NASA TECHNICAL NOTE

NASA TN D-2856

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NASA TN D-2856

THE EVALUATION OF TWO METHODS OF SURVEYING STAR BACKGROUNDS FOR SPACE MISSION SIMULATORS

by Burnett L. Gadeberg

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Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JUNE 1965

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SUMMARY

Two methods have been developed for surveying the star backgrounds of space mission simulators. The first method requires no prealinement or measurement of the simulator of any kind. Sights of a reference cross are used to establish the orientation of the basic coordinate system and the relative dimensions of all pertinent components of the simulator. When these relative dimensions of the simulator are combined with sights of the simulated stars, their relative positions are determined. If the reference cross is constructed with arm dimensions known in terms of a standard arm length, then all dimensions determined by the survey are given in terms of the standard arm length. The second method, which makes use of fewer observations, requires that the survey instrument be prealined with the simulator coordinate system about which the cab angle instrumentation is oriented. In this system all dimensions are given in terms of the distance between the simulator center of rotation and the survey instrument. Both systems produce similar results, whose standard deviations are slightly larger than those of the observations when a minimum of data is used. Both systems may also be used for resurveying without interruption during an extended simulator run.

INTRODUCTION

The value of a space mission simulator depends upon its physical realism and its ability to generate problem situations accurately. One of the purposes of such a simulator is to provide a realistic environment in which methods of navigation and guidance may be evaluated and crews may be trained in navigation techniques. The Midcourse Navigation and Guidance Simulator constructed at the Ames Research Center is provided with a movable spacecraft type cab from which simulated stars and moon may be observed. The "moon" is programmed to move among the stars according to the mission profile. Although the navigation and the guidance computations may be programmed on a digital computer and sight angles for a given simulated trajectory may be computed outside the simulator, the errors introduced by a pilot's sighting may be best generated by taking the difference between a pilot-observed angle and an angle accurately computed from the coordinates of the two simulated celestial bodies sighted. The computed angle is a function of the cab position, because of the parallactic effect of the displacement of the sighting instrument from the center of the air bearing. It is therefore necessary to compute the angle which existed for each position of the cab when a measurement was made.

In order to minimize the bias errors in this pilot-generated noise, one must know the coordinates of the objects sighted with sufficient accuracy that the error of the computed angle is small compared to that of the pilot-generated noise. One may assume that if the expected error of a pilot's sextant sight is of the order of 10 seconds of arc, then the errors of the computed angle should be of the order of 1 or 2 seconds of arc. For a simulator of the type constructed at the Ames Research Center, where the pilot is located approximately 40 feet from the simulated stars, the coordinates of the stars must be known, relative to each other, within approximately 0.004 inch in two coordinates normal to the line of sight. Obviously stadiametric measurements over a distance of 40 feet should be avoided if possible. Professional surveyors employ methods which include precise linear measurements and are designed to give results in dimensional numbers. The problem here was to determine an angle from data of an angular survey. It was consequently decided to develop an independent survey method appropriate to the problem which did not include a stadiametric measurement.

Two methods have been devised by which these star coordinates may be surveyed to the requisite accuracy. One method uses a precise reference cross mounted near the plane of the simulated stars and an accurate theodolite mounted in the cab in a fixed position convenient for the surveyor to operate. The advantage of this system is that the simulator may be assembled without alinement or measurement and all necessary quantities may be derived from the calibration observations made with the theodolite. The second method makes the survey as simple as possible by alining the instrument coordinate system with the simulator coordinate system. The magnitude of the instrument vector relative to the air bearing need not be measured, however, since this quantity is used to make the entire analysis dimensionless.

These survey methods are discussed and some of their advantages and disadvantages are indicated through comparisons with computations of known examples.

NOTATION

The notation used in this report represents three classes of numbers: scalars, vectors, and matrices. Vectors are distinguished with a super-bar and matrices are enclosed in brackets.

A azimuth angle

$\left.\begin{matrix} A \\ B \\ C \end{matrix}\right\}$ coordinates of center of air bearing

$[A_T]$ transformation matrix from instrument coordinate system to simulator coordinate system using cab position angles

$\left.\begin{array}{l} a \\ b \\ c \\ d \\ e \\ f \end{array}\right\}$ coordinates of centers of circles

$\bar{a}$ vector between two instrument positions

h altitude angle

$\left.\begin{array}{l} \bar{I} \\ \bar{J} \\ \bar{K} \end{array}\right\}$ unit vectors of simulator coordinate system established by rotating $\bar{i},\bar{j},\bar{k}$ system 180° about $\bar{k}$

$\left.\begin{array}{l} \bar{I}' \\ \bar{J}' \\ \bar{K}' \end{array}\right\}$ unit vectors of coordinate sytem established by graduated circles of sighting instrument

$\left.\begin{array}{l} \bar{i} \\ \bar{j} \\ \bar{k} \end{array}\right\}$ unit vectors of coordinate system established by reference cross

K dummy constant

$\left.\begin{array}{l} l \\ m \\ n \end{array}\right\}$ direction cosines

[N] transformation matrix from instrument coordinate system to simulator system using reference cross sights

R dummy variable

r magnitude of vector

$\bar{r}$ vector of instrument from air-bearing center

r' radius of circle

s crossarm length

$\bar{u}$ unit vector from instrument to reference cross

$\left.\begin{array}{l} x \\ y \\ z \end{array}\right\}$ coordinates

Z dummy variable

α radius of circle

β radius of circle

γ angle subtended by reference crossarm as seen from instrument

δ angle between $\bar{a}$ and line of sight to a "star"

ϵ parallactic angle of a "star"

$\left.\begin{matrix}\varphi\\ \theta\\ \psi\end{matrix}\right\}$ cab position angles representing roll, pitch, and yaw (see fig. 8)

ANALYSIS OF FIRST METHOD

The Simulator

The Ames Midcourse Simulator cab (fig. 1) is mounted on an air bearing with center of rotation at C. Internally a sighting instrument is located at Inst., with vector $\bar{r}$ locating it relative to the air-bearing center. Approximately 40 feet from the cab is a vertical display containing the illuminated stars, the moon, and a survey cross.

For the first survey method, the position of the survey instrument and the position of the center of the air bearing are determined before the coordinates of the stars are computed. These positions are determined from observations made on a reference cross mounted in the plane of the starboard. It will subsequently be shown that if sights are taken of five scribe marks on the cross, the position of the instrument may be determined relative to the cross. Then if observations are made with the instrument in four different positions (by rotating the cab on the air bearing), the location of the center of the air bearing relative to the cross may be determined, and, in addition, the vector to the instrument from the center of the air bearing may be found. Observing the stars from two of these instrument positions permits the determination of three coordinates of each star relative to the center of the air bearing. All linear quantities may be made dimensionless by dividing by the length of one of the crossarms; consequently, no stadiametric measurements need be made since the final output from the system will be the computation of the angle between two of the stars as seen from some particular location of the sighting instrument.

The survey cross is composed of two straight arms, intersecting orthogonally near their centers and scribed with five cross marks located at the ends of each arm and at their common center. Practical reasons may dictate equidistant crossarms, but this has no bearing on the problem considered here. The vertical and horizontal arms of the cross generate a natural coordinate system. This system has its origin at the center of the cross, $\bar{k}$ axis along the vertical arm with positive direction upward, $\bar{j}$ axis horizontal and positive to the left (when facing the cab), and $\bar{i}$ axis normal and in the direction of the cab. The basic, fixed simulator system $(\bar{I},\bar{J},\bar{K})$, however, is taken with origin at C and is generated by rotating the $(\bar{i},\bar{j},\bar{k})$ system through 180° about the $\bar{k}$ axis.

Analysis of Instrument Position

The geometry involved in the determination of one of the instrument positions by surveying the cross is shown in figure 2. The instrument is located at Inst. and the five scribe marks on the cross are at points 0, 1, 2, 3, and 4. Unit vectors from the instrument position to the sighting marks on the cross are indicated by $\bar{u}_0$, $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$, and $\bar{u}_4$. If the sighting instrument provides an azimuth angle, A, and an altitude angle, h (see fig. 3), then unit vectors are given by

$$\bar{u}_0 = \cos h_0 \cos A_0 \bar{I}' - \cos h_0 \sin A_0 \bar{J}' + \sin h_0 \bar{K}'$$

$$\bar{u}_1 = \cos h_1 \cos A_1 \bar{I}' - \cos h_1 \sin A_1 \bar{J}' + \sin h_1 \bar{K}'$$

$$\bar{u}_2 = \cos h_2 \cos A_2 \bar{I}' - \cos h_2 \sin A_2 \bar{J}' + \sin h_2 \bar{K}'$$

$$\bar{u}_3 = \cos h_3 \cos A_3 \bar{I}' - \cos h_3 \sin A_3 \bar{J}' + \sin h_3 \bar{K}'$$

$$\bar{u}_4 = \cos h_4 \cos A_4 \bar{I}' - \cos h_4 \sin A_4 \bar{J}' + \sin h_4 \bar{K}'$$

where the right-hand triad $\bar{I}'$, $\bar{J}'$, $\bar{K}'$ is the coordinate system of the instrument. Then the angles observed between the points of the cross are defined by

$$\cos \gamma_1 = \bar{u}_1 \cdot \bar{u}_0$$

$$\cos \gamma_2 = \bar{u}_2 \cdot \bar{u}_0$$

$$\cos \gamma_3 = \bar{u}_3 \cdot \bar{u}_0$$

$$\cos \gamma_4 = \bar{u}_4 \cdot \bar{u}_0$$

The analysis may now be separated into horizontal and vertical independent groups. The horizontal group is involved with points 0, 1, and 2 of the cross and the vertical group with points 0, 3, and 4. The horizontal analysis will provide the y component of the instrument position (see fig. 4) and an x_{horiz} component which is normal to the horizontal arm of the cross, but which does not necessarily lie in the xy plane. The vertical analysis provides the z component and an x_{vert} component normal to the vertical arm which does not necessarily lie in the xz plane. The final x component can then be computed from either x_{horiz} and z or x_{vert} and y or it may be advantageous to average the two values. The latter method was found to give the best results for the simulator considered in this study; thus

$$x = \frac{1}{2}\left[\left(x_{vert}^2 - y^2\right)^{1/2} + \left(x_{horiz}^2 - z^2\right)^{1/2}\right]$$

Since the horizontal and vertical analyses are identical, only the horizontal will be considered. In figure 5 points 0, 1, and 2 represent the sighting points on the horizontal arm of the cross and Inst. is the instrument location whose position is required. The lengths of the crossarms s_1 and s_2 and the sight angles γ_1 and γ_2 are known. If the angle γ_1 is known an elementary proposition of plane geometry states that the unknown position lies on a circle through the points 0 and 1, whose center is so located that the central angle between the points 0 and 1 is $2\gamma_1$. Thus two intersecting circles through points 0 and 1 and through points 0 and 2 would give an unambiguous solution, since one of the points of intersection is at the origin (the center of the survey cross which is known to be at a different position than the instrument). However, to facilitate the solution by a digital computer program, a third circle is used which passes through points 1 and 2 and the unknown position, Inst. The equations of three circles are then

$$(x - a)^2 + (y - b)^2 = \alpha^2$$

$$(x - c)^2 + (y - d)^2 = \beta^2$$

$$(x - e)^2 + (y - f)^2 = r'^2$$

and are solved in the following manner. Each is expanded, and it may be seen from figure 5 that

$$a^2 + b^2 = \alpha^2$$
$$c^2 + d^2 = \beta^2$$
$$e^2 + f^2 \neq r'^2$$

Then

$$x^2 + y^2 - 2ax - 2by = 0$$
$$x^2 + y^2 - 2cx - 2dy = 0$$
$$x^2 + y^2 - 2ex - 2fy = r'^2 - (e^2 + f^2)$$

Now let

$$-2Z = x^2 + y^2$$
$$-2K = r'^2 - (e^2 + f^2)$$

Thus

$$ax + by + Z = 0$$
$$cx + dy + Z = 0$$
$$ex + fy + Z = K$$

and the solution is

$$\begin{Bmatrix} x \\ y \\ Z \end{Bmatrix} = \begin{bmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ K \end{Bmatrix}$$

The elements of the matrix and the vector are computed from the following simple equations, which may be deduced from the geometry of figure 5 and the definition of K. They are

$$a = -b/\mathrm{tg}\gamma_1$$
$$b = -s_1/2$$
$$c = d/\mathrm{tg}\gamma_2$$
$$d = s_2/2$$
$$e = (d - b)/\mathrm{tg}(\gamma_1 + \gamma_2)$$
$$f = d + b$$
$$r' = (d - b)/\sin(\gamma_1 + \gamma_2)$$
$$K = -[r'^2 - (e^2 + f^2)]/2 = -s_1 s_2/2$$

The quantity Z is, of course, redundant and is not independent of x and y. However, the proof of the solution may be readily obtained if the expressions for the elements of the matrix are expanded and substituted. The x and y determined in this solution are, in reality, either x_{horiz} and y or x_{vert} and z, depending upon whether we are analyzing the horizontal or vertical sights. After x_{horiz} and x_{vert} have been computed, x is computed from

$$x = \frac{1}{2}\left[\left(x_{vert}^2 - y^2\right)^{1/2} + \left(x_{horiz}^2 - z^2\right)^{1/2}\right]$$

Analysis of Air-Bearing Center Position

Now assume that the instrument has been located with respect to the reference cross for four different positions of the simulator cab. These four points lie on the surface of a sphere, and if they are judiciously chosen, are sufficient to define this sphere and thus locate the center of the air bearing with respect to the center of the cross. The equation of the sphere, relative to the reference cross coordinate system, is given by

$$(x - A)^2 + (y - B)^2 + (z - C)^2 = r^2$$

where A, B, C, and r represent the unknown position of the center and the radius of the sphere, and x, y, z, the known position of the instrument previously determined. Four unknowns are present; hence four equations (from four instrument positions) are required for solution.

If the equations are expanded, they may be put in a symmetrical linear form by the substitutions

$$2K = r^2 - (A^2 + B^2 + C^2)$$
$$2R = (x^2 + y^2 + z^2)$$

and for four positions of the instrument, these are

$$x_1A + y_1B + z_1C + K = R_1$$
$$x_2A + y_2B + z_2C + K = R_2$$
$$x_3A + y_3B + z_3C + K = R_3$$
$$x_4A + y_4B + z_4C + K = R_4$$

Then the solution is

$$\begin{Bmatrix} A \\ B \\ C \\ K \end{Bmatrix} = \begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix}$$

The radius of the sphere generated by the instrument position is then found from

$$r = [2K + A^2 + B^2 + C^2]^{1/2}$$

If greater accuracy is required, observations may be taken from more than four instrument positions. In this case a least-squares reduction may be employed and the observation equations become

$$x_1A + y_1B + z_1C + K = R_1$$
$$x_2A + y_2B + z_2C + K = R_2$$
$$\vdots$$
$$x_nA + y_nB + z_nC + K = R_n$$

These n equations are then reduced to four normalized equations. The first normalized equation is obtained by multiplying each equation above by its own coefficient of A and summing the resulting equations. A similar process using the coefficients of B, C, and K produces the three remaining normalized equations. The matrix solution is then given by

$$\begin{Bmatrix} A \\ B \\ C \\ K \end{Bmatrix} = \begin{bmatrix} \Sigma x^2 & \Sigma xy & \Sigma xz & \Sigma x \\ \Sigma xy & \Sigma y^2 & \Sigma yz & \Sigma y \\ \Sigma xz & \Sigma yz & \Sigma z^2 & \Sigma z \\ \Sigma x & \Sigma y & \Sigma z & n \end{bmatrix}^{-1} \begin{Bmatrix} \Sigma xR \\ \Sigma yR \\ \Sigma zR \\ \Sigma R \end{Bmatrix}$$

where Σ represents the summation from 1 to n of similar terms and n is the number of instrument positions from which observations were made.

The accuracy of this solution will depend, to a certain extent, upon the choice of the instrument locations. The mathematical system makes use of the deviation of the spherical surface from a plane in order to calculate the radius of curvature and the position of the center. Three of the four points required establish the plane and the fourth gives a measure of the deviation from this plane. The greatest accuracy will be attained when the plane and the deviation from that plane are established with the greatest possible accuracy. Hence, when four points are used, three of them should lie at the vertices of an equilateral triangle, with maximum possible distance between them, and the fourth should be located at the center of the triangle (when projected on the plane). When five or more points are used, the best possible configuration is somewhat more obscure. The five positions chosen for the test data used to check the method were predicated on a "maximum neighborhood area" distribution over the portion of the sphere accessible to the observation instrument. Thus, four positions were used to describe a square and the fifth was located in the center of the square (when seen projected).

Analysis of Star Position

The vector positions of the sighting instrument and the center of the air bearing with respect to the reference cross coordinate system have been determined. Consequently, the vector positions of the instrument and the stars may now be determined relative to the simulator coordinate system with the center of the air bearing as origin.

Up to the present we have not concerned ourselves with the orientation of the sighting instrument and its coordinate system. It was not necessary because we worked exclusively with the scalar products of the unit vectors observed with the instrument. Now, however, we are interested in determining the vectors of the stars from the air-bearing center expressed in the simulator system. Thus, the transformation from the instrument coordinate system $(\bar{I}',\bar{J}',\bar{K}')$ to the simulator system $(\bar{I},\bar{J},\bar{K})$ must be determined.

Consider three unit vectors from a given instrument location to the three points located on the cross at 0, 1, and 3. As was done in a previous section, these may be written

$$\bar{u}_0 = l_0\bar{I}' + m_0\bar{J}' + n_0\bar{K}'$$

$$\bar{u}_1 = l_1\bar{I}' + m_1\bar{J}' + n_1\bar{K}'$$

$$\bar{u}_3 = l_3\bar{I}' + m_3\bar{J}' + n_3\bar{K}'$$

where

$$l_0 = \cos h_0 \cos A_0 \qquad m_0 = -\cos h_0 \sin A_0 \qquad n_0 = \sin h_0$$

$$l_1 = \cos h_1 \cos A_1 \qquad m_1 = -\cos h_1 \sin A_1 \qquad n_1 = \sin h_1$$

$$l_3 = \cos h_3 \cos A_3 \qquad m_3 = -\cos h_3 \sin A_3 \qquad n_3 = \sin h_3$$

where h and A are the observed altitude and azimuth angles.

These same unit vectors in the simulator coordinate system may be written in terms of the instrument position x,y,z and the lengths of the crossarms s_1 and s_3 as (see fig. 2)

$$\bar{u}_0 = (x/r_0)\bar{I} + (y/r_0)\bar{J} - (z/r_0)\bar{K}$$

$$\bar{u}_1 = (x/r_1)\bar{I} + [(s_1 + y)/r_1]\bar{J} - (z/r_1)\bar{K}$$

$$\bar{u}_3 = (x/r_3)\bar{I} + (y/r_3)\bar{J} - [(s_3 + z)/r_3]\bar{K}$$

where

$$r_0 = [x^2 + y^2 + z^2]^{1/2}$$

$$r_1 = [x^2 + (s_1 + y)^2 + z^2]^{1/2}$$

$$r_3 = [x^2 + y^2 + (s_3 + z)^2]^{1/2}$$

It should be kept in mind that the coordinates of the center of the reference cross in the simulator coordinate system are the same as the coordinates of the instrument in the cross coordinate system with the sign of the z component reversed.

By equating the vectors written in the two systems we may solve for the transformation matrix

$$[N] = \begin{bmatrix} l_0 & m_0 & n_0 \\ l_1 & m_1 & n_1 \\ l_3 & m_3 & n_3 \end{bmatrix}^{-1} \begin{bmatrix} \frac{x}{r_0} & \frac{y}{r_0} & \frac{-z}{r_0} \\ \frac{x}{r_1} & \frac{(s_1 + y)}{r_1} & \frac{-z}{r_1} \\ \frac{x}{r_3} & \frac{y}{r_3} & \frac{-(s_3 + z)}{r_3} \end{bmatrix}$$

Now any unit vector observed in the instrument system may be written in the simulator system as

$$\bar{u}_s{}^S = [N]^T \bar{u}_s{}^{Inst.}$$

In order to complete the analysis we must be able to compute the magnitude of these new star vectors. Refer now to figure 6 where it may be seen that the vector of a star from the center of the air bearing may be written as

$$\bar{r}_C = \bar{r}_{Inst._1} + r_{s_1}\bar{u}_{s_1}$$

In the figure, C is the center of the air bearing, $\bar{r}_{Inst._1}$ and $\bar{r}_{Inst._2}$ are vectors to two instrument positions 1 and 2, $\bar{u}_{s_1}$ and $\bar{u}_{s_2}$ are two unit vectors of a star observed from the two instrument positions, and r_{s_1} is the magnitude associated with the unit vector $\bar{u}_{s_1}$. The vectors $\bar{r}_{Inst._1}$ and $\bar{r}_{Inst._2}$ were found previously and $\bar{u}_{s_1}$ and $\bar{u}_{s_2}$ are the transformed observed vectors of the star and thus are known. The magnitude of the vector r_{s_1} may be found from the geometry of figure 6 in the following manner

$$\frac{r_{s_1}}{\sin\delta} = \frac{a}{\sin\epsilon}$$

but

$$\sin\delta = \frac{|\bar{u}_{s_2} \times \bar{a}|}{a}$$

and

$$\sin\epsilon = |\bar{u}_{s_2} \times \bar{u}_{s_1}|$$

Hence

$$r_{s_1} = \frac{|\bar{u}_{s_2} \times \bar{a}|}{|\bar{u}_{s_2} \times \bar{u}_{s_1}|}$$

where

$$\bar{a} = \bar{r}_{Inst._2} - \bar{r}_{Inst._1}$$

Thus, the vectors to all of the stars from the center of the air bearing may be found and the analysis of the survey is complete.

TESTS

Test Data

A Fortran program was written which would accept the observed data and, using the equations formulated above (including the least-squares reduction for determining the center position of the air bearing), compute and give the coordinates of the instrument positions relative to both the reference cross and the simulator coordinate systems, and determine the positions of the center of the reference cross and the simulated stars relative to the simulator coordinate system. The angle between two stars was computed directly from the observed data and compared with the angle computed from the final coordinates of the stars determined by the survey. This was considered to be the ultimate check. Since the final purpose of the survey was to determine the coordinates of the individual stars, the angle between any two stars computed with these coordinates was checked to see whether it was correct within a specified accuracy.

In order to check the mathematical formulation and the computer program, a set of test data was hand computed to within 0.01 second of arc for a hypothetical situation for which all the coordinates and dimensions were previously established. These test conditions are shown in figure 7. The center of the cross has coordinates ($x = 40$, $y = 0$, and $z = 4$) from the center of the air bearing and has four equal arm lengths of dimension 8. Two stars, S_2 and S_1, were located at positions ($x = 40$, $y = 4$, $z = 8$) and ($x = 40$, $y = -10$, $z = 12$). The sighting instrument was considered to be located on the surface of a sphere of radius 4. Five instrument positions were used, roll left and roll right 10° (about the $\bar{I}$ axis), pitch up and down 10° (about the $\bar{J}$ axis); all of these being relative to the fifth position which was located on the $\bar{K}$ axis.

The altitudes and azimuths of the unit vectors to the five points of the cross and the two stars were then computed for each of the five instrument positions. These are listed in table I and are the "observed data" used in checking the accuracy of the results.

Accuracy of Results

The precomputed "observed data" were run with the survey program (on an IBM 7094 computer carrying eight significant figures) which computed the groups of coordinates and scalars discussed in the previous section. These were compared with the original quantities used to precompute the input data, and the accuracy of the program was assessed. The assumed quantities and the program computed quantities are listed in table II so that comparisons may be readily made. The quantities listed are: the three components of the positions of the instrument relative to the cross; the three components of the center of the cross relative to the center of the air bearing; the three components of the positions of the instrument relative to the center of the air bearing; the distance of the instrument from the center of the air bearing;

the three components of the stars relative to the center of the air bearing; the computed angle between the stars; and the error in this computed angle.

From table II it is readily seen that the computation of the instrument position relative to the reference cross coordinates gives the components of the position within 0.0003 foot or approximately 0.003 inch. The coordinates of the cross and the stars relative to the air-bearing center are in error by as much as 0.01 foot or approximately 0.1 inch. However, the error made by the program in computing the angle between the two stars, as seen from the roll left position of the instrument, is only 0.73 second of arc. This is well within the acceptable limits originally set.

Some information was also desired on the linearity of the propagation of random observation errors into the computed angle. The program was consequently modified so that random Gaussian errors of any size and zero mean value could be introduced into the observed altitude and azimuth angles. For a given standard deviation of the observation errors, 500 members of the ensemble were considered sufficient to produce a fair representation of Gaussian errors to be applied to any single observed angle. On each pass through the machine, a random error was applied to each of the observed angles, the angle between the two stars was computed, and the deviation from the original angle was determined. At the conclusion of a run, the mean error and the standard deviation of the errors were computed.

Sets of observation errors, characterized by zero mean value and standard deviations of 0.1, 0.25, 1.0, 3.0, and 5.0 seconds of arc, were introduced into the observation angles and the resulting mean errors and standard deviations computed and tabulated in the "First Method" column of table III. The resulting standard deviations are also shown plotted in figure 8 as a function of the standard deviation of the observation errors. The curve shows that the propagation of the error is linear and that the ratio is 1.5 to 1. The mean errors from table III are plotted in figure 9 (on an expanded scale), also as a function of the standard deviation of the observation errors. It is seen that these errors are essentially zero for very low values of the observation errors, but are not zero for sensible values of the observation errors. This indicates that the resulting errors in the computed angle were not strictly Gaussian in character because of nonlinearities in the system. However, the nonlinearities are minor when considered in relation to the problem of estimating output errors produced by a given set of input errors. One may assume that the performance of the survey system may be estimated reasonably from the standard deviation given by the curve of figure 8 with a "mean" error taken as the algebraic sum of the error given in figure 9 and with the computation error (0.73 sec) discussed previously. Thus if the observation errors have a standard deviation of 1 second of arc, the expected resulting error in the computed angle would be

$$0.73 - 0.06 = 0.67$$

second of arc with a standard deviation of 1.5 seconds of arc.

Some of this error is related to the limited angular motion of the cab, which in relation to the distances between the air-bearing center and the instrument and between the air-bearing center and the starboard, contrives to produce relatively long slender triangles which make it difficult to determine linear distances to the desired accuracy. Changing the geometry of the simulator could alleviate some of these difficulties.

It is apparent, from the foregoing, that this method of analysis requires a considerable number of observations. For example, five reference cross observations from at least four instrument positions are required, which is a minimum of twenty observations to determine the constants of the system. Consequently, a formulation was explored which eliminated the use of the reference cross and obtained the instrument vector relative to the center of the air bearing directly from cab position angle measurements. This second survey method is discussed in the following sections.

ANALYSIS OF SECOND METHOD

An alternate method of making the survey was devised which obviated some of the difficulties of the first method but required some additional measurements. The objective of the change was the direct determination of the instrument vector from cab position angle measurements and hence the elimination of the reference cross and its attendant observations. Observations of the stars from two instrument positions are required, as they were in the previous method, to determine the star coordinates. However, in this method the analysis is made dimensionless by dividing all linear quantities by the magnitude of the instrument vector relative to the air-bearing center.

Analysis of Instrument Position

The method used requires the alinement of the instrument coordinate system with the simulator coordinate system, and the determination of the initial vector of the instrument position when the cab position angles are all set to zero. As a result, it is necessary to physically locate the center of the air bearing. This may be done by sighting from an external point to a target placed near the center as the cab is rotated. By successive approximations, the target may be moved until it coincides with the center. The cab may then be leveled and the azimuth angle between the instrument vector and the simulator coordinate system determined. (An accuracy of about 6 minutes of arc was required for the simulator constructed at the Ames Research Center.) Then the sighting instrument may be leveled and by sighting the center of the air bearing with the instrument set at the above determined azimuth, the instrument coordinate system may be brought into alinement with the simulator system. The altitude angle of the instrument vector may now be read directly.

The instrument position vector, for any position of the cab, is then determined from

$$\bar{r}_I = [A_T]\bar{r}_o$$

where $[A_T]$ is the transformation matrix

$$[A_T] = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}$$

Here the angles φ, θ, and ψ are the position angles of the cab relative to the $\bar{I}$, $\bar{J}$, and $\bar{K}$ axes as shown in figure 10. The vector r_o is the initial instrument position vector when the three cab angles are set to zero. Hence, this vector has the form

$$\bar{r}_o = \cos h_o \cos A_o \bar{I} - \cos h_o \sin A_o \bar{J} + \sin h_o \bar{K}$$

where h_o is the altitude angle and A_o is the azimuth angle in simulator coordinates when the cab angles are set to zero.

Analysis of Star Position

If we assume now that we have determined the instrument position, relative to the air-bearing center in the simulator coordinate system, the next problem is to determine the coordinates of the star positions. The method used is almost identical to that described under the first survey method. Observations of the stars from two instrument positions are required. These observations (unit vectors) are transformed from the instrument coordinates system into the simulator coordinate system by use of the transformation matrix, $[A_T]$, described in the previous section.

The vector of a star position, relative to the air-bearing center, is then given by

$$\bar{r}_c = \bar{r}_{Inst._1} + r_{s_1}\bar{u}_{s_1}$$

where

$$r_{s_1} = \frac{|\bar{u}_{s_2} \times \bar{a}|}{|\bar{u}_{s_2} \times \bar{u}_{s_1}|}$$

and

$$\bar{a} = \bar{r}_{Inst._2} - \bar{r}_{Inst._1}$$

as developed in the previous section entitled "Analysis of Star Position," in conjunction with figure 6.

TESTS

Test Data

Determining the coordinates of the stars by the second survey method requires observations from only two positions of the cab. This was indicated in the previous section where the equations for the star survey were developed. It is rather obvious that maximum accuracy will be attained when the two instrument positions have a maximum separation from each other along a line normal to the direction to the simulated star. As a consequence, test data were precomputed for two cab positions only, in the roll left and roll right ($\pm 10^{\circ}$) positions. For each of these positions the altitude and azimuth angles of the two stars were computed to within 0.01 second of arc. The rectangular coordinates were assumed to be 40, 8, and 4 for star number 1 and 40, -8, and 10 for star number 2. The initial altitude and azimuth angles of the sighting instrument, relative to the air-bearing center, were taken to be 90° and 0° and it was assumed that the cab read out instrumentation was accurately alined with the cab coordinate system when these angles were set to zero. All of those input quantities were displayed in table IV.

Accuracy of Results

A computer program was written to accommodate this second method of analysis. The coordinates of the stars were then computed, and the results are listed in table IV along with the input data. It will be noted that the computed coordinates are all one-fourth of the input values. This is because the equations were normalized by dividing by the length of the instrument vector, which was assumed to have a value of 4. Thus, a comparison shows that all of the coordinates of the stars were determined correctly to 4 decimals except the x component of star 2 which is in error by 1 unit in the fourth decimal place. The correct angle between the two stars, the computed angle, and the error between them, as seen from the roll left position of the cab, are also shown in table IV. The error of the computed angle is only 0.02 second of arc.

The effect of random errors was again studied by the same method discussed previously. Again, 500 passes through the computer were used to generate a Gaussian set of errors to be applied to each angle. The resulting mean errors and standard deviations of the errors in the computed angle are listed in table III and shown in figures 8 and 9 as a function of the standard deviation of the input errors.

The results from this second method of analysis are similar to those from the first method. The propagation of the error is linear with the standard deviation of the input errors (fig. 8), and the existence of mean errors indicates the presence of some nonlinearities in the system.

The major performance differences between the two systems are that the second system produces results with slightly lower standard deviations and slightly higher negative mean errors. Hence, if the input to the two methods has a standard deviation of 1 second of arc, the second system will display results with a mean error of -0.13 second and a standard deviation of 1.4 seconds of arc, while the results from the first system will have corresponding values of 0.67 and 1.5 seconds of arc.

CONCLUDING REMARKS

These two theoretical methods of surveying the star backgrounds of space mission simulators discussed in this report have been shown to operate satisfactorily. When evaluated in terms of the computed angle between two stars, their expected accuracies are essentially the same and are somewhat less than the accuracy of the observed data when a minimum of data (such as that of the examples) is used. Using redundant data could improve the expected accuracy, however. The accuracies derived from the tests are also conditioned by the geometrical limitations of the examples used, which were taken from the Ames Midcourse Simulator. Limited cab angular motion, in relation to the distances between the air-bearing center and the instrument and between the air-bearing center and the starboard, results in relatively long slender triangles from which the determination of linear distances to the desired accuracy is difficult. These difficulties could be alleviated by changing the geometry of the simulator.

The two methods, when compared, have both relative advantages and disadvantages. The first method of survey permits the simulator to be assembled without alinement or measurement, as is required by the second method. On the other hand, more survey observations are required in the first method in order to determine the constants of the system and the star coordinates. Nevertheless, the first system may contain some distinct advantages, such as the possibility of a reliable recalibration in the middle of a long-term simulator run without disturbing the progress of the run. On the other hand, recalibration with the second system, which might be less reliable because of the need to orient the sighting instrument, might be accomplished much more readily because of its simplicity. It is thus apparent that the two systems are mutually complementary and both may have a place in the proper operation of a space mission simulator.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Feb. 8, 1965

TABLE I.- PRECOMPUTED TEST DATA FOR USE WITH FIRST METHOD

Reference cross and star no.	Altitude angle	Azimuth angle
	Roll left 10°	
0	0° 5'13".32	0°59'41".39
1	0 5 11.86	349 38 59.45
2	0 5 6.21	12 15 47.97
3	-11 13 28.42	0 59 41.39
4	11 23 30.96	0 59 41.39
S_1	11 0 59.77	14 58 7.52
S_2	5 46 37.94	355 16 33.91
	Roll right 10°	
0	0 5 13.32	359 0 18.61
1	0 5 6.21	347 44 12.03
2	0 5 8.26	10 21 0.54
3	-11 13 28.42	359 0 18.61
4	11 23 30.96	359 0 18.61
S_1	11 6 17.04	13 5 45.89
S_2	5 45 27.05	353 18 22.01
	Pitch up 10°	
0	0 5 8.01	0 0 0.0
1	0 5 2.24	348 52 41.73
2	0 5 2.24	11 7 18.27
3	-11 2 21.64	0 0 0.0
4	11 12 14.73	0 0 0.0
S_1	10 51 29.22	13 45 55.45
S_2	5 40 16.85	354 23 10.50
	Pitch down 10°	
0	0 5 18.90	0 0 0.0
1	0 5 12.50	348 29 43.75
2	0 5 12.50	11 30 16.25
3	-11 25 9.94	0 0 0.0
4	11 35 22.37	0 0 0.0
S_1	11 14 27.57	14 16 27.24
S_2	5 52 6.20	354 14 48.24
	Neutral position	
0	0 0 0.0	0 0 0.0
1	0 0 0.0	348 41 24.24
2	0 0 0.0	11 18 35.76
3	-11 18 35.76	0 0 0.0
4	11 18 35.76	0 0 0.0
S_1	10 58 50.07	14 2 10.48
S_2	5 40 56.78	354 17 21.86

TABLE II.- RESULTS COMPUTED BY FIRST METHOD

Instrument position relative to the reference cross					
Correct			Computed		
x	y	z	x	y	z
40.0000 40.0000 40.6946 39.3054 40.0000	-0.6946 .6946 0 0 0	-0.0608 -.0608 -.0608 -.0608 0	40.0000 40.0000 40.6946 39.3054 39.9999	-0.6948 .6944 -.0001 0 -.0001	-0.0605 -.0605 -.0607 -.0606 0
Center of reference cross from air-bearing center					
40.0000	0	4.0000	39.9902	-0.0002	4.0122
Instrument position relative to the air-bearing center					
0 0 -.6946 .6946 0	0.6946 -.6946 0 0 0	3.9392 3.9392 3.9392 3.9392 4.0000	-0.0097 -.0098 -.7043 .6848 -.0097	0.6946 -.6946 -.0001 -.0001 -.0001	3.9517 3.9517 3.9515 3.9516 4.0122
Distance of instrument from air-bearing center					
4.0000			4.0122		
Coordinates of stars					
40.0000 40.0000	-10.0000 4.0000	12.0000 8.0000	39.9913 39.9905	-10.0004 3.9999	12.0129 8.0120
Computed angle between stars					
20°9'53".39			20°9'54".12		
Error					
0".73					

TABLE III.- STATISTICAL RESULTS

Standard deviation of input error, σ, sec of arc	First method		Second method	
	$\bar{\epsilon}$, sec of arc	σ, sec of arc	$\bar{\epsilon}$, sec of arc	σ, sec of arc
0.1	0.0034	0.1498	-0.0116	0.1383
.25	-.0053	.3747	.0085	.3453
1	-.0632	1.4959	-.1501	1.3836
3	-.2082	4.4881	-.4553	4.1496
5	-.3055	7.4879	.1124	6.9001

TABLE IV.- PRECOMPUTED TEST DATA AND RESULTS FROM SECOND METHOD

$h_o = 90.0^\circ$ $A_o = 0.0^\circ$

Cab position 1

$\theta = 0.0^\circ$ $\psi = 0.0^\circ$ $\varphi = -10.0^\circ$

ID	h	A
1	1°52'19".87	349°49' 6".27
2	6 11 9.09	13 30 57.49

Cab position 2

$\theta = 0.0^\circ$ $\psi = 0.0^\circ$ $\varphi = 10.0^\circ$

ID	h	A
1	-2°1'47".77	347°54'10".86
2	10 8 21.62	8 43 46.47

Star coordinates

ID	x	y	z
1	10.0000	2.0000	1.0000
2	10.0001	-2.0000	2.5000

Correct angle	Computed angle
24°1'22".56	24°1'22".54

Error

0".02

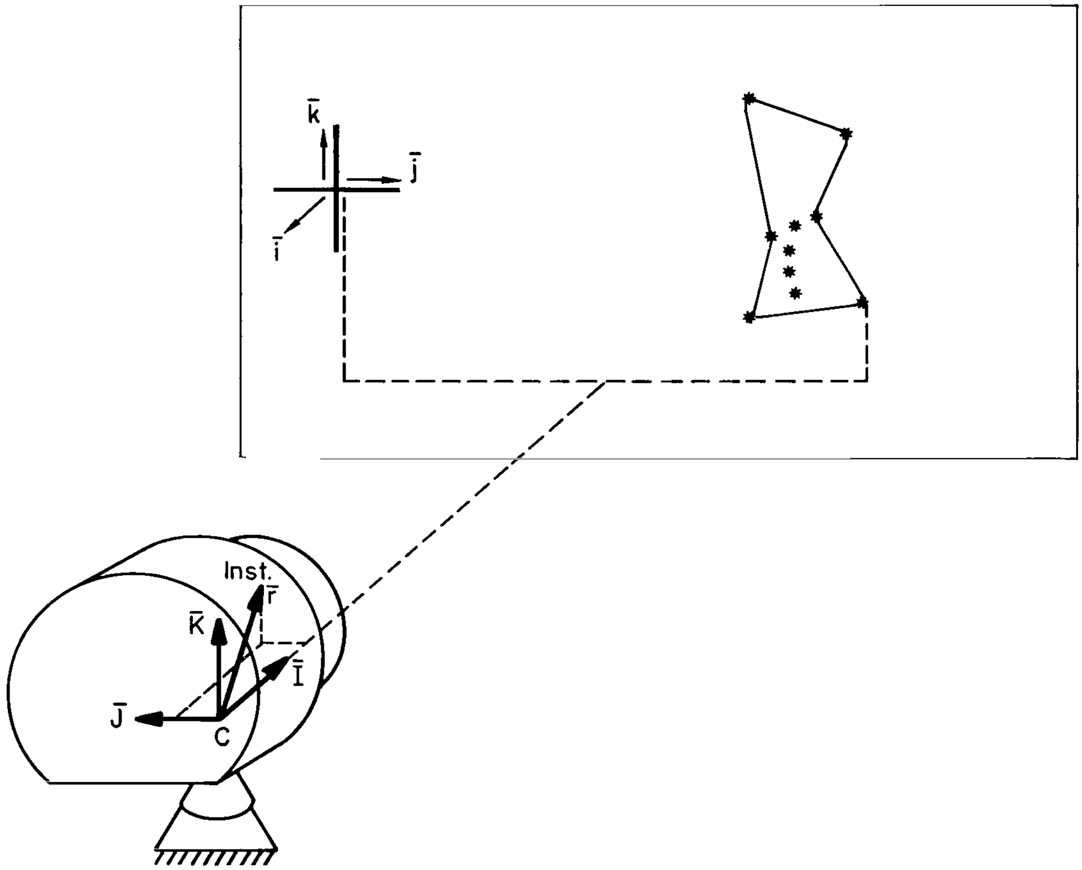

Figure 1.- Basic geometry of the Midcourse Navigation and Guidance Simulator.

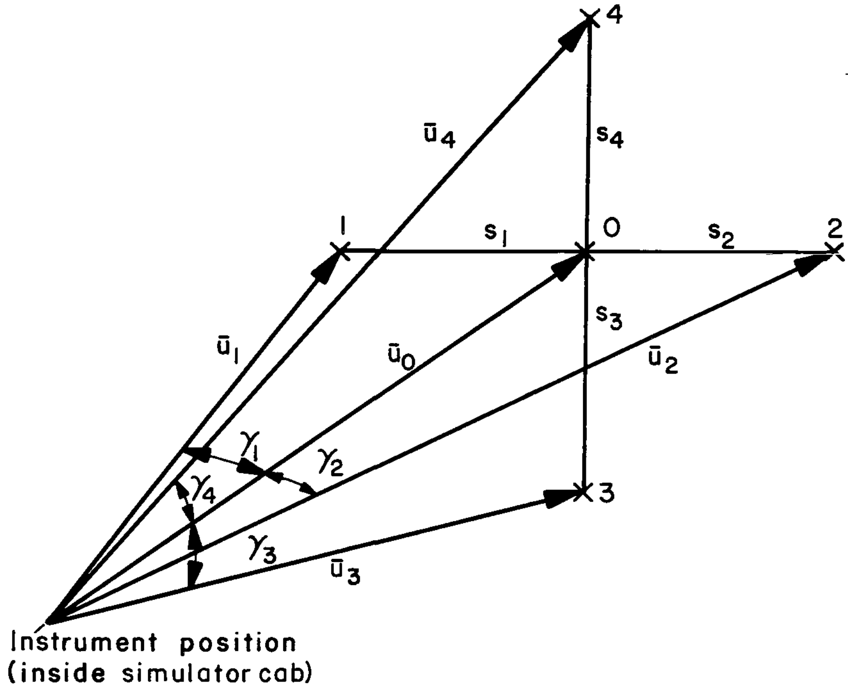

Figure 2.- Sights of the cross from a single position of the survey instrument.

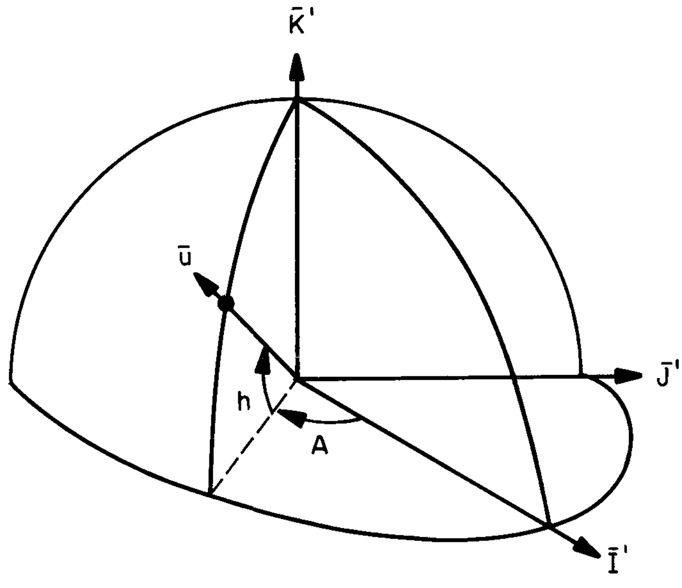

Figure 3.- The instrument coordinate system, showing the altitude and azimuth angles by which the unit vector is described.

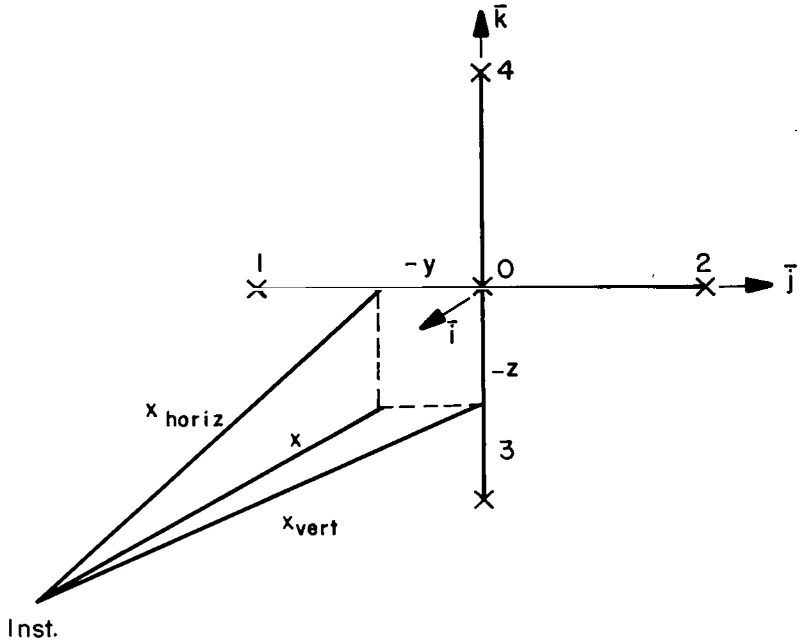

Figure 4.- Resolution of the x component of the instrument position into x_{horiz} and x_{vert}.

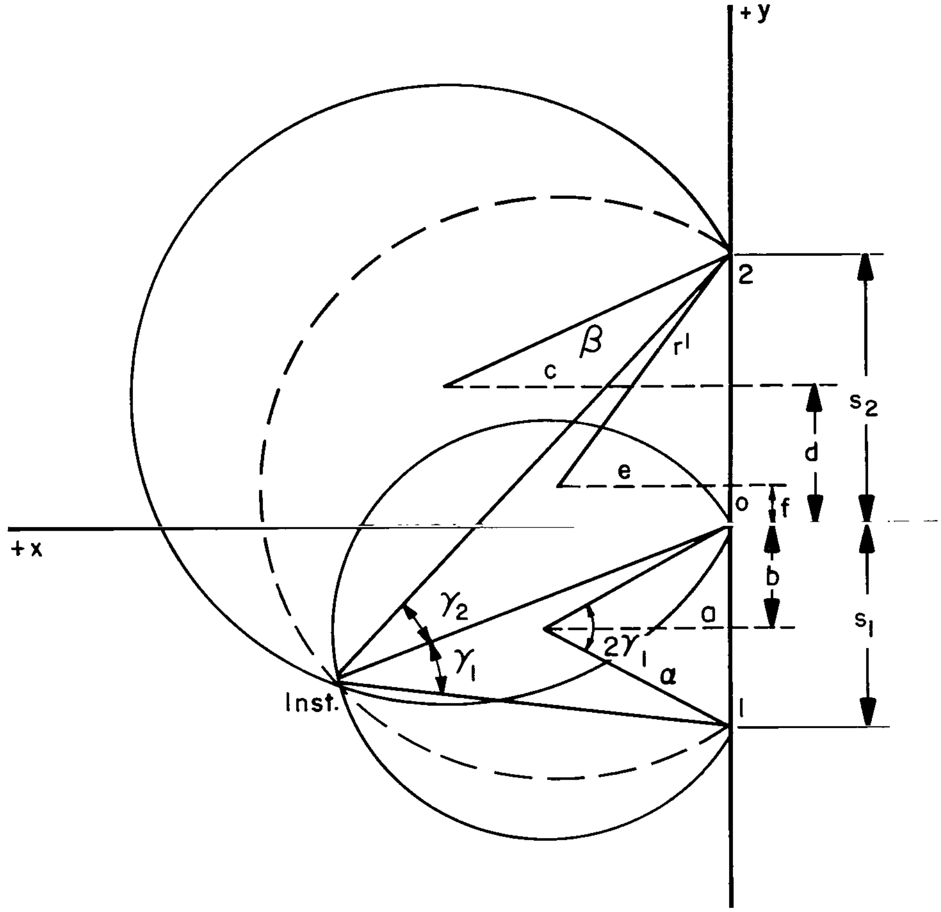

Figure 5.- Geometry of the horizontal survey.

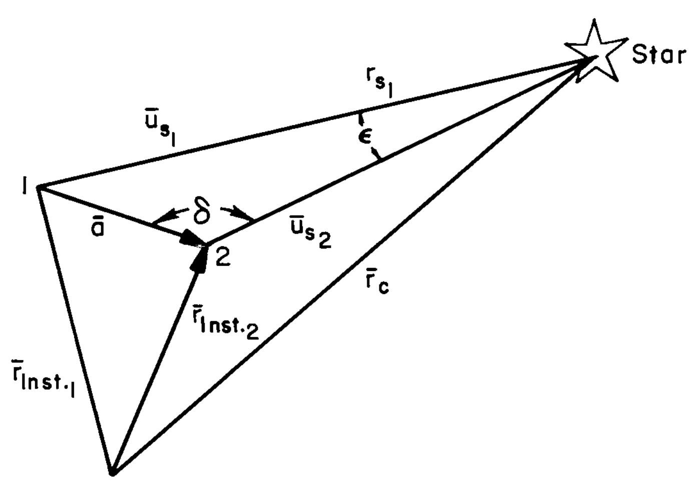

Figure 6.- Sights of a star from two positions of the survey instrument.

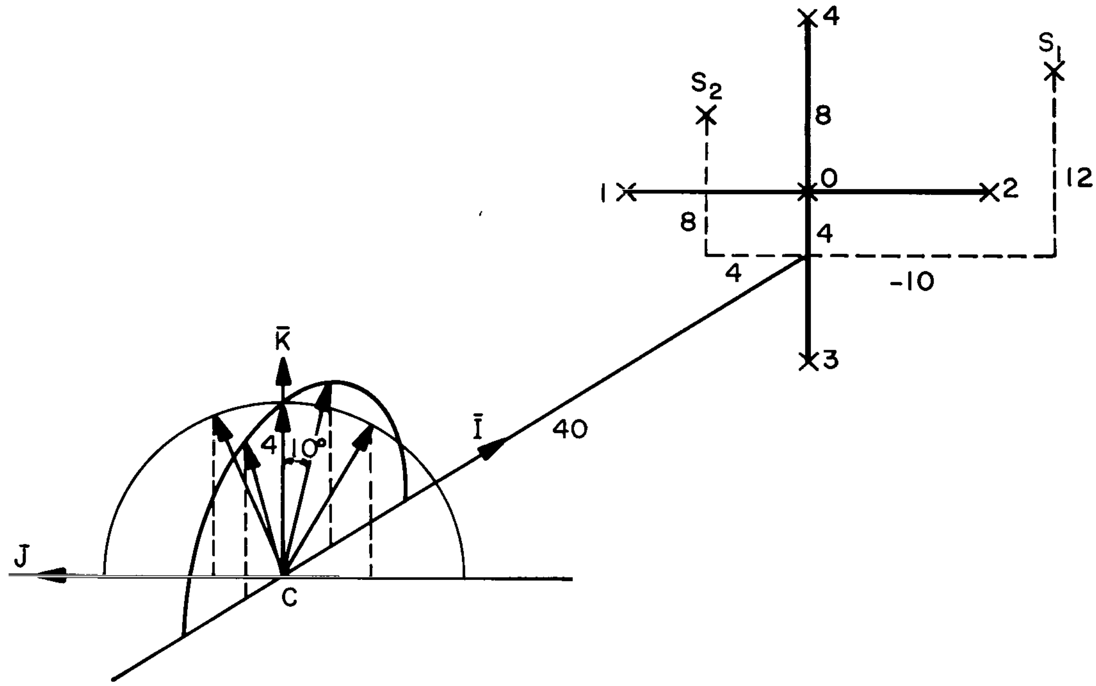

Figure 7.- Setup from which test data were hand computed.

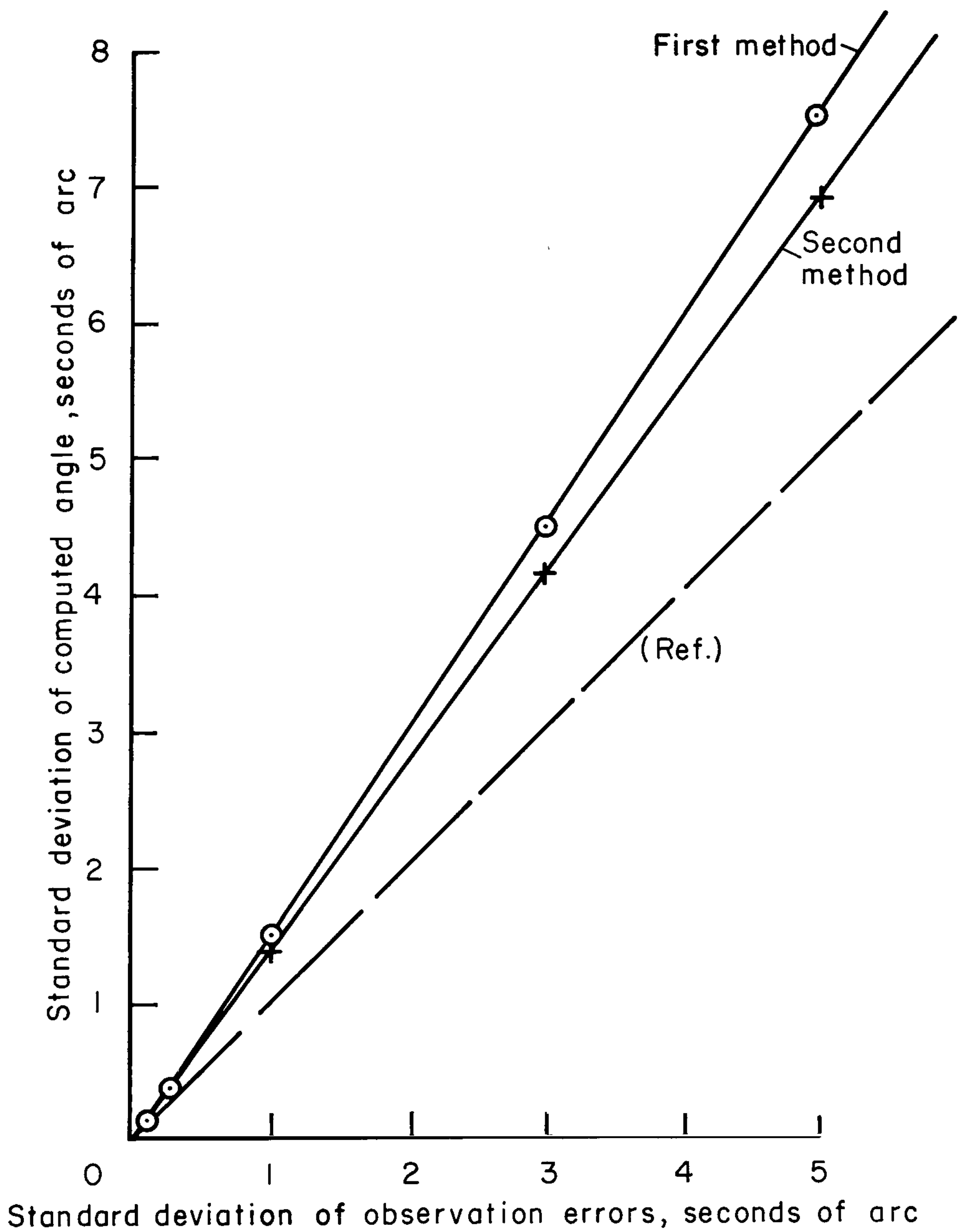

Figure 8.- Standard deviation of the errors in the computed angle due to observation errors.

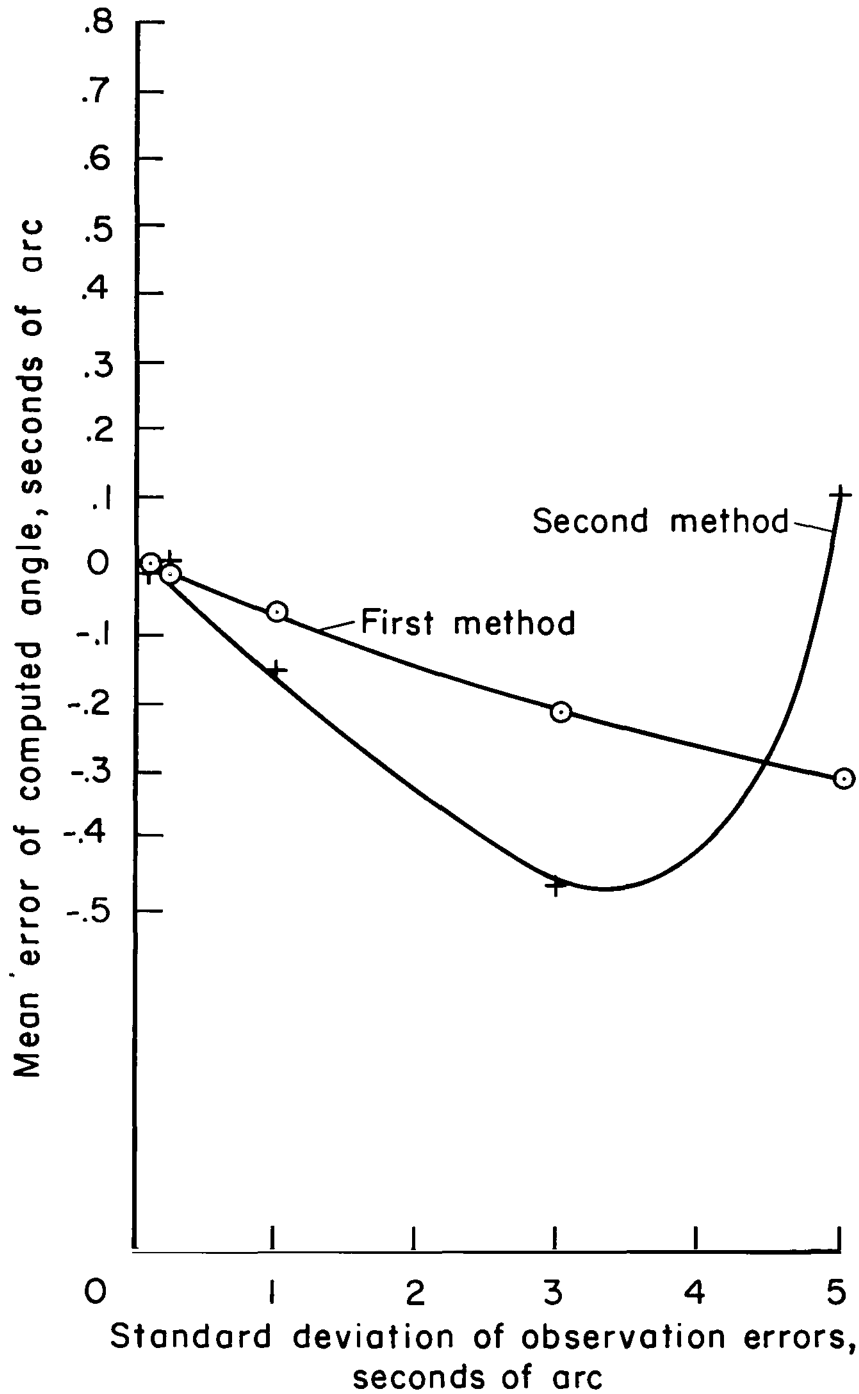

Figure 9.- Mean error in the computed angle due to observation errors.

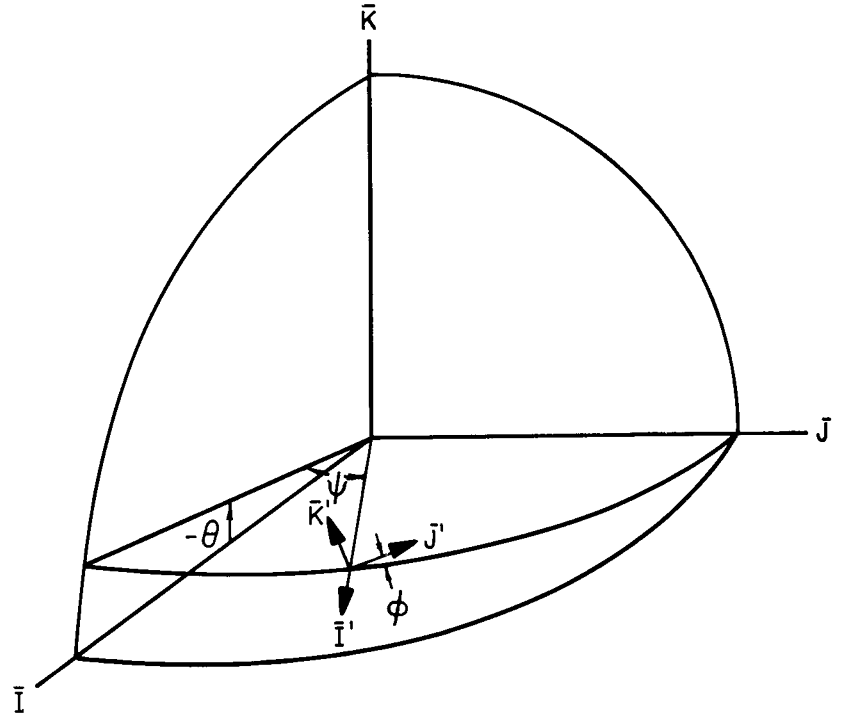

Figure 10.- Cab-position angles.

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