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SUMMARY

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The present experimental study is concerned with the effect of high-velocity fluid flow on the static and dynamic characteristics of a simply supported pipe. The frequency variation of the bending vibrations as a function of fluid velocity, as predicted by G. W. Housner, was verified experimentally. In addition, the critical fluid velocity at which the system becomes statically unstable was also verified.

author

INTRODUCTION

High-performance liquid-propellant launch vehicles require the rapid transfer of large quantities of fluid from tanks to pumps and engines through relatively thin-walled pipes. A study of the influence of the resulting high-velocity fluid on the static and dynamic characteristics of the pipes in launch vehicles is therefore necessary.

Initial work in this field was motivated by vibration problems encountered in the transport of crude oil in pipes as well as possible problems in the feed lines of large water turbines. Ashley and Haviland (ref. 1) were the first to investigate the bending vibrations of a simply supported pipe containing a flowing fluid. Housner (ref. 2) subsequently derived the equation of motion for the same case more completely and developed an equation relating the fundamental bending frequency of a simply supported pipe to the velocity of the internal flow of the fluid. Housner also showed that, at a certain critical velocity (similar to the Euler buckling formula for columns), a statically unstable condition could exist. Long (ref. 3) used an alternate solution to Housner's equation of motion for the simple end conditions and also treated the fixed-free end conditions. In addition, Long attempted to confirm the analysis by comparison with experiment. The experimental results were rather inconclusive since the maximum fluid velocity available for the test was low and the change in the bending frequency was very small. The maximum velocity obtained by Long was only 20 percent of the critical velocity which resulted in a maximum change in the bending frequency of only 3.2 percent. Other treatments of the subject were made by Ta Li (ref. 4), Niordson (ref. 5), and Benjamin (ref. 6).

It is the purpose of this paper to report the results of an experiment to (a) study the effect of fluid velocity on the fundamental bending frequency of a simply supported pipe and (b) determine the existence of a statically unstable condition as predicted in reference 2.

A number of engineering students, while participating consecutively in the cooperative work study program at Langley Research Center, conducted the test program under the general supervision of the second author. The students who planned the experiment, selected and assembled the test apparatus, and conducted the tests were Messrs. Harold L. Dodds, Jr., Richard A. Crocker, Charles H. Fox, Jr., James E. Gardner, and A. David Luckey.

SYMBOLS

A_{2k} amplitude coefficient in series expression (eq. (2)) for $k = 1, 2, 3, \ldots$

A_{2n-1} amplitude coefficient in series expression (eq. (2)) for $n = 1, 2, 3, \ldots$

E modulus of elasticity, lb/ft^2

I moment of inertia, ft^4

k integer

l length, ft

m mass of pipe plus fluid per unit length, slugs/ft

n integer

t time, sec

V average velocity of the fluid relative to x-direction, ft/sec

x coordinate measured along pipe

z coordinate measured perpendicular to pipe

ρ mass of fluid per unit length, slugs/ft

ω angular velocity of pipe relative to its length, rad/sec

Subscripts:

c critical

i mode of vibration, $i = 1, 2, 3, \ldots$

ANALYSIS

The equation of motion of a uniform pipe containing flowing fluid is

$$EI \frac{\partial^4 z}{\partial x^4} = -\rho V^2 \frac{\partial^2 z}{\partial x^2} - 2\rho V \frac{\partial^2 z}{\partial t \, \partial x} - m \frac{\partial^2 z}{\partial t^2} \qquad (1)$$

which was derived in reference 2 by using elementary beam theory. The coordinate system is shown in figure 1. The three terms on the right side of

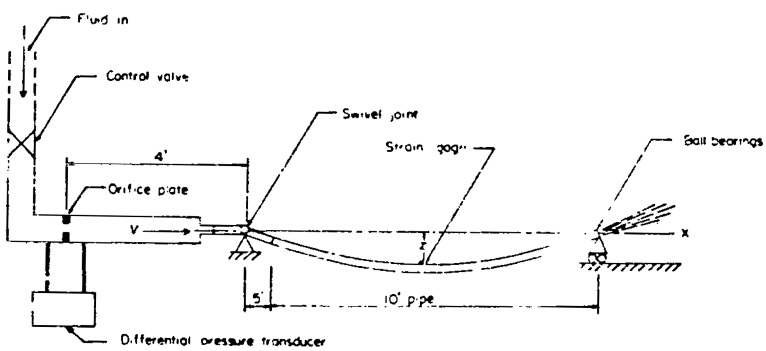


Figure 1.- Schematic diagram of experimental setup.

equation (1) represent the lateral inertial forces which act on the pipe. The first term is the force caused by the change in direction of velocity of the fluid due to the curvature of the pipe. The second term is the force associated with the Coriolis acceleration which occurs because the pipe has an angular velocity with respect to its length while the fluid has a linear velocity with respect to the pipe. The last term represents the force which is related to the vertical acceleration of the pipe. For simplification, structural damping has been omitted in equation (1).

For the case of free vibrations, the general solution of equation (1) for a simply supported pipe with normal modes z_i is given in reference 2 as

$$z_1 = \sum_n A_{2n-1} \sin\left[(2n - 1)\frac{\pi x}{l}\right]\sin \omega_1 t + \sum_k A_{2k} \sin\left[2k \frac{\pi x}{l}\right]\cos \omega_1 t \tag{2}$$

In order to evaluate the coefficients in this equation, equation (2) is substituted into equation (1) and the resulting terms are grouped according to their trigonometric constituents. Equating the coefficients of like trigonometric terms to zero in order to satisfy equation (1), the following set of equations for $n = 1, 2, 3, \ldots$ and $k = 1, 2, 3, \ldots$ are obtained:

$$A_{2n-1}\left[EI(2n - 1)^4\left(\frac{\pi}{l}\right)^4 - \rho V^2(2n - 1)^2\left(\frac{\pi}{l}\right)^2 - m\omega_1^2\right]$$

$$= \frac{8\rho V\omega_1}{l} \sum_k A_{2k} \frac{(2k)^2}{(2k)^2 - (2n - 1)^2} \tag{3}$$

$$A_{2k}\left[EI(2k)^4\left(\frac{\pi}{l}\right)^4 - \rho V^2(2k)^2\left(\frac{\pi}{l}\right)^2 - m\omega_1^2\right]$$

$$= \frac{8\rho V\omega_1}{l} \sum_n A_{2n-1} \frac{(2n - 1)^2}{(2n - 1)^2 - (2k)^2} \tag{4}$$

Supressing all the modes except the first two, the simultaneous solution of equations (3) and (4) results in the following frequency equation for steady-state vibrations:

$$\frac{32\rho V\omega_1}{3l\left[EI\left(\frac{\pi}{l}\right)^4 - \rho V^2\left(\frac{\pi}{l}\right)^2 - m\omega_1^2\right]} = -\frac{3\left[EI\left(\frac{2\pi}{l}\right)^4 - \rho V^2\left(\frac{2\pi}{l}\right)^2 - m\omega_1^2\right]}{\rho V\omega_1} \tag{5}$$

Equation (5) gives the relation between V and ω_1 for the pipe.

By letting $\omega_1 \to 0$ for the case $i = 1$, equation (5) yields a critical velocity at which a static divergence occurs.

$$V_c = \sqrt{\frac{EI\pi^2}{\rho l^2}} \tag{6}$$

As pointed out in reference 2, replacement of ρV^2 in equation (6) by P provides an equation identical to the Euler buckling formula for columns where P represents the axial load.

EXPERIMENTAL INVESTIGATION

The apparatus and testing procedure which were used to study the effect of high-velocity fluid flow on the bending vibrations and stability of a simply supported pipe are described in the following section.

Apparatus

A schematic representation of the experimental setup is shown in figure 1. The test pipe was constructed of aluminum alloy (6061-T6) with a nominal outside diameter of 1.00 inch, a wall thickness of 0.065 inch, and a length of 10 feet. The pipe was mounted horizontally with the downstream end supported by two ball bearings which allowed unrestrained horizontal translation and rotation of the downstream end. The upstream end was connected to a swivel joint. This type of support system provided an accurate simulation of a beam with pinned-pinned boundary conditions. The ball bearings at the end of the pipe prevented axial stresses during lateral deformations. A strain gage was mounted in the center of the pipe at the location of the maximum bending moment from which the frequency and damping of the bending vibrations could be determined.

The average velocity of the fluid (water) was measured by means of an orifice plate located approximately 4 feet upstream from the swivel joint. The instrumentation for measuring the fluid velocity consisted of the orifice plate, a differential-pressure transducer, and a galvanometer recorder. This equipment was accurate to within 2 percent (rms value) of full-scale response.

The fluid for this experiment was obtained from a high-pressure water source located at the Langley landing loads track. This facility, capable of delivering 750 gallons per minute at a maximum pressure of 3200 lb/in^2, is described in detail in reference 7.

Testing Procedure

In order to initiate the free vibrations of the pipe at various fluid velocities, a method of imparting a small disturbance to the pipe was necessary. This disturbance was accomplished by attaching a cord at the midpoint of the pipe which could be manually operated during the test. The procedure for conducting a test was to disturb the pipe by plucking the cord vertically, to record the response of the pipe, and to increase the fluid velocity continuously until instability of the pipe was achieved.

DISCUSSION OF RESULTS

This section includes both the calculated and experimental results and also some remarks concerning the "damping" characteristics of the system.

The values of the constants used in calculating the value of V_c from equation (6) and the relation between ω_1 and V from equation (5) are as follows:

$$\rho = 8.00 \times 10^{-3} \text{ slugs/ft}$$

$$E = 1.44 \times 10^{9} \text{ lbs/ft}^2$$

$$I = 1.0416 \times 10^{-6} \text{ ft}^4$$

$$m = 1.49 \times 10^{-2} \text{ slugs/ft}$$

$$l = 10.5 \text{ ft}$$

A departure from an ideally uniform beam was required for the upstream end of the pipe in order to accommodate part of the swivel joint (fig. 2). The joint provided about 0.5 foot of rigid heavy pipe for an overall test length of 10.5 feet. The effect of this short stiff section (compared with the thin-walled aluminum tube) on the critical velocity is considered negligible since the curvature of the pipe is a maximum at its center and must approach zero at the ends in order to satisfy the boundary conditions. On the other hand, the effect of the swivel joint on the frequency of the pipe could be more pronounced because of the concentrated mass at the end of the pipe. However, comparison of the calculated frequency for a uniform pipe 10.5 feet long with the experimental frequency for the test pipe, which includes the deviation resulting from the swivel joint, indicates no appreciable effect on

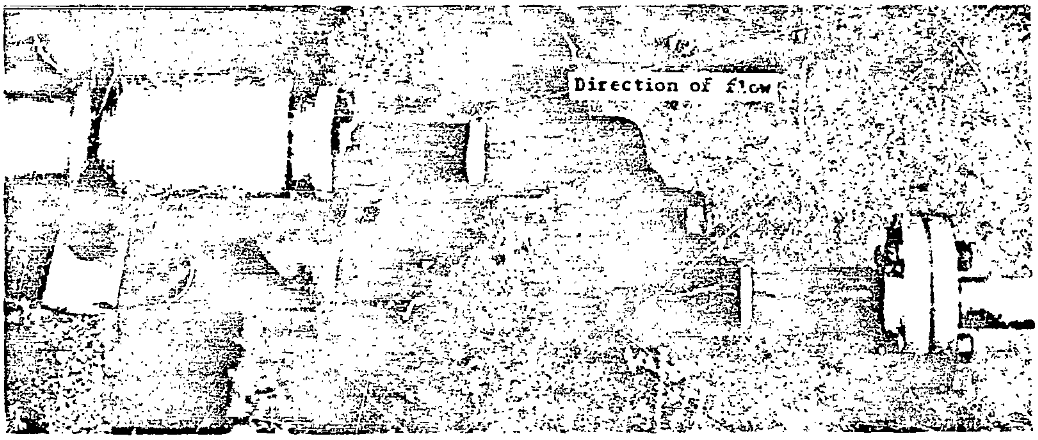


Figure 2.- Swivel joint at upstream end of test pipe. L-64-699L.1

the natural frequency. Consequently, the fundamental equations in reference 2 were not rederived to include the possible effects caused by the swivel joints.

The calculated results from equations (5) and (6) for the fundamental mode of vibration and the experimental data for two identical pipes are shown in figure 3. The specific experimental values pertaining to figure 3, which have been normalized by the theoretical values of ω at $V = 0$ and V_c $\left(\omega_{V=0} = 28.4 \text{ rad/sec} \text{ and } V_c = 129.5 \text{ ft/sec}\right)$, appear in the following table. The overall damping of the system at each data point, as determined from the strain-gage records, also appears in the table:

Pipe	V/V_c	$\omega/\omega_{V=0}$	Log decrement
1	*0	1.042	0.179
	.332	.919	.308
	.595	.849	.409
	.753	.662	.967
2	0	Not obtained	Not obtained
	.167	1.053	0.329
	.354	.958	.337
	.543	.919	.629
	.689	.676	.865
	.752	.739	1.098
	.985	0	Not obtained

*For pipe 1 at $V/V_c = 0$, the data were obtained when the pipe was empty; however, the value, $\omega/\omega_{V=0} = 1.042$, has been corrected to include the mass of the fluid for a full pipe

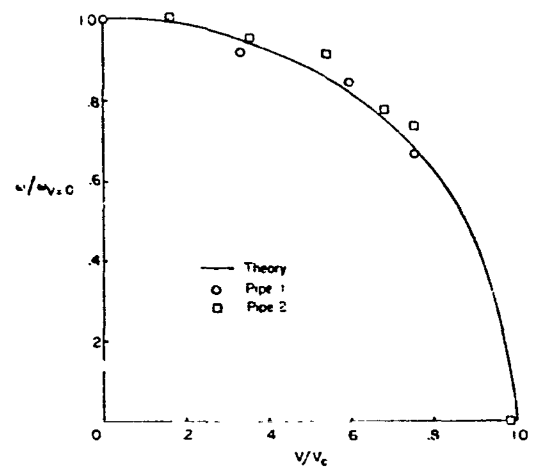


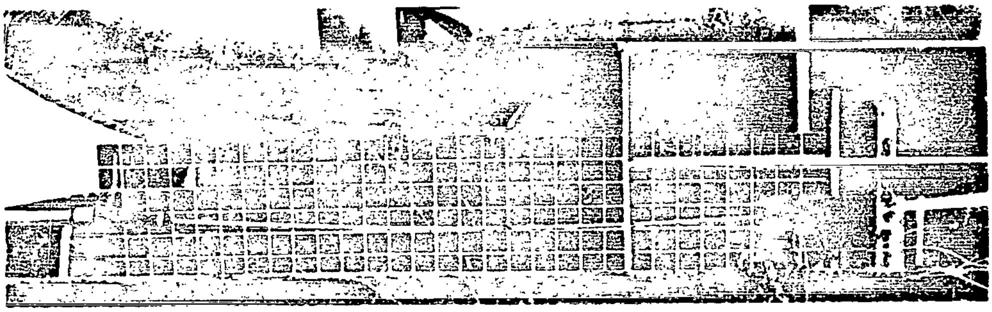
Figure 3.- Comparison of theoretical and experimental results.

As can be observed in figure 3, a good correlation between theory and experiment is achieved. The maximum experimental error relative to the theoretical curve is approximately 7.5 percent.

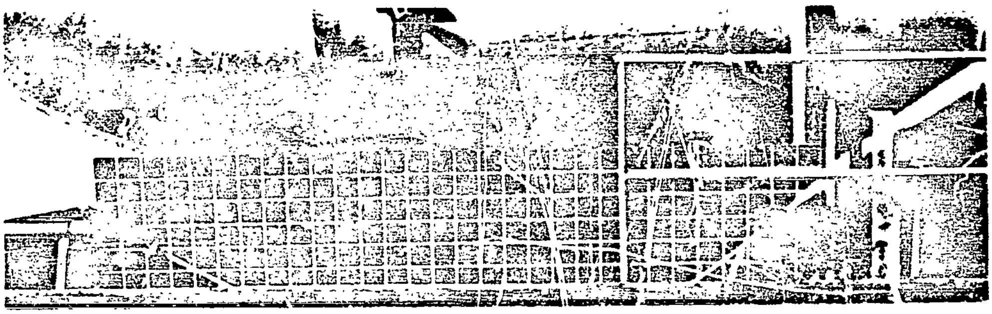
As predicted by the theory, the system became unstable which resulted in a permanent deformation of the pipe. Divergence was achieved with both pipes; however, because of an instrumentation failure during the first test, only the divergence velocity for the second pipe was recorded which was 127.5 ft/sec. The experimental error of the divergence velocity relative to the predicted critical velocity is 1.5 percent. Pictures of the divergence phenomenon are shown in sequence in figure 4. Figure 4(a) shows the curvature of the pipe at V = 120 ft/sec which is just before the pipe diverged. Figure 4(b) shows the curvature of the pipe at the divergence velocity of 127.5 ft/sec. Figure 4(c) shows the permanent deformation of the pipe after the test was completed.

The log decrement, as shown in the table, is a measure of the total damping of the system at each data point. These values were obtained by taking the natural logarithm of the ratio of successive amplitudes for the first completed cycle of each strain-gage record. The log decrement value when the pipe is empty can be considered as a measure of the structural damping of the pipe and its end supports (ref. 3). This value was 0.179 for the first pipe.

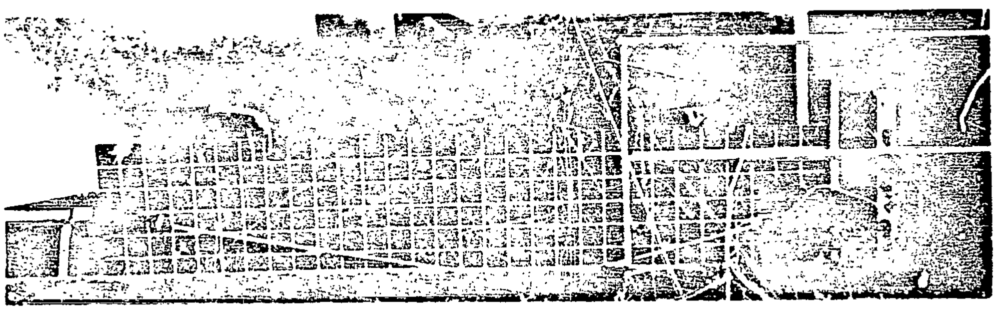
As may be observed in the table, the natural frequency of the pipe decreases as the fluid velocity increases while the log decrement values increase. It should be noted that the log decrement increase is due to the combined effect of the frequency and fluid velocity, although the values differ in some cases for the two pipes.



(a) Curvature of the pipe just before divergence. V = 120 ft/sec. L-64-10020



(b) Curvature of the pipe at divergence. V = 127.5 ft/sec. L-64-10022



(c) Curvature of the pipe after completion of the test. V = 0 ft/sec. L-64-10024

Figure 4.- Photographs of pipe during divergence.

CONCLUDING REMARKS

The present investigation has been concerned with an experimental study of the effect of fluid velocity on the natural frequency of the free vibrations of a simply supported pipe. Tests were conducted with two identical pipes both of which verified the predicted variation of the fundamental bending frequency with fluid velocity. The predicted critical fluid velocity for which the system becomes statically unstable was also verified experimentally. From the results presented, it appears that the theory of G. W. Housner is adequate for predicting the effect of fluid velocity on the natural frequency of simply supported pipes as well as the particular fluid velocity at which a static instability exists.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., March 15, 1965.

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