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KINETIC THEORY APPROACH TO THE STUDY OF A CURVED SHOCK-WAVE

by M. M. Oberai

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UNIVERSITY OF CALIFORNIA
Los Angeles, Calif.
for

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ABSTRACT 26645

Mott-Smith[1] method is extended to derive the first order -- counting Rankine-Hugoniot relations as the zero order -- shock relations for a curved shock-wave formed in a flow when the Reynolds number is not very large. The role of the additive constant that occurs in the solution of the zero order shock-wave structure is discussed. Also, solution of the first order shock-wave structure is obtained.

author

TABLE OF CONTENTS

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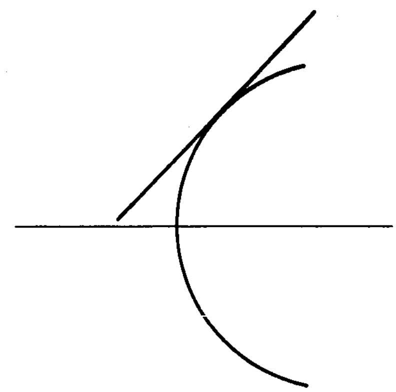
1. INTRODUCTION

For very large Reynolds number supersonic flows, a shock-wave is treated as a discontinuity in the values of flow variables. For a plane shock-wave, Rankine-Hugoniot relations give, directly, the values of these variables behind the shock-wave in terms of those ahead of it. When the shock-wave is curved, it may locally be replaced by the tangent plane and the Rankine-Hugoniot relations for an oblique shock-wave be applied to relate the values of flow variables on the two sides.



However, when the Reynolds number (based on the radius of curvature at the nose of the shock-wave) is only moderately large, the gradients in the region behind the shock-wave necessitate corrections in the Rankine-Hugoniot relations and the problem that we study here is to find the first (in terms of the inverse of Reynolds number) corrective term in the shock relations. This study has already been made in the framework of Navier-Stokes equations.[2,3,4] We propose to approach this problem from the kinetic theory point of view and to this end, extend Mott-Smith[1] method which is known to apply successfully to the problem of a plane shock-wave -- both in a single gas[1] and a mixture.[5] To keep the analysis simple, we shall study a two dimensional shock in the flow of a monatomic gas[6] and assume that the molecules obey the Maxwellian law of force -- viz. $\frac{K}{r^5}$.

2. STATEMENT OF THE PROBLEM

A shock-wave of known shape is formed in a uniform supersonic flow of velocity $\mathbf{V}$, temperature Θ and number density $\mathbf{n}$. If a characteristic length $\boldsymbol{l}$ -- say the radius of curvature at the nose of the shock-wave -- be introduced, we can define a dimensionless parameter ϵ by the relation

$$\epsilon = \frac{\mu}{\mathbf{V} m \mathbf{n} \boldsymbol{l}} \tag{1}$$

where m is the molecular mass and μ the coefficient of viscosity at temperature Θ . As the molecules are supposed to be Maxwellian, we have[7]

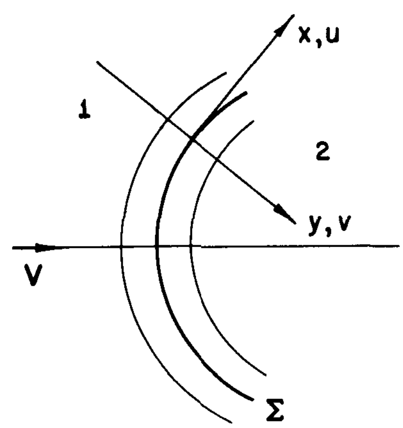
$$\mu = \frac{1}{3\pi A_2} \left(\frac{2m}{K}\right)^{\frac{1}{2}} k\,\Theta \qquad (2)$$

where k is the Boltzmann constant and A_2 the value of the scattering integral. The flow conditions are assumed to be such that ϵ is moderately small and the flow variables behind the shock-wave are expanded in power-series of ϵ. The first terms of these series are those obtained from the Rankine-Hugoniot relations; the problem posed here is to obtain the second terms of the series.

3. METHOD

Velocity components, number density and length are non-dimensionalized with reference to $\mathbf{V}$, $\mathbf{n}$ and ℓ respectively. Temperature θ is non-dimensionalized with reference to $\frac{m}{k}\mathbf{V}^2$ and the non-dimensional quantity $\frac{k}{m}\frac{\theta}{\mathbf{V}^2}$ is denoted by β. In what follows, the above non-dimensionalization is supposed to have been carried out. Mach number M of the flow is given by

$$M^2 = \frac{3}{5}\,\frac{\mathbf{V}^2}{\frac{k}{m}\Theta} \qquad (3)$$



Let $\sum$ be the position in which the shock-wave, postulated as the surface of discontinuity, would be if the terms of order ϵ are not taken into account. Curvilinear coordinates (x, y) are introduced where x is the distance from the nose along the shock-wave and y is normal at the point x; κ is the curvature at this point. Velocity-components along x and y directions are denoted by u and v, respectively. Superscripts (1) and (2) are used to characterize region 1 (ahead of $\sum$) and 2 (behind $\sum$), respectively.

Within the shock region, gradients along the y direction are of $0(1/\epsilon)$; therefore, the coordinate y is stretched as

$$Y = \frac{y}{\epsilon} \qquad (4)$$

so that

$$\frac{\partial}{\partial y} \equiv 0(1)$$

In region 2 (y> 0), various flow variables are expanded as follows:

$$\left.\begin{aligned}
u^{(2)}(x,y;\epsilon) &= u_o^{(2)}(x,y) + \epsilon\, u_1^{(2)}(x,y) + \dots \\
&= u_o^{(2)}(x,o) + \epsilon\left\{ Y\left(\frac{\partial u_o^{(2)}}{\partial y}\right)_{y=0} + u_1^{(2)}(x,o)\right\} + \dots \\
&= U_o^{(2)}(x) + \epsilon\left\{ Y\, U_o^{(2)\prime}(x) + U_1^{(2)}(x)\right\} + \dots \\
v^{(2)}(x,y;\epsilon) &= v_o^{(2)}(x,y) + \epsilon\, v_1^{(2)}(x,y) + \dots \\
&= V_o^{(2)}(x) + \epsilon\left\{ Y\, V_o^{(2)\prime}(x) + V_1^{(2)}(x)\right\} + \dots \\
\beta^{(2)}(x,y) &= \beta_o^{(2)}(x,y) + \epsilon\, \beta_1^{(2)}(x,y) + \dots \\
&= B_o^{(2)}(x) + \epsilon\left\{ Y\, B_o^{(2)\prime}(x) + B_1^{(2)}(x)\right\} + \dots \\
n^{(2)}(x,y) &= n_o^{(2)}(x,y) + \epsilon\, n_1^{(2)}(x,y) + \dots \\
&= N_o^{(2)}(x) + \epsilon\left\{ Y\, N_o^{(2)\prime}(x) + N_1^{(2)}(x)\right\} + \dots
\end{aligned}\right\} \quad (5)$$

In region 1(y < 0), $u^{(1)}$ and $v^{(1)}$ are functions of x alone while $\beta^{(1)}$ and $n^{(1)}$ are constant. However, to preserve symmetry in notation we shall write

$$u^{(1)}(x) = U_o^{(1)}(x), \quad v^{(1)}(x) = V_o^{(1)}(x), \quad \beta^{(1)} = B_o^{(1)} = \left(\frac{3}{5M^2}\right)$$

$$\text{and } n^{(1)} = N_o^{(1)}(=1) \qquad (6)$$

(ξ, η, ζ) denote the components of the molecular velocity and the velocity distribution function in the shock region is postulated to have the form

$$f(x,Y,\xi,\eta,\zeta) = n^{(-)}(x,Y;\epsilon)\left(2\pi\beta^{(1)}\right)^{-3/2} \exp\left\{-\frac{\left(\xi-u^{(1)}\right)^2+\left(\eta-v^{(1)}\right)^2+\zeta^2}{2\beta^{(1)}}\right\}$$
$$+ n^{(+)}(x,Y;\epsilon)\left(2\pi\beta^{(2)}\right)^{-3/2} \exp\left\{-\frac{\left(\xi-u^{(2)}\right)^2+\left(\eta-v^{(2)}\right)^2+\zeta^2}{2\beta^{(2)}}\right\} \qquad (7)$$

with

$$n^{(\pm)}(x, Y; \epsilon) = n_o^{(\pm)}(x, Y) + \epsilon\, n_1^{(\pm)}(x, Y) + \dots \tag{8}$$

As $Y \to -\infty$ $(+\infty)$, the flow in the shock region should match, term by in powers of ϵ, with the upstream flow (downstream flow evaluated at $y = +0$). In other words,

$$\left.\begin{array}{l} \text{Let } Y \to -\infty \begin{cases} n_o^{(-)}(x, Y) = N_o^{(1)} = 1 \quad \text{for all } x \\ n_o^{(+)}(x, Y) = 0 \\ n_1^{(-)}(x, Y) = 0 \\ n_1^{(+)}(x, Y) = 0 \end{cases} \\ \text{and} \\ \text{as } Y \to +\infty \begin{cases} n_o^{(-)}(x, Y) \to 0 \\ n_o^{(+)}(x, Y) \to N_o^{(2)}(x) \\ n_1^{(-)}(x, Y) \to 0 \\ n_1^{(+)}(x, Y) \sim Y\, N_o^{(2)'}(x) + N_1^{(2)}(x) \end{cases} \end{array}\right\} \tag{9}$$

4. MAXWELL TRANSFER EQUATIONS

For any quantity $Q(\xi, \eta, \zeta)$, the Maxwell transfer equation in the curvilinear coordinate system introduced above has the form[8]

$$\frac{\partial}{\partial x}\int f\,\xi\,Q\,d\xi\,d\eta\,d\zeta + \frac{\partial}{\partial y}\left\{(1-\kappa y)\int f\eta\,Q\,d\xi\,d\eta\,d\zeta\right\} + \kappa\int\left(\xi^2\frac{\partial Q}{\partial \eta} - \xi\eta\frac{\partial Q}{\partial \xi}\right) f\,d\xi\,d\eta\,d\zeta$$

$$= \int Q\left(\frac{\partial f}{\partial t}\right)_{\text{collision}} d\xi\,d\eta\,d\zeta$$

or,

$$\epsilon\frac{\partial}{\partial x}\int f\,\xi\,Q\,d\xi\,d\eta\,d\zeta + (1-\epsilon\kappa Y)\frac{\partial}{\partial Y}\int f\eta Q\,d\xi\,d\eta\,d\zeta - \epsilon\kappa\int f\eta\,Q\,d\xi\,d\eta\,d\zeta$$

$$+\epsilon\kappa\int\left(\xi^2\frac{\partial Q}{\partial \eta} - \xi\eta\frac{\partial Q}{\partial \xi}\right)d\xi\,d\eta\,d\zeta = \epsilon\int Q\left(\frac{\partial f}{\partial t}\right)_{\text{collision}} d\xi\,d\eta\,d\zeta \tag{10}$$

When $Q = m$, $m\xi$, $m\eta$ or $\frac{1}{2}m(\xi^2 + \eta^2 + \zeta^2)$ the integral on the right hand side vanishes, so that (substituting for f from Equation 7), we obtain the following conservation equations

$$\epsilon\frac{\partial}{\partial x}\left(n^{(-)}u^{(1)}+n^{(+)}u^{(2)}\right)+(1-\epsilon\kappa Y)\frac{\partial}{\partial Y}\left(n^{(-)}v^{(1)}+n^{(+)}v^{(2)}\right)-\epsilon\kappa\left(n^{(-)}v^{(1)}+n^{(+)}v^{(2)}\right)=0 \tag{11}$$

$$\epsilon\frac{\partial}{\partial x}\left\{n^{(-)}\left(u^{(1)^2}+\beta^{(1)}\right)+n^{(+)}\left(u^{(2)^2}+\beta^{(2)}\right)\right\}+(1-\epsilon\kappa Y)\frac{\partial}{\partial Y}\left(n^{(-)}u^{(1)}v^{(1)}+n^{(+)}u^{(2)}v^{(2)}\right)$$
$$-2\epsilon\kappa\left(n^{(-)}u^{(1)}v^{(1)}+n^{(+)}u^{(2)}v^{(2)}\right)=0 \tag{12}$$

$$\epsilon\frac{\partial}{\partial x}\left(n^{(-)}u^{(1)}v^{(1)}+n^{(+)}u^{(2)}v^{(2)}\right)+(1-\epsilon\kappa Y)\frac{\partial}{\partial Y}\left\{n^{(-)}\left(v^{(1)^2}+\beta^{(1)}\right)+n^{(+)}\left(v^{(2)^2}+\beta^{(2)}\right)\right\}$$
$$+\epsilon\kappa\left\{n^{(-)}\left(u^{(1)^2}-v^{(1)^2}\right)+n^{(+)}\left(u^{(2)^2}-v^{(2)^2}\right)\right\}=0 \tag{13}$$

$$\epsilon\frac{\partial}{\partial x}\left\{n^{(-)}u^{(1)}\left(u^{(1)^2}+v^{(1)^2}+5\beta^{(1)}\right)+n^{(+)}u^{(2)}\left(u^{(2)^2}+v^{(2)^2}+5\beta^{(2)}\right)\right\}$$
$$+(1-\epsilon\kappa Y)\frac{\partial}{\partial Y}\left\{n^{(-)}v^{(1)}\left(u^{(1)^2}+v^{(1)^2}+5\beta^{(1)}\right)\right.$$
$$\left.+n^{(+)}v^{(2)}\left(u^{(2)^2}+v^{(2)^2}+5\beta^{(2)}\right)\right\}$$
$$-\epsilon\kappa\left\{n^{(-)}v^{(1)}\left(u^{(1)^2}+v^{(1)^2}+5\beta^{(1)}\right)+n^{(+)}v^{(2)}\left(u^{(2)^2}+v^{(2)^2}+5\beta^{(2)}\right)\right\}=0 \tag{14}$$

As will become apparent in the subsequent analysis, one more equation is required to completely determine the problem. The choice of Q for this additional equation is arbitrary. We shall not discuss the merits of one choice over another. For reasons of simplicity alone we choose $Q=\eta^2$ and obtain[9]

$$\epsilon\frac{\partial}{\partial x}\left\{n^{(-)}u^{(1)}\left(v^{(1)^2}+\beta^{(1)}\right)+n^{(+)}u^{(2)}\left(v^{(2)^2}+\beta^{(2)}\right)\right\}$$
$$+(1-\epsilon\kappa Y)\frac{\partial}{\partial Y}\left\{n^{(-)}v^{(1)}\left(v^{(1)^2}+3\beta^{(1)}\right)+n^{(+)}v^{(2)}\left(v^{(2)^2}+3\beta^{(2)}\right)\right\}$$
$$+\epsilon\kappa\left\{n^{(-)}v^{(1)}\left(2u^{(1)^2}-v^{(1)^2}-\beta^{(1)}\right)+n^{(+)}v^{(2)}\left(2u^{(2)^2}-v^{(2)^2}-\beta^{(2)}\right)\right\}$$
$$=-\frac{2}{5M^2}n^{(-)}n^{(+)}\left\{\left(v^{(1)}-v^{(2)}\right)^2-\frac{1}{2}\left(u^{(1)}-u^{(2)}\right)^2\right\} \tag{15}$$

5. RANKINE-HUGONIOT RELATIONS AND ZERO ORDER SHOCK STRUCTURE

Substituting the expansions (5) and (8) in Equations (11)-(15), sorting out terms independent of ϵ and making use of (6), the zero order problem may be stated as

$$
\left.
\begin{aligned}
&V_o^{(1)} \frac{\partial n_o^{(-)}}{\partial Y} + V_o^{(2)} \frac{\partial n_o^{(+)}}{\partial Y} = 0 \\
&U_o^{(1)} V_o^{(1)} \frac{\partial n_o^{(-)}}{\partial Y} + U_o^{(2)} V_o^{(2)} \frac{\partial n_o^{(+)}}{\partial Y} = 0 \\
&\left(V_o^{(1)^2} + B_o^{(1)}\right) \frac{\partial n_o^{(-)}}{\partial Y} + \left(V_o^{(2)^2} + B_o^{(2)}\right) \frac{\partial n_o^{(+)}}{\partial Y} = 0 \\
&V_o^{(1)}\left(U_o^{(1)^2} + V_o^{(1)^2} + 5B_o^{(1)}\right) \frac{\partial n_o^{(-)}}{\partial Y} + V_o^{(2)}\left(U^{(2)^2} + V_o^{(2)^2} + 5B_o^{(2)}\right) \frac{\partial n_o^{(+)}}{\partial Y} = 0 \\
&V_o^{(1)}\left(V_o^{(1)^2} + 3B_o^{(1)}\right) \frac{\partial n_o^{(-)}}{\partial Y} + V_o^{(2)}\left(V_o^{(2)^2} + 3B_o^{(2)}\right) \frac{\partial n_o^{(+)}}{\partial Y} \\
&\qquad = -\frac{2}{5M^2} n_o^{(-)} n_o^{(+)} \left\{\left(V_o^{(1)} - V_o^{(2)}\right)^2 - \frac{1}{2}\left(U_o^{(1)} - U_o^{(2)}\right)^2\right\}
\end{aligned}
\right\} \tag{16}
$$

From the first four of Equations (16) and the boundary conditions (9), we obtain the following relations

$$U_o^{(2)} = U_o^{(1)}, \quad V_o^{(2)} = \alpha V_o^{(1)}, \quad B_o^{(2)} = \frac{4\alpha - \alpha^2}{5} V_o^{(1)^2}, \quad N_o^{(2)} = \frac{1}{\alpha} N_o^{(1)} \tag{17}$$

where $\alpha(x)$ is defined by the relation

$$B_o^{(1)} = \frac{4\alpha - 1}{5} V_o^{(1)^2} \tag{18}$$

Relations (17) may be recognized as Rankine-Hugoniot relations obtained by local replacement of the curved shock wave by a plane shock wave of the same slope.

With the help of relations (17) the last equation in (16) yields the following solution for $n_o^{(-)}$ and $n_o^{(+)}$

$$
\left.
\begin{aligned}
n_o^{(-)} &= \frac{1}{1 + e^{A(Y+L)}} \\
n_o^{(+)} &= \frac{1}{\alpha} \frac{e^{A(Y+L)}}{1 + e^{A(Y+L)}}
\end{aligned}
\right\} \tag{19}
$$

where

$$A(x) = \frac{1}{M^2 V_o^{(1)}} \; \frac{1-\alpha}{\alpha(1+\alpha)} \tag{20}$$

and L is constant of integration, the determination of which is discussed in the following section.

<u>Solution (19) gives the zero order shock structure.</u>

6. ADDITIVE CONSTANT THAT OCCURS IN THE SOLUTION OF THE ZERO ORDER SHOCK STRUCTURE

The boundary conditions (9) on $n_o^{(-)}$ and $n_o^{(+)}$ do not determine the constant L that occurs in the solution (19). Every choice of L corresponds to assigning a specified value to the density (and consequently to each flow variable) at the point Y = 0 which is the position of the shock-wave postulated as a mathematical discontinuity.

However, L is involved in the first order (terms containing first power of ϵ) corrections in the shock relations and, therefore, inasmuch that the values assumed, behind the shock-wave, by various flow variables are not arbitrary, the constant L cannot be altogether arbitrary. Chow and Ting[4] point out correctly that L need be specified only if the position of the obstacle -- and there must be an obstacle to produce a curved shock-wave -- has to be determined up to the order ϵ but they are wrong in implying that as far as the determination of the first order corrections in the shock relations is concerned, L may be chosen arbitrarily; in fact, each choice of L leads to different expressions for the first order corrections. Thus, L should be allowed to appear in the first order corrections (Equations 33) and the numerical values of these corrections should be calculated only after determining L by matching with the flow between the shock-wave and the body.

The solution (19) implies that in the zero order shock structure, the density assumes its mean value (arithmetic mean between $N_o^{(1)}$ and $N_o^{(2)}$) at the point Y = -L. <u>Thus $\epsilon \ell L$ is the distance between the position of the shock-wave postulated as a mathematical discontinuity and the point in the zero order shock structure where the value assumed by the density is the mean of the values at the two ends of the shock-wave.</u>

7. FIRST ORDER CORRECTIONS IN THE SHOCK RELATIONS

Substituting expansions (5) and (8) in Equations (11)-(14), sorting out coefficients of ϵ and making use of (17) and the relation $n_o^{(-)} + \alpha n_o^{(+)} = 1$, the first order problem simplifies to

$$\frac{\partial}{\partial x}\left\{U_o^{(1)}\left(n_o^{(-)} + n_o^{(+)}\right)\right\} + \frac{\partial}{\partial Y}\left\{V_o^{(1)}\left(n_1^{(-)} + \alpha n_1^{(+)}\right) + \left(V_1^{(2)} + Y V_o^{(2)'}\right) n_o^{(+)}\right\} - \kappa V_o^{(1)} = 0 \tag{21}$$

$$\frac{\partial}{\partial x}\left\{\left(U_o^{(1)^2} + \frac{4\alpha-1}{5} V_o^{(1)^2}\right) n_o^{(-)} + \left(U_o^{(1)^2} + \frac{4\alpha-\alpha^2}{5} V_o^{(1)^2}\right) n_o^{(+)}\right\}$$
$$+ \frac{\partial}{\partial Y}\left\{U_o^{(1)} V_o^{(1)}\left(n_1^{(-)} + \alpha n_1^{(+)}\right) + U_o^{(1)}\left(V_1^{(2)} + Y V_o^{(2)'}\right) n_o^{(+)} + \alpha V_o^{(1)}\left(U_1^{(2)} + Y U_o^{(2)'}\right) n_o^{(+)}\right\}$$
$$-2\kappa U_o^{(1)} V_o^{(1)} = 0 \tag{22}$$

$$\frac{d}{dx}\left(U_o^{(1)} V_o^{(1)}\right) + \frac{\partial}{\partial Y}\left\{\frac{4}{5}(1+\alpha) V_o^{(1)^2}\left(n_1^{(-)} + \alpha n_1^{(+)}\right) + \left(B_1^{(2)} + Y B_o^{(2)'}\right) n_o^{(+)}\right.$$
$$\left. + 2\alpha V_o^{(1)}\left(V_1^{(2)} + Y V_o^{(2)'}\right) n_o^{(+)}\right\}$$
$$+ \kappa\left\{U_o^{(1)^2}\left(n_o^{(-)} + n_o^{(+)}\right) - V_o^{(1)^2}\left(n_o^{(-)} + \alpha^2 n_o^{(+)}\right)\right\} = 0 \tag{23}$$

$$\frac{\partial}{\partial x}\left\{U_o^{(1)}\left(U_o^{(1)^2} + 4\alpha V_o^{(1)^2}\right)\left(n_o^{(-)} + n_o^{(+)}\right)\right\}$$
$$+ \frac{\partial}{\partial Y}\left\{\begin{array}{l} V_o^{(1)}\left(U_o^{(1)^2} + 4\alpha V_o^{(1)^2}\right)\left(n_1^{(-)} + \alpha n_1^{(+)}\right) + \left(U_o^{(1)^2} + (4\alpha + 2\alpha^2) V_o^{(1)^2}\right)\left(V_1^{(2)} + Y V_o^{(2)'}\right) n_o^{(+)} \\ + 2\alpha U_o^{(1)} V_o^{(1)}\left(U_1^{(2)} + Y U_o^{(2)'}\right) n_o^{(+)} + 5\alpha V_o^{(1)}\left(B_1^{(2)} + Y B_o^{(2)'}\right) n_o^{(+)} \end{array}\right\}$$
$$- \kappa V_o^{(1)}\left(U_o^{(1)^2} + 4\alpha V_o^{(1)^2}\right) = 0 \tag{24}$$

Integrating Equations (21)-(24) with respect to Y from $-\infty$ to $+\infty$, making use of the boundary conditions (9) and replacing Y by $\int_o^Y dY$, the following relations are obtained

$$\alpha V_o^{(1)} N_1^{(2)} + \frac{1}{\alpha} V_1^{(2)} = \int_{-\infty}^{0}\left\{\kappa V_o^{(1)} - \frac{\partial}{\partial x}\left(U_o^{(1)}\left(n_o^{(-)} + n_o^{(+)}\right)\right)\right\} dY$$
$$+ \int_0^{\infty}\left\{\kappa V_o^{(1)} - \frac{\partial}{\partial x}\left(U_o^{(1)}\left(n_o^{(-)} + n_o^{(+)}\right)\right) - \alpha V_o^{(1)} N_o^{(2)'} - \frac{1}{\alpha} V_o^{(2)'}\right\} dY \tag{25}$$

$$\alpha U_o^{(1)}N_1^{(2)}+\frac{1}{\alpha}U_o^{(1)}V_1^{(2)}+V_o^{(1)}U_1^{(2)}=\int_{-\infty}^{0}\left\{2\kappa U_o^{(1)}V_o^{(1)}-\frac{\partial}{\partial x}\left(\left(U_o^{(1)^2}+\frac{4\alpha-1}{5}V_o^{(1)^2}\right)n_o^{(-)}\right.\right.$$

$$\left.\left.+\left(U_o^{(1)^2}+\frac{4\alpha-\alpha^2}{5}V_o^{(1)^2}\right)n_o^{(+)}\right)\right\}dY$$

$$+\int_0^{\infty}\left\{\begin{array}{l}2\kappa U_o^{(1)}V_o^{(1)}-\frac{\partial}{\partial x}\left(\left(U_o^{(1)^2}+\frac{4\alpha-1}{5}V_o^{(1)^2}\right)n_o^{(-)}+\left(U_o^{(1)^2}+\frac{4\alpha-\alpha^2}{5}V_o^{(1)^2}\right)n_o^{(+)}\right)\\ -\alpha U_o^{(1)}V_o^{(1)}N_o^{(2)'}-\frac{1}{\alpha}U_o^{(1)}V_o^{(2)'}-V_o^{(1)}U_o^{(2)'}\end{array}\right\}dY \qquad (26)$$

$$\frac{1}{\alpha}B_1^{(2)}+\frac{4}{5}\alpha(1+\alpha)V_o^{(1)^2}N_1^{(2)}+2V_o^{(1)}V_1^{(2)}=-\int_{-\infty}^{0}\left\{\frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right)+\kappa\left(U_o^{(1)^2}\left(n_o^{(-)}+n_o^{(+)}\right)\right.\right.$$

$$\left.\left.-V_o^{(1)^2}\left(n_o^{(-)}+\alpha^2 n_o^{(+)}\right)\right)\right\}dY$$

$$-\int_0^{\infty}\left\{\begin{array}{l}\frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right)+\kappa\left(U_o^{(1)^2}\left(n_o^{(-)}+n_o^{(+)}\right)-V_o^{(1)^2}\left(n_o^{(-)}+\alpha^2 n_o^{(+)}\right)\right)\\ +\frac{4}{5}\alpha(1+\alpha)V_o^{(1)^2}N_o^{(2)'}+\frac{1}{\alpha}B_o^{(2)'}+2V_o^{(1)}V_o^{(2)'}\end{array}\right\}dY \qquad (27)$$

$$5V_o^{(1)}B_1^{(2)}+\alpha V_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)N_1^{(2)}+\left(\frac{1}{\alpha}U_o^{(1)^2}+(4+2\alpha)V_o^{(1)^2}\right)V_1^{(2)}+2U_o^{(1)}V_o^{(1)}U_1^{(2)}$$

$$=\int_{-\infty}^{0}\left\{\kappa V_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)-\frac{\partial}{\partial x}\left(U_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)\left(n_o^{(-)}+n_o^{(+)}\right)\right)\right\}dY$$

$$+\int_0^{\infty}\left\{\begin{array}{l}\kappa V_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)-\frac{\partial}{\partial x}\left(U_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)\left(n_o^{(-)}+n_o^{(+)}\right)\right)\\ -\alpha V_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)N_o^{(2)'}-\left(\frac{1}{\alpha}U_o^{(1)^2}+(4+2\alpha)V_o^{(1)^2}\right)V_o^{(2)'}\\ -2U_o^{(1)}V_o^{(1)}U_o^{(2)'}-5V_o^{(1)}B_o^{(2)'}\end{array}\right\}dY \qquad (28)$$

Quantities like $U_o^{(2)'}$ etc., can be expressed in terms of x-derivatives of $U_o^{(2)}$ etc., in the following manner. To the zero order approximation, the velocity distribution function for region 2 may be postulated as

$$f^{(2)}(x,y,\xi,\eta,\zeta)=n_o^{(2)}\left(2\pi\beta_o^{(2)}\right)^{-3/2}\exp\left\{-\frac{\left(\xi-u_o^{(2)}\right)^2+\left(\eta-v_o^{(2)}\right)^2+\zeta^2}{2\beta_o^{(2)}}\right\} \qquad (29)$$

from which the following conservation equations may be derived

$$\left.\begin{aligned}
&\frac{\partial}{\partial x}\left(n_o^{(2)}u_o^{(2)}\right)+(1-\kappa y)\frac{\partial}{\partial y}\left(n_o^{(2)}v_o^{(2)}\right)-\kappa n_o^{(2)}v_o^{(2)} = 0\\
&\frac{\partial}{\partial x}\left\{n_o^{(2)}\left(u_o^{(2)^2}+\beta_o^{(2)}\right)\right\}+(1-\kappa y)\frac{\partial}{\partial y}\left(n_o^{(2)}u_o^{(2)}v_o^{(2)}\right)-2\kappa n_o^{(2)}u_o^{(2)}v_o^{(2)} = 0\\
&\frac{\partial}{\partial x}\left(n_o^{(2)}u_o^{(2)}v_o^{(2)}\right)+(1-\kappa y)\frac{\partial}{\partial y}\left\{n_o^{(2)}\left(v_o^{(2)^2}+\beta_o^{(2)}\right)\right\}+\kappa\left\{n_o^{(2)}\left(u_o^{(2)^2}-v_o^{(2)^2}\right)\right\} = 0\\
&\frac{\partial}{\partial x}\left\{n_o^{(2)}u_o^{(2)}\left(u_o^{(2)^2}+5\beta_o^{(2)}\right)\right\}+(1-\kappa y)\frac{\partial}{\partial y}\left\{n_o^{(2)}v_o^{(2)}\left(u_o^{(2)^2}+v_o^{(2)^2}+5\beta_o^{(2)}\right)\right\}\\
&\qquad\qquad -\kappa n_o^{(2)}v_o^{(2)}\left(u_o^{(2)^2}+v_o^{(2)^2}+5\beta_o^{(2)}\right) = 0
\end{aligned}\right\}\quad(30)$$

Equations (30) evaluated at y=+0 give

$$\left.\begin{aligned}
&\frac{1}{\alpha}V_o^{(2)'}+\alpha V_o^{(1)}N_o^{(2)'} = \kappa V_o^{(1)}-\frac{d}{dx}\left(\frac{1}{\alpha}U_o^{(1)}\right)\\
&V_o^{(1)}U_o^{(2)'}+\frac{1}{\alpha}U_o^{(1)}V_o^{(2)'}+\alpha U_o^{(1)}V_o^{(1)}N_o^{(2)'} = 2\kappa U_o^{(1)}V_o^{(1)}-\frac{d}{dx}\left(\frac{1}{\alpha}U_o^{(1)^2}+\frac{4-\alpha}{5}V_o^{(1)^2}\right)\\
&\frac{1}{\alpha}B_o^{(2)'}+2V_o^{(1)}V_o^{(2)'}+\frac{4}{5}\alpha(1+\alpha)V_o^{(1)^2}N_o^{(2)'} = -\kappa\left(\frac{1}{\alpha}U_o^{(1)^2}-\alpha V_o^{(1)^2}\right)-\frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right)\\
&5V_o^{(1)}B_o^{(2)'}+2U_o^{(1)}V_o^{(1)}U_o^{(2)'}+\left(\frac{1}{\alpha}U_o^{(1)^2}+(4+2\alpha)V_o^{(1)^2}\right)V_o^{(2)'}+\alpha V_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)N_o^{(2)'}\\
&\qquad = \kappa V_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)-\frac{d}{dx}\left\{\frac{1}{\alpha}U_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)\right\}
\end{aligned}\right\}$$

(31)

Similarly, the equations governing the upstream flow, evaluated at y=-0 give

$$\left.\begin{aligned}
&\frac{d}{dx}U_o^{(1)} = \kappa V_o^{(1)}\\
&\frac{d}{dx}\left(U_o^{(1)^2}+\frac{4\alpha-1}{5}V_o^{(1)^2}\right) = 2\kappa U_o^{(1)}V_o^{(1)}\\
&\frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right) = -\kappa\left(u_o^{(1)^2}-V_o^{(1)^2}\right)\\
&\frac{d}{dx}\left\{U_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)\right\} = \kappa V_o^{(1)}\left(U_o^{(1)^2}+4\alpha V_o^{(1)^2}\right)
\end{aligned}\right\}\quad(32)$$

Substituting (19), (31) and (32) in the integrals on the right hand side of Equations (25)-(28), and evaluating these integrals, the following algebraic equations giving the first order corrections -- $\epsilon U_1^{(2)}$, etc., -- in the shock relations, are obtained

$$\left.\begin{aligned}
&\alpha V_o^{(1)} N_1^{(2)} + \frac{1}{\alpha} V_1^{(2)} = -\frac{d}{dx}\left(\frac{1-\alpha}{\alpha} L\, U_o^{(1)}\right) \\
&V_o^{(1)} U_1^{(2)} + \alpha U_o^{(1)} V_o^{(1)} N_1^{(2)} + \frac{1}{\alpha} U_o^{(1)} V_1^{(2)} = -\frac{d}{dx}\left\{\frac{1-\alpha}{\alpha} L\left(U_o^{(1)^2} + \alpha V_o^{(1)^2}\right)\right\} \\
&\frac{1}{\alpha} B_1^{(2)} + \frac{4}{5}\alpha(1+\alpha) V_o^{(1)^2} N_1^{(2)} + 2V_o^{(1)} V_1^{(2)} = -\kappa \frac{1-\alpha}{\alpha} L\left(U_o^{(1)^2} + \alpha V_o^{(1)^2}\right) \\
&5V_o^{(1)} B_1^{(2)} + 2U_o^{(1)} V_o^{(1)} U_1^{(2)} + \alpha V_o^{(1)}\left(U_o^{(1)^2} + 4\alpha V_o^{(1)^2}\right) N_1^{(2)} + \left(\frac{1}{\alpha} U_o^{(1)^2} + (4+2\alpha) V_o^{(1)^2}\right) V_1^{(2)} \\
&\qquad = -\frac{d}{dx}\left\{\frac{1-\alpha}{\alpha} L\, U_o^{(1)}\left(U_o^{(1)^2} + 4\alpha V_o^{(1)^2}\right)\right\}
\end{aligned}\right\} \quad (33)$$

It may be noted that the first order corrections obtained above involve only the local slope (through components $U_o^{(1)}$ and $V_o^{(1)}$) and the first arc-derivatives (derivative $\frac{d}{dx}$ and the curvature κ) but neither the second arc-derivatives d^2/dx^2 nor the rate of change of curvature $\frac{d\kappa}{ds}$. Hence the above results remain unaltered if the curved shock is locally replaced by the circle of curvature -- a result stated, without explicit analysis, in Reference 4. Except for the difference in the definition of the coefficients of viscosity, the above results are directly comparable with those obtained from Navier-Stokes theory.[3]

8. FIRST ORDER SHOCK STRUCTURE

Sorting out coefficients of ϵ in Equation (15), the following differential equation is obtained

$$\frac{d}{dx}\left\{\frac{4}{5}(1+\alpha)U_o^{(1)}V_o^{(1)^2}\right\}+\frac{2}{5}\kappa A\, V_o^{(1)^3}(1-\alpha^2)Y\;\frac{e^{A(Y+L)}}{\left(1+e^{A(Y+L)}\right)^2}$$

$$+\frac{12\alpha+2}{5}V_o^{(1)^3}\frac{\partial n_1^{(-)}}{\partial Y}+\frac{12+2\alpha}{5}\alpha^2 V_o^{(1)^3}\frac{\partial n_1^{(+)}}{\partial Y}$$

$$+A\;\frac{e^{A(Y+L)}}{\left(1+e^{A(Y+L)}\right)^2}\left\{\frac{12}{5}(1+\alpha)V_o^{(1)^2}\left(V_1^{(2)}+Y\,V_o^{(2)'}\right)+3V_o^{(1)}\left(B_1^{(2)}+Y\,B_o^{(2)'}\right)\right\}$$

$$+\;\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\left\{\frac{12}{5}(1+\alpha)\;V_o^{(1)^2}V_o^{(2)'}+\;3V_o^{(1)}B_o^{(2)'}\right\}$$

$$+\kappa\left\{V_o^{(1)}\left(2U_o^{(1)^2}-\frac{4}{5}(1+\alpha)V_o^{(1)^2}\right)\frac{1}{1+e^{A(Y+L)}}\;+\;V_o^{(1)}\left(2U_o^{(1)^2}-\frac{4}{5}\alpha(1+\alpha)V_o^{(1)^2}\right)\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\right\}$$

$$=-\frac{2}{5}A\,V_o^{(1)}\,\frac{\alpha(1+\alpha)}{1-\alpha}\left\{\begin{array}{l} V_o^{(1)^2}\dfrac{(1-\alpha)^2}{\alpha}\;\;\dfrac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\;\;n_1^{(-)} \\ +V_o^{(1)^2}(1-\alpha)^2\;\;\dfrac{1}{1+e^{A(Y+L)}}\;\;n_1^{(+)} \\ -2V_o^{(1)}\;\dfrac{1-\alpha}{\alpha}\;\;\dfrac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\;\;\left(V_1^{(2)}+Y\,V_o^{(2)'}\right)\end{array}\right\} \tag{34}$$

Any one out of Equations (21)-(24) may be used to eliminate $n_1^{(+)}$ from Equation (34). We choose Equation (23) as it does not contain x-derivatives of $n_1^{(-)}$ and $n_o^{(+)}$. Equation (23) may be rewritten in the following form

$$\frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right)+\frac{4}{5}(1+\alpha)V_o^{(1)^2}\left(\frac{\partial n_1^{(-)}}{\partial Y}+\alpha\,\frac{\partial n_1^{(+)}}{\partial Y}\right)+\frac{1}{\alpha}\;\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\left(B_o^{(2)'}+2\alpha V_o^{(1)}V_o^{(2)'}\right)$$

$$+\frac{A}{\alpha}\;\frac{e^{A(Y+L)}}{\left(1+e^{A(Y+L)}\right)^2}\;\left\{\left(B_1^{(2)}+Y\,B_o^{(2)'}\right)+2\alpha\,V_o^{(1)}\;\left(V_1^{(2)}+YV_o^{(2)'}\right)\right\}$$

$$+\;\kappa\left\{U_o^{(1)^2}\left(\frac{1}{1+e^{A(Y+L)}}+\frac{1}{\alpha}\;\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\right)\;-V_o^{(1)^2}\left(\frac{1}{1+e^{A(Y+L)}}+\alpha\;\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\right)\right\}=0 \tag{35}$$

Integrating Equation (35) with respect to Y and evaluating the constant of integration by taking limit as $Y\to-\infty$, we obtain

$$Y\frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right)+\frac{4}{5}(1+\alpha)V_o^{(1)^2}\left(n_1^{(-)}+\alpha n_1^{(+)}\right)+\frac{1}{\alpha}\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\left\{\left(B_1^{(2)}+Y\,B_o^{(2)'}\right)\right.$$

$$\left.+2\alpha V_o^{(1)}\left(V_1^{(2)}+YV_o^{(2)'}\right)\right\}$$

$$+\frac{\kappa}{A}\left\{U_o^{(1)^2}\left(\log\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}+\frac{1}{\alpha}\log\left(1+e^{A(Y+L)}\right)\right)-V_o^{(1)^2}\left(\log\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}\right.\right.$$

$$\left.\left.+\alpha\log\left(1+e^{A(Y+L)}\right)\right)\right\}=\kappa L\left(U_o^{(1)^2}-V_o^{(1)^2}\right) \qquad (36)$$

Substituting in (34) values of $\partial n_1^{(+)}/\partial Y$ and $n_1^{(+)}$ from Equations (35) and (36), respectively, we get the following differential equation for $n_1^{(-)}$

$$\frac{\partial n_1^{(-)}}{\partial Y}+A\left(\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}-\frac{1}{1+e^{A(Y+L)}}\right)n_1^{(-)}$$

$$=g_1(x)+g_2\frac{1}{1+e^{A(Y+L)}}+g_3(x)\,Y\frac{1}{1+e^{A(Y+L)}}$$

$$+g_4(x)\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}+g_5(x)\,Y\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}}$$

$$+g_6(x)\frac{e^{A(Y+L)}}{\left(1+e^{A(Y+L)}\right)^2}+g_7(x)\,Y\frac{e^{A(Y+L)}}{\left(1+e^{A(Y+L)}\right)^2}$$

$$+g_8(x)\frac{1}{1+e^{A(Y+L)}}\log\left(1+e^{A(Y+L)}\right)$$

$$+g_9(x)\frac{1}{1+e^{A(Y+L)}}\log\frac{e^{A(Y+L)}}{1+e^{A(Y+L)}} \qquad (37)$$

where

$$g_1(x) = \frac{5}{2(1-\alpha^2)V_o^{(1)^3}}\left\{\frac{\alpha(6+\alpha)}{2(1+\alpha)}\, V_o^{(1)} \frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right) - \frac{d}{dx}\left(\frac{4}{5}(1+\alpha)\,U_o^{(1)}V_o^{(1)^2}\right)\right\}$$

$$g_2(x) = \frac{5\kappa}{2(1-\alpha^2)\,V_o^{(1)^2}}\left\{\frac{\alpha(6+\alpha)}{2(1+\alpha)}\left(U_o^{(1)^2}-V_o^{(1)^2}\right) - \frac{A(1-\alpha)}{2}\, L\left(U_o^{(1)^2}-V_o^{(1)^2}\right) - \left(2U_o^{(1)^2}-\frac{4}{5}\alpha(1+\alpha)\,V_o^{(1)^2}\right)\right\}$$

$$g_3(x) = \frac{5A}{4(1-\alpha)\,V_o^{(1)^2}}\,\frac{d}{dx}\left(U_o^{(1)}V_o^{(1)}\right)$$

$$g_4(x) = \frac{5}{2(1-\alpha^2)\,V_o^{(1)^2}}\left\{\begin{array}{l}\frac{4}{5}A V_o^{(1)}(1+\alpha)\,V_1^{(2)} + \frac{6+\alpha}{2(1+\alpha)}\left(B_o^{(2)'}+2\alpha V_o^{(1)}V_o^{(2)'}\right)\\ -\left(3B_o^{(2)'}+\frac{12}{5}(1+\alpha)V_o^{(1)}V_o^{(2)'}\right)+\kappa\,\frac{6+\alpha}{2(1+\alpha)}\left(U_o^{(1)^2}-\alpha^2V_o^{(1)^2}\right)-\kappa\left(2U_o^{(1)^2}-\frac{4}{5}\alpha(1+\alpha)V_o^{(1)^2}\right)\end{array}\right\}$$

$$g_5(x) = \frac{2A}{(1-\alpha)\,V_o^{(1)}}\,V_o^{(2)'}$$

$$g_6(x) = \frac{5A}{2(1-\alpha^2)V_o^{(1)^2}}\left\{\left(\frac{1-\alpha}{2\alpha}+\frac{6+\alpha}{2(1+\alpha)}-3\right)B_1^{(2)}+\left((1-\alpha)+\frac{\alpha(6+\alpha)}{1+\alpha}-\frac{12}{5}(1+\alpha)\right)V_o^{(1)}\,V_o^{(2)'}\right\}$$

$$g_7(x) = \frac{5A}{2(1-\alpha^2)\,V_o^{(1)^2}}\left\{\begin{array}{l}\left(\frac{6+\alpha}{2(1+\alpha)}+\frac{1-\alpha}{2\alpha}\right)B_1^{(2)}+\left((1-\alpha)+\frac{\alpha(6+\alpha)}{1+\alpha}\right)V_o^{(1)}V_1^{(2)}\\ -\left(\frac{12}{5}(1+\alpha)\;V_o^{(1)}V_o^{(2)'}+3B_o^{(2)'}\right)-\frac{2}{5}\,\kappa\,V_o^{(1)^2}(1-\alpha^2)\end{array}\right\}$$

$$g_8(x) = \frac{5\kappa}{4\alpha(1+\alpha)\,V_o^{(1)^2}}\left(U_o^{(1)^2}-\alpha^2\,V_o^{(1)^2}\right)$$

$$g_9(x) = \frac{5\kappa}{4(1+\alpha)\,V_o^{(1)^2}}\left(U_o^{(1)^2}-V_o^{(1)^2}\right) \qquad (38)$$

Equation (37) admits the solution

$$\left(1+e^{A(Y+L)}\right)\left(1+e^{-A(Y+L)}\right) n_1^{(-)}$$

$$= g_1(x)\left\{2Y + \frac{1}{A} e^{A(Y+L)} - \frac{1}{A} e^{-A(Y+L)}\right\}$$

$$+ g_2(x)\left\{Y - \frac{1}{A} e^{-A(Y+L)}\right\}$$

$$+ g_3(x)\left\{\frac{Y^2}{2} - \frac{Y}{A} e^{-A(Y+L)} - \frac{1}{A^2} e^{-A(Y+L)}\right\}$$

$$+ g_4(x)\left\{Y + \frac{1}{A} e^{A(Y+L)}\right\}$$

$$+ g_5(x)\left\{\frac{Y^2}{2} + \frac{Y}{A} e^{A(Y+L)} - \frac{1}{A^2} e^{A(Y+L)}\right\}$$

$$+ g_6(x) Y + g_7(x) \frac{Y^2}{2}$$

$$+ g_8(x) \left\{\begin{array}{l} \left(Y - \frac{1}{A} e^{-A(Y+L)}\right) \log\left(1+e^{A(Y+L)}\right) \\ + \frac{1}{A} \log \frac{e^{A(Y+L)}}{1+e^{A(Y+L)}} - A \frac{Y^2}{2} + A \int \frac{Y}{1+e^{A(Y+L)}} dY \end{array}\right\}$$

$$+ g_9(x) \left\{\begin{array}{l} \left(Y - \frac{1}{A} - \frac{1}{A} e^{-A(Y+L)}\right) \log \frac{e^{A(Y+L)}}{1+e^{A(Y+L)}} \\ - \frac{1}{A} e^{-A(Y+L)} - A \int \frac{Y}{1+e^{A(Y+L)}} dY \end{array}\right\} \tag{39}$$

$$+ C(x)$$

where C(x) is the constant (with respect to Y) of integration. The boundary conditions on $n_1^{(-)}$ are that as $Y \to \pm\infty$, $n_1^{(-)} \to 0$. But as $Y \to \pm\infty$, the coefficient of $n_1^{(-)} \to \infty$. Therefore, these boundary conditions do not determine C(x). Once again, $n_1^{(-)}(x)$ can be completely determined only if we specify a definite point within the first order shock structure.

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