(GAM-1.)
      CI = SQRT(GAM*AR*TEMP)
      WRITE (6,1050) CI
      KMXM1 = KMX-1
      MXP2 = MX+2
      MXSP1 = MXSP+1
      WTH = WT/2.
      EXPON = 1./(GAM-1.)
      TANBTO = SIN(BETIN)/COS(BETIN)
      TANBT = TANBTO*(ALM-W*R(1)**2)/(ALM-W*R(2)**2)*DN(1)/DN(2)
      DTDM1 = TANBT/R(1)
      DWDM1 = W+ALM/R(1)**2
      ROOT=SQRT(2.)
      SM(1) = -SQRT((Z(2)-Z(1))**2+(R(2)-R(1))**2)
      DO 60 K=1,KMXM1
   60 BA(K)=FLOAT(K-1)*WT/FLOAT(KMXM1-1)
```

```
C
C      CALCULATE ALPHA AND SM
C
      DO 70 I=1,MX
      AB(I)=(Z(I)-R(I))/ROOT
   70 AC(I)=(Z(I)+R(I))/ROOT
      CALL SPLINE(AB,AC,MX,AL,RC)
      DO 80 I=1,MX
      AL(I)=ATAN(AL(I))-.785398
      CAL(I)=COS(AL(I))
   80 SAL(I)=SIN(AL(I))
      DO 85 I=2,MX
      J=I-1
   85 SM(I)=SM(J)+SQRT((Z(I)-Z(J))**2+(R(I)-R(J))**2)
      SMAL(3) = SM(2)+.1*(SM(3)-SM(2))
      SMAL(4) = SM(2)+.5*(SM(3)-SM(2))
   90 ERROR = 1000.
C
C      CALCULATE BETA ON BLADE SURFACES
C  BEGINNING OF LOOP FOR ITERATIONS
C
   91 DO 97 J=1,4
      K = 1
      IF(J.EQ.2) K=KHMX
      IF(J.EQ.3) K=KHP1
      IF(J.EQ.4) K=KMX
      DO 93 I=1,2
      THAL(I,J) = THETA(I,K)
   93 SMAL(I) = SM(I)
      DO 94 I=3,4
   94 THAL(I,J) = THETA(2,K)+(SMAL(I)-SM(2))/(SM(3)-SM(2))*(THETA(3,K)
     1   -THETA(2,K))
      DO 95 I=5,MXP2
      IM2 = I-2
      THAL(I,J) = THETA(IM2,K)
   95 SMAL(I) = SM(IM2)
      CALL SPLIN2(SMAL,THAL(1,J),DTDM1,MXP2,DTDM(1,K),AB)
      DO 96 I=1,2
      CURV(I,K) = AB(I)/12./(1.+(DTDM(I,K)/12.)**2)**1.5
      BETA(I,K) = ATAN(R(I)*DTDM(I,K))
      SBETA(I,K) = SIN(BETA(I,K))
      CBETA(I,K) = COS(BETA(I,K))
   96 CONTINUE
      DO 97 I=3,MX
      IP2 = I+2
      CURV(I,K) = AB(IP2)/12./(1.+(DTDM(IP2,K)/12.)**2)**1.5
      BETA(I,K) = ATAN(R(I)*DTDM(IP2,K))
      SBETA(I,K) = SIN(BETA(I,K))
      CBETA(I,K) = COS(BETA(I,K))
   97 CONTINUE
      PRINT = (ITER.LE.0).OR.(ITNO.LE.NC)
      IF (PRINT) WRITE(6,1060)ITNO
      ERROR1=ERROR
      ERROR=0.
C
```

```
C     START CALCULATION OF PARAMETERS
C
      LAST = KHMX-1
      DO 100 K=2,LAST
      CALL SPLIN2(SM,THETA(1,K),DTDM1,MX,DTDM(1,K),AB)
      DO 100 I=1,MX
      CURV(I,K) = AB(I)/12./(1.+(DTDM(I,K)/12.)**2)**1.5
      BETA(I,K)=ATAN(R(I)*DTDM(I,K))
      SBETA(I,K)=SIN(BETA(I,K))
      CBETA(I,K)=COS(BETA(I,K))
  100 CONTINUE
      DO 101 K=KHP2,KMXM1
      CALL SPLIN2(SM,THETA(1,K),DTDM1,MX,DTDM(1,K),AB)
      DO 101 I=1,MX
      CURV(I,K) = AB(I)/12./(1.+(DTDM(I,K)/12.)**2)**1.5
      BETA(I,K)=ATAN(R(I)*DTDM(I,K))
      SBETA(I,K)=SIN(BETA(I,K))
      CBETA(I,K)=COS(BETA(I,K))
  101 CONTINUE
      DO 110 K=1,KMX
      DO 105 I=1,MX
      WTHETA(I,K)=WA(I,K)*SBETA(I,K)
  105 CONTINUE
      CALL SPLIN2(SM,WTHETA(1,K),DWDM1,MX,DWDM(1,K),AC)
      DO 110 I=1,MX
      SA(I,K) = SAL(I)*SBETA(I,K)*CBETA(I,K)
      SB(I,K) = CBETA(I,K)*R(I)*(2.*W*SAL(I)+DWDM(I,K))
  110 CONTINUE
C
C     END OF PARAMETER CALCULATION
C     CALCULATE VELOCITY DISTRIBUTION,CHECK CONTINUITY
C
      DO 200 I=1,MX
      IND=1
      IND2 = 1
      GO TO 130
  120 WA(I,1)=.5*WA(I,1)
      GO TO 130
  125 WA(I,1)=2.*WA(I,1)
      GO TO 130
  126 WA(I,KHP1) = .5*WA(I,KHP1)
      GO TO 130
  127 WA(I,KHP1) = 2.*WA(I,KHP1)
  130 IF((I.GT.1).AND.(I.LE.MXSP)) GO TO 142
      DO 140 K=2,KMX
      J = K-1
      IF(K.EQ.KHP1) WA(I,K) = WA(I,J)
      IF(K.EQ.KHP1)  GO TO 140
      HT=THETA(I,K)-THETA(I,J)
      WAS = WA(I,J)+(WA(I,J)*SA(I,J)+SB(I,J))*HT
      WASS = WA(I,J)+(WAS*SA(I,K)+SB(I,K))*HT
      WA(I,K) = (WAS+WASS)/2.
  140 CONTINUE
      GO TO 148
  142 DO 144 K=2,KHMX
      J=K-1
      HT=THETA(I,K)-THETA(I,J)
      WAS = WA(I,J)+(WA(I,J)*SA(I,J)+SB(I,J))*HT
      WASS = WA(I,J)+(WAS*SA(I,K)+SB(I,K))*HT
```

```
      WA(I,K) = (WAS+WASS)/2.
  144 CONTINUE
      DO 146 K=KHP2,KMX
      J = K-1
      HT=THETA(I,K)-THETA(I,J)
      WAS = WA(I,J)+(WA(I,J)*SA(I,J)+SB(I,J))*HT
      WASS = WA(I,J)+(WAS*SA(I,K)+SB(I,K))*HT
      WA(I,K) = (WAS+WASS)/2.
  146 CONTINUE
  148 CONTINUE
      DO 150 K=1,KMX
      T1P= 1.-(WA(I,K)**2+2.*W*ALM-(W*R(I)   )**2)/2./CP/TEMP
      IF((T1P.LT..0).AND.(I.GT.1).AND.(I.LE.MXSP).AND.(K.GE.KHP1))
     1  GO TO 126
      IF(T1P.LT..0) GO TO 120
      TPP1P= 1.-          (2.*W*ALM-(W*R(I)   )**2)/2./CP/TEMP
      DENSTY(K) = T1P**EXPON*RHO-(T1P/TPP1P)**EXPON*PLOSS/AR/TPP1P/TEMP
     1 *32.17*SM(I)   /SM(MXBL)
      PRS(K) = DENSTY(K)*AR*T1P*TEMP/32.17/144.
      WM=WA(I,K)*CBETA(I,K)
      AB(K) = DENSTY(K)*WM*DN(I)*R(I)*XN
  150 AC(K)=THETA(I,K)
      CALL INTGRL(AC,AB,KHMX,WTFL)
      IF(WTFL(KHMX).LE..0) GO TO 125
      CALL INTGRL(AC(KHP1),AB(KHP1),KHMX,WTFL(KHP1))
      IF((WTFL(KMX).LE..0).AND.(I.GT.1).AND.(I.LE.MXSP)) GO TO 127
      IF(WTFL( KMX).LE..0) GO TO 125
      IF((I.GT.1).AND.(I.LE.MXSP)) GO TO 155
      DO 152 K=KHP1,KMXM1
      AC(K) = AC(K+1)
  152 WTFL(K) = WTFL(KHMX)+WTFL(K+1)
      IF(ABS(WT-WTFL(KMXM1)).LE.WTOLER) GO TO 160
      CALL CONTIN(WA(I,1),WTFL(KMXM1),IND,I,WT)
      IF(IND.NE.6)GO TO 130
      GO TO 160
  155 IF (ABS(WTH-WTFL(KHMX)).LE.WTOLER) IND=6
      IF (ABS(WTH-WTFL(KMX )).LE.WTOLER) IND2=6
      IF(IND.NE.6) CALL CONTIN(WA(I,1),WTFL(KHMX),IND,I,WTH)
      IF(IND2.NE.6) CALL CONTAL(WA(I,KHP1),WTFL(KMX),IND2,I,WTH)
      IF((IND.NE.6).OR.(IND2.NE.6)) GO TO 130
  160 IF((I.GT.1).AND.(I.LE.MXSP)) GO TO 165
      CALL SPLINT (WTFL,AC,KMXM1,BA,KMXM1,AB)
      GO TO 166
  165 CALL SPLINT(WTFL,AC,KHMX,BA,KHMX,AB)
      CALL SPLINT(WTFL(KHP1),AC(KHP1),KHMX,BA(1)    ,KHMX,AB(KHP1))
  166 CONTINUE
      IF((I.GT.1).AND.(I.LE.MXSP)) GO TO 168
      DO 167 K=1,KHMX
      KA = KMX-K+1
      J = KA-1
  167 AB(KA) = AB(J)
  168 DO 170 K=1,KMX
      DELTA=ABS(AB(K)-THETA(I,K))
  170 IF(DELTA.GT.ERROR)ERROR=DELTA
      IF(.NOT.PRINT) GO TO 178
      A=SM(I)*12.
      C=AL(I   )*57.29577
      D = R(I)*12.
      E = Z(I)*12.
```

```
      F = DN(I)*12.
      WRITE (6,1080) I,A,C,D,E,F
      WRITE (6,1070)
      DO 175 K=1,KHMX,NPRT
      B=BETA(I,K)*57.29577
      C = WTHETA(I,K)+W*R(I)
      WM = WA(I,K)*CBETA(I,K)
      V = SQRT(C**2+WM**2)
      DWDT = WA(I,K)*SA(I,K)+SB(I,K)
      WRITE (6,1090) THETA(I,K),CURV(I,K),B,WA(I,K),WTHETA(I,K),C,WM,V,
     1   PRS(K),DENSTY(K),DTDM(I,K),DWDM(I,K),SA(I,K),SB(I,K),DWDT
  175 CONTINUE
      WRITE  (6,1095)
      DO 176 K=KHP1,KMX,NPRT
      B=BETA(I,K)*57.29577
      C = WTHETA(I,K)+W*R(I)
      WM = WA(I,K)*CBETA(I,K)
      V = SQRT(C**2+WM**2)
      DWDT = WA(I,K)*SA(I,K)+SB(I,K)
      WRITE (6,1090) THETA(I,K),CURV(I,K),B,WA(I,K),WTHETA(I,K),C,WM,V,
     1   PRS(K),DENSTY(K),DTDM(I,K),DWDM(I,K),SA(I,K),SB(I,K),DWDT
  176 CONTINUE
      WRITE (6,1095)
  178 DO 180 K=2,KMXM1
  180 THETA(I,K)=(1.-CORFAC)*THETA(I,K)+CORFAC*AB(K)
  200 CONTINUE
C
C     END OF VELOCITY CALCULATIONS.
C
      WRITE (6,1120) ITNO,ERROR
      IF(ITER.LE.0) GO TO 230
      IF((ERROR.GE.ERROR1).OR.(ERROR.LE.TOLER)) ITER=ITER-1
      ITNO=ITNO+1
      GO TO 91
  230 IF(BCDP.NE.1) GO TO 5
      CALL BCDUMP (THETA(1,1),THETA(21,21))
      CALL BCDUMP (   WA(1,1),    WA(21,21))
      GO TO 5
 1010 FORMAT (4I5,6F10.4)
 1020 FORMAT (8H1RUN NO.I3,10X,25HINPUT DATA CARD LISTING    )
 1021 FORMAT (5(2I2,F8.5),I1)
 1023 FORMAT (32H  THETA-CALCULATED AND/OR INPUT   )
 1030 FORMAT (7F10.4)
 1040 FORMAT (10X24HBCD CARDS FOR THETA,WA     )
 1050 FORMAT (36HK    STAG. SPEED OF SOUND AT INLET = ,F9.2,//)
 1060 FORMAT (///5X13HITERATION NO.I3)
 1070 FORMAT (132HK    THETA    T-CURV   BETA      WA     WTHETA   VTHETA
     1WM       V        PRS      DENSTY    DTDM    DWDM      SA        SB        DW
     2DT        )
 1080 FORMAT (2X16HQUASI-ORTHOGONALI3,6X,4HSM =,F7.4,9H  ALPHA =,F7.2,
     1   5H  R =,F7.4,5H  Z =,F7.4,6H  DN =,F7.4)
 1090 FORMAT (1X,F9.4,2F8.2,5F8.1,F7.2,F9.5,F7.2,F8.0,F8.4,2F9.1)
 1095 FORMAT (1H )
 1120 FORMAT (18HJ    ITERATION NO. I3,10X,24HMAX. STREAMLINE CHANGE = ,
     1F10.6)
 1200 FORMAT(//10X7HNORMAL I4)
 1210 FORMAT(7F18.6)
      END
```

Description of Subroutines

The subroutines SPLINE, SPLINT, INTGRL, and CONTIN are described in reference 1. The subroutine CONTAL is the same as CONTIN and is used so that a continuity calculation across a quasi-orthogonal can be computed on both sides of the splitter blade simultaneously. The subroutine SPLIN2 is based on the spline fit curve, with the derivative specified at the first point of the curve. SPLIN2 gives the first and second derivatives at each point. The calling sequence for SPLIN2 is as follows:

CALL SPLIN2(X, Y, Y1P, N, SLOPE, EM)

where

X	input array
Y	input array, function of X
Y1P,	input, derivative at first point, dY/dX (X(1))
N	input, number of X and Y values given
SLOPE	output array, first derivative, dY/dX
EM	output array, second derivative, d^2Y/dX^2

The subroutine SPLIN2 is as follows:

```
      SUBROUTINE SPLIN2(X,Y,Y1P,N,SLOPE,EM)
      DIMENSION X(50),Y(50),S(50),A(50),B(50),C(50),F(50),W(50),SB(50),
     1G(50),EM(50),SLOPE(50)
      COMMON Q
      INTEGER Q
      DO 10 I=2,N
   10 S(I)=X(I)-X(I-1)
      NO=N-1
      DO 20 I=2,NO
      A(I)=S(I)/6.
      B(I)=(S(I)+S(I+1))/3.
      C(I)=S(I+1)/6.
   20 F(I)=(Y(I+1)-Y(I))/S(I+1)-(Y(I)-Y(I-1))/S(I)
      A(N)=-.5
      B(1)=S(2)/3.
      B(N)=1.
      C(1)=S(2)/6.
      F(1)=(Y(2)-Y(1))/S(2)-Y1P
      F(N)=0.
      W(1)=B(1)
      SB(1)=C(1)/W(1)
```

```
      G(1)=F(1)/W(1)
      DO 30 I=2,N
      W(I)=B(I)-A(I)*SB(I-1)
      SB(I)=C(I)/W(I)
   30 G(I)=(F(I)-A(I)*G(I-1))/W(I)
      EM(N)=G(N)
      DO 40 I=2,N
      K=N+1-I
   40 EM(K)=G(K)-SB(K)*EM(K+1)
      SLOPE(1)=-S(2)/6.*(2.*EM(1)+EM(2))+(Y(2)-Y(1))/S(2)
      DO50 I=2,N
   50 SLOPE(I)=S(I)/6.*(2.*EM(I)+EM(I-1))+(Y(I)-Y(I-1))/S(I)
      IF (Q.EQ.14) WRITE (6,100) N,(X(I),Y(I),SLOPE(I),EM(I),I=1,N)
  100 FORMAT (2X15HNO. OF POINTS =I3/10X5HX     15X5HY     15X5HSLOPE15X5H
     1EM   /(4F20.8))
      RETURN
      END
```

Sample Output from Program

The output given here is the listing for the case used in the numerical example for the mean blade-to-blade surface. It will be noted that there is an exact listing of all input data cards at the beginning of the listing. However, the value for WTOLER should be 0.00001 instead of 0.0000 as rounded off in the listing. The input data listing is followed by the maximum calculated streamline change for each iteration, which is used as the criterion for convergence. After 26 iterations, there is convergence within the specified limit of 0.001-radian maximum streamline change. At this time, streamline coordinates are printed out together with velocities, pressure, and certain parameters used in the calculations. The calculations are based on 20 stream tubes between the blades. Data are printed out only for every other streamline in this listing.

```
RUN NO.  1              INPUT DATA CARD LISTING
  15    22     7     0  4030.0000    0.0305   11.0000    1.6667 1245.0000
   0     0     0    12  1950.0000  221.5000    0.0247    0.0010    0.2500
   0     2     2     0   -23.3800    0.0000    0.1500
 -0.4970   -0.5626   -0.5590   -0.5556   -0.5523   -0.5503   -0.5497
 -0.5552   -0.5828   -0.6674   -0.8503   -1.1139   -1.3724   -1.7284
 -2.0844
  0.0742   -0.0086   -0.0122   -0.0156   -0.0190   -0.0207   -0.0222
 -0.0284   -0.0558   -0.1452   -0.3347   -0.5842   -0.8012   -1.1572
 -1.5132
 -0.2091   -0.2942   -0.2978   -0.3012   -0.3014   -0.3002   -0.3016
 -0.2091   -0.2770   -0.2734   -0.2700   -0.2712   -0.2767   -0.2880
  0.3795    0.3955    0.4576    0.5551    0.7143    0.8720    1.0467
  1.2370    1.4921    1.7906    2.1000    2.3500    2.5500    2.8500
  3.1500
  3.3100    3.0100    2.6770    2.4028    2.1541    2.0036    1.8931
  1.8120    1.7460    1.6908    1.6262    1.5767    1.5606    1.5527
  1.5510
  0.0380    0.0378    0.0406    0.0429    0.0448    0.0457    0.0458
  0.0489    0.0522    0.0562    0.0602    0.0639    0.0658    0.0666
  0.0668

     STAG. SPEED OF SOUND AT INLET =  2011.55

     ITERATION NO.  1          MAX. STREAMLINE CHANGE =   0.047759
     ITERATION NO.  2          MAX. STREAMLINE CHANGE =   0.066704
     ITERATION NO.  3          MAX. STREAMLINE CHANGE =   0.056437
     ITERATION NO.  4          MAX. STREAMLINE CHANGE =   0.046788
     ITERATION NO.  5          MAX. STREAMLINE CHANGE =   0.037940
     ITERATION NO.  6          MAX. STREAMLINE CHANGE =   0.030659
     ITERATION NO.  7          MAX. STREAMLINE CHANGE =   0.024828
     ITERATION NO.  8          MAX. STREAMLINE CHANGE =   0.020120
     ITERATION NO.  9          MAX. STREAMLINE CHANGE =   0.016351
     ITERATION NO. 10          MAX. STREAMLINE CHANGE =   0.013294
     ITERATION NO. 11          MAX. STREAMLINE CHANGE =   0.010829
     ITERATION NO. 12          MAX. STREAMLINE CHANGE =   0.008846
     ITERATION NO. 13          MAX. STREAMLINE CHANGE =   0.007244
     ITERATION NO. 14          MAX. STREAMLINE CHANGE =   0.005955
     ITERATION NO. 15          MAX. STREAMLINE CHANGE =   0.004932
     ITERATION NO. 16          MAX. STREAMLINE CHANGE =   0.004065
     ITERATION NO. 17          MAX. STREAMLINE CHANGE =   0.003352
     ITERATION NO. 18          MAX. STREAMLINE CHANGE =   0.002934
     ITERATION NO. 19          MAX. STREAMLINE CHANGE =   0.002352
     ITERATION NO. 20          MAX. STREAMLINE CHANGE =   0.002057
     ITERATION NO. 21          MAX. STREAMLINE CHANGE =   0.001669
     ITERATION NO. 22          MAX. STREAMLINE CHANGE =   0.001507
     ITERATION NO. 23          MAX. STREAMLINE CHANGE =   0.001218
     ITERATION NO. 24          MAX. STREAMLINE CHANGE =   0.001095
     ITERATION NO. 25          MAX. STREAMLINE CHANGE =   0.000985
```

ITERATION NO. 26

QUASI-ORTHOGONAL 1 SM =-0.3004 ALPHA = -89.17 R = 3.3100 Z = 0.3795 DN = 0.0380

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.4970	3.49	-49.09	378.7	-286.2	825.4	248.0	861.9	11.20	0.02267	-4.18	6941.	0.4949	-201.9	-14.5
-0.4405	4.12	-49.09	377.9	-285.6	826.0	247.5	862.3	11.20	0.02267	-4.18	6941.	0.4949	-201.9	-14.9
-0.3838	1.23	-49.09	377.0	-284.9	826.7	246.9	862.8	11.21	0.02267	-4.18	6941.	0.4949	-201.9	-15.4
-0.3271	0.13	-49.09	376.1	-284.3	827.3	246.3	863.2	11.21	0.02268	-4.18	6941.	0.4949	-201.9	-15.8
-0.2702	0.92	-49.09	375.2	-283.6	828.0	245.7	863.7	11.21	0.02268	-4.18	6941.	0.4949	-201.9	-16.2
-0.2131	-5.02	-49.09	374.3	-282.9	828.7	245.1	864.2	11.21	0.02268	-4.18	6941.	0.4949	-201.9	-16.7
-0.2131	4.56	-49.09	374.3	-282.9	828.7	245.1	864.2	11.21	0.02268	-4.18	6941.	0.4949	-201.9	-16.7
-0.1560	4.24	-49.09	373.3	-282.1	829.5	244.5	864.8	11.21	0.02268	-4.18	6941.	0.4949	-201.9	-17.2
-0.0986	0.76	-49.09	372.3	-281.4	830.2	243.8	865.3	11.21	0.02268	-4.18	6941.	0.4949	-201.9	-17.7
-0.0412	-0.71	-49.09	371.3	-280.6	831.0	243.2	865.9	11.22	0.02269	-4.18	6941.	0.4949	-201.9	-18.2
0.0164	-0.01	-49.09	370.2	-279.8	831.8	242.5	866.4	11.22	0.02269	-4.18	6941.	0.4949	-201.9	-18.7
0.0742	-6.09	-49.09	369.1	-279.0	832.6	241.7	867.0	11.22	0.02269	-4.18	6941.	0.4949	-201.9	-19.3

QUASI-ORTHOGONAL 2 SM = 0. ALPHA = -83.93 R = 3.0100 Z = 0.3955 DN = 0.0378

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5626	22.92	-1.68	364.9	-10.7	1000.1	364.7	1064.6	10.69	0.02201	-0.12	7859.	0.0292	-39.1	-28.4
-0.5165	12.80	-20.40	358.7	-125.0	885.8	336.2	947.5	10.70	0.02203	-1.48	6501.	0.3249	-355.9	-239.3
-0.4673	13.56	-24.95	343.0	-144.7	866.2	311.0	920.3	10.72	0.02206	-1.85	5734.	0.3803	-518.8	-388.3
-0.4139	13.35	-27.39	319.9	-147.2	863.7	284.1	909.2	10.76	0.02211	-2.07	5342.	0.4062	-595.3	-465.3
-0.3557	12.55	-27.23	292.7	-133.9	876.9	260.3	914.7	10.80	0.02215	-2.05	5434.	0.4046	-575.5	-457.1
-0.2942	30.73	-6.49	270.0	-30.5	980.3	268.3	1016.4	10.83	0.02219	-0.45	7038.	0.1117	-243.5	-213.3
-0.2770	21.71	-1.50	369.9	-9.7	1001.2	369.8	1067.3	10.68	0.02200	-0.10	7721.	0.0260	-73.6	-64.0
-0.2315	12.90	-19.95	362.0	-123.5	887.4	340.2	950.4	10.69	0.02202	-1.45	6328.	0.3189	-397.7	-282.3
-0.1827	14.17	-24.82	344.4	-144.5	866.3	312.6	921.0	10.72	0.02206	-1.84	5575.	0.3788	-555.4	-425.0
-0.1294	14.19	-27.57	319.7	-148.0	862.9	283.4	908.2	10.76	0.02211	-2.08	5217.	0.4080	-622.1	-491.7
-0.0709	13.31	-27.72	291.1	-135.4	875.5	257.7	912.6	10.80	0.02216	-2.10	5340.	0.4095	-593.9	-474.7
-0.0086	31.89	-6.70	267.0	-31.2	979.7	265.2	1015.0	10.83	0.02220	-0.47	6947.	0.1153	-266.0	-235.2

QUASI-ORTHOGONAL 3 SM = 0.3387 ALPHA = -75.01 R = 2.6770 Z = 0.4576 DN = 0.0406

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5590	-0.02	1.82	496.1	15.7	914.8	495.8	1040.5	9.90	0.02100	0.23	-1234.	-0.0306	-2011.0	-2026.2
-0.5213	-2.90	3.10	437.3	23.7	922.7	436.7	1020.8	10.02	0.02116	0.24	2007.	-0.0522	-1287.3	-1310.1
-0.4791	-2.66	0.94	386.4	6.3	905.4	386.4	984.4	10.12	0.02128	0.07	2774.	-0.0158	-1117.9	-1124.0
-0.4314	-2.25	-1.08	334.3	-6.3	892.8	334.3	953.3	10.20	0.02139	-0.08	2939.	0.0181	-1080.9	-1074.9
-0.3750	-1.93	-2.04	271.4	-9.7	889.4	271.3	929.8	10.29	0.02150	-0.16	2476.	0.0343	-1183.7	-1174.4
-0.2978	-0.70	-1.76	159.2	-4.9	894.1	159.1	908.2	10.40	0.02164	0.01	-956.	0.0297	-1949.2	-1944.4
-0.2734	0.17	1.92	493.8	16.6	915.6	493.5	1040.1	9.90	0.02101	0.22	-980.	-0.0324	-1954.3	-1970.3
-0.2356	-3.27	2.94	436.6	22.4	921.5	436.1	1019.4	10.02	0.02116	0.23	2191.	-0.0496	-1246.3	-1268.0
-0.1934	-3.43	0.78	387.1	5.3	904.3	387.1	983.6	10.12	0.02128	0.06	2859.	-0.0131	-1098.9	-1104.0
-0.1458	-3.07	-1.14	335.5	-6.7	892.3	335.4	953.3	10.20	0.02139	-0.09	2911.	0.0192	-1087.1	-1080.7
-0.0895	-2.44	-1.92	271.9	-9.1	889.9	271.7	930.5	10.29	0.02150	-0.15	2380.	0.0324	-1205.3	-1196.5
-0.0122	-0.25	-1.50	157.8	-4.1	894.9	157.7	908.7	10.41	0.02164	0.01	-1033.	0.0253	-1966.6	-1962.6

QUASI-ORTHOGONAL 4 SM = 0.6298 ALPHA = -64.78 R = 2.4028 Z = 0.5551 DN = 0.0429

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5556	0.01	1.60	559.4	15.6	822.6	559.2	994.6	9.37	0.02030	0.10	236.	-0.0253	-1412.1	-1426.2
-0.5196	2.22	1.66	502.5	14.6	821.5	502.2	962.9	9.50	0.02047	0.14	-597.	-0.0262	-1578.9	-1592.1
-0.4795	3.51	2.26	440.7	17.4	824.3	440.3	934.6	9.63	0.02064	0.20	-83.	-0.0357	-1475.4	-1491.1
-0.4335	4.41	2.65	373.9	17.3	824.2	373.5	904.9	9.75	0.02080	0.23	258.	-0.0417	-1406.8	-1422.4
-0.3781	4.48	2.43	295.3	12.5	819.4	295.0	870.9	9.87	0.02095	0.21	213.	-0.0383	-1416.1	-1427.4
-0.3014	1.15	-0.63	183.5	-2.7	804.3	183.5	824.9	9.99	0.02110	-0.16	126.	0.0131	-1434.6	-1432.2
-0.2700	-0.55	1.09	578.0	11.0	818.0	577.9	1001.5	9.32	0.02024	0.10	-1279.	-0.0173	-1715.8	-1725.8
-0.2349	2.41	1.21	513.5	10.8	817.7	513.4	965.5	9.48	0.02044	0.11	-1701.	-0.0190	-1800.1	-1809.9
-0.1954	3.98	1.62	445.5	12.6	819.5	445.3	932.7	9.62	0.02063	0.14	-841.	-0.0255	-1627.8	-1639.2
-0.1496	4.72	1.72	373.8	11.2	818.2	373.6	899.4	9.75	0.02080	0.15	-191.	-0.0272	-1497.5	-1507.7
-0.0939	4.24	1.28	291.6	6.5	813.5	291.5	864.1	9.88	0.02095	0.11	28.	-0.0202	-1454.1	-1459.9
-0.0156	0.03	-1.72	176.8	-5.3	801.7	176.7	820.9	10.00	0.02111	-0.17	197.	0.0271	-1419.9	-1415.1

QUASI-ORTHOGONAL 5 SM = 0.9250 ALPHA = -49.65 R = 2.1541 Z = 0.7143 DN = 0.0448

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5523	-0.12	1.26	544.7	12.0	735.4	544.5	915.0	9.10	0.01992	0.14	-160.	-0.0167	-1131.1	-1140.2
-0.5125	-1.52	2.56	501.3	22.4	745.8	500.8	898.3	9.20	0.02005	0.25	178.	-0.0340	-1069.6	-1086.6
-0.4693	-2.60	3.41	453.8	27.0	750.4	453.0	876.5	9.30	0.02018	0.33	73.	-0.0452	-1087.5	-1108.0
-0.4213	-3.47	3.79	400.5	26.5	749.9	399.7	849.8	9.40	0.02031	0.37	58.	-0.0503	-1089.9	-1110.0
-0.3666	-3.74	3.30	340.7	19.6	743.0	340.1	817.2	9.49	0.02044	0.32	276.	-0.0438	-1051.3	-1066.2
-0.3012	-0.67	-0.02	275.7	-0.1	723.3	275.7	774.1	9.58	0.02055	-0.14	1056.	0.0003	-913.1	-913.0
-0.2712	-1.64	-2.64	538.6	-24.8	698.6	538.1	881.8	9.11	0.01994	0.15	1086.	0.0350	-906.7	-887.9
-0.2312	-3.96	-1.27	501.4	-11.1	712.3	501.3	871.0	9.20	0.02005	-0.12	499.	0.0168	-1012.8	-1004.3
-0.1882	-4.67	0.09	455.6	0.7	724.2	455.6	855.6	9.29	0.02018	0.01	-36.	-0.0012	-1109.0	-1109.6
-0.1404	-4.91	1.26	401.5	8.8	732.2	401.5	835.1	9.39	0.02031	0.12	-231.	-0.0167	-1143.8	-1150.6
-0.0857	-3.92	1.64	338.6	9.7	733.1	338.5	807.4	9.50	0.02044	0.16	-178.	-0.0218	-1134.2	-1141.5
-0.0190	0.19	-1.22	264.5	-5.6	717.8	264.4	764.9	9.60	0.02057	-0.12	68.	0.0162	-1090.1	-1085.8

QUASI-ORTHOGONAL 6 SM = 1.1430 ALPHA = -37.60 R = 2.0036 Z = 0.8720 DN = 0.0457

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5503	-0.09	0.95	566.5	9.4	682.2	566.4	886.7	8.88	0.01961	0.14	-378.	-0.0101	-884.2	-889.9
-0.5096	0.60	1.42	527.7	13.1	685.9	527.5	865.3	8.97	0.01973	0.15	-1009.	-0.0151	-989.4	-997.4
-0.4659	1.25	1.76	482.8	14.8	687.7	482.6	840.1	9.07	0.01986	0.18	-1338.	-0.0187	-1044.0	-1053.1
-0.4180	1.92	1.91	431.4	14.4	687.2	431.2	811.3	9.17	0.01999	0.20	-1563.	-0.0203	-1081.6	-1090.3
-0.3640	2.61	1.89	371.9	12.3	685.2	371.7	779.5	9.27	0.02013	0.20	-1688.	-0.0201	-1102.4	-1109.8
-0.3002	2.95	2.36	301.8	12.4	685.3	301.5	748.7	9.37	0.02026	-0.07	-1465.	-0.0251	-1064.9	-1072.5
-0.2767	2.40	-1.86	585.9	-19.1	653.8	585.6	877.7	8.83	0.01955	0.10	-5109.	0.0198	-1673.3	-1661.7
-0.2365	3.39	-1.77	525.3	-16.2	656.6	525.0	840.7	8.97	0.01974	-0.19	-3301.	0.0189	-1371.6	-1361.7
-0.1921	3.57	-1.28	469.3	-10.4	662.4	469.2	811.7	9.09	0.01990	-0.13	-2166.	0.0136	-1182.5	-1176.1
-0.1426	3.20	-0.61	414.1	-4.4	668.5	414.1	786.3	9.20	0.02003	-0.06	-1454.	0.0065	-1063.9	-1061.3
-0.0864	2.32	-0.15	357.2	-0.9	671.9	357.2	761.0	9.29	0.02016	-0.02	-867.	0.0016	-966.0	-965.4
-0.0207	0.32	-0.61	298.1	-3.2	669.7	298.1	733.0	9.38	0.02027	-0.15	-7.	0.0065	-822.3	-820.4

QUASI-ORTHOGONAL 7 SM = 1.3498 ALPHA = -27.49 R = 1.8931 Z = 1.0467 DN = 0.0458

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5497	-1.67	-0.76	596.0	-7.9	627.9	595.9	865.7	8.69	0.01934	0.12	-1780.	0.0061	-867.7	-864.0
-0.5084	-3.46	-1.33	558.9	-13.0	622.8	558.7	836.7	8.77	0.01946	-0.15	-2231.	0.0107	-938.6	-932.6
-0.4644	-5.29	-2.11	516.4	-19.0	616.7	516.1	804.2	8.87	0.01959	-0.23	-2652.	0.0170	-1004.5	-995.8
-0.4166	-7.38	-3.31	467.5	-27.0	608.8	466.7	767.1	8.97	0.01972	-0.37	-3061.	0.0266	-1068.0	-1055.6
-0.3635	-10.24	-5.35	409.4	-38.2	597.6	407.6	723.4	9.08	0.01986	-0.59	-3619.	0.0429	-1152.9	-1135.3
-0.3015	-15.15	-9.24	334.2	-53.7	582.1	329.9	669.1	9.20	0.02002	-0.00	-4841.	0.0732	-1333.1	-1308.6
-0.2880	-17.58	-16.08	582.8	-161.4	474.4	560.0	733.9	8.72	0.01538	-0.26	-5015.	0.1228	-1324.2	-1252.6
-0.2440	-13.84	-11.51	530.0	-105.7	530.0	519.4	742.1	8.84	0.01955	-1.29	-4005.	0.0902	-1194.2	-1146.4
-0.1966	-10.84	-8.03	478.4	-66.9	568.9	473.7	740.3	8.95	0.01969	-0.89	-3102.	0.0639	-1065.8	-1035.2
-0.1448	-8.24	-5.30	427.6	-39.5	596.3	425.8	732.7	9.05	0.01982	-0.59	-2290.	0.0424	-944.2	-926.0
-0.0872	-5.47	-3.10	377.5	-20.4	615.4	377.0	721.7	9.14	0.01593	-0.34	-1499.	0.0249	-822.2	-812.8
-0.0222	-1.32	-1.51	329.1	-8.7	627.1	328.9	708.1	9.21	0.02003	-0.12	-610.	0.0122	-682.9	-678.9

QUASI-ORTHOGONAL 8 SM = 1.5566 ALPHA = -18.68 R = 1.8120 Z = 1.2370 DN = 0.0489

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5552	-3.47	-5.32	634.8	-58.9	549.7	632.0	837.6	8.50	0.01907	0.10	-4376.	0.0296	-1046.1	-1027.3
-0.5165	-2.43	-6.53	595.8	-67.7	540.8	591.9	801.8	8.60	0.01920	-0.76	-4129.	0.0362	-1006.6	-985.1
-0.4754	-1.22	-7.81	556.4	-75.6	533.0	551.3	766.8	8.69	0.01933	-0.91	-3790.	0.0431	-953.1	-929.1
-0.4315	0.36	-9.38	517.2	-84.2	524.3	510.3	731.7	8.78	0.01945	-1.09	-3352.	0.0515	-883.9	-857.3
-0.3843	2.77	-11.64	478.9	-96.5	512.0	469.1	694.4	8.86	0.01955	-1.36	-2779.	0.0633	-792.7	-762.4
-0.3331	7.16	-15.58	443.0	-118.9	489.6	426.8	649.5	8.93	0.01965	0.25	-2037.	0.0829	-671.7	-635.0
-0.3331	12.78	-19.25	443.0	-145.9	462.6	418.3	623.6	8.93	0.01965	-0.19	3062.	0.0997	68.6	112.7
-0.2786	7.10	-16.67	437.5	-125.4	483.1	419.1	639.6	8.94	0.01966	-1.98	312.	0.0880	-328.2	-289.7
-0.2229	3.43	-14.08	414.6	-100.8	507.8	402.1	647.7	8.98	0.01971	-1.66	-1150.	0.0755	-546.4	-515.1
-0.1642	0.93	-11.48	380.7	-75.8	532.8	373.1	650.4	9.04	0.01979	-1.35	-1808.	0.0625	-649.4	-625.6
-0.1003	-1.04	-8.74	339.3	-51.6	557.0	335.4	650.2	9.10	0.01987	-1.02	-1948.	0.0481	-675.9	-659.5
-0.0284	-2.94	-5.25	292.7	-26.8	581.8	291.5	650.7	9.17	0.01996	-0.06	-1662.	0.0292	-638.1	-629.5

QUASI-ORTHOGONAL 9 SM = 1.8201 ALPHA = -11.52 R = 1.7460 Z = 1.4921 DN = 0.0522

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.5828	-7.26	-16.81	679.8	-196.6	389.7	650.7	758.5	8.30	0.01878	-0.08	-7963.	0.0553	-1333.2	-1295.6
-0.5456	-7.67	-17.28	634.2	-188.4	398.0	605.6	724.6	8.42	0.01895	-2.14	-6999.	0.0566	-1196.0	-1160.1
-0.5060	-8.15	-17.74	590.6	-179.9	406.4	562.5	694.0	8.53	0.01909	-2.20	-6181.	0.0579	-1079.6	-1045.4
-0.4637	-8.71	-18.17	548.6	-171.0	415.3	521.3	666.5	8.63	0.01923	-2.26	-5407.	0.0591	-970.0	-937.6
-0.4184	-9.39	-18.48	508.8	-161.3	425.1	482.6	643.1	8.72	0.01934	-2.30	-4539.	0.0600	-848.4	-817.9
-0.3698	-10.30	-18.50	473.0	-150.1	436.3	448.6	625.8	8.79	0.01944	-1.03	-3291.	0.0601	-676.2	-647.8
-0.3698	-11.81	-17.50	473.0	-142.2	444.2	451.1	633.1	8.79	0.01944	-1.83	-3599.	0.0572	-722.7	-695.6
-0.3179	-9.77	-19.05	439.1	-143.3	443.1	415.0	607.1	8.86	0.01953	-2.37	-3118.	0.0616	-650.2	-623.1
-0.2617	-8.86	-19.52	404.5	-135.1	451.3	381.3	590.8	8.92	0.01961	-2.44	-3028.	0.0629	-635.9	-610.5
-0.2005	-8.40	-19.21	367.0	-120.7	465.6	346.6	580.5	8.98	0.01969	-2.39	-3051.	0.0620	-640.4	-617.6
-0.1329	-8.10	-18.37	324.9	-102.3	484.0	308.3	573.9	9.04	0.01977	-2.28	-3081.	0.0597	-647.6	-628.2
-0.0558	-8.27	-17.28	275.7	-81.9	504.5	263.2	569.0	9.10	0.01985	-0.17	-3203.	0.0566	-668.5	-652.9

QUASI-ORTHOGONAL 10 SM = 2.1237 ALPHA = -10.61 R = 1.6908 Z = 1.7906 DN = 0.0562

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.6674	-7.95	-33.66	795.1	-440.6	127.2	661.7	673.9	7.89	0.01820	-0.62	-11299.	0.0849	-1499.0	-1431.5
-0.6314	-7.05	-33.42	745.3	-410.3	157.6	622.1	641.7	8.04	0.01841	-4.68	-10423.	0.0846	-1400.3	-1337.2
-0.5935	-6.30	-33.41	696.4	-383.3	184.5	581.3	609.9	8.18	0.01860	-4.68	-9573.	0.0846	-1300.4	-1241.5
-0.5533	-5.70	-33.63	648.3	-358.8	209.0	539.8	578.8	8.31	0.01878	-4.72	-8801.	0.0849	-1206.5	-1151.5
-0.5102	-5.24	-34.04	600.4	-335.9	231.9	497.5	548.9	8.44	0.01894	-4.79	-8161.	0.0854	-1126.0	-1074.7
-0.4638	-4.96	-34.61	551.9	-313.3	254.6	454.2	520.7	8.55	0.01910	-1.85	-7744.	0.0860	-1070.0	-1022.5
-0.4638	-4.62	-34.81	551.9	-314.8	253.0	453.1	519.0	8.55	0.01910	-2.31	-7921.	0.0863	-1087.9	-1040.3
-0.4131	-5.37	-35.05	502.1	-288.2	279.7	411.0	497.2	8.66	0.01924	-4.98	-6937.	0.0865	-971.2	-927.8
-0.3574	-6.12	-35.44	453.0	-262.5	305.3	369.1	479.0	8.76	0.01937	-5.05	-6156.	0.0869	-876.9	-837.6
-0.2954	-7.00	-35.76	403.5	-235.6	332.2	327.4	466.4	8.85	0.01949	-5.11	-5491.	0.0873	-797.5	-762.3
-0.2254	-7.89	-35.79	352.7	-206.2	361.6	286.1	461.1	8.93	0.01960	-5.12	-4808.	0.0873	-719.0	-688.3
-0.1452	-8.05	-35.22	301.3	-173.8	394.1	246.2	464.6	9.00	0.01969	-0.61	-3869.	0.0867	-616.1	-590.0

QUASI-ORTHOGONAL 11 SM = 2.4397 ALPHA = -12.82 R = 1.6262 Z = 2.1000 DN = 0.0602

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-0.8503	-10.36	-53.07	895.4	-716.4	-170.3	537.9	564.2	7.49	0.01762	-2.08	-7117.	0.1065	-725.0	-629.6
-0.8074	-10.12	-51.57	865.9	-678.9	-132.8	538.1	554.3	7.59	0.01776	-9.30	-8154.	0.1080	-837.3	-743.8
-0.7646	-9.77	-50.30	832.0	-640.6	-94.5	531.4	539.8	7.70	0.01791	-8.89	-8904.	0.1090	-925.5	-834.8
-0.7213	-9.34	-49.30	794.3	-602.6	-56.5	518.0	521.1	7.82	0.01808	-8.58	-9397.	0.1097	-988.5	-901.4
-0.6770	-8.88	-48.60	753.4	-565.5	-19.4	498.2	498.6	7.94	0.01825	-8.37	-9669.	0.1100	-1026.7	-943.8
-0.6311	-8.42	-48.25	709.6	-529.7	16.4	472.4	472.7	8.07	0.01842	-2.30	-9761.	0.1102	-1042.1	-963.9
-0.6311	-8.49	-48.22	709.6	-529.4	16.7	472.7	473.0	8.07	0.01842	-2.17	-9679.	0.1102	-1035.2	-957.1
-0.5828	-7.89	-48.35	662.7	-495.4	50.8	440.4	443.4	8.20	0.01860	-8.30	-9925.	0.1102	-1055.0	-982.0
-0.5312	-7.50	-48.89	611.7	-461.0	85.1	402.3	411.2	8.33	0.01878	-8.45	-10059.	0.1099	-1055.7	-988.5
-0.4745	-7.43	-49.89	555.8	-425.2	120.9	358.1	378.0	8.46	0.01896	-8.76	-10185.	0.1093	-1045.3	-984.6
-0.4105	-7.95	-51.40	493.1	-385.4	160.8	307.6	347.1	8.60	0.01914	-9.24	-10398.	0.1082	-1030.2	-976.9
-0.3347	-9.58	-53.42	419.3	-336.7	209.4	249.8	326.0	8.74	0.01932	-2.14	-10781.	0.1061	-1014.9	-970.4

QUASI-ORTHOGONAL 12 SM = 2.6946 ALPHA = -7.55 R = 1.5767 Z = 2.3500 DN = 0.0639

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-1.1139	-4.10	-62.70	860.9	-765.1	-235.6	394.8	459.7	7.55	0.01768	-4.73	2617.	0.0535	93.9	140.0
-1.0562	-4.83	-61.82	866.4	-763.7	-234.2	409.2	471.5	7.53	0.01766	-14.20	1205.	0.0547	9.1	56.5
-1.0042	-5.37	-60.89	867.0	-757.5	-228.0	421.8	479.5	7.53	0.01766	-13.67	-261.	0.0558	-84.4	-36.0
-0.9516	-5.71	-59.97	862.6	-746.8	-217.3	431.7	483.3	7.55	0.01768	-13.17	-1719.	0.0569	-182.6	-133.5
-0.9000	-5.83	-59.10	853.2	-732.2	-202.6	438.1	482.7	7.58	0.01772	-12.72	-3117.	0.0579	-281.7	-232.4
-0.8491	-5.72	-58.34	838.9	-714.1	-184.6	440.3	477.4	7.62	0.01778	-4.90	-4420.	0.0587	-377.9	-328.6
-0.8491	-5.71	-58.34	838.9	-714.1	-184.6	440.3	477.4	7.62	0.01778	-4.93	-4448.	0.0587	-379.7	-330.5
-0.7984	-5.38	-57.74	820.0	-693.4	-163.9	437.7	467.4	7.68	0.01787	-12.06	-5555.	0.0593	-463.9	-415.3
-0.7474	-4.74	-57.37	796.7	-671.0	-141.5	429.6	452.3	7.76	0.01797	-11.89	-6596.	0.0597	-542.3	-494.8
-0.6954	-3.72	-57.35	769.1	-647.6	-118.1	414.9	431.3	7.84	0.01809	-11.88	-7599.	0.0597	-613.7	-567.8
-0.6415	-2.24	-57.83	736.6	-623.5	-94.0	392.2	403.3	7.94	0.01822	-12.10	-8646.	0.0592	-679.0	-635.4
-0.5842	-0.16	-59.03	698.3	-598.7	-69.2	359.4	366.0	8.05	0.01837	-5.01	-9909.	0.0580	-741.7	-701.2

QUASI-ORTHOGONAL 13 SM = 2.8952 ALPHA = -2.68 R = 1.5606 Z = 2.5500 DN = 0.0658

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-1.3724	2.69	-63.23	781.5	-697.5	-173.4	351.9	392.3	7.78	0.01798	-9.82	2958.	0.0188	151.2	165.9
-1.3113	1.88	-63.18	791.7	-706.3	-182.2	357.2	401.0	7.75	0.01794	-15.21	2973.	0.0188	152.3	167.2
-1.2511	1.08	-63.00	801.5	-714.0	-189.9	363.8	410.4	7.72	0.01790	-15.09	2758.	0.0189	140.6	155.8
-1.1919	0.31	-62.73	810.1	-719.9	-195.8	371.2	419.7	7.69	0.01786	-14.91	2340.	0.0190	117.0	132.4
-1.1338	-0.42	-62.37	816.8	-723.6	-199.5	378.8	428.1	7.67	0.01783	-14.69	1751.	0.0192	82.9	98.6
-1.0767	-1.06	-61.97	821.3	-724.9	-200.8	385.9	435.0	7.65	0.01781	-8.27	1031.	0.0194	40.0	55.9
-1.0767	-1.07	-61.97	821.3	-724.9	-200.8	385.9	435.0	7.65	0.01781	-8.26	1042.	0.0194	40.7	56.6
-1.0205	-1.62	-61.56	823.0	-723.7	-199.6	392.0	439.9	7.65	0.01780	-14.20	207.	0.0196	-10.5	5.6
-0.9651	-2.06	-61.15	821.9	-720.0	-195.9	396.6	442.3	7.65	0.01781	-13.96	-639.	0.0197	-63.7	-47.5
-0.9102	-2.35	-60.79	817.8	-714.0	-189.9	399.1	442.0	7.66	0.01783	-13.75	-1460.	0.0199	-116.6	-100.3
-0.8557	-2.47	-60.51	811.0	-706.1	-182.0	399.2	438.7	7.69	0.01786	-13.60	-2220.	0.0200	-166.2	-150.0
-0.8012	-2.33	-60.38	801.6	-697.1	-173.0	396.1	432.3	7.72	0.01790	-9.95	-2884.	0.0201	-209.5	-193.4

QUASI-ORTHOGONAL 14 SM = 3.1954 ALPHA = -0.71 R = 1.5527 Z = 2.8500 DN = 0.0666

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-1.7284	-0.71	-60.96	793.5	-694.0	-172.6	385.2	422.1	7.71	0.01787	-14.75	-133.	0.0052	-14.6	-10.4
-1.6721	-0.05	-61.19	793.3	-695.3	-173.9	382.3	420.0	7.71	0.01787	-14.05	78.	0.0052	-1.3	2.8
-1.6154	0.42	-61.43	793.9	-697.4	-175.9	379.7	418.5	7.71	0.01787	-14.19	304.	0.0052	12.7	16.8
-1.5584	0.72	-61.64	795.2	-700.0	-178.5	377.7	417.8	7.71	0.01786	-14.32	517.	0.0051	25.7	29.8
-1.5010	0.90	-61.82	797.2	-702.9	-181.5	376.5	417.9	7.70	0.01785	-14.42	697.	0.0051	36.5	40.6
-1.4435	0.96	-61.95	799.8	-706.0	-184.6	376.1	419.0	7.69	0.01784	-12.34	826.	0.0051	44.2	48.3
-1.4435	0.96	-61.95	799.8	-706.0	-184.6	376.1	419.0	7.69	0.01784	-12.34	822.	0.0051	44.0	48.1
-1.3860	0.94	-62.02	802.7	-709.1	-187.6	376.6	420.7	7.68	0.01783	-14.55	896.	0.0051	48.4	52.5
-1.3285	0.87	-62.05	805.7	-711.9	-190.5	377.7	423.0	7.67	0.01781	-14.57	887.	0.0051	47.8	51.9
-1.2712	0.76	-62.03	808.6	-714.3	-192.8	379.2	425.4	7.66	0.01780	-14.55	799.	0.0051	42.5	46.6
-1.2141	0.62	-61.97	811.0	-716.0	-194.6	381.0	427.9	7.66	0.01779	-14.52	642.	0.0051	33.0	37.2
-1.1572	0.46	-61.87	812.7	-716.9	-195.5	383.1	430.1	7.65	0.01778	-12.68	432.	0.0051	20.3	24.4

QUASI-ORTHOGONAL 15 SM = 3.4954 ALPHA = -0.02 R = 1.5510 Z = 3.1500 DN = 0.0668

THETA	T-CURV	BETA	WA	WTHETA	VTHETA	WM	V	PRS	DENSTY	DTDM	DWDM	SA	SB	DWDT
-2.0844	-0.33	-61.91	736.7	-649.9	-129.0	346.9	370.1	7.85	0.01804	-15.25	3283.	0.0002	199.6	199.7
-2.0240	-0.03	-61.24	748.0	-655.7	-134.8	359.9	384.3	7.82	0.01800	-14.10	2792.	0.0002	173.4	173.6
-1.9653	0.22	-60.80	757.4	-661.2	-140.3	369.6	395.3	7.79	0.01796	-13.84	2365.	0.0002	149.0	149.1
-1.9078	0.39	-60.55	765.4	-666.5	-145.6	376.3	403.5	7.77	0.01793	-13.70	1997.	0.0002	126.7	126.8
-1.8510	0.49	-60.46	772.0	-671.7	-150.8	380.6	409.4	7.75	0.01790	-13.65	1690.	0.0002	107.5	107.6
-1.7945	0.53	-60.49	777.6	-676.8	-155.9	383.0	413.5	7.73	0.01788	-14.45	1446.	0.0002	91.8	92.0
-1.7945	0.53	-60.49	777.6	-676.8	-155.9	383.0	413.5	7.73	0.01788	-14.45	1449.	0.0002	92.1	92.2
-1.7383	0.52	-60.59	782.5	-681.6	-160.7	384.2	416.5	7.72	0.01786	-13.73	1260.	0.0002	79.8	79.9
-1.6821	0.48	-60.73	786.7	-686.2	-165.3	384.6	418.6	7.71	0.01784	-13.81	1140.	0.0002	71.8	71.9
-1.6259	0.41	-60.89	790.6	-690.6	-169.7	384.7	420.5	7.69	0.01782	-13.89	1066.	0.0002	66.8	67.0
-1.5696	0.33	-61.03	794.3	-694.7	-173.9	384.7	422.1	7.68	0.01781	-13.98	1019.	0.0002	63.6	63.7
-1.5132	0.24	-61.19	797.8	-698.7	-177.9	384.5	423.7	7.67	0.01779	-13.53	963.	0.0002	59.8	59.9

ITERATION NO. 26 MAX. STREAMLINE CHANGE = 0.000736

REFERENCES

1. Katsanis, Theodore: Use of Arbitrary Quasi-Orthogonals for Calculating Flow Distribution in the Meridional Plane of a Turbomachine. NASA TN D-2546, 1964.

2. Stanitz, John D.; and Prian, Vasily D.: A Rapid Approximate Method for Determining Velocity Distribution on Impeller Blades of Centrifugal Compressors. NACA TN 2421, 1951.

3. Kramer, J. J.: Analysis of Incompressible, Nonviscous Blade-to-Blade Flow in Rotating Blade Rows. Trans. ASME, vol. 80, no. 1, Jan. 1958, pp. 263-275.

NASA-Langley, 1965 E-2811