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RADIATION GEOMETRY FACTOR BETWEEN THE EARTH AND A SATELLITE

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SUMMARY

In solving thermal problems of satellites, the geometry factors must be known. A brief discussion of geometry factors, an analytical expression, and computer procedure is given.

Methods for computing these geometry factors from basic relations require lengthy computer programs with many inputs. A method was devised in the Space Thermodynamics Branch of the Research Projects Laboratory of MSFC for obtaining these factors for space thermal calculations from a subroutine similar to the common trigonometric subroutines. It requires only 600 octal locations. This is less than 10 percent of the total memory of the IBM 7094 Mod. II used by the Computation Laboratory. Maximum computer time required per factor obtained is .0015 seconds on the IBM 7094 Mod. II. This subroutine has been used for over two years without any apparent problems.

INTRODUCTION

The geometry factor, F, between two bodies of various geometrical configurations must be known in order to solve most heat problems encountered in space. A brief description of geometry factors follows.

The Stefan-Boltzmann radiation law (fourth power) relates the amount of radiative flux emitted from a black body to its absolute temperature. A non-black body radiates a fraction of the amount of energy flux radiated by a black body at the same temperature. This fraction is known as the total hemispherical emittance (or emissivity) of the non-black body at the given temperature.

Assuming opaque bodies and Lambert's cosine law, the angular distribution of the flux radiated from an elemental area on a body, (a), can be calculated. If another body, (b), is placed in the vicinity of (a), each elemental area on (b) has incident upon it some fraction of the flux from each elemental area on (a). It follows that the amount of incident flux absorbed on (b) can be calculated by a double integration. This amount is a fraction of the total flux emitted, and this fraction is called the geometry factor, F.

Reflected parallel radiation from an outside source, or albedo, also enters into orbital problems. Assuming diffuse reflection, an albedo-F can be obtained by a similar double integration. Except for the simplest geometrical configurations, the integrations for F and albedo-F must be carried out by numerical techniques.

A fact which is often overlooked is that, by using the reciprosity theorem, the $F_{A \to B}$ (the geometry factor from a body, A, to another body, B) can be calculated if the $F_{B \to A}$ (the geometry factor from the body, B, to the body, A) is known. Obviously, the simplest route in computation should always be chosen. For example, the F is much more easily calculated between the earth and a smaller object, such as a satellite, by thinking in terms of the satellite radiating to the earth instead of vice versa, although in practical problems the F is used to compute earth radiation to a satellite.

Convair Corporation performed the necessary integrations, previously mentioned, for several geometrical configurations. Their results were published in graphical form [1,2].

Presently, the Space Thermodynamics Branch of Research Projects Division is using these results for two of these geometrical configurations for both albedo and the so-called IR (earth's infrared), which is the earth's Stefan-Boltzmann radiation. The two configurations are: (1) a sphere (earth) and a flat plate (on a satellite), and (2) a sphere (earth) and a cylinder (on a satellite).

Since most thermal problems of orbiters require many tedious arithmetic operations, the IBM 7090 computer in the Computation Division has been utilized. An analytical expression for F as a function of the known geometrical parameters (such as attitude, etc.) is imperative for computing purposes. Calculating the factors from basic assumptions by numerical integration is too lengthy to use for a subroutine as visualized by R-RP-T. Since polynomials are very adaptive to computers in that they require a minimum of memory storage, minimum computing time, and are easily programmed, an emperical polynomial was generated with a "sum of the least squares polynomial curve fitting" routine at the Computation Laboratory.

PROCEDURE

A. F FOR EARTH'S IR TO CYLINDER

The following parameters are geometrically defined in figure 1 for IR between a planet and a cylinder:

r = altitude (km)

γ = attitude angle (deg)

Figure 2 shows the F versus altitude for a cylinder as plotted in [2] for several representative attitudes. Figure 3 shows the same curves as figure 2 on Cartesian coordinates over the range 182 km $\leq r \leq$ 3500 km for $\gamma = 0°, 20°, 40°, 60°$, and $90°$.

Letting F be a function of h for each curve in figure 3, a least square curve fit equation was obtained of degrees 2 through 9. The 5th degree equations were determined to be the smallest degree acceptable. Thus, for each γ value ($0°, 20°, 40°, 60°, 90°$), a 5th degree polynomial equation was obtained:

$$(F)_j = \sum_{i=0}^{5} B_{ij} \, r^i$$

where,

$j = 0$ when $\gamma = 0°$,

$j = 1$ when $\gamma = 20°$,

$j = 2$ when $\gamma = 40°$,

$j = 3$ when $\gamma = 60°$,

and $j = 4$ when $\gamma = 90°$.

The B_{ij}'s are listed in table I.

Other least square curve fit processes were used to obtain the B_{ij} as a function of γ. Of these, the 4th degree polynomial equations were best:

$$B_{ij} = \sum_{k=0}^{4} A_{ik} \, \gamma^k ,$$

with the A_{ij} given in table II. Hence,

$$F = \sum_{i=0}^{5} \sum_{k=0}^{4} A_{ik} \, \gamma^k \quad r^i \quad ,$$

for a cylinder $0° \leq \gamma \leq 90°$, since the cylinder is symmetrical about its axis. The above equations are not valid outside the limits $0° \leq \gamma \leq 90°$ and $182 \text{ km} \leq r \leq 3500 \text{ km}$.

B. F FOR EARTH'S IR TO FLAT PLATE

Similar figures (4, 5 & 6) and tables (III & IV) are shown for a flat plate radiating on one side. The resulting equation for the F of a flat plate is:

$$F = \sum_{i=0}^{4} \sum_{k=0}^{7} A_{ik} \gamma^{k} r^{i}$$

where $0° \leq \gamma \leq 110°$ and $182 \text{ km} \leq r \leq 3500 \text{ km}$.

Since γ can range from 0° to 180° for a flat plate, and the case 110° to 180° is not covered in the polynomial, most of the F's in this range are zero for all practical purposes. The non-zero F's are obtained by a linear interpolation included in the subroutine for F. The interpolation routine is not as adequate as desired, due to the fact that F is not a linear function of γ or h and $F \to 0$ at different h values for a given γ. This fact makes it necessary to interpolate differently over specified increments of altitude. This causes a discontinuity in interpolative values of F at the value of h where any two interpretative groups are separated. A discontinuity (less than $\pm 3\% \ F_{max}$) in computation occurs upon crossing one of the boundaries. It should be noted that this occurs when $0 < F < .2$.

C. ALBEDO-F FOR BOTH CYLINDER AND FLAT PLATE

Figures 7 and 8 show a graphical definition of the parameters necessary to compute the albedo-F.

The albedo-F is not only a function of parameters γ and h, but of θ_s and ϕ_c also. These angles did not enter into the IR case because of symmetry. Figures 9 through 18 (taken from [2]) are typical examples of graphs for albedo-F.

Intuitively, the dependence of the albedo-F upon θ_s is thought to be approximately proportional to $\cos \theta_s$ and that $F \cong$ albedo-F when $\theta_s = 0$. Also, albedo-F is a week function of ϕ_c. These assumptions are illustrated in figures 17 and 18 where the solid line curves are typical integrated curves and the dotted line curves are the corresponding curves obtained by using the above assumption. Hence,

$$\text{albedo-F} \cong F \cos \theta_s, \text{ if } \cos \theta > 0$$

and $$\text{albedo-F} = 0, \text{ if } \cos \theta \leq 0 .$$

This subroutine is coded to calculate F for the flat plate or the cylinder. Only F is in the output; if the albedo-F is desired, one must multiply by $\cos \theta_s$.

CONCLUSION

As far as the IR is concerned, there is one basic type of error (neglecting the interpolation routine). Assuming the data from the Convair reports to be valid, this is the F obtained for values of γ not used in the curve fitting process. Since the B_{ij} and A_{ij} are continuous, the error is estimated to be within ±2% of F_{max}. Over an entire orbit, it is believed that these errors will usually have a cancelling effect. For the albedo-F the same errors occur as in IR, plus an error, brought about by using the assumption albedo-$F \cong F \cos \theta_s$, when $\cos \theta_s > 0$ and albedo-F = 0, when $\cos \theta_s \leq 0$, which is estimated to be less than ± 5% of F_{max}.

Methods for computing these geometry factors from basic relations require lengthy computer programs with many inputs. A method was devised in R-RP-T for obtaining these factors for space thermal calculations from a subroutine similar to the common trigonometric subroutines. It requires only 600 octal locations. This is less than 10 percent of the total memory of the IBM 7094 Mod. II used by the Computation Laboratory. Maximum computer time required per factor obtained is .0015 seconds on the IBM 7094 Mod. II. This subroutine has been used for over two years without any apparent problems.

In making typical computations of the radiation, it must be remembered that the factors are defined such that one uses the actual area of one side of the plate and the projected area (length x diameter) of the cylinder.

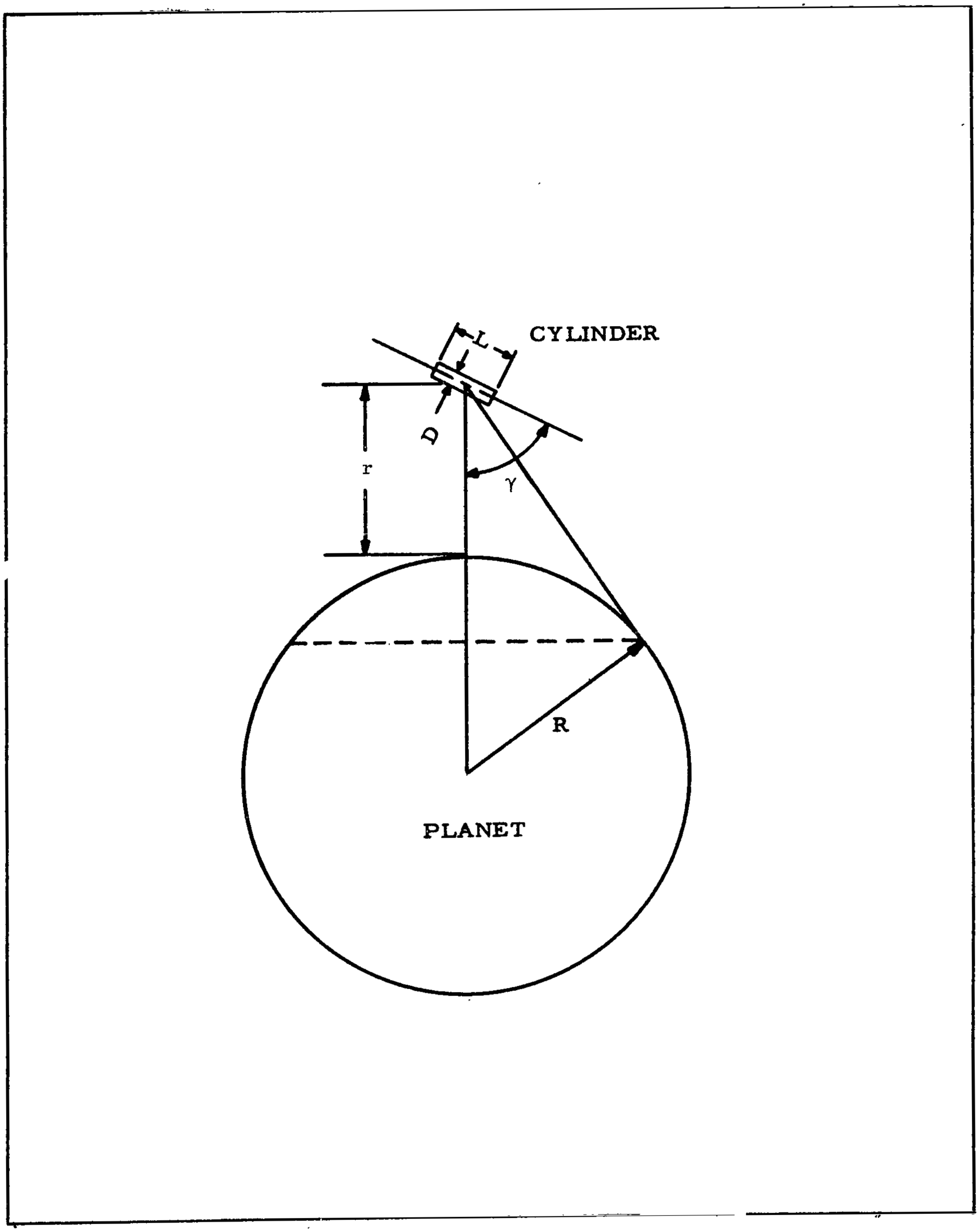

FIGURE 1. GEOMETRY FOR PLANETARY THERMAL RADIATION TO A CYLINDER

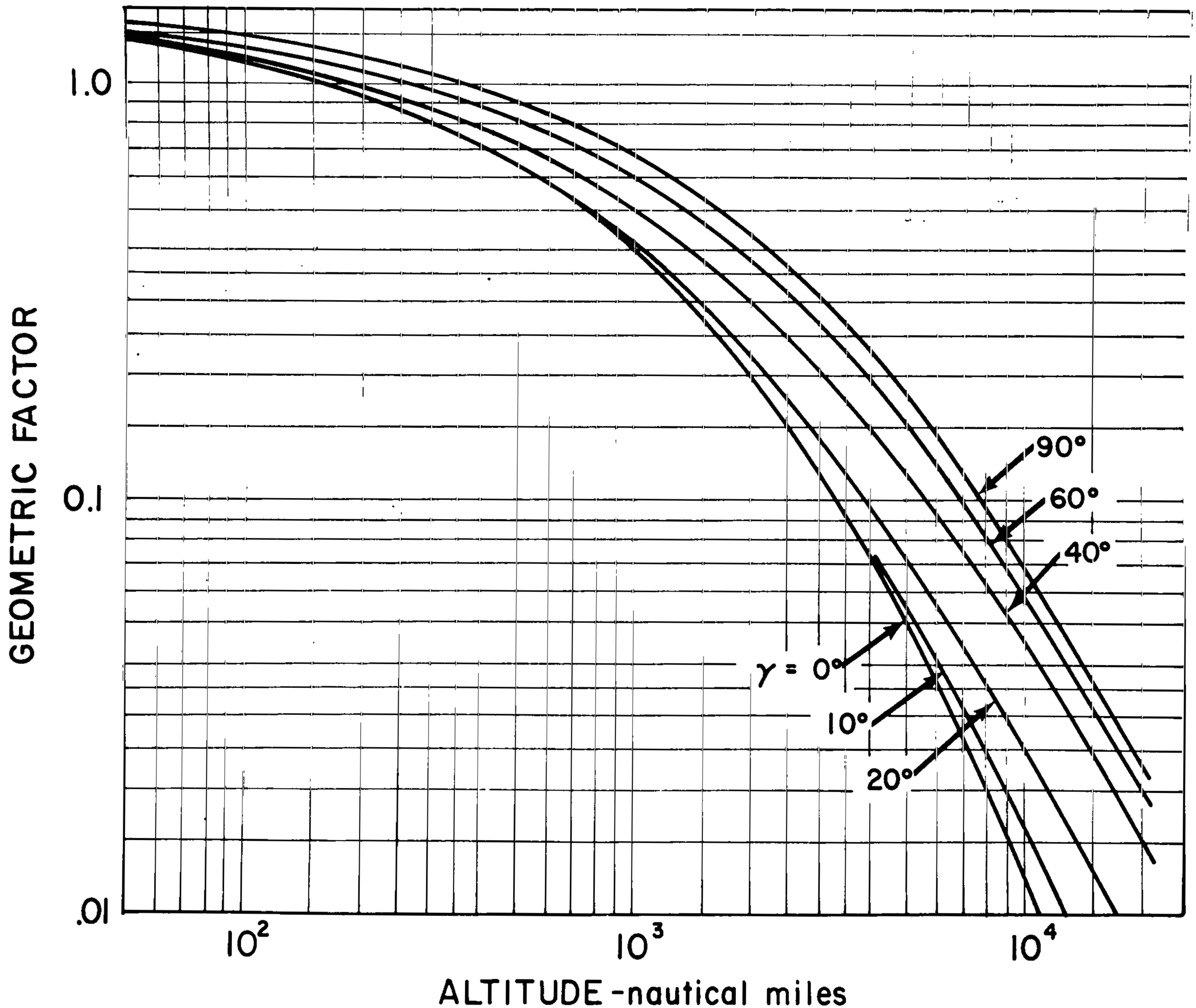

FIGURE 2. GEOMETRIC FACTOR FOR EARTH THERMAL RADIATION INCIDENT TO A CYLINDER VS. ALTITUDE, WITH ATTITUDE ANGLE AS A PARAMETER

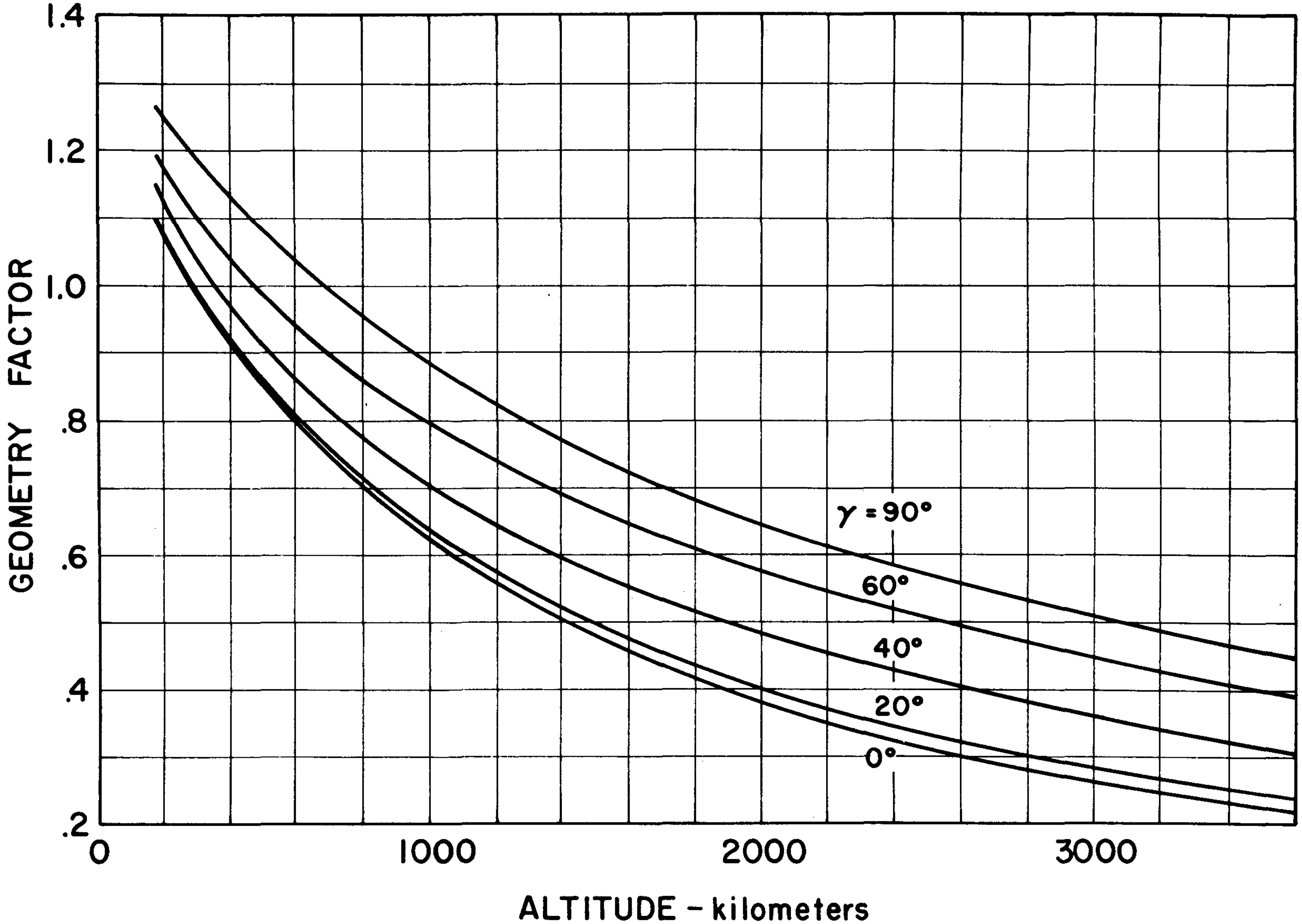

FIGURE 3. GEOMETRY FACTOR VS. ALTITUDE FOR IR TO A CYLINDER

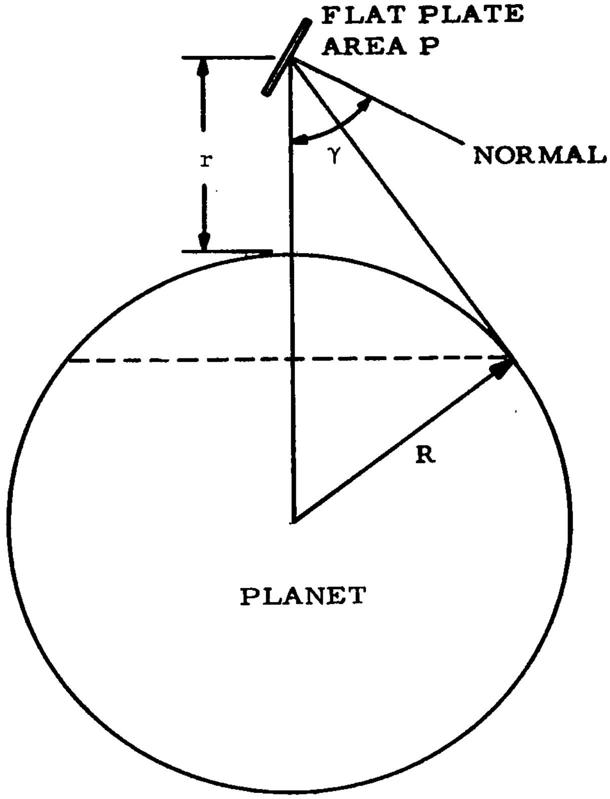

FIGURE 4. GEOMETRY FOR PLANETARY THERMAL RADIATION TO A FLAT PLATE

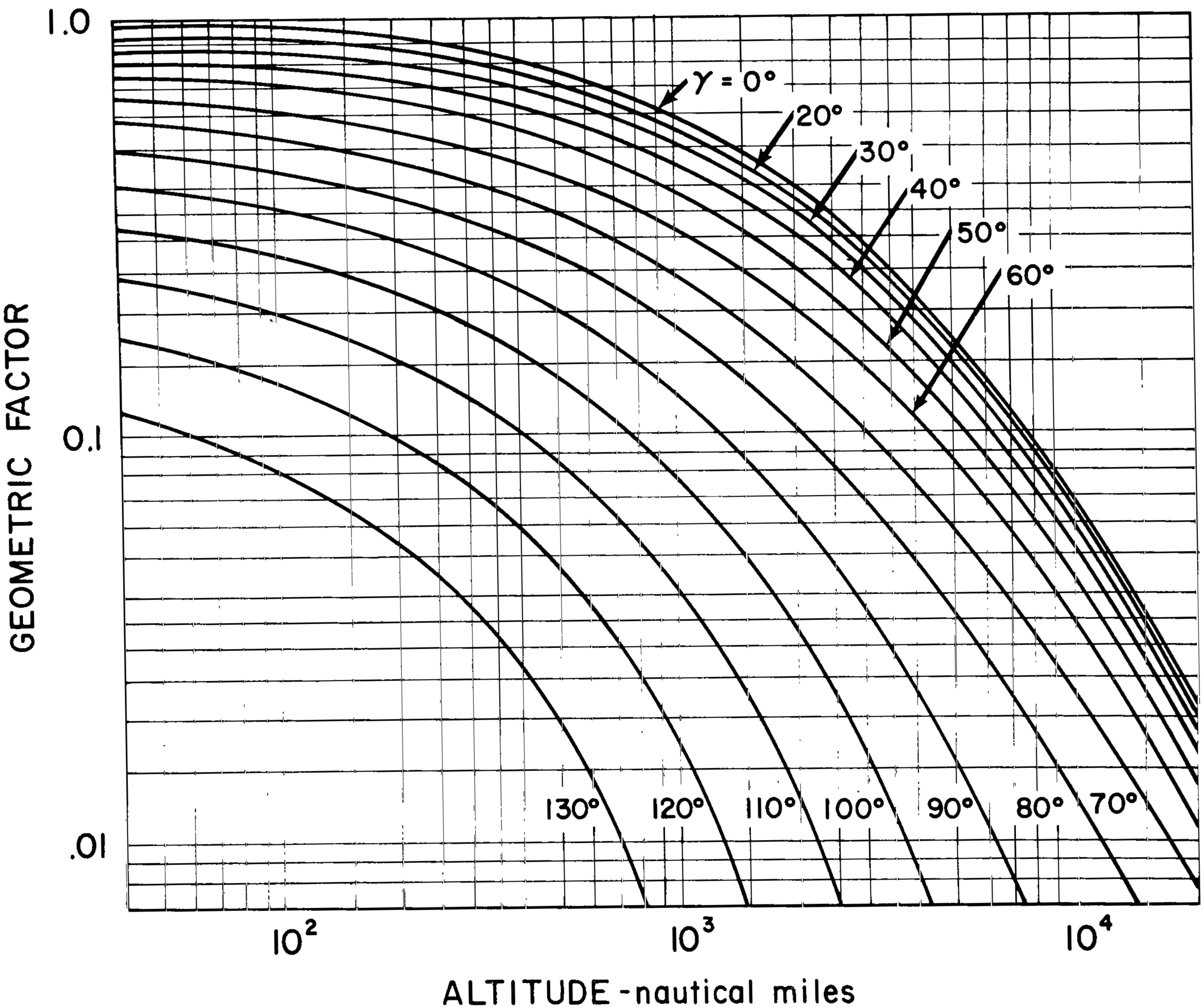

FIGURE 5. GEOMETRIC FACTOR FOR EARTH THERMAL RADIATION INCIDENT TO A FLAT PLATE VS. ALTITUDE WITH ATTITUDE ANGLE AS A PARAMETER

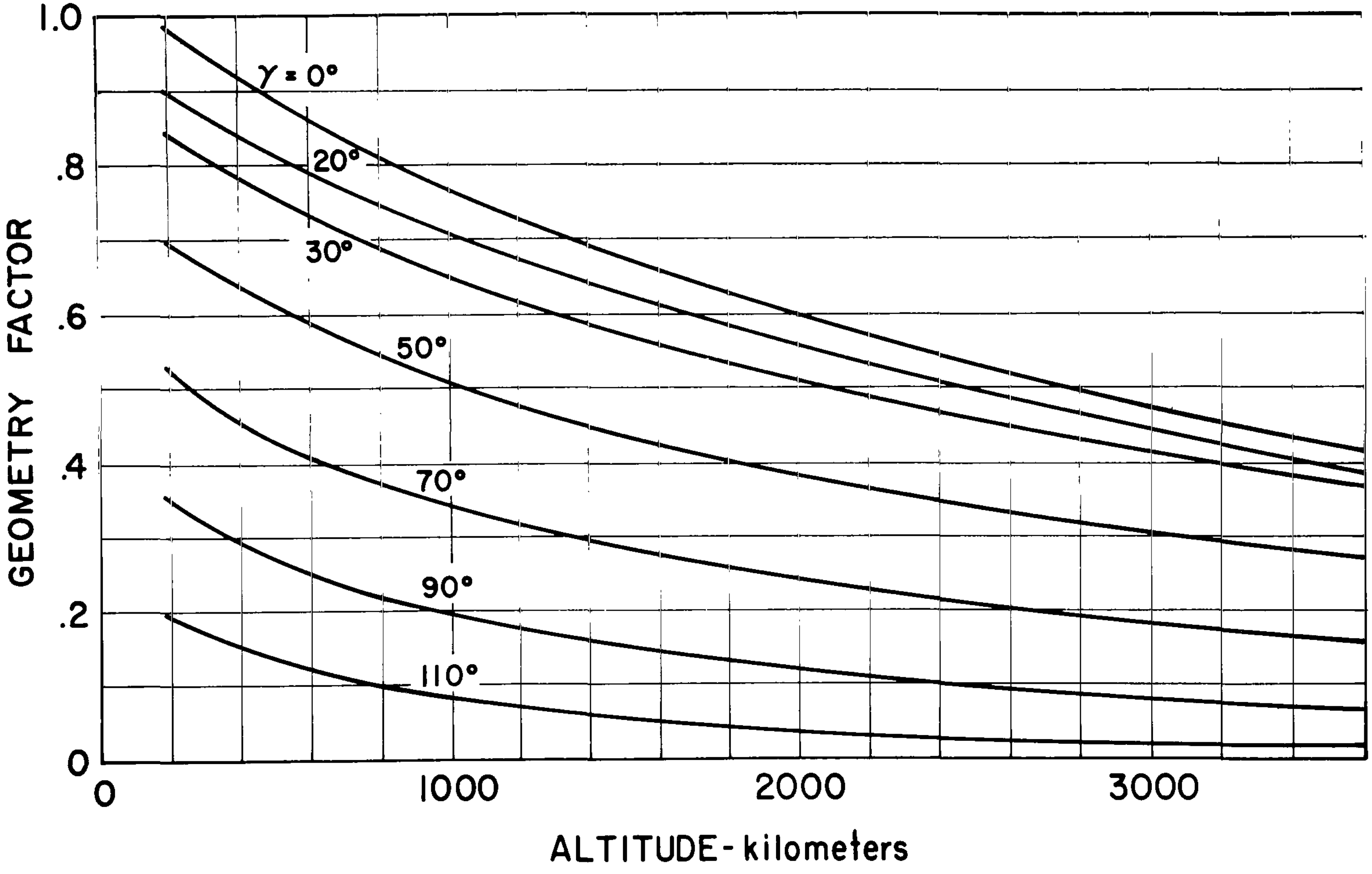

FIGURE 6. GEOMETRY FACTOR VS. ALTITUDE FOR IR TO A FLAT PLATE

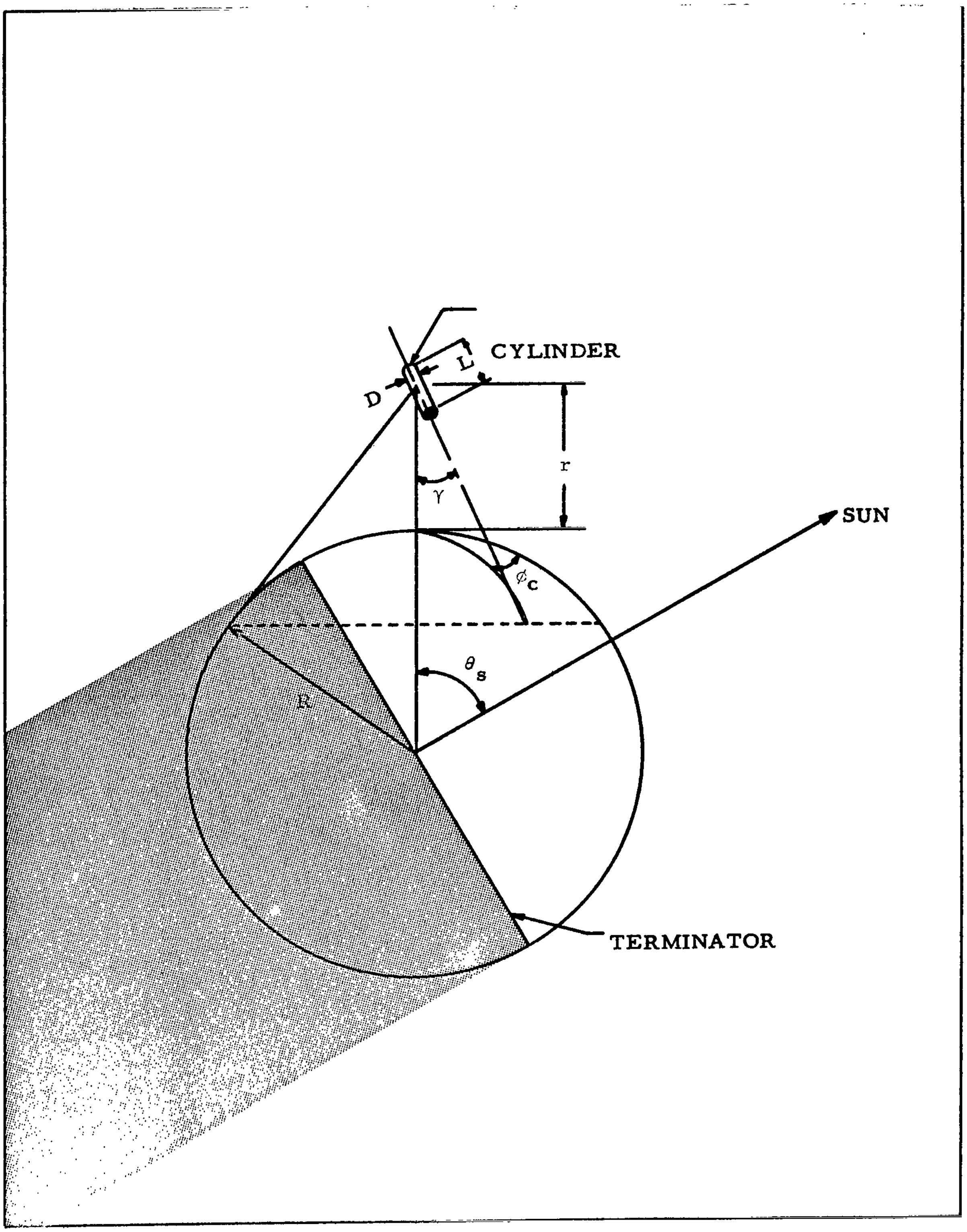

FIGURE 7. GEOMETRY FOR PLANETARY ALBEDO TO A CYLINDER

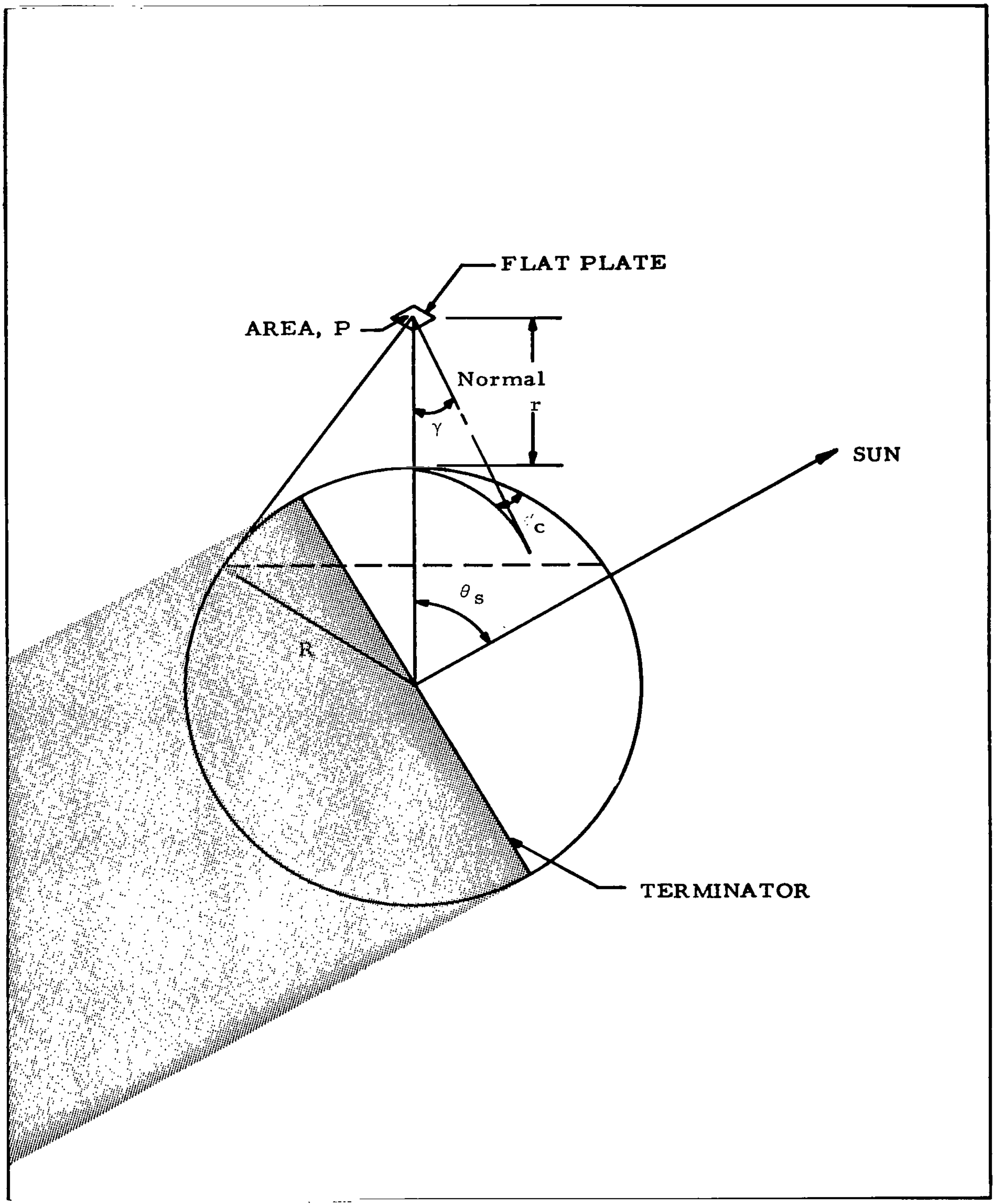

FIGURE 8. GEOMETRY FOR PLANETARY ALBEDO TO A FLAT PLATE

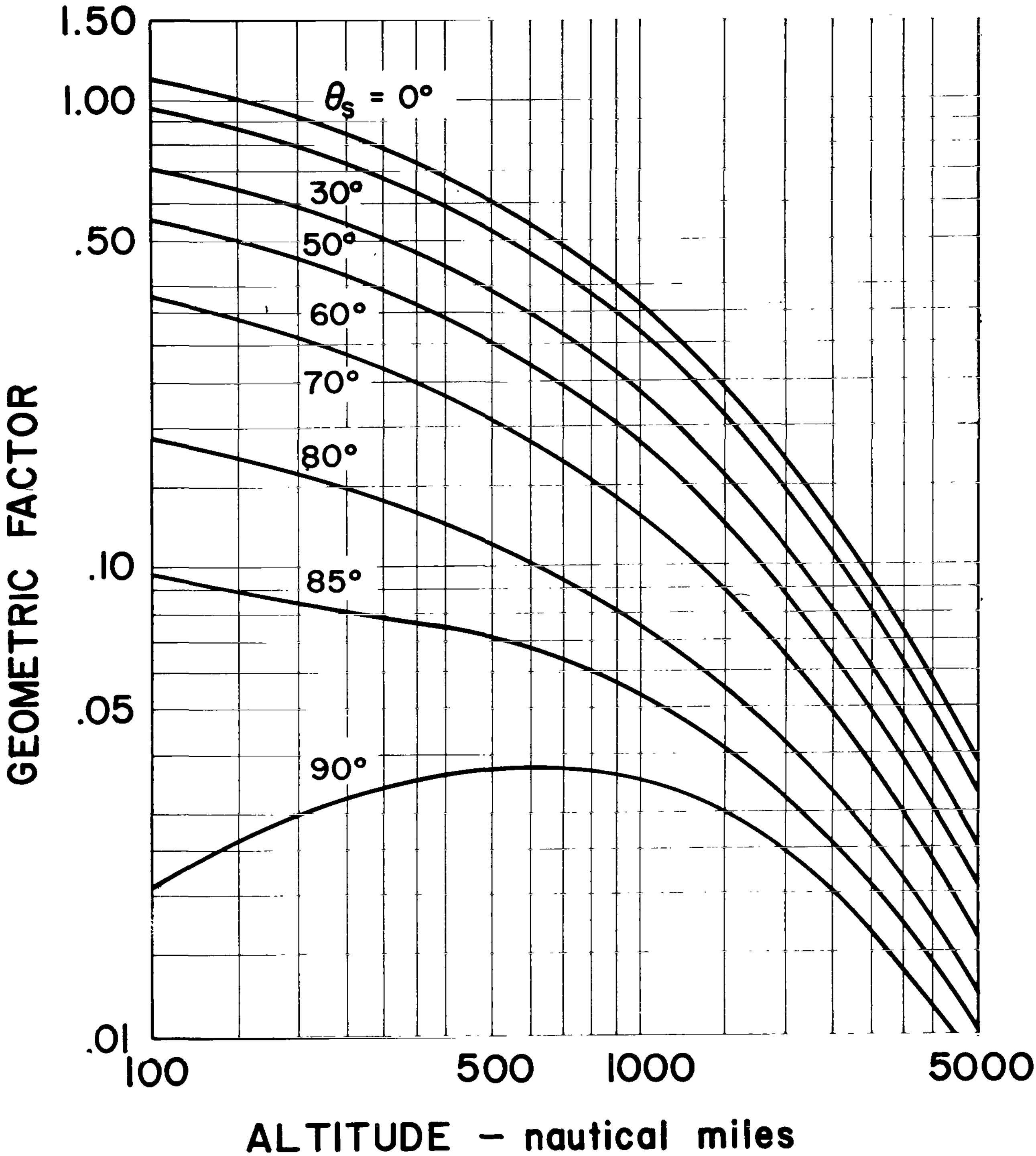

FIGURE 9. GEOMETRIC FACTOR FOR ALBEDO TO A CYLINDER VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN CYLINDER AND SUN AS A PARAMETER $\gamma = 0°$, $\phi_c = 0° - 180°$

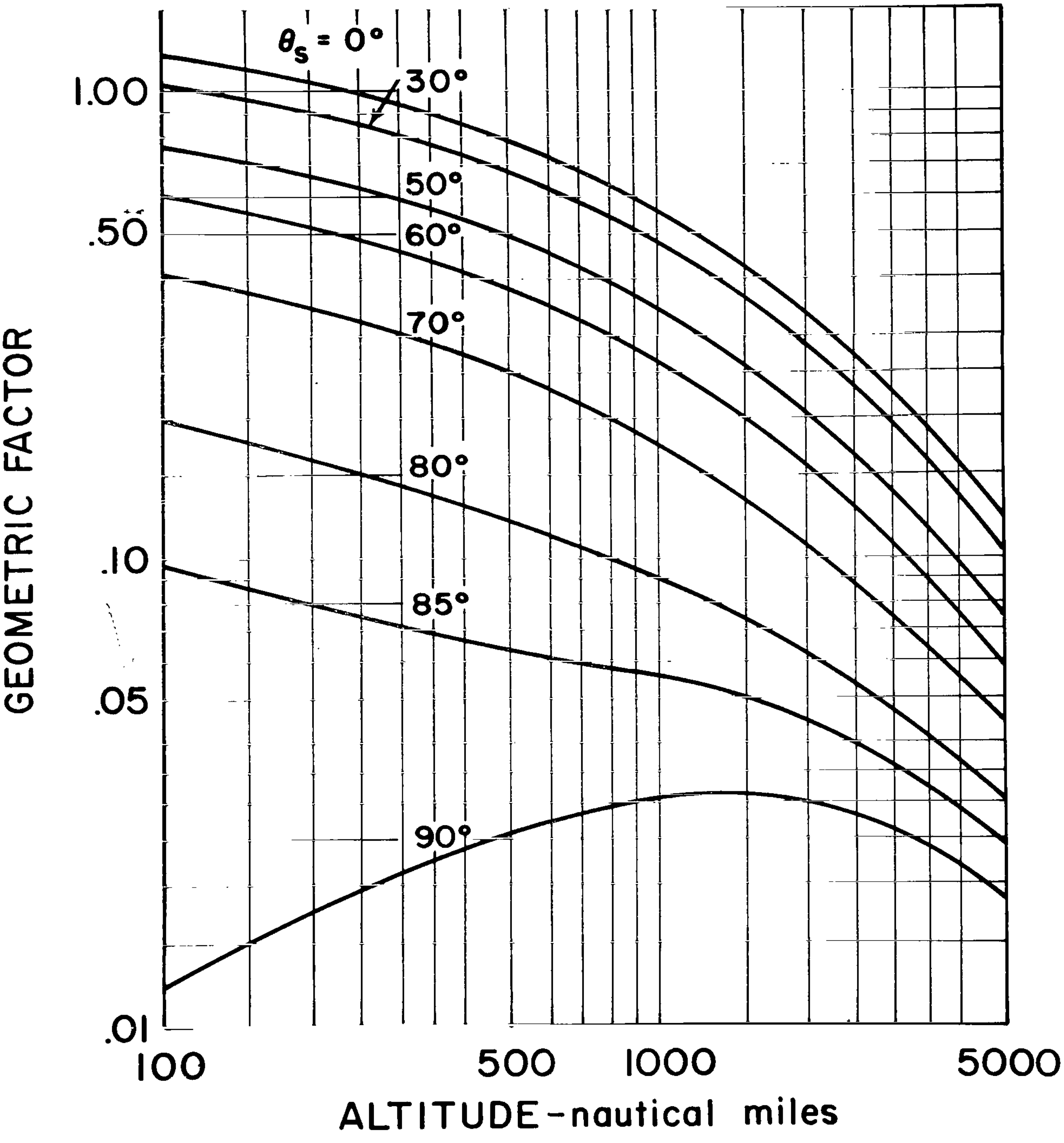

FIGURE 10. GEOMETRIC FACTOR FOR ALBEDO TO A CYLINDER VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN CYLINDER AND SUN AS PARAMETER
$\gamma = 60°$, $\phi_C = 0°$

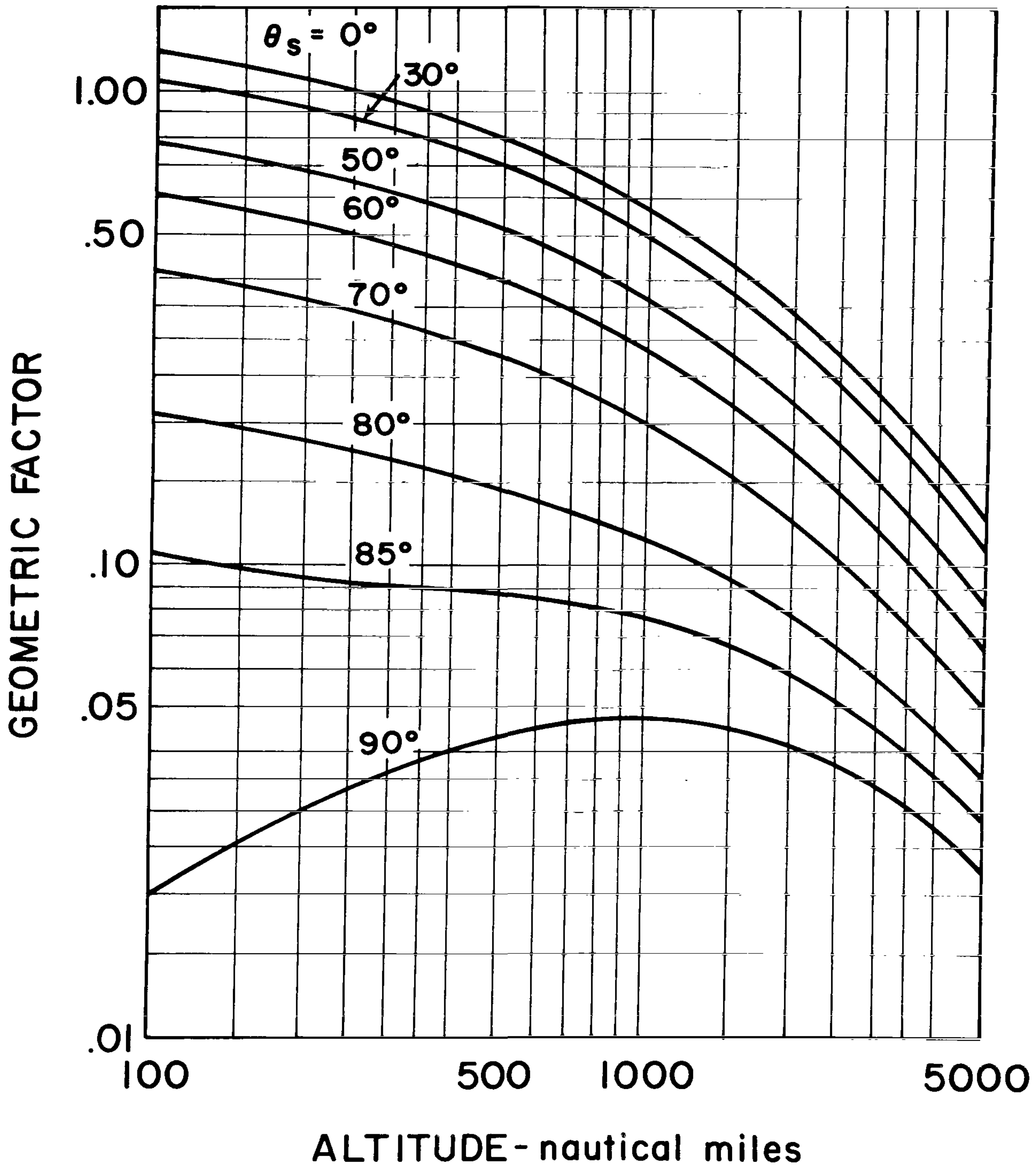

FIGURE 11. GEOMETRIC FACTOR FOR ALBEDO TO A CYLINDER VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN CYLINDER AND SUN AS A PARAMETER $\gamma = 60°$, $\phi_c = 90°$

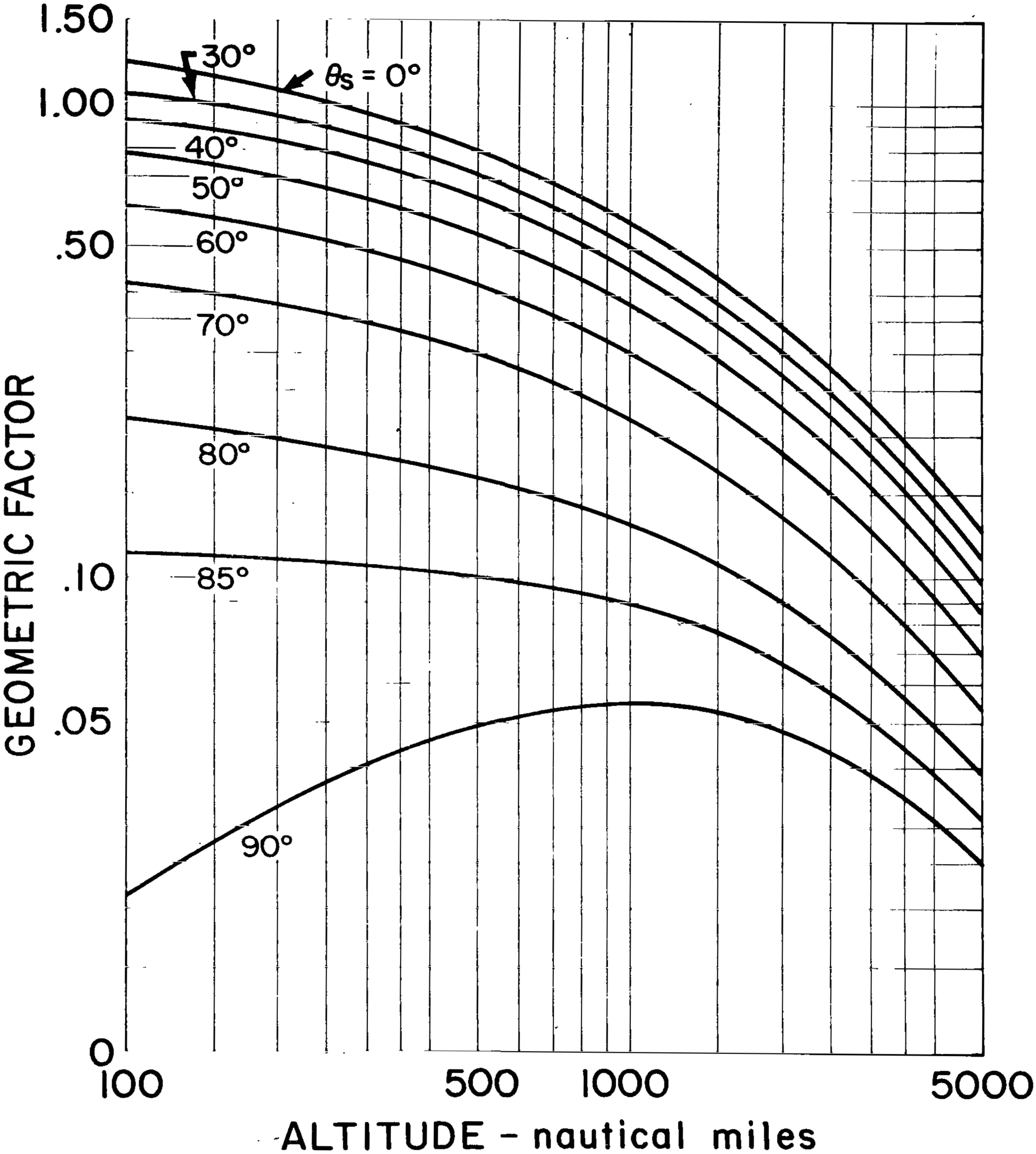

FIGURE 12. GEOMETRIC FACTOR FOR ALBEDO TO A CYLINDER VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN CYLINDER AND SUN AS A PARAMETER $\gamma = 60°$, $\phi_C = 180°$

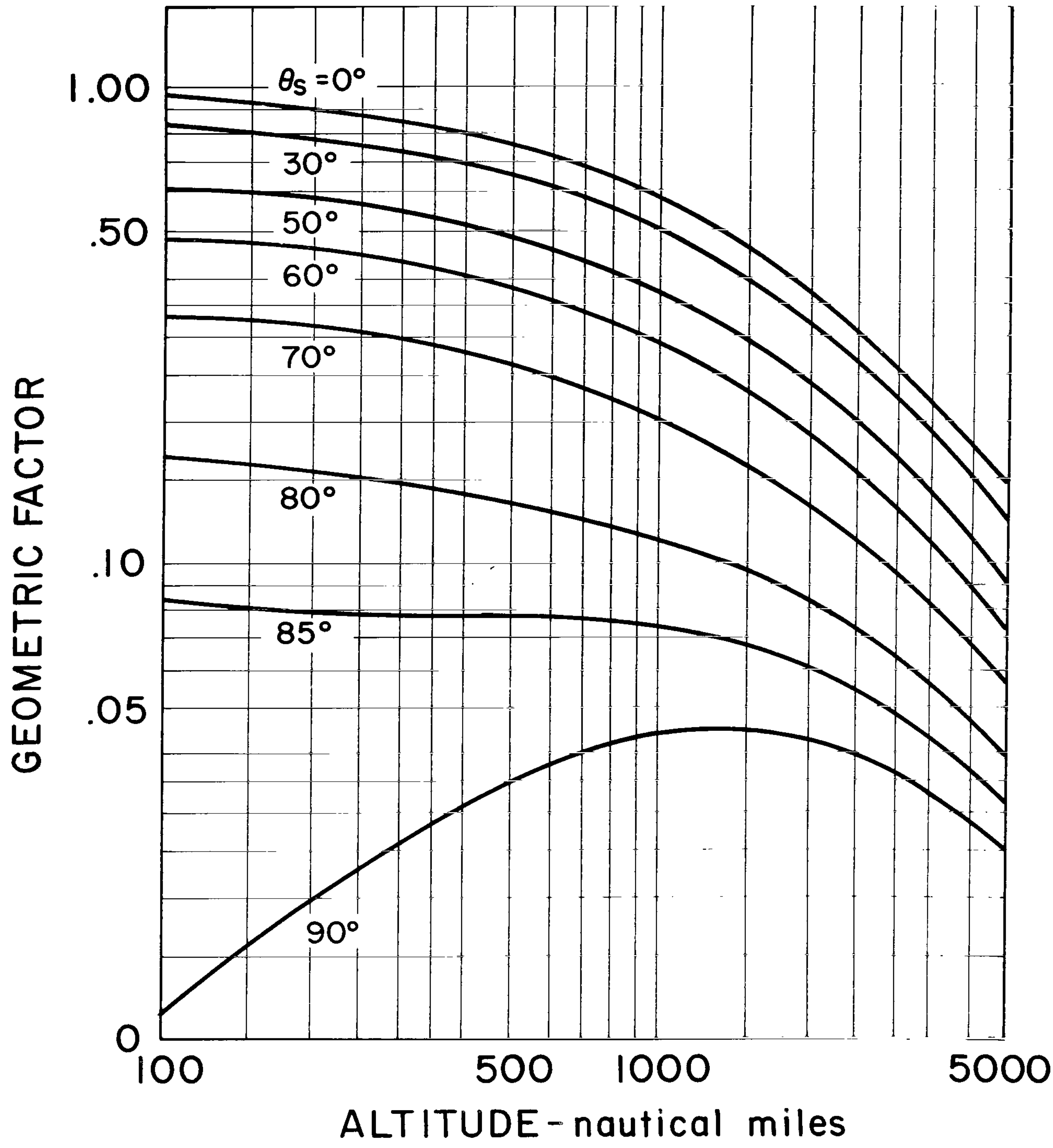

FIGURE 13. GEOMETRIC FACTOR FOR ALBEDO TO ONE SIDE OF A FLAT PLATE VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN FLAT PLATE AND SUN AS PARAMETER $\gamma = 0°$, $\phi_c = 0° - 180°$

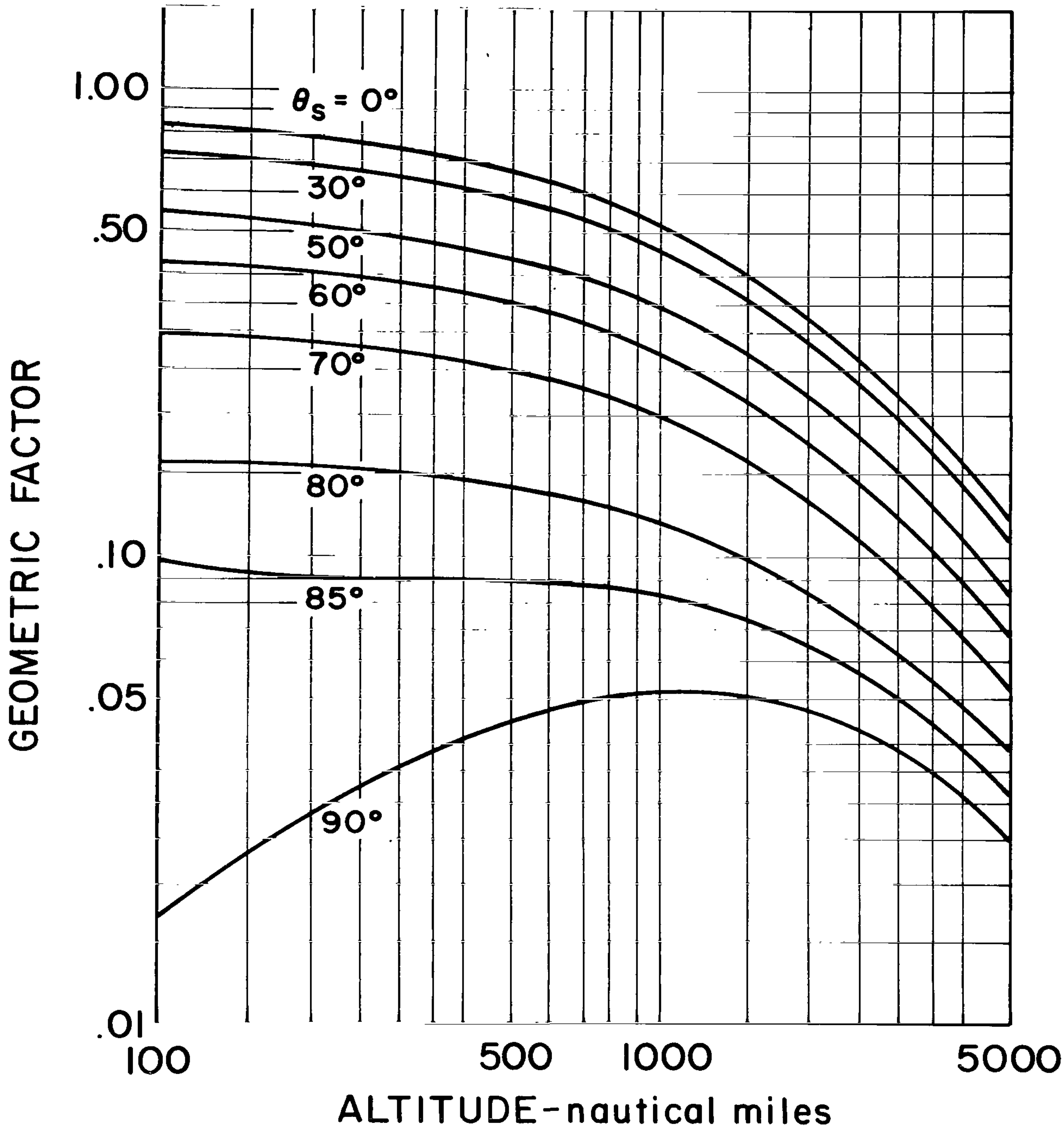

FIGURE 14. GEOMETRIC FACTOR FOR ALBEDO TO ONE SIDE OF A FLAT PLATE VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN FLAT PLATE AND SUN AS PARAMETER $\gamma = 30°$, $\phi_c = 0°$

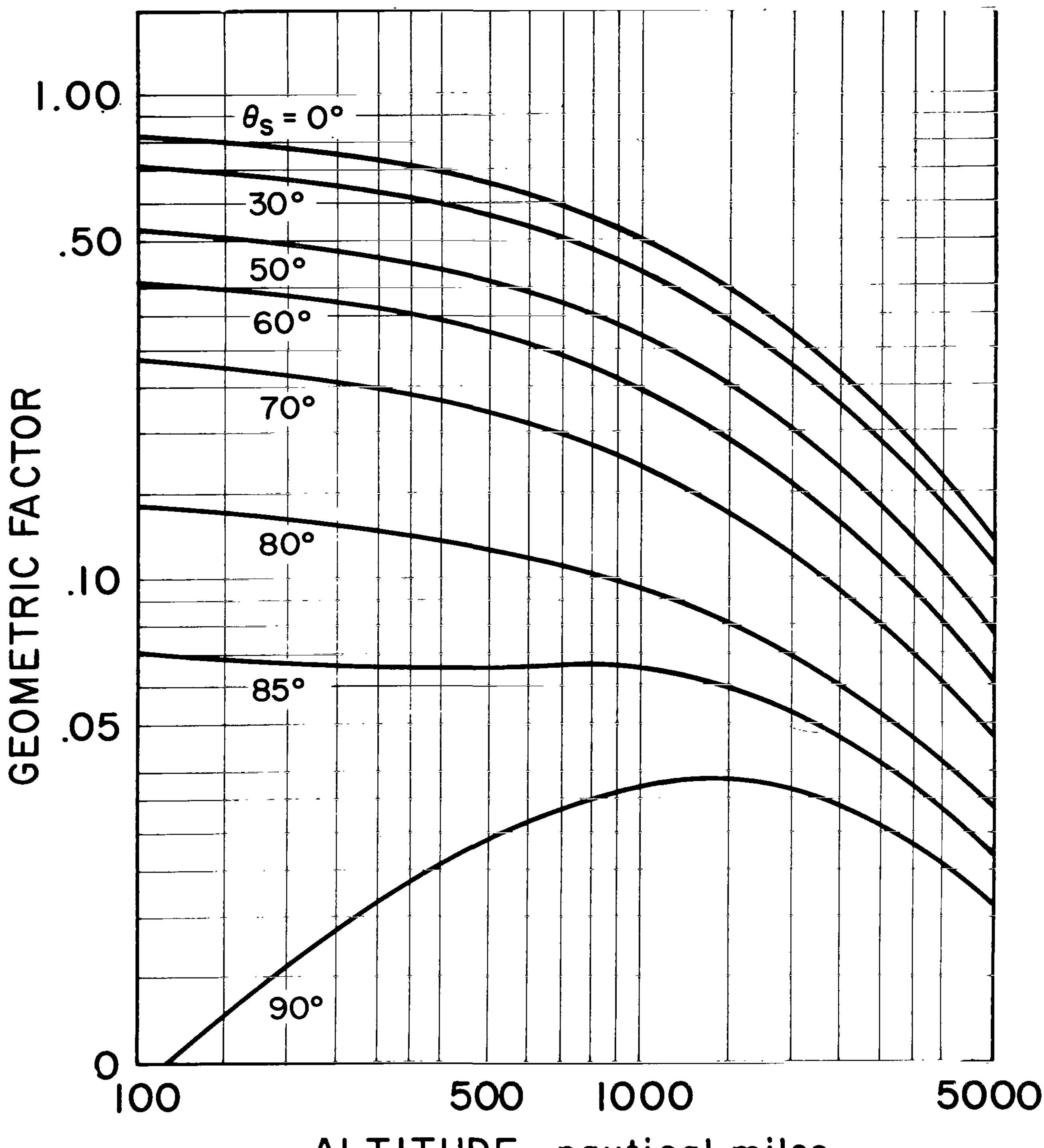

FIGURE 15. GEOMETRIC FACTOR FOR ALBEDO TO ONE SIDE OF A FLAT PLATE VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN FLAT PLATE AND SUN AS PARAMETER $\gamma = 30°$, $\phi_c = 90°$

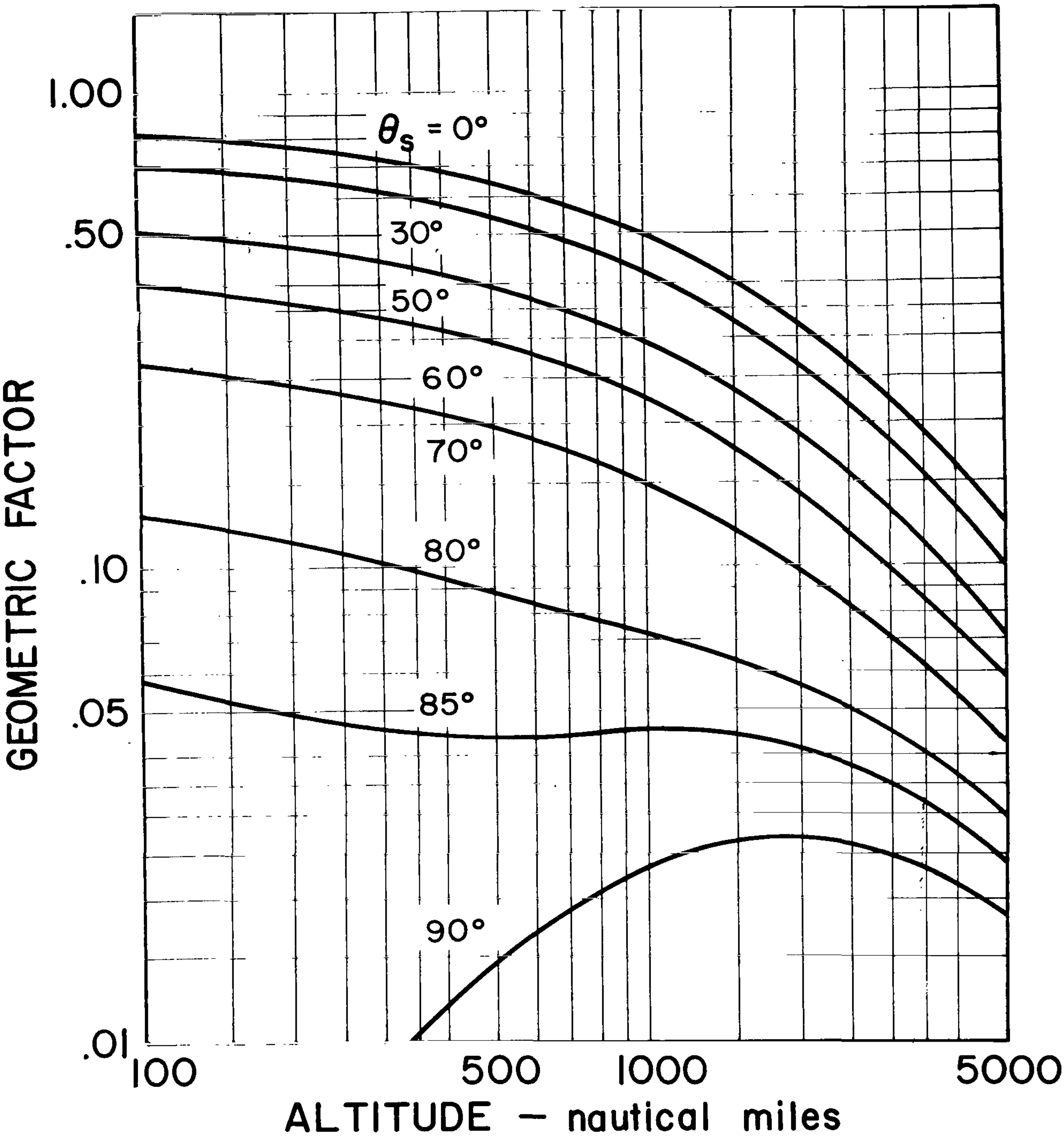

FIGURE 16. GEOMETRIC FACTOR FOR ALBEDO TO ONE SIDE OF A FLAT PLATE VS. ALTITUDE WITH ZENITH DISTANCE BETWEEN FLAT PLATE AND SUN AS PARAMETER $\gamma = 30°$, $\phi_c = 180°$

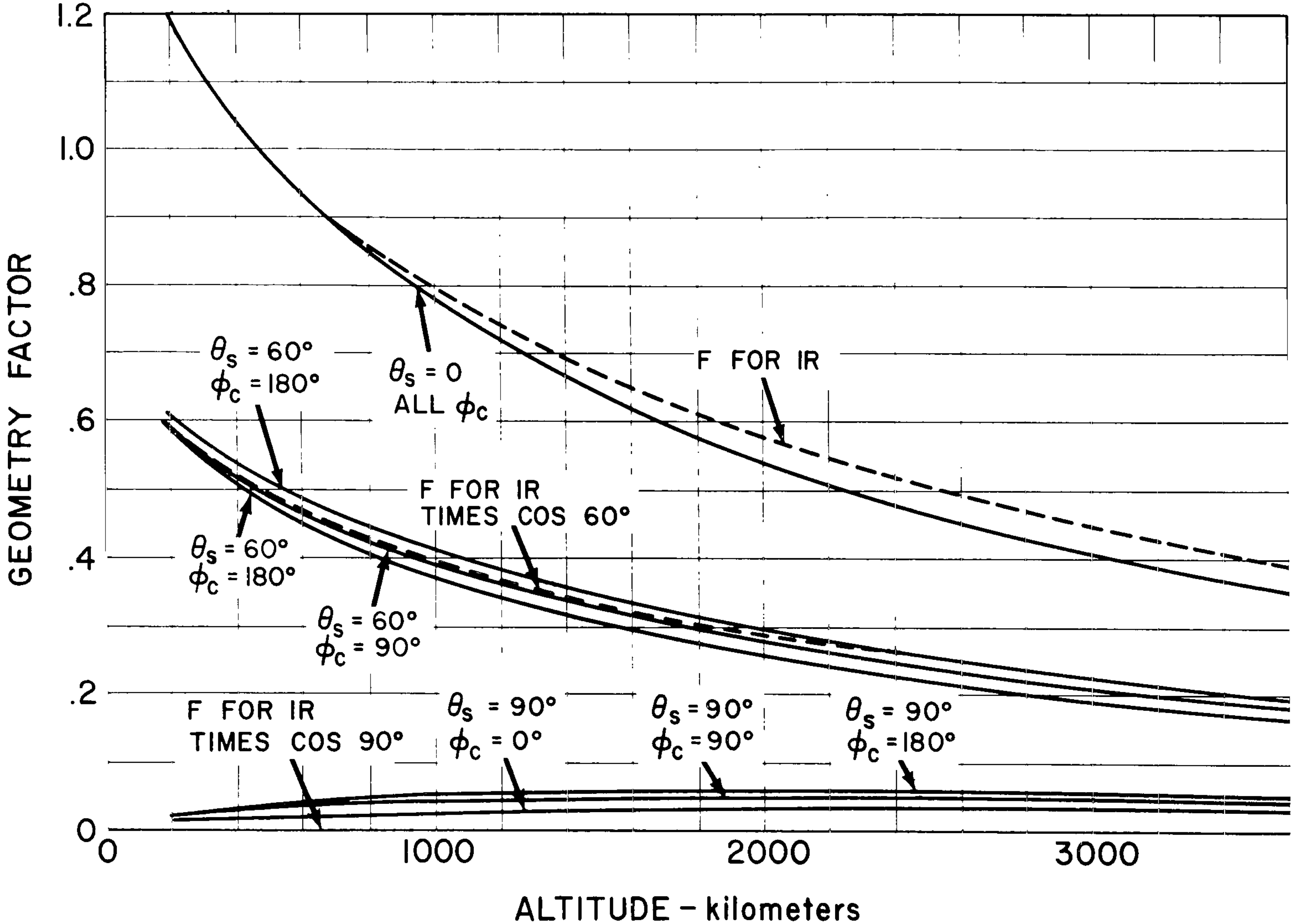

FIGURE 17. GEOMETRY FACTOR VS. ALTITUDE FOR ALBEDO TO A CYLINDER (γ = 60°)

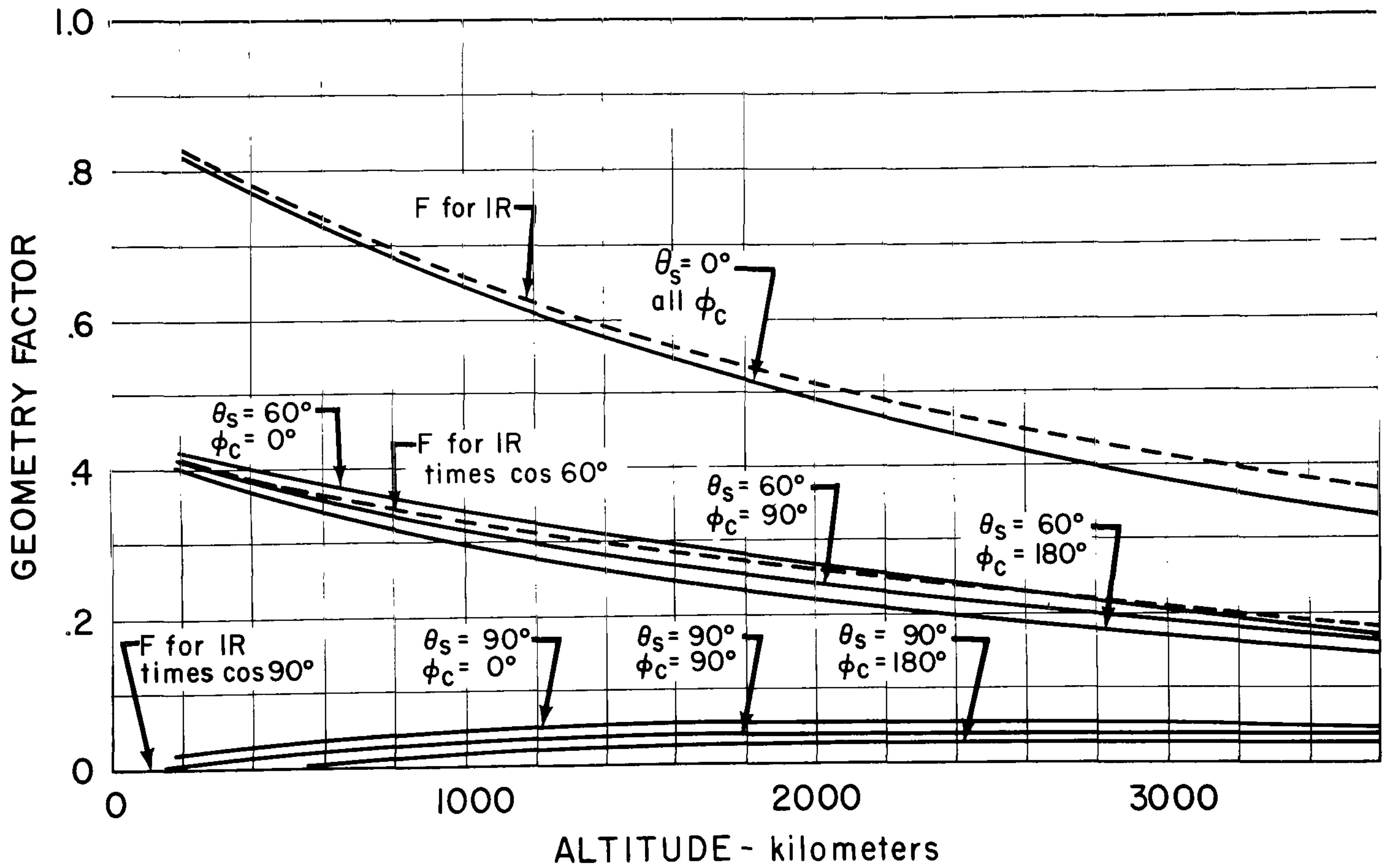

FIGURE 18. GEOMETRY FACTOR VS. ALTITUDE FOR ALBEDO TO A FLAT PLATE ($\gamma = 30°$)

TABLE I

B_{ij} FOR CYLINDER

	j=0 (γ = 0°)	j=1 (γ = 20°)	j=2 (γ = 40°)	j=3 (γ = 60°)	j=4 (γ = 90°)
i = 0	$+.12912 \times 10^{1}$	$+.12988 \times 10^{1}$	$+.13303 \times 10^{1}$	$+.13449 \times 10^{1}$	$+.14086 \times 10^{1}$
i = 1	$-.11546 \times 10^{-2}$	$-.12129 \times 10^{-2}$	$-.11584 \times 10^{-2}$	$-.95335 \times 10^{-3}$	$-.84191 \times 10^{-3}$
i = 2	$+.69623 \times 10^{-6}$	$+.81600 \times 10^{-6}$	$+.79503 \times 10^{-6}$	$+.58425 \times 10^{-6}$	$+.44063 \times 10^{-6}$
i = 3	$-.29237 \times 10^{-9}$	$-.33986 \times 10^{-9}$	$-.33629 \times 10^{-9}$	$-.22681 \times 10^{-9}$	$-.15707 \times 10^{-9}$
i = 4	$+.53083 \times 10^{-13}$	$+.73911 \times 10^{-13}$	$+.73500 \times 10^{-13}$	$+.46419 \times 10^{-13}$	$+.30839 \times 10^{-13}$
i = 5	$-.43402 \times 10^{-17}$	$-.63521 \times 10^{-17}$	$-.63232 \times 10^{-17}$	$-.37997 \times 10^{-17}$	$-.24892 \times 10^{-17}$

TABLE II

A_{ik} FOR CYLINDER

	k = 0	k = 1	k = 2	k = 3	k = 4
i = 0	$+.12912\mathrm{x}10^{1}$	$-.17242\mathrm{x}10^{-2}$	$+.15665\mathrm{x}10^{-3}$	$-.29166\mathrm{x}10^{-5}$	$+.17221\mathrm{x}10^{-7}$
i = 1	$-.11546\mathrm{x}10^{-2}$	$-.22624\mathrm{x}10^{-5}$	$-.16699\mathrm{x}10^{-6}$	$+.79023\mathrm{x}10^{-8}$	$-.59320\mathrm{x}10^{-10}$
i = 2	$+.69695\mathrm{x}10^{-6}$	$+.53858\mathrm{x}10^{-8}$	$+.18843\mathrm{x}10^{-9}$	$-.92692\mathrm{x}10^{-11}$	$+.68441\mathrm{x}10^{-13}$
i = 3	$-.26237\mathrm{x}10^{-9}$	$-.37223\mathrm{x}10^{-11}$	$-.91473\mathrm{x}10^{-13}$	$+.49288\mathrm{x}10^{-14}$	$-.36760\mathrm{x}10^{-16}$
i = 4	$+.53083\mathrm{x}10^{-13}$	$+.10402\mathrm{x}10^{-14}$	$+.20723\mathrm{x}10^{-16}$	$-.12171\mathrm{x}10^{-17}$	$+.91996\mathrm{x}10^{-20}$
i = 5	$-.43403\mathrm{x}10^{-17}$	$-.10267\mathrm{x}10^{-18}$	$-.18091\mathrm{x}10^{-20}$	$+.11289\mathrm{x}10^{-21}$	$-.86196\mathrm{x}10^{-24}$

TABLE III

B_{ij} FOR FLAT PLATE

	j=0 ($\gamma = 0°$)	j=1 ($\gamma = 20°$)	j=2 ($\gamma = 30°$)	j=3 ($\gamma = 50°$)	j=4 ($\gamma = 70°$)	j=5 ($\gamma = 90°$)	j=6 ($\gamma = 110°$)
i = 0	$+.10507\text{x}10^{1}$	$+.96610\text{x}10$	$+.90240\text{x}10$	$+.77200\text{x}10$	$+.58590\text{x}10$	$+.41410\text{x}10$	$+.2375\text{x}10$
i = 1	$-.37475\text{x}10^{-3}$	$-.34413\text{x}10^{-3}$	$-.32520\text{x}10^{-3}$	$-.38502\text{x}10^{-3}$	$-.37334\text{x}10^{-3}$	$-.36150\text{x}10^{-3}$	$-.26265\text{x}10^{-3}$
i = 2	$+.11106\text{x}10^{-6}$	$+.10817\text{x}10^{-6}$	$+.91757\text{x}10^{-7}$	$+.15112\text{x}10^{-6}$	$+.17036\text{x}10^{-6}$	$+.18877\text{x}10^{-6}$	$+.14473\text{x}10^{-6}$
i = 3	$-.23451\text{x}10^{-10}$	$-.24013\text{x}10^{-10}$	$-.16535\text{x}10^{-10}$	$-.34398\text{x}10^{-10}$	$-.44416\text{x}10^{-10}$	$-.51021\text{x}10^{-10}$	$-.39877\text{x}10^{-10}$
i = 4	$+.21995\text{x}10^{-14}$	$+.22752\text{x}10^{-14}$	$+.12802\text{x}10^{-14}$	$+.31408\text{x}10^{-14}$	$+.46773\text{x}10^{-14}$	$+.53038\text{x}10^{-14}$	$+.42302\text{x}10^{-14}$

TABLE IV

A_{ik} FOR A FLAT PLATE

	k = 0	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7
i = 0	$+.10507\text{x}10^{1}$	$+.74328\text{x}10^{-2}$	$-.13397\text{x}10^{-2}$	$+.58132\text{x}10^{-4}$	$-.12748\text{x}10^{-5}$	$+.14561\text{x}10^{-7}$	$-.82937\text{x}10^{-10}$	$+.18615\text{x}10^{-12}$
i = 1	$-.37475\text{x}10^{-3}$	$-.25359\text{x}10^{-4}$	$+.32820\text{x}10^{-5}$	$-.14765\text{x}10^{-6}$	$+.31718\text{x}10^{-8}$	$-.35445\text{x}10^{-10}$	$+.19886\text{x}10^{-12}$	$-.44198\text{x}10^{-15}$
i = 2	$+.11106\text{x}10^{-6}$	$+.23673\text{x}10^{-7}$	$-.28046\text{x}10^{-8}$	$+.12167\text{x}10^{-9}$	$-.25507\text{x}10^{-11}$	$+.28049\text{x}10^{-13}$	$-.15570\text{x}10^{-15}$	$+.34371\text{x}10^{-18}$
i = 3	$-.23451\text{x}10^{-10}$	$-.78885\text{x}10^{-11}$	$+.90018\text{x}10^{-12}$	$+.76986\text{x}10^{-13}$	$-.83013\text{x}10^{-15}$	$-.83013\text{x}10^{-17}$	$+.45444\text{x}10^{-19}$	$-.99389\text{x}10^{-22}$
i = 4	$+.21995\text{x}10^{-14}$	$+.83341\text{x}10^{-15}$	$-.92378\text{x}10^{-16}$	$+.37480\text{x}10^{-17}$	$-.74129\text{x}10^{-19}$	$+.77824\text{x}10^{-21}$	$-.41737\text{x}10^{-23}$	$+.89956\text{x}10^{-26}$

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