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APOLLO EXTENSION SYSTEM STUDIES REPORT ON

NAVIGATION SYSTEM FOR A MOBILE LABORATORY

Prepared under Contract No. NAS8-11096 by

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NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

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R-ASTR-A
ASTRIONICS LABORATORY

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ABSTRACT

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A typical dead reckoning system utilizing a vehicle attitude and heading reference package is studied. The distance information is resolved into coordinate axes which yield vehicle position. System error models are developed which provide equations for position error versus time and distance. Constant velocity operation is assumed in the analysis however the error equations may be developed for any velocity profile where vehicle velocity is expressed as a function of time. A linear, single degree of freedom vertical loop, and a free directional gyro reference are the principal items studied in the error analysis. Component errors and vehicle disturbances are described and their effect presented in a mathematical relationship.

An error analysis, where typical vehicle parameters are assumed, is shown, with results presented for both rotational errors of the coordinate system and for resultant position errors.

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SECTION 1.0

INTRODUCTION

This report is a summary of the study effort during Task Order N-47. The Scope of Work was to include an analysis of a dead reckoning navigation system for a lunar roving vehicle. To provide the required information a series of items are studied, relating to the use of a stabilized platform as a basic heading and attitude reference device. The principal part of the report is a description of the system studied and an analysis of the errors obtained from the use of this system. Results show a cross track error of 300 meters and an altitude error of 70 meters for a mission time of six hours and for typical velocities of 5 to 10 kilometers per hour.

The dead reckoning equipment is to be utilized by the vehicle driver during portions of lunar travel between position fixes. The guidance problem is not considered here inasmuch as guidance of the vehicle will be achieved primarily by visual observations for both the manned and the unmanned missions. The dead reckoning system is to be utilized intermittently by the driver to indicate his postion on a lunar chart. This relation is used to provide an indication of his position in relation to the terrain, which is the primary pilotage data source.

SECTION 2.0

SYSTEM REPRESENTATION

A dead reckoning navigation system for a land vehicle utilizes distance and direction to compute present position in relation to a known starting point. The distance measurement is resolved into components along some selected set of axes. Since the driving of a vehicle along an arbitrary path over rough terrain involves frequent changes in heading, as well as pitch angle (as referred to the isogalic plane), it is necessary that distances along the coordinate axes be the result of an accumulation of incremental distances.

The reference coordinate system will be a selenographic fixed location system as shown in Figure 1. The dead reckoning system will utilize starting coordinates consisting of a latitude and longitude reading. Distance traversed is to be related back to this starting point. In Figure 1 the XYZ coordinates represent the inertial reference frame. The $x_t y_t z_t$ system represents the selenographic system in which the vehicle navigates. The $x_t y_t z_t$ system is rotating about the Z axis of the inertial system with angular velocity Ω.

Now if we consider the vehicle moving on the surface of a sphere of radius R, which is assumed to remain constant, then the vehicle is located at any time by coordinates α, β where α represents the latitude change and β, the longitude change from the reference point. The mechanism for obtaining α and β is contained within the vehicle. α and β are of course, equal to x_t/R and y_t/R where:

- x_t is the distance along an arc of the great circle path in the direction the vehicle is traveling.
- y_t is the distance along the y_t direction, or the arc length of a direction transverse to the path of the vehicle.
- R is the radius of the body.

The orthogonal coordiante system x_t, y_t, z_t is constrained to the surface of the body hence, everywhere on the body the z_t axis is directed along the radius vector. The system measures the rotation of the x_t, y_t, z_t system from a inertial set at the starting point. Hence the object of the dead reckoning system studied herein is to yeld the x_t, y_t, and z_t distances along the axes. Since these resolved distances are the result of an accumulation of incremental distances along the path they can be expressed mathematically as:

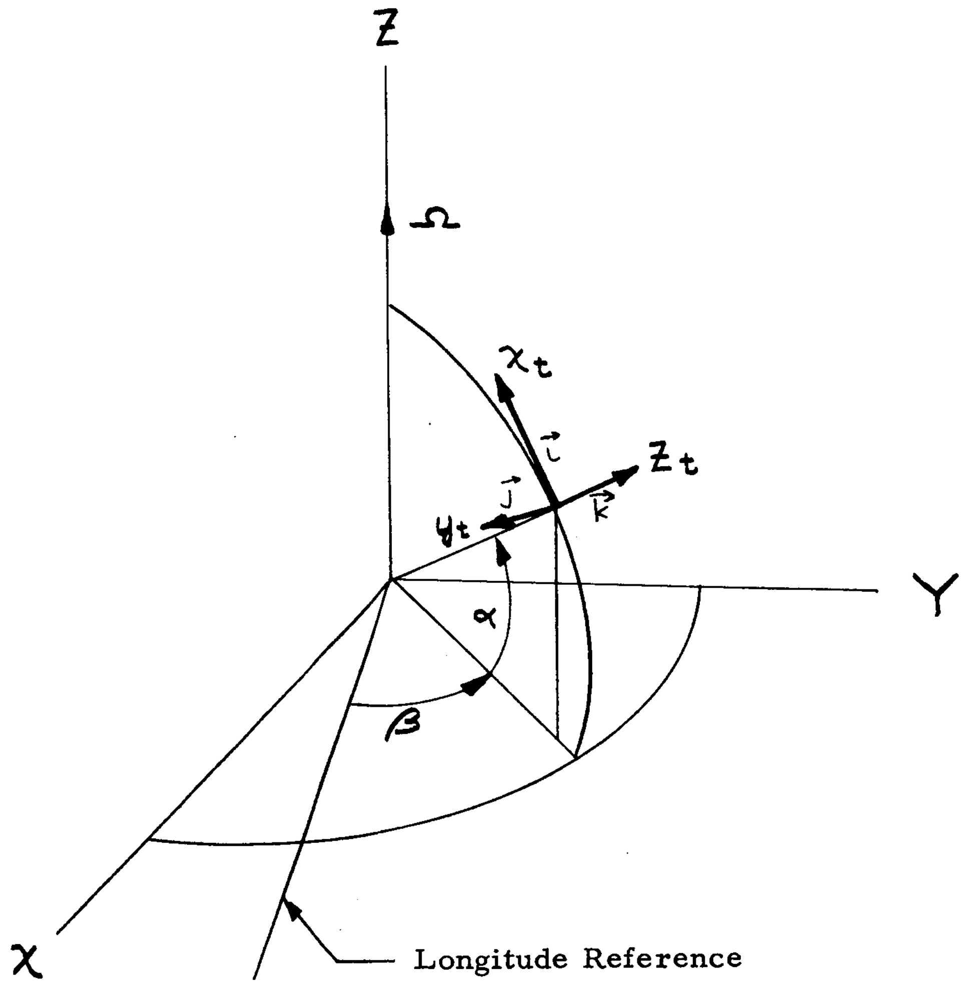


FIGURE 1
REFERENCE COORDINATE SYSTEM

$$x_t = \int (\cos \psi)(\cos p)\, ds$$

$$y_t = \int (\sin \psi)(\cos p)\, ds$$

$$z_t = \int (\sin p)\, ds$$

where

ψ = heading of vehicle with respect to north

p = pitch of the vehicle relative to an isogalic plane

ds = incremental path length

One method of mechanizing these equations is shown in block diagram in Figure 2. Such a mechanization would utilize vehicle velocity as an input and perform the integration with respect to time. The outputs, when converted to angles α and β can be used to provide the latitude and longitude coordinates.

2.1 MEASUREMENT OF ANGLES

Assume that a coordinate system is established as shown in Figure 3. In this diagram x_v is directed along the longitudinal axis of the vehicle, y_v is along the traverse axis, and z_v toward the vehicle's zenith.

The relationship of the vehicle system to the selenographic, t, system is given by the Euler angles which relate the transformation between coordinate systems. This is shown in Figure 4.

The vehicle relationship to the selenographic system is given by an azimuth angle, a, a roll angle, r, and a pitch angle, p.

Figure 5 represents a line diagram of a gimbal arrangement for measurement of vehicle pitch angle with respect to the vehicle coordinate system. The vehicle will pitch and roll in its own coordinate system x_v, y_v, z_v. To obtain the desired pitch angle with respect to the reference system x_t, y_t, z_t the readouts of the synchros are fed through an appropriate transformation network.

The measurement of the azimuth angle requires an additional gimbal. Azimuth is the angle between the x_t and the x_a axis. A

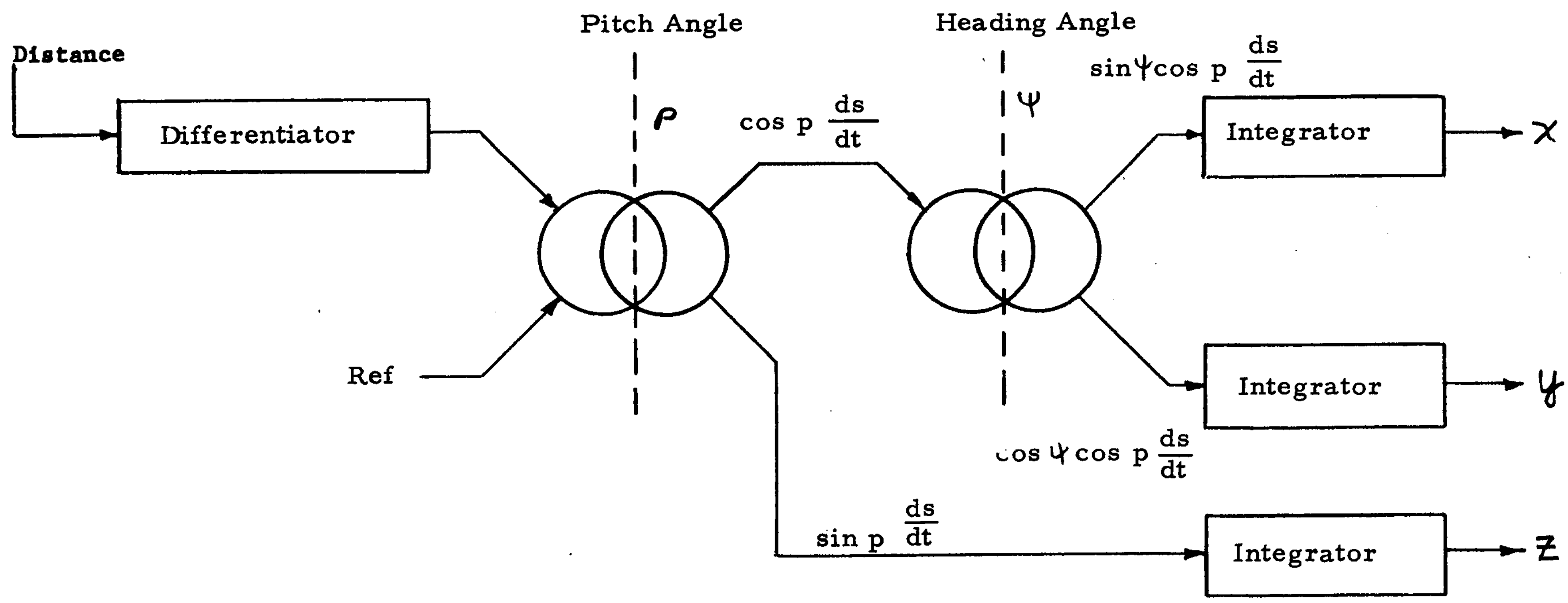


FIGURE 2

DEAD RECKONING COMPUTER MECHANIZATION

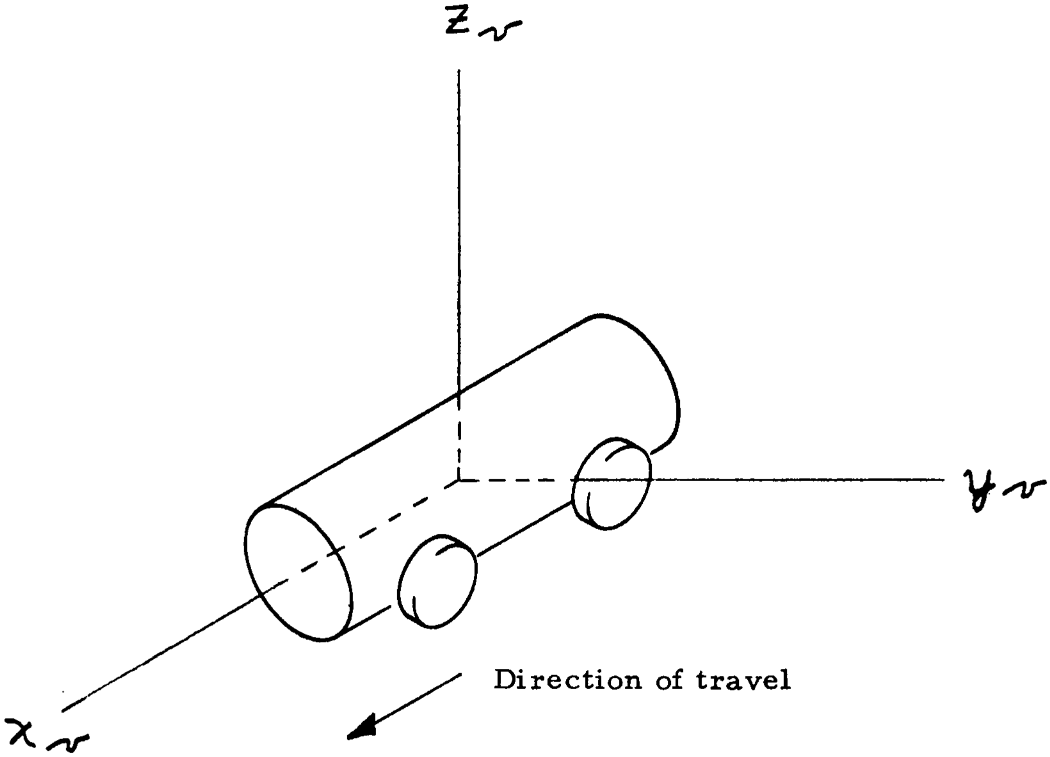
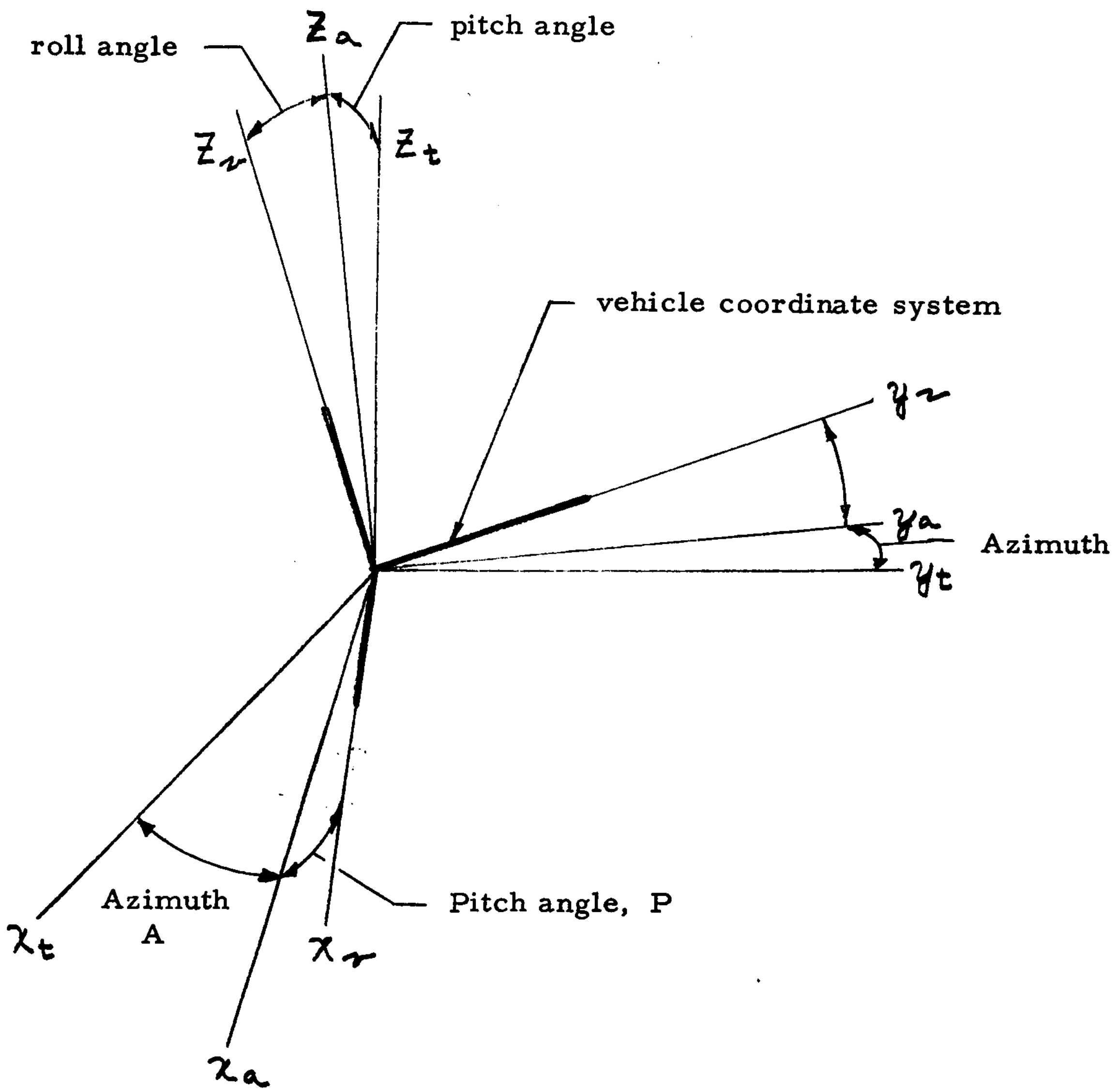


FIGURE 3

VEHICLE COORDINATE SYSTEM



1st Rotation about z_t angle A
2nd Rotation about y_a angle P
3rd Rotation about x_v angle R

FIGURE 4

ANGULAR RELATIONSHIP OF COORDINATE SYSTEMS

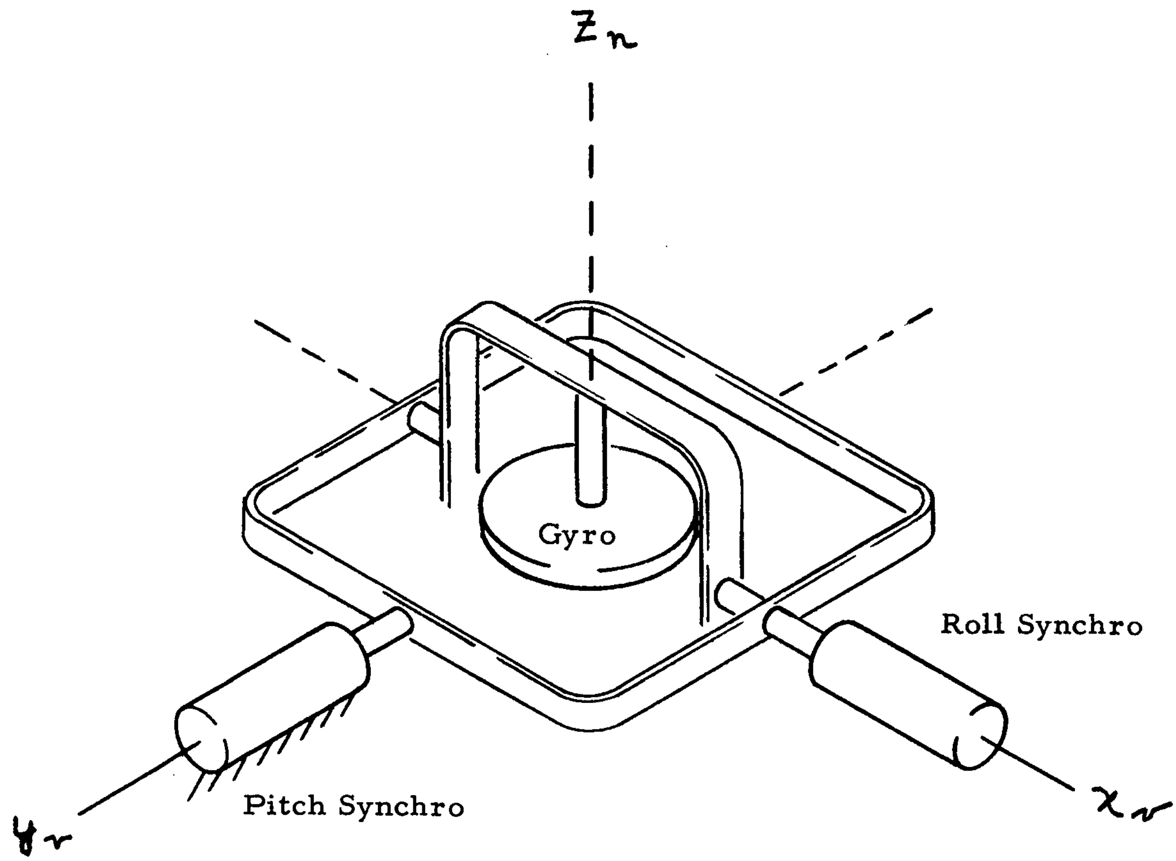


FIGURE 5

TYPICAL GIMBAL SYSTEM

directional gyro or platform will normally be aligned toward north and measurements of the vehicle direction are made with respect to this reference. The directional gyro may be a separate instrument, or may be incorporated in a three axis platform. The required measurements are then the distance, the pitch angle, and the heading angle. If these quantities could be determined accurately then the mechanization of Figure 2 would yield the correct location of the vehicle. Errors in the measured quantities will cause the computed position to be in error, the total system error being the r. s. s. of the errors in measurement due to all the errors sources, each taken individually. The classification of errors to be considered is given in the next section.

SECTION 3.0

CLASSIFICATION OF ERRORS

The three error source quantities d_r, dP, and $d\psi$ may each be thought of as consisting of several factors which may be categorized into groups. Each error source is then to be classified as one of the following types.

Systematic Error. An error that is a function of a system parameter susceptible to calibration and removal either at the initial alignment or at a later date.

Noise Error. An error that occurs in a random fashion or the source of which is of a random nature.

Bias Error. An error which is non-varying over short periods of time but which cannot be compensated for.

The classification of the errors and the listing of errors sources for a navigation system is given in Table 1.

Distance errors are not considered in this report inasmuch as information required to perform such an analysis is not available. For purposes of completness the estimate for accuracy of distance is taken as one percent of the distance traveled. Odometer systems in use by the U. S. Army are reported to have a system accuracy of this magnitude (Reference 1).

TABLE 1
CLASSIFICATION OF DEAD RECKONING
SYSTEM ERRORS

	Type of Error		
Source of Error	Systematic	Bias	Noise
Distance	Wheel diameter to meter Ratio	Wheel distortion	Wheel Slippage
Initial Position		Gravity	Readout Operator Interpolation Tables Vertical Sensor
Pitch Angle	Velocity Error Moon Rate Error Coriolis Acceleration Vehicle Turn Rate	Gravity Servo Dead Zone	Vehicle Dynamics Readout Error Gyro Drift Vertical Sensor Linearity
Azimuth Angle		Initial Alignment Gyro drift	Gyro drift
Final Position			Map Inaccuracies Display System

SECTION 4.0

SELECTION OF A SYSTEM

The measurement of the pitch and roll angle of the vehicle is made with respect to a gyro held vertical, and a directional gyro aligned toward north. The reference direction for the vertical is the direction of a plum bob.

A vertical directional reference is necessary for two reasons; (1) to provide an axis from which vehicle pitch may be measured, and (2) to maintain the spin axis of the directional gyro in a horizontal plane. Gyros are inertial instruments, therefore due to the rotation of the moon, and due to motion of the vehicle over the lunar surface, the inertial axes will not maintain their correct relationship with the selenographic coordinate system. To maintain the directions over useable periods of time requires compensation methods. In the case of the vertical, the gyro spin axis can be made to maintain its direction along the Z_t axis by use of a vertical loop. The direction of the gravity vector is determined by a vertical sensing element, and the error measured is used to torque the vertical gyro.

The north direction cannot be slaved to an external reference due to the slow spin rate of the moon and the absence of a magnetic field, and a north reference must be compensated for in an open loop manner.

In order to study the accuracy of the system it is necessary to obtain equations which give the performance of both the vertical loop and the directional gyro.

4.1 OPERATION OF THE VERTICAL LOOP

The vertical sensing element in an erection loop is an accelerometer or a pendulum. Such a device of course is sensitive to vehicle accelerations as well as the acceleration due to gravity. If the pendulum on the stabilized element is tilted by some angle, ϕ, then the acceleration sensed by the device can be expressed as

$$sV(s) - g\phi(s)$$

where

s	-	Laplace Operator
V	-	Vehicle Velocity
g	-	Acceleration due to gravity

The total angle through which the axis must be moved is equal to the tilt angle plus the angle due to rotation about the body of radius R. Since the total sensed acceleration should be made proportional to the rate of change of the correction angle, the following expression may be written (see Ref. 2).

$$K(sV(s)-g\phi(s) = \phi\,(s) + \frac{V(s)}{R}$$

$$\phi(s)\,(s + K_g) = (Ks - 1/Rs)V(s)$$

$$\frac{\phi(s)}{V(s)} = \frac{Ks - 1/R}{s + Kg}$$

Expressed in block diagram form

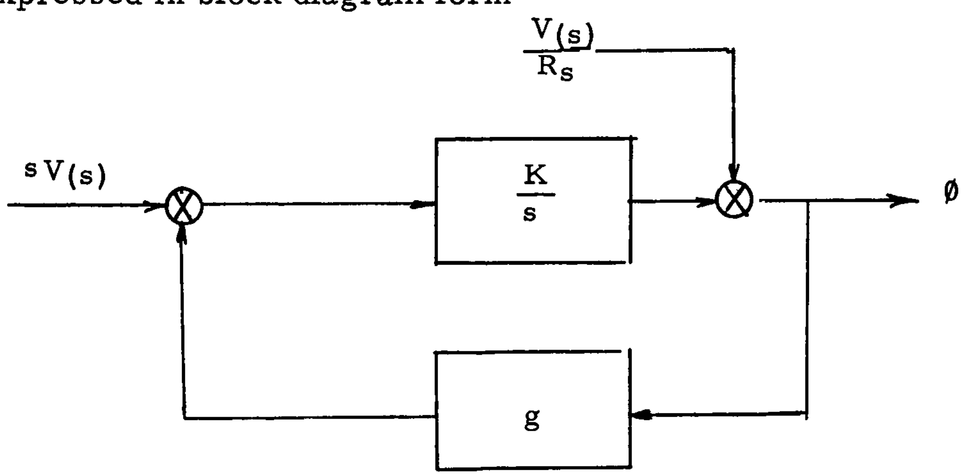


FIGURE 6

SIMPLIFIED LOOP DIAGRAM

Note that the erection loop has two inputs, one consisting of the vehicle acceleration $sV(s)$, and the other the turn rate around the body $\frac{V(s)}{Rs}$ the first of these terms, $sV(s)$, may be considered as a noise source. The bandwidth of the servo loop is to be adjusted so that this term, vehicle acceleration, is to be effectively filtered. This requires a reasonably low value of gain K. At the same time, due to input $\frac{V(s)}{Rs}$, a large value of gain is desired in order to reduce the tilt angle due to vehicle velocity. This tilt angle error is given by the final value theorem of the transfer function $\frac{\phi(s)}{V(s)}$.

Let $V(s) = \frac{Vo}{s}$

$$\phi_o \Big|_{t \to \infty} = \lim_{s \to 0} sG(s) = \frac{Vo}{KgR}$$

where

ϕ_o.	=	steady state tilt error
V_o	=	velocity of vehicle
K	=	gain in the erection loop
R	=	radius of the body

The adjustment of the gain K is then a compromise between filtering of vehicle induced perturbations and reduction of the velocity error.

The velocity of the lunar vehicle is established between limits. A velocity of 20 ft/sec is a possible upper limit, while much lower velocity levels will generally be the case.

Vehicle induced perturbations are those generated by acceleration of the vehicle along the direction of travel and transverse to the direction of travel, as caused by vehicle turn rates. It is difficult to establish values and times associated with this input, and more knowledge of the vehicle and the terrain is necessary before this item can be determined.

4.2 SCHULER TUNED VERTICAL LOOP

By proper choice of system gain, it is possible to reduce the velocity lag to a minimum, and make the system not susceptible to acceleration errors. This is obtained by the addition of a second integration to the loop and the choice of gain K equal to $1/R$. When these two conditions are met the vertical loop becomes Schuler tuned and the vertical direction, at least theoretically, is maintained independent of vehicle motion.

The problems associated with a Schuler tuned vertical loop are complexity with the attendant increase in weight and power. To achieve the second integration requires typically an electro-mechanical integrator due to the accuracy requirements and long integration times.

Additionally the characteristic second order servo, resulting from another integration in the loop, generates an oscillatory response when disturbed. To be useful this oscillation must be damped out. Proper damping requires the measurement of vehicle velocity to a reasonably high accuracy.

The use of a Schuler tuned loop is not recommended for the reasons stated above. Further, the increased accuracy of the vertical will not result in a significantly improved system since the more critical items are the directional gyro for north direction and the accuracy of the distance measurement.

4.3 SYSTEM GAIN

A rather arbitrary decision is made that the vertical loop must be fast enough, i.e., bandwidth, such that the velocity lag resulting from a continuous input of maximum vehicle velocity shall not exceed 5 minutes of arc. This value is commensurate with other errors in the system and can be effectively tolerated.

Since the system proposed is a Type 1 servo we can adjust the velocity constant, K_v, to satisfy the error at maximum angular input rate. By definition (Reference 3)

$$K_v = \lim_{s \to 0} s\ G(s)$$

For the system proposed (Figure 6)

$$K_v = Kg$$

The maximum rate of the input angle, R, is that rate corresponding to a vehicle speed of 17 km/hr.

$$\therefore\ R = \frac{17}{1700} = .01 \text{ Rad per hour}$$

Therefore for a maximum tolerable error angle of 5 arc minutes (.00145 Radians) we obtain

$$K_v = \frac{R}{E}$$

$$= \frac{.01 \text{ Rad per hour}}{.00145 \text{ rad}}$$

$$= 6.9\ hr^{-1}$$

or $$Kg = 6.9\ hr^{-1}$$

4.4 SUITABILITY OF TIME CONSTANT SELECTION

The value of the time constant, or loop gain, has been selected as 6.9 hr^{-1}. The time constant, evaluated in minutes is 8.7 minutes or 520 seconds. A rule of thumb for selecting a suitable time constant is that the time constant of the vertical loop should be at least ten times longer than the longest steady state disturbance. Thus a disturbance lasting longer than 52 seconds would be long enough to cause a measureable error in the vertical.

The principal disturbance is acceleration, either longitudinally or traverse, to the vehicle direction. Consider longitudinal acceleration initially. It is difficult to evaluate the driver actions in relation to the terrain, however we should be able to get some feel for the problem based upon the probable distribution of obstacles that would affect the pendulum. Let the vehicle stability be predominately 1/2 cps motion or 2 sec. A pendulum with a time constant in excess of 2 sec would reduce the effect from vehicle dynamics and only be affected by accelerations that persist for longer than this period of time.

The length of time that these accelerations persist depends on the driver and the terrain. Suppose that a constant velocity is called for to the control system and the vehicle is to travel across the portion of the terrain shown in the close up photo in Ranger 7. It is difficult to draw a parallel pair of lines 10 feet apart, (i. e., vehicle wheel base) that are not affected by the small craters. Based on this we can assume a maximum distance the vehicle can travel without having to change direction or speed is on the order of the time represented by say, 10 to 20 feet. Even at extremely slow velocities of 1 to 2 ft/sec this corresponds to relatively short time interval. Therefore the accelerations associated with these changes in velocity are rather short, on the order of 1 to 2 seconds. While it is theoretically possible for the vehicle to accelerate from a stopped position to a maximum velocity of 16 ft per sec at an acceleration level of .1 ft/sec^2 (time of 160 seconds) the probability of its so doing is very low due to the frequent occurrence of obstacles.

Another justification for the selection of the system is found in considering the operation of the vehicle speed control system. A vehicle speed control loop is a surety in the case of the unmanned automatic control phase, and likely in the manned phase of the mission. Thus errors in velocity are to be corrected in a short period of time, since the error is amplified and used to drive the motors at the maximum rate as set by the speed control loop bandwidth.

Further information on the vehicle and terrain characteristics may reveal that the low level acceleration will persist for long periods of time, resulting in errors in position determination. In this event it is proposed that acceleration measurements of the vehicle along the x_V and y_V axis be made. These can effectively be measured from body mounted accelerometers. The output of these accelerometers will then be fed to a torquer on the pendulum which will function to maintain the pendulum in a vertical direction during periods of acceleration and deceleration. Such a technique has been used with good results (Reference 5).

SECTION 5.0

LONG TERM ERROR SOURCES

The error sources for which external compensation might be required are those whose frequency components are long with respect to the time constant of the erection loop. The following are those generally compensated for:

(1) Coriolis Error

(2) Centrifugal

(3) Constant Velocity

(4) Effect of rotation of the moon

(5) Turning Errors

(6) Component

The magnitude of these quantities was estimated by Perkins (Reference 4). The Coriolis and Centrifugal errors were considered negligible, while correction was suggested for the velocity error, and latitude error. Those due to long term acceleration and turning of the vehicle may be ignored if the time constants for these maneuvers is short in comparison to the servo response time constant.

We can now examine the magnitude of the errors due to error sources 3 and 4 above.

<u>Velocity Error Magnitude</u>

The transfer function for velocity error to tilt angle is derived from the block diagram.

$$\frac{\emptyset}{V} = \frac{1}{sR} \times \frac{1}{K} \times \frac{Ks}{s + Kg} = \frac{1/R}{s + Kg}$$

which has a final value, $\emptyset_o = V_o/KRg$ as derived above.

The magnitude of Kg was estimated as 6.9 hr^{-1}. Therefore for a vehicle of V_o = 5 Km/hr = 4.56 ft/sec.

$$\phi_o = \frac{4.56 \text{ ft/sec}}{(.00192/\text{sec})(1728)(3381) \text{ ft}} = 4.08 \times 10^{-4} \text{ Rad}$$

$$= 1.4 \text{ minutes of arc}$$

Thus this error need not be compensated for and can be considered in the error analysis as a contributing error source.

Effect of Relation of the Moon

The last term which may affect the accuracy of the vertical is that due to the latitude correction due to spin rate of the moon. The maximum sensed spin rate is .536 o/hr which occurs at the lunar equator. The effect of this rate is equivalent to the velocity error, i.e., a vehicle moving at a rate corresponding to .536 o/hr would generate a tilt error of this magnitude as a maximum.

Since $\dot{\phi} = \frac{V}{R}$

$$V = R(.536 \text{ o/hr}) = \frac{.536 \text{ o/hr } (1728)\ (3281) \text{ ft}}{57.3 \text{ o/rad } (3600 \text{ sec/hr}} = 14.7 \text{ ft/sec}$$

$$\therefore \phi_o = \frac{(14.7)\ (57)\ (60)}{(.00192)\ (1728)\ (3281)} = 4.63 \text{ arc minutes}$$

It is suggested that an error source of this type be removed from the system by compensation. The compensation method is simple. The only input necessary for determining the rate of torquing the gyro is the latitude of the vehicle.

SECTION 6.0

ERROR MODEL

The coordinates of the error are taken to be along local vertical coordinate system axes with the x axis directed along the great circle path of the vehicle, the y direction along the direction transverse to the path, and the z axis in the direction of the local vertical. Thus a component of error exist along each of these axes, a track error, ex, cross track error, ey, and vertical error, ez.

The first portion of the error model will be to determine the error angles of the reference coordinate system. The error angles represents a tilt of the reference axis in a direction given by the vector rotation of the axis. Thus an error angle ϕ_y exist due to the motion of the vehicle along a great circle path in the x direction. The ϕy error angle defines the deflection of the true vertical in the pitch plane of the vehicle path and results in an altitude postion error, ez. Likewise a error ϕz exist which relates the drift of the azimuth reference and results in a cross track error, ey. The manner in which these error angles generate position errors is shown in Figure 8. The second order error sources which involve the product of error quantities are neglected.

The determination of the error angles is made due to component error sources. Those error sources considered are as shown below

Error angle ϕy

Accelerometer scaling error, a_{1x}

Accelerometer bias, a_{ox}

Initial platform tilt, ϕ_{xo}

Gyro drift rate, ϵy

Error angle ϕz

Initial alignment error, ϕ_{zo}

Gyro drift rate, ϵ_z

Body spin rate, Ω

The error angle ϕx is not determined inasmuch as its contribution is of a second order nature as shown in Figure 8.

6.1 VERTICAL LOOP COMPONENT ERRORS

A block diagram of the vertical loop showing the manner that the component errors affect the error angle, ϕy, is shown in Figure 7. In appendix A, the transfer functions relating the error angle to each component error is derived. The results are shown in Table 2, using Laplace transform notation.

TABLE 2

RESPONSE FUNCTION FOR TYPICAL ERROR INPUTS

Item	Input	Transfer Function	Input	Response
Vehicle Velocity	V(s)	$-\frac{1/R}{s + Kg}$	$\frac{v_o}{s}$	$-\frac{v_o/R}{s(s + Kg)}$
Accelerometer Bias	a_o	$\frac{K}{s(s + Kg)}$	$\frac{a_o}{s}$	$\frac{a_o K}{s(s + Kg)}$
Gyro Drift	$\epsilon y(s)$	$\frac{1}{s + Kg}$	$\frac{\epsilon_o}{s}$	$\frac{\epsilon_o}{s(s + Kg)}$
Initial Tilt	$\frac{\phi_o}{s}$	$\frac{1}{s + Kg}$	$\frac{\phi_o}{s}$	$\frac{\phi_o}{S(s + Kg)}$

The effect of a scale factor error in the accelerometer results in a gain change in the system which affects both the time constant and the tracking accuracy. To determine the effect of a gain change it is necessary to find the error angle at the correct gain which will be subtracted from the error angle with the scale factor error. This is shown in Appendix A. From Appendix A and Table 2 the expression for the total error $\phi y(t)$ may be written as the sum of the response due to each individual error source.

Converting to Laplace transform to the time function the following expression results for $\phi y(t)$.

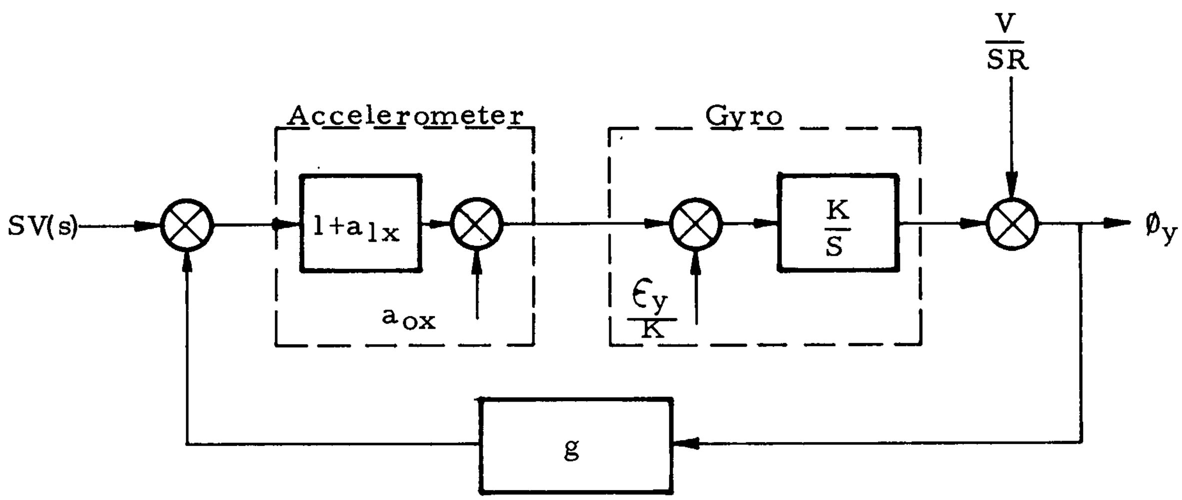


FIGURE 7

VERTICAL LOOP SHOWING COMPONENT ERRORS

$$\phi y(t) = \frac{a_o}{g}(1-e^{-Kgt}) + \frac{\epsilon_y}{Kg}(1-e^{-Kgt})$$

$$+ \phi_o e^{-Kgt} + (b^2K + \frac{1}{R w_1}) v_o e^{-w_1 t}$$

$$+ \frac{a_1 v_o}{Rw_1} - (K + \frac{1}{RKg}) v_o e^{-Kgt} \qquad (1)$$

Where $b^2 = a_1 + 1$

$w_1 = b^2 Kg$

and the vehicle velocity is asumed constant. The last three terms represent the error angle due to a scale factor error.

6.2 HEADING COMPONENT ERROR EFFECTS

The system proposed utilizes a directional gyro north reference. The directional gyro may be a two degree of freedom separately housed instrument or a single degree of freedom gyro mounted on a stable element. In either case the spin axis is to be maintained in the horizontal plane by means of the vertical gyro. For the separately housed instrument, slaving signals must be supplied from the vertical gyro to the platform. It is recommended that the single degree of freedom instrument be utilized since the accuracy obtained from this type of instrument is superior to that from a two degree of freedom device. (Reference 6).

The response function for the ϕ z error angle is derived in reference 2. The results is equally applicable to the case studied here. The resulting response function for $\phi z(t)$ is given by

$$\phi_z(t) = \phi_{oz} \cos\left(\frac{V_{ox}}{R}\right)t - \phi_{ox} \sin\left(\frac{V_{ox}}{R}\right)(t) - \frac{R}{V_{ox}} \epsilon_{ox} \left(1-\cos \frac{V_{ox}}{R}t\right)$$

$$+ \frac{R}{V_{ox}} \epsilon_{oz} \left(\sin \frac{V_{ox}}{R}\right)(t)$$

Note that for a lunar vehicle considered the term $\frac{v_o t}{R}$ is small (~ 1 rad at 10 hours and v_o = 17 km per hour) hence the expression may be linearized by the small angle approximation.

$$\therefore \; \phi_z(t) = \phi_{oz} - \left(\frac{\phi_{ox} V_{ox}}{R}\right) t - (\epsilon_{oz})\, t \qquad (2)$$

The second term on the right hand side is negligible. Thus for the time and velocities considered the expression for $\phi_z(t)$ shows an increasing error with time.

6.3 Nature of the Position Error

Assume that a vehicle is traveling along a path chosen by the vehicle driver. The distance that the vehicle is from the starting point is given by x, say, where $x(t) = v(t) \cdot t$ where

$x(t)$ is the distance at any time, t

$v(t)$ is the velocity function of time

t is the time of computation.

Length AB represents the actual path of the vehicle and represents the distance along the vehicle track. Let the path be broken up into n segments each Δt apart.

Now as the vehicle is traversing the length AB the vehicles computer is continuously computing its position with respect to the starting point. This computation is carried out for the cross path directions, z and y.

If path direction is given as along a heading ψ then the computer heading is given as $\psi + \phi$. Then the error in the computed position is a function of ϕ, the error angle. As shown in Figure 8 there exist a relationship between the error along each axis and the subsequent position error. This is expressed as

$$ey = (\phi_z)\, x(t)$$

$$ez = (\phi_y)\, x(t)$$

Now let $\phi_z = \phi_z(t)$ a function of time and $x(t) = v(t) \cdot t$ where $v(t)$ is a function of time

$$\therefore ey = \sum_{i=1}^{n} \phi_z(t)_i \cdot v(t)_i \cdot \Delta t_i$$

where Δ t are the increments of time along the path

$v(t)_i$ the velocity during increment i

and $\phi_z(t)_i$ is the error angle during increment i

By definition

$$\int_0^t \phi_z(t) \cdot v(t)\, dt = \lim_{n=\infty} \sum_{i=1}^{n} \phi_z(t)_i \cdot v(t)_i \cdot \Delta t_i$$

For any arbitrary velocity profile and error angle, both expressable as functions of time, the total error is given by

$$E_y = \int \phi_z(t) \cdot v(t)\, dt$$

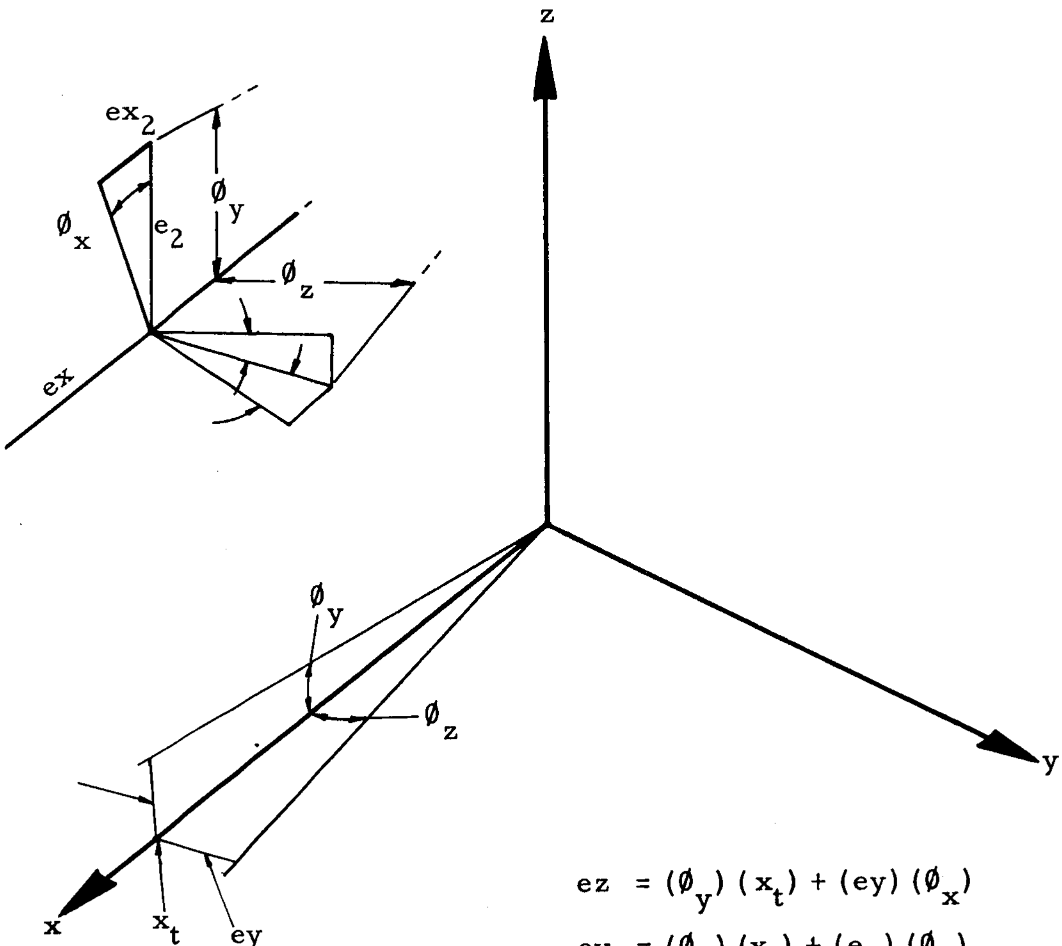
$$E_z = \int \phi_y(t) \cdot v(t)\, dt$$

The velocity of the vehicle is expected to vary dependent on the terrain and driver actions. However it is assumed that a vehicle speed control loop will be employed. Such a loop will tend to keep the velocity of the vehicle bounded within certain limitations, this being a matter of how accurately the speed control loop is maintained. It is considered a good assumption to let the velocity v(t) be given as

$$v(t) = v_o + v_1(t)$$

where v_o is the constant value of velocity about the mean value. $v_1(t)$ is then a function of vehicle speed and frequency of occurance of speed distrubances such as slopes.

From photographs of the lunar surface that have been made it can safely be assumed that a high frequency of occurance of slope changes



$$ez = (\emptyset_y)\,(x_t) + (ey)\,(\emptyset_x)$$

$$ey = (\emptyset_z)\,(x_t) + (e_z)\,(\emptyset_x)$$

$$ex = (\emptyset_z)\,(e_y) + (e_z)\,(\emptyset_y)$$

RELATION BETWEEN ANGLE AND POSITION ERROR

FIGURE 8

will occur, say at least one every 10 to 20 meters. At a mean velocity of 5Km/hr (1.39 m/sec), the slope changes causing a load on the velocity servo will occur at a frequency of

$$f = \frac{1.39 \text{ m/sec}}{10 \text{ m/slope variation}} = .139 \text{ slope variation/sec}$$

This is to be compared with the bandwidth of the servo loop which was chosen as

$$6.9 \text{ Rad/hr} = \frac{6.9 \text{ rad}}{1\text{v}} \quad \frac{6.28 \text{ cycle (hr)}}{\text{rad} \quad 3600 \text{ sec}} = .012 \text{ cycles/sec.}$$

Thus the perturbation for the assumed speed and distance between slope variations are outside the bandwidth of the velocity servo and hence to do affect the lag angle of the gyro.

From another standpoint the distance traveled over a period of time will be equal to

$$d = v_o (t) + v_1 (t)$$

If v_1 is assumed to have a statistical distribution about the steady state velocity v_o then the distances represented by v_1 (t) are equally probable for substracting from or adding to the distance computed $v_o \cdot t \therefore$ for practical purposes $v_1(t)$ may be neglected.

For a servo controlled speed loop then the v(t) may be represented as $v(t) = v_o t$.

SECTION 7.0

ERROR EVALUATION

The system evaluated is a Type 1 single degree of freedom vertical loop with a separate directional gyro for an azimuth reference. The value of the error terms and parameter values of the error terms is given in Table 3 below.

TABLE 3

PARAMETER VALUES FOR SYSTEM CONSIDERED

Item	Symbol	Values	Units
Accelerometer Bias	a_o	10^{-3}	g
Gyro Drift	ϵ_y, ϵ_z	.05 .1	degrees per hour degrees per hour
Loop Gain	Kg	6.9	hr^{-1}
Body Radius	R	1728	kilometers
Vehicle Velocity	v_o	5 10	kilometer per hour kilometer per hour
Scale Factor Error	a_1	5	percent
Initial Tilt (vertical)	ϕ_{oy}	6	minutes of arc
Initial Alignment (azimuth)	ϕ_{oz}	2	minutes of arc
time	t	6	hours maximum

7.1 INITIAL ALIGNMENT ERRORS

The initial tilt of the vertical, ϕ_{oy}, is dependent on the alignment method and the time allotted for the initial tilt to converge to a minimum value. Since the vertical loop is equipped with a servo to erect the spin axis then a matter of 3 time constants (say 20 minutes) is required to have the loop converge to its minimum value. For the error analysis an initial error of 6 minutes of arc is assumed. This would correspond to some manual method of rotating the platform to a nominal level condition prior to energizing the erection loop.

The initial heading alignment is much more critical. Since the heading gyro operates in a free condition the initial misalignment will result in position errors that become significant with time and distance.

Three basic techniques may be used to initially align the axis of the direction memory. These are:

(1) Surveying (landmarks or celestially)

(2) Gyro compassing

(3) Radio Celestial in cooperation with CSM

For the case of surveying by celestial observations. The accuracy with which an azimuth may be determined by star sightings is dependent on the following items:

(a) Unknown libration of moon 1/2 min.

(b) Accuracy of knowledge of the location of observation (1 min)

(c) Accuracy of the reading (1/5 min)

(e) Sight reduction (2 min)

Considering each of these error sources as random variables the rss of the resultant accuracy is

$$= \sqrt{1/4 + 1 + 1/25 + 4}$$

$$= 2.4 \text{ minutes of arc}$$

The second method is that of obtaining north by a north seeking gyro. Estimates have been made on the accuracy of a two axis spherical north seeker as 2 minutes of arc after a period of 2 hours at the lunar equator.

The third method is subject to an interface problem with availability of the CSM. Additional equipment would also be required to compute the direction to the orbit ground track. Hence this third method is disregarded.

In summary the initial alignment accuracy of the azimuth direction memory can be taken as a one sigma deviation of 2 minutes of arc.

7.2 EVALUATION OF ϕ_y NOISE ERROR SOURCES

The following error sources are considered:

Accelerometer Bias

Accelerometer Scale factor

Gyro drift rate

Initial Tilt

All the above are random in nature and hence the root sum of the squares of the error from each source will represent the desired accuracy. Expressed as a sum, the expression for the tilt of the vertical $\phi_y(t)$ is given by equation (in Section 6-1)

7.2.1 Vertical Tilt due to Accelerometer Bias

The error due to accelerometer bias is expressed as:

$$\phi_y(t) = \frac{a_o}{g} \left(1-e^{-at} \right)$$

The results of this expression for an accelerometer bias of 10^{-3} g is given in Table 4. The error is given versus time. The third column gives the tilt error in milliradians as one standard deviation (1σ value.) The $1\sigma^2$ value is given in column 4.

TABLE 4

VERTICAL TILT ERROR DUE TO ACCELEROMETER BIAS

time (hours)	$(1-e^{-at})$	ϕ_y 1σ value (millirad)	ϕ_y $1\sigma^2$ value
0	0	0	0
.1	.498	.498	.249
.2	.749	.749	.56
.4	.936	.936	.876
.6	.984	.984	.98
.8	.996	.996	.992
1	1	1	1
2	1	1	1
4	1	1	1
6	1	1	1

7.2.2 Vertical Tilt due to Gyro Drift

The error angle due to gyro drift rate is expressed as

$$\phi_y(t) = \frac{\epsilon_o}{Kg} \left(1-e^{-at} \right)$$

The results of this expression for gyro drift rates of .05 and .1 degree per hour gyros is given in Table 5. The angle is given in terms of 1σ and $1\sigma^2$ values for each of the drift rates considered:

TABLE 5

VERTICAL TILT DUE TO GYRO DRIFT RATE

time (hours)	ϵ_o = .05 deg/hr		ϵ_o = .1 deg/hr	
	1σ mr	$1\sigma^2$	1σ mr	$1\sigma^2$
0	0	0	0	0
.1	.126	.0159	.063	.0044
.2	.189	.0357	.0945	.00893
.4	.236	.0558	.118	.0139
.6	.248	.0613	.124	.0154
.8	.252	.0635	.126	.0159
1	.253	.064	.1265	.016
2	.253	.064	.1265	.016
4	.253	.064	.1265	.016
6	.253	.064	.1265	.016

7.2.3 Vertical Tilt Due to Initial Alignment Error

The error angle due to an initial alignment error is given by

$$\phi_y(t) = \phi_o\, e^{-at}$$

The values of ϕ_y as a function of time are computed and listed in Table 6.

TABLE 6

VERTICAL ERROR DUE TO INITIAL ALIGNMENT ERROR

time (hours)	e^{-at}	ϕ_y 1σ value mr	$1\sigma^2$
0	1	1.74	3.04
.1	.502	.376	.778
.2	.2502	.437	.191
.4	.0635	.108	.0157
.6	.016	.0279	0
.8	.004	.007	0
1	0	0	0
2	0	0	0
4	0	0	0
6	0	0	0

7.2.4 Vertical Tilt Due to Accelerometer Scale Factor Error

The scale factor error is determined for a change of 5% in the system gain. This error may result from environmental changes of the equipment and is to be expected. In this system the scale factor change results in some random change of the vertical and contributes indirectly to a position error. This error source may be thought of as any random distrubance which may cause a change in vertical loop gain. The error was computed for a velocity of 5 km per hour and 10 km per hour and was found to be completely negligible.

7.3 EVALUATION OF ϕ_y SYSTEMATIC AND BIAS ERROR SOURCES

Two bias errors sources are considered which result in tilt of the vertical. These are the velocity lag and the latitude error.

7.3.1 Velocity Error

$$\phi_{(t)} = (K - \frac{1}{Rkg}) v_o e^{-k_g t} + \frac{v_o}{Rkg}$$

The assumed velocities were 5 & 10 Km/hr. In each case the final steady state error will exist as $\frac{v_o}{KgR}$ which is a maximum peak.

For v_o=5 km per hour ϕ_t = 1.44 minutes = .42 mr

For v_o = 10 km per hour ϕ_t = 2.88 minutes =.82 mr

7.3.2 Latitude Error

$$\text{Spin Rate of Moon} = \frac{14.7 \text{ ft/sec km}}{(3381 \text{ ft})} \left(\frac{3600 \text{ sec}}{\text{hr}}\right)$$

$$= 15.6 \text{ km/hr}$$

$\therefore \phi_t$ = 4.5 minutes max = (1.28 mr)

7.4 ERROR SUMMATION FOR ϕ_y TERMS

Table 7 summarizes the errors in the following manner. The total peak error is expressed as

$$\text{Total Peak Error} = \sqrt{3\,\overline{(\text{Noise Error})}^2 + \overline{(\text{Bias Errors})}^2}$$

The results of this tabulation are shown in Figure 9.

TABLE 7

SUMMATION OF ERRORS IN INDICATED VERTICAL

Error	$1\sigma^2$ Values Time (Hour)									
	0	.1	.2	.4	.6	.8	1	2	4	6
Accelerometer Bias	0	.249	.56	.876	.98	.992	1	1	1	1
Gyro Drift $\epsilon_o = .05$ o/hr	0	.016	.036	.056	.061	.063	.064	.064	.064	.064
Initial Tilt	3.04	.778	.191	.0157	0	0	0	0	0	0
Scale Factor	0	0	0	0	0	0	0	0	0	0
Σ Noise Factors	3.04	1.043	.787	.1477	1.041	1.055	1.064	1.064	1.064	1.064
3Σ Noise Factors	10.12	3.129	2.361	.443	3.123	3.165	3.192	3.192	3.192	3.192
Velocity Error ^{2}	.672	.672	.672	.672	.672	.672	.672	.672	.672	.672
Latitude Error ^{2}	1.57	1.570	1.570	1.570	1.570	1.570	1.570	1.570	1.570	1.570
Sum	21.362	5.371	4.603	2.685	5.365	5.407	5.434	5.434	5.434	5.434
TOTAL PEAK ERROR	4.62	2.305	2.145	1.64	2.31	2.325	2.33	2.33	2.33	2.33

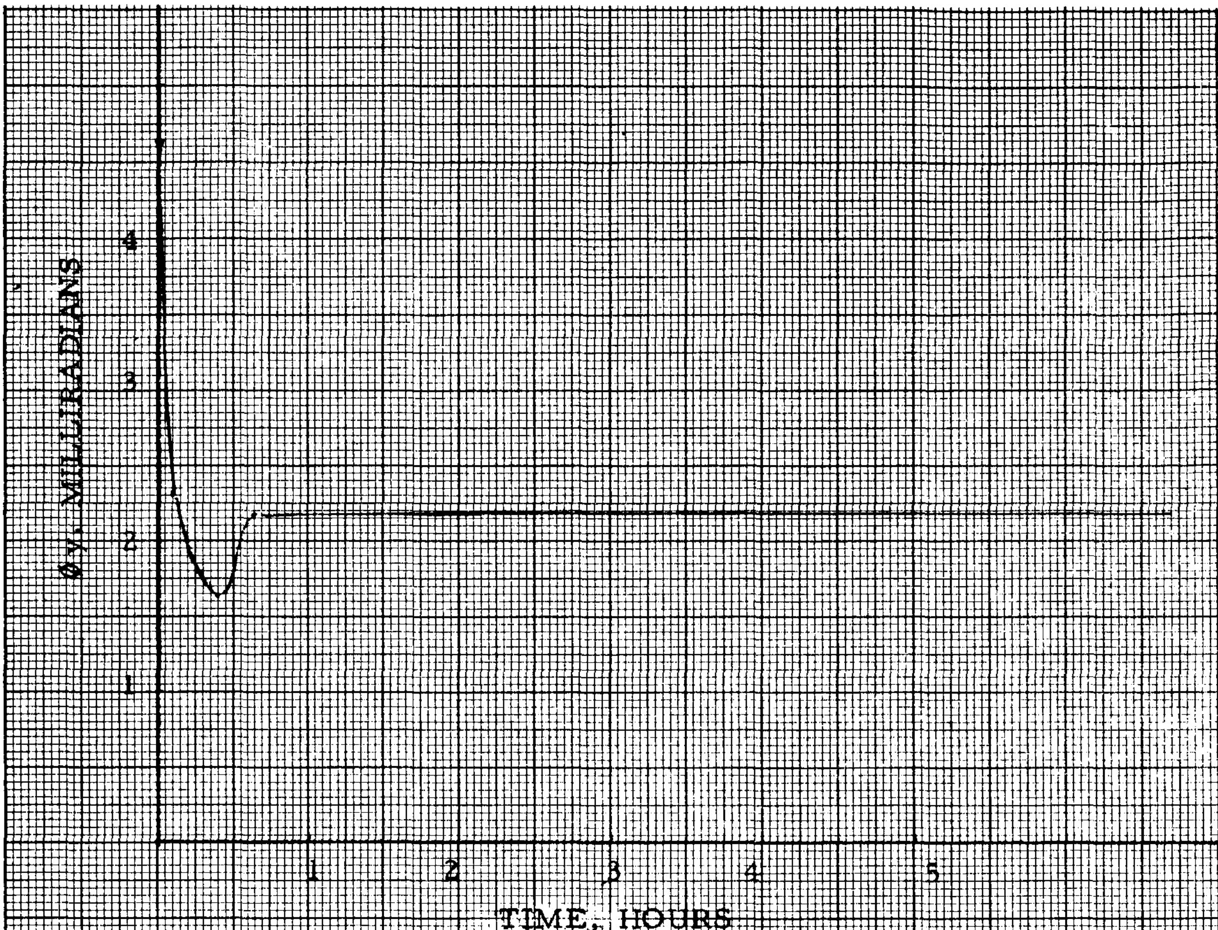


FIGURE 9
TOTAL PEAK ERROR OF INDICATED VERTICAL

7.5 EVALUATION OF ϕ_z VERSUS TIME

The equation for $\phi_z(t)$ was shown in section 6.2.as equation (2). There are three principal error sources that reflect in an azimuth error angle. These are gyro drift, uncompensated latitude rate due to the spin of the moon, and the initial alignment. These are considered as random variables.

7.5.1 Initial Alignment Error

The initial alignment error of the azimuth reference ϕ_{oz} is a constant value. It was estimated in section 7.1 as a one sigma value of 2 minutes of arc.

$$\phi_{oz} = 2 \text{ minutes of arc} = .581 \text{ mr}$$

$$1\sigma^2 \text{ value of } \phi_{oz} = .338$$

7.5.2 Azimuth Gyro Drift

The azimuth gyro drift is estimated at between .05 and .1 degree per hour. The assumption is made that these drift rates are constant. The resulting error angle after a period of time for the stated drift rates is shown in Table 8.

TABLE 8

DRIFT ANGLE VERSUS TIME FOR AXIMUTH GYRO

1 Values, ϕ Milliradiaus 1 ^{2} Values

	$(\epsilon_q)\, x(t)$		ϕ_z milliradiaus	
time (hours)	v_o = 5 km/hr	v_o = 10 km/hr	v_o = 5 km/hr	v_o = 10km/hr
0	0	0	0	0
.1	.0872	.1744	.00762	.0304
.2	.1744	.3488	.0304	.122
.4	.3488	.6976	.122	.487
.6	.5232	1.0464	.273	1.09
.8	.6976	1.3952	.487	1.94
1	.872	1.744	.761	3.04
2	1.744	3.488	3.04	12.22
4	3.488	6.976	12.22	48.7
6	5.232	10.464	27.3	109.

7.6 ERROR SUMMATION FOR ϕ_z TERMS

Table 9 summaries the two contributing errors. The total peak error in the indicated azimuth is three times the v.s.s. sum of the two independent random errors. The results of the tabulation is shown in Figure 10.

TABLE 9

SUMMATION OF ERRORS IN INDICATED AZIMUTH

Error	$1\sigma^2$ Values Time (Hour)									
	0	.1	.2	.4	.6	.8	1	2	4	6
Initial Tilt	.3380	.3380	.3380	.3380	.3380	.3380	.338	.338	.338	.338
Gyro Drift ϵ_o = .05 o/hr	.0	.0076	.0304	.1220	.2730	.4870	.7610	3.040	12.220	27.300
$\Sigma 1\sigma^2$ Values	.3380	.3456	.3684	.4600	.6110	.8250	1.0990	3.378	12.558	27.638
$3\Sigma 1\sigma^2$ Values	1.0140	1.0368	1.1052	1.3800	1.8330	2.475	3.27	10.134	37.65	83
Total Peak Error (Milliradians)	1.005	1.016	1.05	1.174	1.352	1.572	1.81	3.18	6.14	9.11

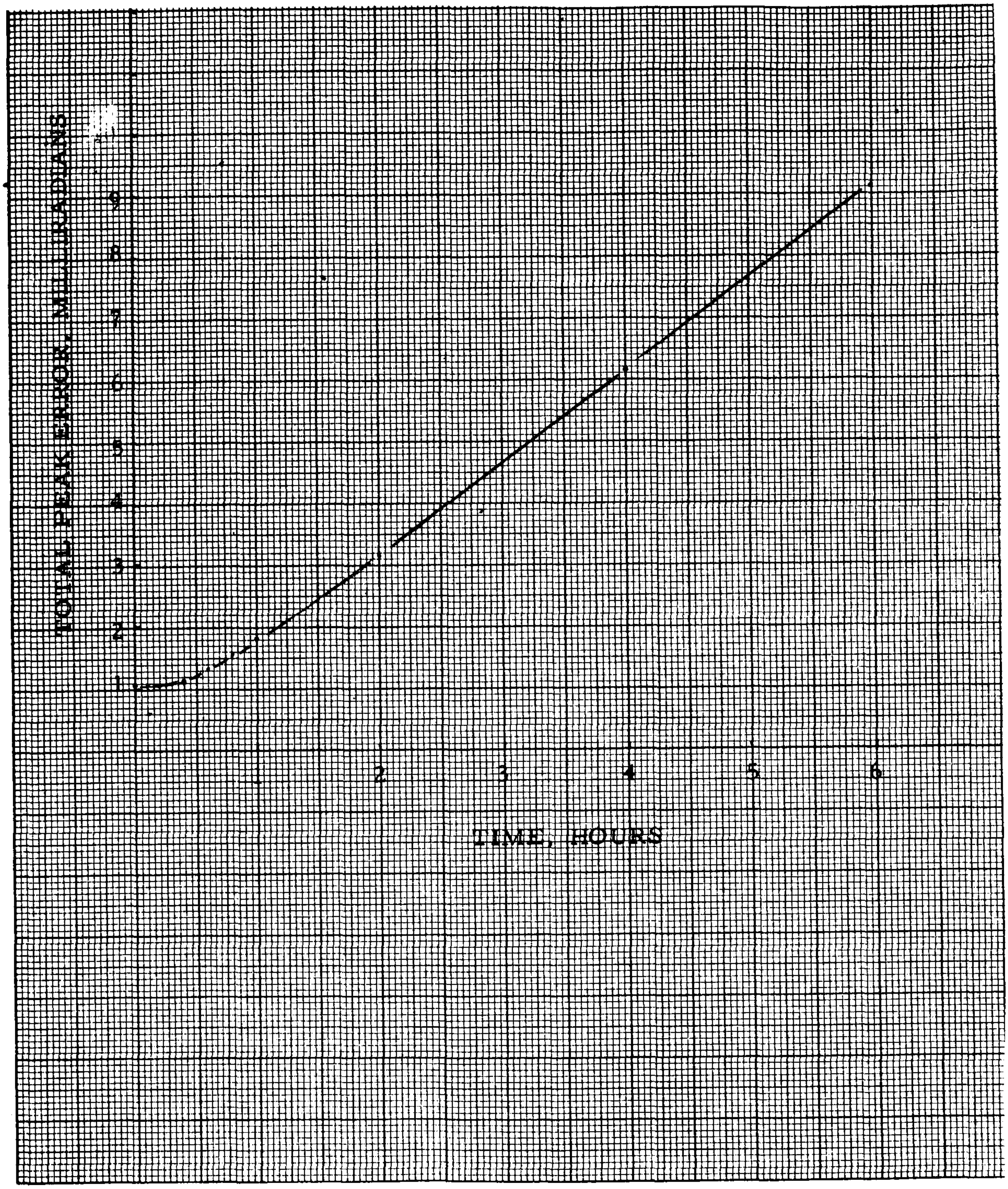


FIGURE 10. TOTAL PEAK ERROR OF INDICATED AZIMUTH REFERENCE

7.7 DETERMINATION OF ALTITUDE POSITION ERROR, ez, FROM NOISE SOURCES

The position error is to be calculated for each error source in turn, utilizing the formula given in section 6.0. The total position error for the noise sources is then given by the r.s.s. sum of the error due to each individual error source. The error contribution for each source is given in the following paragraphs.

7.7.1 Position Error Due to Accelerometer Bias

The tilt angle for an accelerometer bias error, a_o, for the assumed system, is given by.

$$\phi_y(t) = \frac{a_o}{g}\left(1-e^{-Kgt}\right)$$

The position error is given by

$$e_z = \int_0^t v_o\, \phi_y(t)\, dt$$

$$= \frac{v_o\ a_o}{g} \int_0^t \left(1-e^{-Kgt'}\right) dt'$$

$$= \frac{v_o\ a_o}{g}\left[t + \frac{1}{Kg} e^{-Kgt}\right] - \frac{v_o\ a_o}{g\,(Kg)}$$

The values of e_z versus time are shown in Table 10 and presented graphically in Figure 11.

TABLE 10

POSITION ERROR DUE TO ACCELEROMETER BIAS

time (hours)	$t + \frac{1}{Kg} e^{-Kgt}$	v_o = 5 km/hr e_z meters	v_o = 10 km/hr e_z meters
0	.145	0	0
.1	.1728	.139	.278
.2	.2363	.456	.913
.4	.4092	1.3210	2.64
.6	.602	2.285	4.57
.8	.800	3.275	6.55
1	1	4.275	8.55
2	2	9.275	18.55
4	4	19.275	38.55
6	6	29.275	58.55

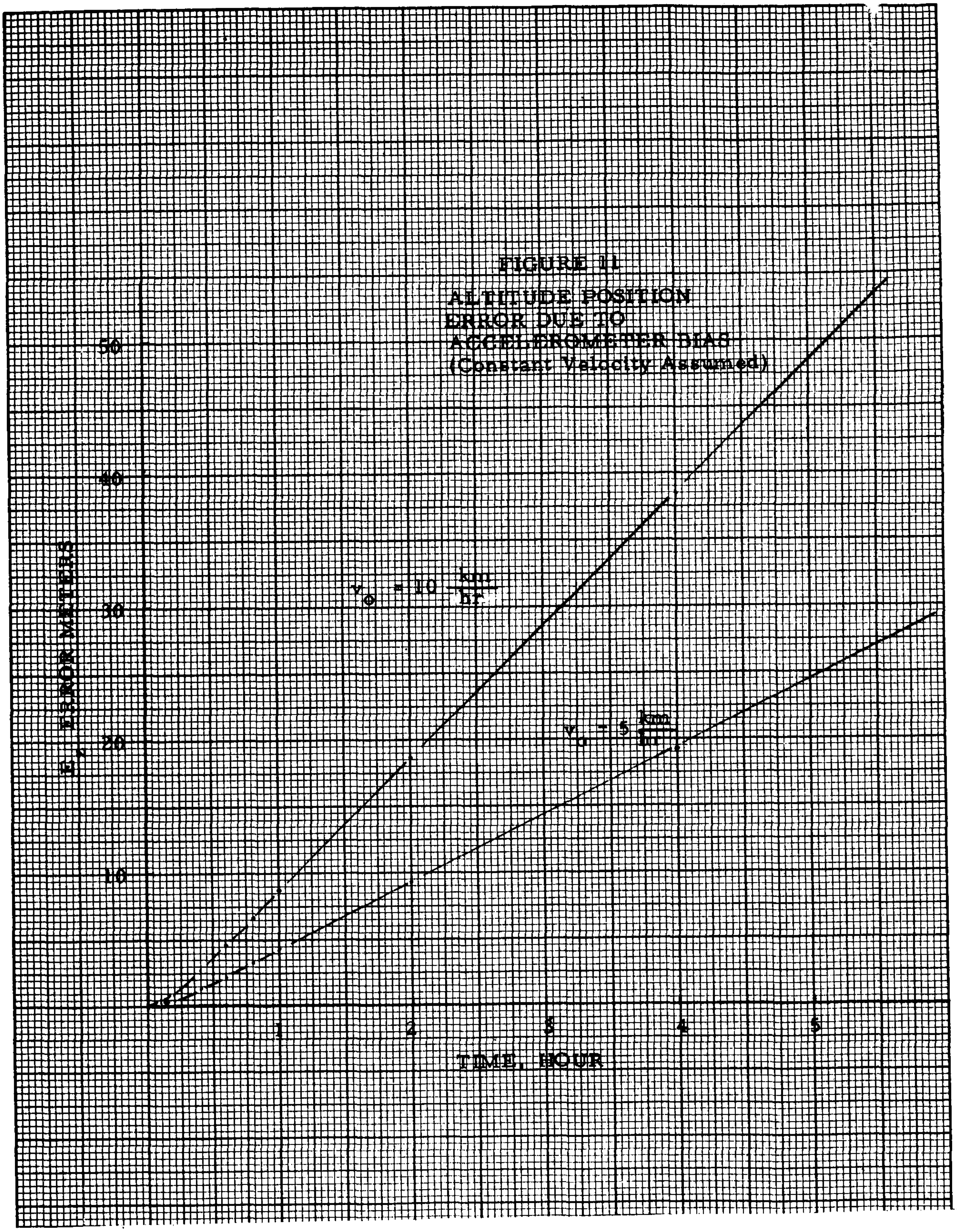


FIGURE 11. ALTITUDE POSITION ERROR DUE TO ACCELEROMETER BIAS (Constant Velocity assumed)

7.7.2 <u>Position Error Due to Gyro Drift</u>

The tilt angle for the vertical gyro, due to gyro drift, ϵ_o, for the assumed system is given by:

$$\phi_y(t) = \frac{\epsilon_o}{Kg} \left(1 - e^{-Kgt} \right)$$

The position error is given by

$$e_z = \int_o^t v_o \ \phi_y(t') \, dt$$

which reduces to

$$e_z = \frac{v_o \ \epsilon_o}{kg} \left[t + \frac{1}{Kg} \ e^{-Kgt} \right] - \frac{v_o \epsilon_o}{(Kg)^2}$$

The values of e_z versus time are shown in Table 11a and 11b plotted in Figure 12.

TABLE 11a

POSITION ERROR DUE TO GYRO DRIFT (ϵ_o = .1 o/hr)

		ϵ_z, meter	
time (hours)	$t + \frac{1}{Kg} e^{-Kgt}$	$v_o = 5$	$v_o = 10$
0	.145	0	0
.1	.1728	.035	.070
.2	.2363	.115	.232
.4	.4092	.335	.670
.6	.602	.578	1.156
.8	.800	.826	1.652
1	1	1.084	2.168
2	2	2.446	5.892
4	4	4.876	9.752
6	6	7.396	14.792

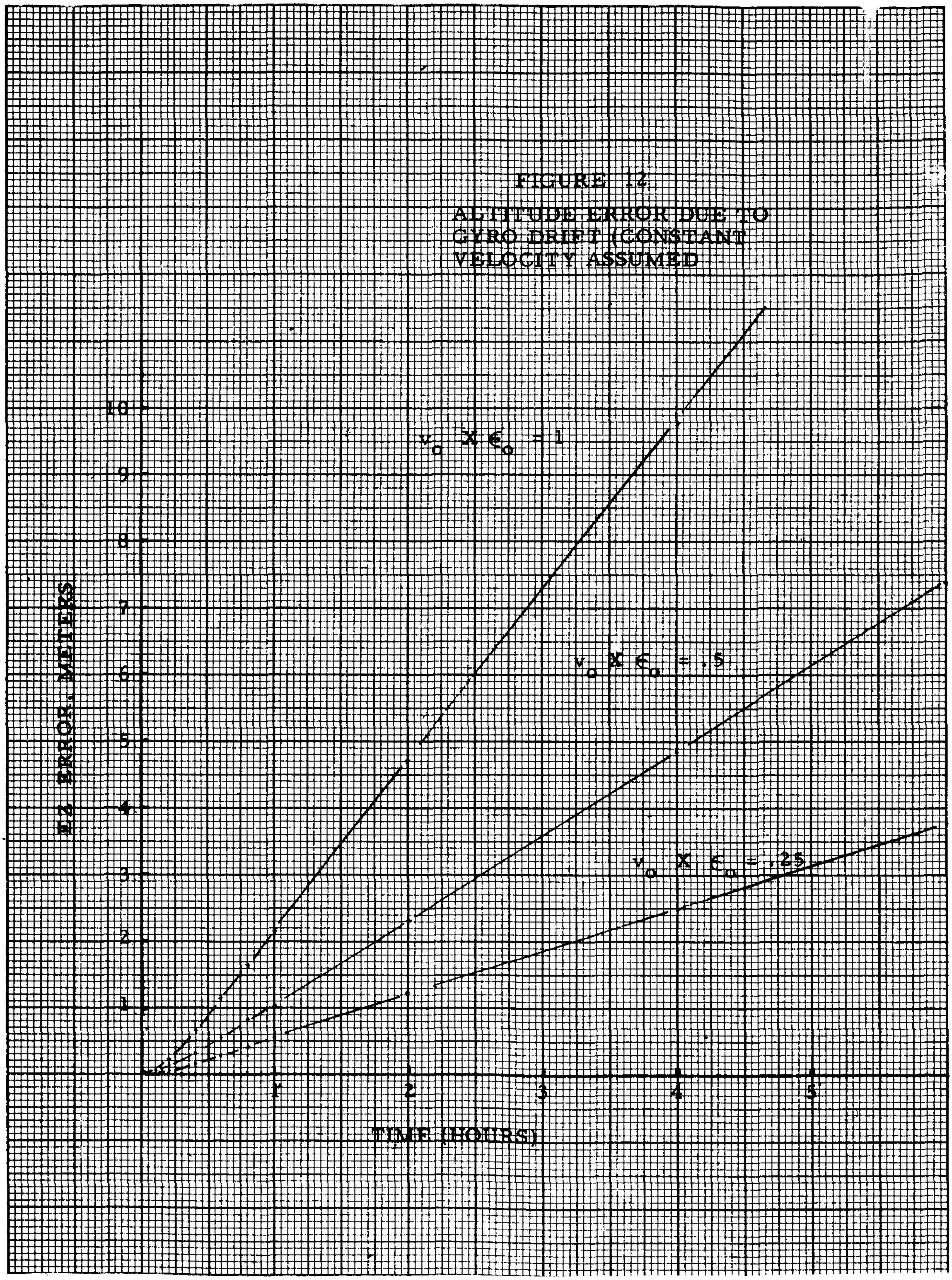


FIGURE 12. ALTITUDE ERROR DUE TO GYRO DRIFT
(Constant Velocity Assumed)

TABLE 11b

POSITION ERROR DUE TO GYRO DRIFT (ϵ_o = .05 o/hr)

time (hours)	$t + \frac{1}{Kg} e^{-Kgt}$	e_{z_1}, Meters	
		$v_o = 5$	$v_o = 10$
0	.145	0	0
.1	.1728	.017	.135
.2	.2363	.057	.115
.4	.4092	.1675	.335
.6	.602	.289	.578
.8	.8005	.413	.826
1	1	.542	1.084
2	2	1.223	2.446
4	4	2.488	4.876
6	6	3.798	7.396

7.7.3 Altitude Position Error Due to Initial Tilt

The initial alignment error of the vertical gyro will converge when the loop is energized. The effect of the initial tilt error will cause a position error that is constant with time. The expression for the error angle and subsequent position error are as given below

$$\phi_y(t) = \phi_o e^{-Kgt}$$

$$e_z = \int_o^t v_o \phi_o e^{-Kgt'} dt'$$

$$= \left[\frac{v_o \phi_o}{Kg} e^{-Kgt} \right] + \frac{v_o \phi_o}{Kg}$$

The position error resulting from this error source is shown in Table 13 & Figure 13.

TABLE 12

POSITION ERROR DUE TO INITIAL ALIGNMENT TILT

time (Hours)	$\frac{1}{Kg} e^{-Kgt}$	E_z, Meters v_o = 5 km/hr	E_z, Meters v_o = 10 km/hr
0	.145	0	0
.1	.0728	.91	1.82
.2	.0363	1.371	2.742
.4	.0092	1.613	3.326
.6	.00232	1.8	3.604
.8	.00058	1.82	3.64
1	0	1.83	3.66
2	0	1.83	3.66
4	0	1.83	3.66
6	0	1.83	3.66

7.7.4 Position Error Due to Scale Factor Error

The position error resulting from a scale factor change of five percent was found to introduce no significant position error.

7.7.4 Position Error Due to Readout Error

The position computer will calculate the vehicle's position with reference to the angle on the readout. It is assumed for this sytem that the readout accuracy is correct to within 3 minutes of arc. This will also be a random function.

The error in the readout angle is denoted Θ_R. Θ_R has a 1σ value of 3 minutes of arc = .873 milliradians. The position error e_z is given by

$$e_z = \int_0^t \Theta_R v_o \, dt = v_o \Theta_R t$$

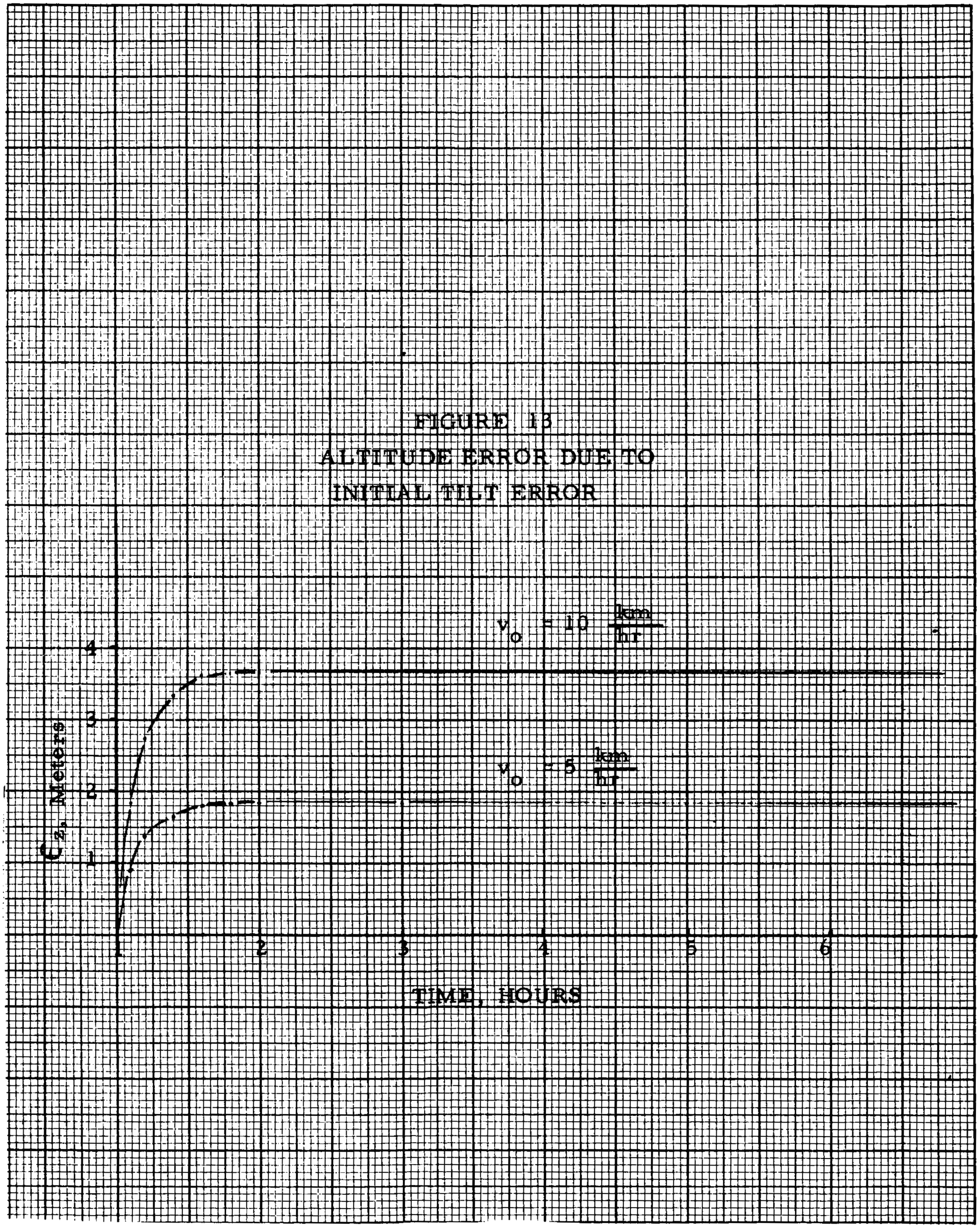


FIGURE 13. ALTITUDE ERROR DUE TO INITIAL TILT ERROR

TABLE 13

POSITION ERROR DUE TO READOUT ERROR

Time (Hours)	Error, Meter	
	v_o = 5 km per hr	v_o = 10 km per hr
0	0	0
.1	.436	.872
.2	.872	1.745
.4	1.745	3.49
.6	2.62	5.24
.8	3.49	6.98
1	4.36	8.72
2	8.72	17.45
4	17.45	34.9
6	26.2	52.4

7.7.5 Position Error Due to Vehicle Acceleration

The effect of vehicle acceleration and deceleration is to deflect the pendulum. This will result in an error in the indicated vertical which affects a position error. The magnitude of this postion error is determined in Appendix B for typical acceleration levels. The position error is generated during the portion of time that the acceleration level persist, and the total error becomes the summation of the errors from each acceleration or deceleration period.

It is recommended that the acceleration generated errors be compensated for. It is a simple matter to measure vehicle acceleration. This acceleration is then to be used to torque the gyro back to the true vertical, as done in the system in Reference 5.

7.8 ALTITUDE POSITION ERROR RESULTING FROM BIAS ERRORS

Altitude postion errors due to bias sources arise from the velocity lag for a type 1 vertical loop, and due to the Corolios error. The Corolios error is to be removed by compensation.

Velocity Error

The expression for the error angle due to an initial step velocity input is

$$\emptyset(t) = \frac{v_o}{RK_g}(1-e^{-Kgt})$$

The resultant expression for the position error becomes

$$e_z = \frac{v_o}{RKg}\left[(t + \frac{1}{Kg} e^{-Kgt}\right] - \frac{v_o}{R\,(Kg)^2}$$

The value of this expression for the assumed case is given in tabular form in Table 14 and graphically in Figure 14.

TABLE 14

POSITION ERROR RESULTING FROM VELOCITY LAG

Time (Hours)	$(t + \frac{1}{Kg} e^{-Kgt})$	E_z Error, Meter	
		$v_o = 5$	$v_o = 10$
0	.145	0	0
.1	.1728	.059	.232
.2	.2363	.192	.764
.4	.4092	.556	2.22
.6	.602	.961	3.84
.8	.800	1.376	5.50
1	1	1.796	7.18
2	2	3.896	15.58
4	4	8.096	32.38
1	6	12.296	49.18

Gravity Bias Error

The deflection of the vertical due to gravity anamolies may be considered as a bias error rather than a noise error because of the limited range of operation of the vehicle in relation to the moon's size. The period of operation of the dead reckoning function is approximately six hours or less and covering an area not greater than 40 km across. Over this area, large measureable vertical deflections, as may be caused by mountain masses, will likely remain constant.

The total vertical deflection will be distributed along the ϕ_y direction or the ϕ_x direction dependent on the heading of the vehicle. The maximum one sigma deviation of gravity caused vertical errors has been assumed as one arc minute. If the heading of the vehicle can be assumed as an arbritrary random function then the deflection along the ϕ_y axis, or the ϕ_x axis, can be assumed as the rms values of the sine function, or in this case, as .707 arc minutes.

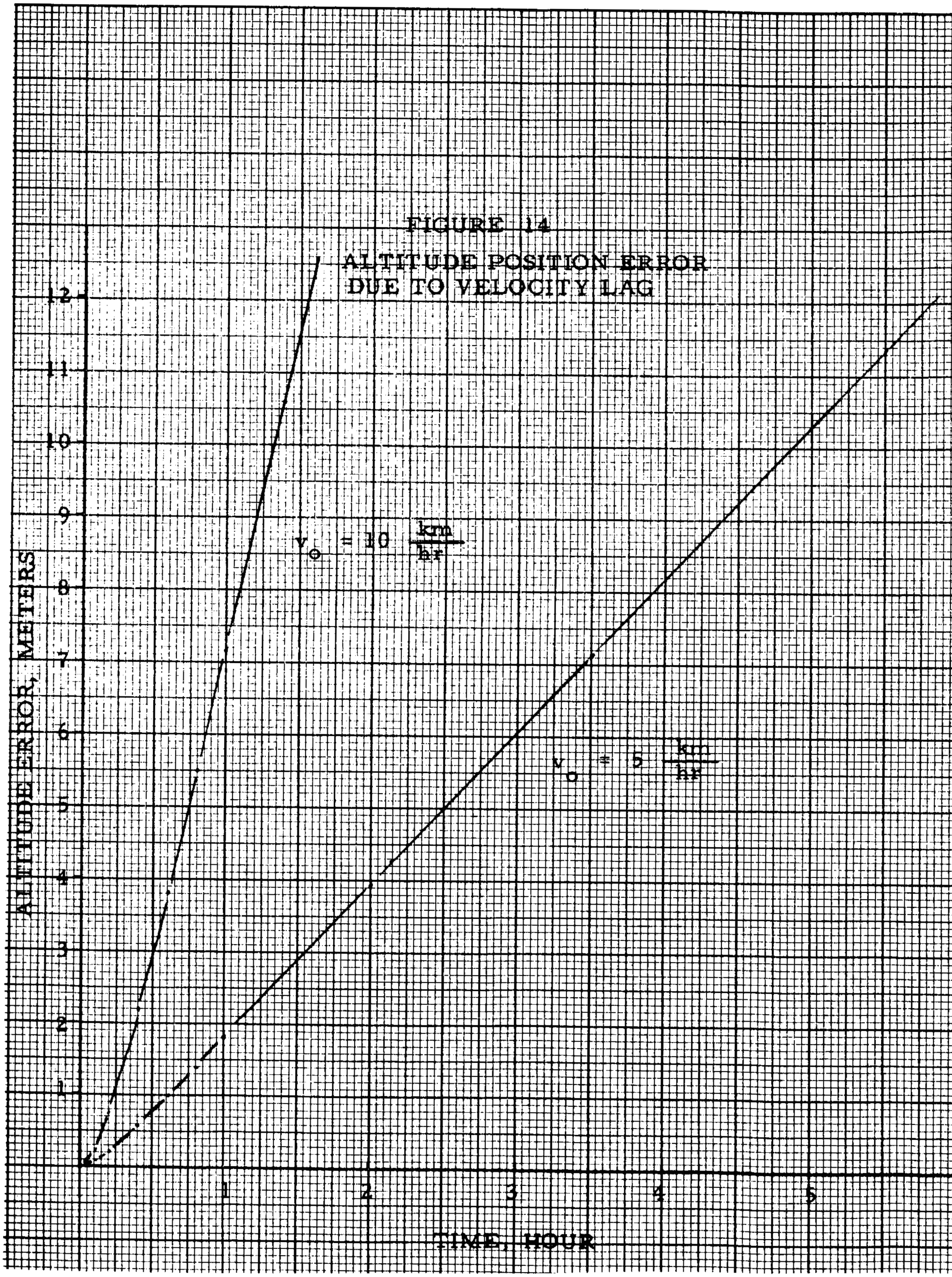


FIGURE 14. ALTITUDE POSITION ERROR DUE TO VELOCITY LAG

The position error resulting from this source, governed by the assumptions made above, is given by

$$e_z = v_o (t) (\phi_{y_g})$$

where ϕ_{y_g} is the deflection of the vertical due to gravity effects. Table 15 lists this error as function of time

TABLE 15

ALTITUDE ERROR DUE TO GRAVITY ANAMOLIES

Time (Hours)	e_z Error, Meters	
	$v_o = 5 \frac{km}{mr}$	$v_o = 10 \frac{km}{hr}$
0	0	0
.1	.103	.206
.2	.206	.412
.4	.412	.824
.6	.618	1.236
.8	.823	1.648
1	1.03	2.06
2	2.06	4.12
4	4.12	8.24
6	6.18	12.36

7.9 ERROR SUMMATION FOR ALTITUDE ERROR

Table 16 and 17 contain the results of an error summation of the various error sources into an overall peak error for an assumed velocity of 5 km/hr, and 10 km/hr. The results of this are shown in Figure 15.

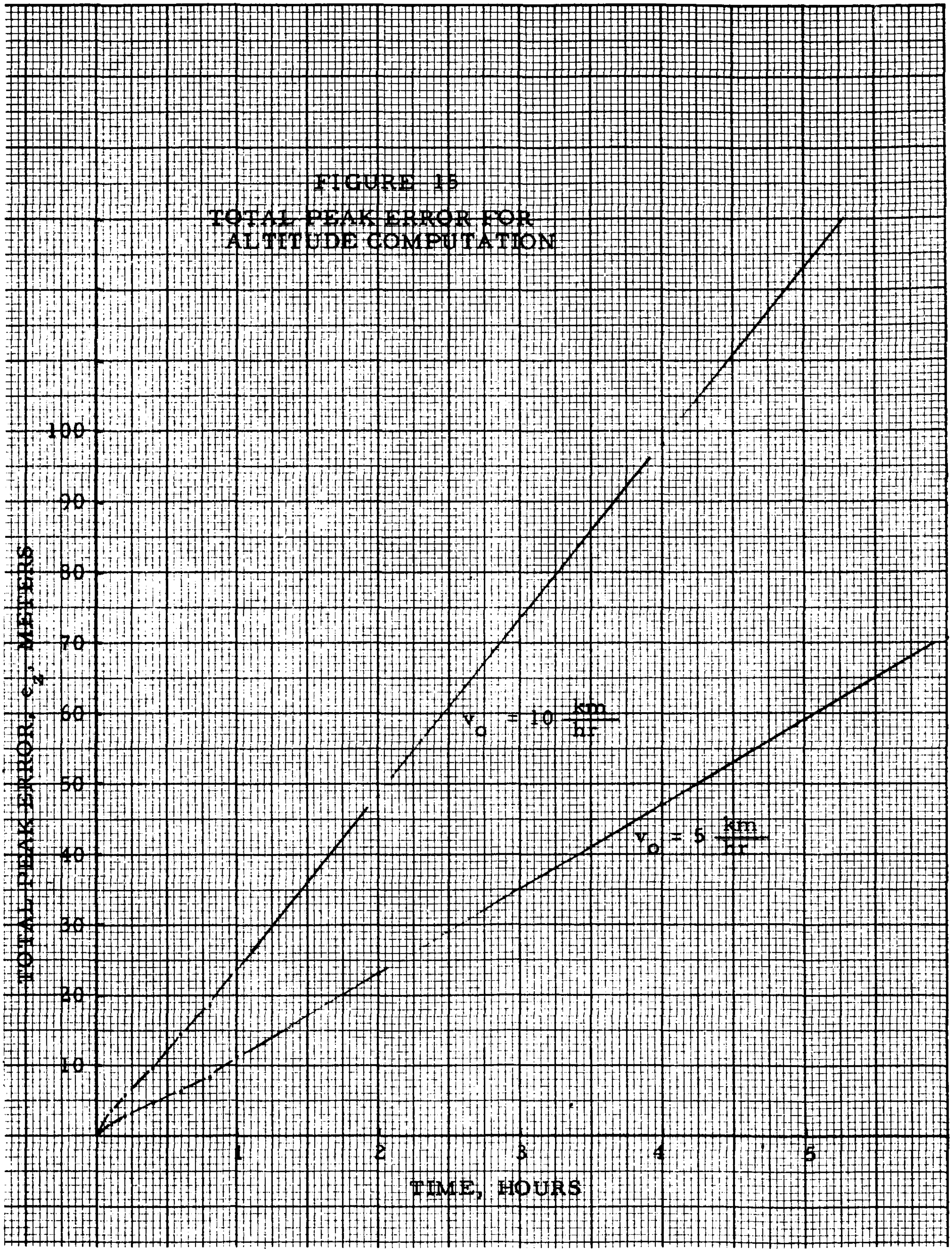


FIGURE 15. TOTAL PEAK ERROR FOR ALTITUDE COMPUTATION

TABLE 16

TABULATION FOR ALTITUDE ERROR, E_z , v_o = 5 km PER HOUR

Time	0		1		2		4		6		8		1		2		4		6	
Type Error	Noise	Bias	N	B	N	B	N	B	N	B	N	B	N	B	N	B	N	B	N	B
ERROR																				
Accelerometer Bias $1\sigma^2$ Value	0		.0194		.208		1.74		5.22		10.7		18.3		85.8		380		860	
Gyro Drift $1\sigma^2$ Value	0		.00126		.0132		.112		.334		.686		1.09		6.0		23.8		54.5	
Initial Tilt $1\sigma^2$ Value	0		.83		1.88		2.6		3.24		3.32		3.32		3.32		3.32		3.32	
Scale Factor Error $1\sigma^2$ Value			0		0		0		0		0		0		0		0		0	
Readout Error $1\sigma^2$ Value	0		.191		.761		3.05		6.88		12.2		19.1		76.2		304		688	
$\sum$ Noise Sources			1.0417		2.8622		7.502		10.976		26.902		41.81		171.32		711.12		1605.82	
3 ($\sum$ Noise Sources)			3.1251		8.5866		22.506		37.428		80.706		125.43		513.96		2133.36		4817.46	
Velocity Lag $3\sigma^2$		0		.00349		.0368		.309		.923		1.89		3.23		15.2		65.3		148
Corolios $3\sigma^2$		0		0		0		0		0		0		0		0		0		0
Gravity Bias $3\sigma^2$		0		.0106		045		.17		.382		.678		1.06		4.25		38.2		38.2
Total Peak Error	0		1.77		2.94		4.79		6.12		9.12		11.39		23.1		47.1		70.73	

TABLE 17

TABULATION FOR ALTITUDE ERROR, E_z, v_o = 10 km PER HOUR

	0		.1		.2		.4		.6		.8		1		2		4		6	
ERROR	Noise	Bias	N	B	N	B	N	B	N	B	N	B	N	B	N	B	N	B	N	B
Accelerometer Bias $1\sigma^2$ Values	0		.0773		.834		7.0		20.8		42.9		73		344		1500		3430	
Gyro Drift $1\sigma^2$ Values	0		0049		.054		.45		1.32		2.73		4.71		34.7		95		218	
Initial Tilt $1\sigma^2$ Values			3.31		7.52		11.1		13		13.2		13.2		13.2		13.2		13.2	
Scale Factor Error $1\sigma^2$ Values	0		0		0		0		0		0		0		0		0		0	
Readout Error $1\sigma^2$ Values	0		.76		3.05		12.2		27.4		48.8		76		305		1220		2740	
$\sum$ Noise Sources	0		4.152		11.458		30.75		62.52		107.63		166.91		696.9		2828.2		6401.2	
3 ($\sum$ Noise Sources)	0		12.456		34.374		92.25		187.56		322.89		500.73		2090.7		8484.6		19203.6	
Velocity Lag, $3\sigma^2$		0		.054		.583		4.93		15.8		30.2		51.5		233		1.050		2420
Corolios $3\sigma^2$		0		0		0		0		0		0		0		0		0		0
Gravity Bias $3\sigma^2$		0		.0425		.17		.68		1.525		2.71		4.25		17		68		153
Total Peak Error	0		3.54		5.926		9.89		14.31		18.87		23.59		48.38		98.0		147.56	

SECTION 8.0

DETERMINATION OF CROSS TRACK POSITION ERROR, ey

The cross track error is computed in the same manner as the altitude error. The position error is given by

$$ey = \int_o^t \phi_z \cdot v_o \cdot dt$$

8.1 CROSS TRACK ERROR, ey, DUE TO INITIAL ALIGNMENT ERROR

The cross track error is given by

$$\phi_z = \phi_{oz}$$

The resulting position error becomes

$$\phi_{oz} \; v_o t$$

This error is tabulated in Table 18.

TABLE 18
CROSS TRACK ERROR, ey, DUE TO INITIAL ALIGNMENT ERROR
Error ey, Meters

Time (Hrs)	v_o = 5 km/hr	v_o = 10 km/hr
.0	0	0
.1	.291	.582
.2	.582	1.165
.4	1.165	2.33
.6	1.75	3.5
.8	2.32	4.66
1	2.91	5.82
2	5.82	11.65
4	11.65	23.3
6	17.5	35

8.2 CROSS TRACK ERROR, ey, DUE TO GYRO DRIFT

The cross track error angle is given by

$$\phi_z(t) = \epsilon_{oz} t$$

The resulting position error is

$$ey(\epsilon) = \int_o^{\epsilon} v_o \epsilon_{oz} t' dt$$

$$= \frac{v_o \epsilon_o t^2}{2}$$

The value of this error is given in Table 19.

TABLE 19
CROSS TRACK ERROR, ey, DUE TO GYRO DRIFT, $\epsilon_{oz} = .05^o/hr$

Time (Hrs)	ey error, meters	
	v_o = 5 km/hr	v_o = 10 km/hr
0	0	0
.1	.0436	.0872
.2	.1745	.348
.4	.7	1.395
.6	1.58	3.14
.8	2.79	5.58
1	4.36	8.72
2	17.4	34.9
4	69.7	139.5
6	157.	316.

8.3 CROSS TRACK ERROR, ey, DUE TO READOUT ERROR

The position computer will calculate the vehicle's position with reference to the angle on the azimuth readout. It is assumed for this system that the readout accuracy is correct within three minutes of arc. This will also be a random function.

The error in the azimuth angle readout is denoted θ_A. θ_A has a one sigma value of three minutes of arc = .873 milliradians. The position error is given by

$$e_y = \int_0^t \theta_A v_o \, dt = \theta_A v_o t$$

This error is listed in Table 20.

TABLE 20
POSITION ERROR, ey, DUE TO READOUT ERROR

Time (Hrs)	ey Error, Meters	
	v_o = 5 km/hr	v_o = 10 km/hr
0	0	0
.1	.436	.873
.2	.873	1.745
.4	1.74	3.59
.6	2.62	5.24
.8	3.49	6.98
1	4.36	8.73
2	8.73	17.45
4	17.4	35.9
6	26.2	52.4

8.4 POSITION ERROR SUMMATION FOR CROSS TRACK, ey, ERROR TERMS

Tables 21 and 22 summarizes the major contributing error to cause a cross track error. The total peak error is three times the r. s. s. of the individual error sources. The resultant error is plotted in Figure 16.

TABLE 21
SUMMATION OF ERROR TO AFFECT CROSS TRACK ERROR,
v_o = 5 (Time Hours - 1 σ^2 Values)

Sources	0	.1	.2	.4	.6	.8	1	2	4	6
Initial Alignment	0	.0848	.339	1.36	3.06	5.39	8.5	34	136	306
Gyro Drift	0	~ 0	.029	.49	2.5	7.8	19	304	4870	24600
Readout	0	.19	.77	3.04	6.88	12.2	19	762	306	688
Total Peak Error	0	.908	1.85	3.83	6.11	8.72	11.81	34.12	126.24	277

TABLE 22

SUMMATION OF ERRORS TO AFFECT CROSS TRACK ERROR,
v_o = 10 km/hr

	$1\sigma^2$ Values									
	Time - Hour									
Sources	0	.1	.2	.4	.6	.8	1	2	4	6
Initial Alignment	0	.339	1.36	5.41	13.25	21.7	33.9	136	543	1225
Gyro Drift	0	~0	.121	1.96	9.9	31.1	76	1220	19400	100000
Readout	0	.77	3.04	12.8	27.5	49	76.2	306	1290	2750
Total Peak Error	0	1.82	3.69	7.77	12.32	17.48	23.48	70.61	252.4	558.5

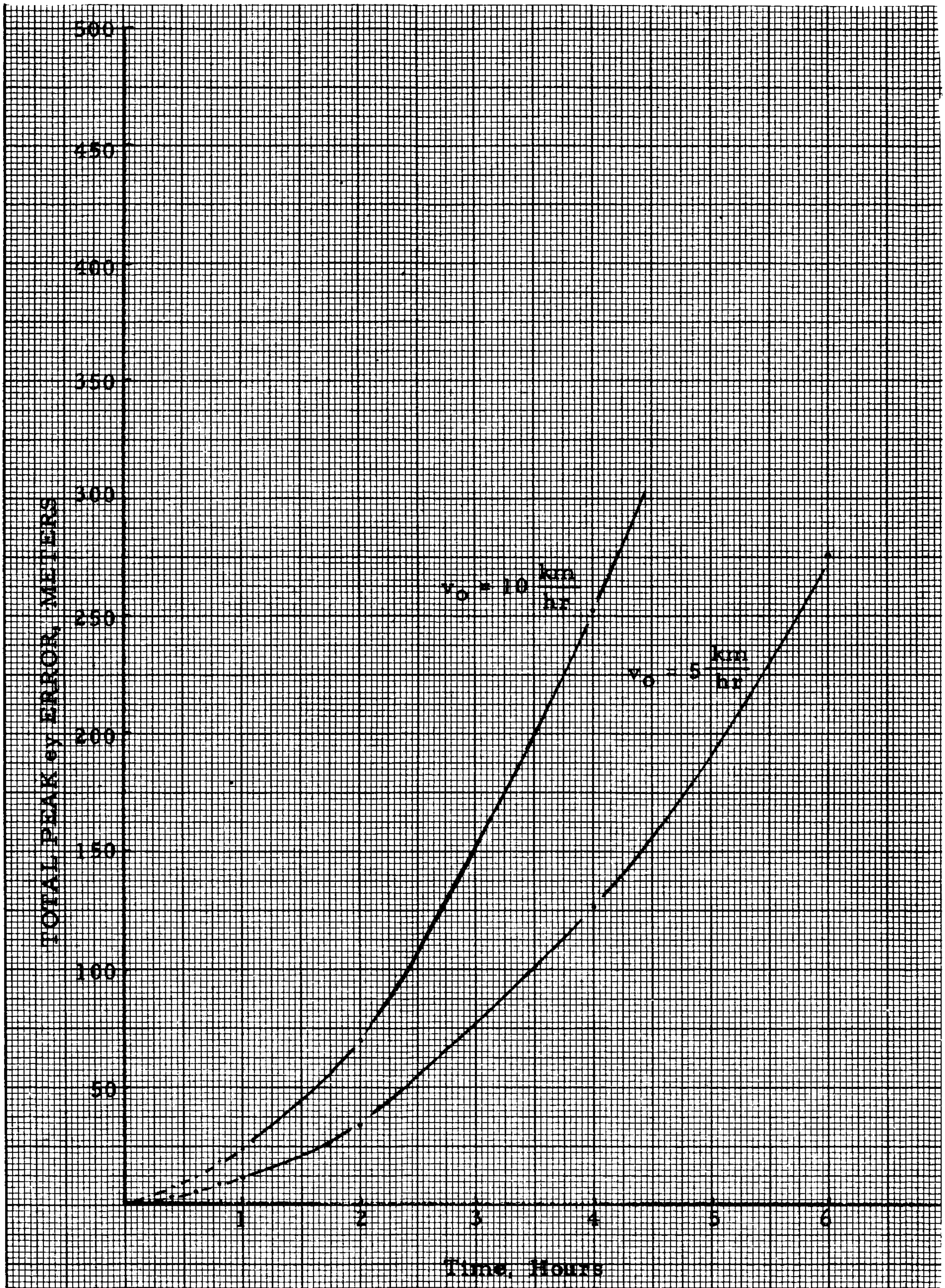


FIGURE 16
TOTAL PEAK CROSS TRACK ERROR

SECTION 9.0

CONCLUSIONS AND RECOMMENDATIONS

9.1 CONCLUSIONS

A navigation system of the type discussed herein will provide sufficiently accurate position information to allow safe conduct of a lunar vehicle for missions involving times up to six hours. Beyond six hours, the error in the track and cross track directions for typical velocities of 5 to 10 kilometers per hour will become excessive; on the order of 300 meters in the track direction (due to distance measurement inaccuracy) and 300 meters in the cross track direction (due to component inaccuracies). This error is sufficiently large that the error in absolute position of the vehicle approaches one kilometer. This is obtained when combining the dead reckoning error with the error in the initial position fix. At this point a new position fix should be made.

The theme of this investigation was to study a relatively simple system configuration, not relying on high accuracy gyros and complicated compensation methods. The drift rate of the azimuth gyro was selected as .05 degrees per hour. It is not anticipated that a small, lightweight, rugged gyro, with higher accuracy, suitable for use on the moon will be available during the time frame of the first lunar exploration.

Relatively simple vertical gyro instrumentation will achieve altitude computation to an accuracy of 70 meters.

The acceleration compensation is the singlemost critical item affecting system error. If the acceleration error induced by the vehicle can satisfactorily be compensated for then a system of the sort discussed in this report is entirely satisfactory. If the acceleration compensation cannot be mechanized then a more complicated system, i.e., Type II vertical loop leaves the only alternative for the maintaining of an accurate vertical.

9.2 RECOMMENDATIONS

It is recommended that a dead reckoning system be breadboarded and tests performed on its accuracy. This should be in cross country terrain situation to simulate vehicle accelerations that may be encountered on the moon. This will allow the determination of the velocity and acceleration profile of a vehicle. When this information is available an analysis may be performed on a computer to determine the best system.

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APPENDIX A

RESPONSE FUNCTION OF VERTICAL LOOP TO TYPICAL INPUT ERROR SOURCES

From the block diagram, Figure 8, the loop gain function for each input is obtained.

(a) Velocity Input, v_o

$$\frac{\phi y(s)}{\frac{V(s)}{RS}} = \frac{1}{1 + \frac{Kg}{s}} = -\frac{s}{s + Kg}$$

$$V(s) = v_o/s$$

$$\therefore \quad \phi y(s) = -\frac{v_o/R}{s(s + Kg)}$$

(b) Accelerometer Bias, a_x

$$\frac{\phi y(s)}{a_o(s)} = \frac{K/s}{1 + Kg/s} = -\frac{K}{s + Kg}$$

$$a_o(s) = \frac{a_x}{s}$$

$$\therefore \quad \phi y(s) = \frac{a_x K}{s(s + Kg)}$$

(c) Initial Tilt, ϕ_o

$$\frac{\phi y(s)}{\frac{\phi_o}{s}} = \frac{1}{1 + \frac{Kg}{s}} = \frac{s}{s + Kg}$$

$$\therefore \quad \phi y(s) = \frac{\phi_o}{s + Kg}$$

(d) Scaling Error, a_1

The error angle in the presence of scaling error is determined as follows:

$$\phi y(s) = s\,V(s)\ \frac{b^2 k}{1 + \frac{b^2 K}{s}} - \frac{V(s)}{R\,(s)}\ \frac{1}{1 + \frac{b^2 K}{s}}$$

$$= s\,V(s)\ \frac{b^2 K}{s + w_1} - \frac{V(s)}{R}\ \frac{1}{s + w_1}$$

letting $b^2 = a_1 + 1$

$$\phi\ y\ (s) = \frac{s\,V(s)\,a_1 K + s\,V(s)\,K - V(s)\,/R}{s + w_1}$$

The error angle with the correct gain is

$$\phi y(s) = \frac{s\,V(s)\ K - V(s)/R}{s + Kg}$$

Thus the error angle due to scale factor error becomes

$$\phi_{(s)} = \frac{s\,V(s)\,a_1\ K + s\,V(s)\ K - V(s)/R}{s + w_1} - \frac{s\,V(s)\ K - V(s)/R}{s + Kg}$$

(e) Gyro drift, ϵ_y

$$\frac{\phi y(s)}{\epsilon y\ (s)} = \frac{1}{K}\ \frac{\frac{K}{s}}{1 + \frac{Kg}{s}} = \frac{1}{s + Kg}$$

$$\epsilon y(s) = \frac{\epsilon o}{s}$$

$$\therefore\ \phi y(s) = \frac{\epsilon o}{s\ (s + Kg)}$$

APPENDIX B

RESPONSE FUNCTION OF TYPE I VERTICAL GYRO SYSTEM TO A CONSTANT ACCELERATION

The error angle response of a type 1 vertical loop is given by

$$\phi(s) = \frac{Ks - 1/R}{s + Kg} V(s) \qquad (1)$$

Let $V(s) = \frac{a_o}{s^2}$ for a constant acceleration, equivalent to a ramp input of vehicle velocity.

$$\phi(s) = \frac{Ks - 1/R}{s + Kg} \frac{a_o}{s^2}$$

$$\phi(s) = \frac{a_o K}{s(s + Kg)} - \frac{a_o/R}{s^2(s + Kg)} \qquad (2)$$

Evaluation of 1st term in equation (2) by partial fraction expansion:

$$\frac{a_o K}{s(s + Kg)} = \frac{A}{s} + \frac{B}{s + Kg} \qquad (3)$$

$$A = \lim_{s \to o} s \left(\frac{a_o K}{s(s + Kg)} \right) = \frac{a_o}{g} \qquad (4)$$

$$B = \lim_{s \to o} (s + Kg) \left(\frac{a_o K}{s(s + Kg)} \right) = -\frac{a_o}{g} \qquad (5)$$

$$\frac{a_o K}{s(s + Kg)} = \frac{a_o}{g} \left(\frac{1}{s} \right) - \frac{a_o}{g} \left(\frac{1}{s + Kg} \right) \qquad (6)$$

Evaluation of 2nd term in equation (2) by partial fraction expansion.

$$\frac{a_o/R}{s^2(s + Kg)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s + Kg} \qquad (7)$$

multiply by s^2

$$\frac{a_o/R}{s + Kg} = A + Bs + \frac{Cs^2}{s + Kg} \qquad (8)$$

let $s = O \therefore \quad A = \dfrac{a_o}{RKg}$ (9)

differentiate equation (8) with respect to s

$$-\frac{a_o/R}{(s + Kg)^2} = B + \frac{2Cs}{s + Kg} - \frac{2Cs^2}{(s + Kg)^2} \qquad (10)$$

let $s = O \therefore \quad B = -\dfrac{a_o}{R(Kg)^2}$ (11)

multiply equation (7) by (s + Kg)

$$\frac{a_o/R}{s^2} = \frac{s + Kg}{s^2} A + \frac{s + Kg}{s} B + C \qquad (12)$$

let $s = -Kg \therefore \quad C = \dfrac{a_o}{RK^2g^2}$ (13)

$$\therefore \frac{a_o/R}{s^2(s + Kg)} = \frac{a_o}{RKg}\left(\frac{1}{s^2}\right) + \left(\frac{-a_o}{RK^2g^2}\right)\left(\frac{1}{s}\right) + \left(\frac{a_o}{R(Kg)^2}\right)\frac{1}{(s + Kg)} \qquad (14)$$

substituting equations (6) and (14) into equation (2) the Laplace expression for the total ϕ (s) is obtained. This then reduces to the following time expression for ϕ (t) due to a constant acceleration.

$$\phi(t) = \frac{a_o}{g}\left(1 - e^{-Kgt}\right) + \frac{a_o}{RKg}(t) + \frac{a_o}{R(Kg)^2}\left(1 - e^{-Kgt}\right) \qquad (15)$$

The first and third terms of equation (15) reduce to constant values after several time constants, leaving the dominate error as term two which increases linearily with time.

<u>Position error resulting from a constant acceleration.</u>

The position error is given by

$$e_z = \int \phi(t)\ v(t)\ dt \qquad (16)$$

$\phi(t)$ is given by expression (15)

$v(t)$ for a constant acceleration = $a_o t$

$$\therefore e_z = \left[\frac{a_o^{\,2}}{g} + \frac{a_o^{\,2}}{R(Kg)^2}\right] \int_0^t \left(t^1\right)\left(1 - 3^{-kgt}\right) dt + \frac{a_o^{\,2}}{RKg} \int_0^t \left(t^1\right)^2 dt \qquad (17)$$

Evaluation of this expression between the limits of o and t results in the following expression for the position error.

$$e_z = a_o^{\,2} \left\{ \left[\frac{1}{g} + \frac{1}{R(Kg)^2}\right]\left(\frac{t^2}{2} + \frac{te^{-Kgt}}{3RKg}\right) + \frac{t^3}{3RKg} \right\} \qquad (18)$$

This term is evaluated for three acceleration levels, .5 ft/sec^2, 1 ft/sec^2, and 2 ft/sec^2. The maximum velocity capability is assumed to be 20 ft/sec. Therefore, maximum times for evaluation are, respectively, 40 sec, 20 sec, and 10 sec. The results of this evaluation is shown in Table 1 .

TABLE 1

ALTITUDE POSITION ERROR VERSUS TIME

(Type 1 vertical loop with loop gain, Kg, = 6.9 hr^{-1}, for constant acceleration)

Time (Sec.)	Resultant Postion Error in feet		
	a_o = .5 ft per sec^2	a_o = 1 ft per sec^2	a_o = 2 ft per sec^2
5	151.5	606	1212
10	306	1225	2450
15	457	1828	3656
20	606	2425	
25	756		
30	903		
35	1060		
40	1196		

DISTRIBUTION

INTERNAL

DIR
DEP-T
R-DIR
R-AERO-DIR
-S
-SP (23)
R-ASTR-DIR
-A (13)
R-P& VE-DIR
-A
-AB (15)
-AL (5)
R-RP-DIR
-J (5)
R-FP-DIR
R-FP (2)
R-QUAL-DIR
-J (3)
R-COMP-DIR
R-ME-DIR
-X
R-TEST-DIR
I-DIR
MS-IP
MS-IPL (8)

EXTERNAL

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