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NASA CR - 61075

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FACILITY FORM 808	N65 28000	
	(ACCESSION NUMBER)	(THRU)
	100	1
	(PAGES)	(CODE)
	CW 61075	30
	(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

PARAMETRIC STUDY OF LUNAR LANDING TECHNIQUES
USING A PREDETERMINED THRUST ORIENTATION

Prepared under Contract No. NAS 8-20073 by

Sam H. Harlin, Jr.

BROWN ENGINEERING COMPANY

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Microfiche (MF) .25¢

For

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

June 1965

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TECHNIQUES USING A PREDETERMINED
THRUST ORIENTATION

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Prepared under Contract No. NAS 8-20073

BROWN ENGINEERING COMPANY
300 Sparkman Drive
Huntsville, Alabama

For

R-P&VE-AA
PROPULSION AND VEHICLE ENGINEERING
LABORATORY

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

PARAMETRIC STUDY OF LUNAR LANDING
TECHNIQUES USING A PREDETERMINED
THRUST ORIENTATION

ABSTRACT

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A parametric study has been made to show the variation in trajectory parameters, for a predetermined thrust orientation angle to perform lunar descents from an initial parking orbit. Although the analytical methods used in this study are quite common, the presentation of these data in parametric form is unique. To study the actual deviation in the descent parameters the thrust orientation angle is varied. In previous parametric studies most of the data presented have been based on a decelerated stage with a zero angle of thrust orientation. A single propulsive stage was used in this study. Specific impulses and parking orbit altitudes are varied for comparison. Powered braking was considered both before and after passing through pericenter of the Hohmann orbit.

The results of the study show the variations of descent parameters for earth thrust-to-weight ratio. These parameters include energy requirements, traversed altitude, initial injection parameters, burning time and range.

Author

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DEFINITION OF SYMBOLS

Symbol	Definition
F/W_0	Initial Earth Thrust-to-Weight Ratio (based on weight at sea level)
g	Earth Gravitational Acceleration, 9.80665 m/sec^2
g_e	Lunar Gravitational Acceleration, 1.60862 m/sec^2
h	Altitude, km
h_p	Pericenter Altitude, km
h_{park}	Parking Orbit Altitude, km
Δh	Altitude Change, $h-h_0$, km
I_{sp}	Specific Impulse, sec
m	Mass, kg
r	Radius, km
t	Time, sec
Δt	Time Increment, sec
V_{ex}	Exhaust Velocity, m/sec ($I_{sp} g$)
V	Velocity, m/sec
V^*	Comparative Velocity, m/sec
V_{imp}	Stage Characteristic Impulse Velocity, m/sec
V_{char}	Stage Characteristic Velocity, m/sec
X	Range, km
α	Thrust vector orientation angle measured from the velocity to the thrust vector (positive down), deg

DEFINITION OF SYMBOLS (concluded)

Symbol	Definition
α'	$\alpha - \pi$ (Used for simplification in report)
ζ	Propellant mass fraction, W_{prop}/W_o
θ	Flight path angle from local vertical, deg
μ_{a}	Gravitational constant for the moon, $4906 \text{ km}^3/\text{sec}^2$
ψ	Central Angle from arbitrary reference, deg
Δ	Denote change or increment
Subscript	
a	Apocenter
B	Burnout
f	Final
id	Ideal
o	Initial
a	Moon
P	Pericenter
park	Parking Orbit
prop	Propellant

PARAMETRIC STUDY OF LUNAR LANDING
TECHNIQUES USING A PREDETERMINED
THRUST ORIENTATION

SUMMARY

A parametric study has been made to show the variation of trajectory parameters, for a predetermined thrust orientation angle, in performing lunar descents from an initial parking orbit. A single propulsive stage was used in all instances. The specific impulses chosen varied from 280 to 320 seconds and from 400 to 440 seconds. The parking orbit altitudes selected for comparison were 150, 185 and 300 kilometers. Powered braking was considered both before and after passing through pericenter of the Hohmann orbit. The thrust orientation angle was varied from +30 to -30 degrees where applicable.

The results of the study show the variations of descent parameters for earth thrust-to-weight ratio from 0.2 to 0.6. These parameters include energy requirements, traversed altitude, initial injection parameters, burning time, and range.

SECTION I. INTRODUCTION

As a part of the Saturn program involving lunar landing studies, specific emphasis is being directed toward a study of the powered trajectory phase of the lunar braking stage. The detailed and conceptual design of vehicle systems must reflect consideration of the interrelation of mission profile and stage design.

The purpose of this study is to establish flight profiles and energy requirements for stages having various thrust-to-weight ratios, specific impulses, altitudes, and Hohmann pericenters utilizing predetermined pitch programs.

The analytical methods used in this study are quite common; however, the presentation of the data in parametric form is unique. To study the actual deviation in the descent parameters the thrust orientation angle is varied. In previous parametric studies most of the data presented have been based on a decelerated stage with a zero angle of thrust orientation. This study relates to the overall powered phase of the lunar braking stage and reflects the parameters of energy requirements, range traversed, altitude traversed, along with the general ignition conditions.

The approach used for the descent maneuver was to apply an impulse to the spacecraft in a parking orbit to attain a Hohmann transfer ellipse. At the proper point on the transfer ellipse, powered braking was initiated to arrive at a 300-meter altitude with zero velocity. The analysis was limited to planar trajectories with a constant, predetermined tilt program neglecting all perturbative forces.

SECTION II. ASSUMPTIONS

A summary of the basic assumptions used in this analysis follows:

1. Deceleration of a single propulsion stage from a transfer ellipse to a 300-meter (hover) altitude and zero velocity using constant thrust.

2. Initial parking orbit altitudes:

$$h_{\text{park}} = 150 \text{ kilometers}$$

$$h_{\text{park}} = 185 \text{ kilometers}$$

$$h_{\text{park}} = 300 \text{ kilometers}$$

3. Initial pericenter altitudes for transfer ellipse:

$$h_p = 10 \text{ kilometers}$$

$$h_p = 15 \text{ kilometers}$$

$$h_p = 20 \text{ kilometers}$$

$$h_p = 25 \text{ kilometers}$$

4. Specific impulses of 280, 300, 320, 400, 420 and 440 seconds were used.

5. The initial earth thrust-to-weight ratio for a chemical stage was varied parametrically from 0.2 to 0.6.

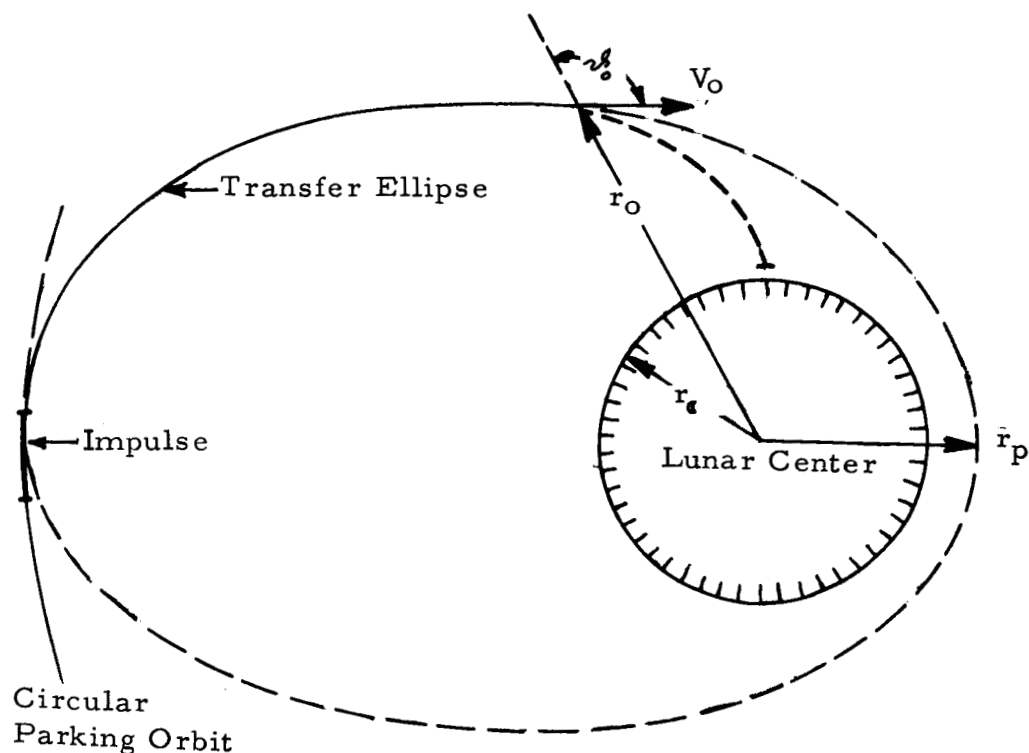
6. Mean spherical moon:

$$\mu_e = 4906 \text{ km}^3/\text{sec}^2$$

$$r_e = 1738.3 \text{ km}$$

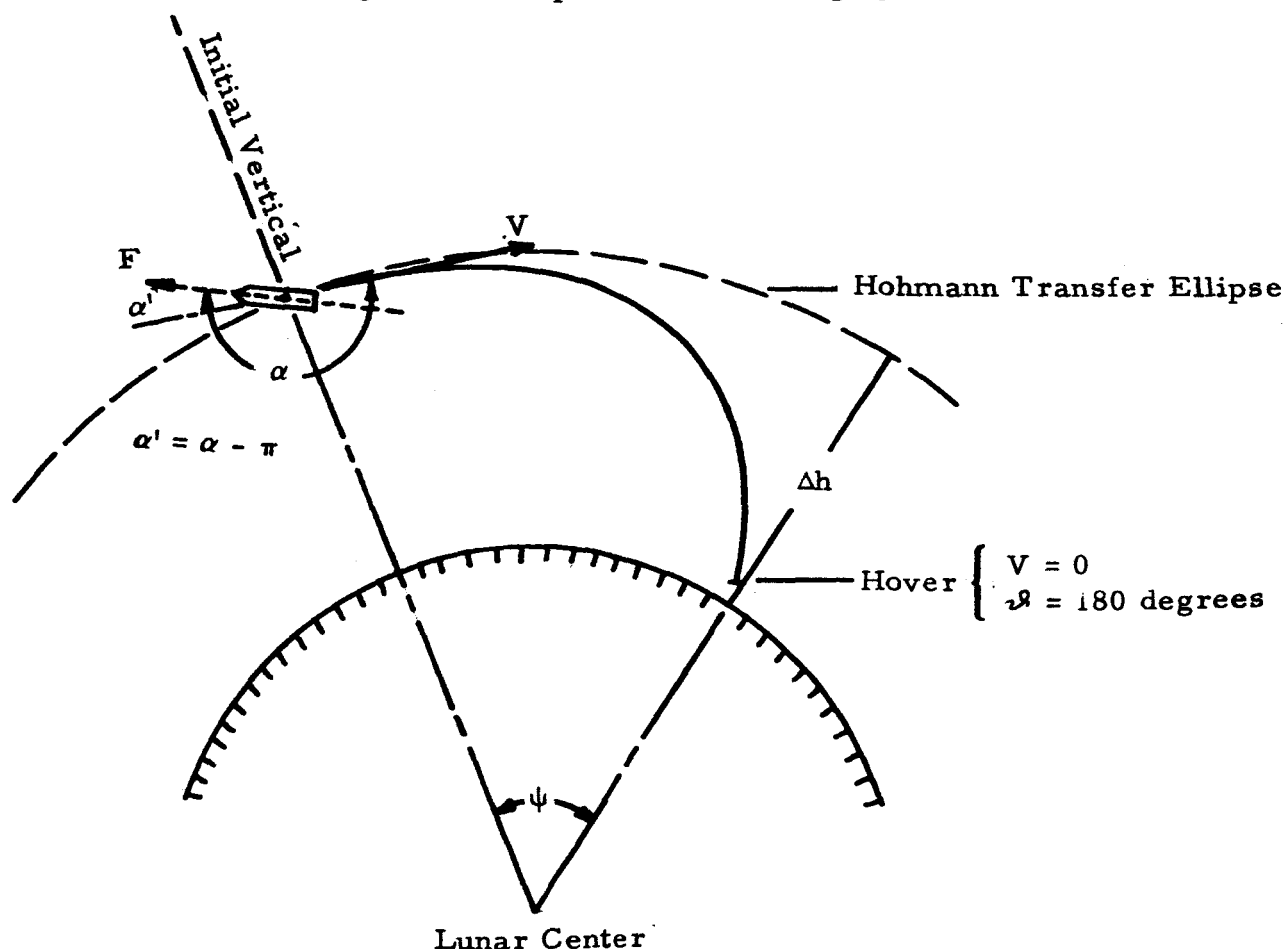
SECTION III. ANALYSIS

One method of descent from a lunar parking orbit is to direct an impulse opposite the velocity vector (i.e. thrust orientation of $\alpha = 180^\circ$) to attain a low-pericenter transfer ellipse. Figure 1 shows the impulse velocity requirement to attain the Hohmann transfer ellipse. At a proper point on this transfer ellipse, powered braking is initiated to arrive at a very low lunar altitude, on the order of 300 meters, with zero velocity. The sketch below illustrates the geometric procedure.



In initiating the powered braking, the optimum thrust orientation angle varies with the initial thrust-to-weight ratio of the spacecraft, the specific impulse of the engine and the descent altitude. To minimize the velocity requirement, this thrust angle should be as close to optimum as possible. In this study, numerous descent trajectories were computed varying the angles of attack until the optimum angle was

determined for each condition. The sketch below shows the represented thrust orientation angle with respect to descending spacecraft.



The thrust orientation angle is measured positively downward from the velocity vector to the thrust vector. In case the angle of attack is 180° the thrust vector is directed opposite the velocity vector.

Parking orbit altitudes of 150, 185, and 300 kilometers were selected for this study. The pericenter altitudes of the transfer ellipse were varied from 10 to 25 kilometers. Such factors as surface irregularities and guidance errors should be considered when selecting a pericenter altitude for the transfer ellipse.

The powered descent in this study was initiated from a point both prior to pericenter and on the return leg after pericenter. The thrust vector orientation angle was varied until the optimum angle was determined for each Earth thrust-to-weight ratio, specific impulse, and transfer altitude. A constant thrust was applied in all instances.

The following point mass equations of motion were used:

$$\dot{V} = \frac{F \cos \alpha}{m} - \frac{g_e r_e^2}{r^2} \cos \vartheta \quad (1)$$

$$V \dot{\vartheta} = \frac{F \sin \alpha}{m} + \left[\frac{g_e r_e^2}{r^2} - \frac{V^2}{r} \right] \sin \vartheta \quad (2)$$

$$\dot{r} = V \cos \vartheta \quad (3)$$

$$\dot{\psi} = \frac{V \sin \vartheta}{r} \quad (4)$$

where

$$m = m_0 + \int \dot{m} dt \quad (5)$$

and

$$\dot{m} = - \frac{F}{V_{ex}} \quad (6)$$

The velocity and flight path angle may be obtained by integrating the equation of motion

$$V = \int \dot{V} dt \quad (7)$$

$$\vartheta = \int \dot{\vartheta} dt \quad (8)$$

The range and altitude can then be calculated by the relations

$$X = r_e \int \frac{V \sin \vartheta}{r} dt = r_e \int \dot{\psi} \quad (9)$$

$$h = h_0 + \int \dot{r} dt \quad (10)$$

and the central angle

$$\psi = \int \dot{\psi} dt \quad (11)$$

The energy expended by the vehicle is equivalent to the characteristic velocity or

$$V_{\text{char}} = V_{\text{ex}} \ln \left(\frac{1}{1 - \zeta} \right) \quad (12)$$

and may be determined from Figures 2A through 3L. Consequently, the velocity losses are the difference between the characteristic velocity and the change in comparative velocity, or

$$\Delta V_{\text{loss}} = V_{\text{char}} - \Delta V^* \quad (13)$$

where the comparative velocity is

$$V^* = \sqrt{V_o^2 + 2\mu \left(\frac{1}{r} - \frac{1}{r_o} \right)} \quad (14)$$

and the change in comparative velocity during descent from $r = r_o$ to $r = r_f$ is

$$\Delta V^* = \sqrt{V_o^2 + 2\mu \left(\frac{1}{r_f} - \frac{1}{r} \right)} - V_f \quad (15)$$

By substituting for velocity losses in the original equation

$$\Delta V_{\text{loss}} = V_{\text{ex}} \ln \left(\frac{1}{1 - \zeta} \right) - \sqrt{V_o^2 + 2\mu \left(\frac{1}{r_f} - \frac{1}{r_o} \right)} - V_f \quad (16)$$

SECTION IV. RESULTS

The results of this analysis are presented graphically in Section VI. The data were separated according to pericenter altitudes and specific impulse for ignition both before and after pericenter passage. For convenience, the energy requirements, initial ignition velocity and altitude traversed during descent are plotted as functions of the initial vehicle thrust-to-weight ratio.

The impulse velocity, shown in Figure 1, is the velocity necessary to attain a Hohmann transfer ellipse from the circular parking orbit. An impulse velocity of less than 75 m/sec is sufficient to perform all of the Hohmann transfers; therefore, all velocity losses are extremely small and negligible.

The characteristic velocity requirements to perform each of the missions considered are presented in Figures 2A through 3L. Figures 4A through 5L show the altitude traversed during the burning phase of the descent maneuver. The traversed altitude is also the initial altitude of engine ignition. The range is shown as a function of initial altitude at engine ignition in Figures 6A through 7L. The graphic presentation of range is not legible as a function of thrust-to-weight ratio because the curves are sometimes closer than plotting accuracy will allow; therefore, for clarity, the range angle was presented as a function of altitude.

The propellant mass fraction and burning time can be obtained from Figures 8 and 9, respectively. The initial Hohmann orbital parameters at engine ignition may be found in Figures 10 and 11. These parameters were presented in this form for simplicity. The change in flight path angle during descent is $180^\circ - \theta_0$.

SECTION V. CONCLUSIONS

The results of this parametric study show that the thrust orientation angle has an effect on the parameters of the descent mode. To assume a zero angle of attack for all lunar descents is by no means optimum. In many cases of higher thrust-to-weight ratio (i.e., above 0.55), a thrust orientation of zero degrees would not be applicable for the mission. From 0.2 to 0.55 thrust-to-weight ratio, however, zero degrees is an adequate approximation for preliminary calculations. The trend, as indicated in the graphic presentation of the data, indicates that as the thrust-to-weight decreases to 0.2, the velocity requirement is minimized with a positive angle of attack.

Descent initiation at points either before or after pericenter is found to be a factor in minimizing the actual energy requirements for each mission. It is shown that descents after pericenter require less energy, thus less propellant for the maneuver. This could be a basic consideration in vehicle design and planning.

SECTION VI. GRAPHIC PRESENTATION

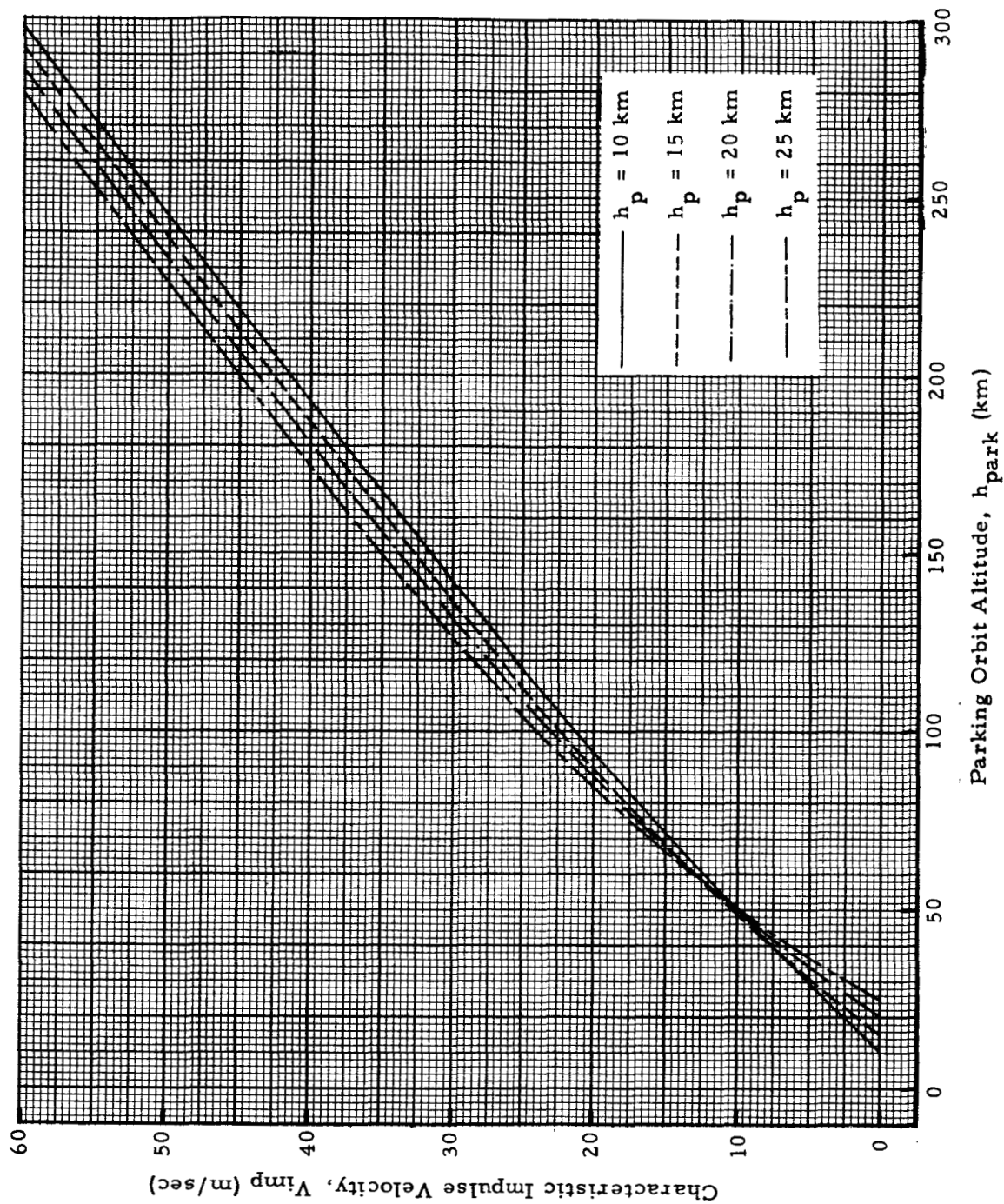


FIGURE 1. IMPLUSE VELOCITY REQUIRED TO ATTAIN HOHMANN TRANSFER ELLIPSE

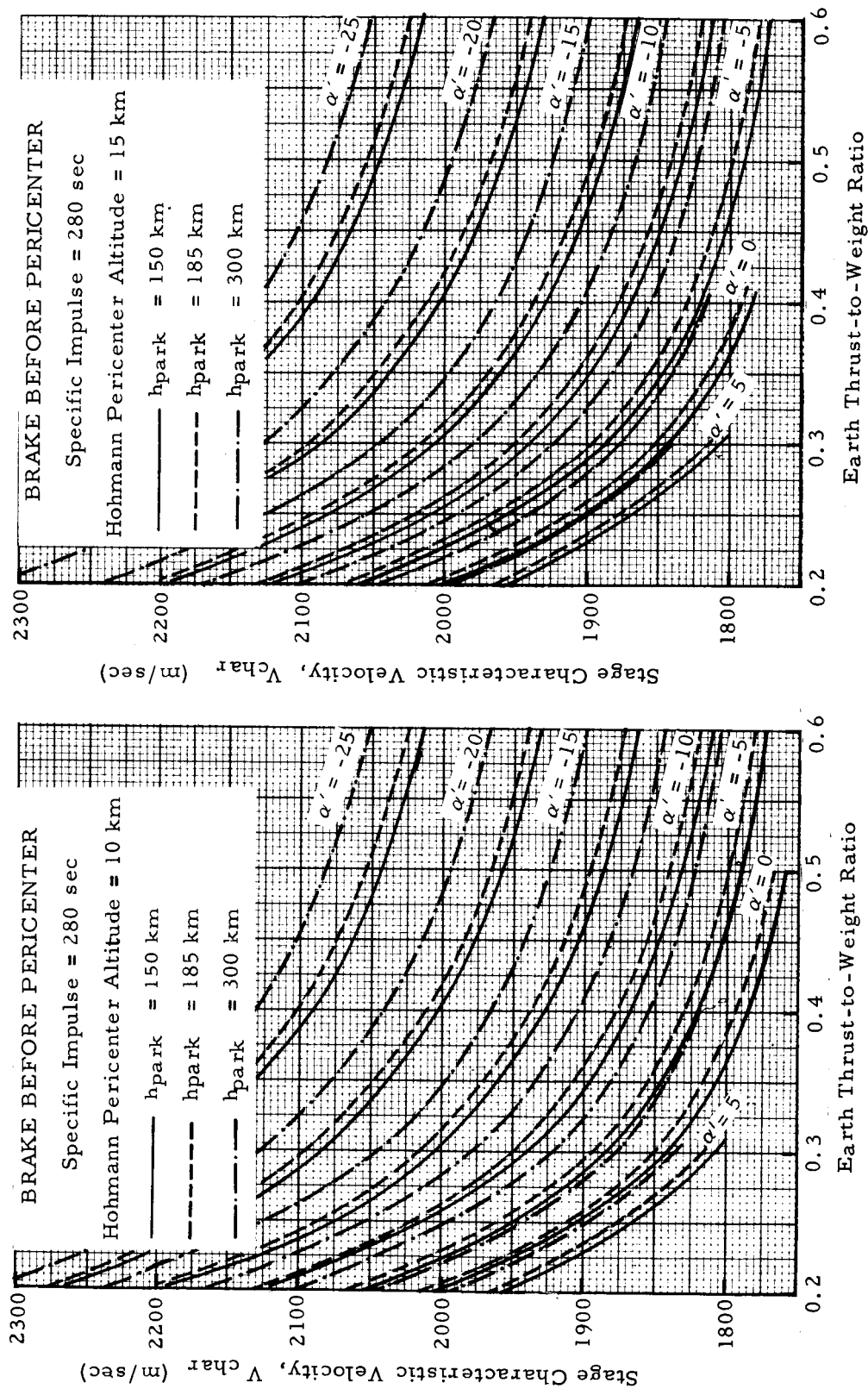


FIGURE 2 a. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

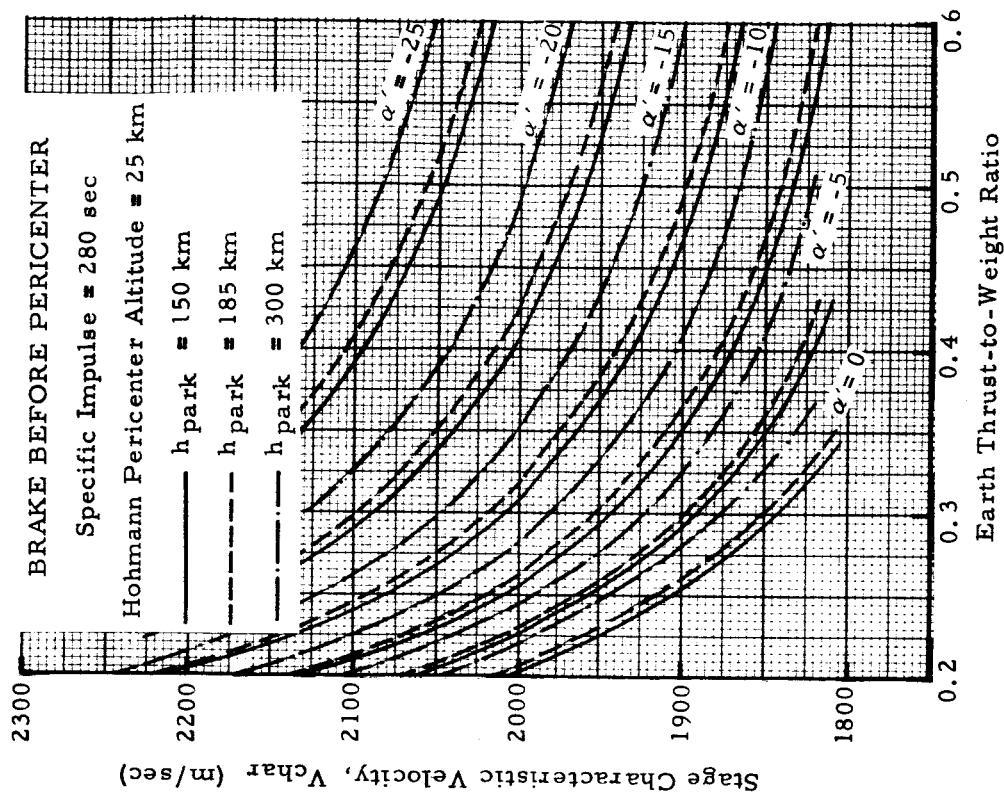
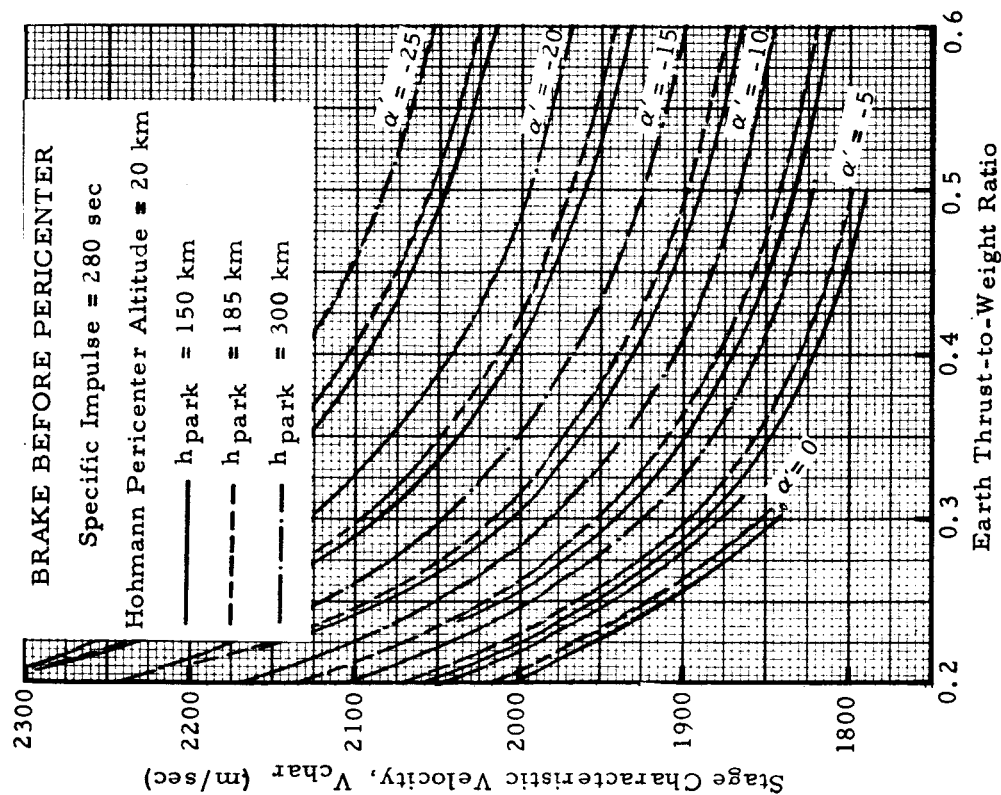


FIGURE 2 b. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

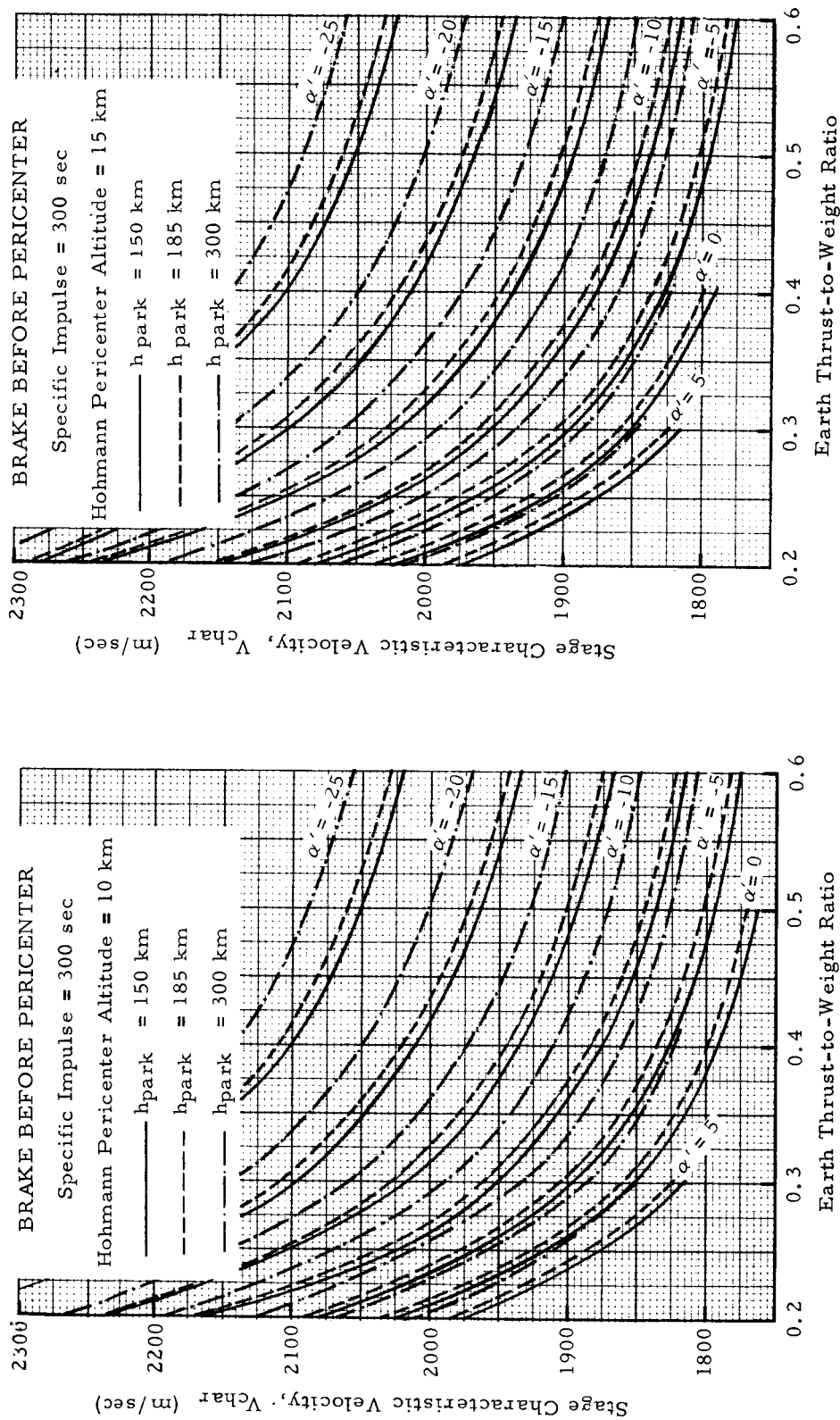


FIGURE 2 c. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

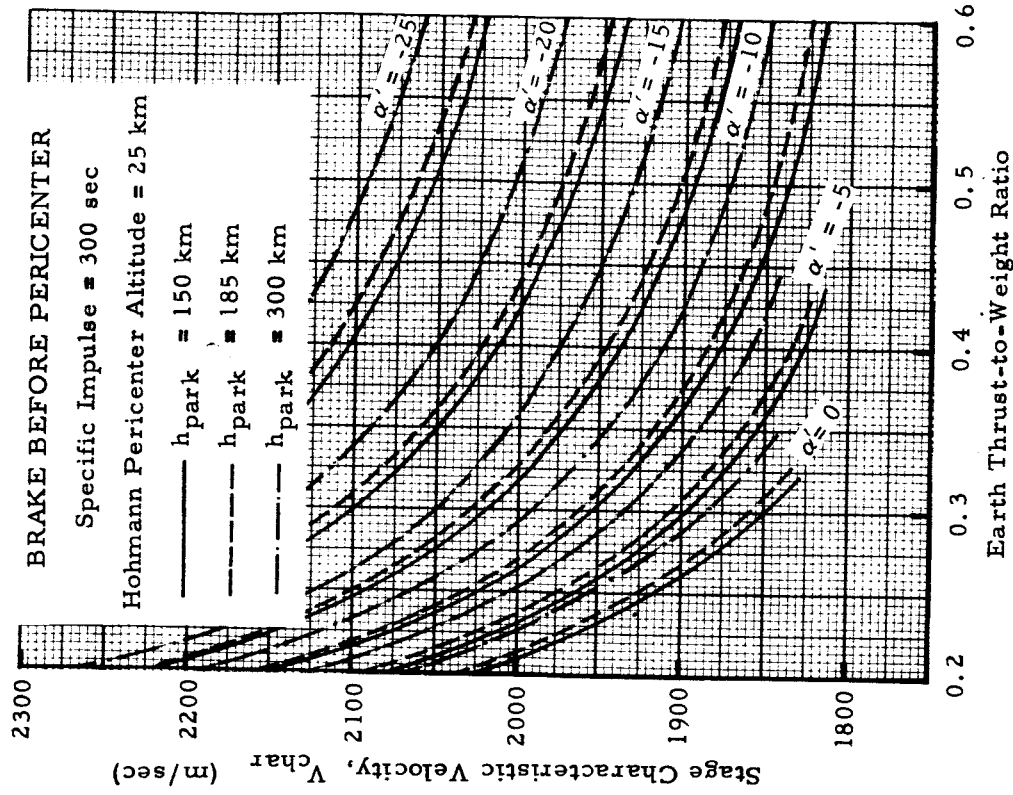
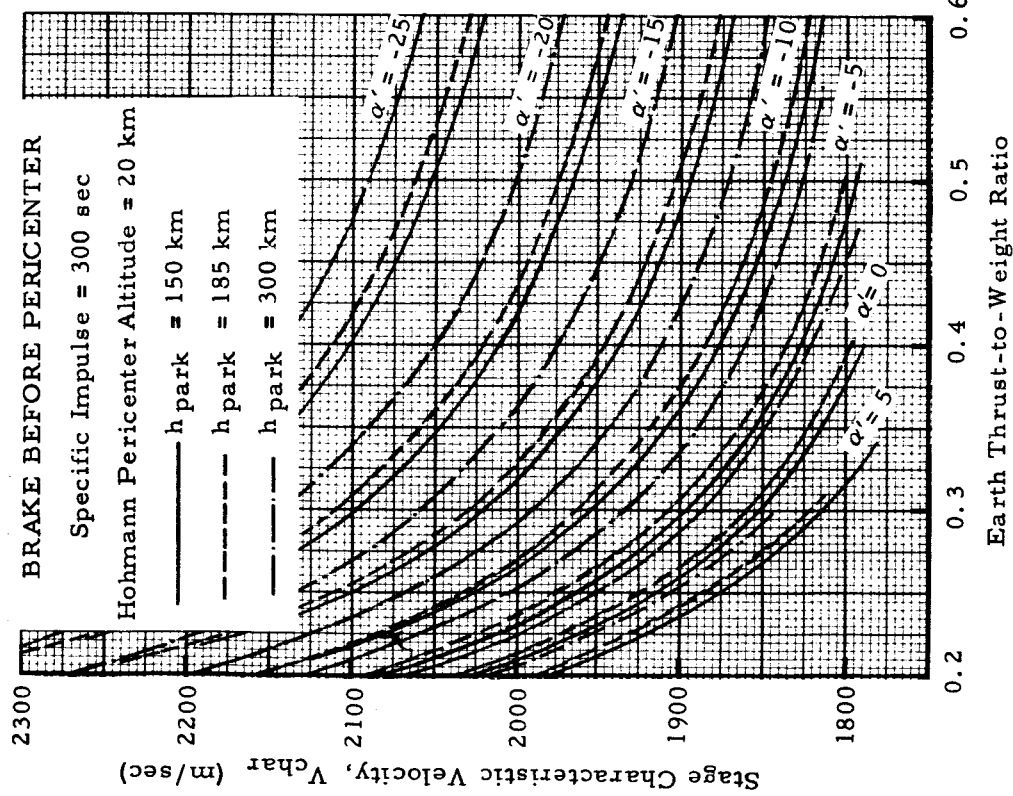


FIGURE 2 d. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE
 PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

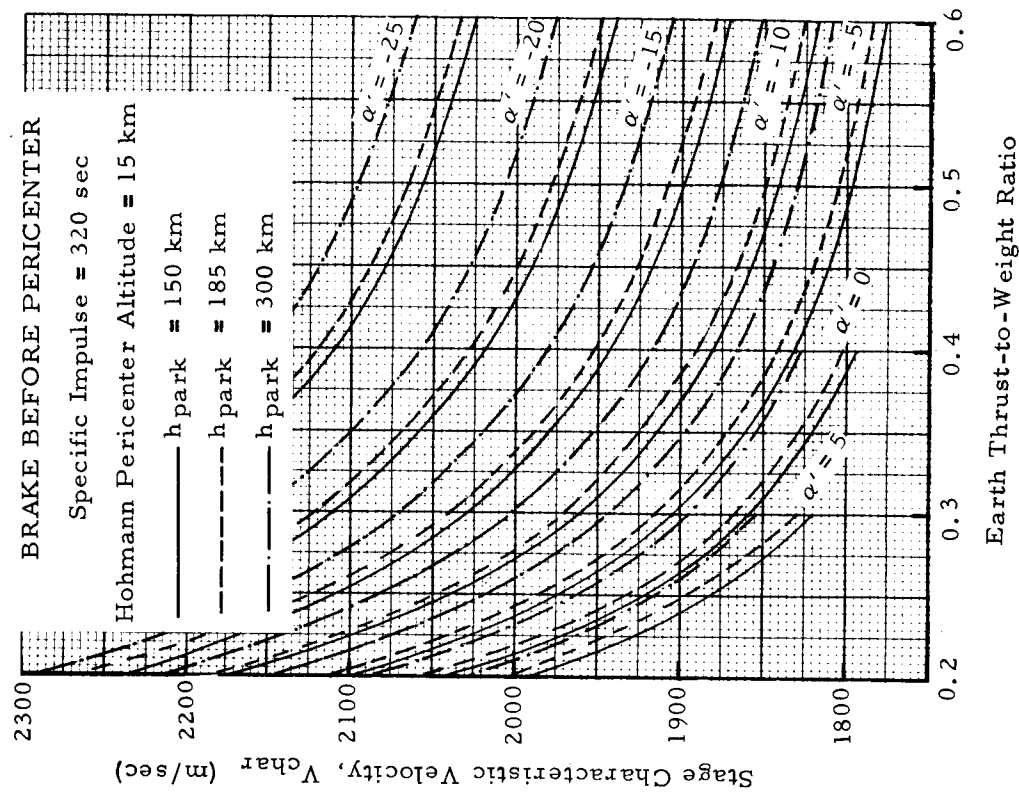
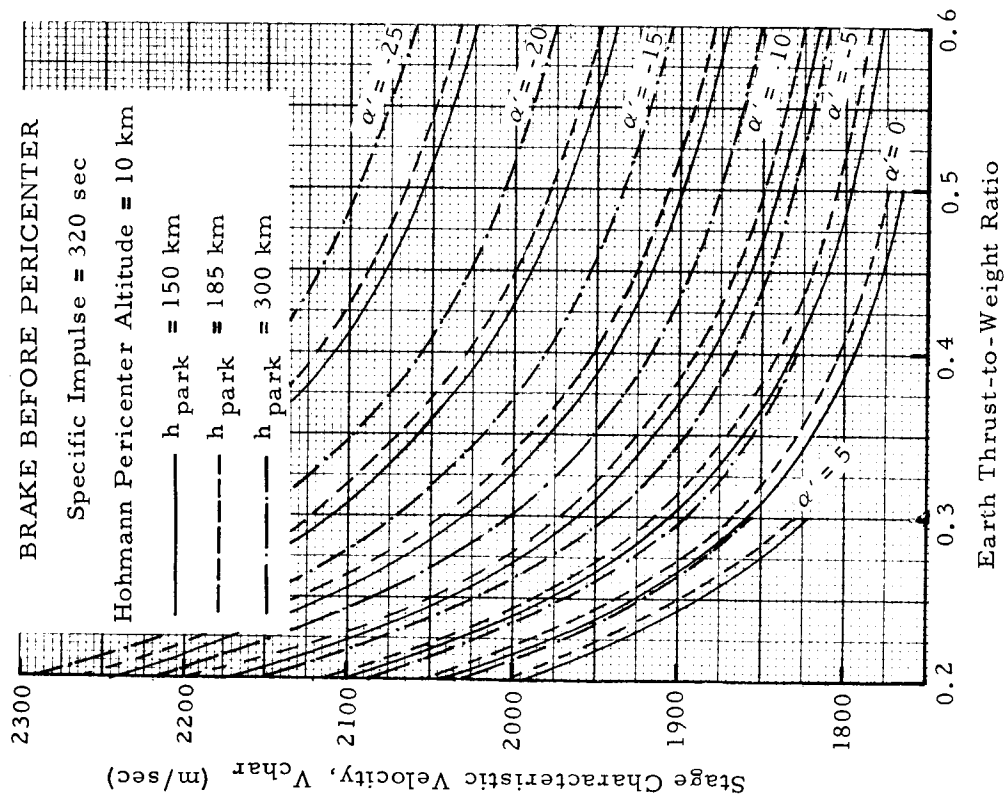


FIGURE 2 e. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

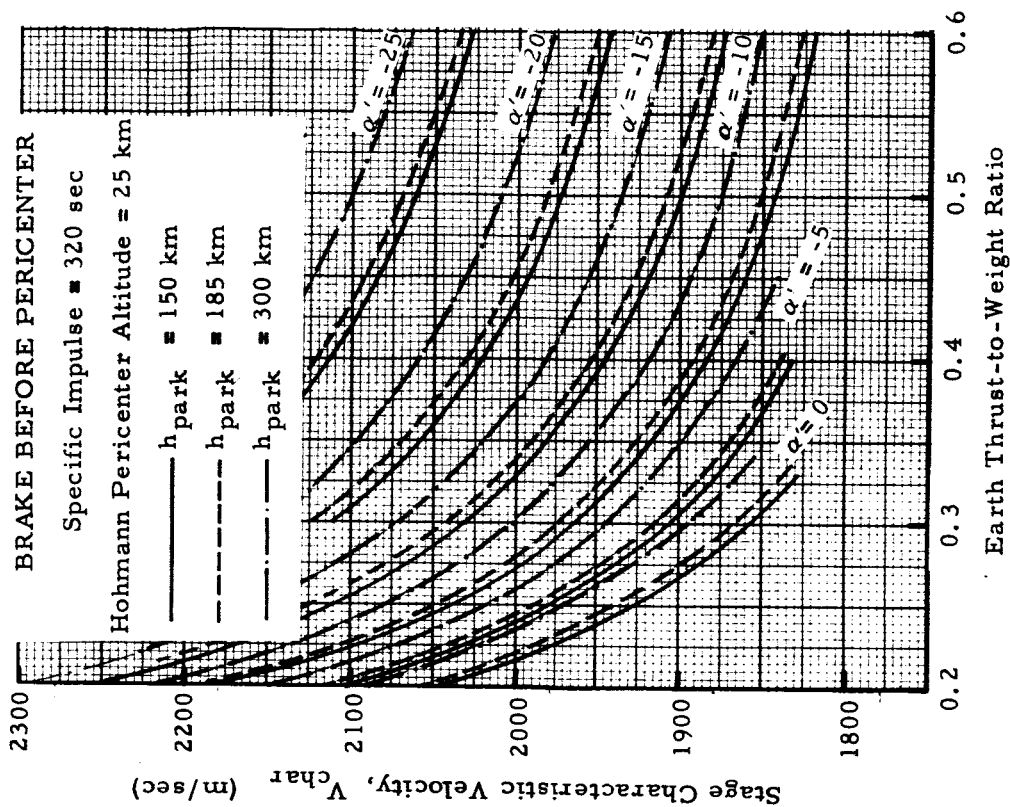
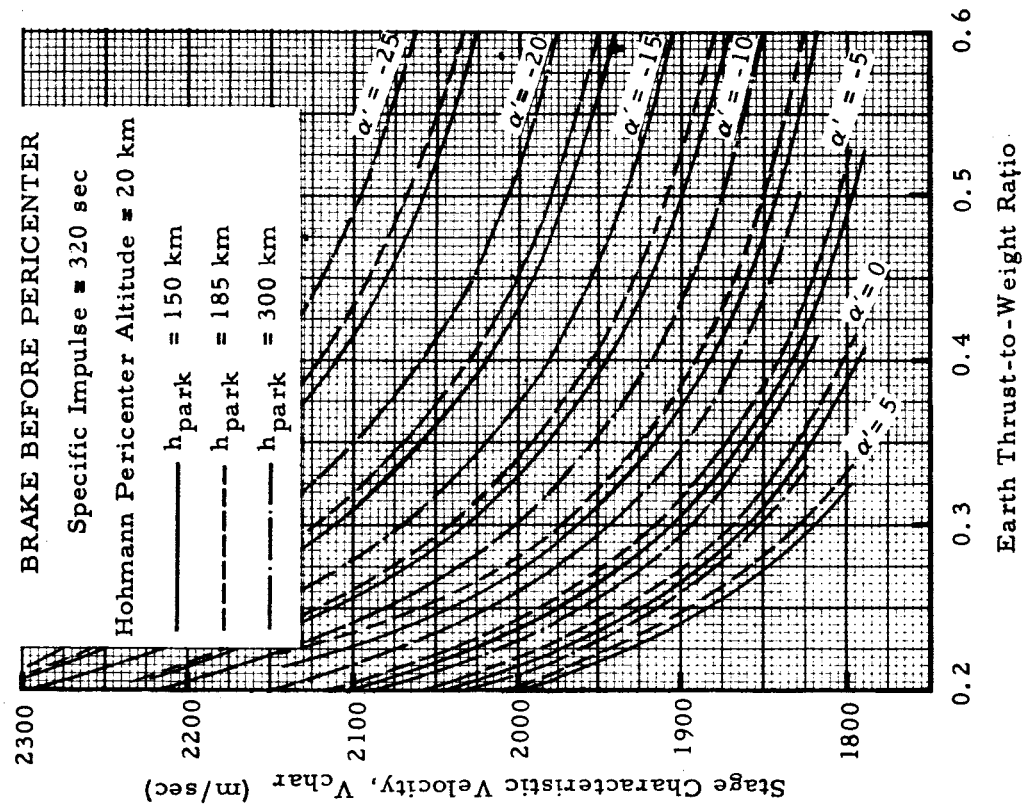


FIGURE 2 f. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

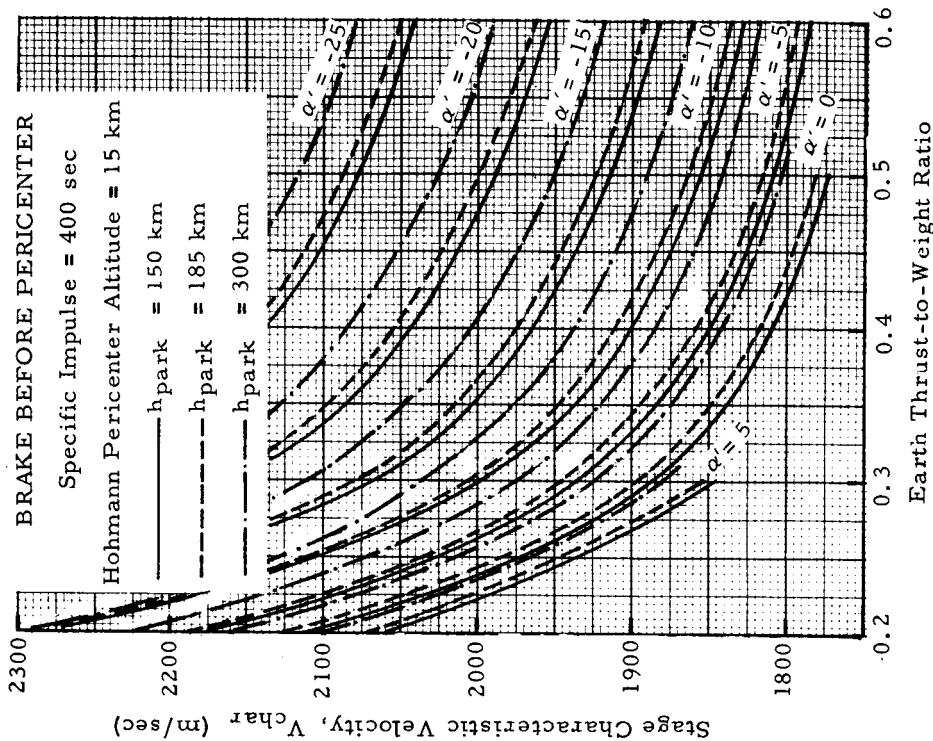
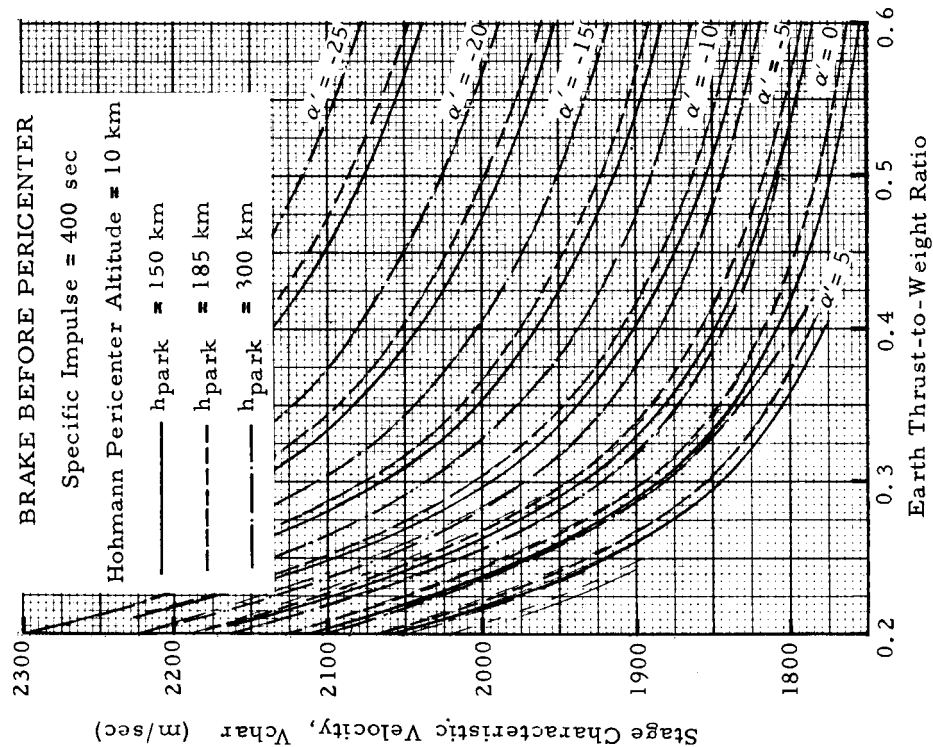


FIGURE 2 g. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

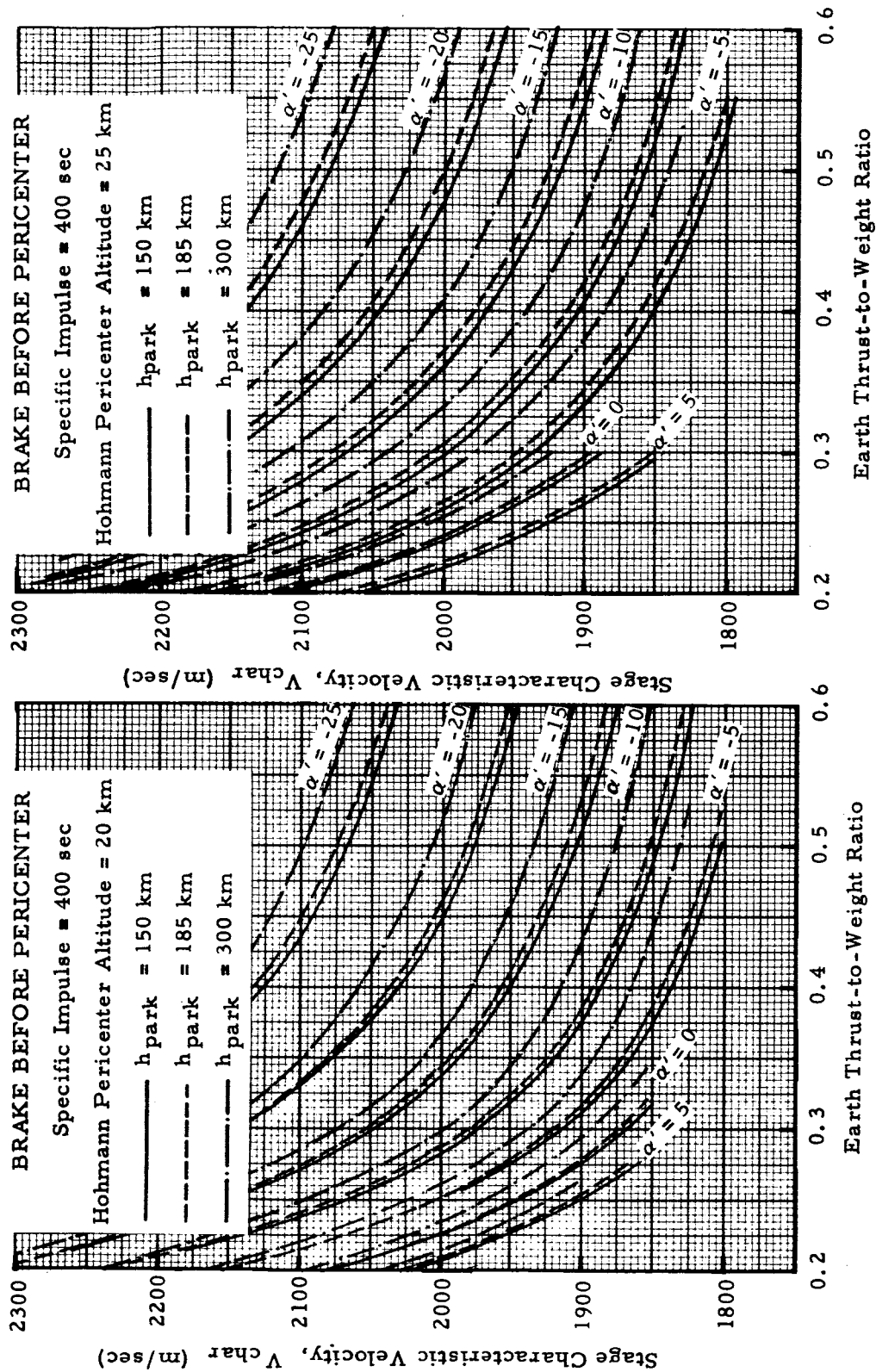


FIGURE 2 h. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 20, AND 25 KM

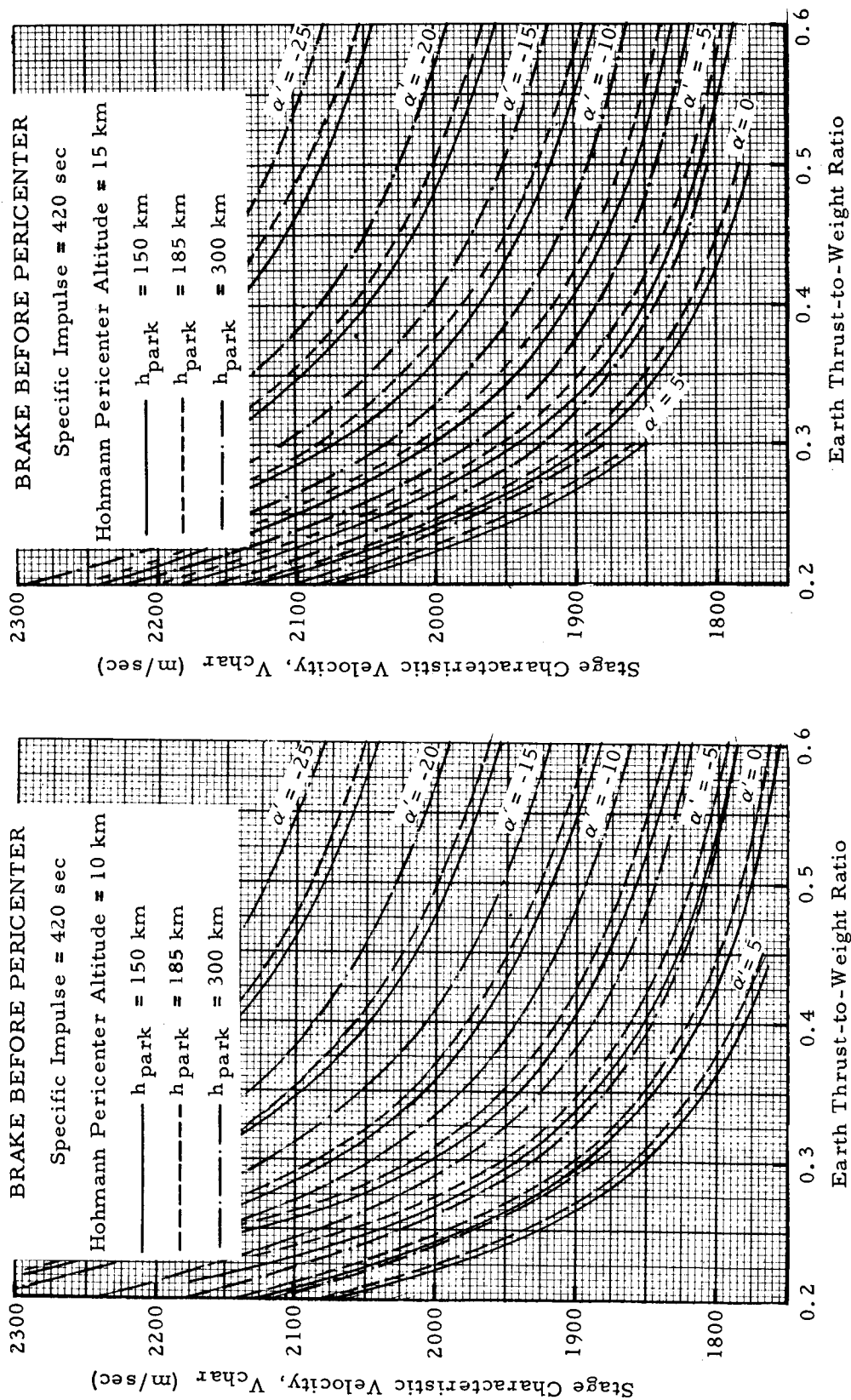


FIGURE 2 i. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

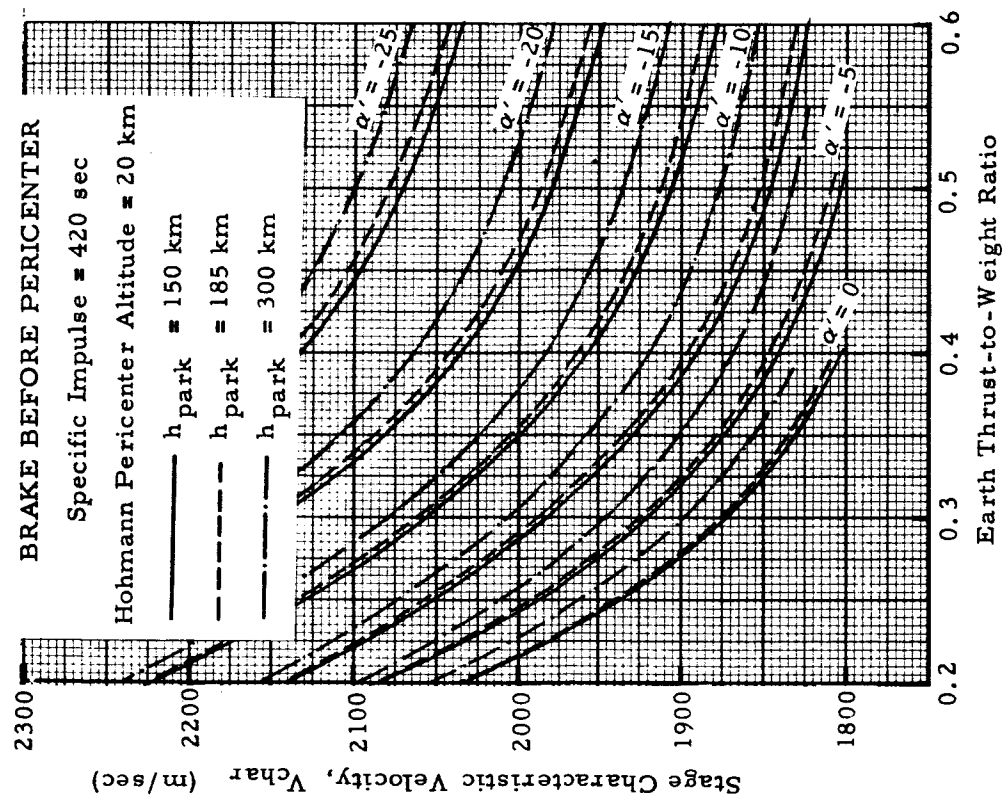
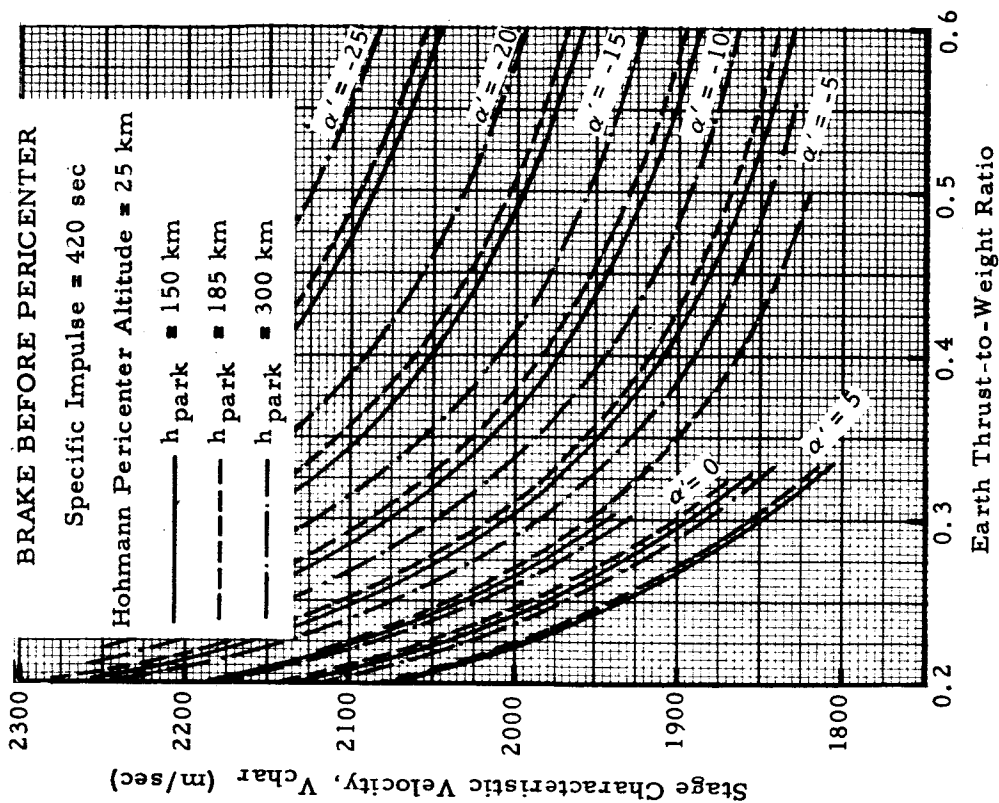


FIGURE 2 j. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

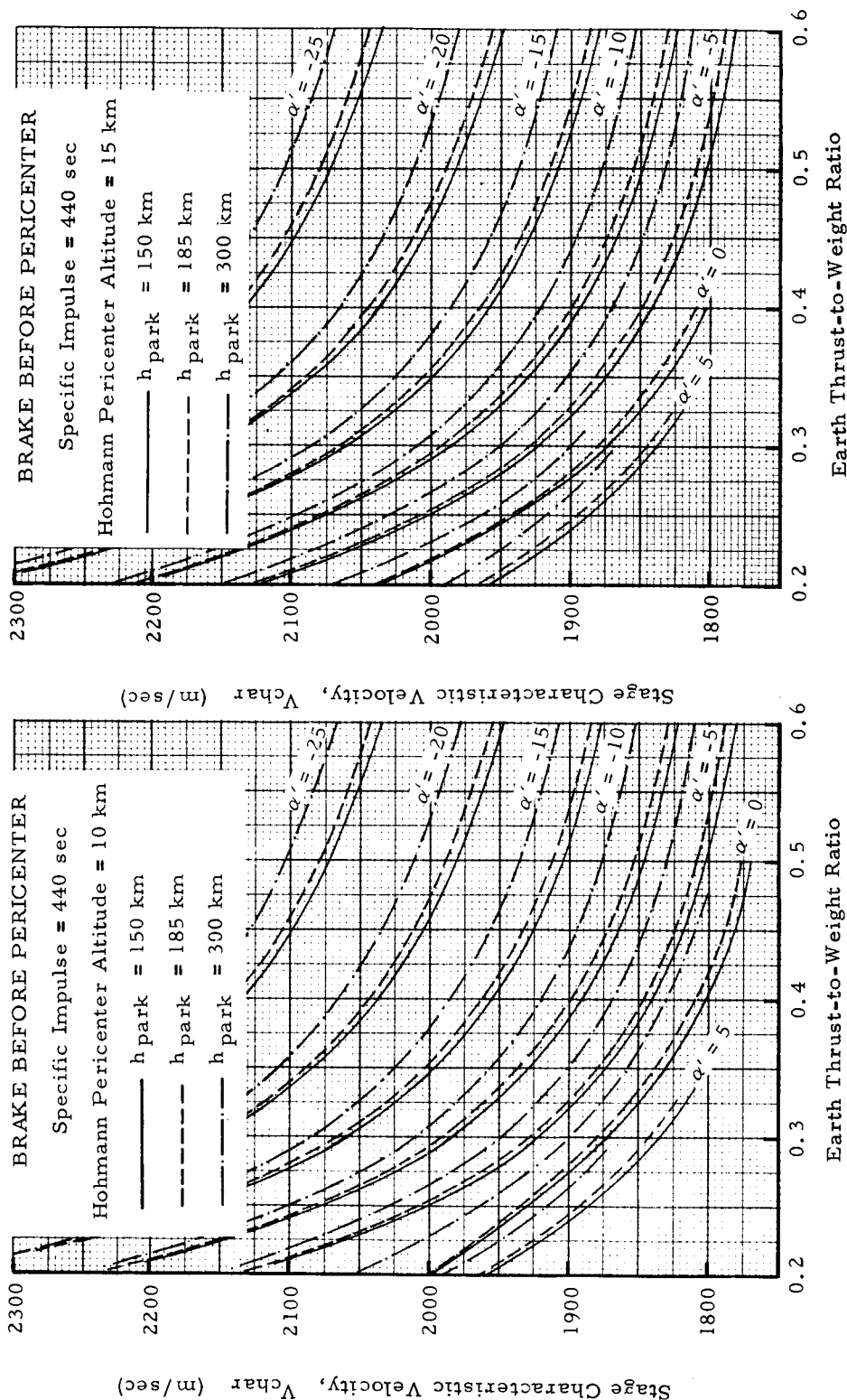


FIGURE 2 k. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

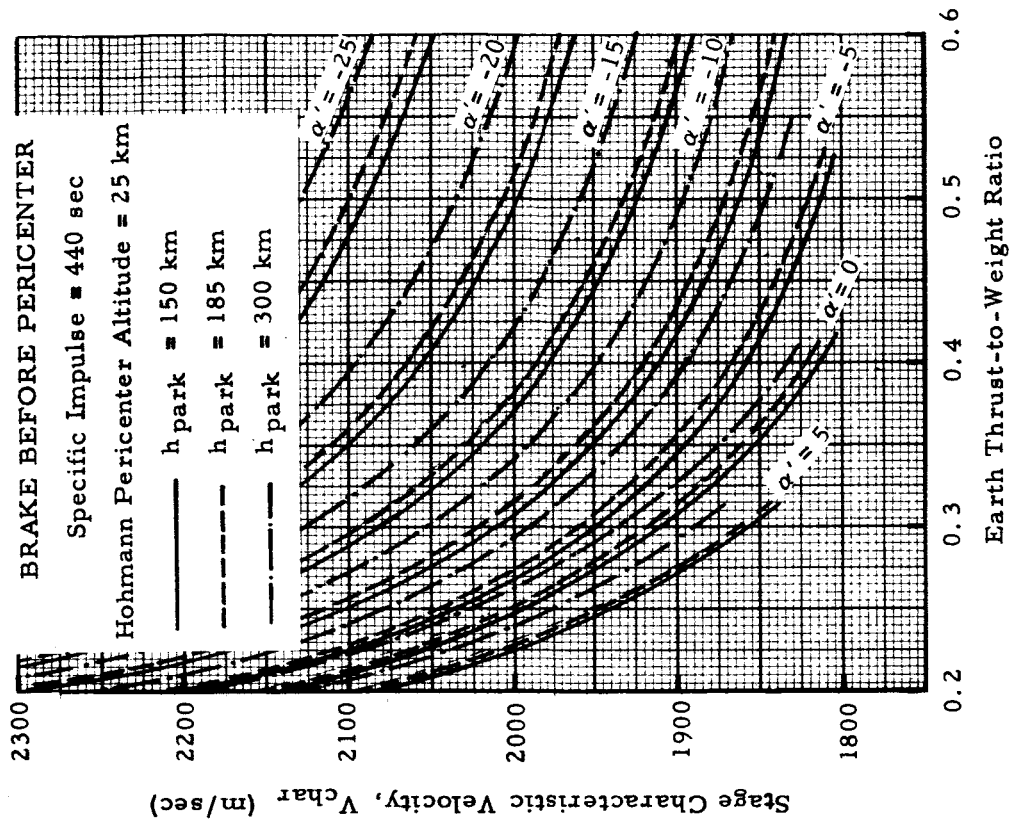
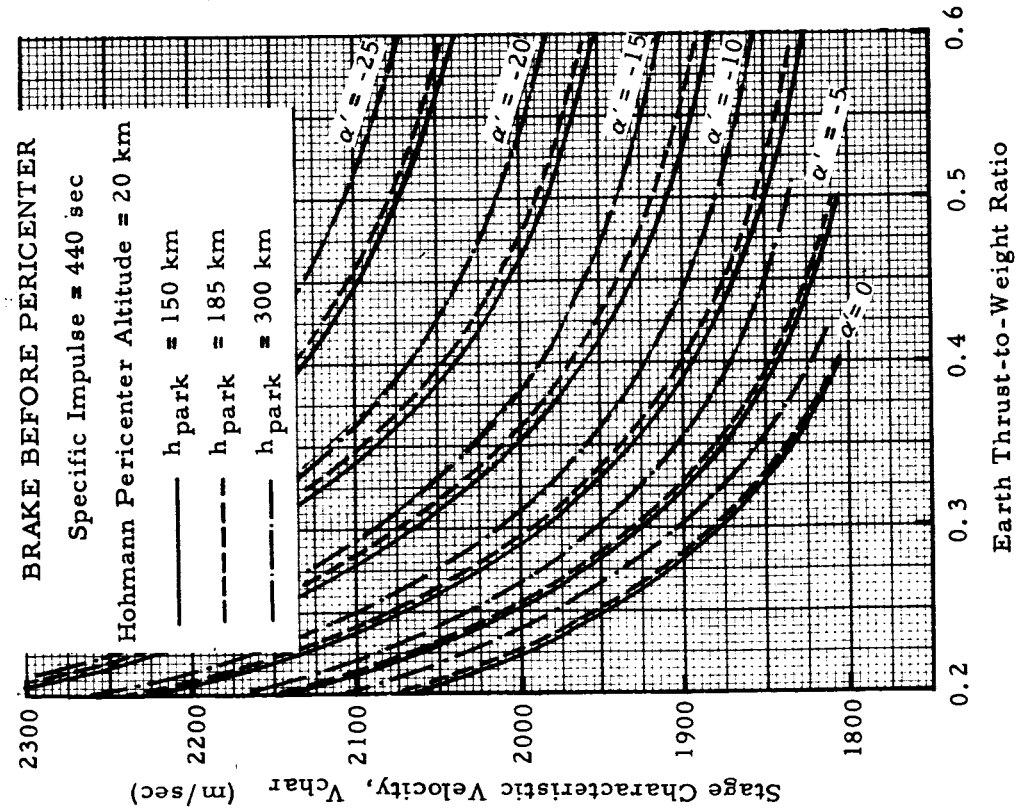


FIGURE 2 1. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

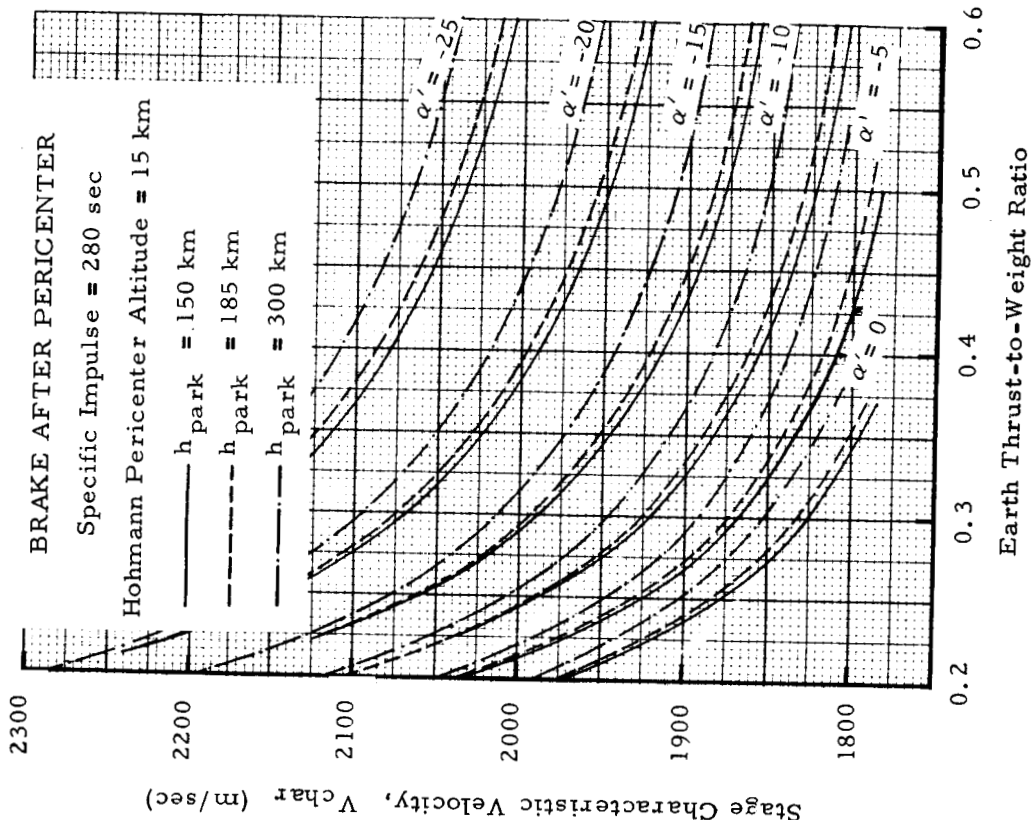
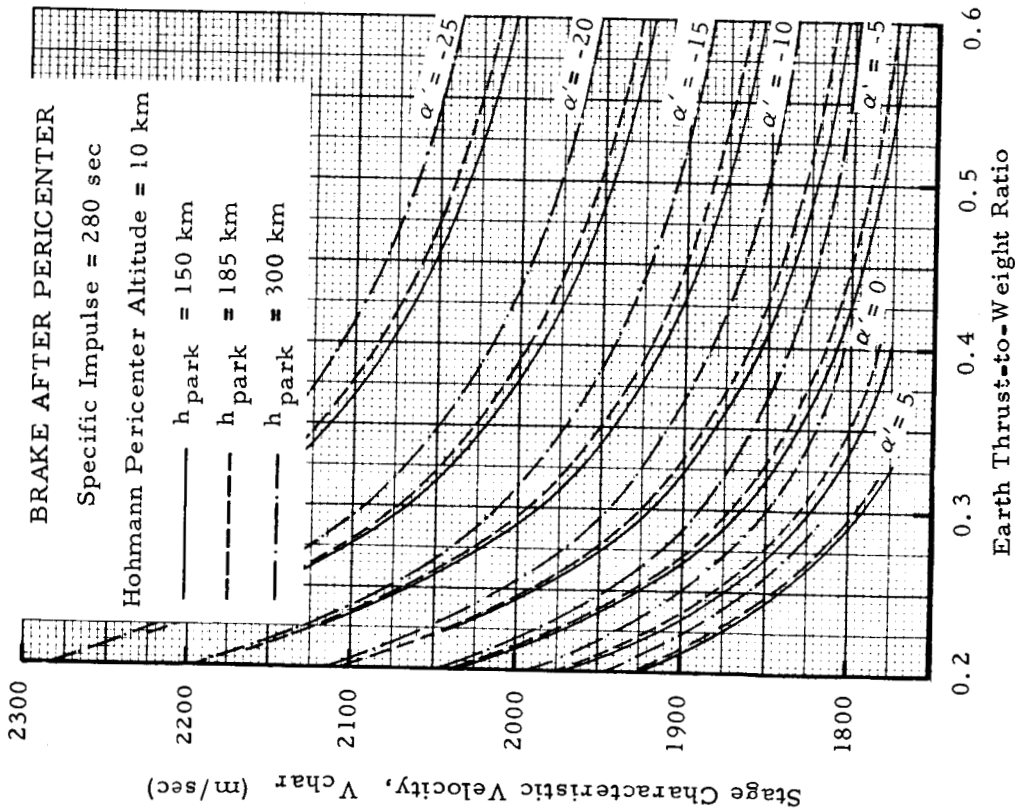


FIGURE 3 a. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER
 PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
 HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

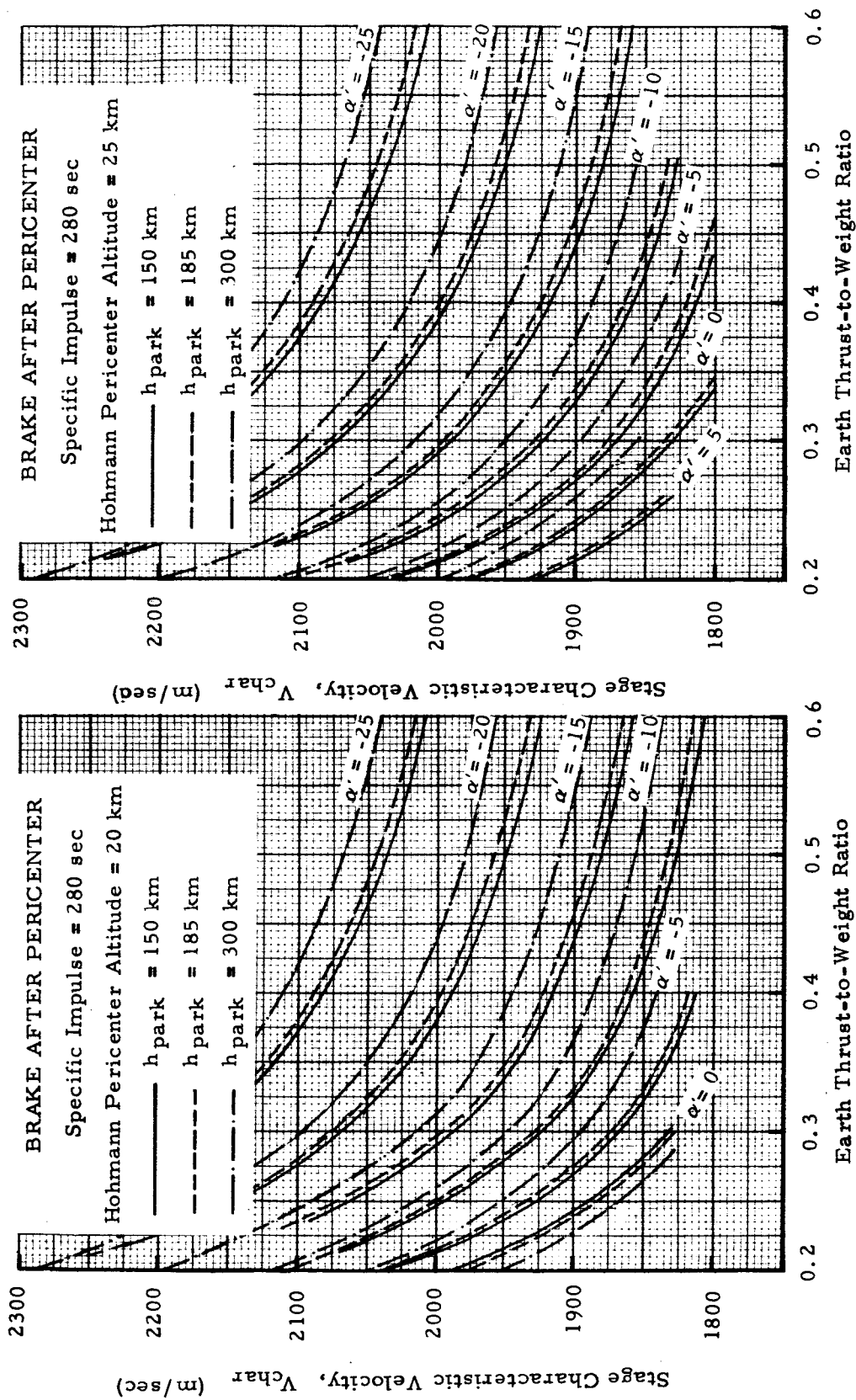


FIGURE 3 b. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
 HOHMMANN PERICENTER ALTITUDE = 20 AND 25 KM

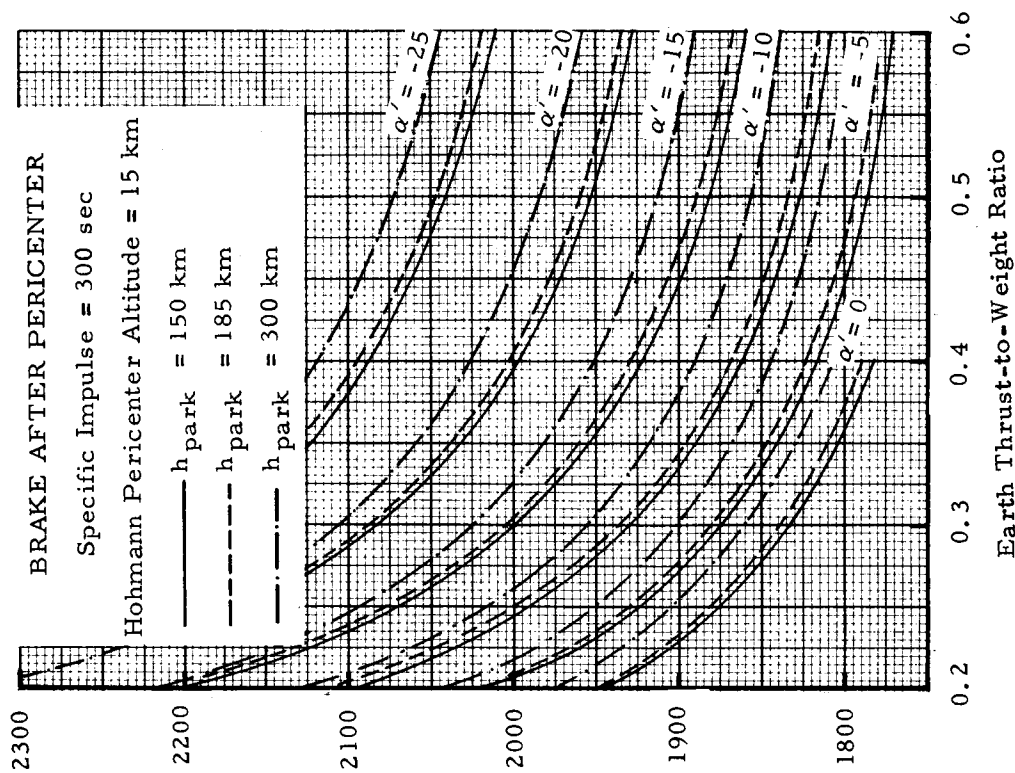
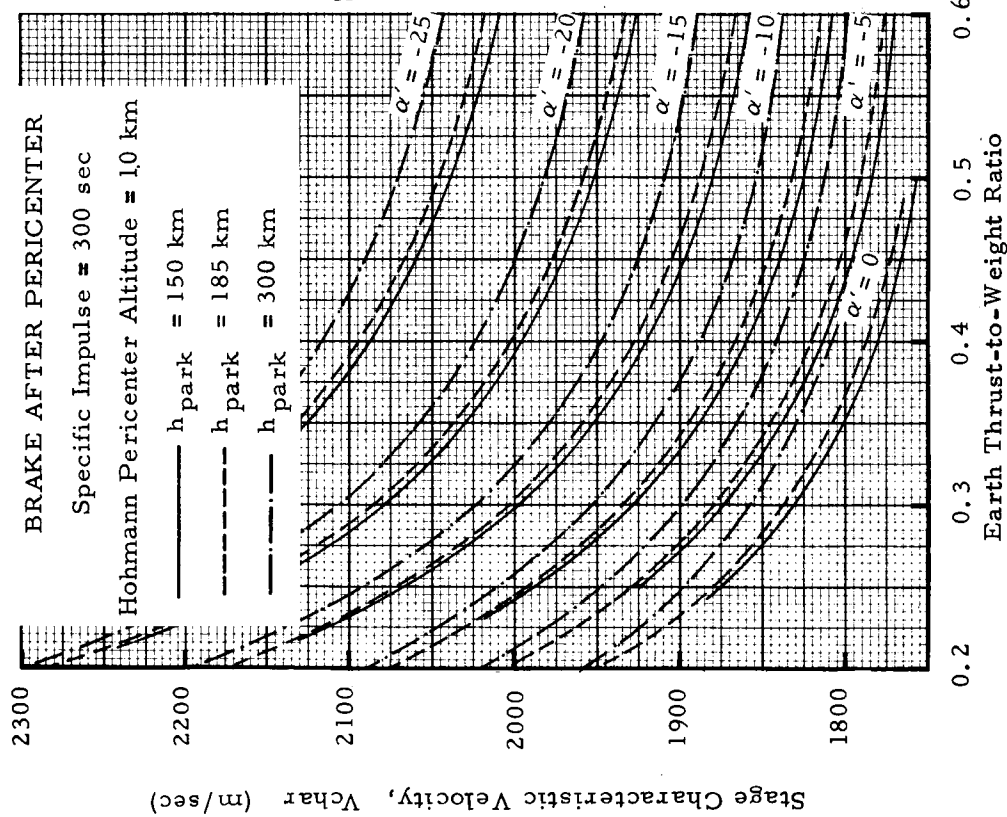


FIGURE 3 c. . CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER
PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

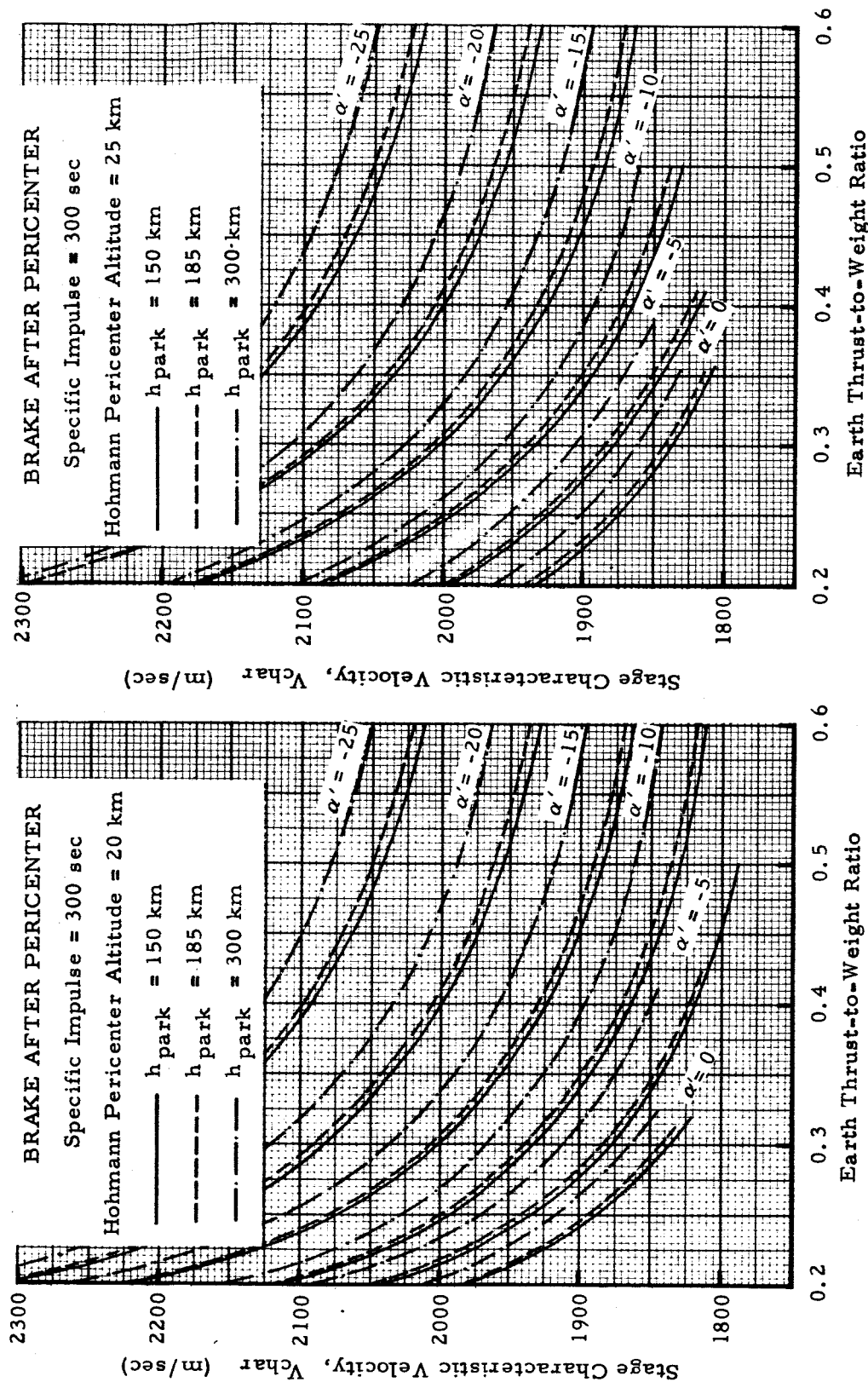


FIGURE 3 d. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

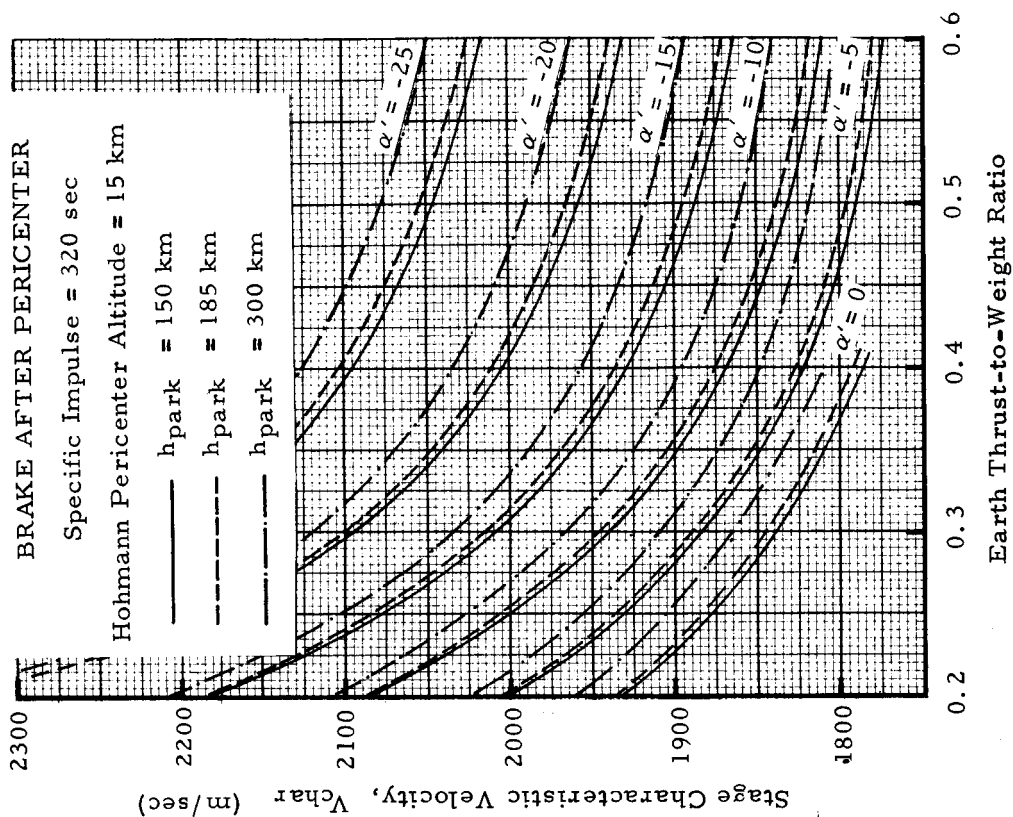
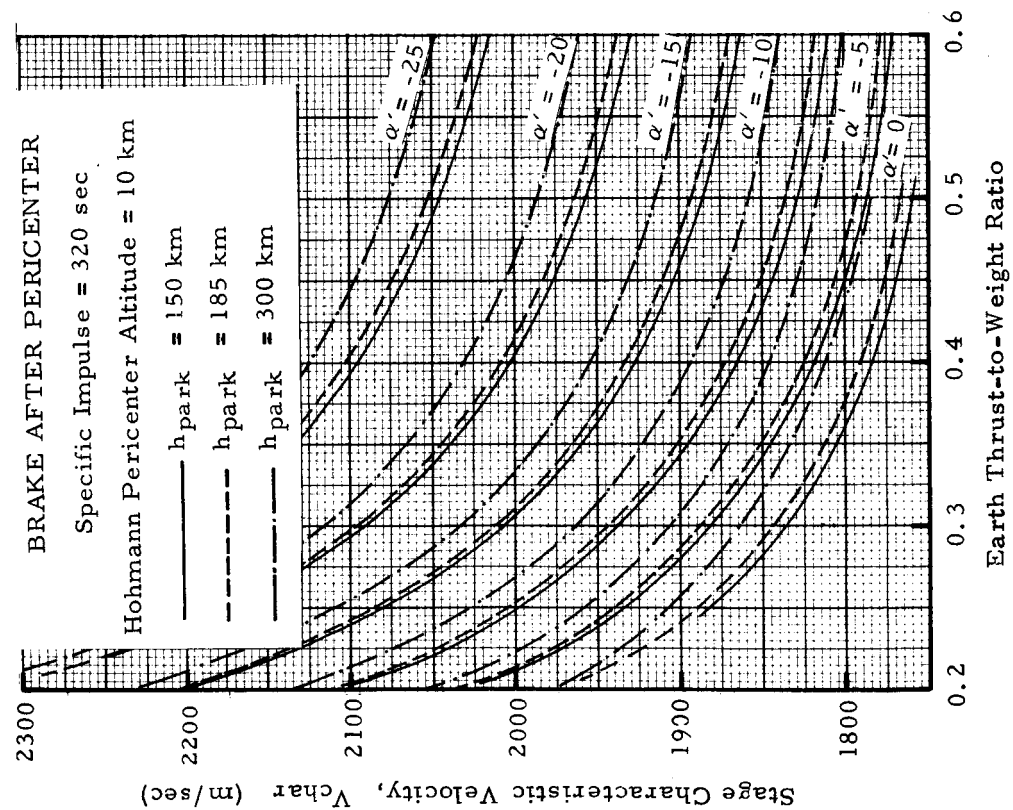


FIGURE 3 e. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

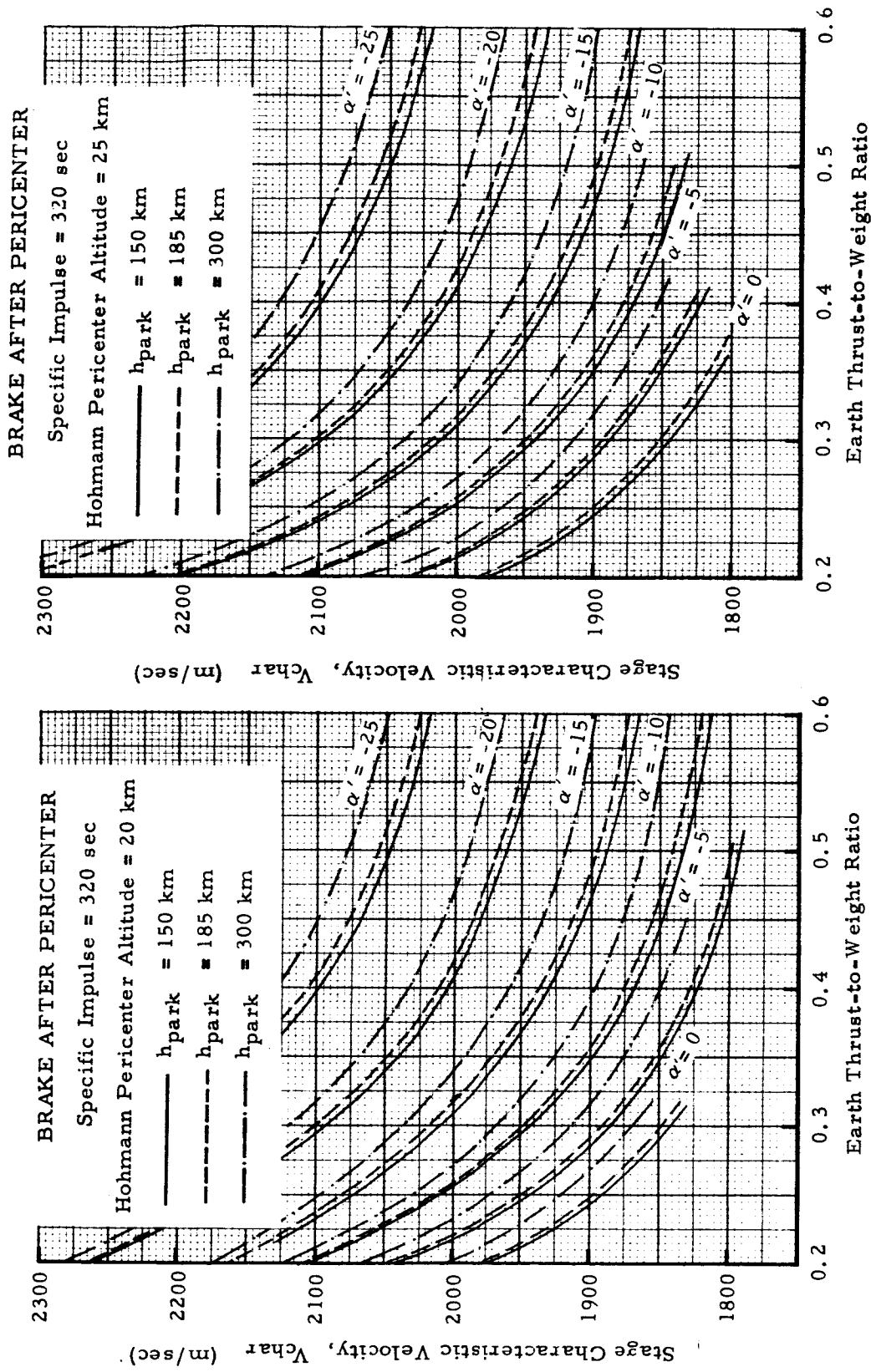


FIGURE 3 f. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

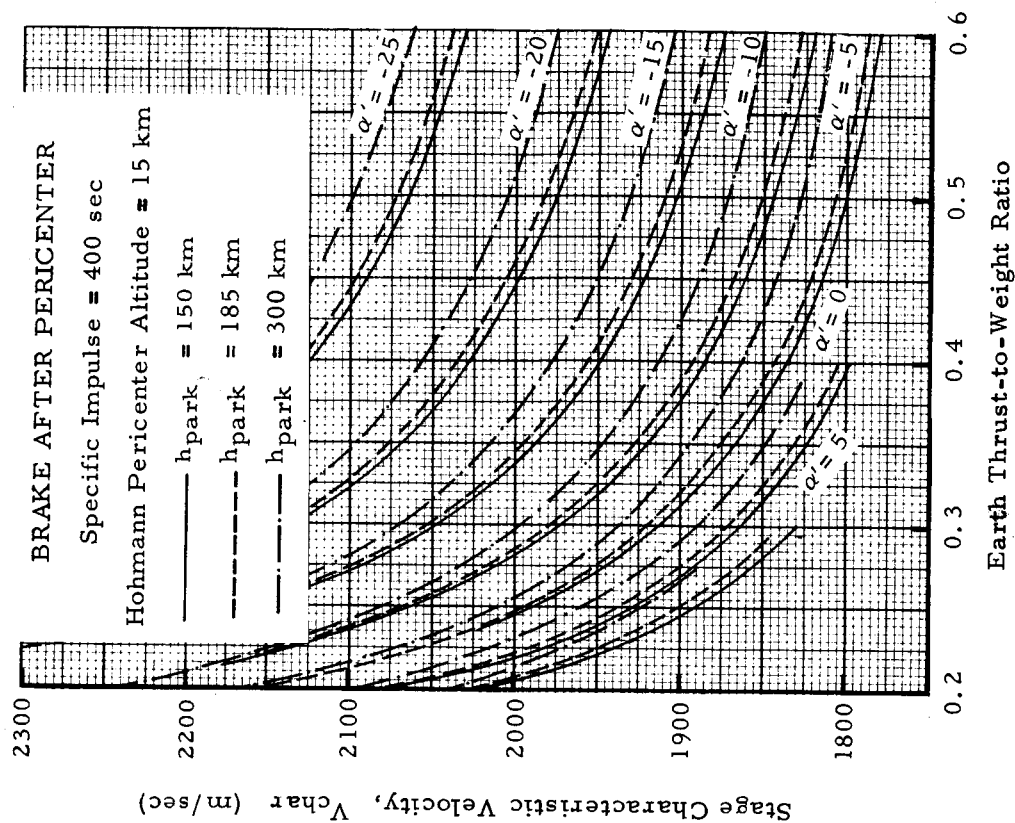
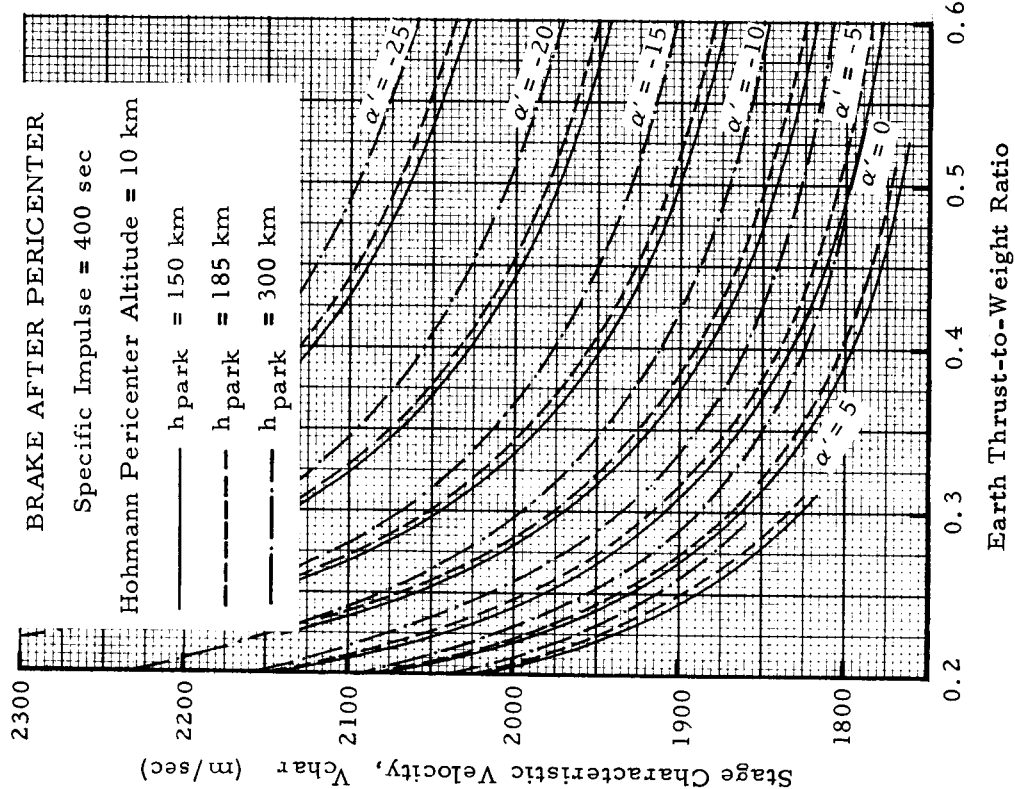


FIGURE 3 g. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER
PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

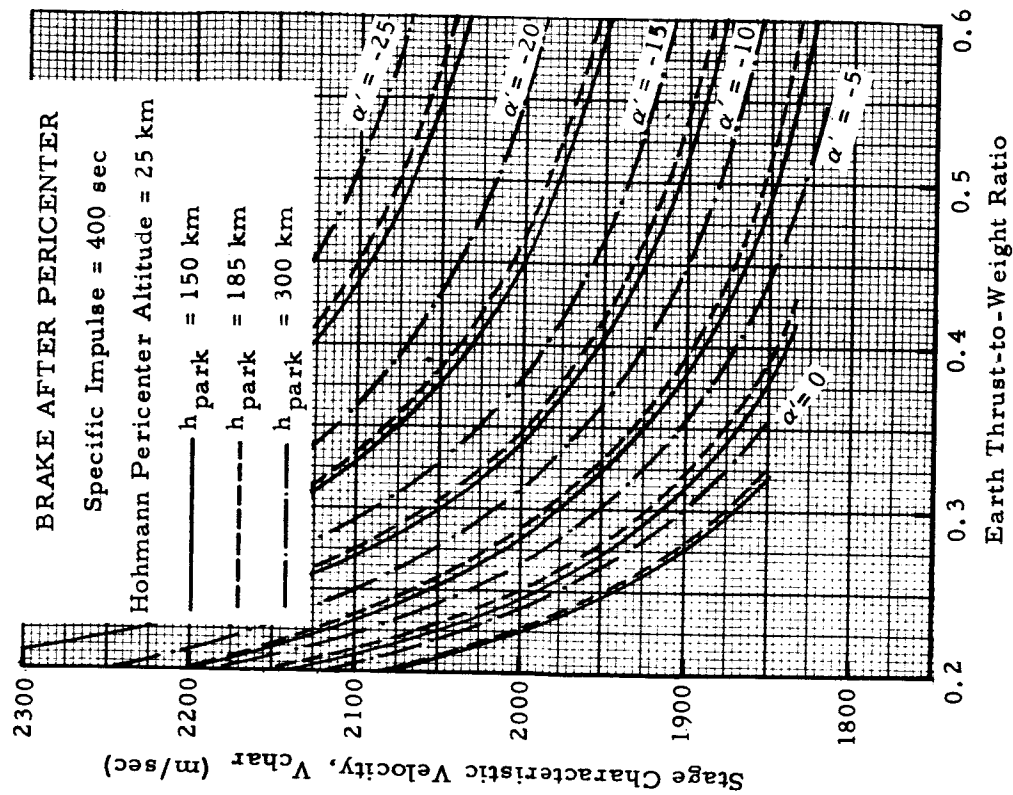
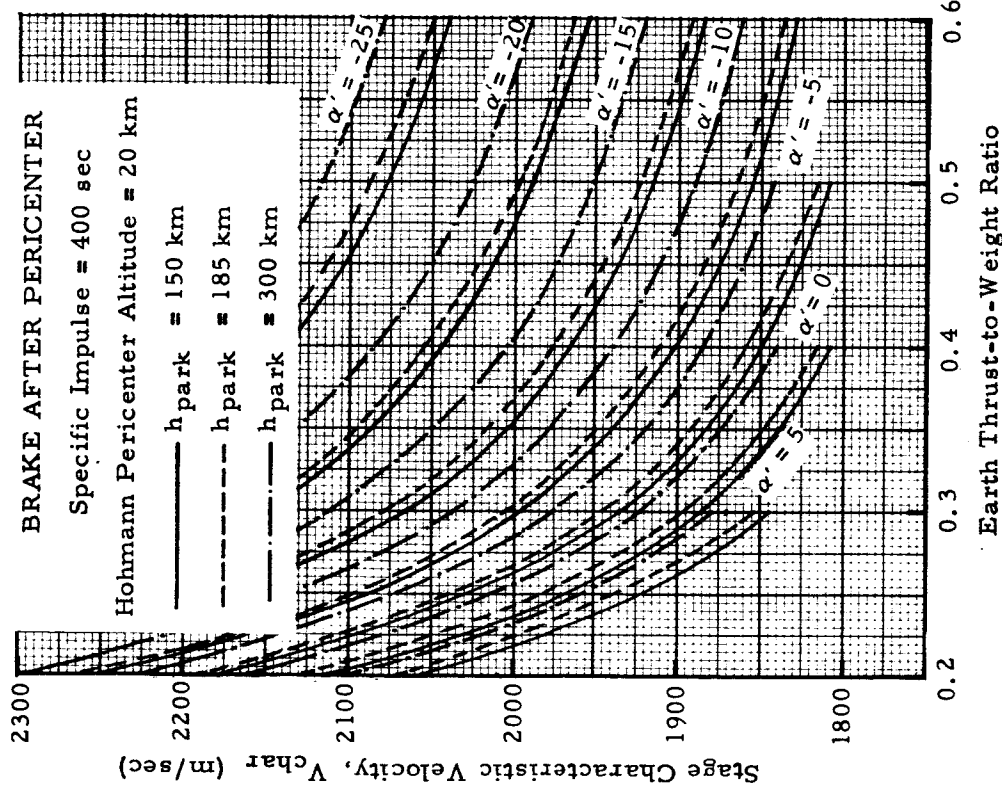


FIGURE 3 h. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

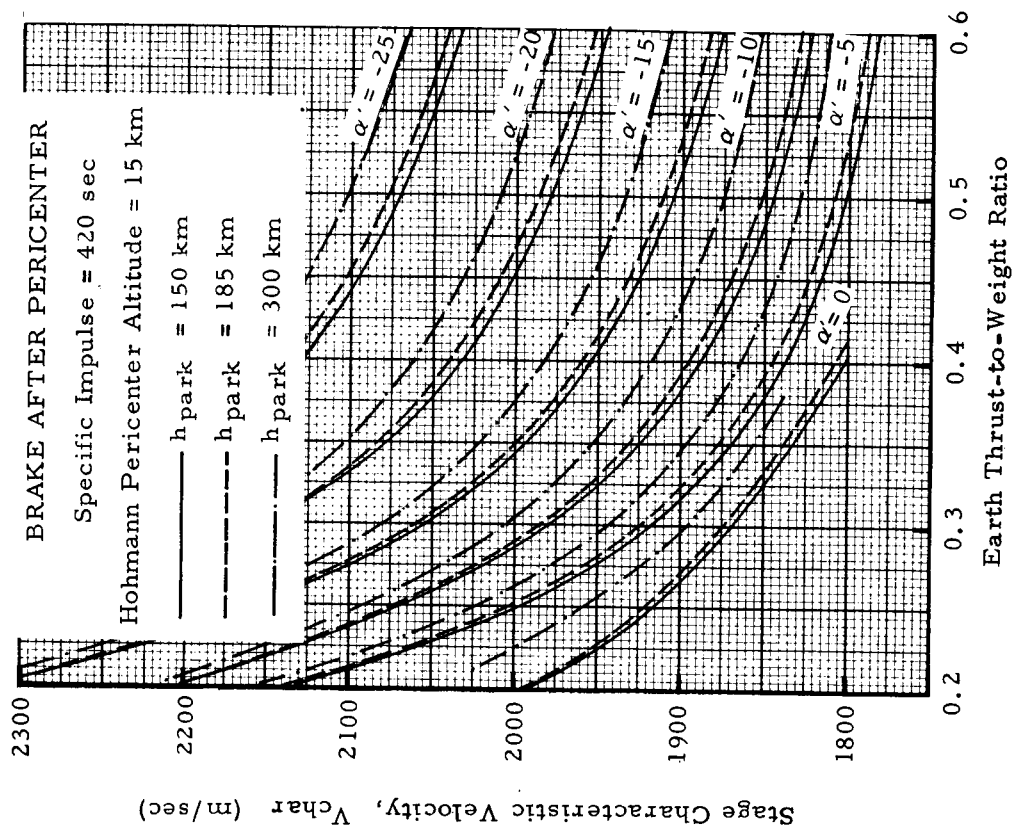
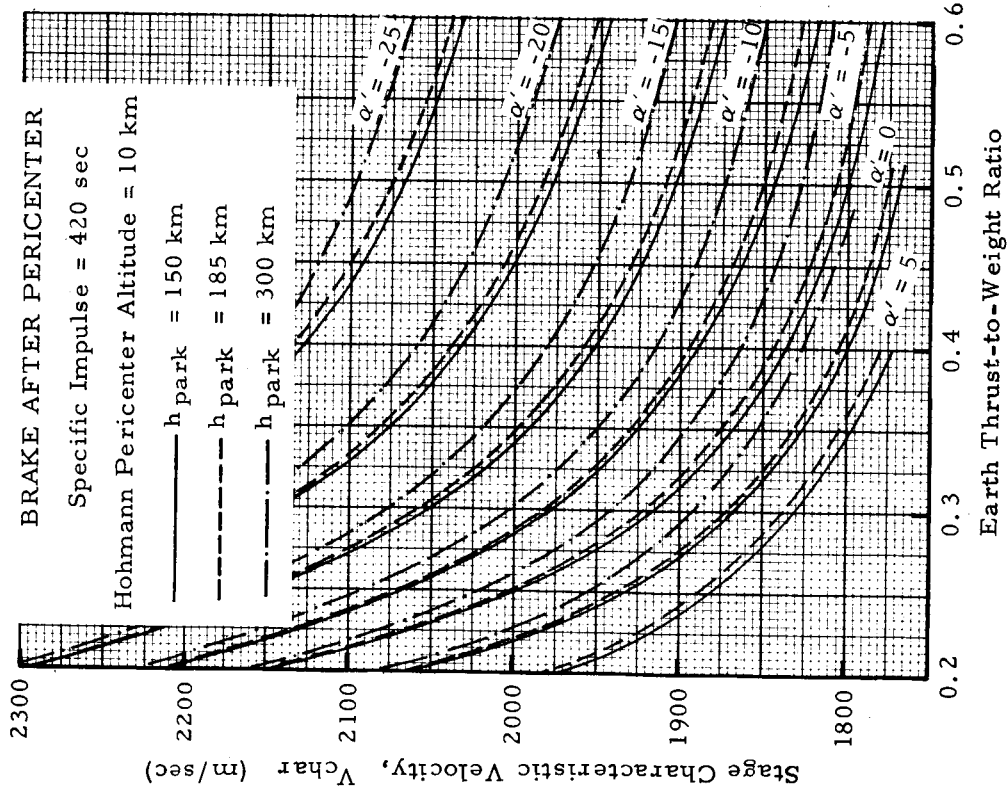


FIGURE 3 i. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER
PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

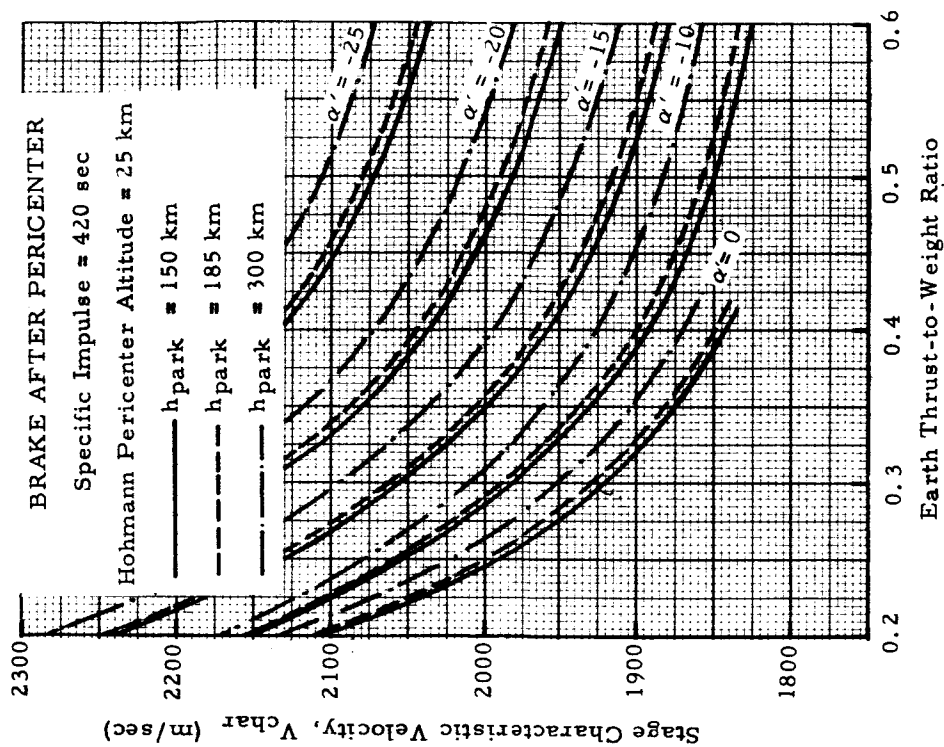
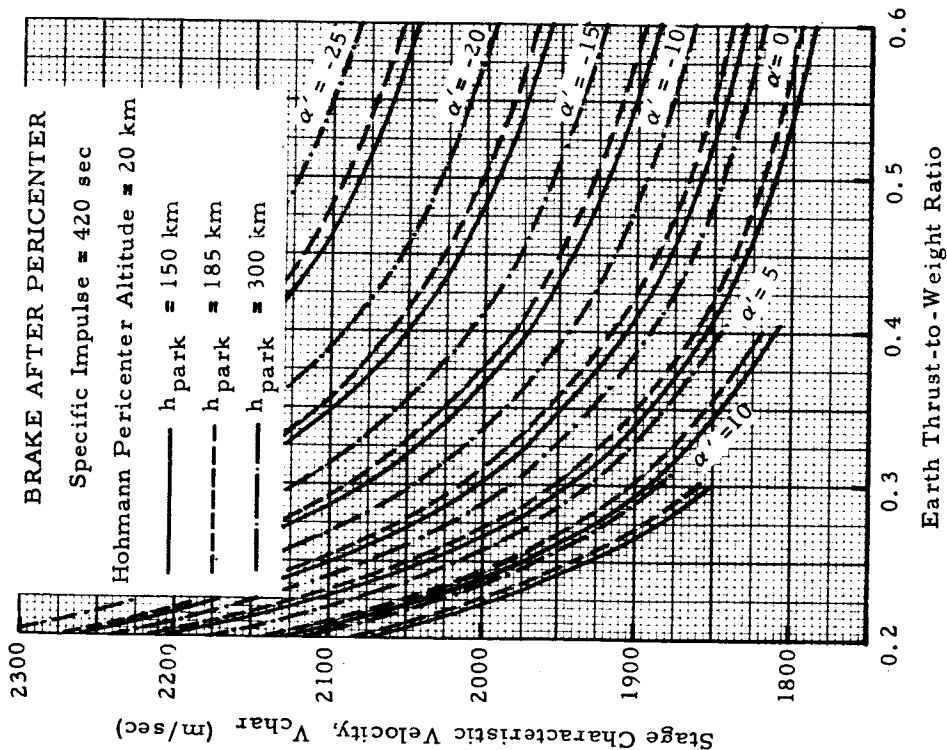


FIGURE 3 j. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

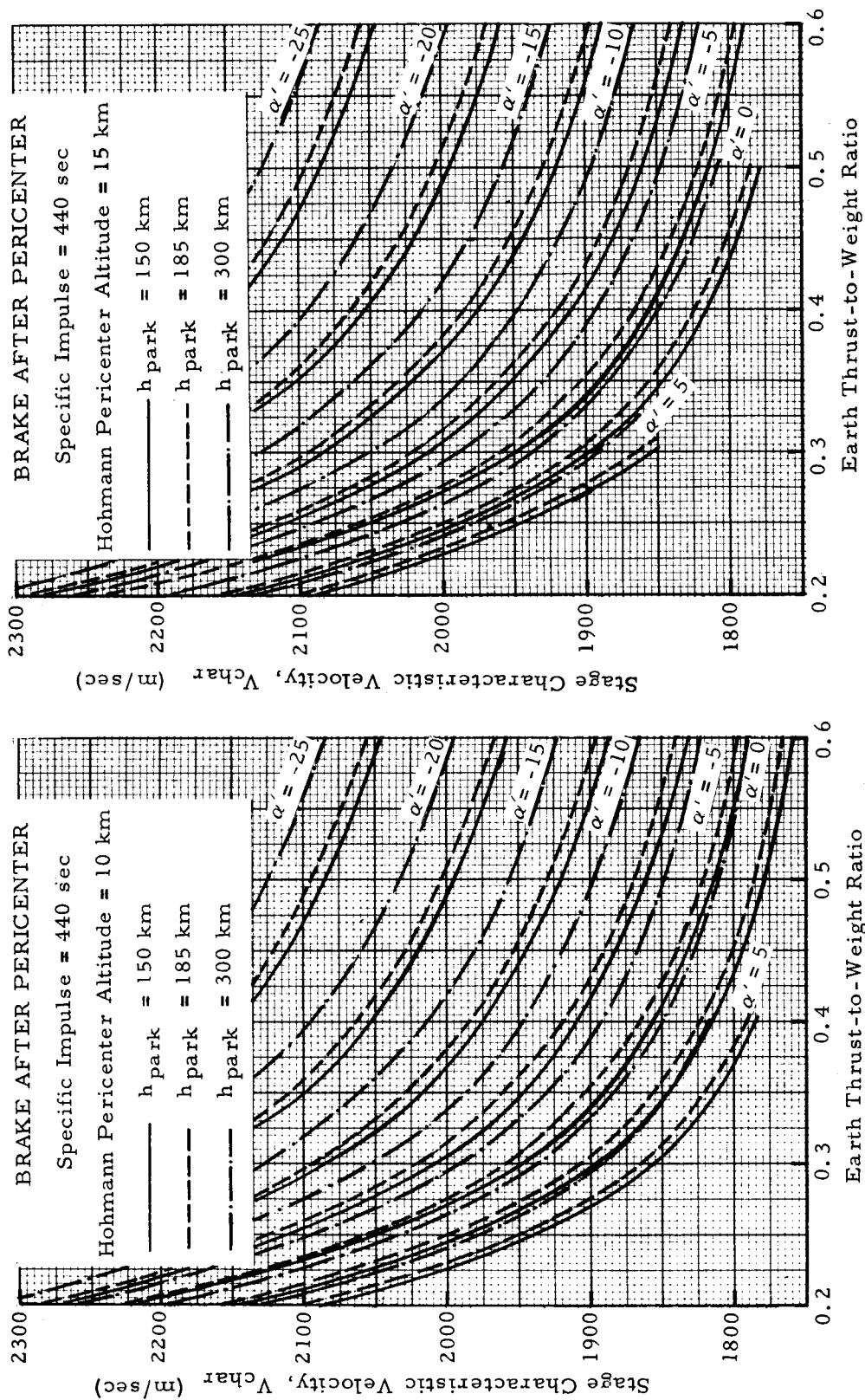


FIGURE 3 k. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 25 KM

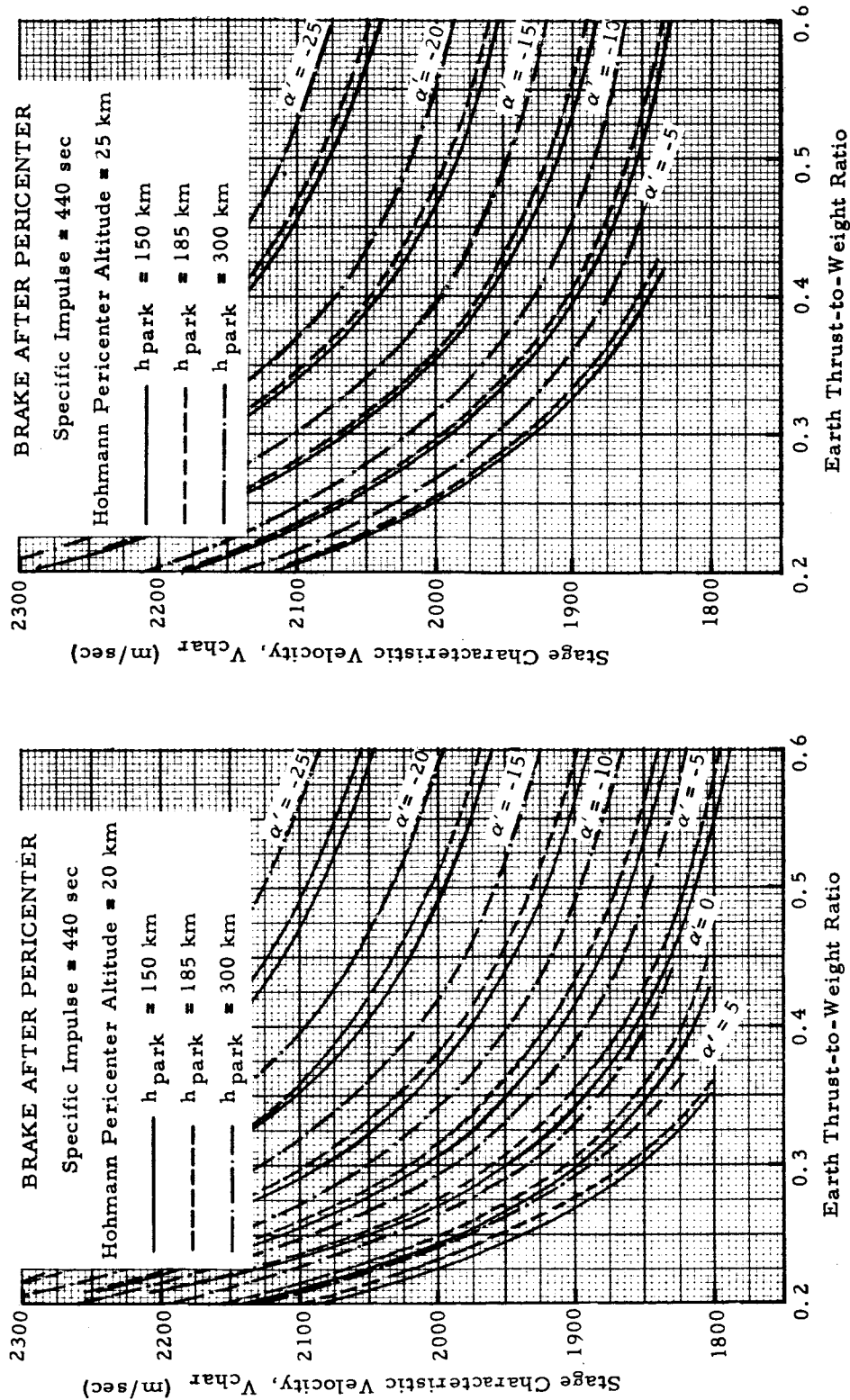


FIGURE 3 1. CHARACTERISTIC VELOCITY REQUIREMENT FOR DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

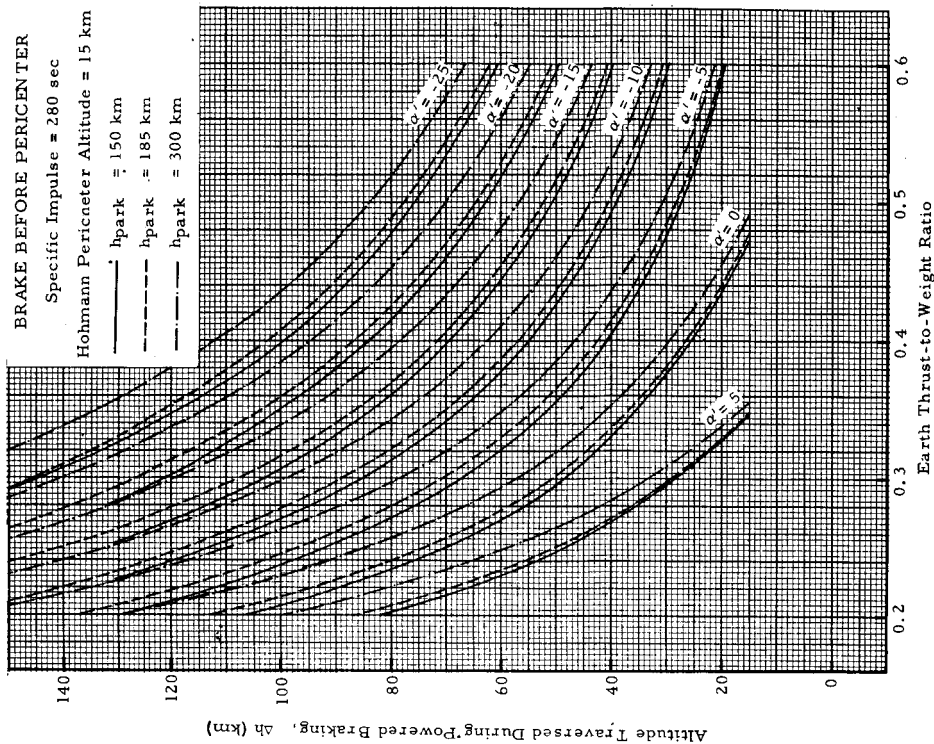
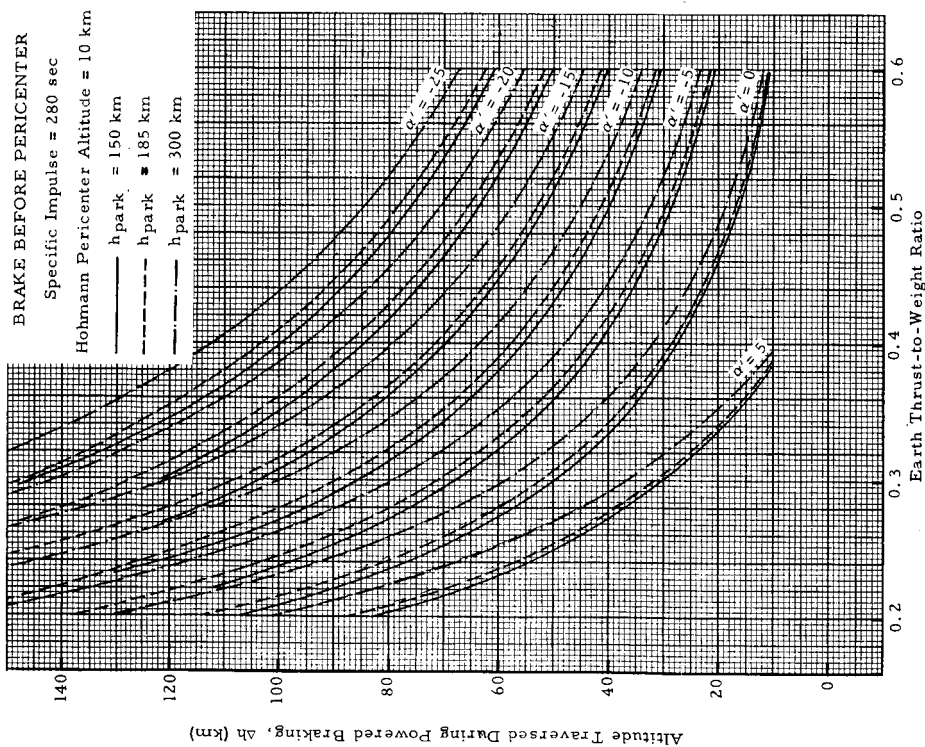


FIGURE 4 a. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

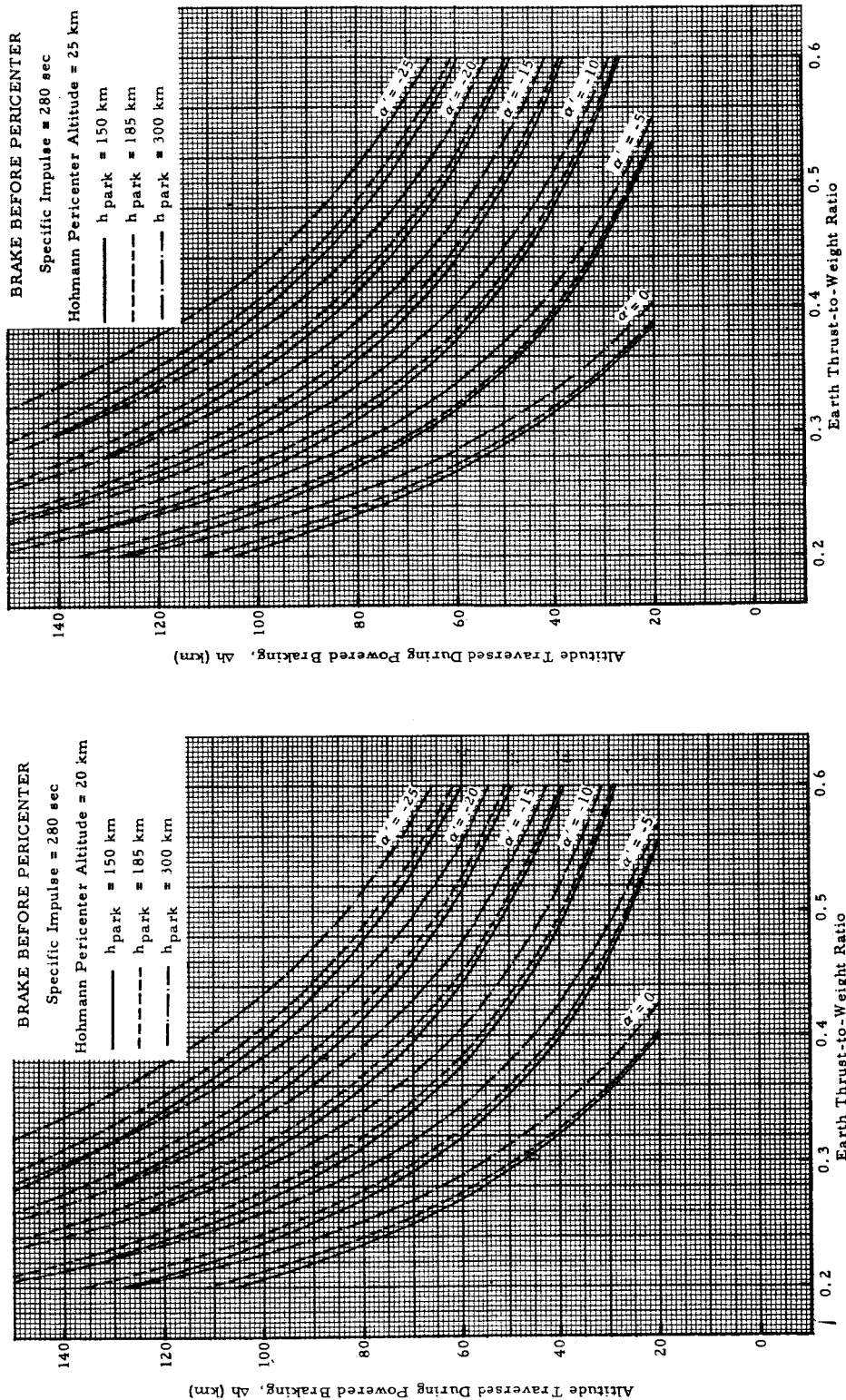


FIGURE 4 b. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

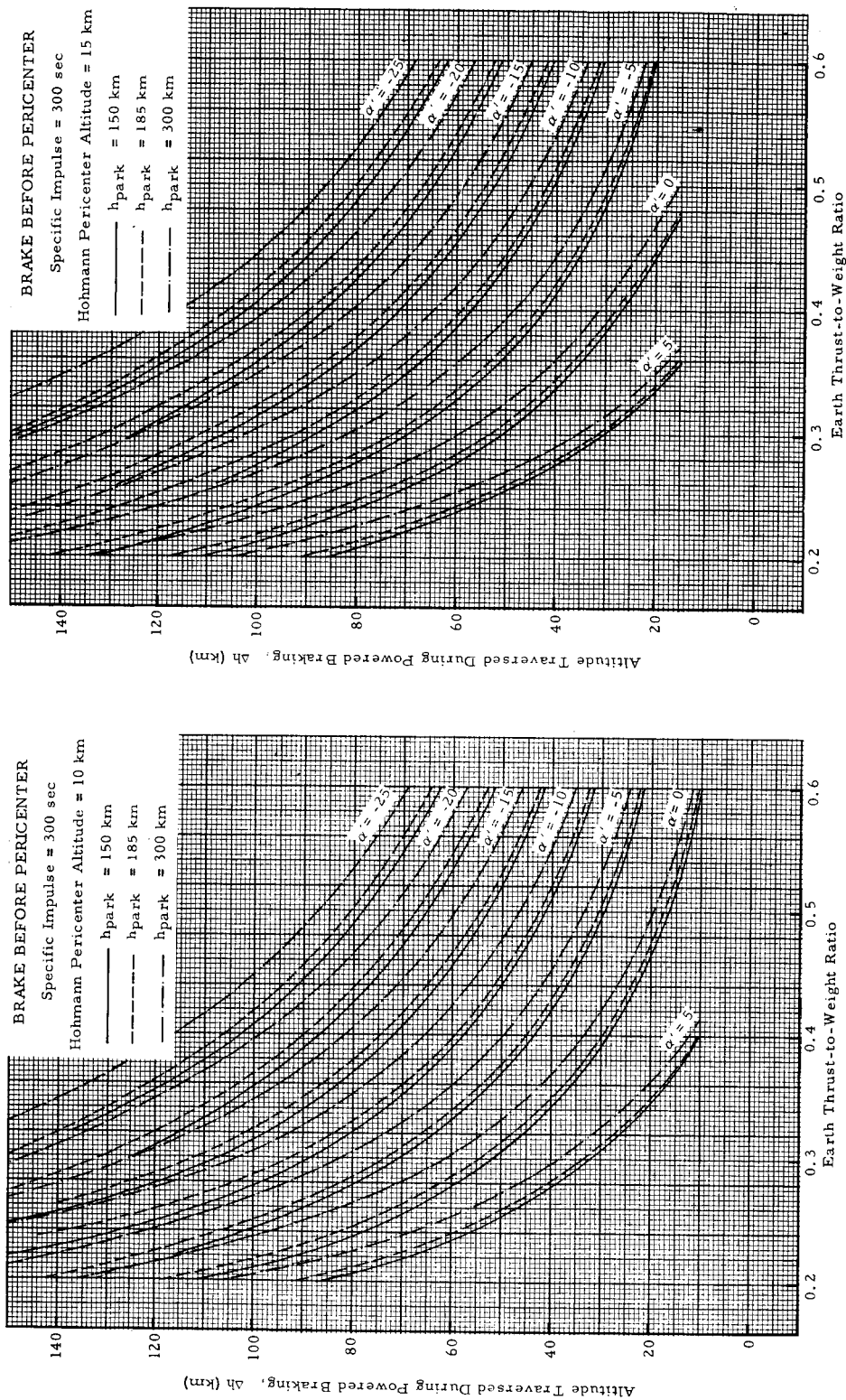


FIGURE 4 c. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

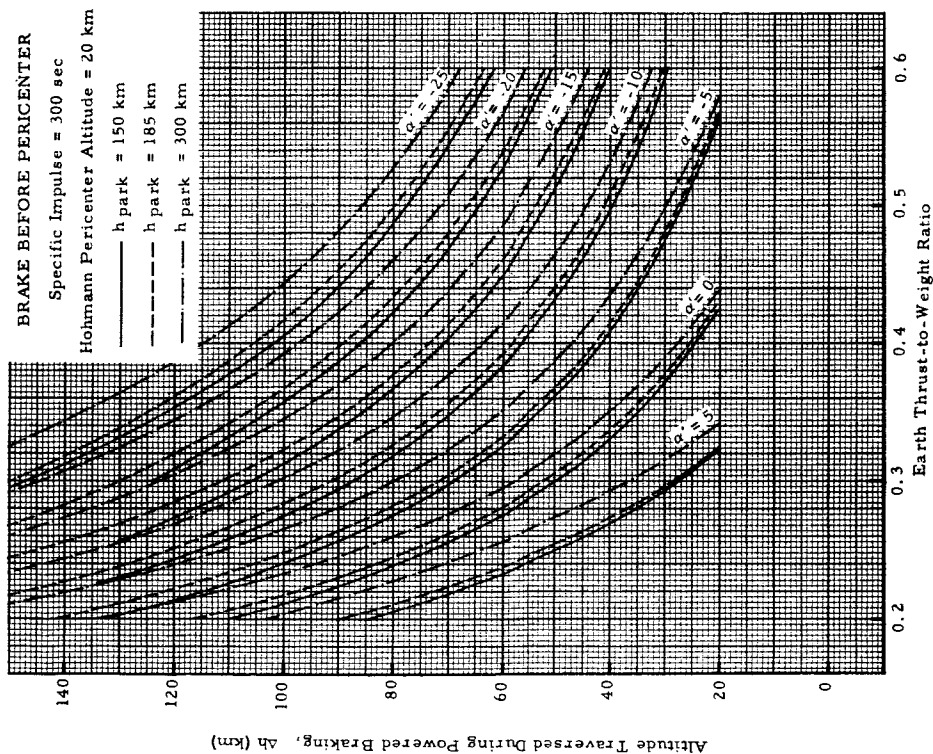
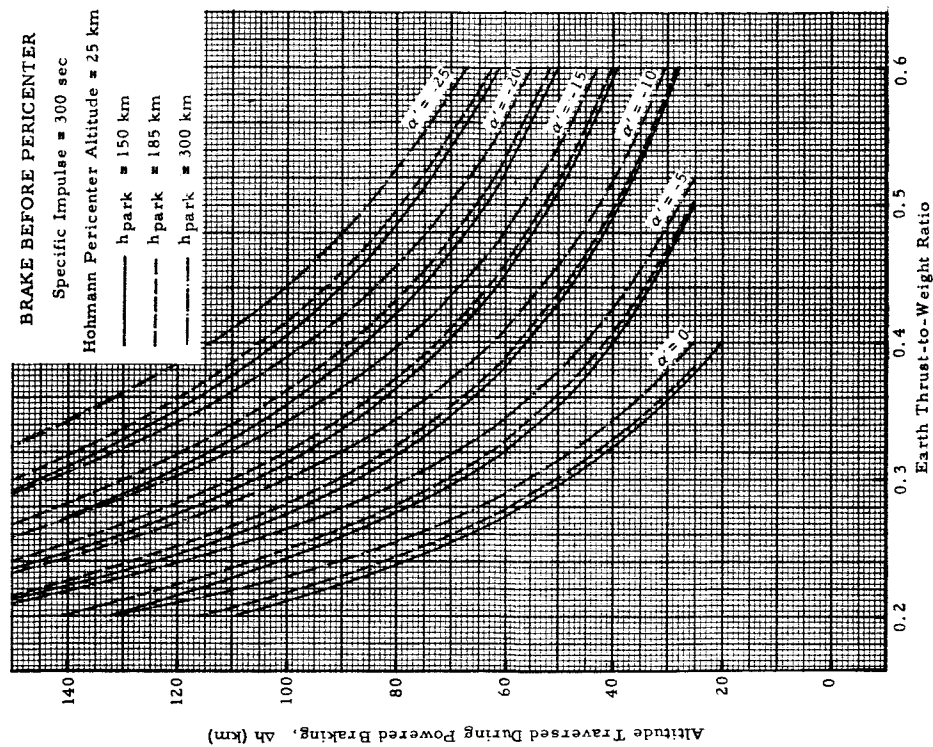


FIGURE 4 d. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

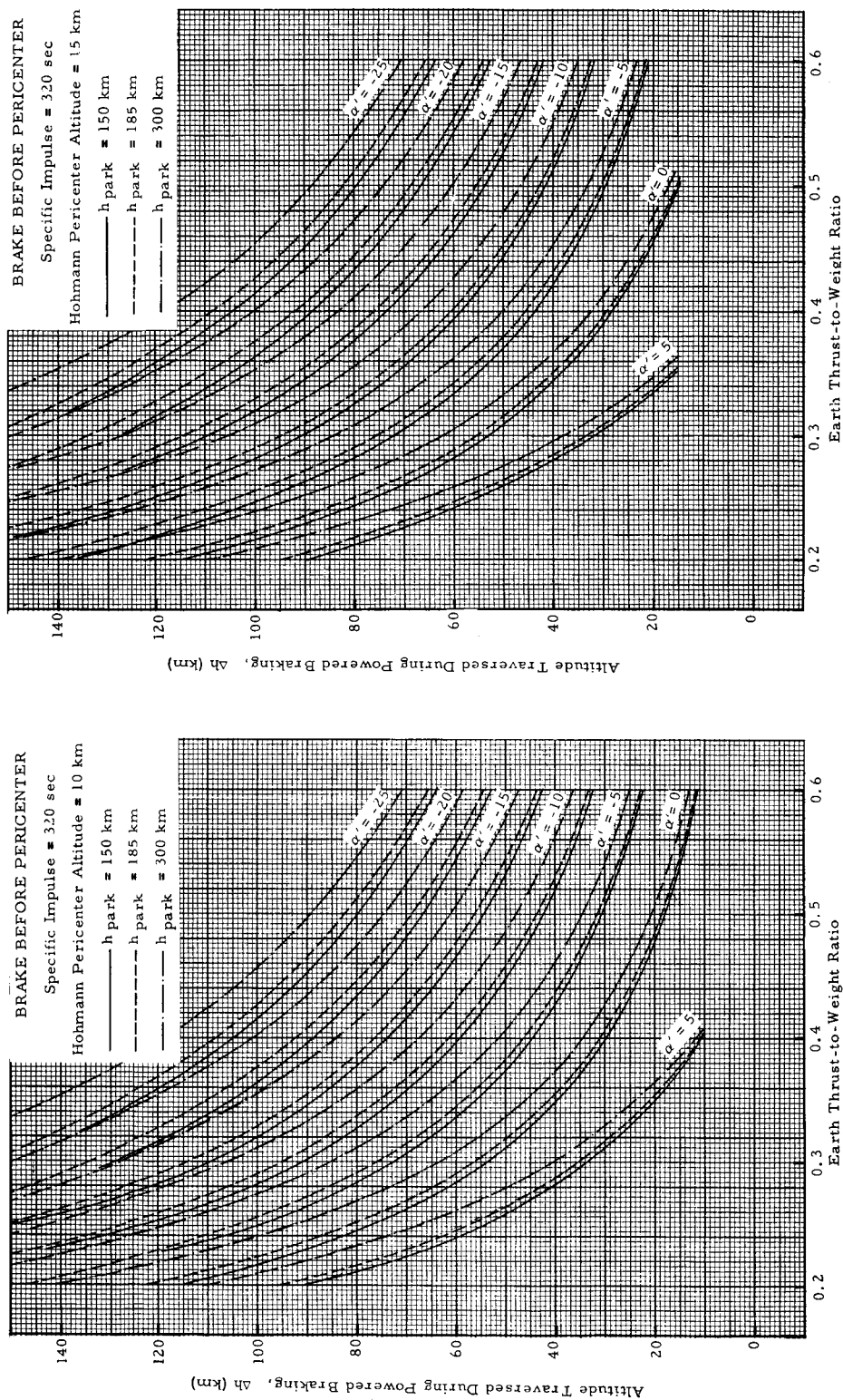


FIGURE 4 e. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

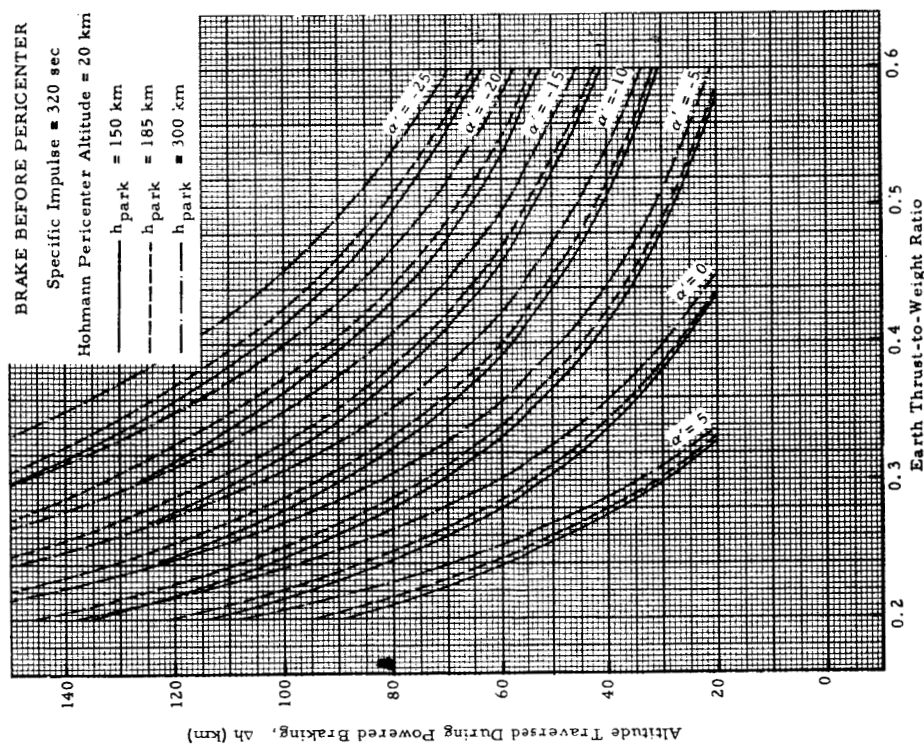
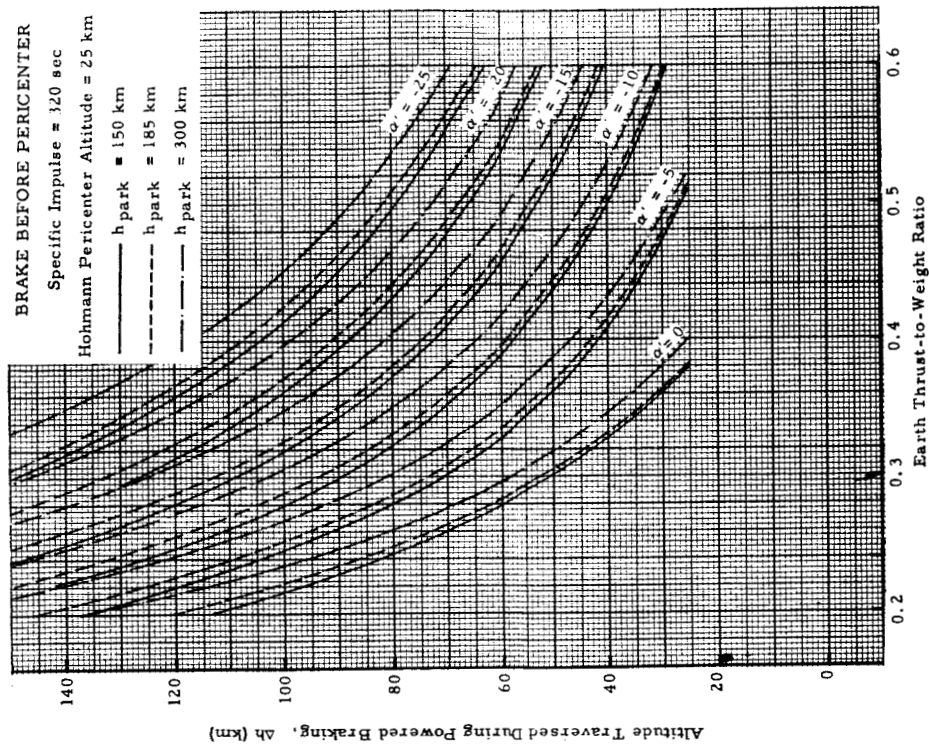


FIGURE 4 f. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

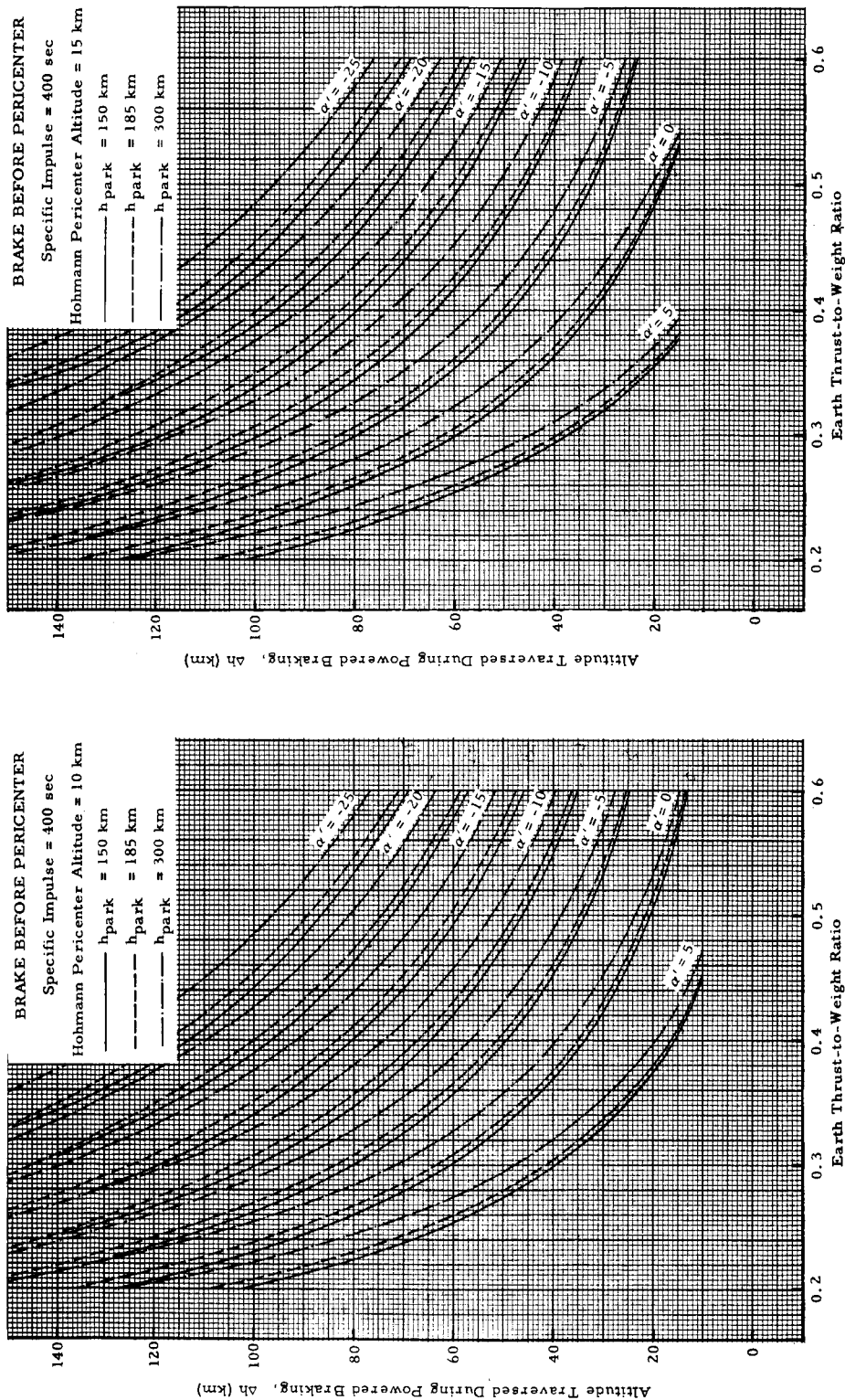


FIGURE 4 g. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

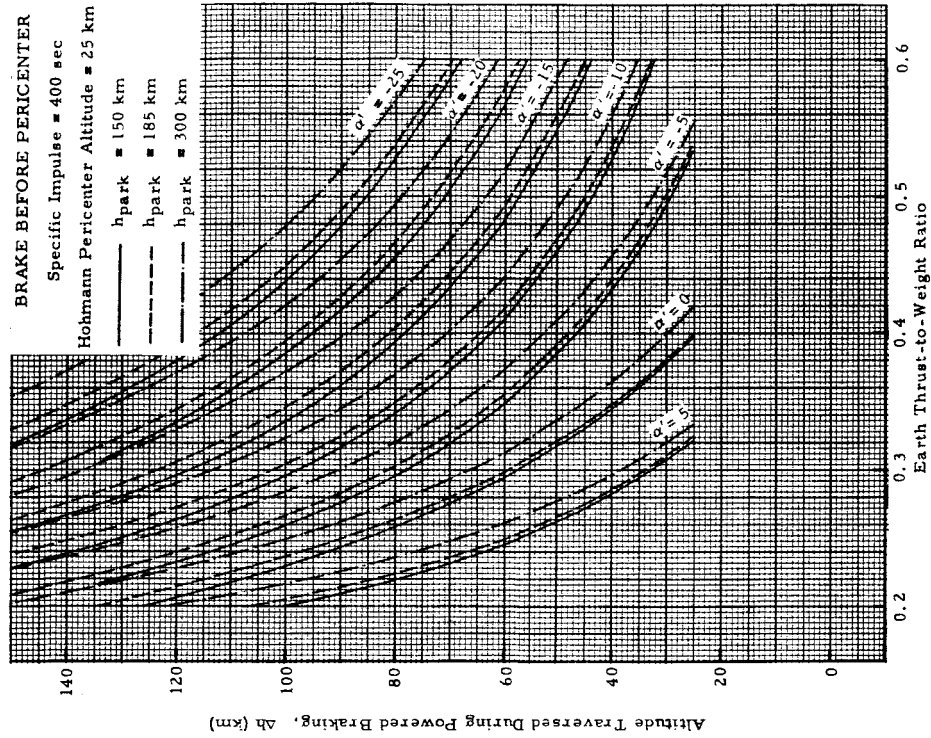
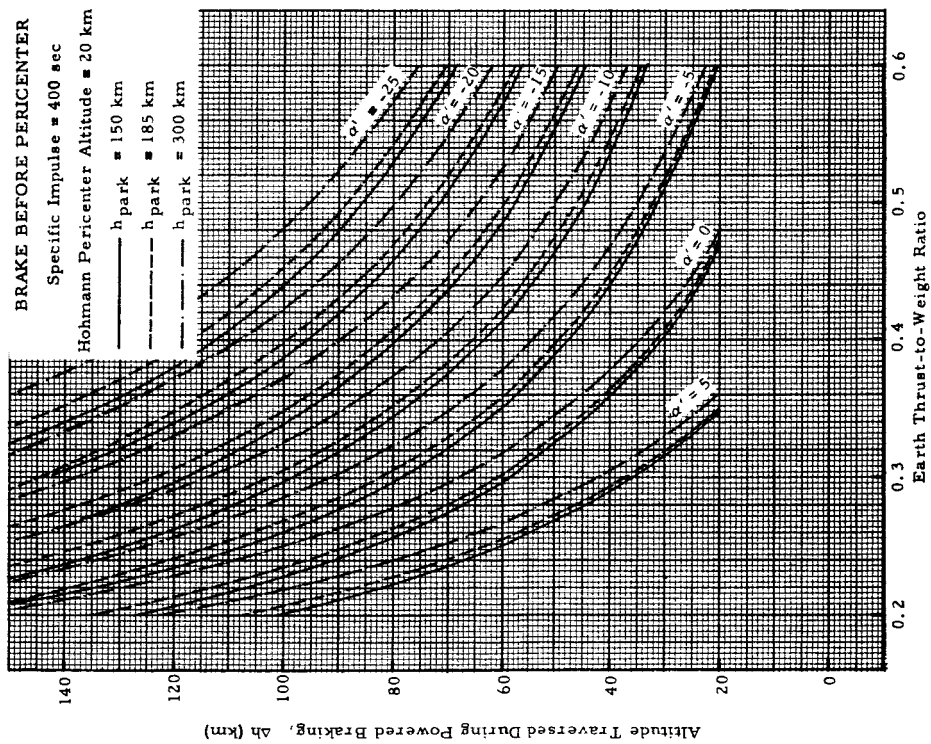


FIGURE 4 h. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE
 PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

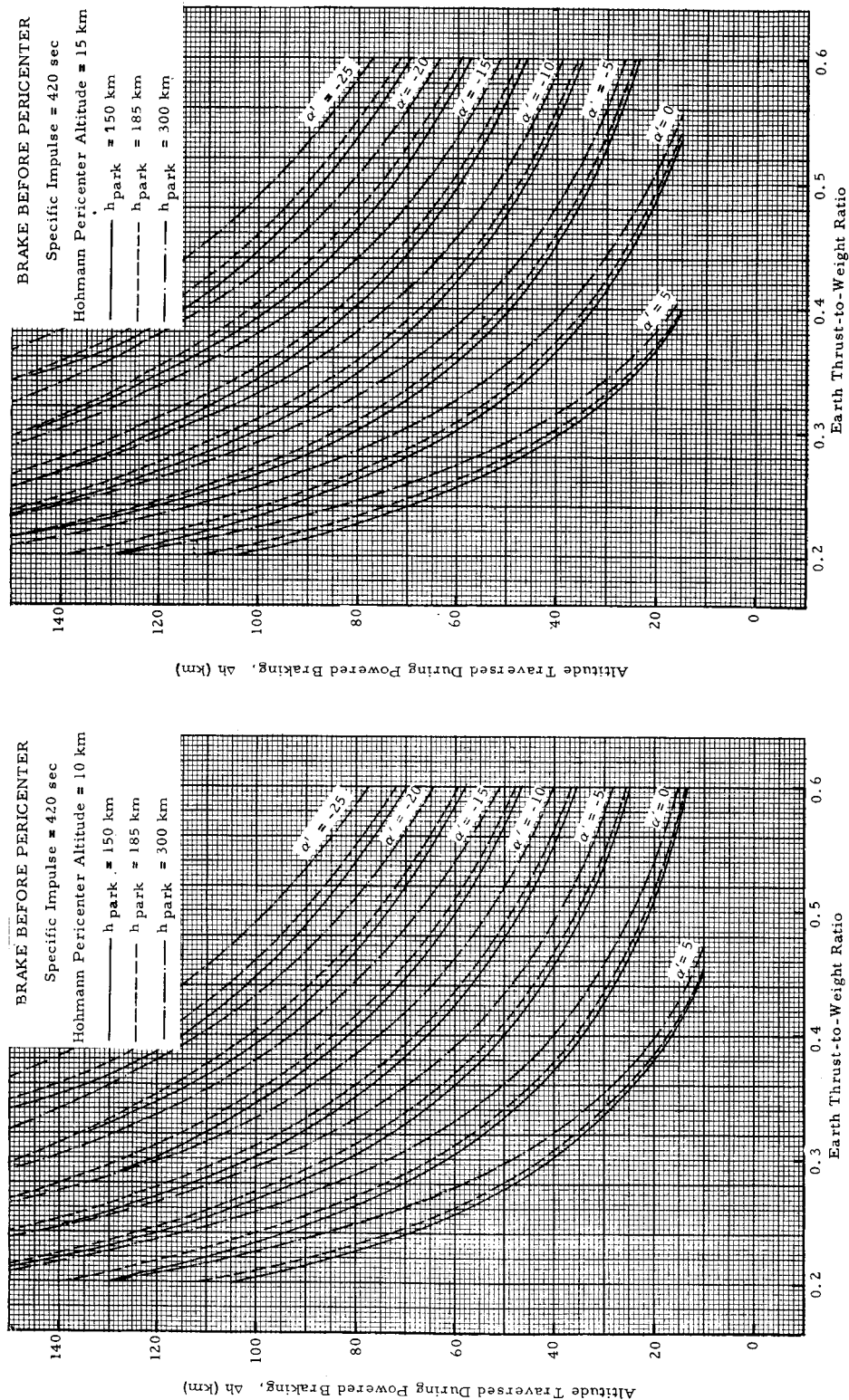


FIGURE 4 i. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

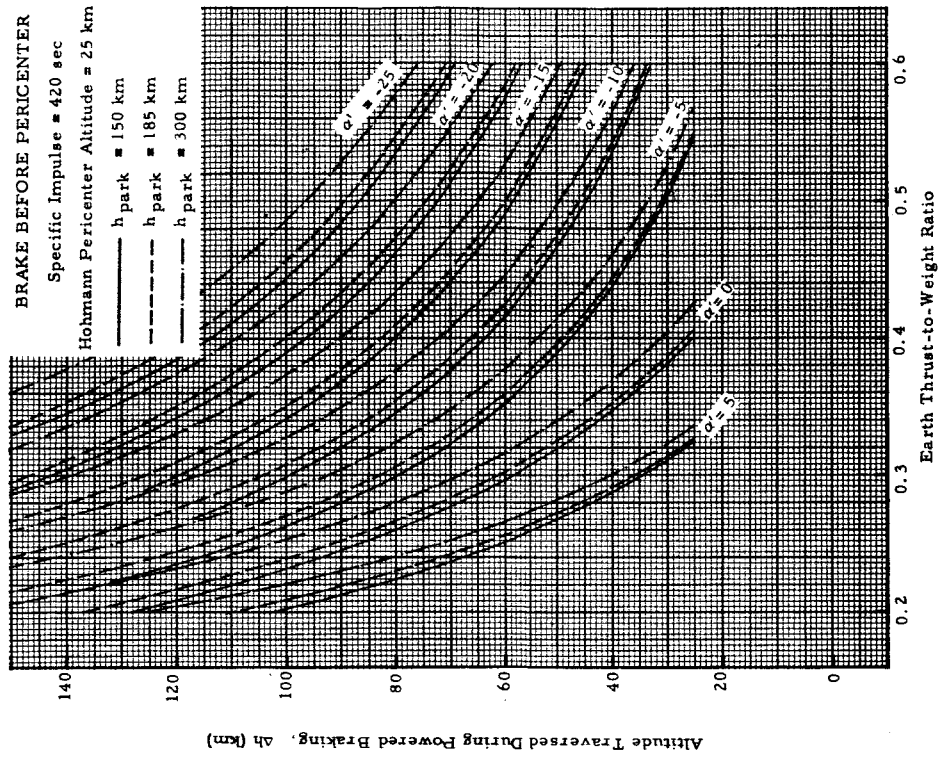
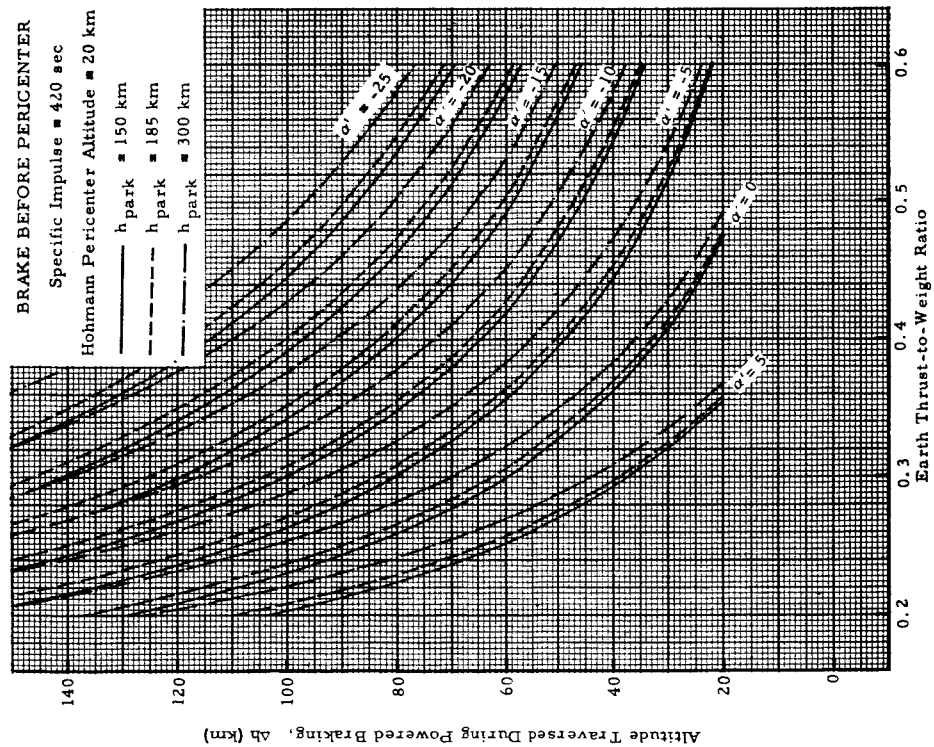


FIGURE 4 j. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE
PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

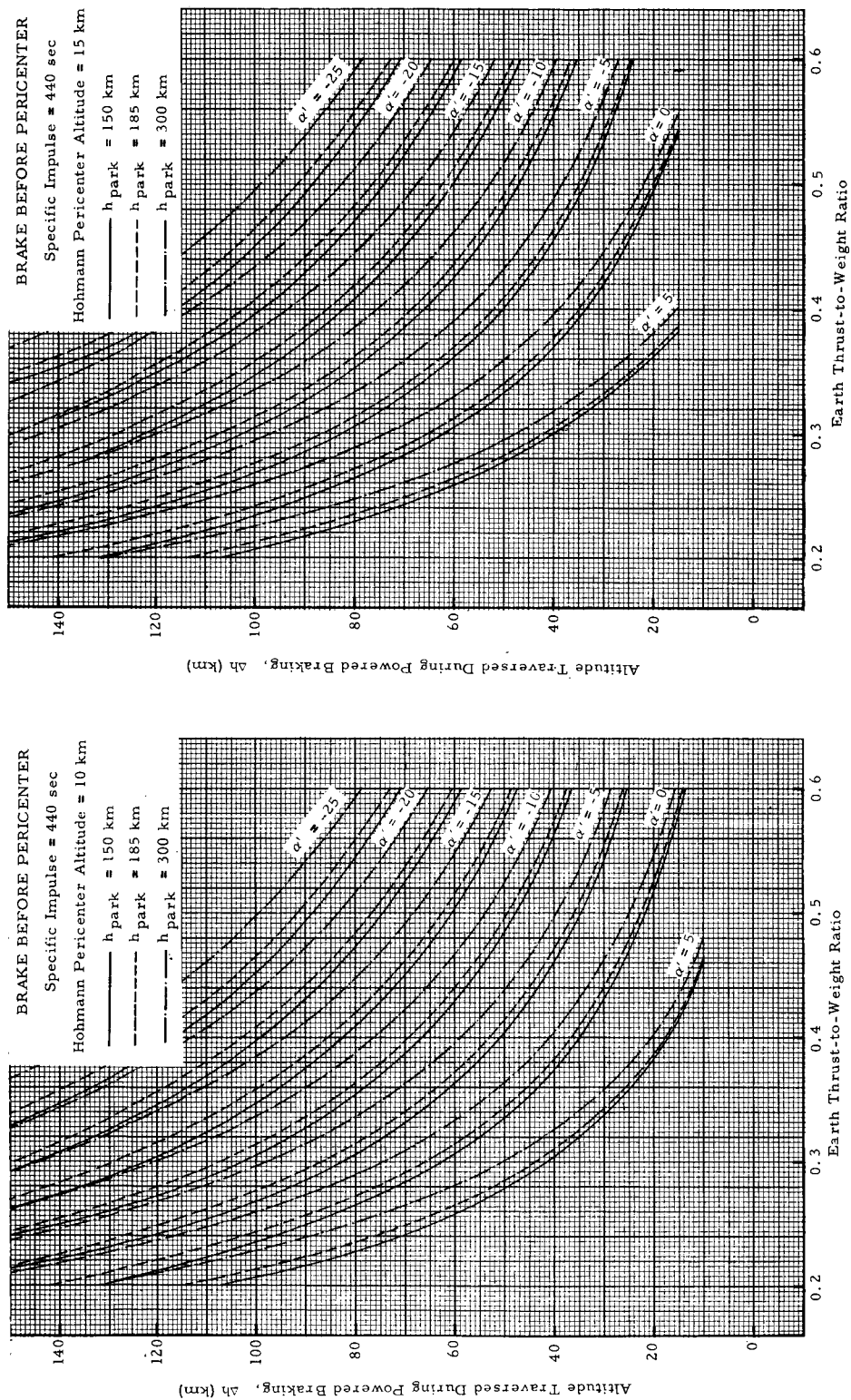


FIGURE 4 k. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

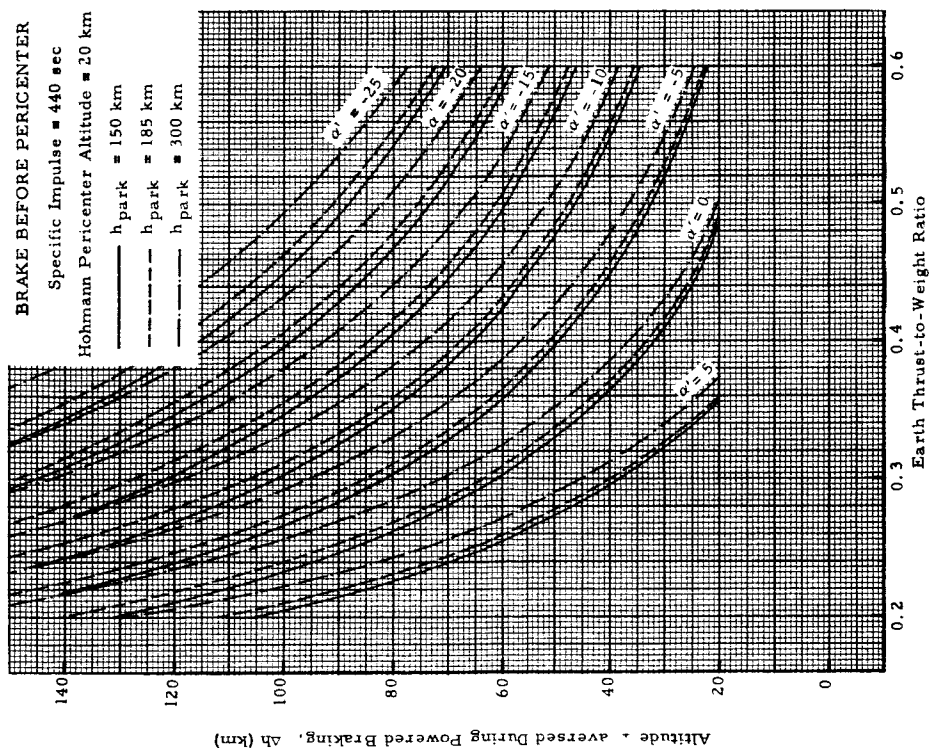
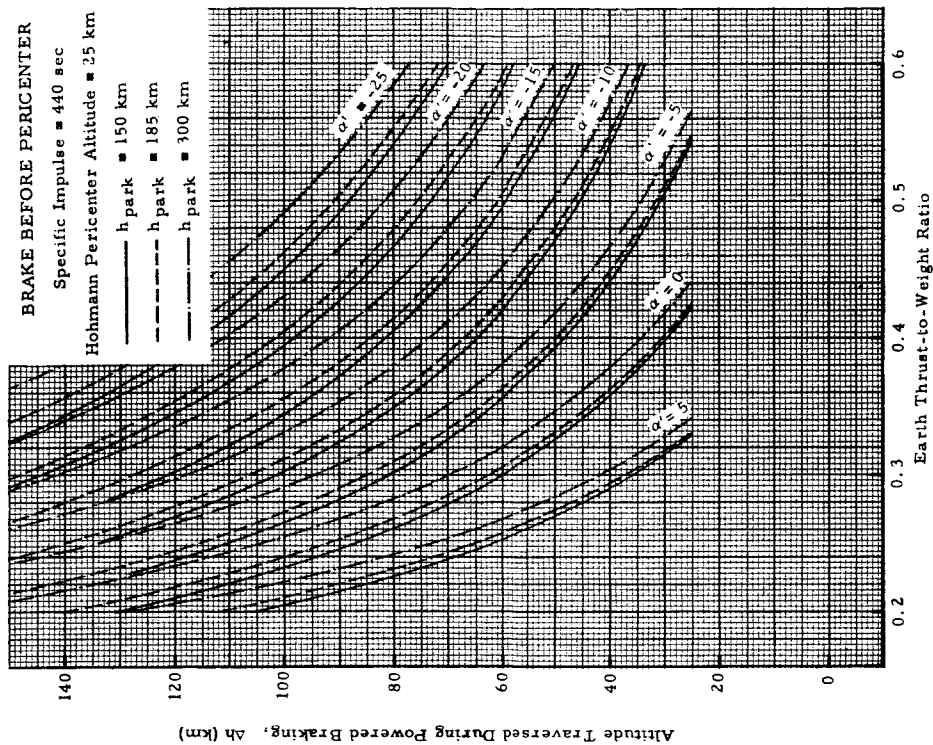


FIGURE 4.1. ALTITUDE TRAVERSED DURING POWERED DESCENT BEFORE
 PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

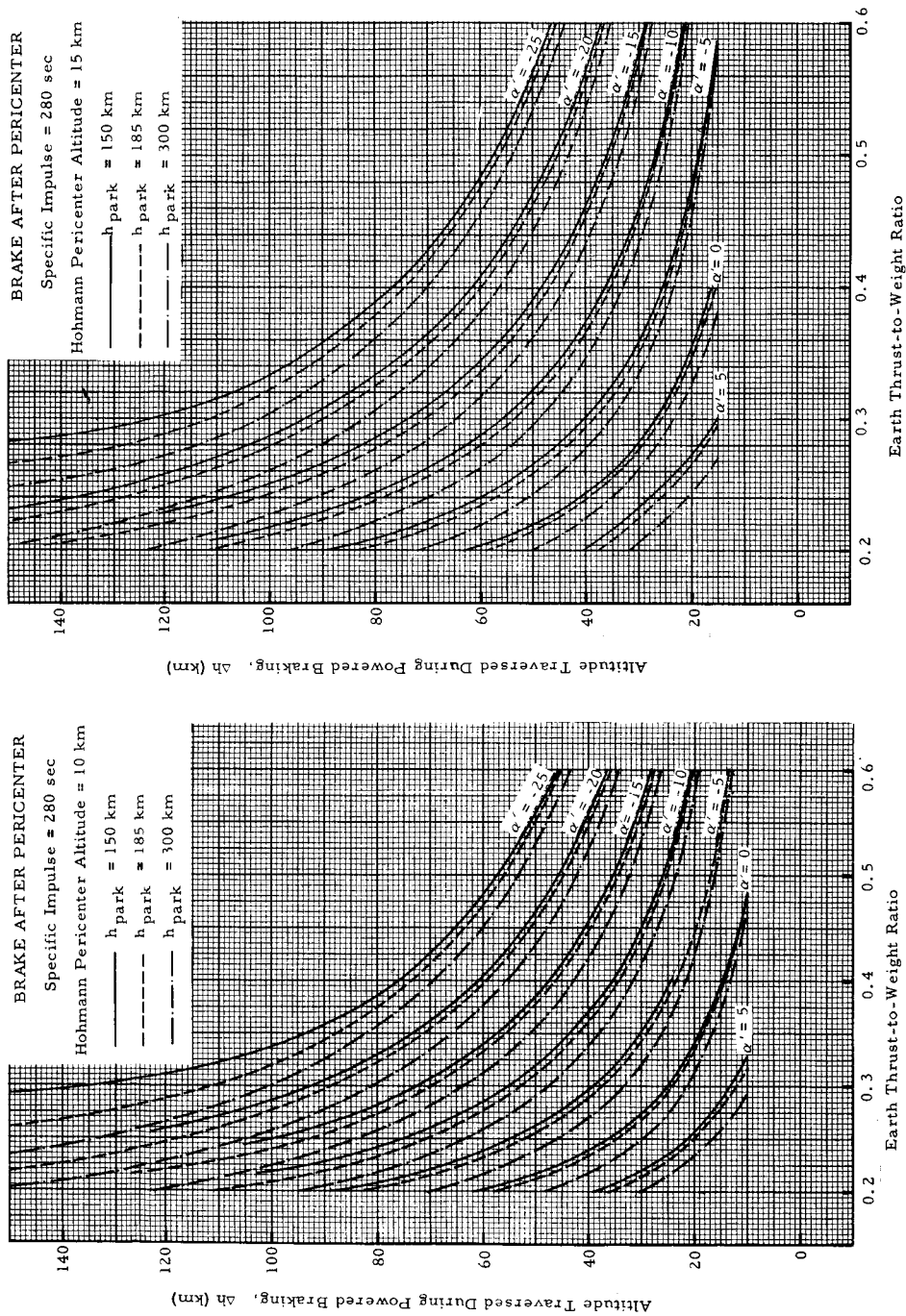


FIGURE 5 a. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER
 PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
 HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

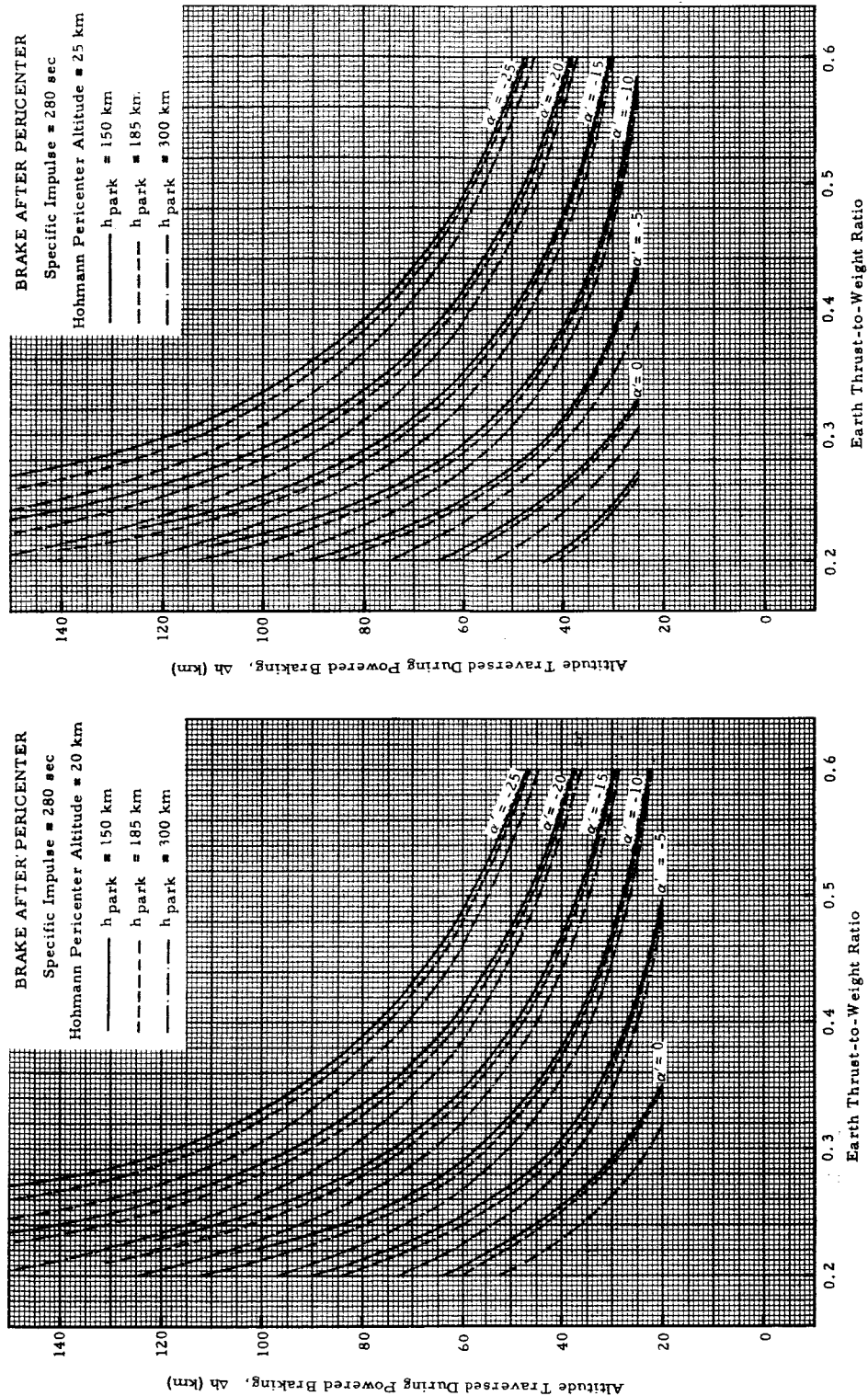


FIGURE 5 b. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

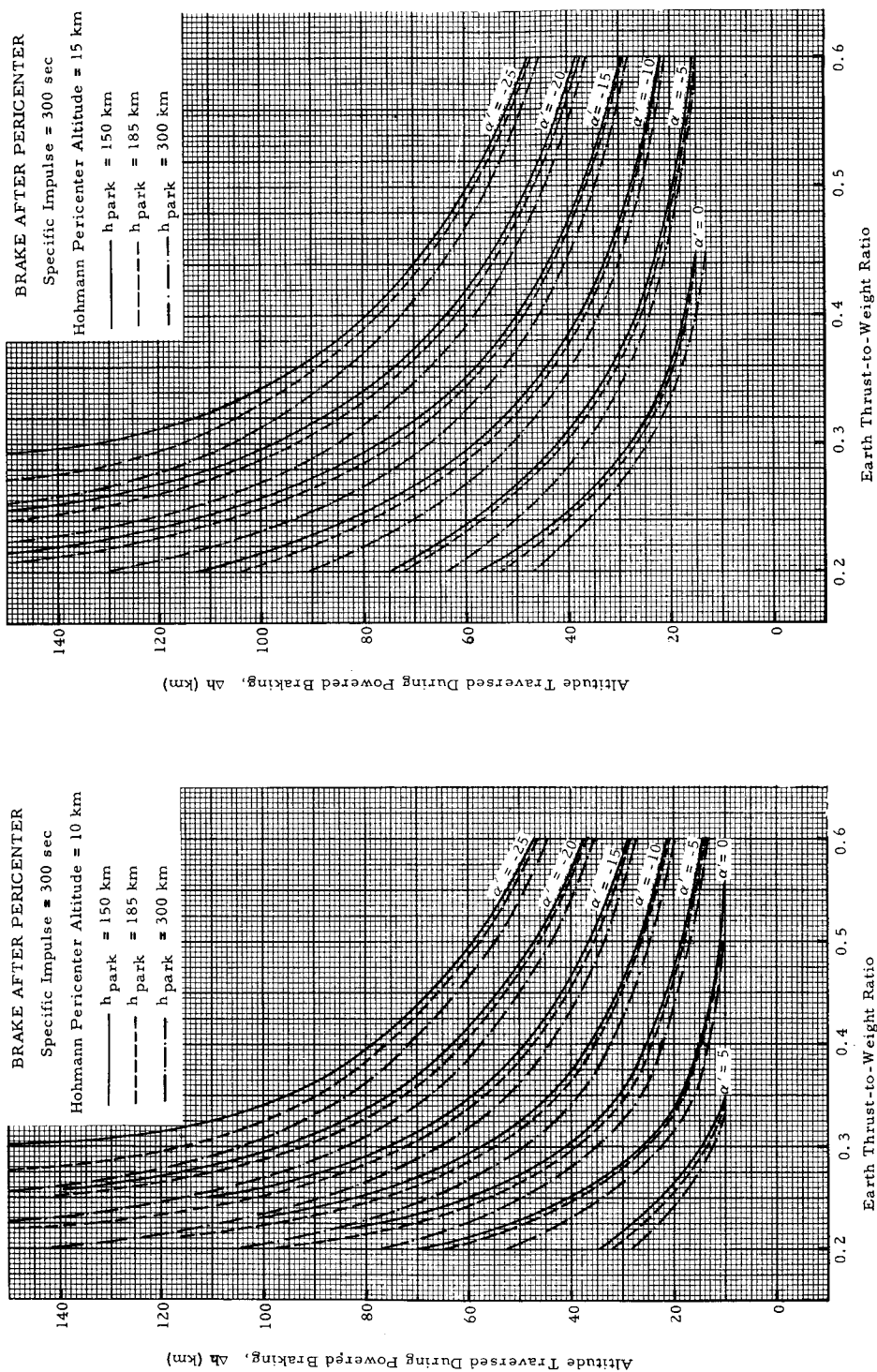


FIGURE 5 c. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

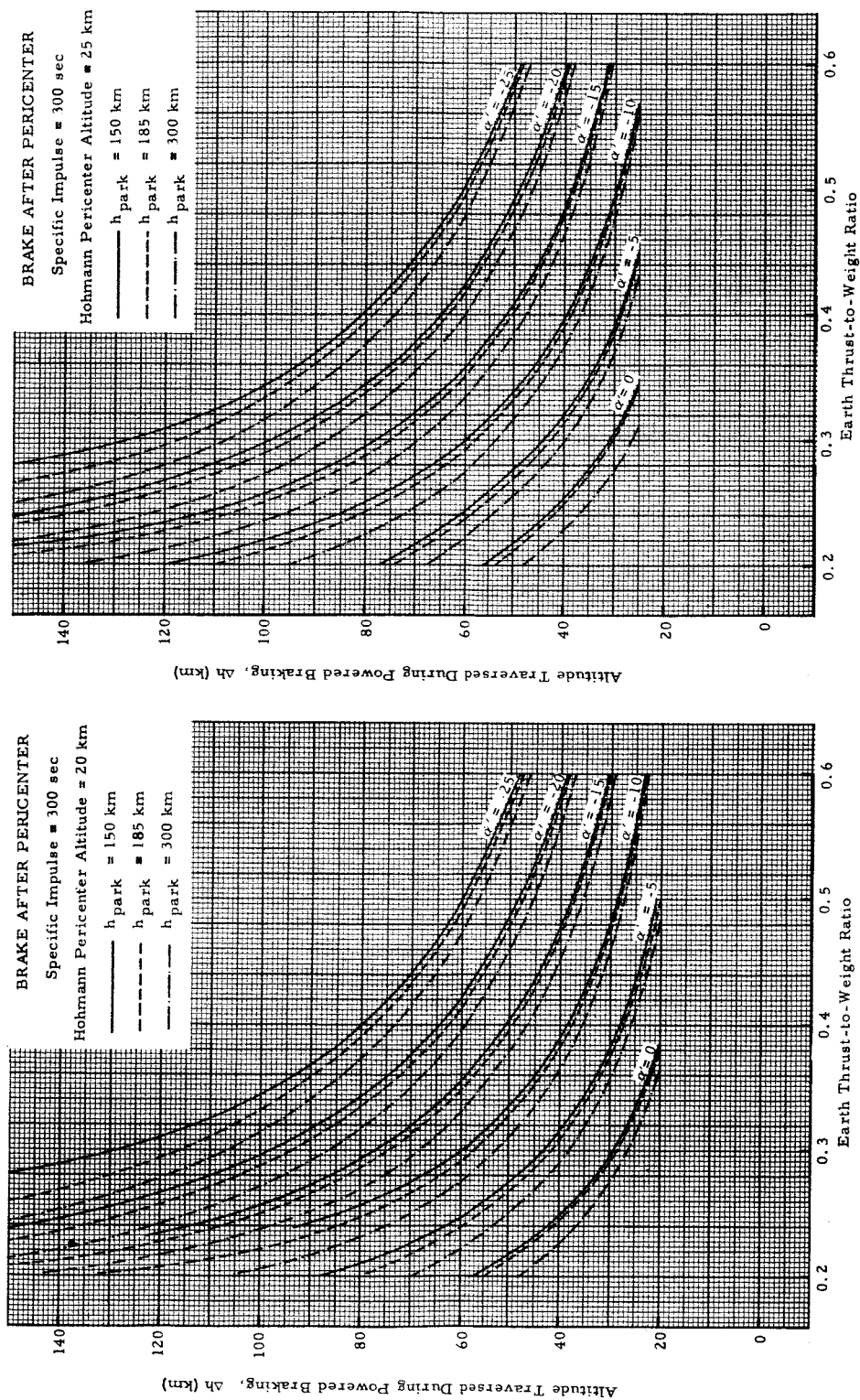


FIGURE 5 d. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER
PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

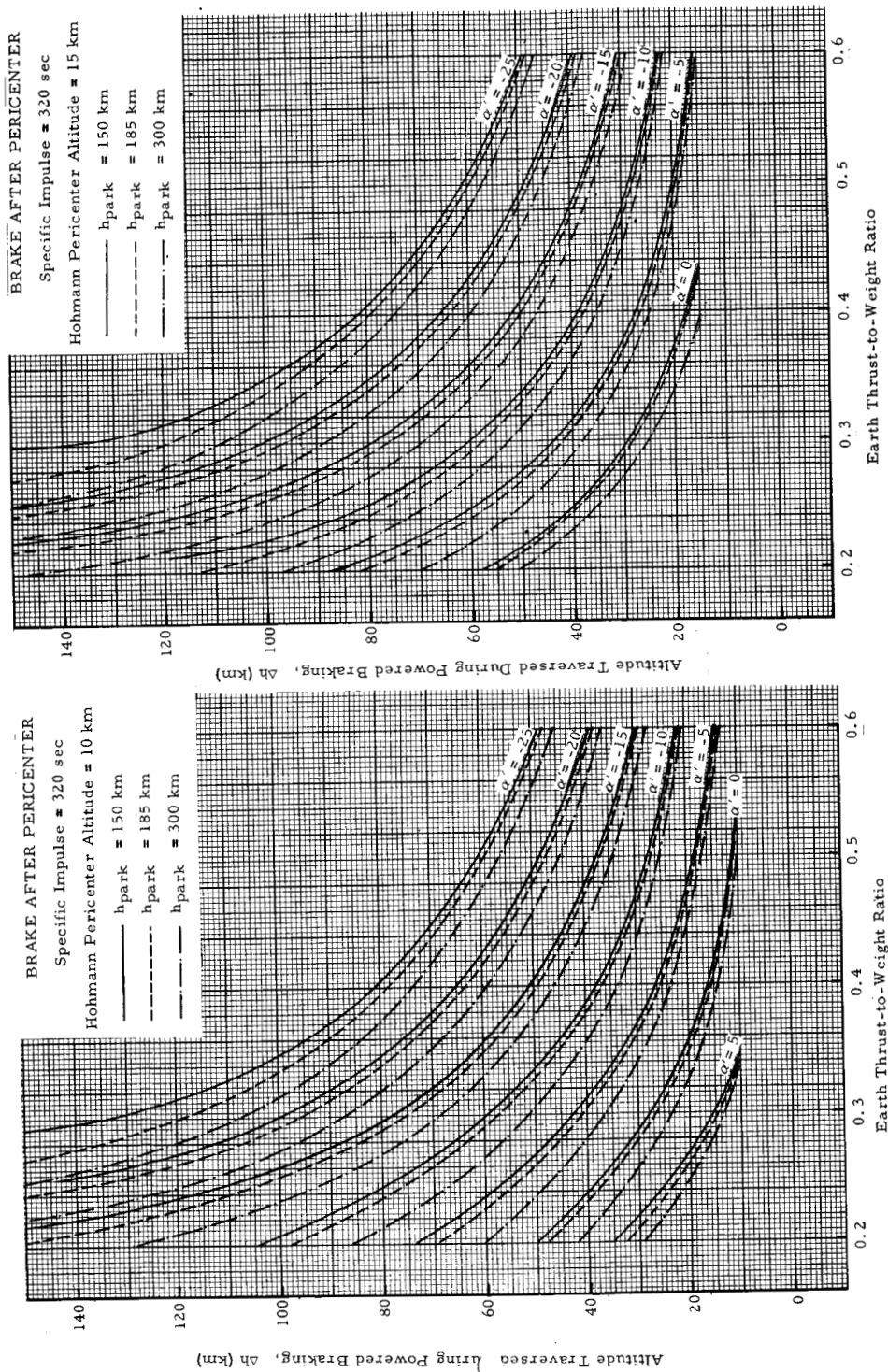


FIGURE 5 e. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

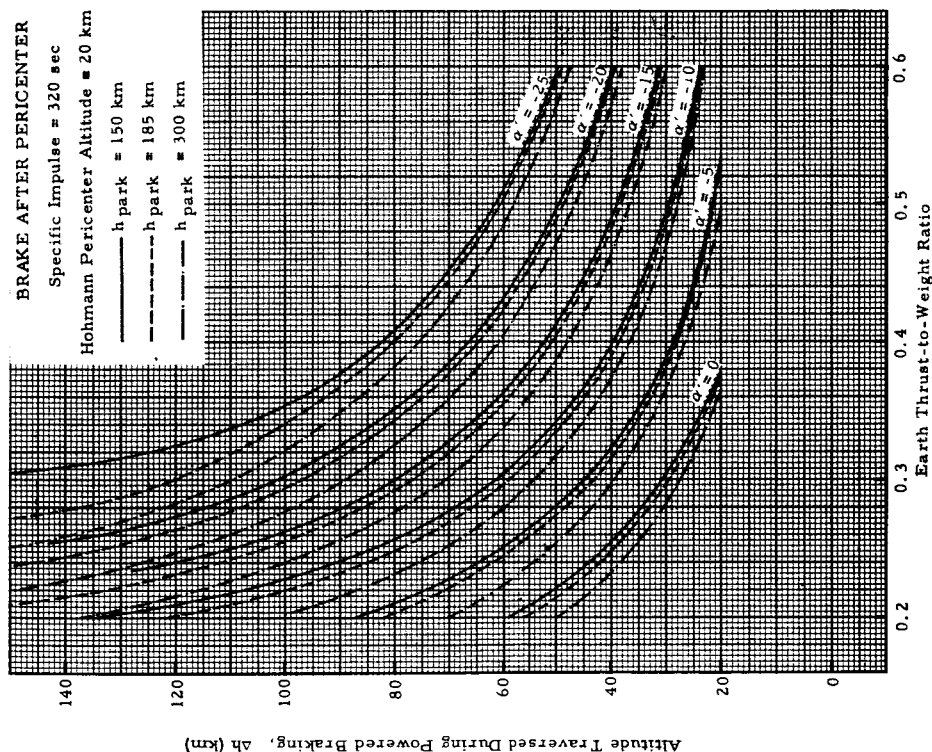
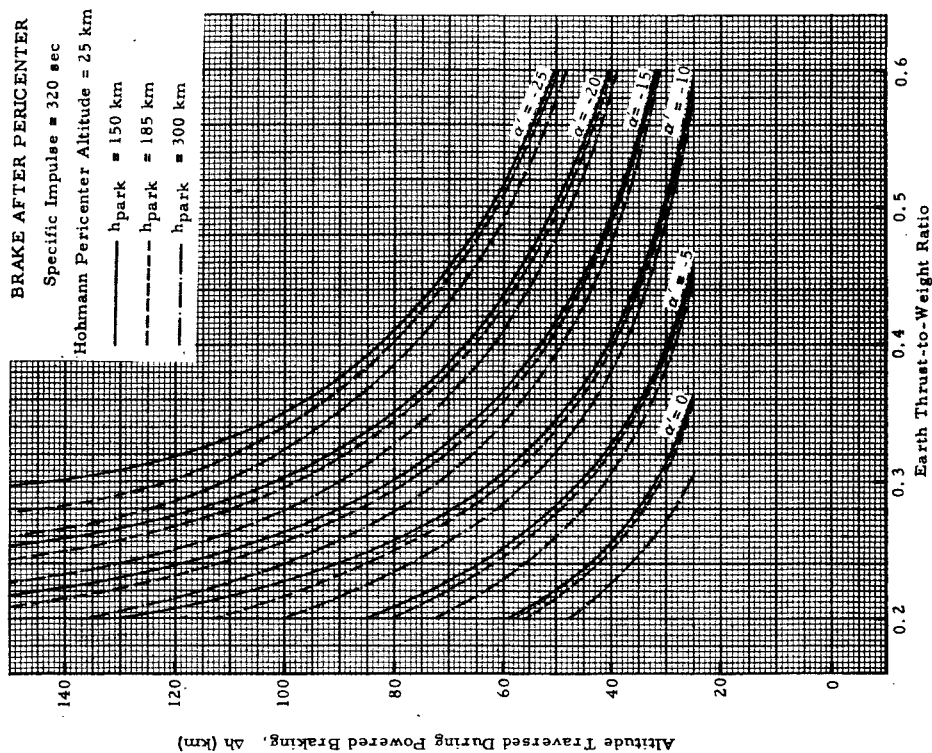


FIGURE 5 f. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

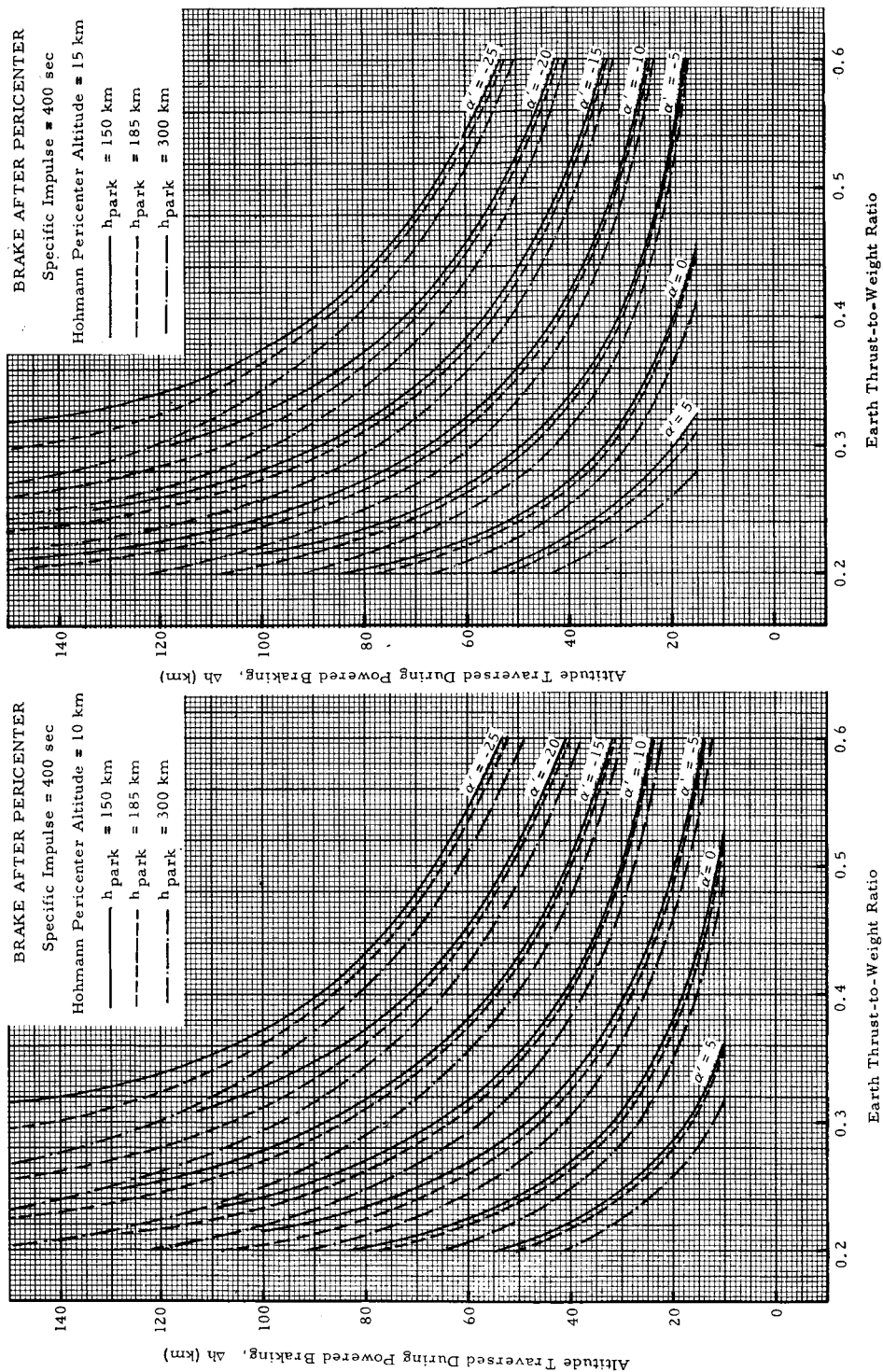


FIGURE 5 g. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

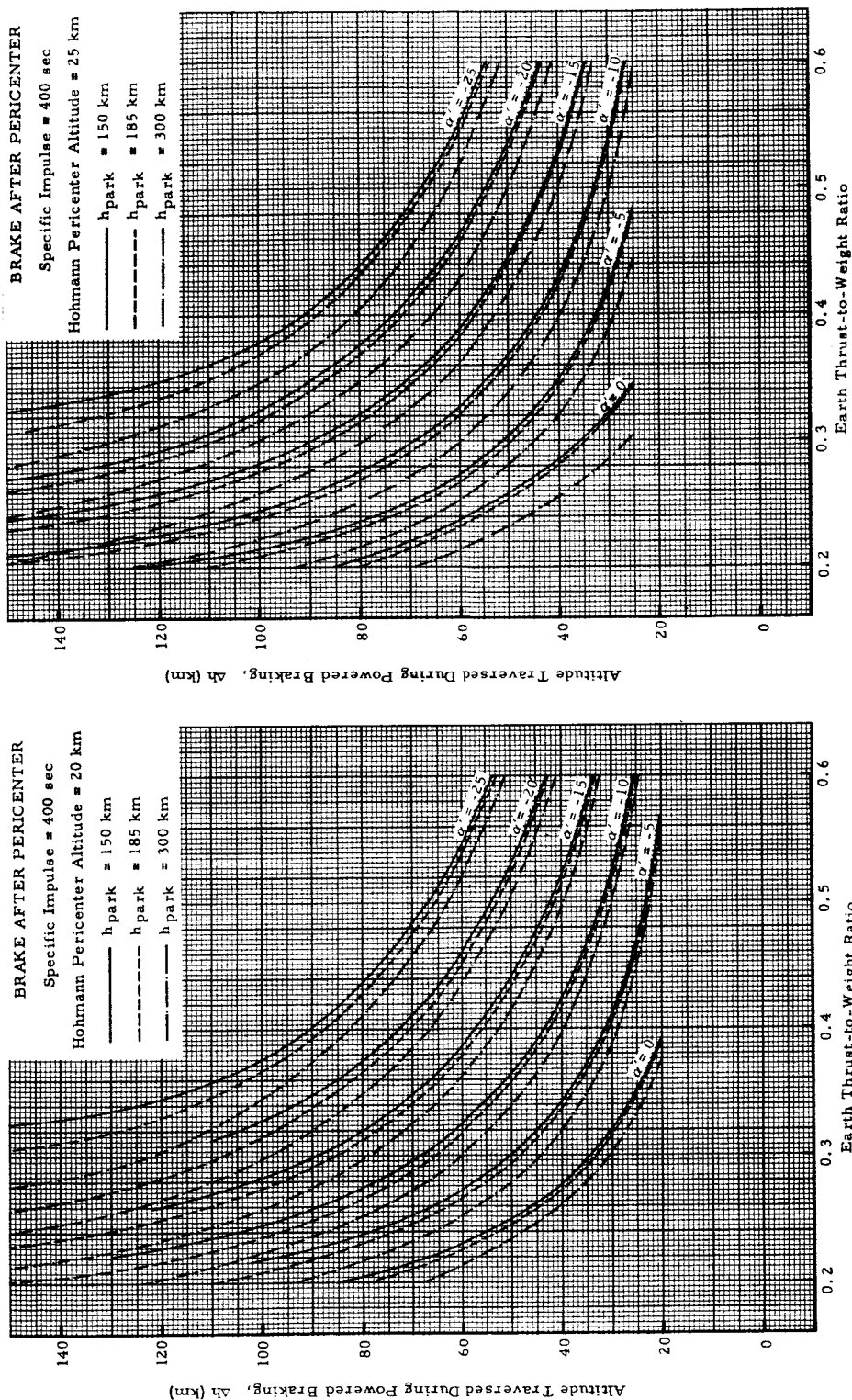


FIGURE 5 h. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

FIGURE 5. i. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS HOHMANN PERICENTER ALTITUDE $\approx$ 10 AND 15 KM

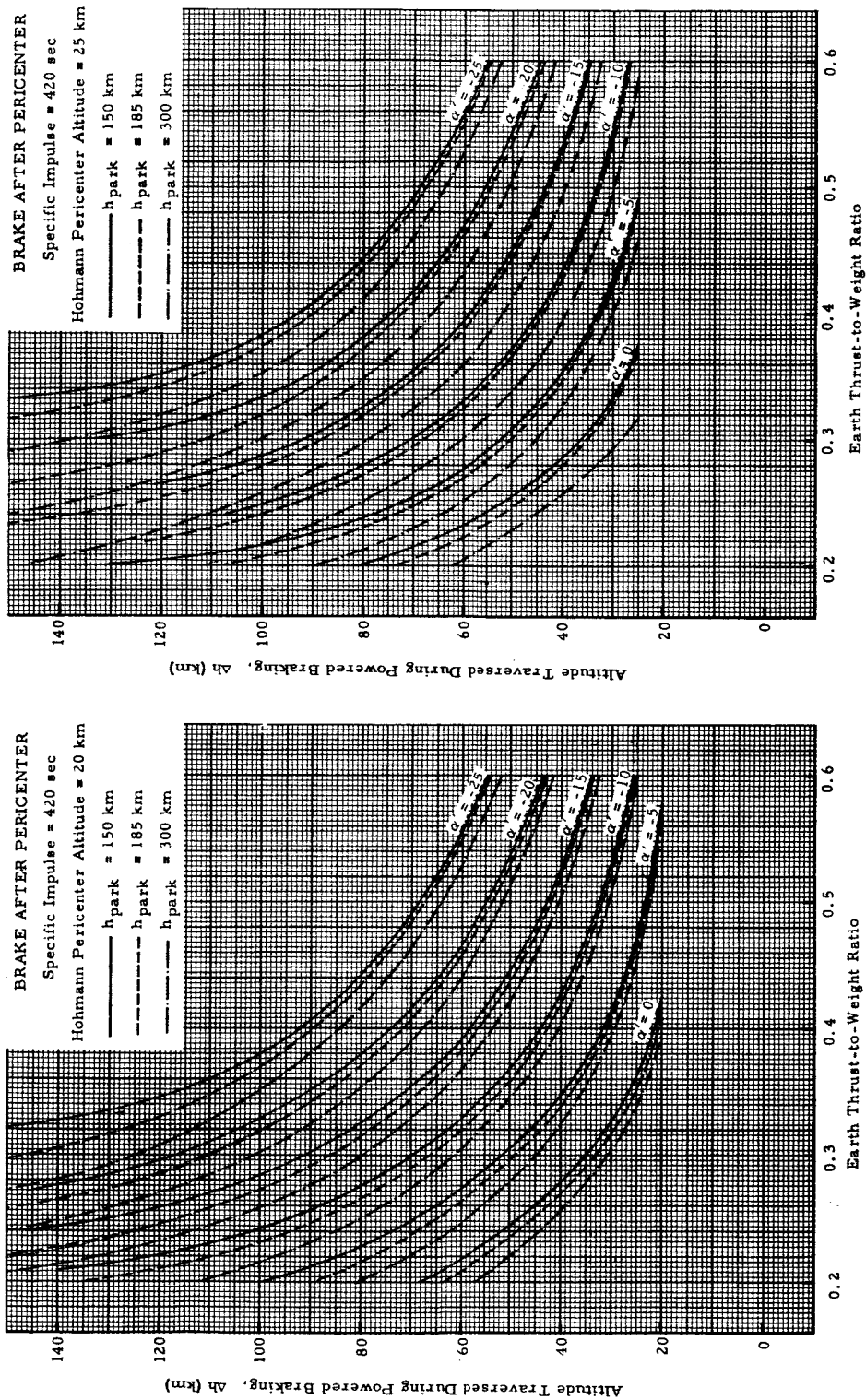


FIGURE 5 j. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

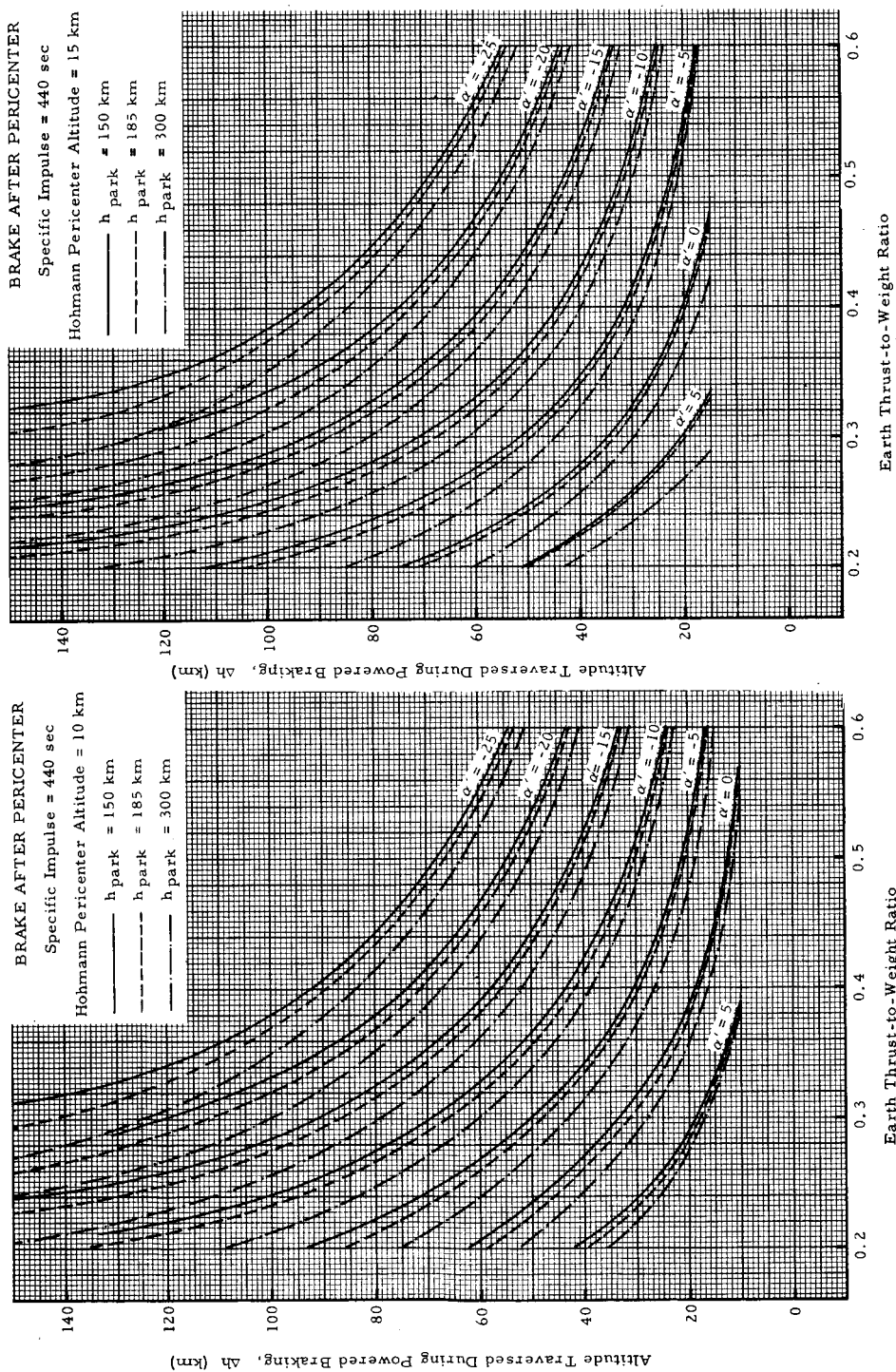


FIGURE 5 k. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

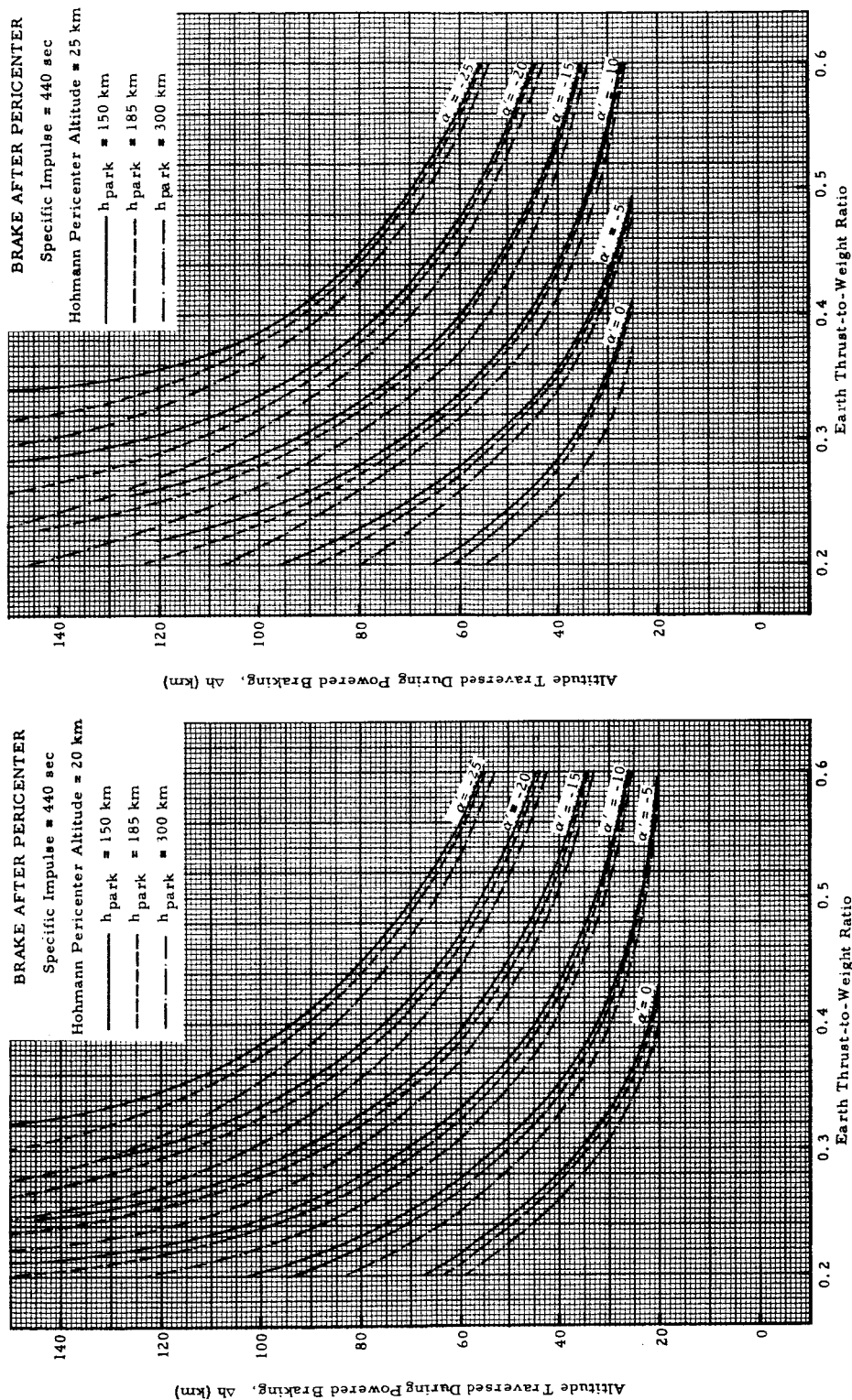
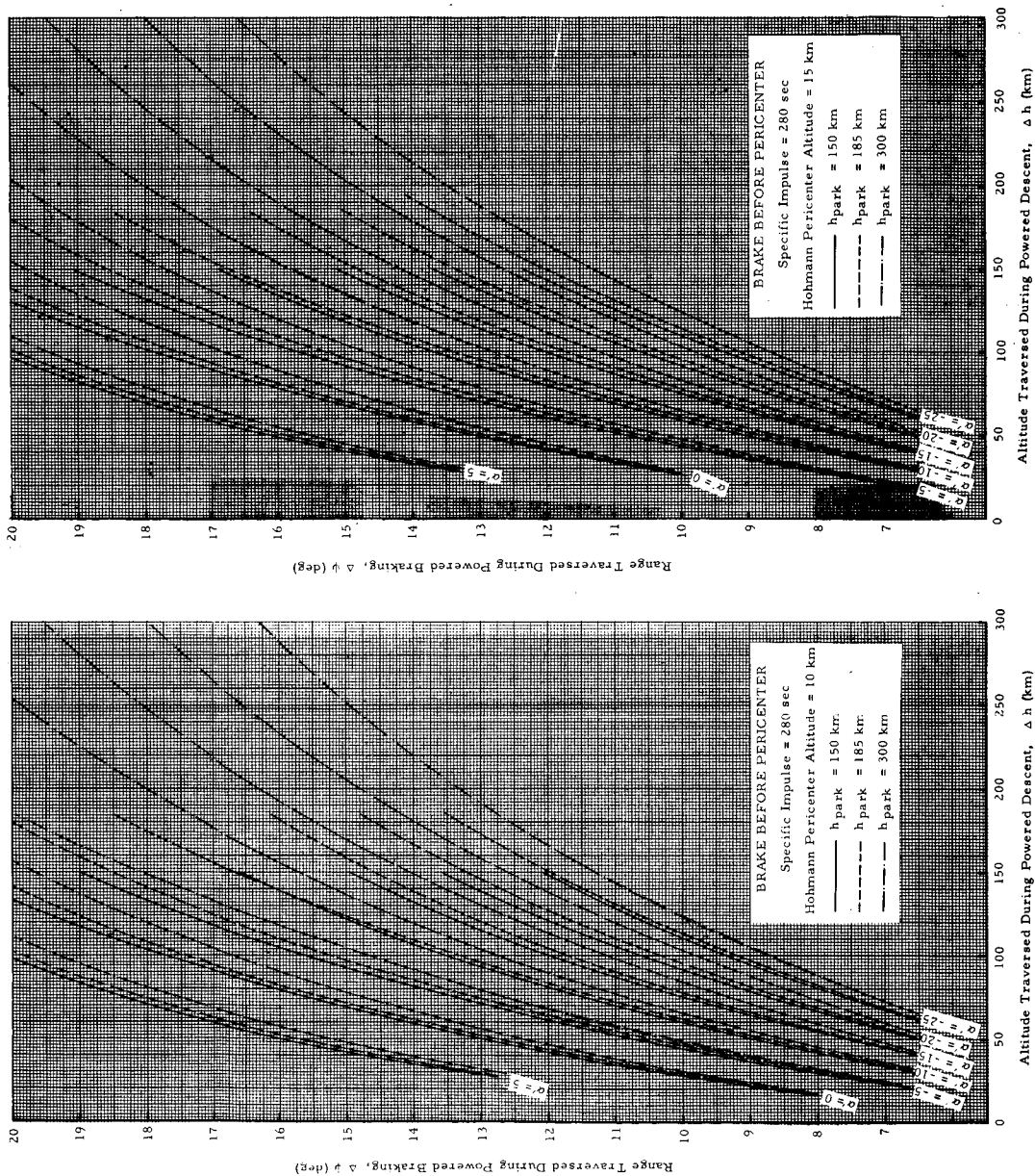


FIGURE 5 1. ALTITUDE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM



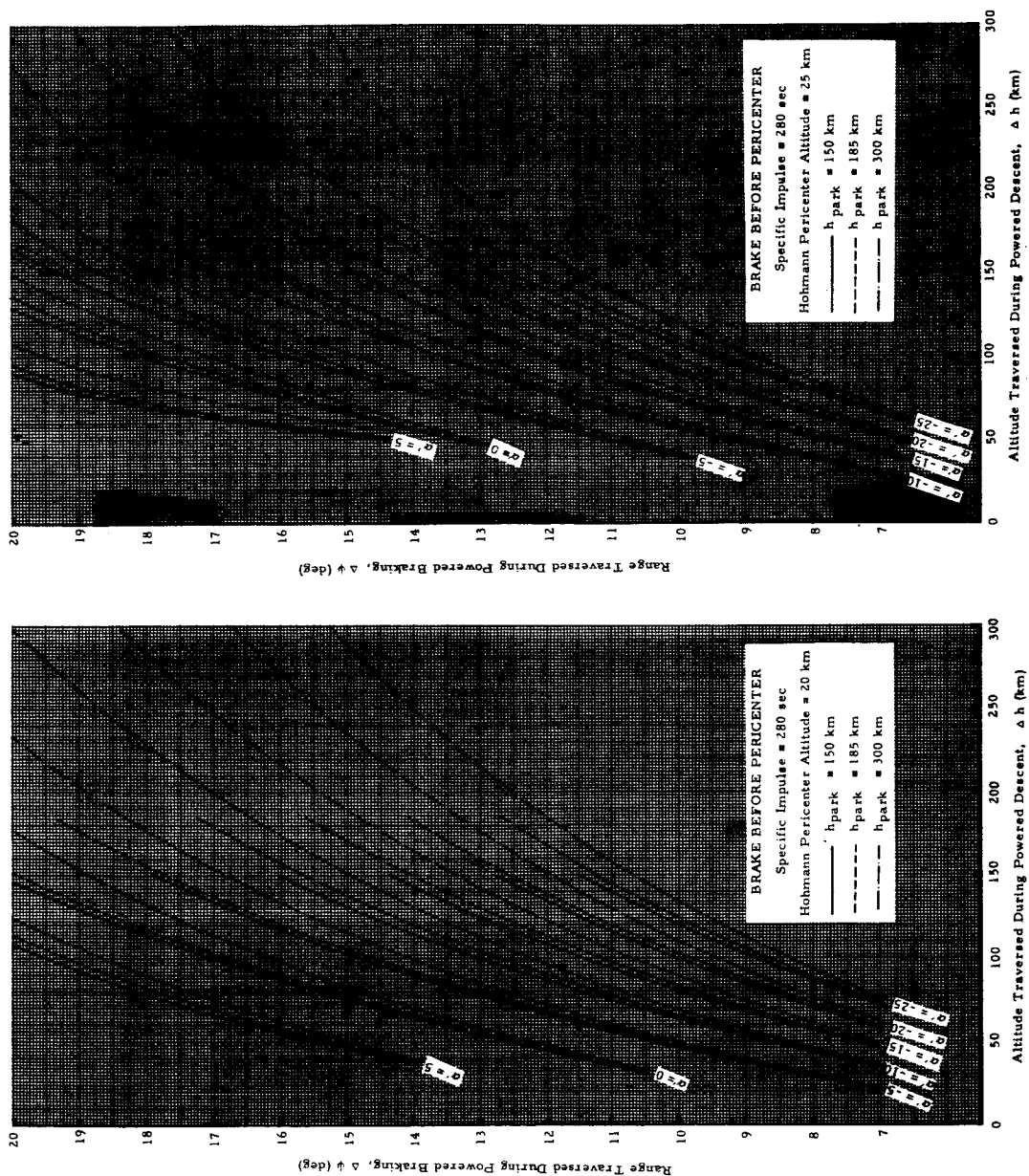


FIGURE 6 b. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

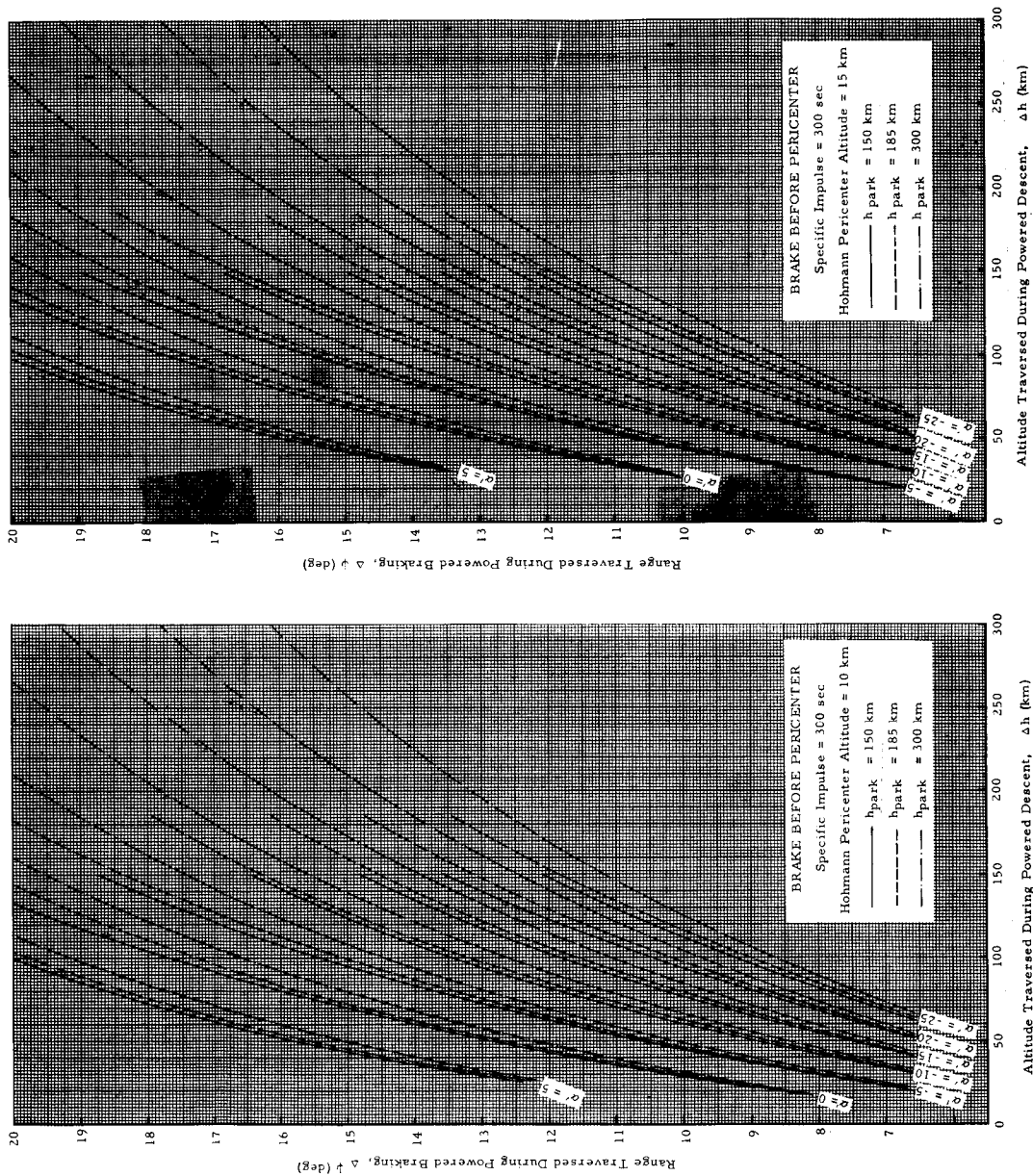


FIGURE 6 c. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
 HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

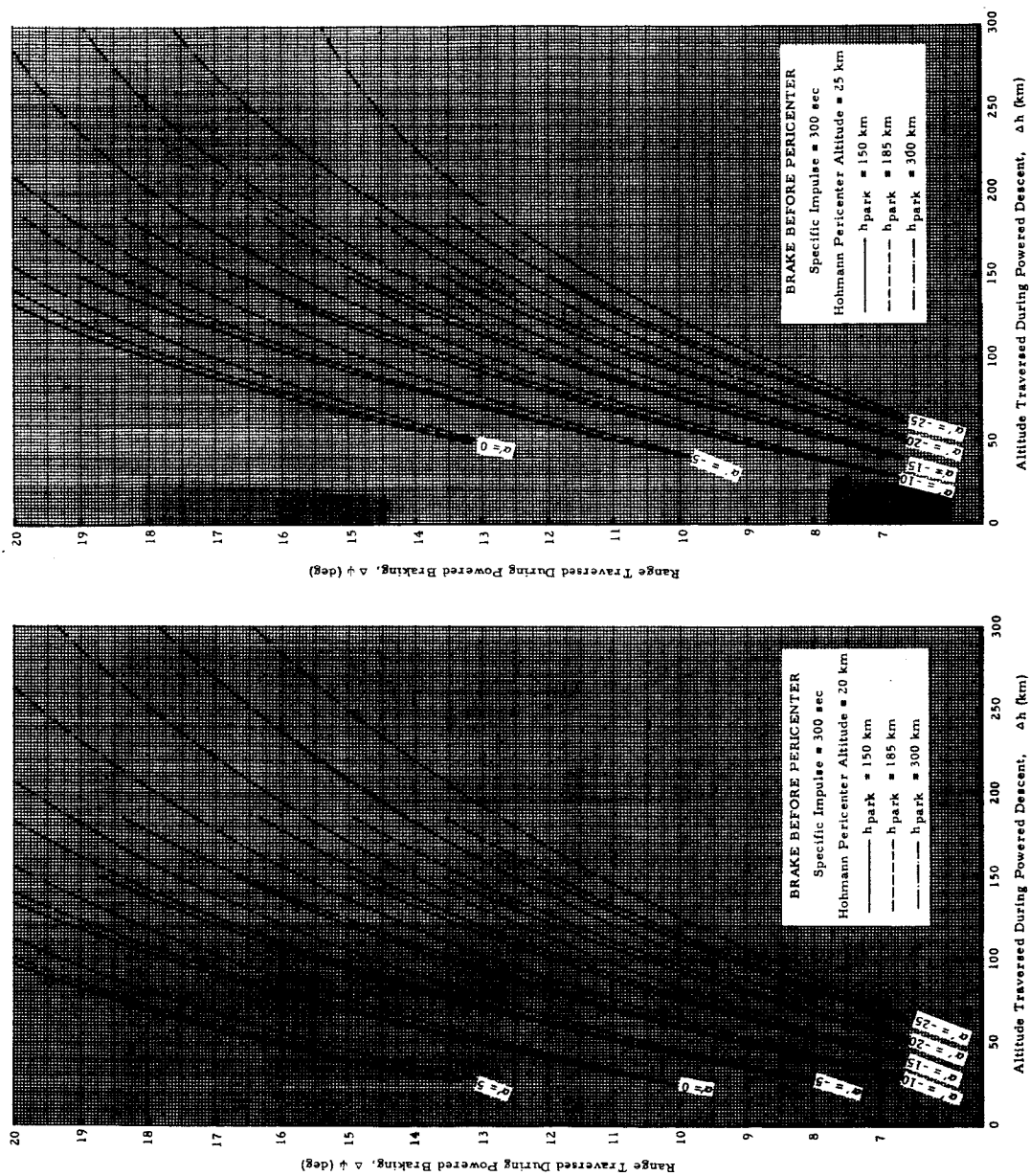


FIGURE 6 d. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

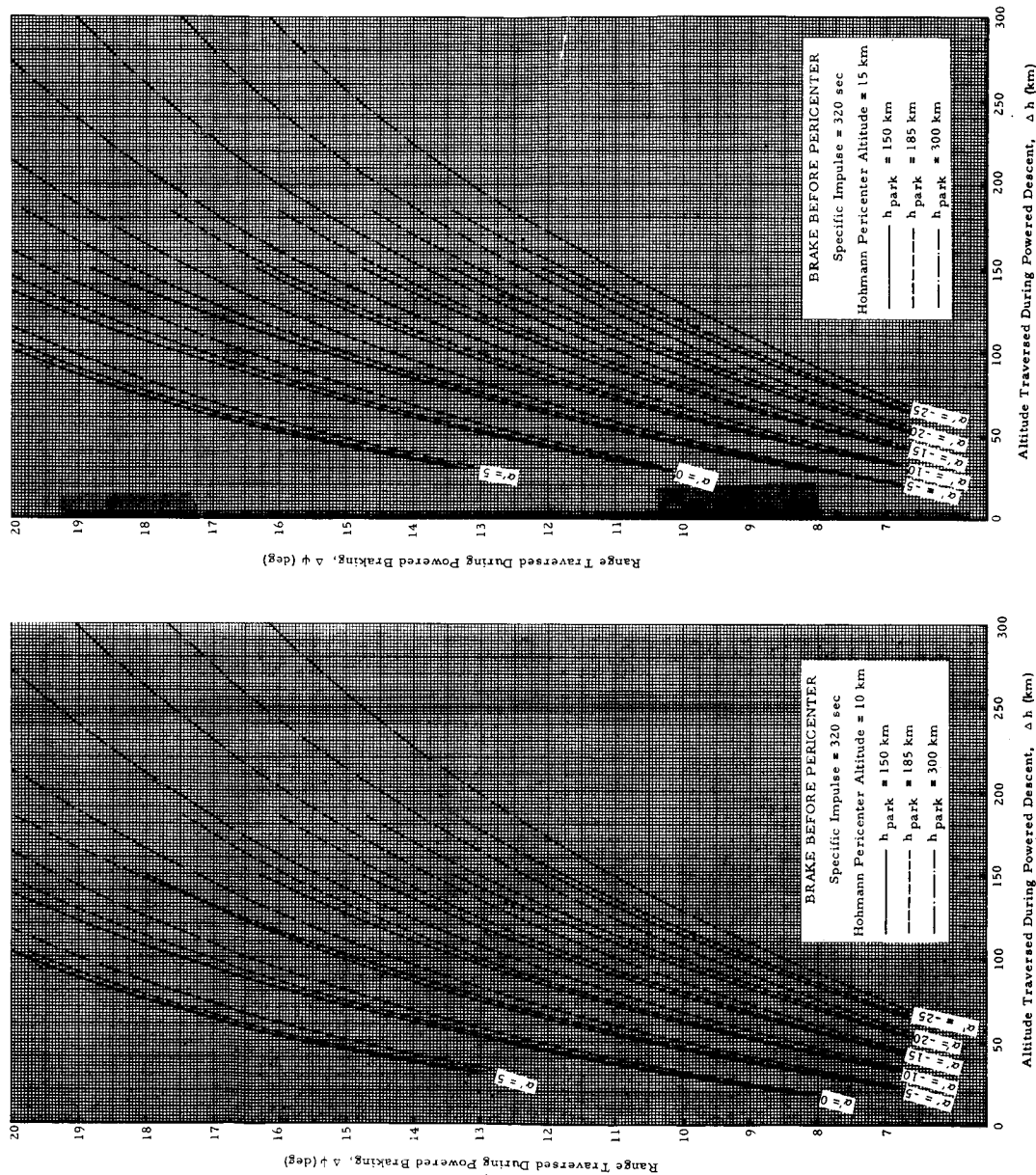


FIGURE 6 e. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

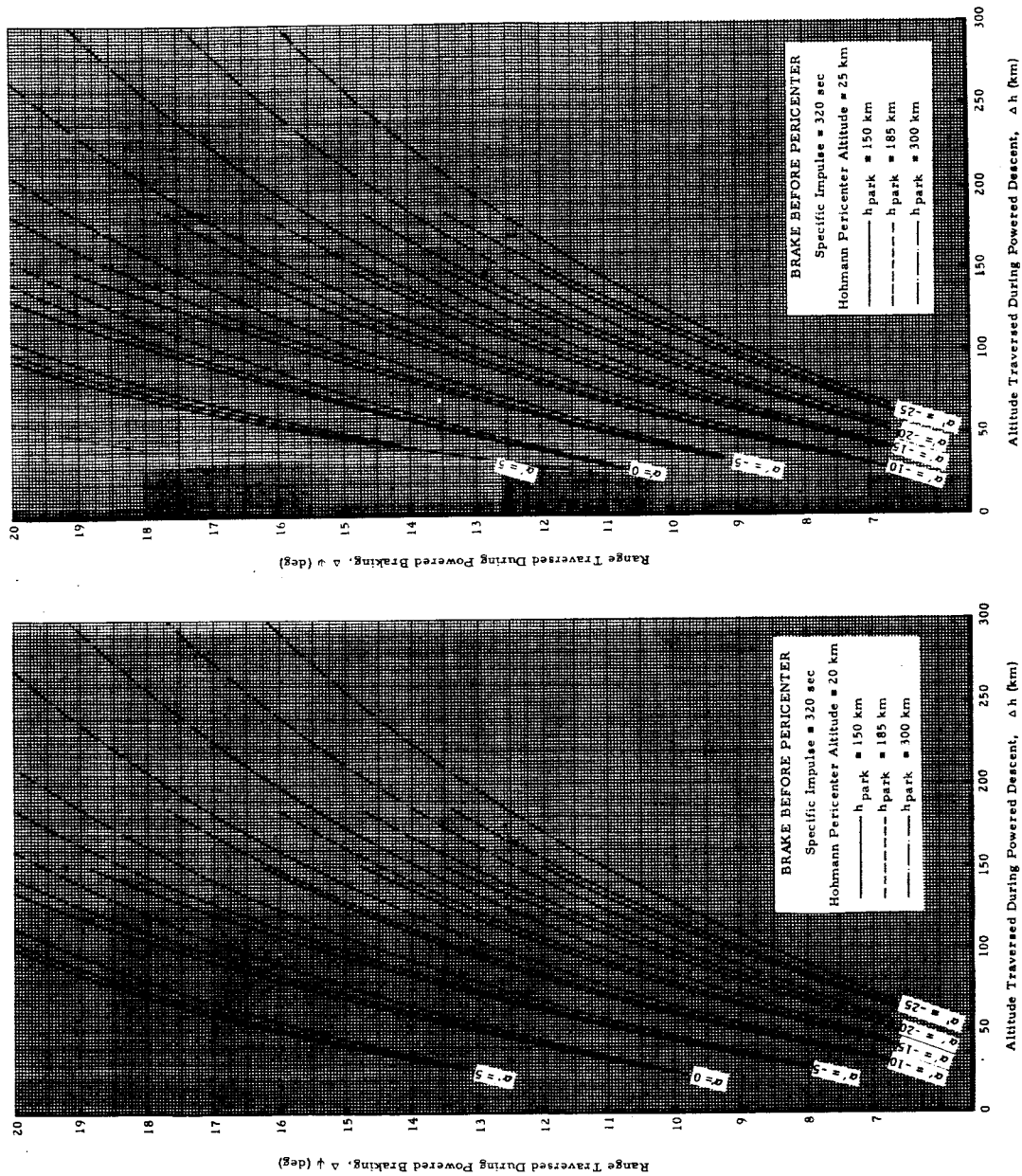


FIGURE 6 f. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

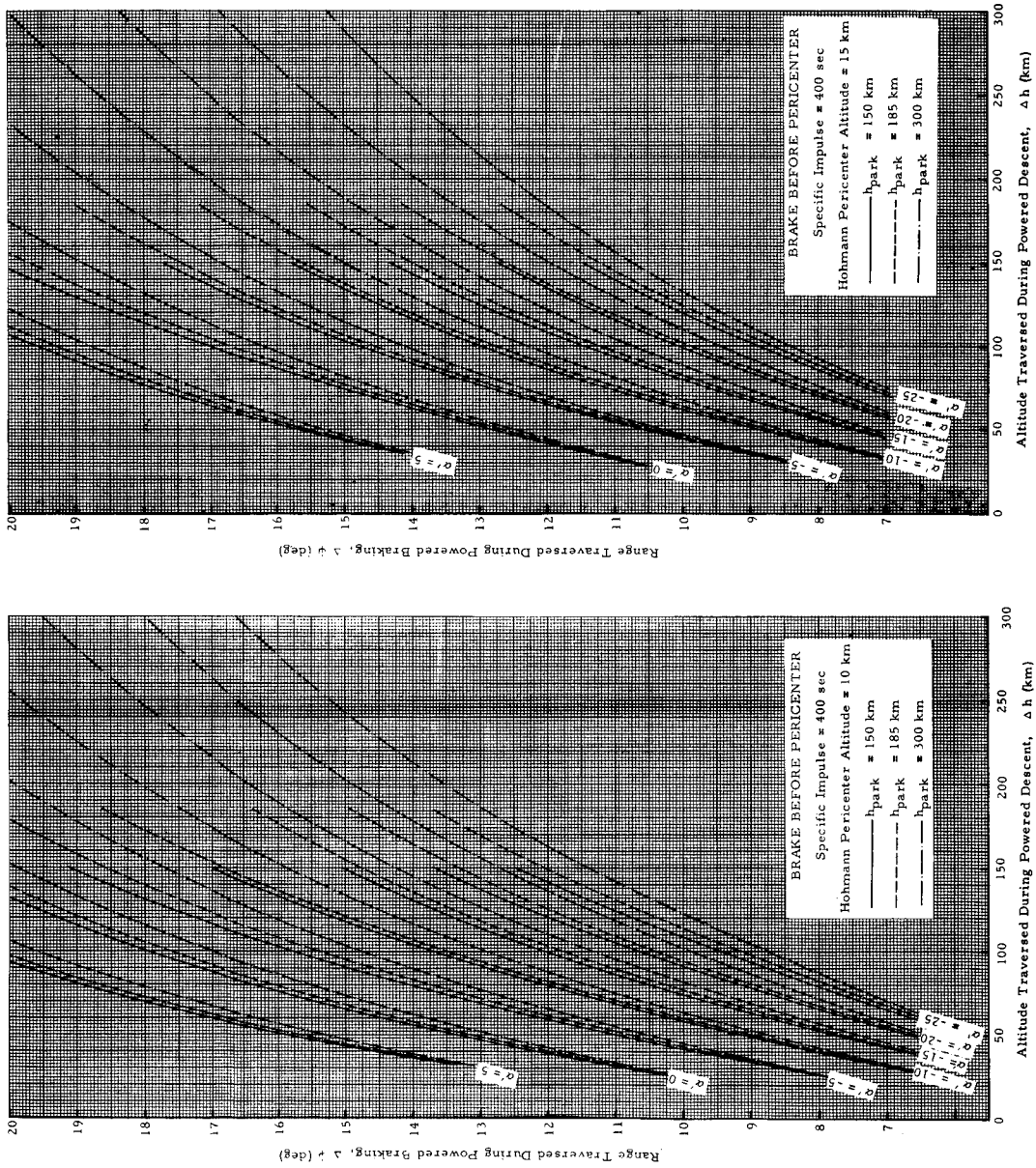


FIGURE 6 g. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

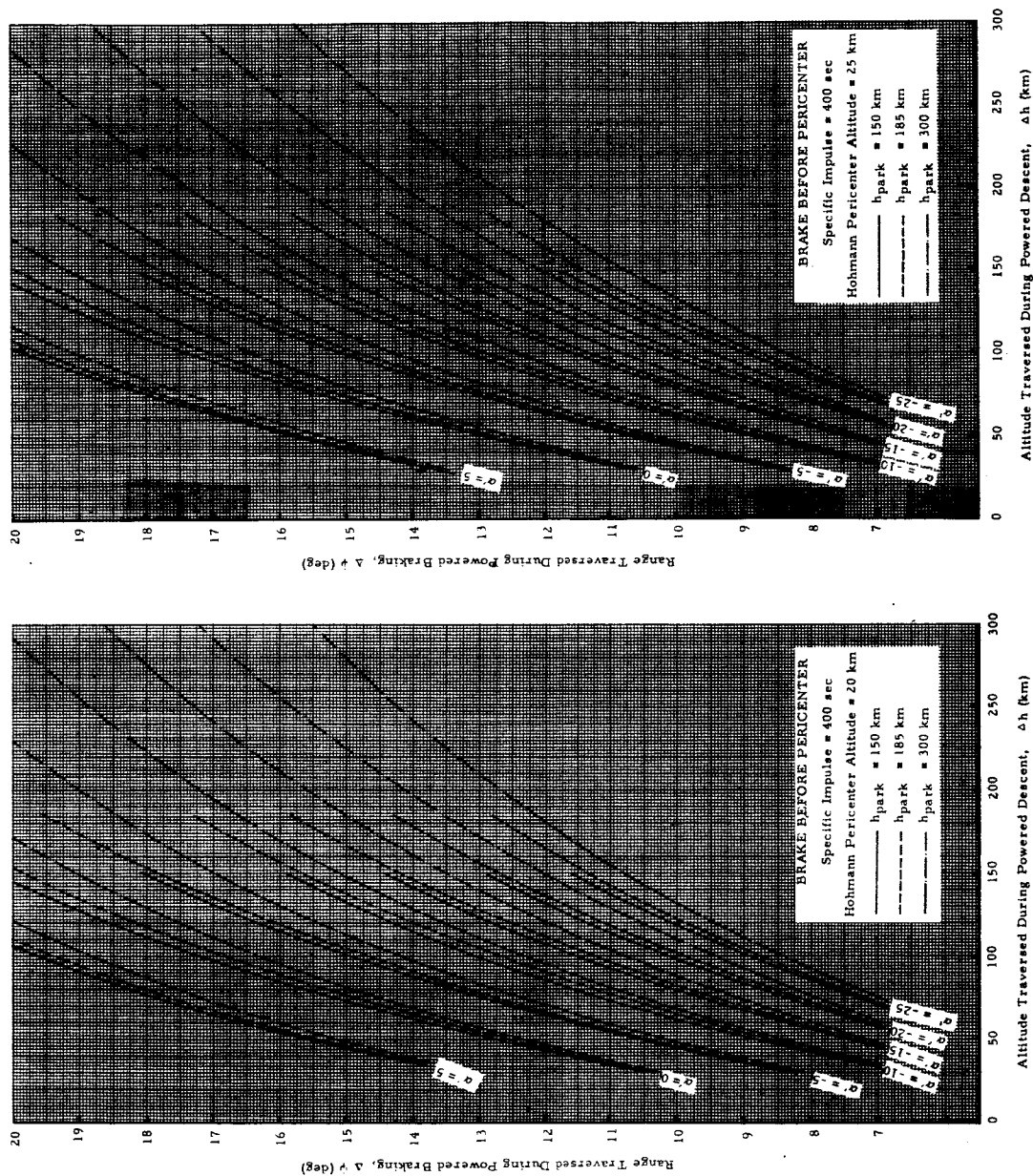


FIGURE 6 h. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

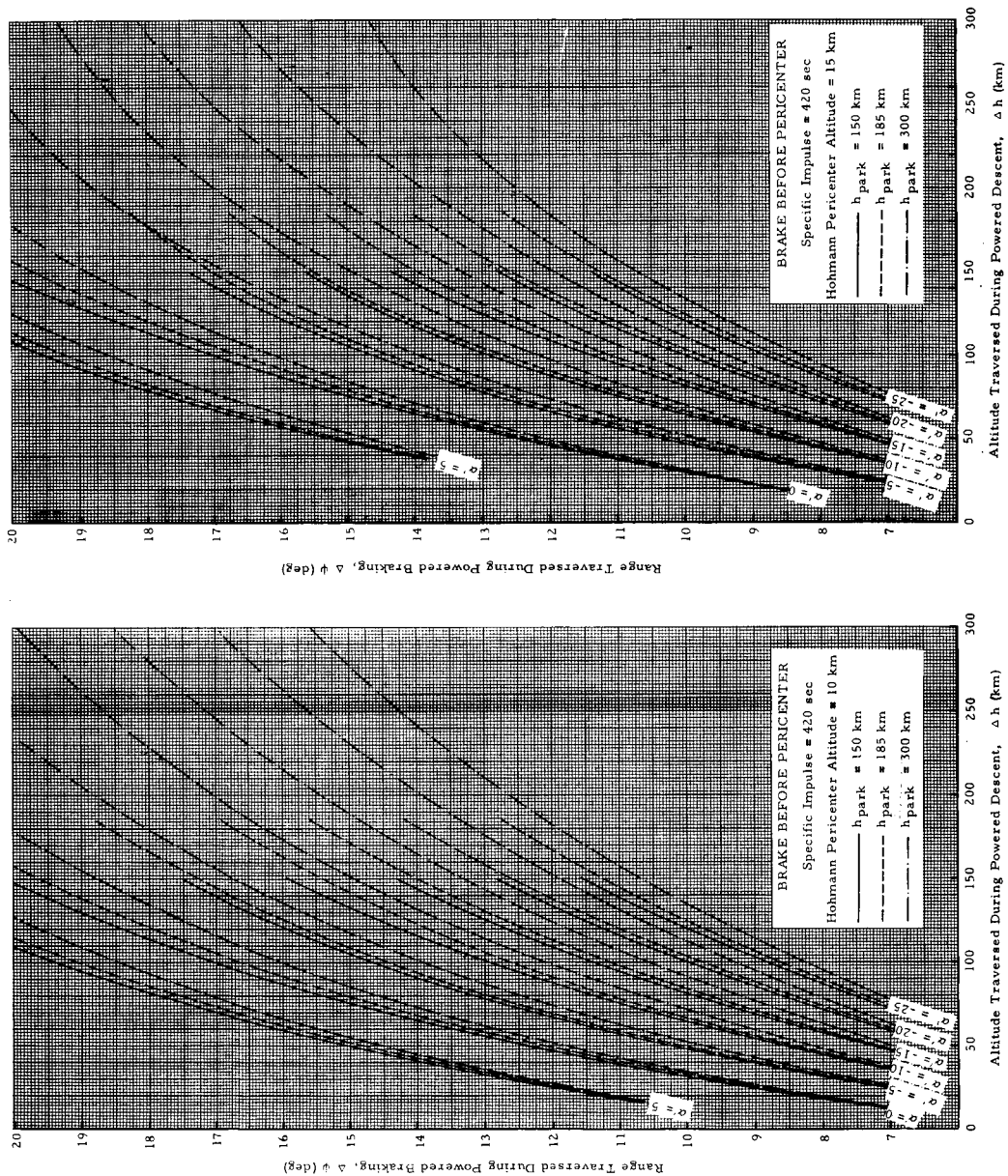


FIGURE 6 i. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

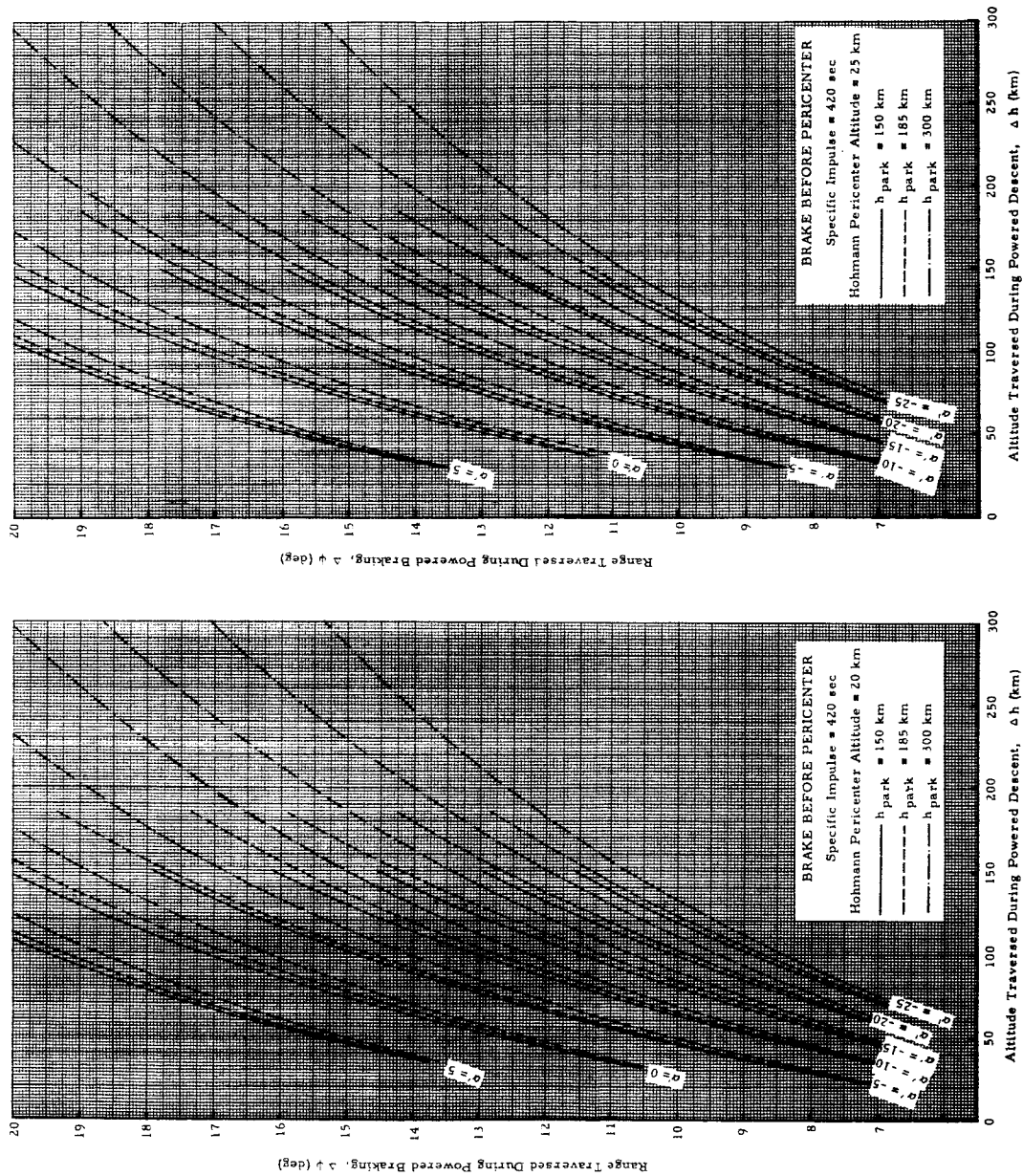


FIGURE 6 j. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

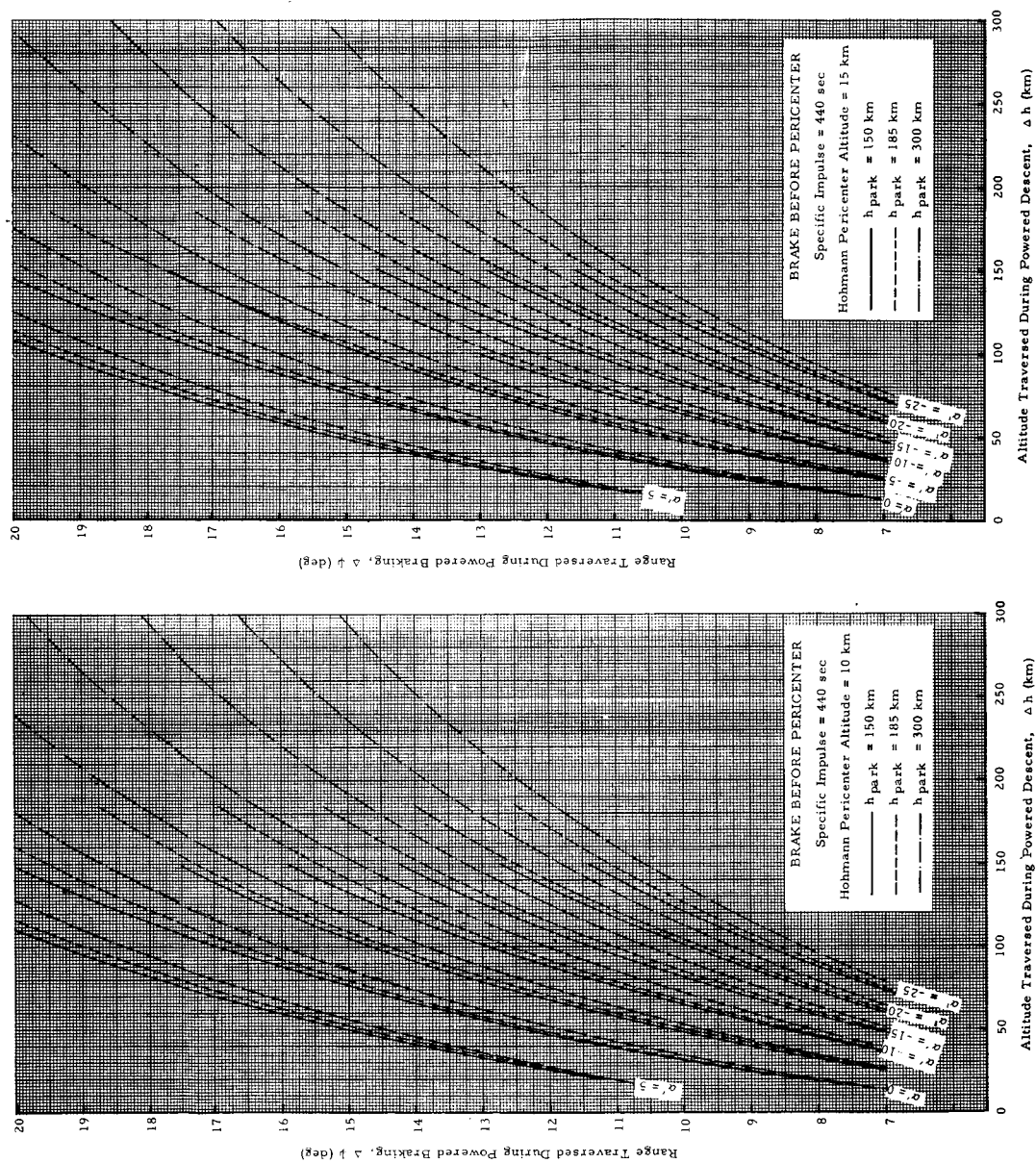


FIGURE 6 k. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

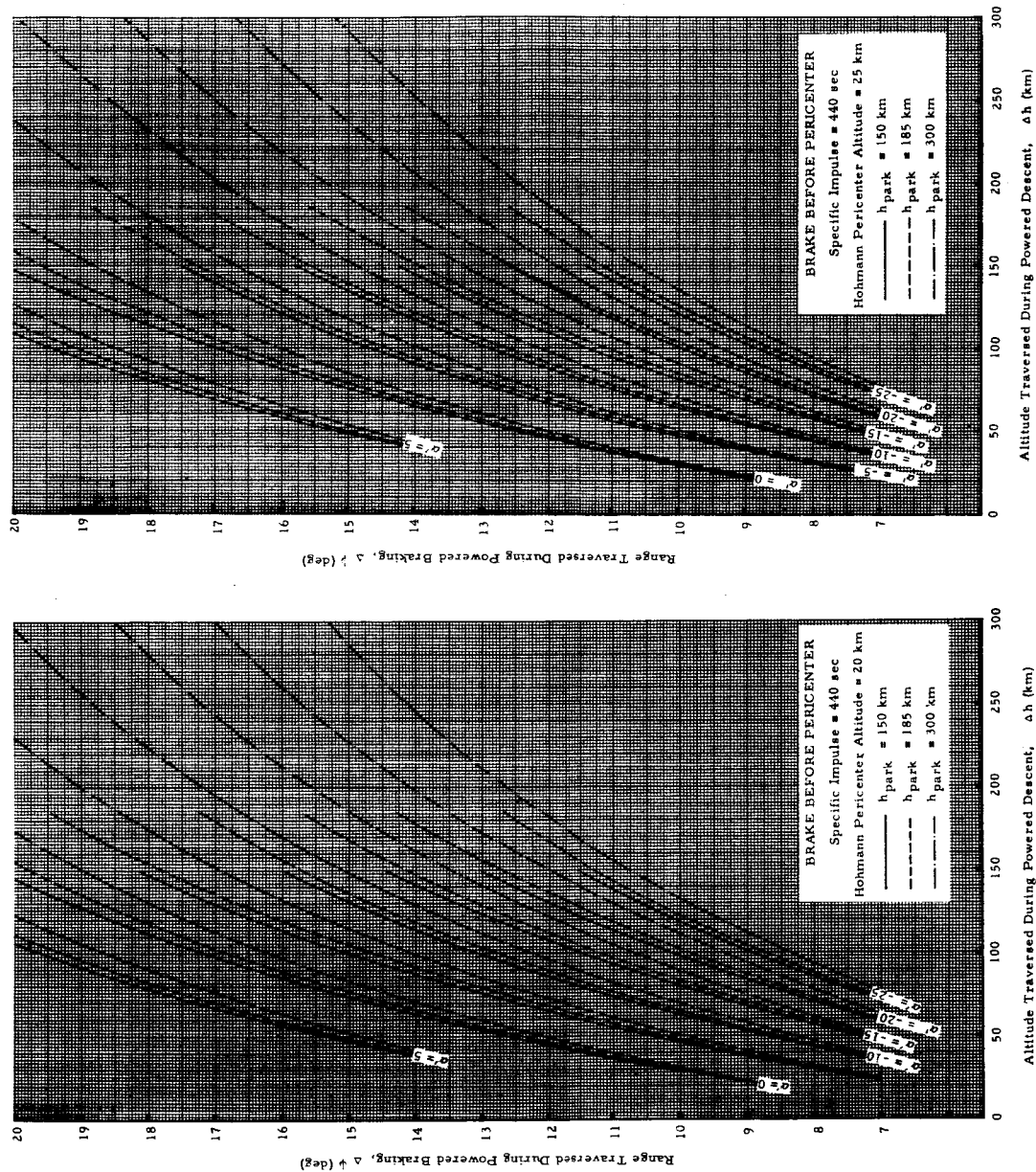


FIGURE 6 1. RANGE TRAVERSED DURING POWERED DESCENT BEFORE PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

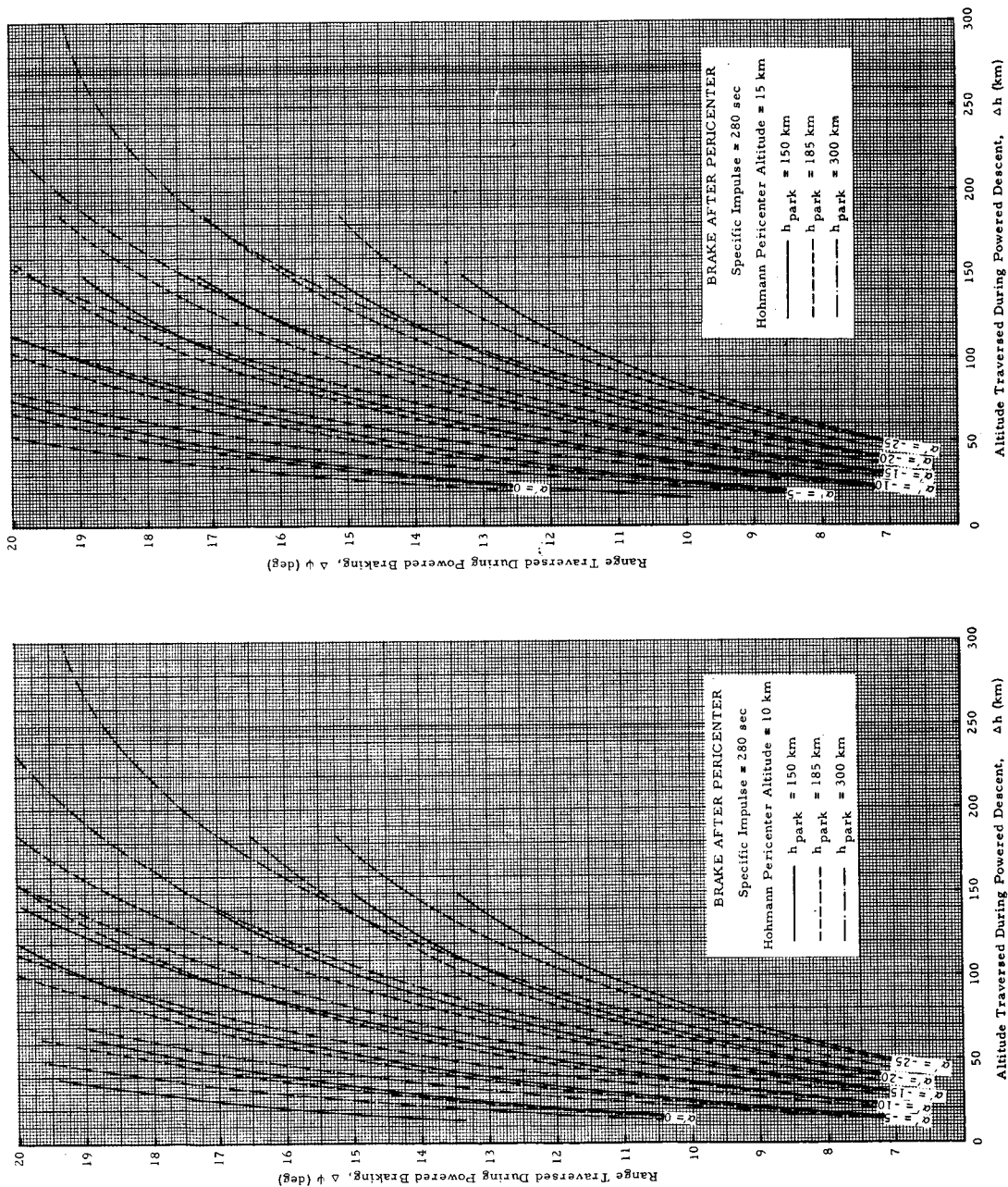


FIGURE 7 a. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

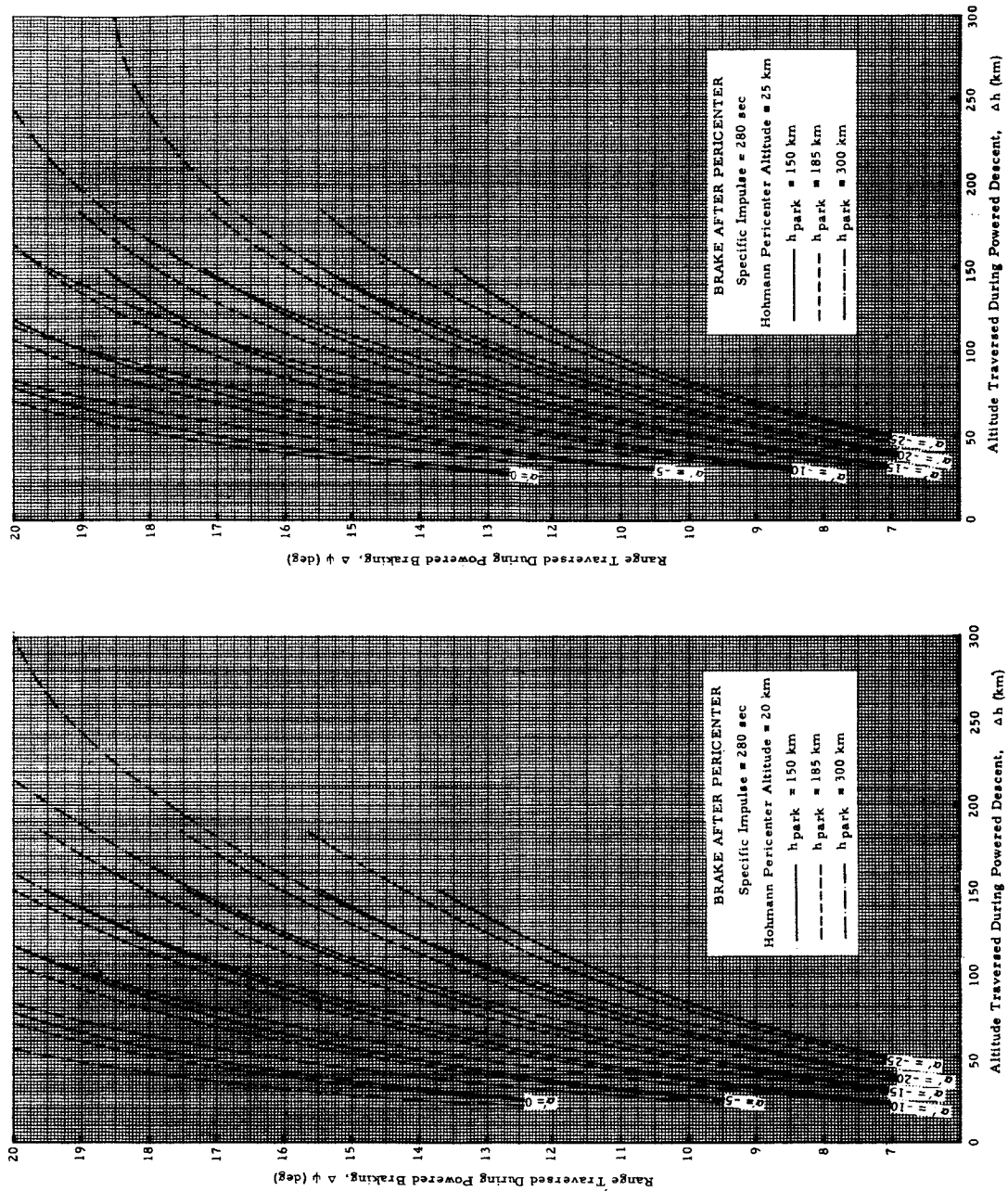


FIGURE 7 b. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 280 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

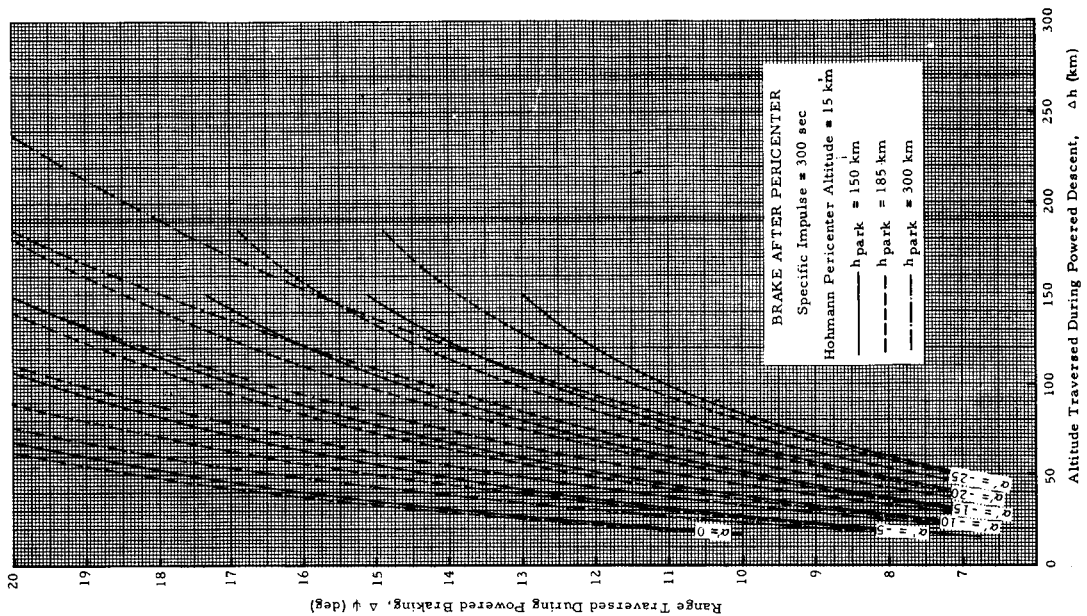
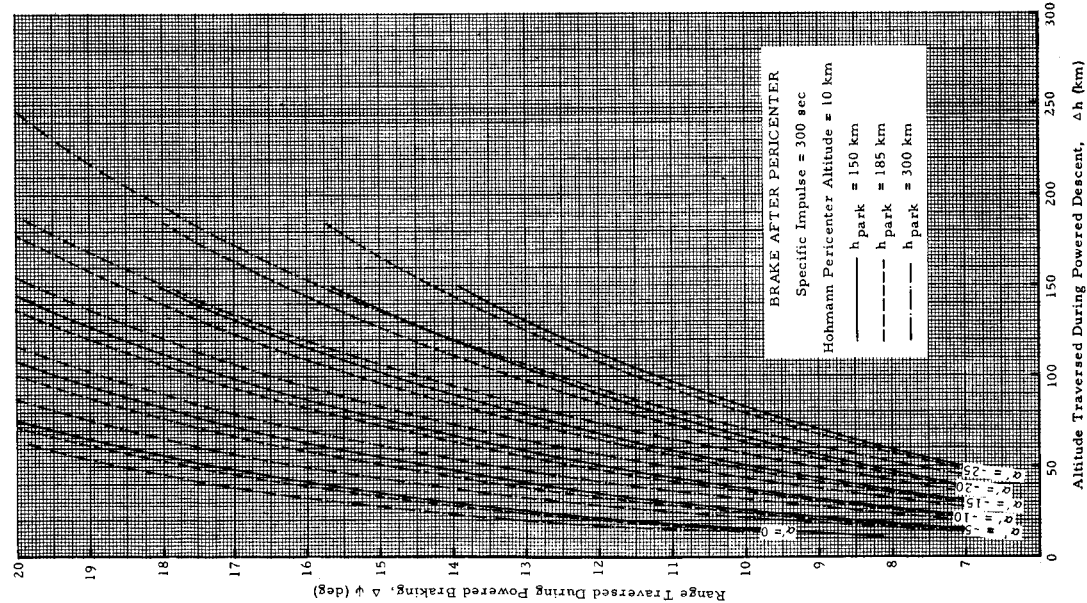


FIGURE 7 c. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

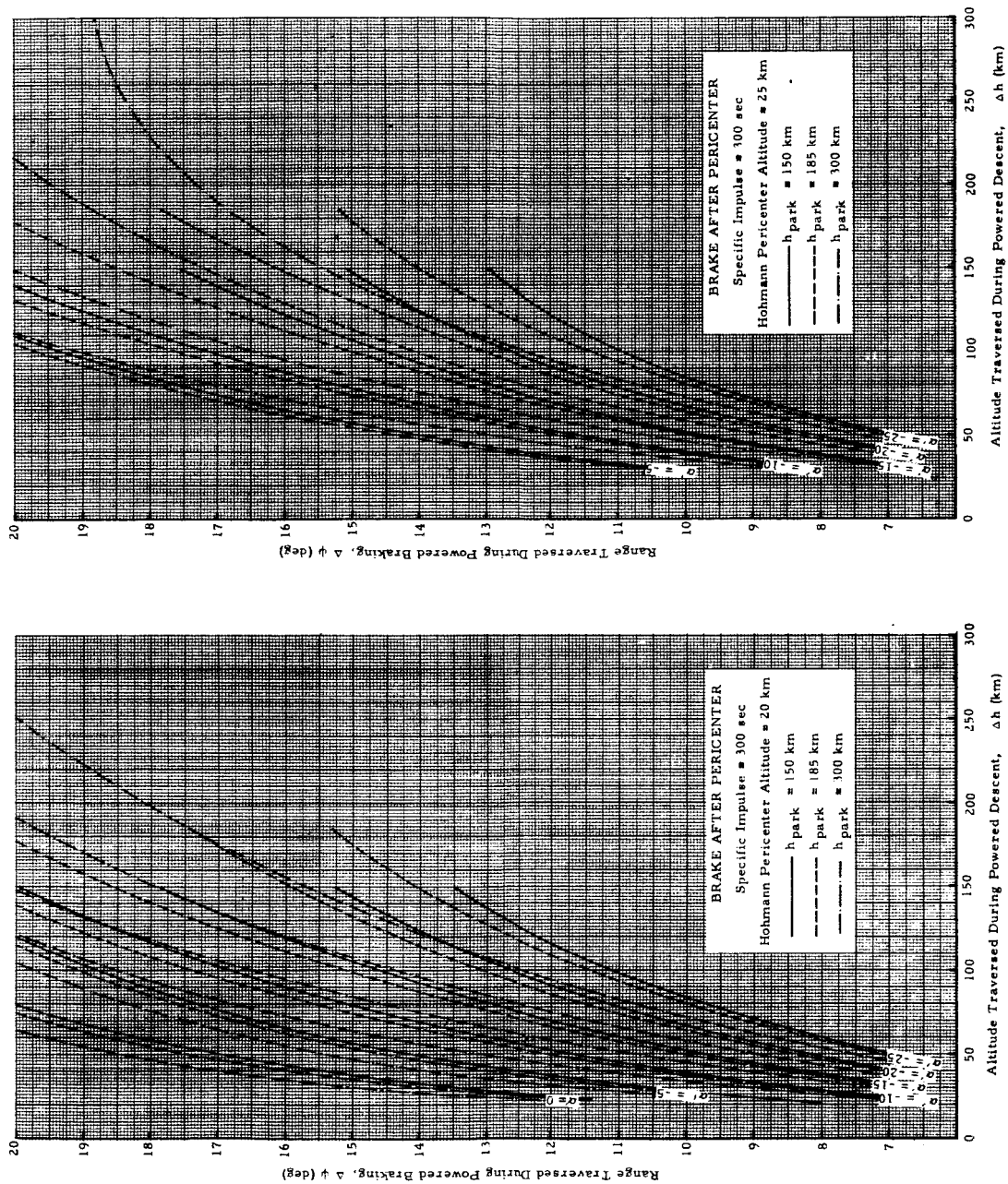


FIGURE 7 d. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 300 SECONDS

HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

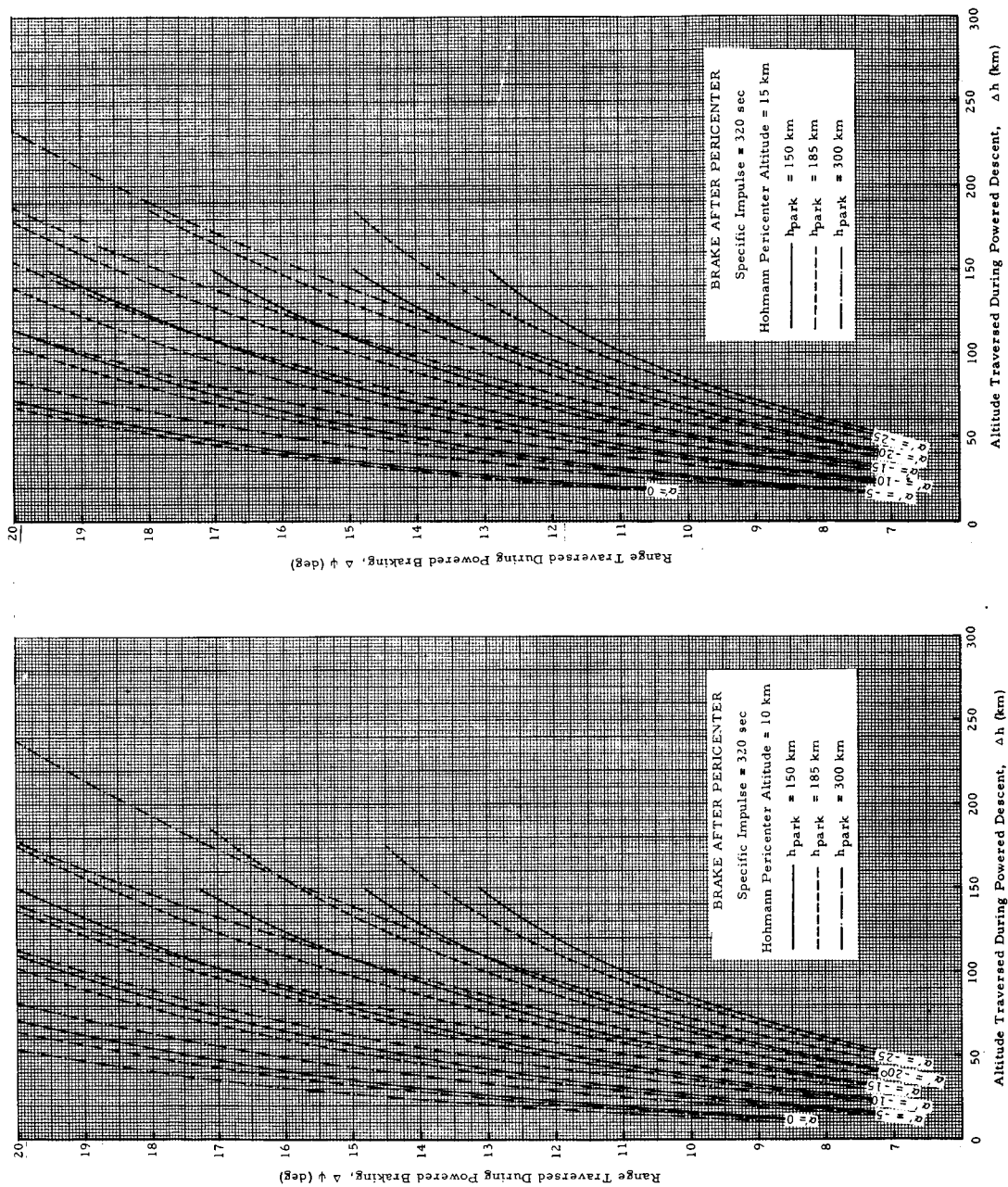


FIGURE 7 e. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

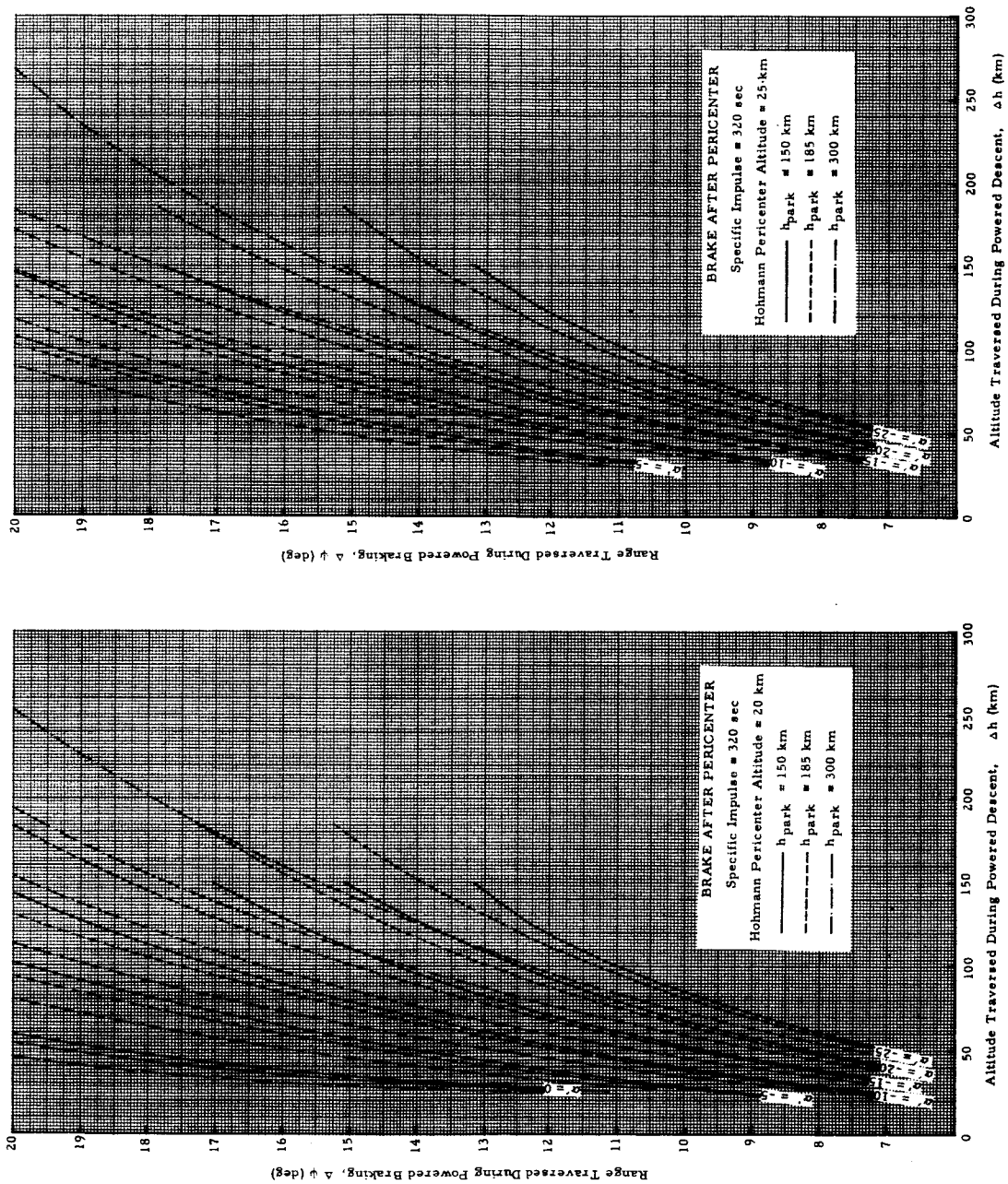


FIGURE 7 f. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 320 SECONDS
 HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

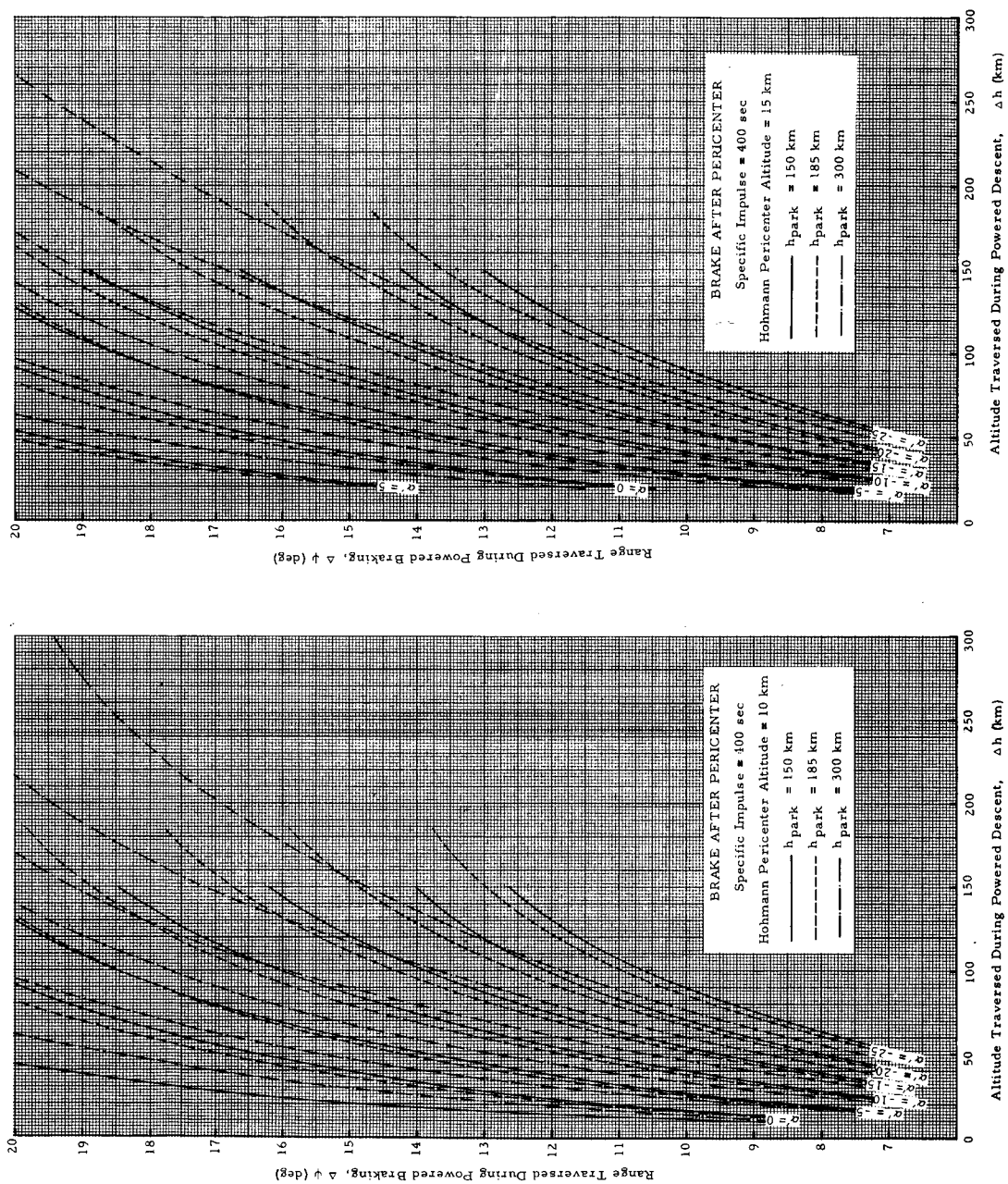


FIGURE 7 g. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

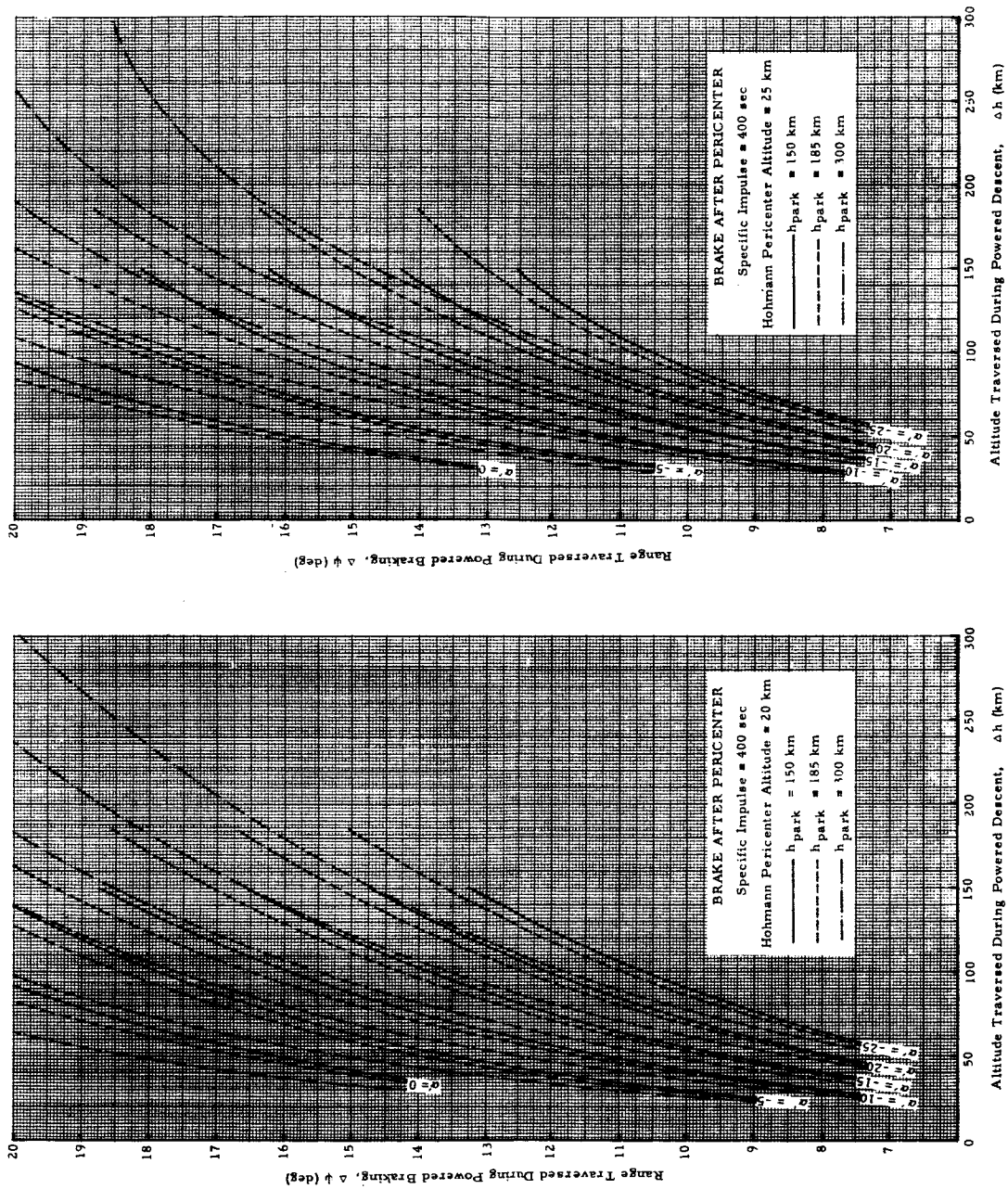


FIGURE 7 h. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 400 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

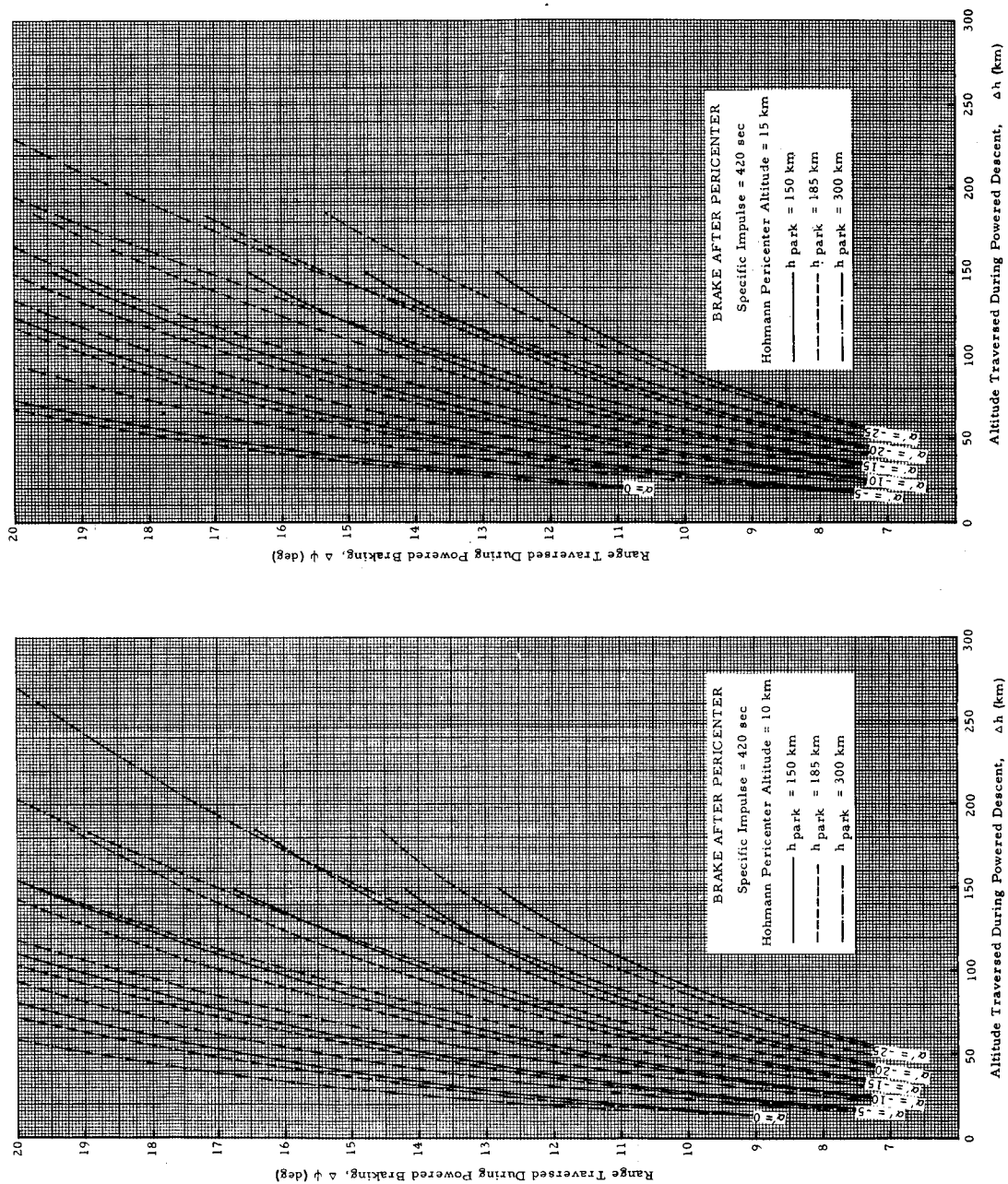


FIGURE 7 i. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

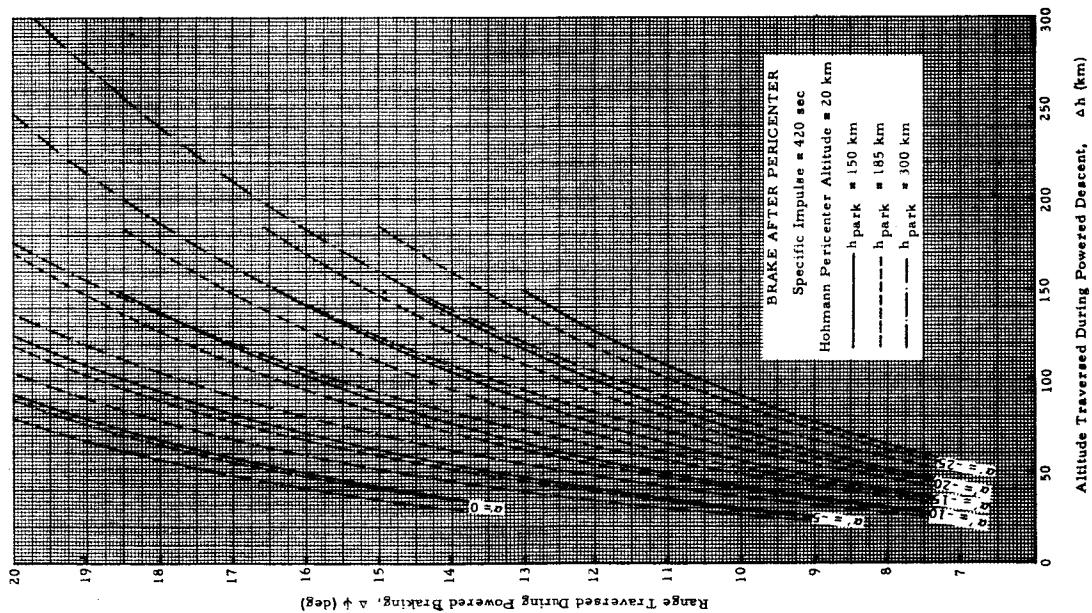
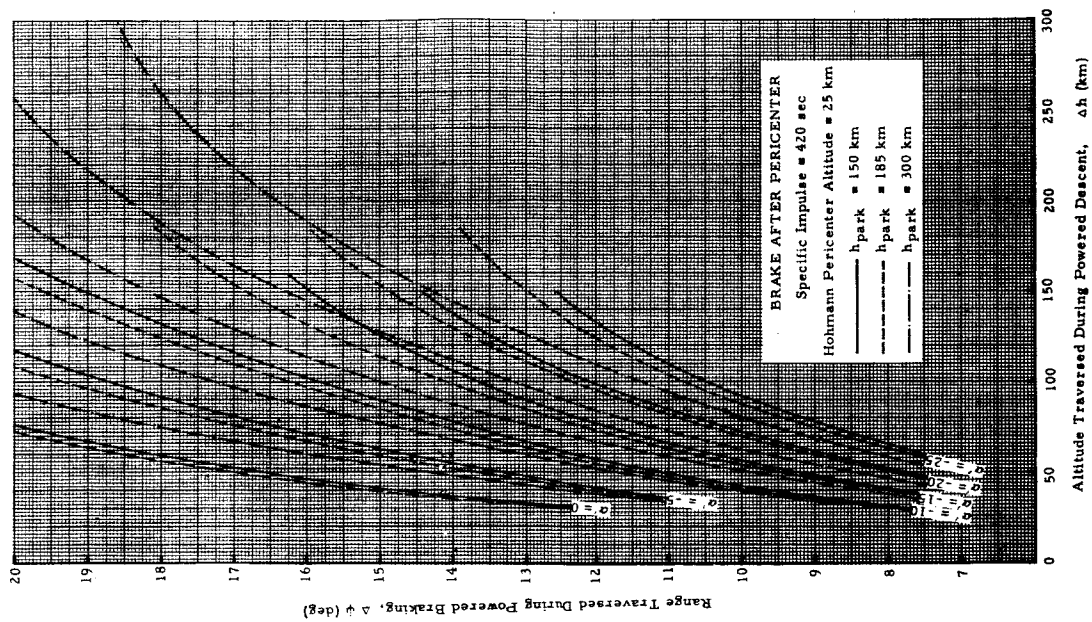


FIGURE 7 J. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 420 SECONDS
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

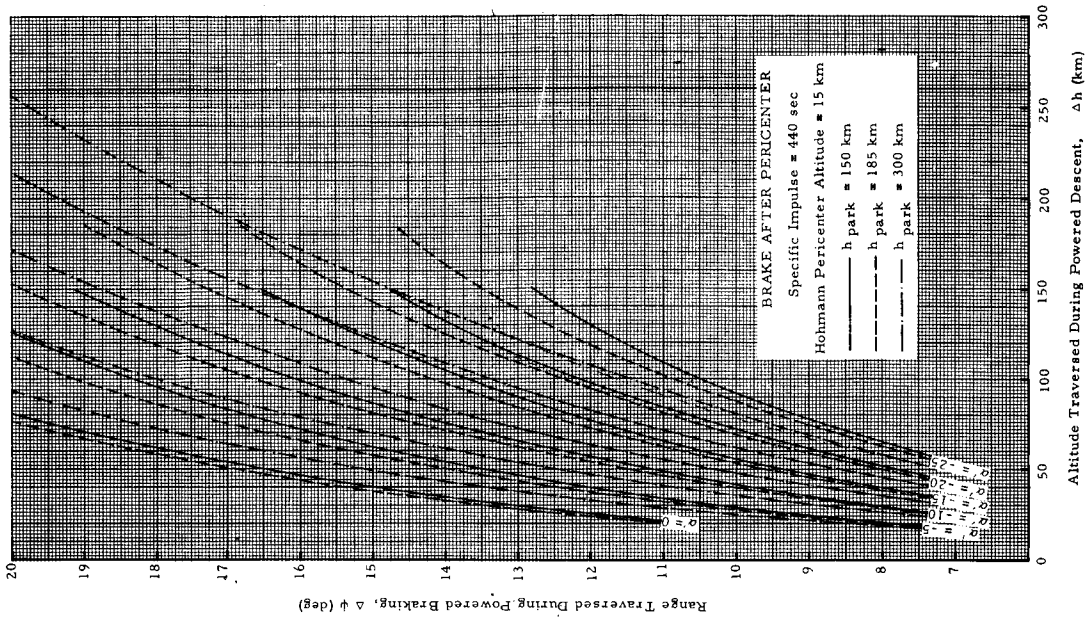
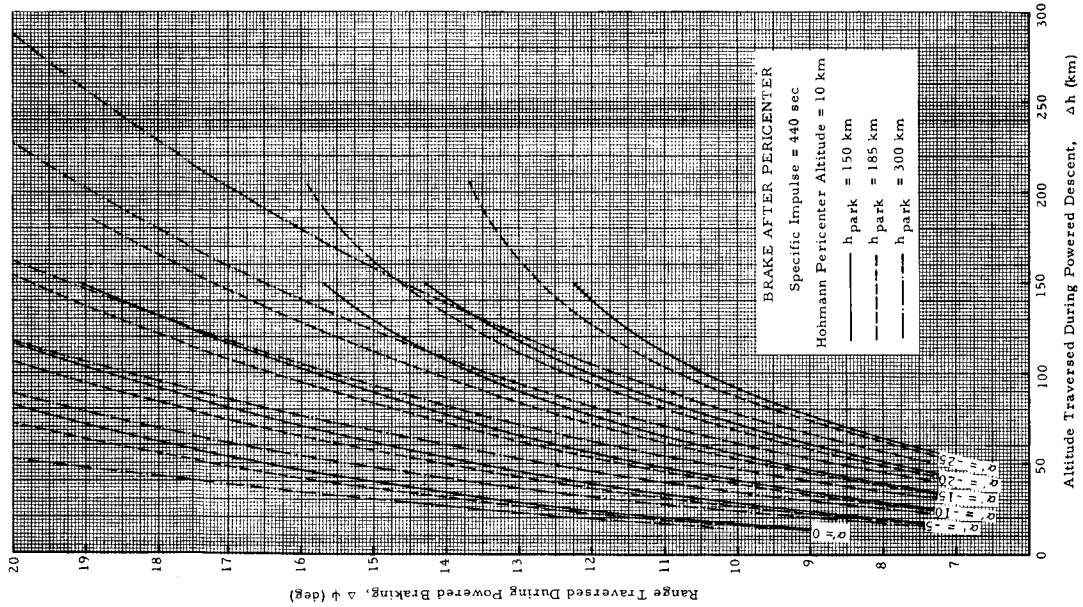


FIGURE 7k. RANGE TRAVERSED DURING POWERED DESCENT AFTER PERICENTER WITH SPECIFIC IMPULSE OF 440 SECONDS
HOHMANN PERICENTER ALTITUDE = 10 AND 15 KM

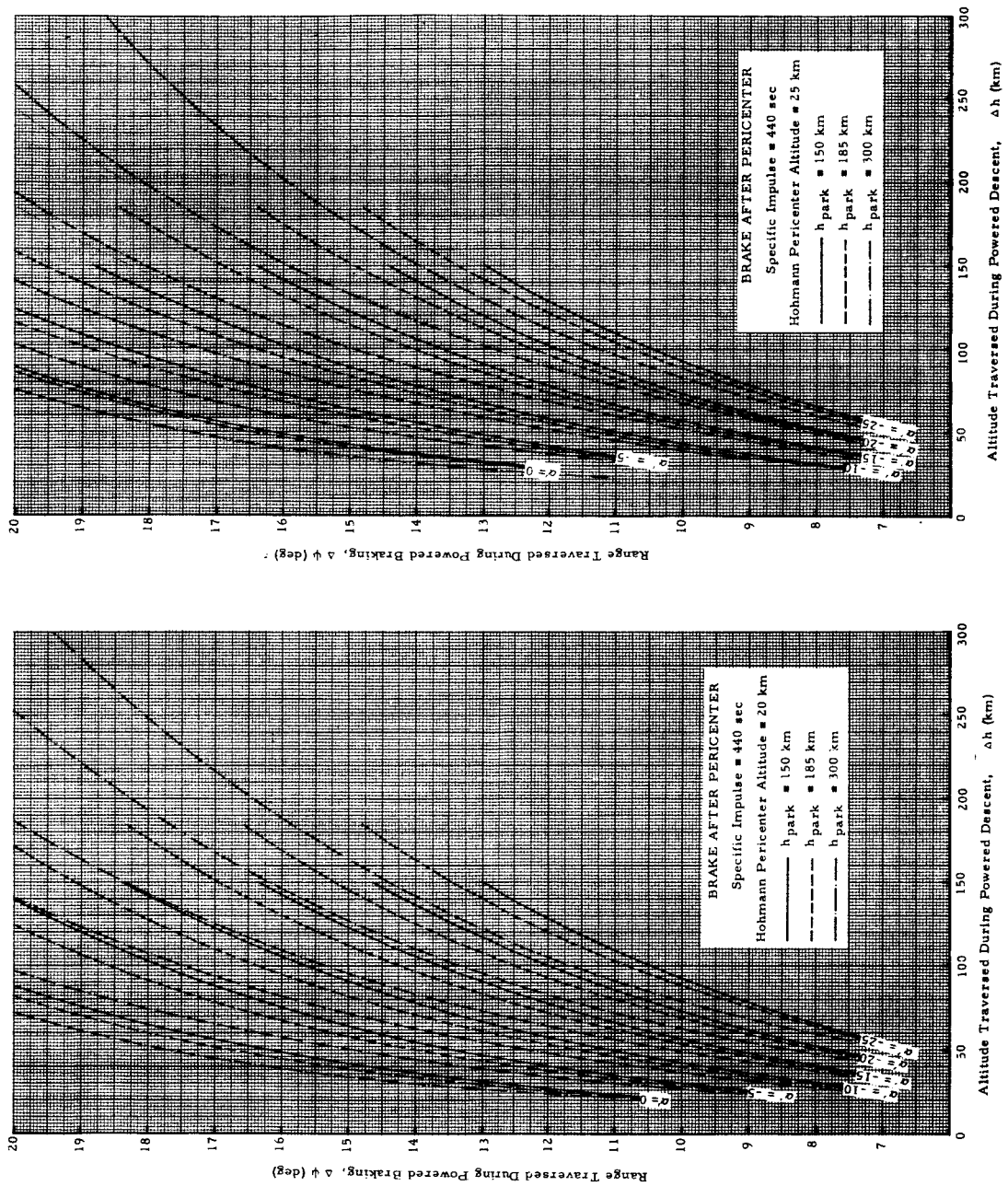


FIGURE 7.1.

RANGE TRAVERSED DURING POWERED DESCENT, Δh (km)
HOHMANN PERICENTER ALTITUDE = 20 AND 25 KM

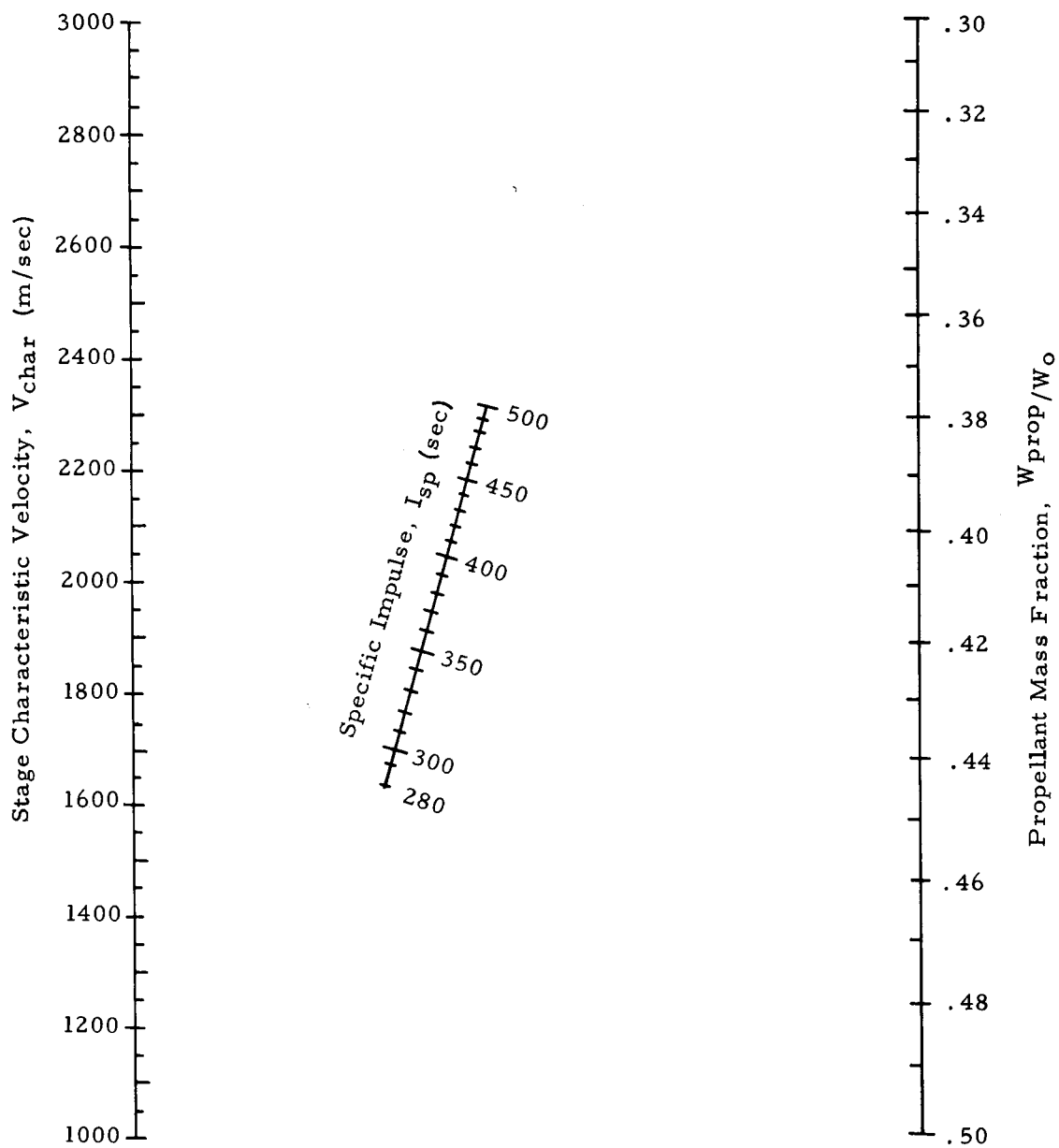


FIGURE 8. CHARACTERISTIC VELOCITY AS A FUNCTION OF PROPELLANT MASS FRACTION AND SPECIFIC IMPULSE

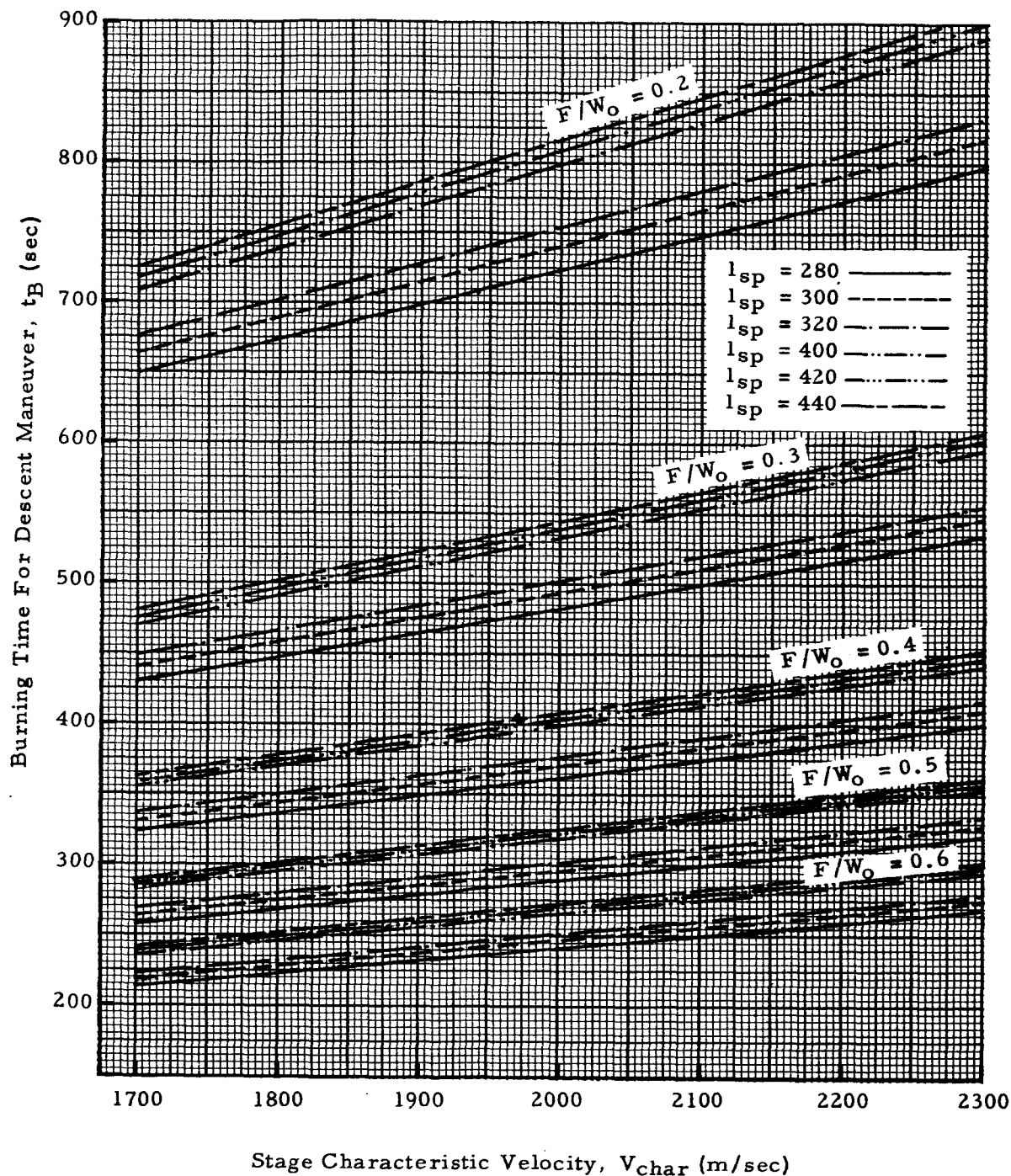


FIGURE 9. BURNING TIME FOR POWERED DESCENT AS A FUNCTION OF CHARACTERISTIC VELOCITY AND SPECIFIC IMPULSE

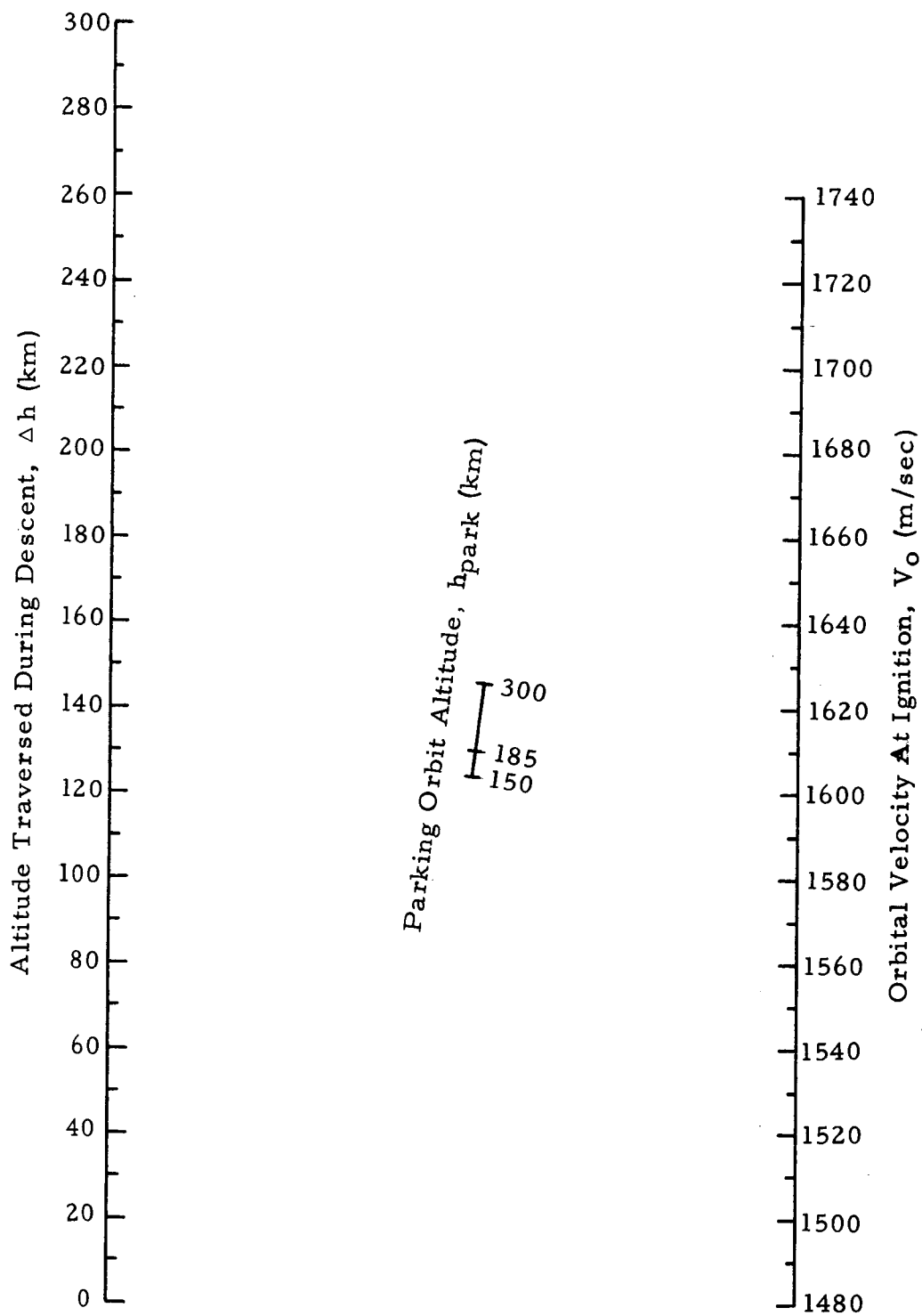


FIGURE 10. INITIAL ORBITAL VELOCITY ON HOHMANN TRANSFER ELLIPSE
AS A FUNCTION OF INITIAL ALTITUDE

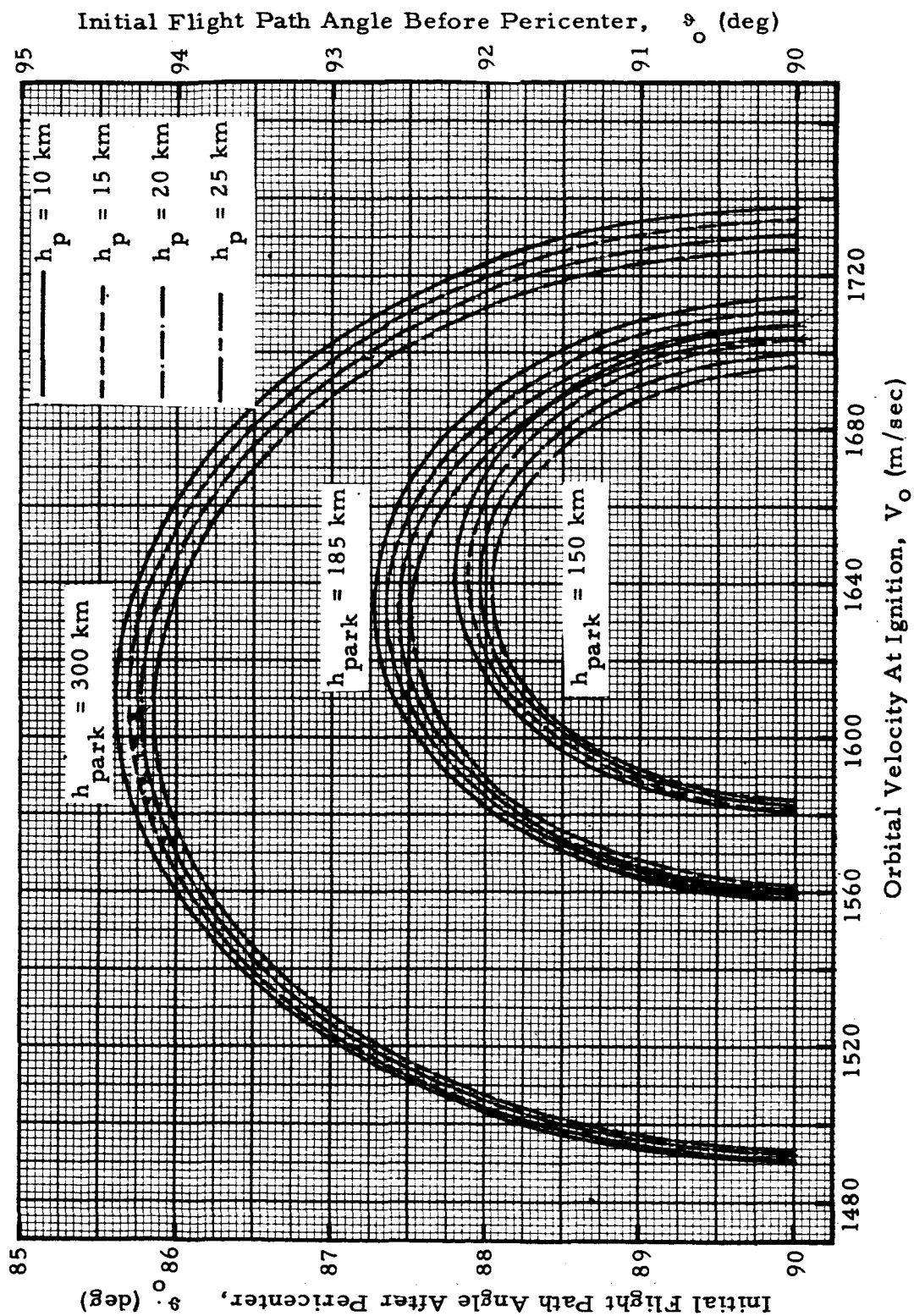


FIGURE 11. INITIAL FLIGHT PATH ANGLE ON HOHMANN TRANSFER ELLIPSE
AS A FUNCTION OF INITIAL VELOCITY

BIBLIOGRAPHY

Akridge, C. M. and Sam H. Harlin, Parametric Performance Analysis for Lunar Orbit Braking and Descent, NASA TN D-2643, February 1965.

Brehears, Robert R., Spacecraft Propulsion Requirements for Lunar Missions, JPL-TR-32-628, April 1, 1965.

Steiner, P., Lunar Powered Flight, STL Report No. 8976-0004-RU-000, August 10, 1961.

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