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# An Interplanetary Diffusion Model for the Time Behavior of Intensity in a Solar Cosmic Ray Event*

by

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## ABSTRACT

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It is found possible to give a coherent interpretation of the time history of the intensity of several solar cosmic ray events in terms of a unified interplanetary diffusion model which assumes that the diffusion coefficient $D = M r^{\beta}$, where $r$ is the heliocentric radial distance and $M$ and $\beta$ are parameters, which may be dependent on particle energy $E$. The model is particularly successful in accounting for the decay phases. For the September 28, 1961 event $M$ varies as $E^{0.33}$ and, assuming spherical geometry, $\beta$ is equal to one for $55 < E < 500$ MeV and decreases rapidly at lower energies. At $r = 1$ A.U. the mean free path $\lambda$ is about 0.081 A.U. for $E \sim 200$ MeV and 0.15 for $E = 23$ MeV. The energy dependences of $\beta$ and $\lambda$ suggest 0.006 A.U. as the order of magnitude of the scale size of irregularities in the interplanetary magnetic field. The source spectrum is inferred. The possibility of non-spherical geometry of diffusion is considered, as is the implication of the results with respect to the solar modulation of galactic cosmic radiation.

Author

# INTRODUCTION

Many studies of the time dependence of the intensity of solar cosmic rays [Meyer et al., 1956; Bryant et al., 1962, 1964; Charakhchyan et al., 1962; Hoffman and Winckler, 1963; Winckler and Bhavsar, 1963] have suggested that diffusion plays an essential role in the interplanetary propagation of such particles. It is usually possible to fit observed intensity data up to and somewhat beyond the time of maximum intensity by assuming an infinitely extended, diffusive medium consisting of a uniform distribution of isotropically-scattering centers. Plausible values of diffusion coefficients are found but this simple model is strikingly inadequate in accounting for the decay phase of observed intensity-versus-time data. Hence, it has become customary to add an additional ad hoc feature to the model, namely a perfectly transmitting outer boundary to the diffusive region in the form of sphere of radius $r_o$ centered in the sun. This feature causes a more rapid decay and tends to give a reasonably satisfactory fit to observed data, with values of $r_o$ typically equal to 2 or 3 astronomical units. Such large values of $r_o$ do not affect the fit to early-time data, given by the first phase of the calculation [Hoffman and Winckler, 1963; Winckler and Bhavsar, 1963; Krimigis and Lin, 1964].

The foregoing two-phase model predicts a strictly exponential decay of intensity. Such decays are known to occur but are by no means the only type observed. Power law decays have been seen [Arnoldy et al., 1960; Anderson and Enemark, 1960; Winckler et al., 1961; Lin and Van Allen, 1963], as have decays which fit neither an exponential nor a power law [Earl, 1961; Krimigis and Van Allen, to be published]. The present paper analyzes the time dependence of intensity of several solar cosmic ray events in terms of a somewhat different model, which appeals to the author, at least, as having a more natural physical character, and which has sufficient parametric flexibility to account for various laws of decay.

## THE MODEL

There is not yet sufficient direct knowledge of the interplanetary magnetic field to permit the calculation of particle trajectories in a "microscopic" manner. It is probable, however, that the effective diffusion coefficient for a charged particle is a function of particle energy, of position in space, of direction, and of time.

No attempt at full generality is made here. Only a modest generalization of the uniform isotropic model is attempted.

Specifically, it is assumed

(a) That particles diffuse in a random walk through a medium of static scattering centers,

(b) That the scattering is isotropic, and

(c) That particles neither gain nor lose energy in the scattering process;

but that

(d) The density of scattering centers diminishes with increasing radial distance r from the sun, but is not a function of heliographic latitude or longitude.

This type of model appeared first, to the author's knowledge, as one of a number of models set forth in E. N. Parker's book "Interplanetary Dynamical Processes" [1961, p. 217]. We propose to examine it in detail and check its predictions with the experimental data at hand.

In order to obtain an explicit solution to the diffusion equation we must assume a specific form for the spatial dependence of the diffusion coefficient. Let us assume, following Parker, that the diffusion coefficient D varies as a power of r, that is,

$$D = M r^{\beta} ,$$

where $\beta$ and M are parameters independent of r but possibly dependent on particle energy. Then the diffusion equation for isotropic scattering of particles of a given energy is

$$\frac{\partial \rho}{\partial t} = \frac{1}{r^{\alpha}} \frac{\partial}{\partial r} \left( M r^{\alpha+\beta} \frac{\partial \rho}{\partial r} \right) . \qquad (1)$$

Here $\rho$ = the number of particles per unit volume at position r, at time t, of energy E.

$\alpha$ = parameter specifying the dimensionality of the space to be used.

A solution to an equation of this form is written down without proof by Parker in the forementioned book. We feel that the detailed solution is of interest and have therefore worked it out with the necessary modifications for the present purpose and have presented it in Appendix I.

In order to transform solution (15A) in Appendix I to directly observable quantities we use the relation

$$\rho = \frac{4\pi I}{v}$$

I = directional intensity
  = no. (unit area, unit time, unit solid angle)$^{-1}$ .

v = velocity of the particle of given energy.

In writing down the above relation we have assumed explicitly that the intensity is isotropic. It should also be noted that the development has been for particles of a given energy only. Then (15A) becomes

$$I(r,t) = \frac{N\,v}{4\pi(2-\beta)^{(2\alpha+\beta)/(2-\beta)}} \; \frac{1}{\Gamma\,[(\alpha+1)/(2-\beta)]} \left(\frac{1}{M}\right)^{\frac{\alpha+1}{2-\beta}}$$

$$\times \; \frac{1}{t^{(\alpha+1)/(2-\beta)}} \; \exp\left[ -\frac{1}{M} \, \frac{r^{2-\beta}}{(2-\beta)^2} \, \frac{1}{t} \right] \qquad (2)$$

where $\beta < 2$ .

N = no. of particles per unit solid angle emitted at t=0.

Multiplying both sides by $t^{(\alpha+1)/(2-\beta)}$ and taking logarithms we have

$$\ell n I\, t^{(\alpha+1)/(2-\beta)} = \ell n \left[ \frac{Nv}{(2-\beta)^{(2\alpha+\beta)/(2-\beta)}} \; \frac{1}{\Gamma[(\alpha+1)/(2-\beta)]} \times \left( \frac{1}{M} \right)^{(\alpha+1)/(2-\beta)} \right] - \frac{1\, r^{2-\beta}}{M\,(2-\beta)^2} \; \frac{1}{t} \; . \qquad (3)$$

Now, if we plot $\ell n\ I\ t^{(\alpha+1)/(2-\beta)}$ vs $t^{-1}$ we have a family of straight lines corresponding to various numerical values of the ratio $\left( \frac{\alpha+1}{2-\beta} \right)$. The slope of a particular straight line is given by

$$\text{slope} = m = -\frac{r^{2-\beta}}{(2-\beta)^2\, M} \; . \qquad (4)$$

the intercept of the line with the $t=\infty$ axis gives us the term

intercept = b

$$= \ell n \left[ \frac{Nv}{(2-\beta)^{(2\alpha+\beta)/(2-\beta)} \; \Gamma[(\alpha+1)/(2-\beta)]} \left( \frac{1}{M} \right)^{(\alpha+1)/(2-\beta)} \right] . \qquad (5)$$

By taking the derivative of I in equation (2) with respect to time and setting it equal to zero, we find the time of maximum intensity $t_{max}$. Thus setting,

$$\frac{\partial I}{\partial t} = 0 ,$$

we find

$$\frac{1}{M} \frac{r^{2-\beta}}{(2-\beta)^2\, t_{max}} - \frac{\alpha+1}{2-\beta} = 0$$

or

$$t_{max} = \frac{1}{M} \frac{r^{2-\beta}}{(2-\beta)(\alpha+1)} . \tag{6}$$

In terms of the slope, the last equation can be rewritten as follows:

$$t_{max} = m \frac{\beta-2}{\alpha+1} . \tag{7}$$

For the case of radial diffusion away from a point source in three-dimensional space, $\alpha=2$ and equation (7) becomes:

$$t_{max} = m \frac{\beta-2}{3} . \tag{8}$$

## DISCUSSION

To summarize, we have equation (1) whose solution in terms of intensity is equation (2), with the supplemental expressions (4), (5), and (7). These expressions give us a complete description of the variation of the intensity as a function of time and position in space. A set of calculated curves showing the time variation for various distances is given in Figure 1. It is clear from equation (2) that the decay is not a negative exponential in time, though segments of the decay curve may be so approximated over periods of time whose lengths depend on the values of r and M. As seen in Figure 1 for the cases r = 3 or 4 A.U., the decay is approximately an exponential over the time period 15 to 40 hours. But from equation (2) it is seen that the intensity approaches asymptotically a $t^{-(\alpha+1)/(2-\beta)}$ decay at large t. If we plot the calculated intensity vs time in a log-log plot (Figure 2) we observe that even as late as fifty hours after the beginning of the event, the intensity at r = 1 A.U. has not yet reached the asymptotic decay law which in this exemplary case is $t^{-3}$.

It is clear from the above examples, that the apparent law of decay of intensity is a function of M and of the radial distance (as well as of the values of $\alpha$ and $\beta$). Moreover, it should be recalled that M and $\beta$, in particular, are probably functions of particle energy. Thus, it is seen that this relatively simple model does provide a new level of flexibility and physical insight in understanding observational data.

It is of further interest to examine the intensity as a function of position for various times (Figure 3). We observe that we start with a distribution strongly peaked at the origin at t=1 hr, but at t=100 hrs the intensity is nearly constant out to at least 6 A.U. Such curves offer an additional basis for the interpretation of data obtained with deep space probes at various heliocentric distances.

It is clearly desirable to derive values of M and of $\beta$ from the detailed configuration of the interplanetary magnetic field. For example, the intuitive expectation is that the scattering of a charged particle by an inhomogeneity in a magnetic field is a maximum when the cyclotron radius of the particle's trajectory is comparable to the scale of the inhomogeneity and diminishes for both greater and lesser values of the cyclotron radius. This expectation is confirmed by detailed calculation [Parker, 1964]. In due course, it may be possible to use observed field configurations for more comprehensive calculation. Meanwhile, we continue to use the macroscopic point of view.

## ANALYSIS OF OBSERVATIONAL DATA

The usefulness of the above theory will now be checked by analyzing several bodies of observational data on I vs t at $r \sim 1$ A.U. in the framework of equation (3) and its consequences. Specifically, the model implies that a plot of $\ln I\, t^{(\alpha+1)/(2-\beta)}$ vs $1/t$ will be a straight line for any value of the quantity $(\alpha+1)/(2-\beta)$. It should be noted that observational data at a single value of r cannot yield separate determinations of $\alpha$ and $\beta$, unless M is known. An assumption about the effective geometry is required (e.g., $\alpha=2$ corresponds to diffusion away from a point source in three-dimensional space) for a unique determination of $\beta$. This assumption ($\alpha=2$) has been used in the following analysis. Insofar as observational data permit we have taken the impulsive emission of solar cosmic rays (t=0) to have occurred at the moment of emission of a burst of hard X-rays during the course of the flare, since it is plausible that energetic particles are generated at this time. In the cases where no solar X-rays were observed we have taken the beginning of the optical flare as t=0. It is noted that errors in selection of zero time of the order of a few minutes have little effect on the values of $\beta$ and M which are derived from the full analysis of the I vs t data.

(a) $\ln I\, t^{(\alpha+1)/(2-\beta)}$ vs $t^{-1}$ Plots.

Figure 4 shows a plot of data for the September 28, 1961 event, as observed by a 302 G.M. tube on Explorer XII outside the magnetosphere [Van Allen et al., 1962]. The data have been plotted for $\alpha$=2 and for several assumed values of $\beta$. It is seen that the best approximation to a straight line is obtained for $\beta = 2/3$; the value of $\frac{M}{r^{2-\beta}}$ is 0.0771 $(\text{hrs})^{-1}$. (By dividing M by $r^{2-\beta}$ we have dimensions in $(\text{hours})^{-1}$ for the above quotient; the numerical value of $r^{2-\beta}$ is equal to one at r = 1 A.U. for any $\beta$.) Note that the intensities used are the integral intensities ($E_p$ > 23 MeV), and are only approximately those of a monoenergetic assemblage of particles. In agreement with earlier analyses as discussed in the Introduction, the model gives a decisively inferior representation of the data for $\beta$=0, that is, for a diffusion coefficient which is independent of r.

Figure 5 shows a similar analysis for the solar cosmic ray event of July 18, 1961. We find $\beta = 4/5$ and $\frac{M(E)}{r^{2-\beta}}$ is 0.056 $(\text{hours})^{-1}$. The case for $\beta$=0 is again shown for comparison.

Another case is shown in Figure 6 for the event of April 15, 1963 [data from L. A. Frank, 1965]. The data are well organized by this treatment with $\beta$=1, $\frac{M(E)}{r^{2-\beta}} = 0.104$ $(\text{hours})^{-1}$.

The intensity-time profile of the event is shown in Figure 7. It is possible that the observed particles are due to more than one flare, as suggested by the observed structure in the intensity-time profile.

From study of these three cases, we observe that satisfactory interpretation of the data can be obtained for similar but somewhat different values of β and M (see Table I). These variations in values of the parameters are presumably attributable to variations in the interplanetary medium and, perhaps, to differences in the energy spectra of the particles.

(b) Dependence of β on E.

The next question that presents itself is the dependence of β and of M on particle energy. We use additional data from the September 28, 1961 event obtained by Explorer XII with an array of detectors by Bryant et al. [1964]. Figures 8 through 13 show data of these authors replotted in the manner previously described. It is seen that all of these data for different energy ranges are well represented by a single choice of β, namely $\beta=1.0$. See also Table I.

A plot of β(E) vs E is given in Figure 15 for the event of September 28, 1961. It appears that β is independent of E for E > 55 MeV but drops markedly between 55 and 23 MeV.

The data point for $E > 40$ MeV (Figure 14) has the indicated uncertainty due to the poor quality of the data late in the event. However, it is possible to state that $\beta$ is definitely not as great as one and it is somewhat greater than 2/3.

From the fact that the radial dependence of D is a strong function of particle energy for $23 < E_p < 87$ MeV it is evident that magnetic irregularities of different scale dimensions exist. Furthermore, the spectrum of dimensions must include ones comparable to the cyclotron radii of protons of energy of the order of several hundred MeV or less (e.g., ~ 0.004 A.U.). (See also the discussion associated with Figure 18.)

### (c) The Diffusion Coefficient as a Function of Distance and Energy.

In Figure 16 we have plotted from the data of Table I the diffusion coefficient D (E, r) as a function of r for various energies. We observe that D (E, r) increases linearly with distance above 55 MeV. We shall return later to the question of the spatial dependence of the diffusion coefficient.

In Figure 17 the diffusion coefficient is plotted as a function of energy for a given position in space. It should be noted that the energy dependence of D is independent of position in space if $\beta$ is independent of energy. It is seen that in this particular case for $E \gtrsim 55$ MeV ($\beta$=1.0) the diffusion coefficient can be written as

$$D(E, r) = 1.56 \times 10^{-2} E^{0.335} r, \qquad E \gtrsim 55 \text{ MeV} \tag{9}$$

from which we have that

$$M(E) = 1.56 \times 10^{-2} E^{0.335} \tag{10}$$

where E is in MeV and M (E) has the dimensions of (A.U.)/hr. It appears from Figure 17 that D approaches a constant value as the energy decreases below 55 MeV. In order to understand the physical significance of this, we have plotted in Figure 18 the mean-free-path $\lambda = \frac{3D}{v}$ as a function of energy. It is seen that $\lambda$ at 1 A.U. approaches a constant value of ~ 0.081 A.U. at high energies and that it increases rapidly at low energies. (The cyclotron radius of a 23 MeV proton is smaller than that of a 450 MeV proton by a factor of 4.8.) The apparent meaning of this result is that the cyclotron radius of a 23 MeV proton is considerably smaller than the dimensions of the most effective scattering centers. In this vein of thought it would be expected that $\lambda$ would increase again as E goes to higher values than those shown in Figure 18.

A crude estimate of the dimensions of effective scattering centers [cf. McCracken, 1962, and Parker, 1964] can be made by finding the cyclotron radius of, say, a 450 MeV proton (one for which $\lambda$ has a minimum value). In a magnetic field of 4 gamma, this radius is 0.006 A.U.

### (d) Total Number of Particles and Energy Spectrum.

We have seen from expression (5) that, knowing the values of $\dot{M}$ and $\beta$, we can calculate the total number of particles emitted at t=0. In the case where data are available for various energies, one is able to construct the energy spectrum of the particles emitted at t=0 at the sun. The results of this computation are shown in Figures 19, 20, 21, and 22.

In these figures the data are plotted in four commonly assumed forms, namely, a power law in energy, an exponential in energy, a power law in magnetic rigidity, and an exponential in magnetic rigidity, respectively. An inspection of the graphs shows that any one of these laws can be used to describe the spectrum within the indicated experimental errors. The best fit appears to be the exponential rigidity spectrum in Figure 22. The vertical error bars represent estimated maximum errors; the true errors are probably less. The differential spectra for the three laws (omitting the exponential in energy case) are as follows:

$$\frac{dN}{dE} = 2.23 \times 10^{32} \, E^{-(2.41 \pm 0.3)} \qquad (11)$$

$$\frac{dN}{dP} = 8.87 \times 10^{36} \, P^{-(3.7 \pm 0.3)} \qquad (12)$$

$$\frac{dN}{dP} = 1.53 \times 10^{28} \, e^{-P/(173 \pm 30)} \qquad (13)$$

In the above expressions the units are:

$\frac{dN}{dE}$ in number/MeV,

E in MeV,

$\frac{dN}{dP}$ in number/MV,

P in MV.

It should be noted that an analysis of the same data by Bryant et al. [1964] using normalization factors in an intensity vs distance plot resulted in an energy spectrum at t=0 which varied as $E^{-1.7}$, in disagreement with the present analysis.

A useful check on the above spectral forms is provided by computing the number of particles of $E > 23$ MeV or $P > 209$ MV, and comparing it with the "observed" number from the 302 G.M. tube. Using equation (13) we have

$$N_p > 209 \text{ MV} = \int_{209}^{\infty} 1.53 \times 10^{28} \; e^{-P/173} \, dP$$

$$= 1.53 \times 10^{28} \times 173 \times e^{-209/173}$$

$$\simeq 0.79 \times 10^{30} \text{ particles.}$$

This is to be compared with $N \simeq 1 \times 10^{30}$, computed directly from equation (5) by use of the slope and intercept from Figure 4. The corresponding numbers predicted by the power law in energy and power law in rigidity are $1.88 \times 10^{30}$ and $1.82 \times 10^{30}$, respectively.

An elementary calculation shows that such numbers of particles carry a very small fraction of the total energy of a flare. This, if we assume the typical dimension of a flare to be $\sim 10^4$ km with magnetic fields of about 1000 gauss, then the total energy stored in the field is of the order of $10^{31}$ ergs. On the other hand the total energy of $12.6 \times 10^{30}$ particles of $E \simeq 23$ MeV is about $4.6 \times 10^{26}$ ergs, that is, the energy carried by the particles is less than $10^{-4}$ of that contained in the flare region.

It is of special interest to observe the time evolution of the differential energy spectrum. We can do this by use of equation (2) with the appropriate constants from Table I. The energy spectrum "folds over" at early times and becomes gradually softer as time increases. These results are consistent with the observation of Bryant et al. [1962] and provide an additional test of the internal consistency of the model.

## RECAPITULATION AND COMMENTS

In the foregoing, the analysis of the time history of the intensity of several solar cosmic ray events (at 1 A.U.) has been made in terms of a unified model of interplanetary diffusion which assumed that the effective diffusion coefficient is a power law function of heliocentric radial distance and is also a function of particle energy. It is found possible to give a coherent interpretation of the time history* of the intensity of each monoenergetic component with specific values for the two parameters M and $\beta$. Sets of the empirical values of these parameters yield, directly, the energy and radial dependence of the diffusion coefficient and of the diffusion mean free path and, indirectly, the order of magnitude of the scale size of inhomogeneities of the interplanetary magnetic field. Predictions are made on the radial dependence of the time history of solar cosmic ray intensity. Also spectra for

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*The present treatment is relatively inadequate to explain the early data, that is, those during the first half-hour or so after the flare. This inadequacy is presumably connected with the early anisotropy and with possible delays in emission from the influence of the sun [Reid, 1964].

the source function at the sun are derived. Our analysis was based mostly on the September 28, 1961 event because of the large body of observational data available. However, all of the other events shown here, as well as those in process of analysis, receive a coherent interpretation in terms of the model proposed. It is important to note that the fit to experimental data determines only the value of the quantity

$$\delta = \frac{\alpha+1}{2-\beta} \tag{14}$$

and not $\alpha$ and $\beta$ separately. In the foregoing analysis it has been assumed that $\alpha=2$. This value of $\alpha$ corresponds to spherical symmetry about the sun and immediately raises the issue that particles from "back-side" flares rarely reach the earth in measurable intensities. The value $\alpha=2$ is, however, equally applicable to diffusion in a conical region whose vertex is at the source and whose sidewall is bounded by radial lines through the source. If the diffusion region is of a flared-trumpet shape then $\alpha > 2$, and a different value of $\beta$ is implied. Thus, taking $\delta$ as the known quantity

$$\beta = 2 - (\alpha+1)/\delta \quad . \tag{15}$$

A typical, observed value of $\delta$ is 3. Hence for $\alpha > 2$

(trumpet diffusion), $\beta < 1$; for $\alpha = 2$ (spherical or conical diffusion), $\beta = 1$; and for $\alpha = 1$ (cylindrical diffusion in, say, the ecliptic plane), $\beta = 4/3$. Parker [1963] points out that the value of $\beta = 1$ is a critical one in the following sense: The number of mean free paths between inner and outer radii $r_1$ and $r_2$ is

$$q = \int_{r_1}^{r_2} \frac{dr}{\lambda} \propto \int_{r_1}^{r_2} \frac{dr}{r^{\beta}} .$$

Thus

$$q \propto \frac{(r_2)^{1-\beta} - (r_1)^{1-\beta}}{(1 - \beta)} \qquad \text{for } \beta < 1$$

$$q \propto \ell n \left( \frac{r_2}{r_1} \right) \qquad \text{for } \beta = 1$$

$$q \propto \left( \frac{1}{\beta-1} \right) \left[ \frac{1}{(r_1)^{\beta-1}} - \frac{1}{(r_2)^{\beta-1}} \right] \qquad \text{for } \beta > 1$$

and $q \to \infty$ as $r_2 \to \infty$ for $\beta = 1$ or $\beta < 1$ but is finite for $\beta > 1$. Parker remarks that for $\beta \leq 1$, galactic cosmic rays would experience an "infinite diffusive resistance" from infinity and would therefore enter the inner solar system at an infinitely slow rate, contrary to the evidence from the 11-year modulation.

This line of argument might be taken to favor the possibility that interplanetary diffusion is more nearly cylindrical (as in the ecliptic plane) than spherical and hence that $\alpha < 2$ and $\beta > 1$.

The argument may be turned another way, however, by noting that our values of $\beta \leq 1$ for $E_p < 400$ MeV during September 1961 would imply the absence of particles of such energies in the galactic cosmic radiation at the earth. This implication does indeed receive some support from the facts [Meyer and Vogt, 1963]. Also, it is not unreasonable to suppose that the value of $\beta$ increases for greater particle energies; and it may vary during the solar cycle, corresponding to different states of turbulence of the interplanetary medium. Thus, even if $r_2$ is taken to be infinite, it is not clear that any physical contradiction exists. On the contrary, we suggest that the empirical value of $\beta$ as determined herein may provide a significant insight on the entire question of the interplanetary modulation of the galactic cosmic ray spectrum. Specifically, it is of interest to study the variation of $\beta$ (E) during a solar cycle.

Table I

Relevant Constants for the Various Events

| Event | $E_p$ (MeV) | $\beta$ for $\alpha$=2 | Slope = m | Intercept = b | N ( $\frac{\text{particles}}{\text{sterad}}$ ) | $\frac{M}{r^{2-\beta}}$ $(\text{hrs})^{-1}$ |
|---|---|---|---|---|---|---|
| 18 July 1961 | > 40 | 4/5 | -12.4 | 10.9 | $5.58 \times 10^{30}$ | 0.056 |
| 28 Sept. 1961 | > 23 | 2/3 | - 7.30 | 8.30 | $1.02 \times 10^{30}$ | 0.0771 |
| 28 Sept. 1961 | > 40 | 4/5 | - 8.38 | 7.9 | -- | 0.0828 |
| 28 Sept. 1961 | $55 \leq E_p \leq 118$ | 1 | -12.0 | 4.99 | $2.34 \times 10^{29}$ | 0.0833 |
| 28 Sept. 1961 | $118 \leq E_p \leq 150$ | 1 | -10.16 | 3.99 | $6.01 \times 10^{28}$ | 0.0984 |
| 28 Sept. 1961 | $150 \leq E_p \leq 200$ | 1 | - 9.81 | 3.14 | $3.98 \times 10^{28}$ | 0.102 |
| 28 Sept. 1961 | $200 \leq E_p \leq 255$ | 1 | - 8.58 | 2.49 | $3.11 \times 10^{28}$ | 0.1165 |
| 28 Sept. 1961 | $255 \leq E_p \leq 335$ | 1 | - 7.93 | 1.44 | $1.85 \times 10^{28}$ | 0.126 |
| 28 Sept. 1961 | $335 \leq E_p \leq 500$ | 1 | - 7.03 | 0.24 | $1.47 \times 10^{28}$ | 0.142 |
| 15 April 1963 | > 23 | 1 | - 9.6 | 7.4 | $1.59 \times 10^{29}$ | 0.104 |

## APPENDIX

### The Detailed Solution of Equation (1)

We have that

$$\frac{\partial \rho}{\partial t} = \frac{1}{r^\alpha} \frac{\partial}{\partial r} \left[ M r^\beta r^\alpha \frac{\partial \rho}{\partial r} \right] . \tag{1A}$$

Here $\rho$ = the number of particles per unit volume at position r, at time t, of energy E.

$\alpha$ = parameter specifying the dimensionality of the space to be used.

The boundary condition is that $\rho$ must vanish at infinity.

Let $\tau = M t$. Then

$$\frac{\partial \rho}{\partial \tau} = \frac{1}{r^\alpha} \frac{\partial}{\partial r} \left( r^{\alpha+\beta} \frac{\partial \rho}{\partial r} \right)$$

and, carrying out the differentiation

$$\frac{\partial \rho}{\partial \tau} = r^\beta \frac{\partial^2 \rho}{\partial r^2} + (\alpha+\beta)\, r^{\beta-1} \frac{\partial \rho}{\partial r} . \tag{2A}$$

Let us assume a solution of the form

$$\rho\,(r,\tau) = R\,(r)\, T\,(\tau) . \tag{3A}$$

By making the appropriate substitutions into (2A) we have

$$\frac{1}{T}\frac{\partial T}{\partial \tau} = \frac{r^{\beta}}{R}\frac{\partial^2 R}{\partial r^2} + (\alpha+\beta)\ r^{\beta-1}\ \frac{1}{R}\frac{\partial R}{\partial r}\ .$$

Since each side of the last equation is a function of only one variable, we must have

$$\frac{1}{T}\frac{\partial T}{\partial \tau} = \frac{r^{\beta}}{R}\frac{\partial^2 R}{\partial r^2} + (\alpha+\beta)\ r^{\beta-1}\ \frac{1}{R}\frac{\partial R}{\partial r} = -k^2$$

where $-k^2$ is a constant. We then have immediately

$$\frac{1}{T}\ \frac{\partial T}{\partial \tau} = -k^2$$

$$T = e^{-k^2 \tau}\ . \qquad (4A)$$

Now

$$r^{\beta}\ \frac{d^2 R}{dr^2} + (\alpha+\beta)\ r^{\beta-1}\frac{dR}{dr} + k^2\ R = 0$$

or

$$\frac{d^2 R}{dr^2} + \frac{\alpha+\beta}{r}\ \frac{dR}{dr} + \frac{k^2}{r^{\beta}}\ R = 0\ . \qquad (5A)$$

The existence of the solution to this equation was proved by Malmstén [1850]. This is a modified form of the Bessel equation and it is satisfied by the following expression [see e.g., Watson, 1922]:

$$R\ (r) = \frac{1}{r^{(\alpha+\beta-1)/2}}\ J_p\ \left[\ \frac{2k}{(\beta-2)\ r^{(\beta-2)/2}}\ \right] \qquad (6A)$$

where $p = (\alpha+\beta-1)/(2-\beta)$.

The complete solution is a superposition of solutions (4A) and (6A), namely

$$\rho\ (r,\tau) = \frac{1}{r^{(\alpha+\beta-1)/2}} \int_0^\infty dk\ k\ f(k) \exp\ (-k^2\ \tau)$$

$$\times \quad J_p \left[ \frac{2k}{(\beta-2)} \frac{1}{r^{(\beta-2)/2}} \right] . \qquad (7A)$$

At $\tau = 0$, we have

$$\rho\ (r,o) = \frac{1}{r^{(\alpha+\beta-1)/2}} \int_0^\infty dk\ k\ f(k)\ J_p \left[ \frac{2k}{(\beta-2)} \frac{1}{r^{(\beta-2)/2}} \right]$$

or

$$r^{(\alpha+\beta-1)/2}\ \rho\ (r,o)$$

$$= \int_0^\infty dk\ k\ f\ (k)\ J_p \left[ \frac{2k}{(\beta-2)} \frac{1}{r^{(\beta-2)/2}} \right] . \qquad (8A)$$

At this point we need to express $f(k)$ in terms of $\rho\ (r,o)$. The proper inversion theorem to be used is the Fourier-Bessel theorem [see, e.g., Sneddon, 1951, p. 65].

$$\phi\,(r) \;=\; \int_0^\infty k\, f(k)\, J_\nu\,(kr)\, dk\;. \tag{9A}$$

$$f\,(k) \;=\; \int_0^\infty r\, \phi\,(r)\, J_\nu\,(kr)\, dr\;. \tag{10A}$$

By carrying out the algebra we find that

$$f\,(k) \;=\; \frac{2}{2-\beta} \int_0^\infty dr\; r^{(\alpha-\beta+1)/2}\, J_p\left[\frac{2k}{\beta-2}\, \frac{1}{r^{(\beta-2)/2}}\right] \rho\,(r,o)\;. \tag{11A}$$

To evaluate f (k), we must now specify the function ρ (r,o). If N particles per unit solid angle are released at t = 0, ρ (r,o) may be represented by

$$\rho\,(r,o) \;=\; \lim_{\epsilon \to o} \; N\, \frac{\delta\,(r-\epsilon)}{\epsilon^{\alpha}} \;. \tag{12A}$$

[See, e.g., Morse and Feshbach for the generalized form of the Dirac delta function.]

Then we have

$$f\,(k) \;=\; \lim_{\epsilon \to o} \frac{2}{2-\beta} \int_0^\infty \frac{r^{(\alpha-\beta+1)/2}}{\epsilon^{\alpha}}\; J_p\left[\frac{2k}{(\beta-2)}\, \frac{1}{r^{(\beta-2)/2}}\right] \times N\, \delta\,(r-\epsilon)\, dr$$

$$= \lim_{\epsilon \to o} \frac{2N}{2-\beta}\, \epsilon^{(1-\alpha-\beta)/2} \left[\frac{2k}{2(\beta-2)}\, \frac{1}{\epsilon^{(\beta-2)/2}}\right] \times \sum_{j=o}^{\infty} \frac{(-1)^j}{j!\,\Gamma(j + \frac{\alpha+\beta-1}{2-\beta} + 1)} \left(\frac{2k}{(\beta-2)\,\epsilon^{(\beta-2)/2}}\right)^{2j} \tag{13A}$$

where the definition of the Bessel function was used. Now, by combining terms and expanding, we have

$$f(k) = \frac{2N}{2-\beta} \lim_{\epsilon \to o} \epsilon^{(1-\alpha-\beta)/2+\frac{1}{2}(\alpha+\beta-1)} \left(\frac{k}{\beta-2}\right)^{\frac{\alpha+\beta-1}{2-\beta}}$$

$$\times \left[ \frac{1}{\Gamma(1+\frac{\alpha+\beta-1}{2-\beta})} - \frac{1}{\Gamma(1+\frac{\alpha+\beta-1}{2-\beta}+1)} \left(\frac{2k}{\beta-2}\right)^2 \epsilon^{-\beta+2} \right.$$

$$\left. + \text{consts } \epsilon^{2(-\beta+2)} + \ldots \right]$$

$$f(k) = \frac{2N}{2-\beta} \lim_{\epsilon \to o} \epsilon^{o} \left(\frac{k}{\beta-2}\right)^{\frac{\alpha+\beta-1}{2-\beta}} \left[ \frac{1}{\Gamma(1+\frac{\alpha+\beta-1}{2-\beta})} \right.$$

$$- \frac{1}{\Gamma(1+\frac{\alpha+\beta-1}{2-\beta}+1)} \left(\frac{2k}{\beta-2}\right)^2 \epsilon^{-\beta+2}$$

$$\left. + \text{consts } \epsilon^{2(-\beta+2)} + \ldots \right] .$$

From the last expression we observe that as we let $\epsilon \to o$, we must restrict ourselves to $\beta < 2$, otherwise the expression becomes undefined. With this restriction in mind we finally have

$$f(k) = \frac{2N}{2-\beta} \left(\frac{k}{\beta-2}\right)^{\frac{\alpha+\beta-1}{2-\beta}} \frac{1}{\Gamma[\frac{(\alpha+1)}{(2-\beta)}]}, \quad \beta < 2 . \qquad (14A)$$

It is not obvious to the author why such a restriction on $\beta$ should exist; the physical meaning of it is not immediately apparent. Now let us substitute (14A) into (7A).

$$\rho(r,\tau) = \frac{1}{r^{(\alpha+\beta-1)/2}} \int_0^\infty dk\, k\, \frac{2N}{2-\beta} \left(\frac{k}{\beta-2}\right)^{(\alpha+\beta-1)/(2-\beta)}$$

$$\times \frac{1}{\Gamma[(\alpha+1)/(2-\beta)]}$$

$$\times \exp(-k^2 \tau)\, J_p\left[\frac{2k}{(\beta-2)} \frac{1}{r^{(\beta-2)/2}}\right]$$

$$= \frac{2N}{2-\beta} \frac{1}{\Gamma[(\alpha+1)/(2-\beta)]} \frac{1}{(\beta-2)^{(\alpha+\beta-1)/(2-\beta)}}$$

$$\times \frac{1}{r^{(\alpha+\beta-1)/2}}$$

$$\times \int_0^\infty dk\, k^{(\alpha+\beta-1)/(2-\beta)+1}\, J_p\left[\frac{2k}{\beta-2} \frac{1}{r^{(\beta-2)/2}}\right] \exp(-k^2 \tau) .$$

An integral of this general form can be evaluated [see Watson, 1922].

$$\int_0^\infty J_\nu(at) \exp(-p^2 t^2)\, t^{\nu+1}\, dt$$

$$= \frac{a^\nu}{(2p^2)^{\nu+1}} \exp\left(-\frac{a^2}{4p^2}\right) .$$

Therefore, we have

$$\rho\ (r,t) = \frac{N}{(2-\beta)^{(2\alpha+\beta)/(2-\beta)}} \frac{1}{\Gamma[\ (\alpha+1)/(2-\beta)\ ]} \frac{1}{\tau^{(\alpha+1)/(2-\beta)}}$$

$$\times \exp\left( \frac{-\ r^{2-\beta}}{(2-\beta)^2\ \tau} \right) .$$

By substitution of M t for $\tau$, we have finally

$$\rho\ (r,t) = \frac{N}{(2-\beta)^{(2\alpha+\beta)/(2-\beta)}} \frac{1}{\Gamma[\ (\alpha+1)/(2-\beta)\ ]} \left( \frac{1}{M} \right)^{(\alpha+1)/(2-\beta)}$$

$$\times \frac{1}{t^{(\alpha+1)/(2-\beta)}} \exp\left( -\frac{1}{M} \frac{r^{2-\beta}}{(2-\beta)^2} \frac{1}{t} \right) . \quad (15A)$$

This is the desired solution of equation (1A). To see whether it reduces to the proper form in special cases, let us take $\alpha=2$ (spherical geometry), $\beta=0$ (ordinary diffusion equation with constant diffusion coefficient). Then equation (15A) becomes

$$\rho\ (r,t) = \frac{N}{2^2\ \Gamma[3/2]} \frac{1}{[M\ t)^{3/2}} \exp\left( -\frac{r^2}{4M\ t} \right)$$

$$= \frac{N}{2\,\pi^{1/2}\ [M\ t]^{3/2}} \exp\left( -\frac{r^2}{4M\ t} \right) .$$

We recognize this as the solution of the diffusion equation with constant diffusion coefficient M, in the case of an infinitely extensive medium. Our solution then reduces to the proper form, as it should.

## ACKNOWLEDGEMENTS

The author is indebted to Professor J. A. Van Allen for his continuous encouragement and illuminating comments on many essential points of this work. Thanks are due to Dr. W. C. Lin for valuable discussions and Dr. D. Venkatesan for encouragement. The diligent work of Mr. Lee Burns on the long computations contributed significantly to the successful completion of this work.

## REFERENCES

Anderson, K. A., and D. C. Enemark, Observations of solar cosmic rays near the north magnetic pole, J. Geophys. Res., 65, 2657-2671, 1960.

Arnoldy, R. L., R. A. Hoffman, and J. R. Winckler, Solar cosmic rays and soft radiation observed at 5,000,000 kilometers from earth, J. Geophys. Res., 65, 3004-3007, 1960.

Bryant, D. A., T. L. Cline, U. D. Desai, and F. B. McDonald, Explorer 12 observations of solar cosmic rays and energetic storm particles after the solar flare of September 28, 1961, J. Geophys. Res., 67, 4983-5000, 1962.

Bryant, D. A., T. L. Cline, U. D. Desai, and F. B. McDonald, Studies of solar protons with Explorers XII and XIV, Goddard Space Flight Center Report X-611-64-217, July 1964.

Charakhchyan, A. N., V. E. Tulinov, and T. N. Charakhchyan, The energy spectrum and time dependence of the intensity of solar cosmic ray protons in flares, Proc. Intern. Conf. Cosmic Rays and The Earth Storm--Kyoto, 2, 365-372, September 1961.

Earl, J. A., Balloon measurements of solar cosmic rays at Fort Churchill, Canada, during July, 1959, J. Geophys. Res., 66, 3095-3102, 1961.

Frank, L. A., A survey of electrons E > 40 keV beyond 5 earth radii with Explorer 14, J. Geophys. Res. (April 1, 1965).

Hoffman, D. J., and J. R. Winckler, Simultaneous balloon observations at Fort Churchill and Minneapolis during the solar cosmic ray events of July 1961, J. Geophys. Res., 68, 2067-2098, 1963.

Krimigis, S. M., and W. C. Lin, On the interplanetary propagation of solar cosmic rays, paper presented at the AGU Meeting in Washington, D. C., April 21-24, 1964.

Krimigis, S. M., and J. A. Van Allen, Two low energy solar proton events during September 1961 (to be published).

Lin, W. C., and James A. Van Allen, Observation of solar cosmic rays from October 13, 1959 to February 17, 1961 with Explorer VII, Proc. International School of Physics Enrico Fermi Course XXIV, "Space Exploration and the Solar System", Academic Press: New York 1964, pp. 194-235.

Malmsten, C. J., De l'equation differentielle, Journal fur Math., XXXIX (1850), pp. 108-115.

McCracken, K. G., The cosmic-ray flare effect. 3. Deductions regarding the interplanetary magnetic field, J. Geophys. Res., 67, 447-458, 1962.

Meyer, P., E. N. Parker, and J. A. Simpson, Solar cosmic rays of February 1956 and their propagation through interplanetary space, Phys. Rev., 104, 768-783, 1956.

Meyer, P., and R. Vogt, Primary cosmic rays and solar protons, 2, Phys. Rev., 129, 2275-2279, 1963.

Morse, P. M., and H. Feshbach, Methods of Theoretical Physics, 2 Parts, McGraw-Hill, New York, 1953.

Parker, E. N., 1963, Interplanetary Dynamical Processes, Interscience Publishers, New York.

Parker, E. N., The scattering of charged particles by magnetic irregularities, J. Geophys. Res., 69, 1755-1758, 1964.

Reid, George C., A diffusive model for the initial phase of a solar proton event, J. Geophys. Res., 69, 2659-2667, 1964.

Sneddon, I. N., Fourier Transforms, McGraw-Hill, New York, 1951.

Van Allen, J. A., L. A. Frank, G. F. Pieper, W. A. Whelpley, B. J. O'Brien, and W. G. V. Rosser, Coordinated Injun I--Explorer XII observations of solar cosmic rays, 28 September to 4 October 1961 (presented at Explorer XII Symposium, Goddard Space Flight Center, Greenbelt, Maryland, January 18, 1962).

Watson, G. N., A Treatise on the Theory of Bessel Functions, Cambridge University Press, 1922.

Winckler, J. R., and P. D. Bhavsar, The time variations of solar cosmic rays during the September 3, 1960 event, J. Geophys. Res., 68, 2099-2115, 1963.

Winckler, J. R., P. D. Bhavsar, and L. Peterson, The time variations of solar cosmic rays during July, 1959, at Minneapolis, J. Geophys. Res., 66, 995-1022, 1961.

## FIGURE CAPTIONS

Figure 1. Semilogarithmic plots of the time variation of intensity of monoenergetic particles for various heliocentric radial distances.

Figure 2. A check on the asymptotic approach to a $t^{-3}$ decay at r = 1 A.U.; as late as 50 hours after the event, the asymptotic value has not been reached. The constants are those of Figure 1.

Figure 3. Variation of the intensity as a function of position in space at selected times. Fifty hours after the event the intensity is essentially constant over the range of r shown.

Figure 4. The best approximation to a straight line is found for $\beta = 2/3$ (for an assumed $\alpha = 2$). I is in (particles) x $(cm^2 \text{ sec sterad})^{-1}$ and t in hours.

Figure 5. The July 18, 1961 event.

Figure 6. The event of April 15, 1963 [data courtesy of L. A. Frank].

Figure 7. The complex structure of the intensity time profile of April 15, 1963 event.

Figure 8. Analysis of the data of Bryant et al. [1964] for the September 28, 1961 event. I is in $(m^2 \text{ sec sterad})^{-1}$.

Figure 9. Same as Figure 8 for different energy range.

Figure 10. Same as Figure 8 for different energy range.

Figure 11. Same as Figure 8 for different energy range.

Figure 12. Same as Figure 8 for different energy range.

Figure 13. Same as Figure 8 for different energy range.

Figure 14. The data for E > 40 MeV from the September 28, 1961 event. The value of β is uncertain due to the poor quality of the data late in the event.

Figure 15. The dependence of β on energy for the event of September 28, 1961.

Figure 16. The dependence of the diffusion coefficient on position in space for the event of September 28, 1961.

Figure 17. The diffusion coefficient as a function of energy at r = 1 A.U. The data are taken from Table I.

Figure 18. The mean-free-path as a function of energy for the event of September 28, 1961 at r = 1 A.U.

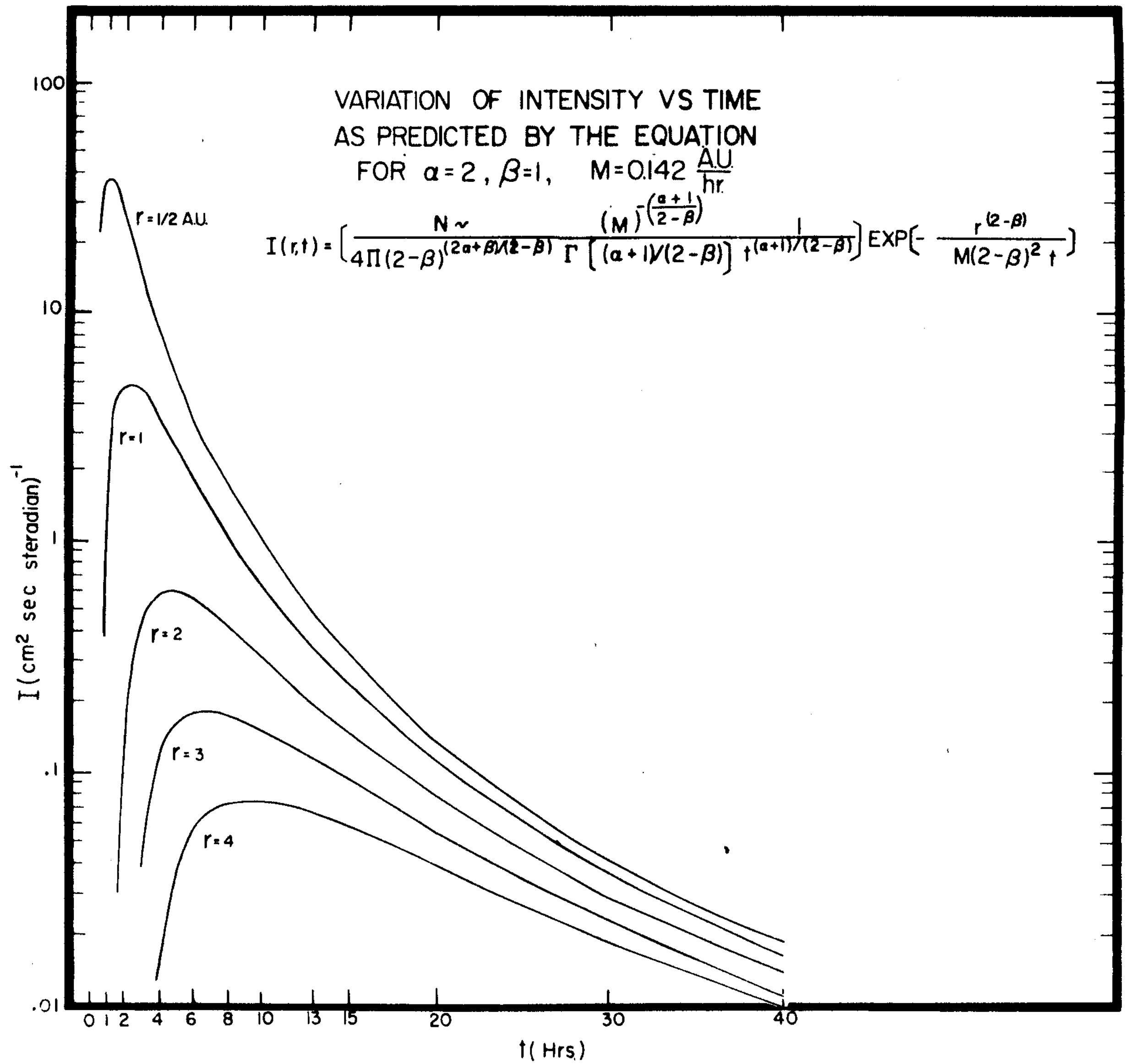
Figure 19. The differential energy spectrum at t=0 as a power law in energy for the 28 September event 1961.

Figure 20. The differential energy spectrum at t=0 as an exponential in energy.

Figure 21. The differential rigidity spectrum at t=0 as a power law in rigidity.

Figure 22. The differential rigidity spectrum at t=0 as an exponential in rigidity.

Figure 23. The differential energy spectrum for chosen times for the event of September 28, 1961 as predicted by the model.



VARIATION OF INTENSITY VS TIME
AS PREDICTED BY THE EQUATION
FOR $\alpha = 2$, $\beta = 1$, $M = 0.142 \frac{A.U.}{hr.}$
$I(r,t) = \left[\frac{N \sim}{4\Pi(2-\beta)^{(2\alpha+\beta)/(2-\beta)}} \frac{(M)^{-\left(\frac{\alpha+1}{2-\beta}\right)}}{\Gamma\left[(\alpha+1)/(2-\beta)\right]} \frac{1}{t^{(\alpha+1)/(2-\beta)}}\right] EXP\left(-\frac{r^{(2-\beta)}}{M(2-\beta)^2 t}\right)$
r = 1/2 A.U.
r = 1
r = 2
r = 3
r = 4
100
10
1
.1
.01
I (cm² sec steradian)⁻¹
0 1 2 4 6 8 10 13 15 20 30 40
t (Hrs.)


**FIGURE 1**

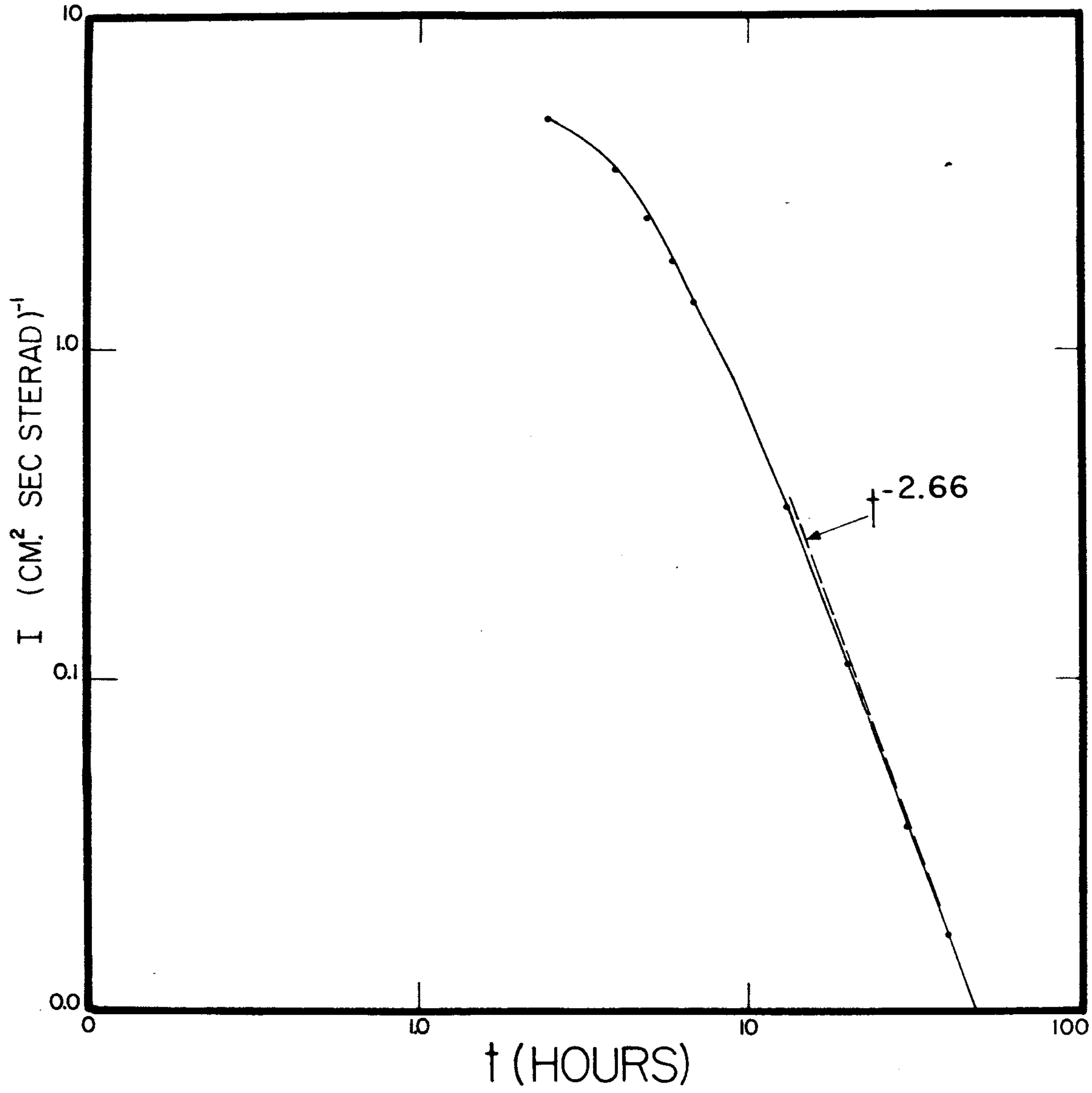
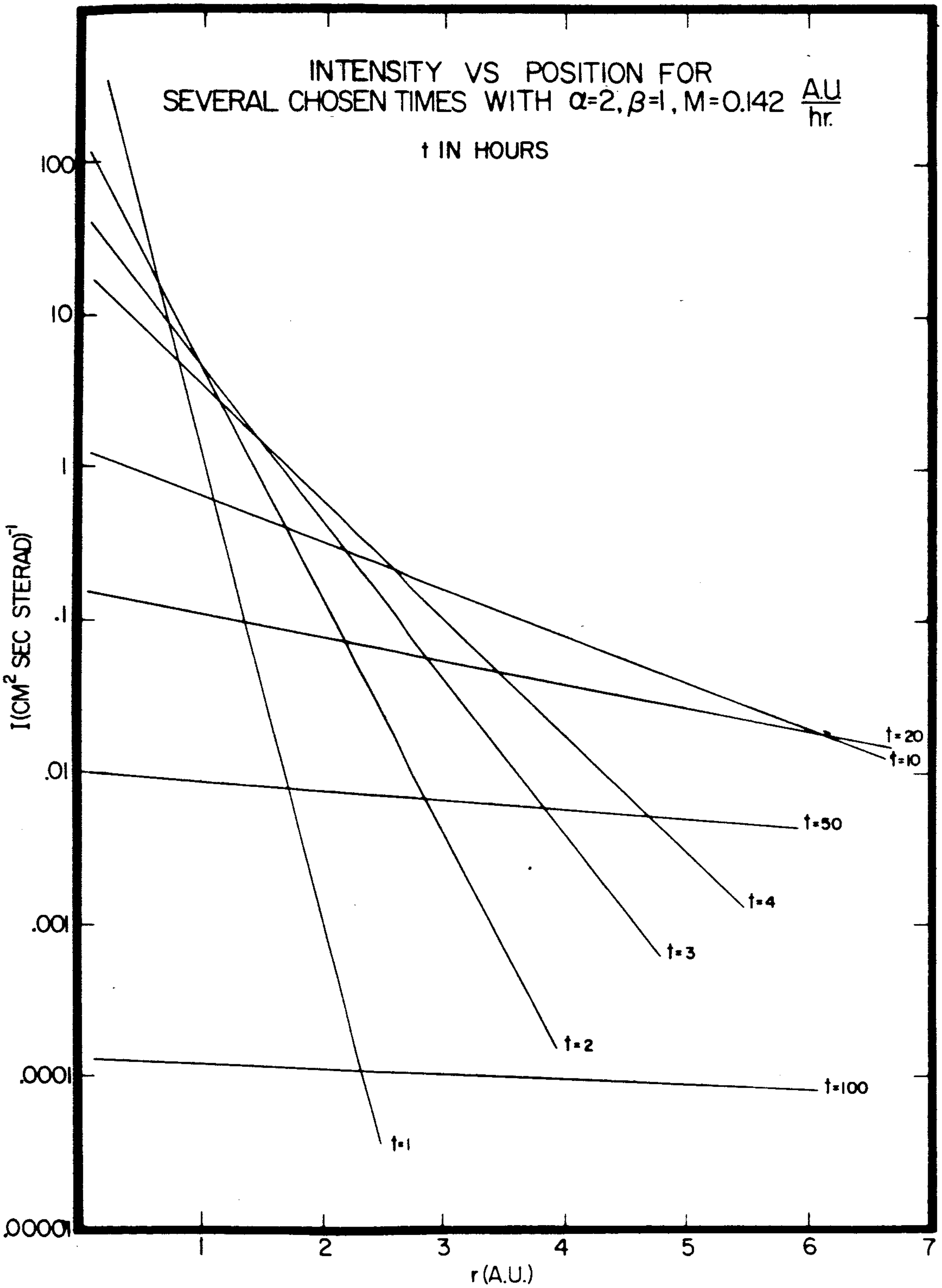
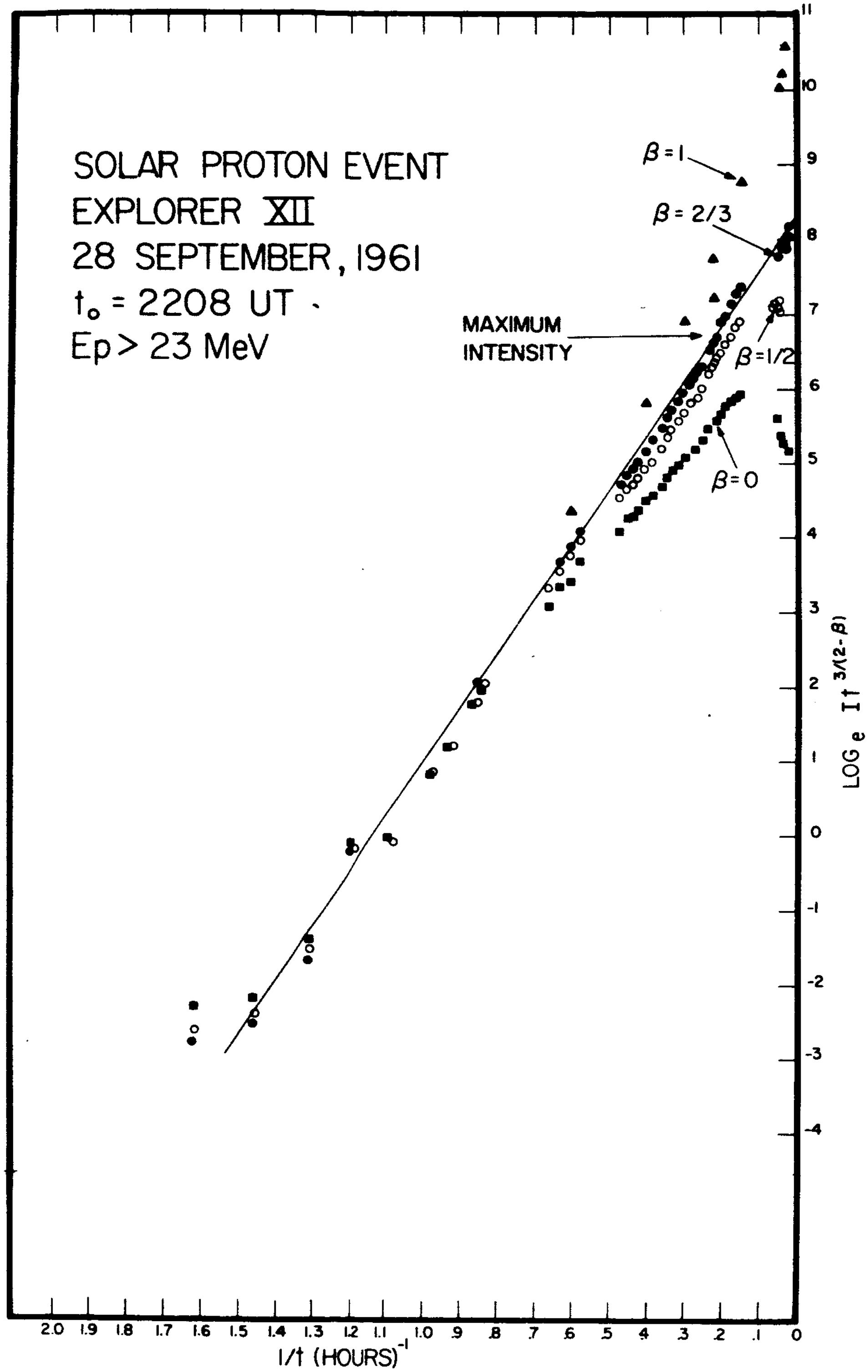



FIGURE 2



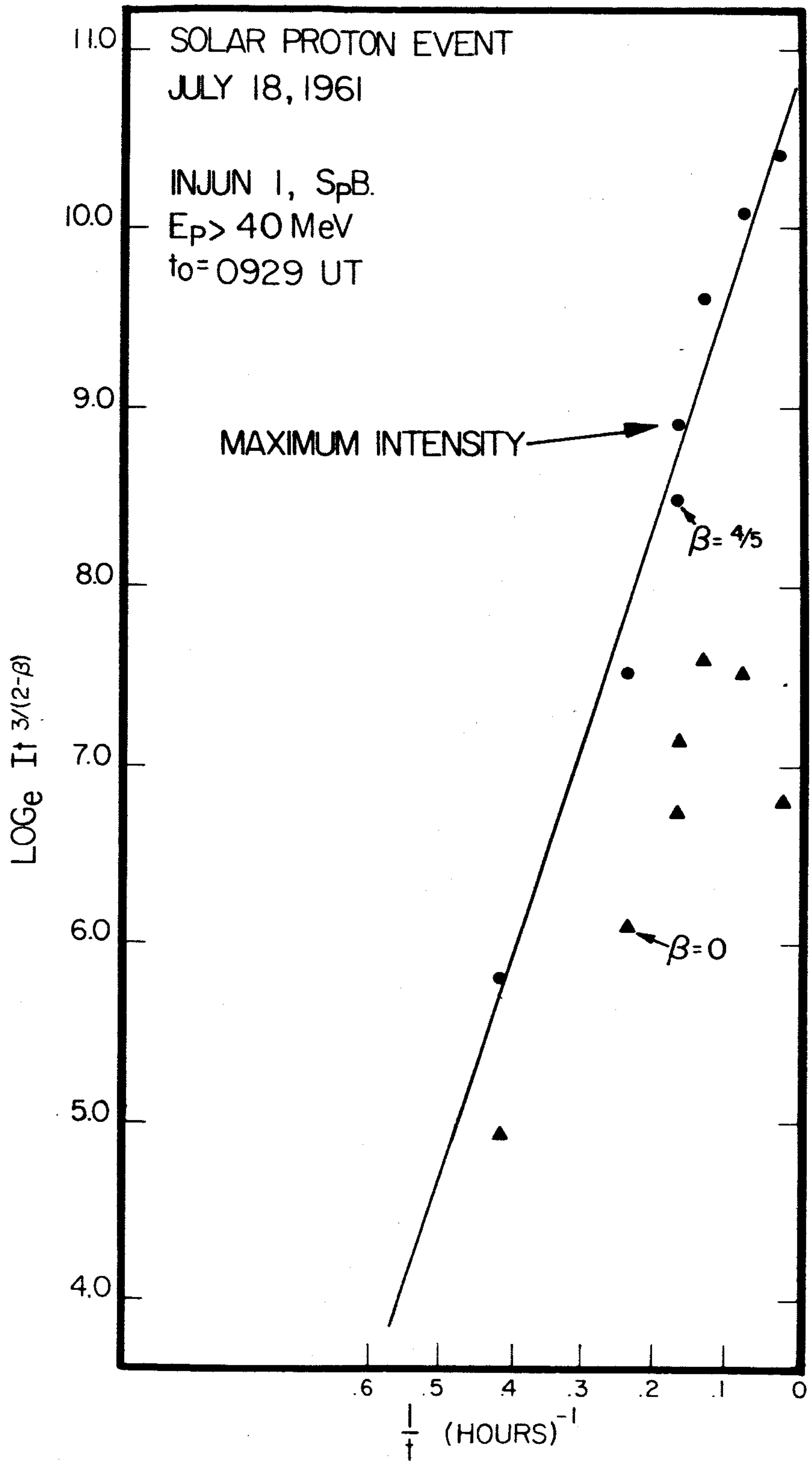
INTENSITY VS POSITION FOR
SEVERAL CHOSEN TIMES WITH α=2, β=1, M=0.142 A.U/hr.
t IN HOURS
100
10
1
.1
.01
.001
.0001
.00001
I (CM² SEC STERAD)⁻¹
1
2
3
4
5
6
7
r (A.U.)
t=20
t=10
t=50
t=4
t=3
t=2
t=100
t=1


FIGURE 3



SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER, 1961
$t_o$ = 2208 UT
Ep > 23 MeV
MAXIMUM INTENSITY
β = 1
β = 2/3
β = 1/2
β = 0
$\text{LOG}_e \; I t^{3/(2-\beta)}$
1/t (HOURS)$^{-1}$
2.0 1.9 1.8 1.7 1.6 1.5 1.4 1.3 1.2 1.1 1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1 0
11 10 9 8 7 6 5 4 3 2 1 0 -1 -2 -3 -4


FIGURE 4



SOLAR PROTON EVENT
JULY 18, 1961
INJUN 1, SpB.
EP> 40 MeV
to=0929 UT
MAXIMUM INTENSITY
β= 4/5
β=0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
LOGe It 3/(2-β)
.6
.5
.4
.3
.2
.1
0
1/t (HOURS)-1


FIGURE 5

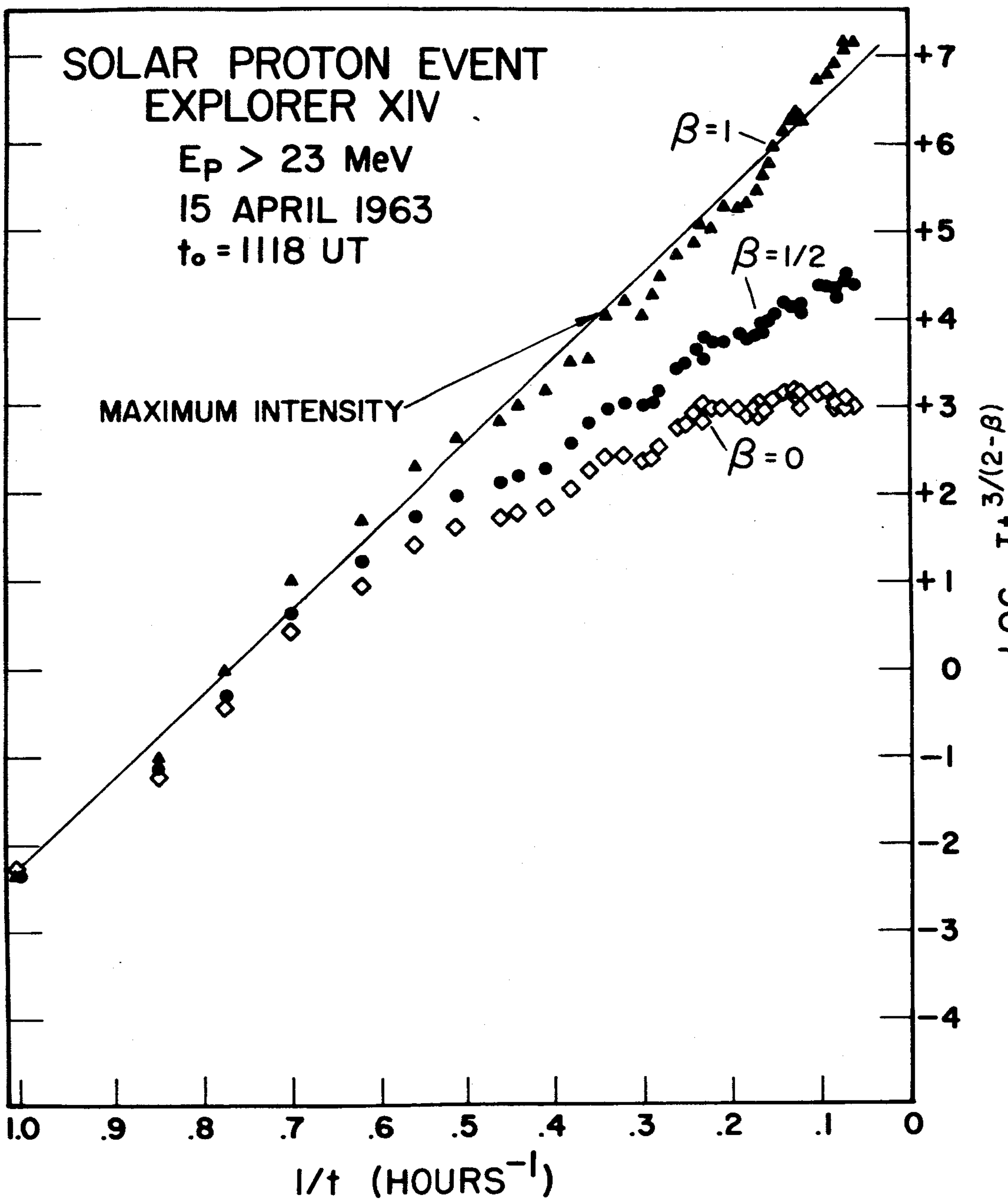
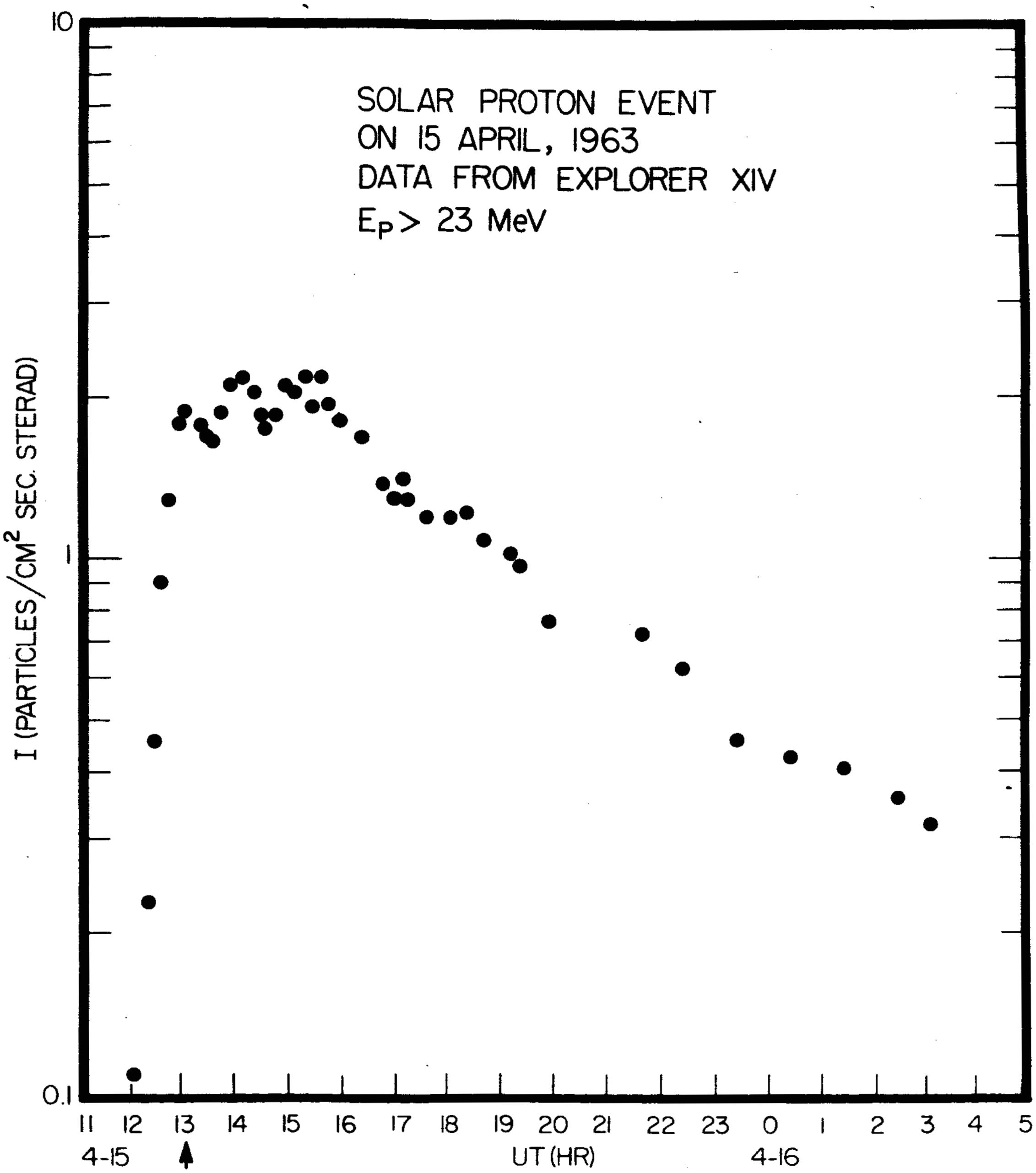
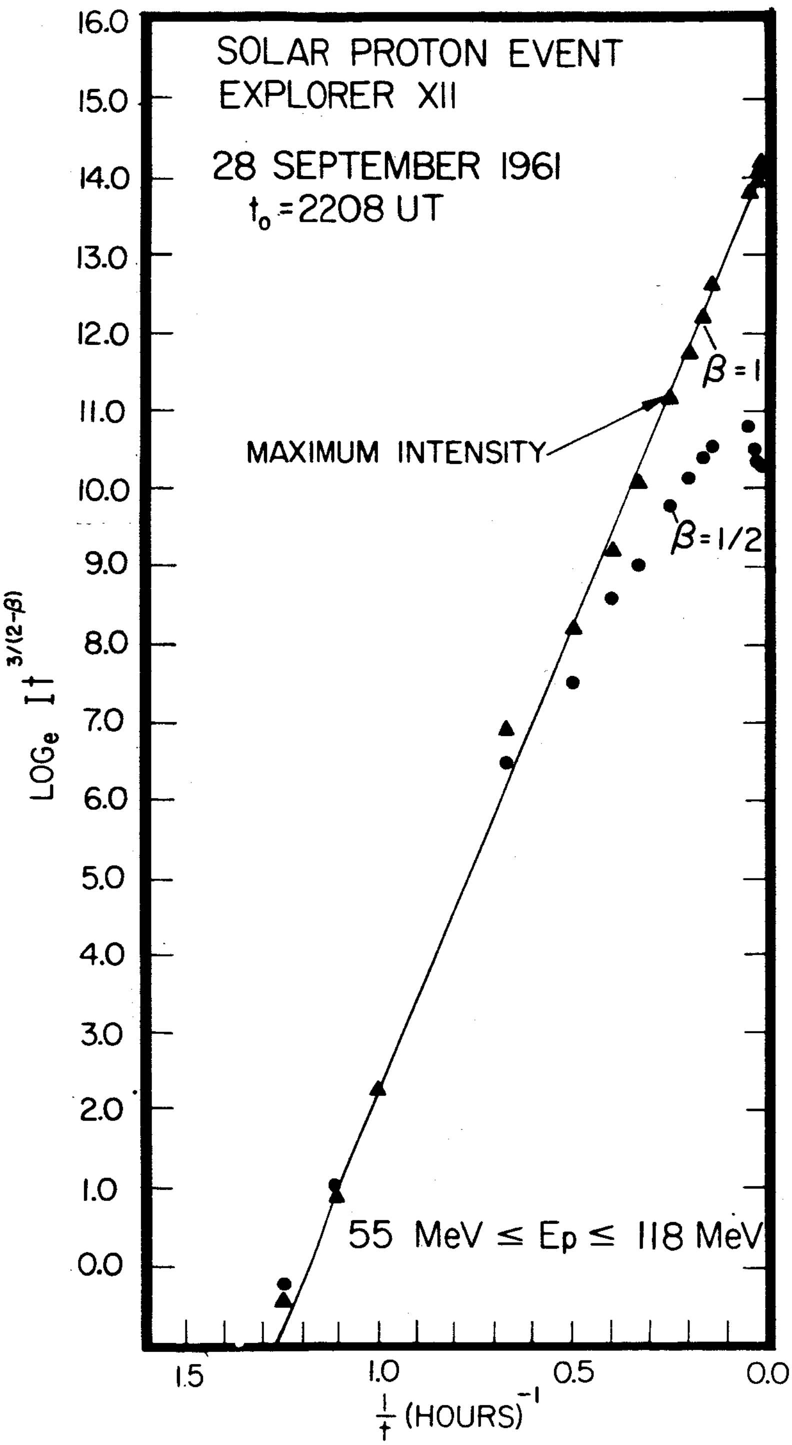

SOLAR PROTON EVENT
EXPLORER XIV
$E_P > 23$ MeV
15 APRIL 1963
$t_o$ = 1118 UT
MAXIMUM INTENSITY
β = 1
β = 1/2
β = 0
+7
+6
+5
+4
+3
+2
+1
0
−1
−2
−3
−4
$LOG_e \, It^{3/(2-\beta)}$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1/t (HOURS$^{-1}$)

FIGURE 6



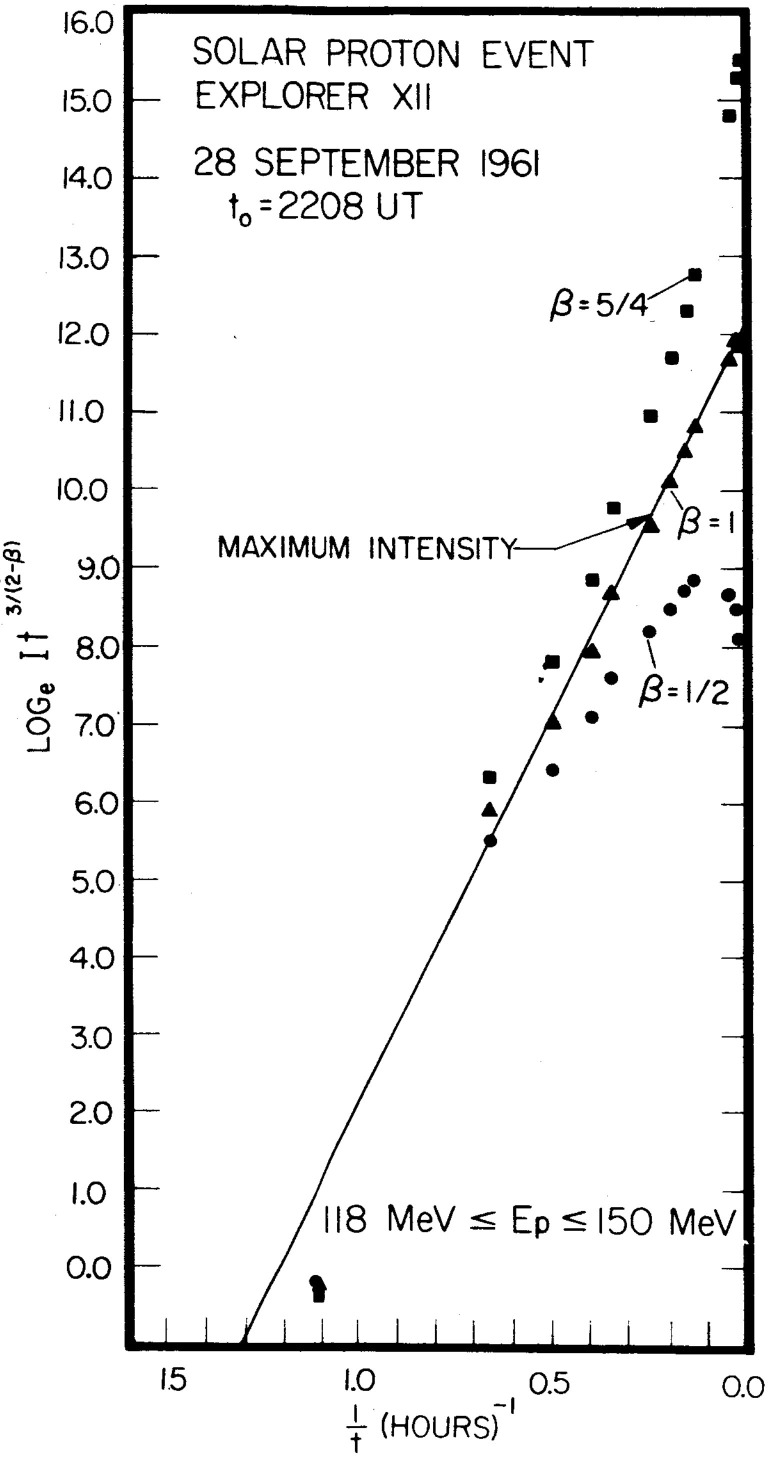
SOLAR PROTON EVENT
ON 15 APRIL, 1963
DATA FROM EXPLORER XIV
Ep > 23 MeV
I (PARTICLES/CM² SEC. STERAD)
10
1
0.1
11 12 13 14 15 16 17 18 19 20 21 22 23 0 1 2 3 4 5
4-15
UT (HR)
4-16


FIGURE 7



SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER 1961
$t_0$ = 2208 UT
$\beta$ = 1
MAXIMUM INTENSITY
$\beta$ = 1/2
55 MeV ≤ Ep ≤ 118 MeV
$\mathrm{LOG}_e\ I t^{3/(2-\beta)}$
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
1.5
1.0
0.5
0.0
$\frac{1}{t}$ (HOURS)$^{-1}$


FIGURE 8



SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER 1961
$t_o$ = 2208 UT
$\beta$ = 5/4
MAXIMUM INTENSITY
$\beta$ = 1
$\beta$ = 1/2
118 MeV ≤ Ep ≤ 150 MeV
$LOG_e\ I\,t^{3/(2-\beta)}$
$\frac{1}{t}$ (HOURS)$^{-1}$
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
1.5
1.0
0.5
0.0


FIGURE 9

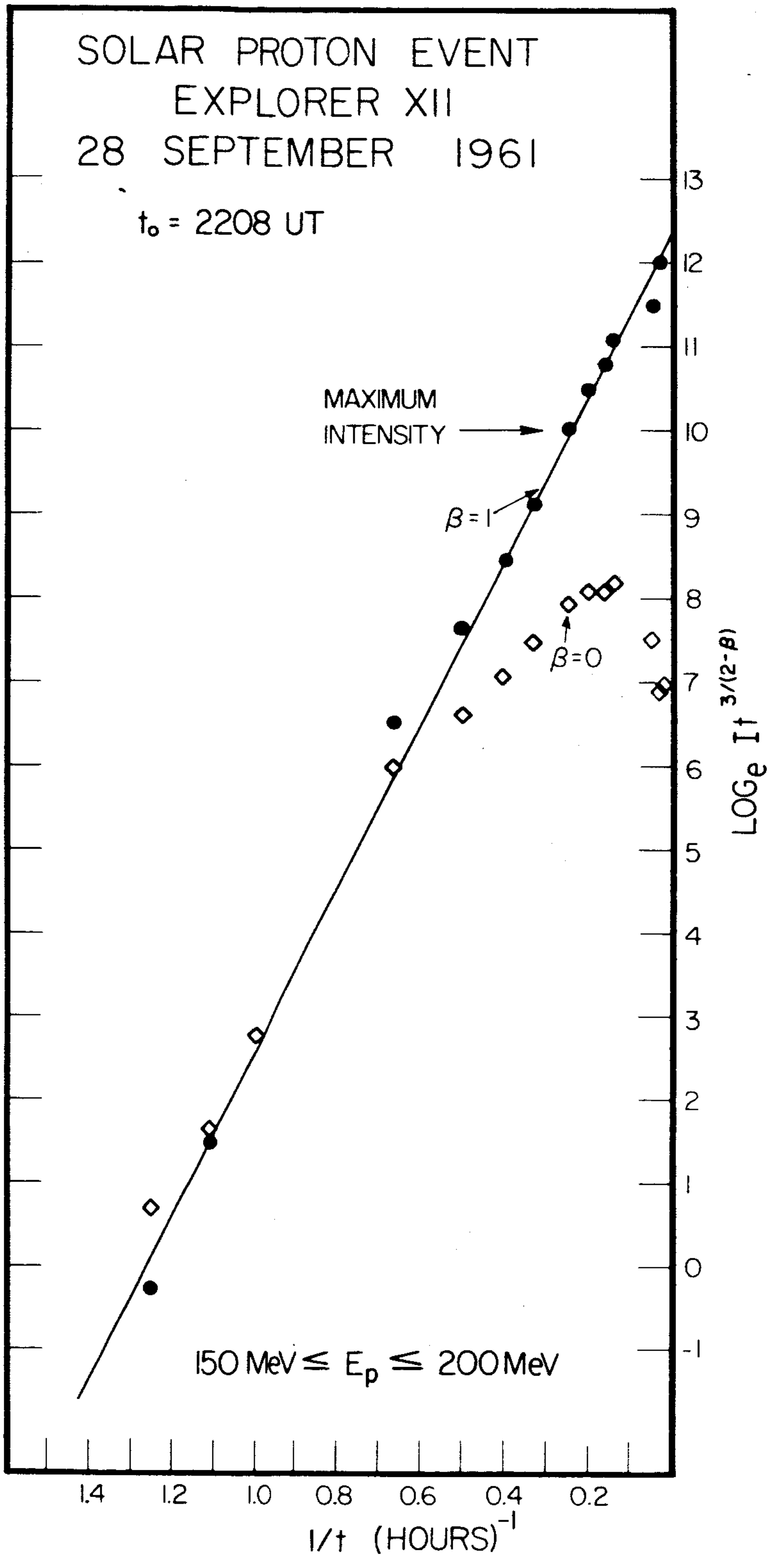
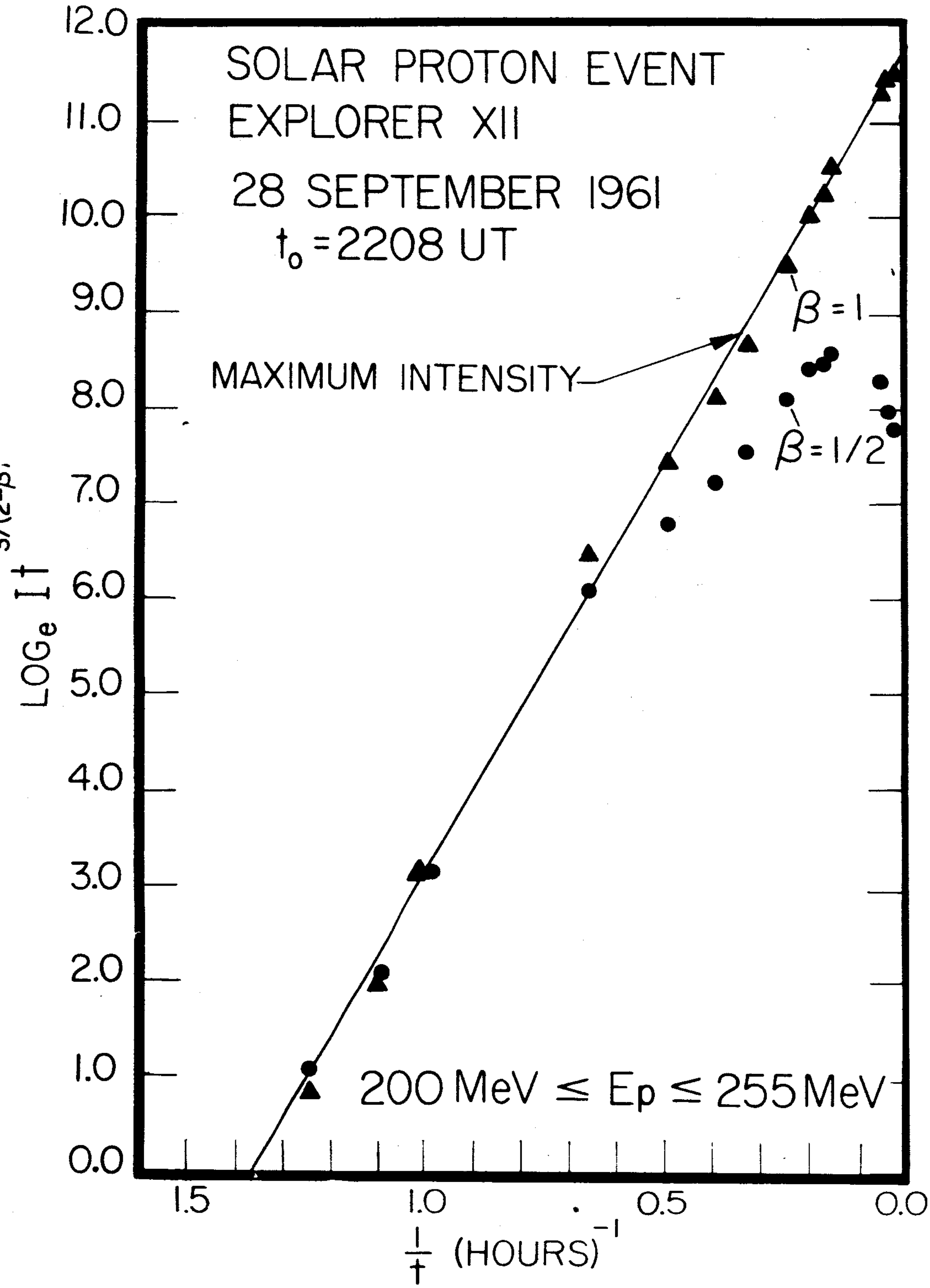
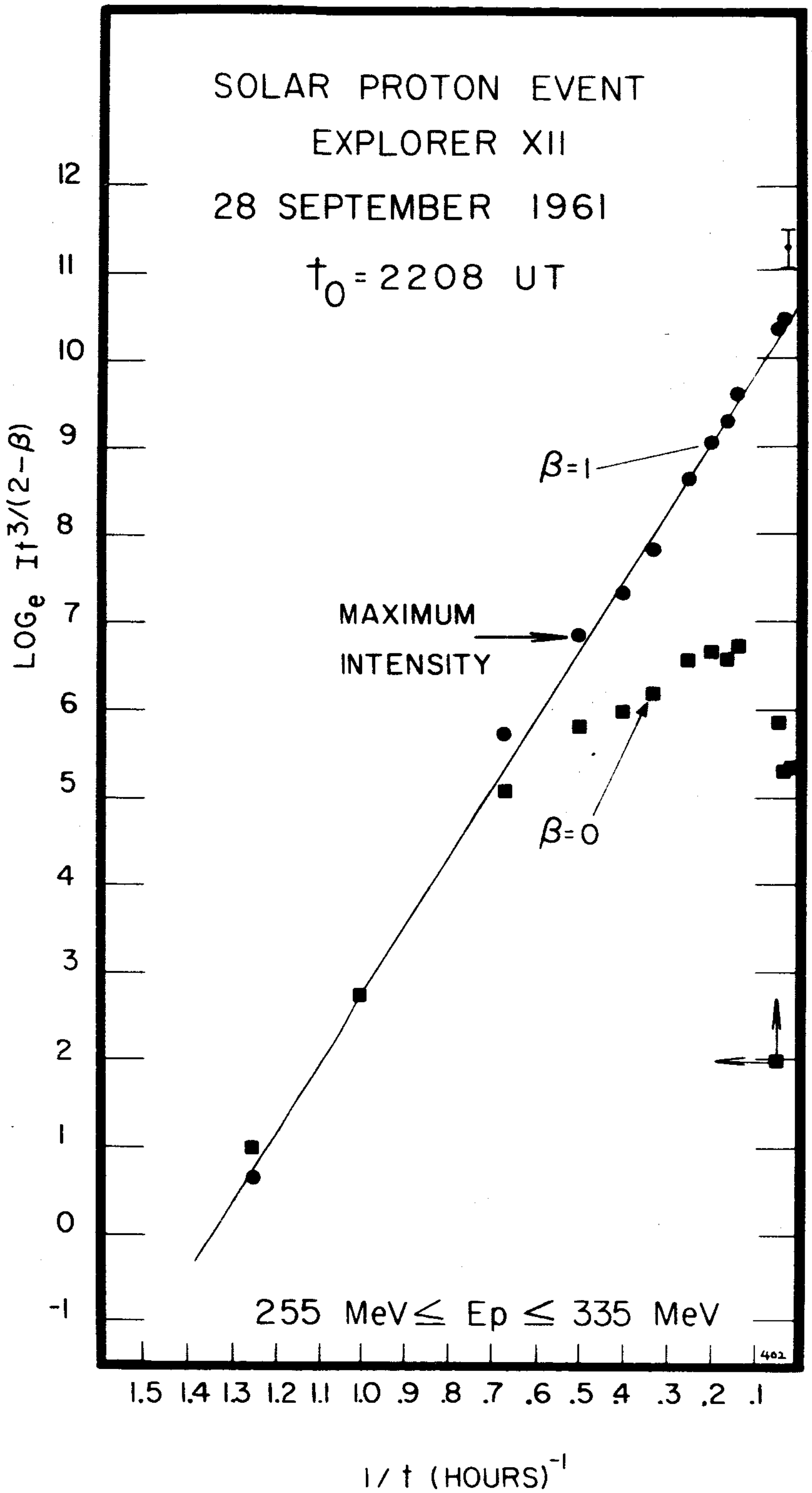

SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER 1961
t₀ = 2208 UT
MAXIMUM INTENSITY
β = 1
β = 0
150 MeV ≤ E_p ≤ 200 MeV
LOG_e I t^{3/(2-β)}
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
1.4
1.2
1.0
0.8
0.6
0.4
0.2
1/t (HOURS)^-1

FIGURE 10



SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER 1961
$t_0$ = 2208 UT
MAXIMUM INTENSITY
β = 1
β = 1/2
200 MeV ≤ Ep ≤ 255 MeV
$LOG_e\ I t^{3/(2-\beta)}$
$\frac{1}{t}$ (HOURS)$^{-1}$
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
1.5
1.0
0.5
0.0


FIGURE 11



SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER 1961
$t_0$ = 2208 UT
β=1
MAXIMUM INTENSITY
β=0
255 MeV ≤ Ep ≤ 335 MeV
LOG$_e$ It$^{3/(2-β)}$
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
1.5 1.4 1.3 1.2 1.1 1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1
1/t (HOURS)$^{-1}$


FIGURE 12

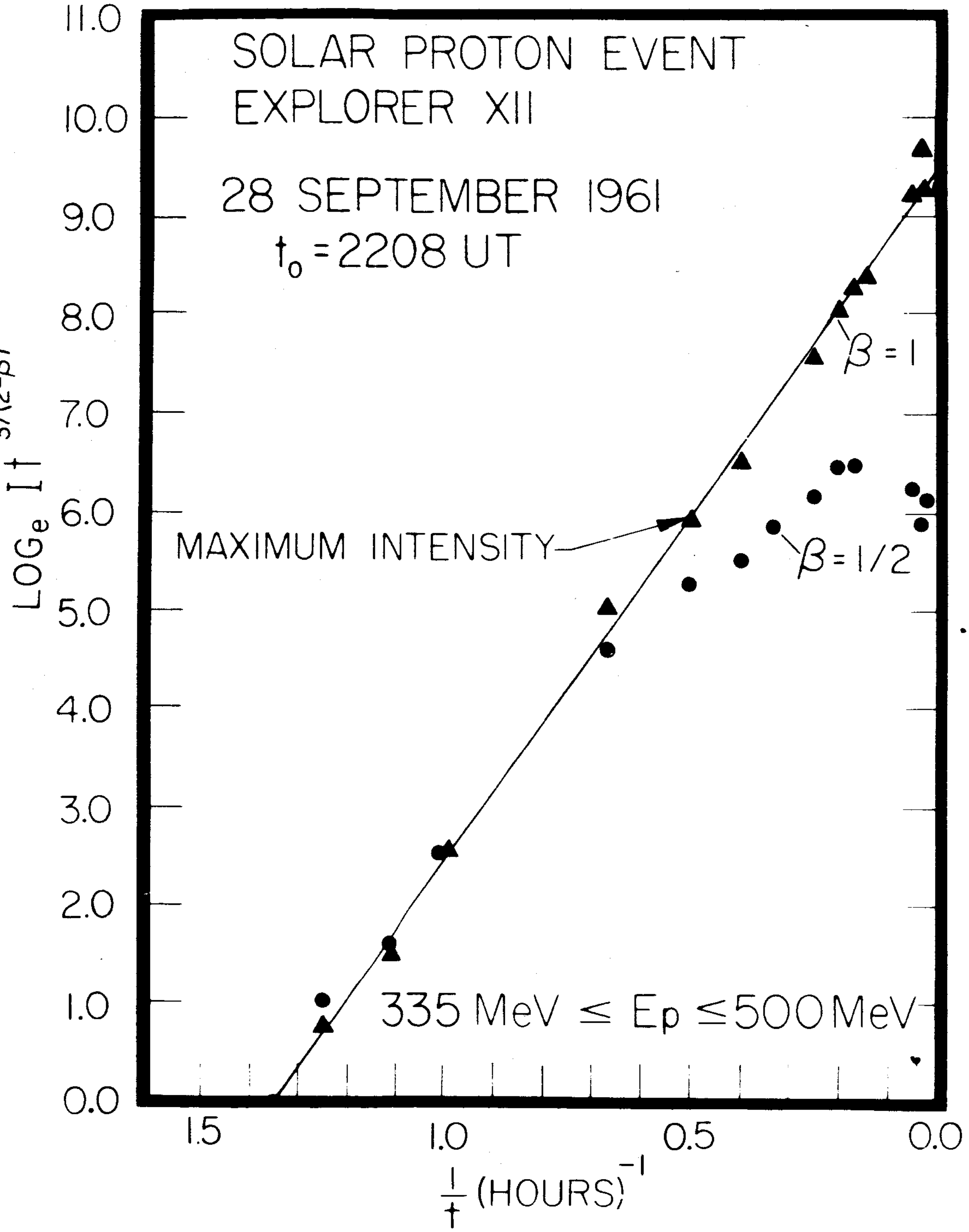
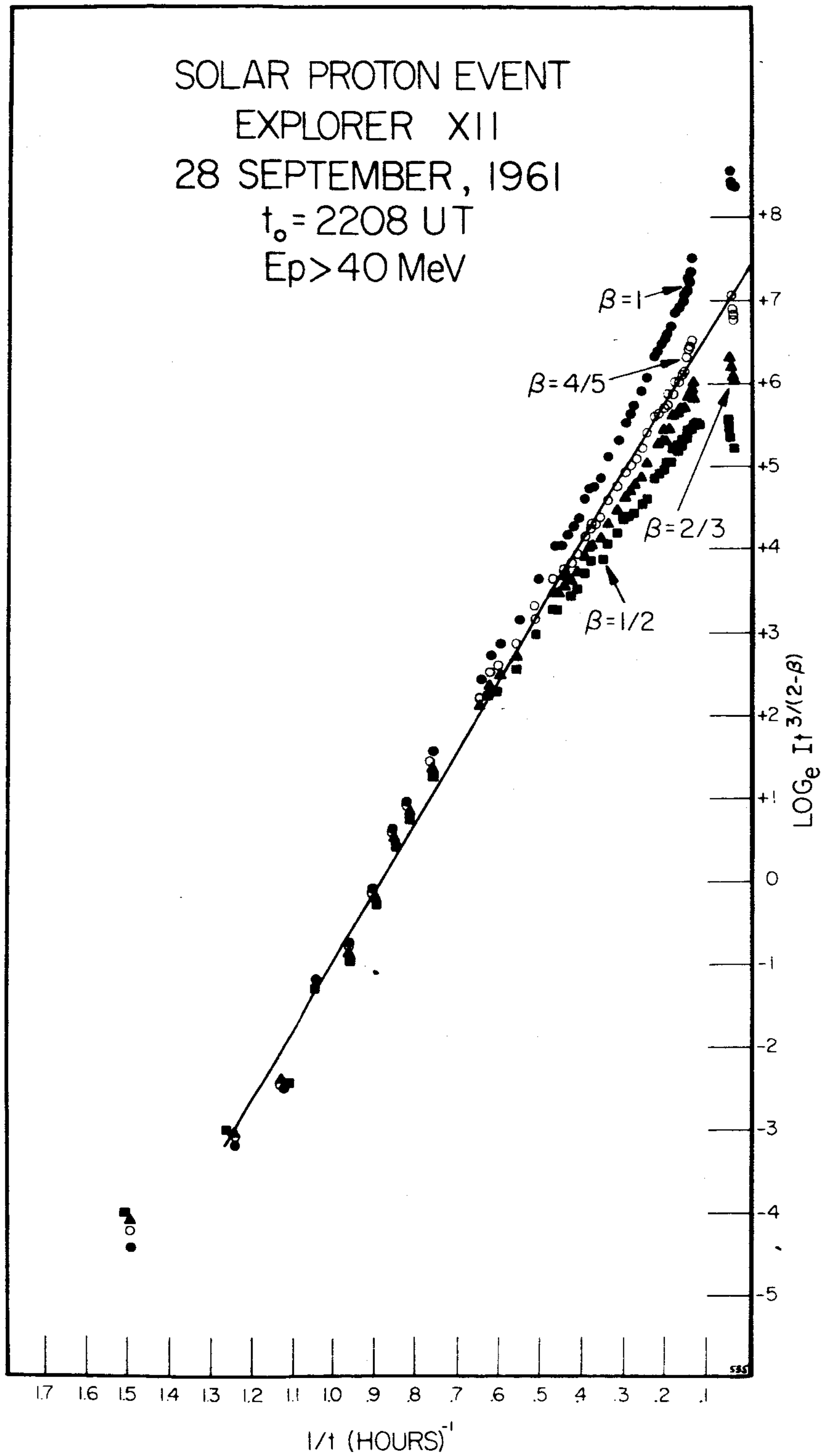
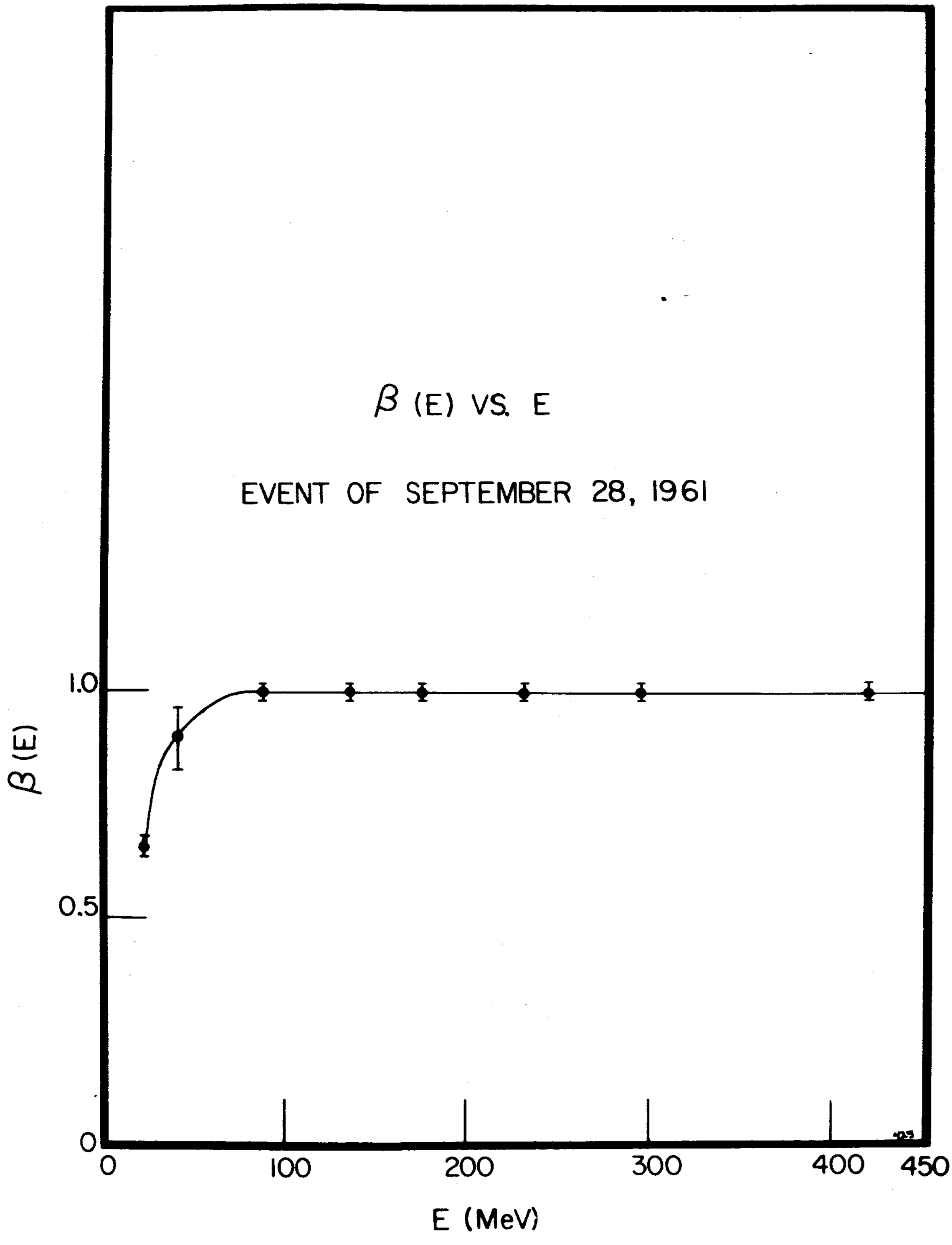

SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER 1961
$t_0$ = 2208 UT
MAXIMUM INTENSITY
β = 1
β = 1/2
335 MeV ≤ Ep ≤ 500 MeV
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
$LOG_e\ I\ t^{3/(2-\beta)}$
1.5
1.0
0.5
0.0
$\frac{1}{t}$ (HOURS)$^{-1}$

FIGURE 13



SOLAR PROTON EVENT
EXPLORER XII
28 SEPTEMBER, 1961
$t_o$ = 2208 UT
Ep>40 MeV
β=1
β=4/5
β=2/3
β=1/2
+8
+7
+6
+5
+4
+3
+2
+1
0
-1
-2
-3
-4
-5
$LOG_e\ I_t^{\ 3/(2-\beta)}$
1.7 1.6 1.5 1.4 1.3 1.2 1.1 1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1
1/t (HOURS)$^{-1}$


FIGURE 14



β (E) VS. E
EVENT OF SEPTEMBER 28, 1961
1.0
0.5
0
β (E)
0
100
200
300
400
450
E (MeV)


FIGURE 15

THE DIFFUSION COEFFICIENT AS A FUNCTION OF r FOR VARIOUS ENERGIES
THE DATA ARE FOR THE SEPTEMBER 28, 1961 EVENT

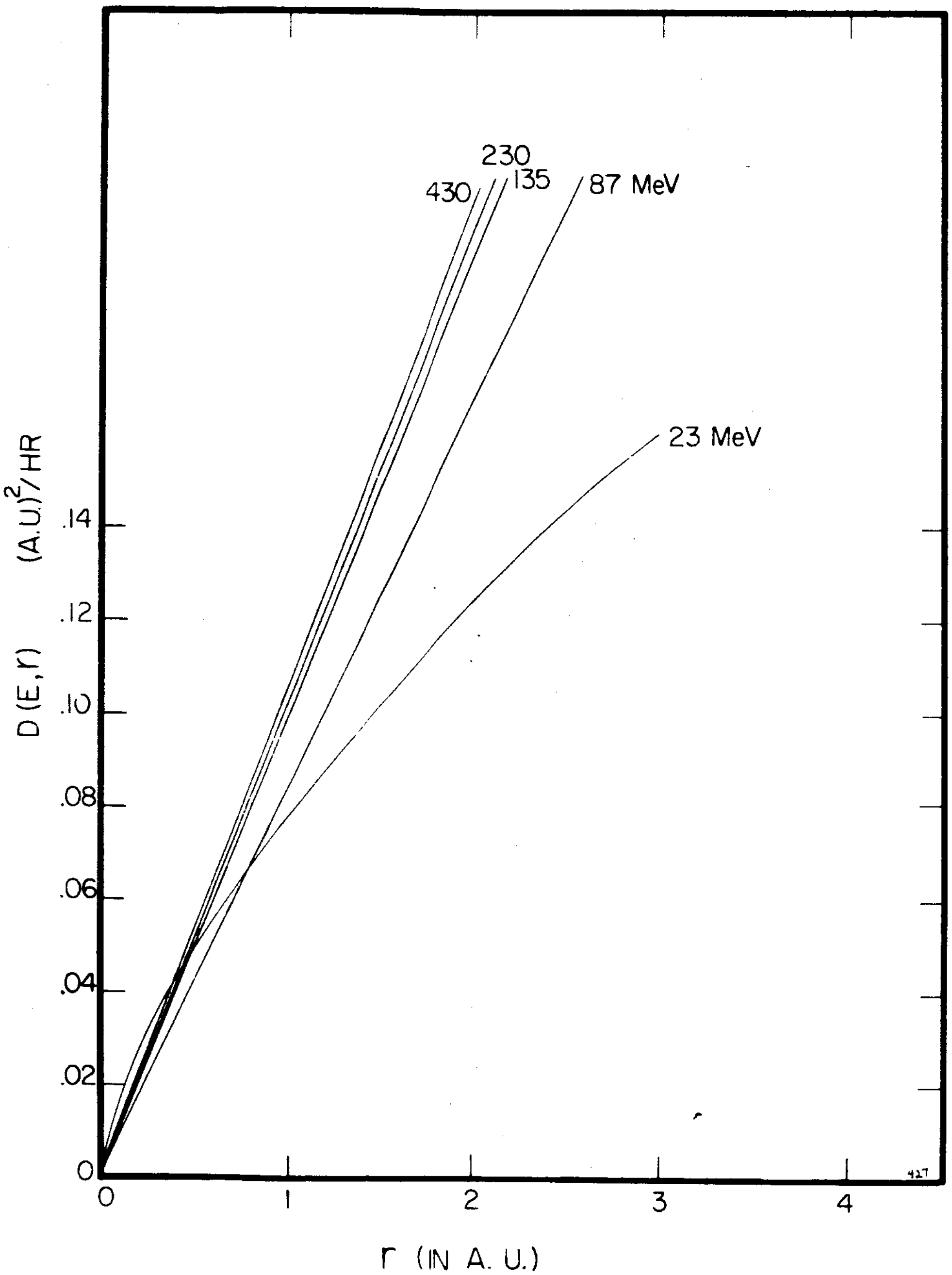



FIGURE 16

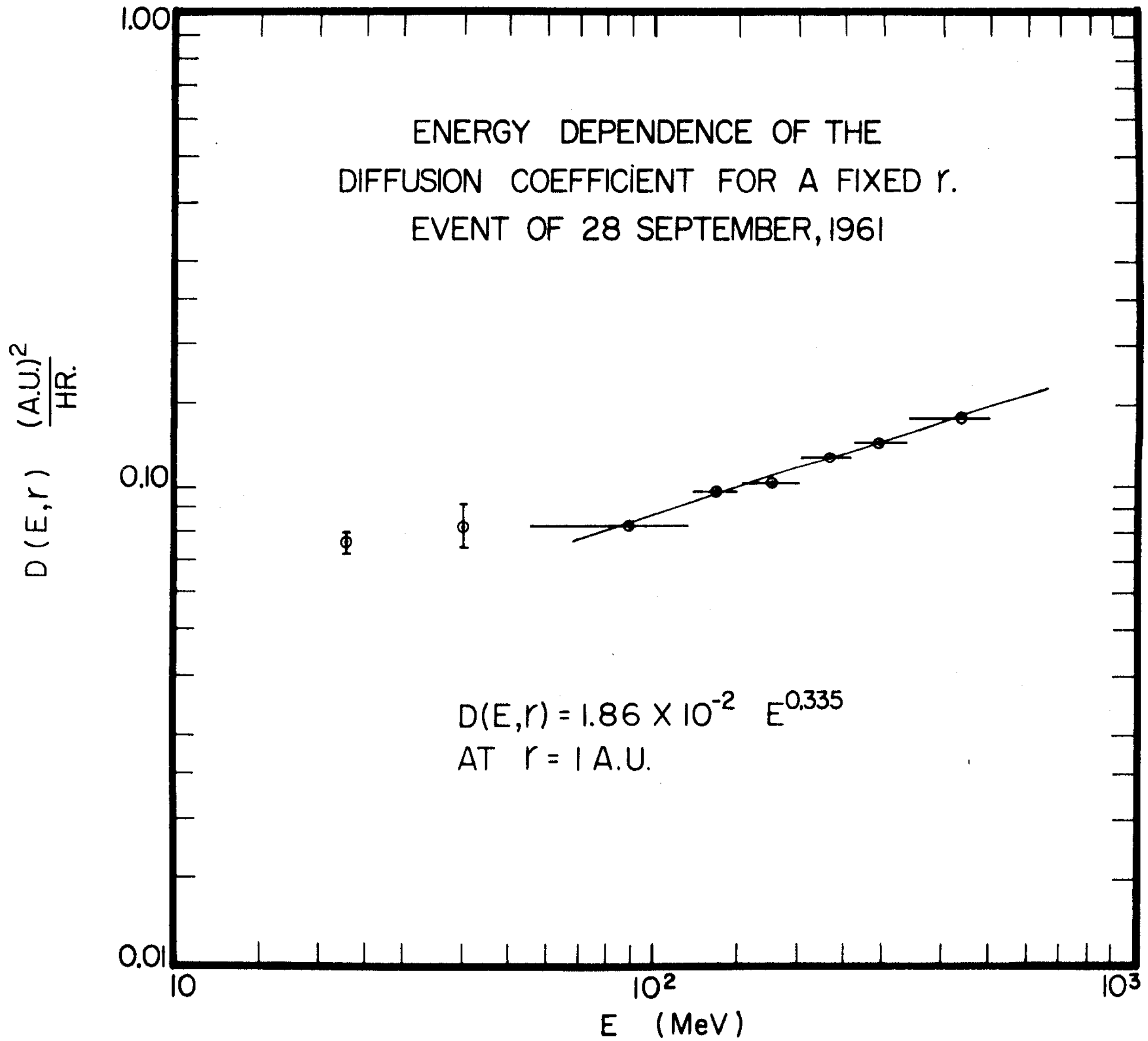
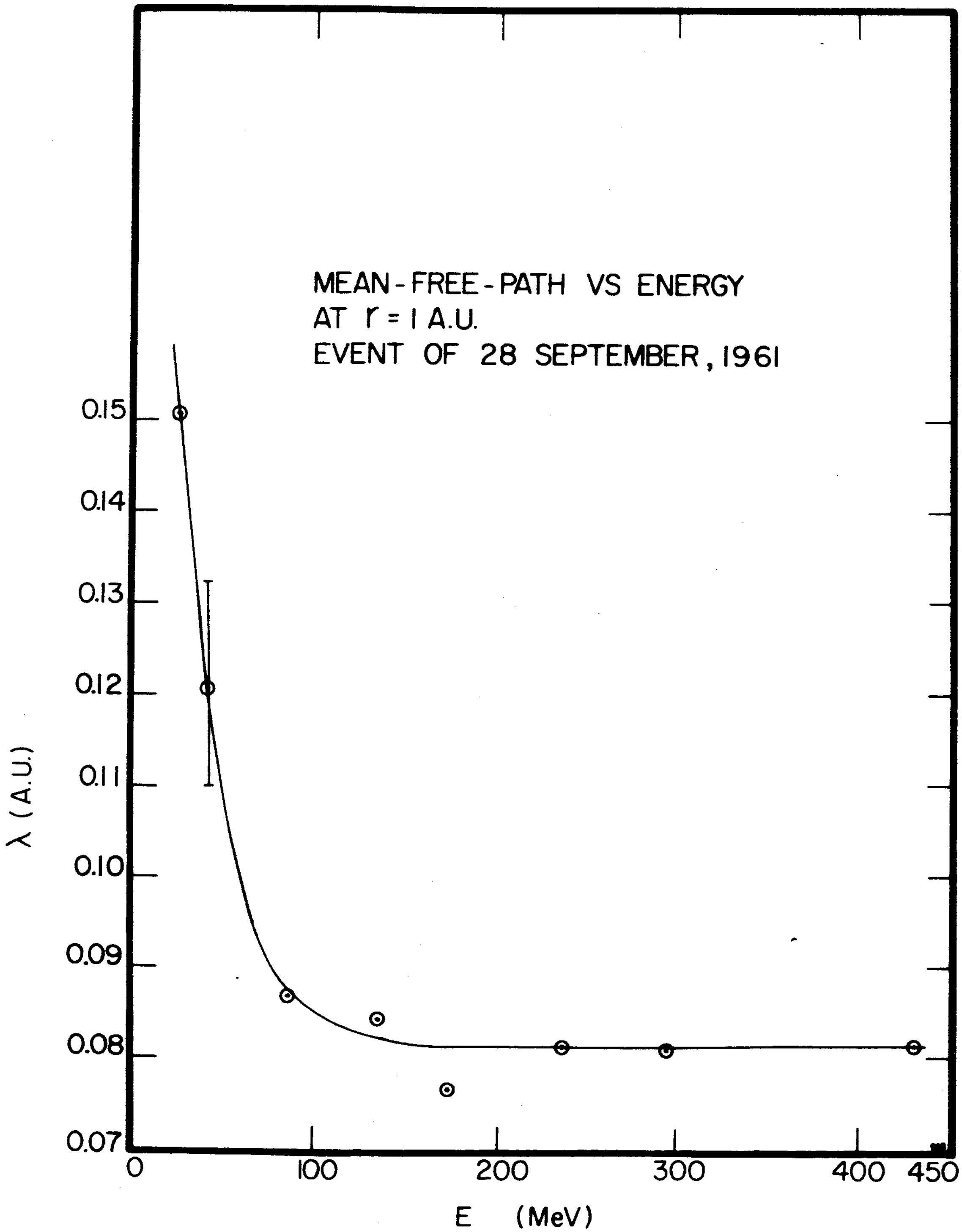

ENERGY DEPENDENCE OF THE
DIFFUSION COEFFICIENT FOR A FIXED r.
EVENT OF 28 SEPTEMBER, 1961
D(E,r) = 1.86 X 10^-2 E^0.335
AT r = 1 A.U.
D (E, r) (A.U.)^2 / HR.
1.00
0.10
0.01
10
10^2
10^3
E (MeV)

FIGURE 17



MEAN-FREE-PATH VS ENERGY
AT r = 1 A.U.
EVENT OF 28 SEPTEMBER, 1961
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
λ (A.U.)
0
100
200
300
400
450
E (MeV)


FIGURE 18

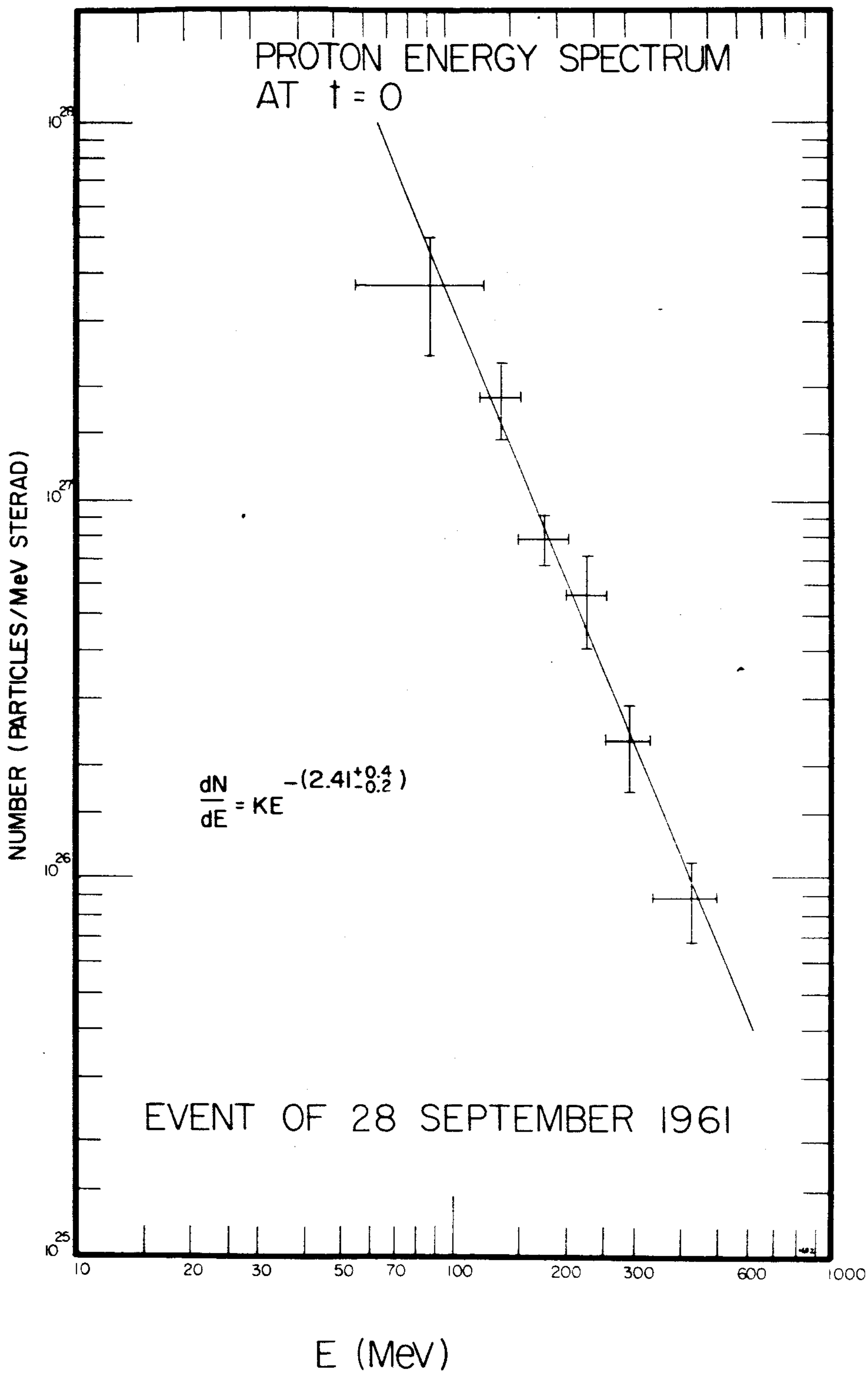
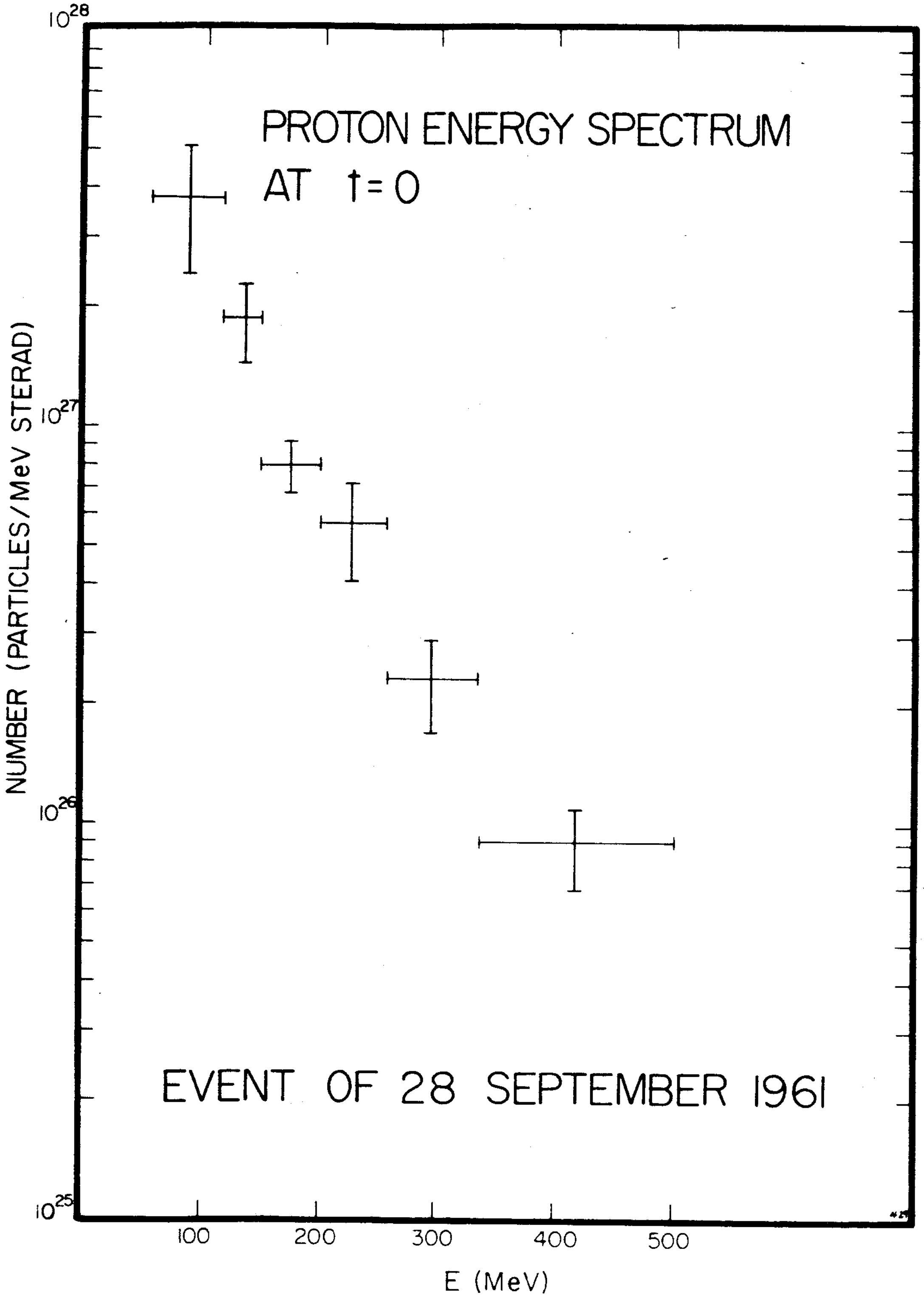

PROTON ENERGY SPECTRUM
AT t = 0
$\frac{dN}{dE} = KE^{-(2.41^{+0.4}_{-0.2})}$
EVENT OF 28 SEPTEMBER 1961
NUMBER (PARTICLES/MeV STERAD)
$10^{28}$
$10^{27}$
$10^{26}$
$10^{25}$
10
20
30
50
70
100
200
300
600
1000
E (MeV)

**FIGURE 19**



PROTON ENERGY SPECTRUM
AT t=0
EVENT OF 28 SEPTEMBER 1961
NUMBER (PARTICLES/MeV STERAD)
$10^{28}$
$10^{27}$
$10^{26}$
$10^{25}$
100
200
300
400
500
E (MeV)


**FIGURE 20**

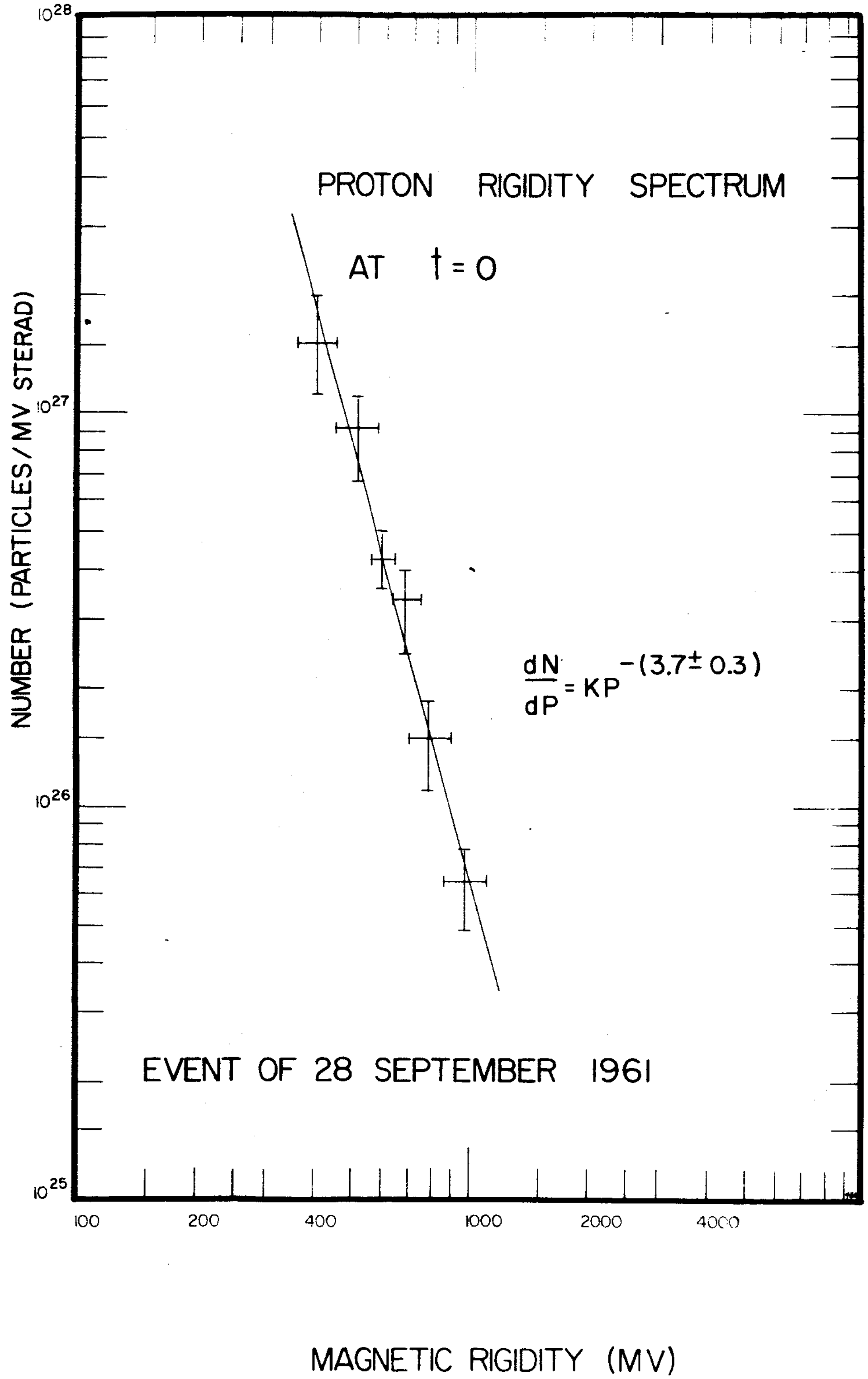

PROTON RIGIDITY SPECTRUM
AT t = 0
$\frac{dN}{dP} = KP^{-(3.7 \pm 0.3)}$
EVENT OF 28 SEPTEMBER 1961
NUMBER (PARTICLES / MV STERAD)
$10^{28}$
$10^{27}$
$10^{26}$
$10^{25}$
100
200
400
1000
2000
4000
MAGNETIC RIGIDITY (MV)

**FIGURE 21**

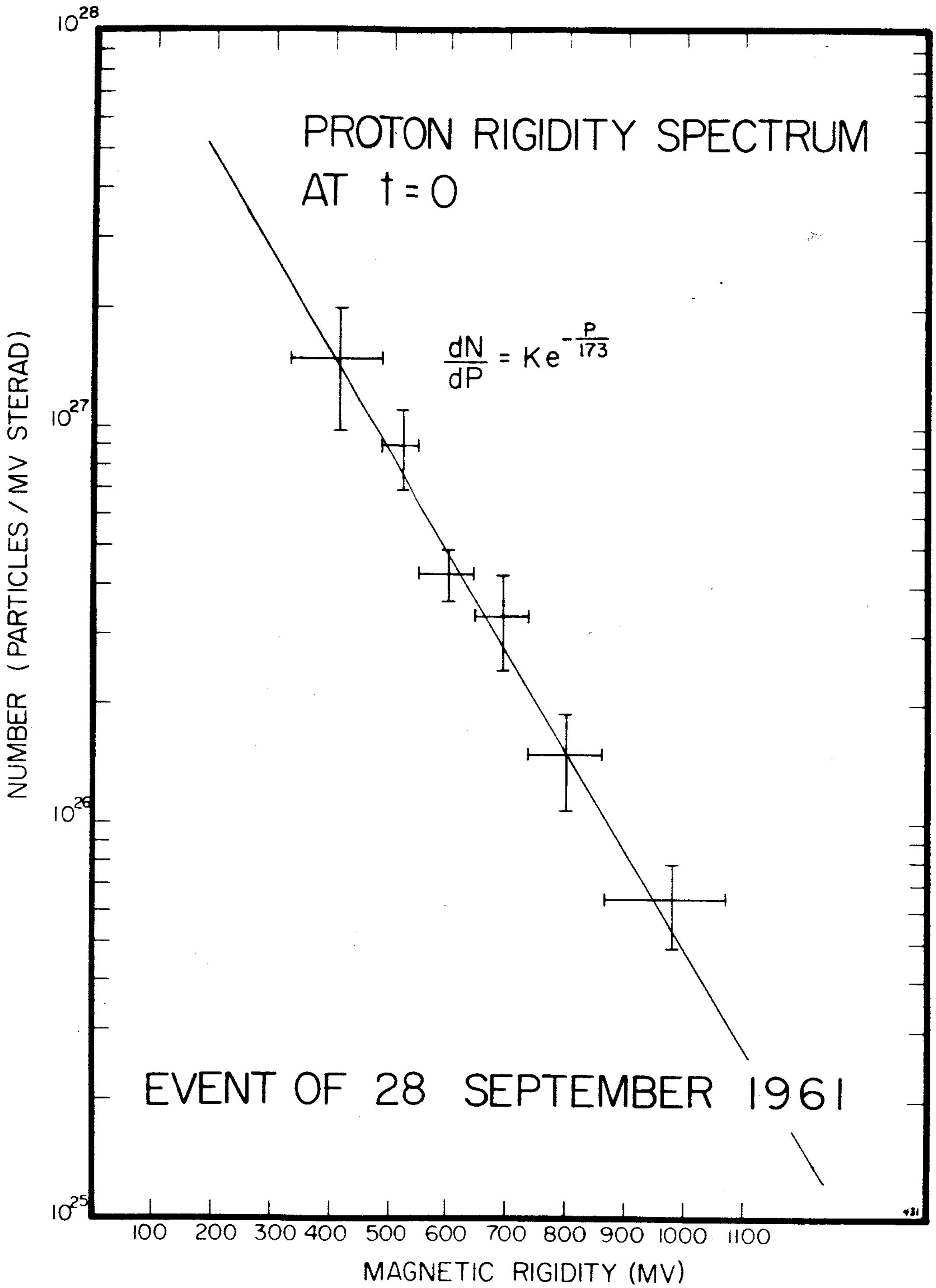

PROTON RIGIDITY SPECTRUM
AT t = 0
$\frac{dN}{dP} = Ke^{-\frac{P}{173}}$
EVENT OF 28 SEPTEMBER 1961
$10^{28}$
$10^{27}$
$10^{26}$
$10^{25}$
NUMBER (PARTICLES / MV STERAD)
100 200 300 400 500 600 700 800 900 1000 1100
MAGNETIC RIGIDITY (MV)

**FIGURE 22**

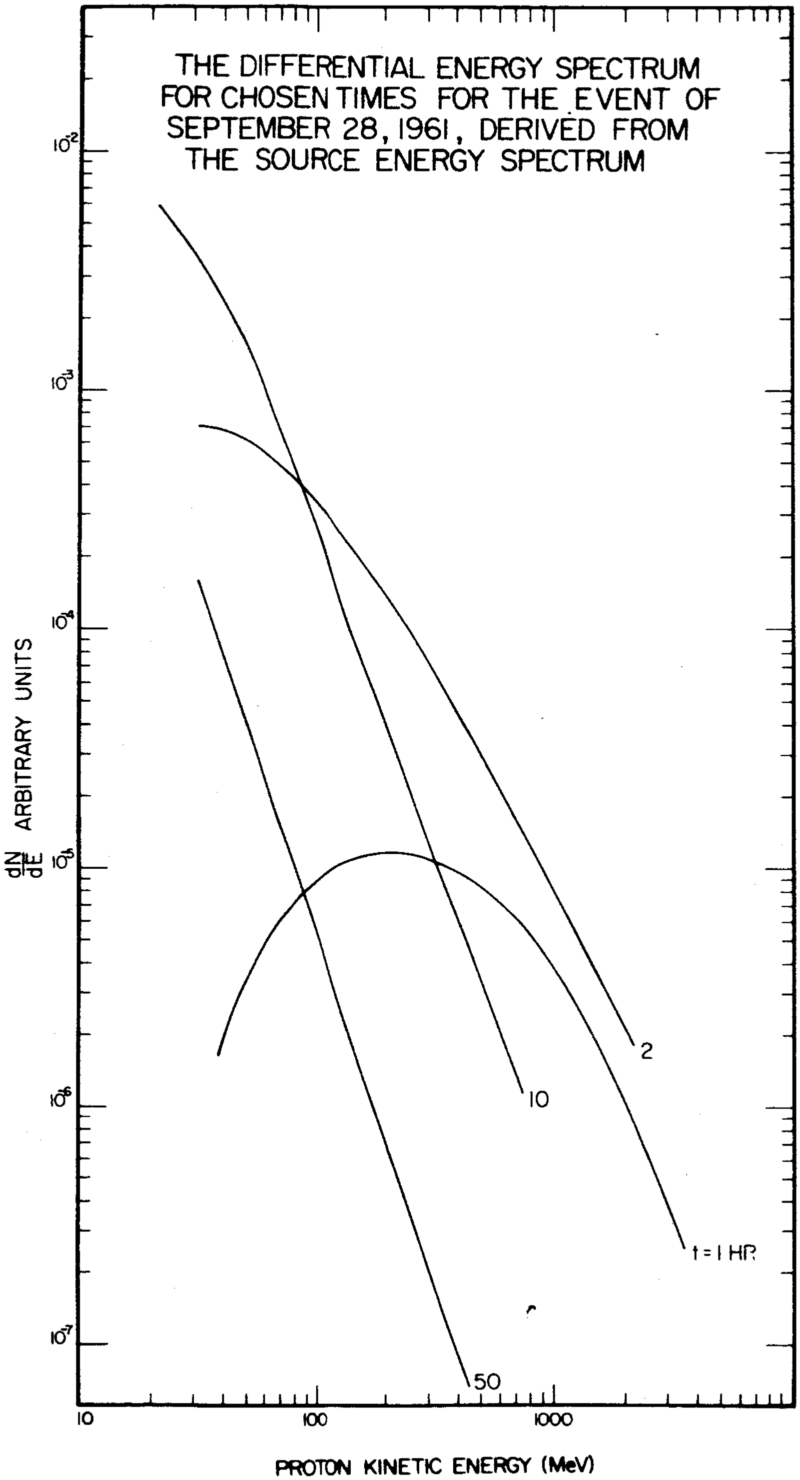

THE DIFFERENTIAL ENERGY SPECTRUM
FOR CHOSEN TIMES FOR THE EVENT OF
SEPTEMBER 28, 1961, DERIVED FROM
THE SOURCE ENERGY SPECTRUM
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
dN/dE ARBITRARY UNITS
2
10
t = 1 HR
50
10
100
1000
PROTON KINETIC ENERGY (MeV)

FIGURE 23