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ANALYSIS OF THE ERRORS ASSOCIATED WITH THE DETERMINATION OF PLANETARY ATMOSPHERE STRUCTURE FROM MEASURED ACCELERATIONS OF AN ENTRY VEHICLE

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SUMMARY

An analytic method is developed for estimating the errors in the density and pressure structure of a planetary atmosphere constructed from measurements of accelerations experienced by an entry body. Among the sources of error considered both singly and collectively are the measuring accuracy of the accelerometers, the uncertainties in entry speed, entry angle, and aerodynamic coefficients, the frequency of data measurements, and the uncertainty of the attitude of entry bodies having nonzero lift-to-drag ratios.

Results of sample calculations are presented to show the accuracy with which the extremes of a range of postulated model atmospheres for Mars can be defined from measurements of accelerations experienced by a spherically shaped entry body. It is found that if the accelerometer method were used only over that portion of the atmosphere traversed at speeds greater than sonic speed, then any of the atmospheres postulated for Mars can be defined reasonably well even when errors from all sources are combined in the most unfavorable manner. Some of the results obtained by the analytic method are compared with those from more precise numerical calculations and the agreement is found to be excellent.

INTRODUCTION

Properties of planetary atmospheres can be determined from observations on board an entry vehicle passing through the atmosphere. Experiments involving a number of forms of observations, such as acceleration, spectral distribution and intensity of shock-layer radiation, oscillation frequency, etc., were proposed in reference 1 and the proposals were developed in more detail in reference 2. Considerable attention has been focused on one of these experiments in which knowledge of the acceleration experienced by a body, having known aerodynamic characteristics and entering an atmosphere on a ballistic trajectory, is used to construct the density and pressure structure of the atmosphere. A detailed procedure for obtaining atmosphere structure from measurements made with on-board accelerometers was given in reference 3; it was also shown that from the standpoint of flight mechanics, the ideal entry body for the atmosphere-structure experiment is an aerodynamically stabilized sphere.

A number of sources of error in atmosphere definition are associated with the proposed method. Among these are the measuring accuracy of the accelerometers, the uncertainties in entry speed, entry angle, and aerodynamic coefficients, the frequency of data measurements, and the uncertainty in knowledge of the attitude of lifting entry bodies. Reference 2 gives an analytic method for predicting the possible inaccuracies in the definition of postulated Mars' atmospheres due to just one of these sources, errors in measured accelerations. The method of reference 2 is limited, however, since it only allows for errors originating with the accelerometers and since assumptions of small errors and constant drag coefficients are inherent in the method. The effects of errors from this same source on the definition of the same postulated Mars atmospheres were estimated by a more precise numerical method in reference 3. Subsequent to these studies, general methods have been devised for assessing the effects of errors from the probable sources regardless of their magnitudes. These are presented herein and their use is demonstrated by some examples pertinent to the problem of determining the atmosphere of Mars.

NOTATION

A reference area for aerodynamic coefficients

a_R resultant acceleration due to aerodynamic loads

a_s component of resultant acceleration due to aerodynamic loads directed along the flight path

C_D coefficient of aerodynamic drag, drag$/(1/2)\rho V^2 A$

C_{D_o} coefficient of aerodynamic drag evaluated at body attitude corresponding to zero lift

C_L coefficient of aerodynamic lift, lift$/(1/2)\rho V^2 A$

C_R coefficient of resultant aerodynamic force, $\sqrt{C_D^2 + C_L^2}$

g local acceleration due to gravity

g_o acceleration due to gravity evaluated at surface of planet

H_ρ local atmosphere density scale height, defined by equation (4)

h altitude above planet surface

m mass of entry body

p ambient pressure in atmosphere

Re Reynolds number

r distance from center of planet to mass center of entry body

t time

V flight speed

$\Delta(\ )$ difference between quantity evaluated from measured data at a given instant of time and exact value of quantity evaluated at the same instant, $(\tilde{\ }) - (\ )$

θ flight-path angle measured from local horizontal, positive down

ρ ambient density in atmosphere

σ resultant angle of attack of entry body

τ dummy variable of integration

φ $\tan^{-1} C_L/C_D$

$(\tilde{\ })$ quantity evaluated from measured data

Subscripts

E quantity evaluated at entry

F quantity evaluated at time corresponding to the final usable measurement of acceleration

max maximum value of function occurring in a specified entry

min minimum value of function

ANALYSIS

Sources of error in constructing a planetary atmosphere from a time history of acceleration experienced by an entry body are identified and methods for assessing their influence on the desired results are developed in this section. Specifically, equations are developed for calculating the density, pressure, and density scale height variations with altitude. The equations are cast in forms that clearly show the roles played by the various factors in degrading precision of the results.

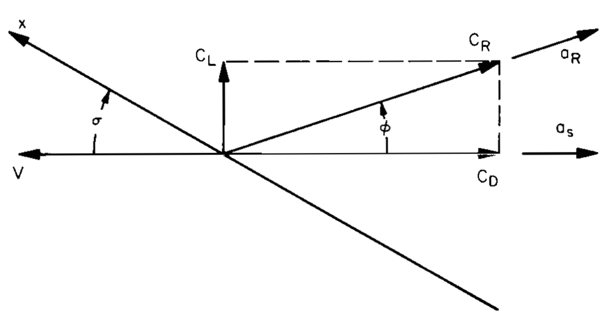
The problem can be approached in a number of ways. For example, an error in the calculated density variation with altitude can be interpreted as being due entirely to errors in density at given altitudes, or entirely to errors in altitude at given densities. On the other hand, it might be interpreted as the result of errors in density at given times in the trajectory plus errors in altitude at those times. The latter interpretation will be adopted here because it enables the prominent factors governing the problem to be clearly identified. It also allows for the possibility of small errors in density and pressure at a given time, but yet for large errors in these quantities at a given altitude caused by a poor determination of altitude. It will be found that the equations for density, pressure, and altitude all have time as a common variable. The assumption is made that time is known exactly at the instants when acceleration is measured. The approach will be first to show how errors in density and pressure at a given instant of time are related to errors in knowledge of the trajectory at that same time. Then equations will be derived for the errors in trajectory variables in terms of time.

Equations for Errors in Atmosphere Properties in Terms of Errors in Trajectory Variables

An equation for the density ρ in terms of the drag coefficient C_D and the component along the trajectory of resultant acceleration due to aerodynamic loads can be written as follows:

$$\rho = 2\left(\frac{m}{C_D A}\right)\frac{a_s}{V^2} \tag{1}$$

Accelerometers located at the mass center of an entry body can be alined to measure components of acceleration along three mutually perpendicular axes. The vector sum of these components is the resultant acceleration due to aerodynamic loads. Except when the lift of the entry body is zero, the magnitude and direction of this resultant vector are different from the component of acceleration along the flight path. This is illustrated in the sketch in which the longitudinal axis of the body is identified by the symbol x. The magnitude of the acceleration component along the flight path is given in terms of the resultant acceleration by the following equation



$$a_s = a_R \cos \varphi$$

where

$$\cos \varphi = \frac{1}{\sqrt{1 + (C_L/C_D)^2}}$$

This result and equation (1) are combined to give the following expression for the density in terms of the resultant acceleration a_R

$$\rho = 2\left(\frac{m}{C_D A}\right)\frac{a_R \cos \varphi}{V^2} \tag{2}$$

To obtain the ambient pressure, the weight of a vertical column of atmosphere is integrated over the trajectory:

$$p = \int_0^t g\rho V \sin \theta \, d\tau \tag{3}$$

where it has been assumed that the altitude at time zero is high enough that the pressure at this instant is zero. The local density scale height is defined as

$$H_\rho = \frac{p}{\rho g} \tag{4}$$

Quantities obtained from measured data, and therefore subject to error, will be denoted by a tilda over the symbol while those without a tilda will denote exact values. Equations for the ratios of measured values to exact values are formed by means of equations (2), (3), and (4) and are given below.

$$\left.\begin{aligned}
\frac{\tilde{\rho}}{\rho} &= \frac{C_D}{\tilde{C}_D}\frac{\tilde{a}_R}{a_R}\frac{\cos\tilde{\varphi}}{\cos\varphi}\left(\frac{V}{\tilde{V}}\right)^2 \\
\frac{\tilde{p}}{p} &= \frac{1}{p}\int_0^t \tilde{g}\tilde{\rho}\tilde{V}\sin\tilde{\theta}\,d\tau \\
\frac{\tilde{H}_\rho}{H_\rho} &= \frac{\tilde{p}}{p}\frac{\rho}{\tilde{\rho}}\frac{g}{\tilde{g}}
\end{aligned}\right\} \tag{5}$$

A better form for these equations is obtained if a symbol is defined for the difference between values deduced from measurements and exact values. Let

$$\Delta(\ ) = (\tilde{\ }) - (\ )$$

With this definition, equations (5) take the following form

$$\left.\begin{aligned}
\frac{\tilde{\rho}}{\rho} &= \frac{\left(1 + \frac{\Delta a_R}{a_R}\right)\left(1 + \frac{\Delta\cos\varphi}{\cos\varphi}\right)}{\left(1 + \frac{\Delta C_D}{C_D}\right)\left(1 + \frac{\Delta V}{V}\right)^2} \\
\frac{\tilde{p}}{p} &= \frac{1}{p}\int_0^t g\rho V\sin\theta\,\frac{\left(1 + \frac{\Delta g}{g}\right)\left(1 + \frac{\Delta a_R}{a_R}\right)\left(1 + \frac{\Delta\cos\varphi}{\cos\varphi}\right)\left(1 + \frac{\Delta\sin\theta}{\sin\theta}\right)}{\left(1 + \frac{\Delta C_D}{C_D}\right)\left(1 + \frac{\Delta V}{V}\right)}\,d\tau \\
\frac{\tilde{H}_\rho}{H_\rho} &= \frac{\tilde{p}}{p}\frac{\rho}{\tilde{\rho}}\frac{g}{\tilde{g}}
\end{aligned}\right\} \tag{6}$$

Equations (6) relate the values of density, pressure, and scale height deduced from measurements to errors in acceleration, speed, path angle, gravity, lift-to-drag ratio, and coefficient of resultant force. The problem of determining the errors in trajectory variables, speed, path angle, and altitude from the measured resultant accelerations will be considered next.

Equations for Errors in Trajectory Variables

The trajectory is governed by the following set of equations (ref. 3).

$$\left.\begin{aligned}
&\frac{dV}{dt} + \frac{C_D A}{m}\left(\frac{1}{2}\rho V^2\right) - g \sin\theta = 0 \\
&\frac{V}{g}\frac{d\theta}{dt} + \left(\frac{V^2}{gr} - 1\right)\cos\theta + \frac{1}{g}\left(\frac{C_L A}{m}\right)\left(\frac{1}{2}\rho V^2\right) = 0 \\
&\frac{dh}{dt} = -V\sin\theta
\end{aligned}\right\} \tag{7}$$

These equations written in terms of the resultant acceleration, a_R, with the aid of equation (2) are

$$\left.\begin{aligned}
&\frac{dV}{dt} + a_R \cos\varphi - g\sin\theta = 0 \\
&\frac{V}{g}\frac{d\theta}{dt} + \left(\frac{V^2}{gr} - 1\right)\cos\theta + \frac{1}{g}\frac{C_L}{C_D} a_R \cos\varphi = 0 \\
&\frac{dh}{dt} = -V\sin\theta
\end{aligned}\right\} \tag{8}$$

The set of trajectory equations can be simplified if the flight-path angle is assumed to remain constant at the entry angle, that is, $\theta \equiv \theta_E$. The error resulting from this assumption diminishes with increasing entry angle and becomes zero for a 90° entry. It has been shown in references 1, 2, and 3 that steep entries (50° to 90°) are preferred for this experiment so that assuming a constant path angle is not seriously restrictive, as verified by results of numerical calculations presented later. It is further assumed that the acceleration due to gravity is constant at the planet surface value. With these approximations, the equations for the trajectory deduced from measurements can be written as follows:

$$\left.\begin{aligned}
&\frac{d\tilde{V}}{dt} + \tilde{a}_R \cos\tilde{\varphi} - g_o \sin\tilde{\theta}_E = 0 \\
&\frac{d\tilde{h}}{dt} = -\tilde{V}\sin\tilde{\theta}_E
\end{aligned}\right\} \tag{9a}$$

A similar pair of equations can be written for the trajectory which would be obtained under the same approximations ($\theta = \theta_E$, $g = g_o$) if there were no errors in either a_R or the entry conditions. The latter trajectory, which will be called the reference trajectory, is written as

$$\left.\begin{aligned}\frac{dV}{dt} + a_R \cos \varphi - g_o \sin \theta_E = 0 \\ \frac{dh}{dt} = -V \sin \theta_E\end{aligned}\right\} \tag{9b}$$

An equation for the difference ΔV between the computed speed and the reference speed is obtained when the first of equations (9b) is subtracted from the first of equations (9a). This gives

$$\frac{d(\Delta V)}{dt} + \tilde{a}_R \cos \tilde{\varphi} - a_R - g_o \Delta \sin \theta_E = 0$$

Further algebraic manipulation puts the above equation into the following form

$$\frac{d(\Delta V)}{dt} + (\cos \varphi + \Delta \cos \varphi)\Delta a_R + a_R \Delta \cos \varphi - g_o \Delta \sin \theta_E = 0 \tag{10}$$

Equation (10) can be integrated analytically if $\cos \varphi$, $\Delta \cos \varphi$, Δa_R, and $\Delta \sin \theta_E$ are assumed constant for all time. The implications of making such assumptions will now be examined in detail. The assumption that $\cos \varphi$ and $\Delta \cos \varphi$ are constant is exact for a spherical body for which C_L is always zero, for with $C_L = 0$, both φ and $\tilde{\varphi}$ are identically zero. For nonspherical bodies undergoing planar oscillatory motion, the quantity $\cos \varphi$ will also be oscillatory with time, and for spinning nonspherical bodies undergoing coning motion, $\cos \varphi$ will not be oscillatory but rather will vary with the envelope of the total angle of attack. The present theory can be used to estimate the errors associated with the most extreme assumptions for the cases involving nonspherical bodies. For example, if the rate at which the components of resultant acceleration were measured was not sufficient to allow the variation of angle of attack through the trajectory to be reconstructed, then the worst case, from the standpoint of errors, would be to assume that the angle of attack was identically zero for all time. This is equivalent to assuming that the lift-to-drag ratio associated with the measured data is zero so that $\Delta \cos \varphi = 1 - \cos \varphi$. Now, for $\Delta \cos \varphi$ to be constant, the quantity $\cos \varphi$ must be invariant. Again, the worst case can be assumed. For all bodies of revolution, except a sphere, the absolute value of the lift-to-drag ratio $|C_L/C_D|$ increases from zero as the resultant angle of attack is either increased or decreased from zero. This resultant angle can never be larger at any point in the trajectory than the value at entry if the body is dynamically stable. The maximum possible value at entry would be known with reasonable precision so that, for any chosen probe configuration, an estimate could be made of the maximum value of $|C_L/C_D|$ (hence, φ) obtainable in the range of possible attitudes. A value so chosen gives a value of φ denoted as φ_{max}. Now consider the implication of assuming a constant error in measured acceleration Δa_R. Measurement accuracy of accelerometer systems is often given as a fraction of the maximum value measurable by the instrument, but it is likely that such errors would be random with time. For this analysis, however, they will be assumed constant at the maximum value. This means that the measured accelerations would be biased from the true accelerations by a constant maximum amount. This bias produces the largest errors in calculated atmospheres. Finally, assuming a constant error in path angle is consistent with assuming a constant path angle throughout the trajectory. The assumptions discussed above allow equation (10) to be rewritten in the following form:

$$\frac{d(\Delta V)}{dt} + \Delta a_R + (\sec \varphi_{max} - 1)a_s - g_o \, \Delta \sin \theta_E = 0 \tag{11}$$

Equation (11) can be integrated analytically if the following substitution is made:

$$a_s = g_o \sin \theta_E - \frac{dV}{dt}$$

The result is

$$\Delta V = \Delta V_E + \left[\Delta \sin \theta_E - (\sec \varphi_{max} - 1)\sin \theta_E - \frac{\Delta a_R}{g_o}\right]g_o t - (\sec \varphi_{max} - 1)(V_E - V) \tag{12}$$

where

$$\sec \varphi_{max} = \sqrt{1 + (C_L/C_D)^2_{max}}$$

An equation for the difference Δh between the calculated altitude and the reference altitude at any instant of time is obtained when the second of equations (9b) is subtracted from the second of equations (9a). The result is

$$\frac{d(\Delta h)}{dt} = -V\Delta \sin \theta_E - \Delta V(\sin \theta_E + \Delta \sin \theta_E) \tag{13}$$

This equation can be integrated directly, with the aid of equation (12), to give

$$\begin{aligned}\Delta h = \Delta h_F &+ \frac{\Delta \sin \theta_E}{\sin \theta_E}(h - h_F) \\ &+ (\sin \theta_E + \Delta \sin \theta_E)\left\{\frac{g_o}{2}\left[\Delta \sin \theta_E - (\sec \varphi_{max} - 1)\sin \theta_E - \frac{\Delta a_R}{g_o}\right](t_F^2 - t^2)\right. \\ &\left. + \left[\Delta V_E - (\sec \varphi_{max} - 1)V_E\right](t_F - t) + \frac{(\sec \varphi_{max} - 1)}{\sin \theta_E}(h - h_F)\right\}\end{aligned} \tag{14}$$

Finally, equations (6) must be rewritten to be consistent with the assumptions just introduced. Two factors are affected. One is the ratio $\Delta \cos \varphi/\cos \varphi$. This can be written

$$\frac{\Delta \cos \varphi}{\cos \varphi} = \sec \varphi_{max} - 1$$

The other factor that requires explanation is $[1 + (\Delta C_D/C_D)]$. Since $\tilde{C}_D$ must be taken equal to $\tilde{C}_{D_O}$ to be consistent with the assumption that the body is in a nonlifting attitude,

$$\left(1 + \frac{\Delta C_D}{C_D}\right) = \frac{C_{D_O} + \Delta C_{D_O}}{C_D} \tag{15}$$

Since the aerodynamic characteristics of most simple bodies with thin shock layers are not strongly dependent on gas composition, it can be assumed that C_{D_O} is a function only of Reynolds number and speed so that

$$\Delta C_{D_O} = C_{D_O}(\tilde{R}e, \tilde{V}) - C_{D_O}(Re, V)$$

Note that C_D in equation (15) should be evaluated at the body attitude selected for determining φ_{max}. The equations for the density, pressure, and scale height become

$$\left.\begin{aligned}
\frac{\tilde{\rho}}{\rho} &= \frac{\left(1 + \dfrac{\Delta a_R}{a_R}\right) \sec \varphi_{max}}{\dfrac{C_{D_O} + \Delta C_{D_O}}{C_D}\left(1 + \dfrac{\Delta V}{V}\right)^2} \\
\frac{\tilde{p}}{p} &= \frac{g_o}{p}\int_0^t \rho V \sin \theta_E \frac{\left(1 + \dfrac{\Delta a_R}{a_R}\right)\left(1 + \dfrac{\Delta \sin \theta_E}{\sin \theta_E}\right) \sec \varphi_{max}}{\dfrac{C_{D_O} + \Delta C_{D_O}}{C_D}\left(1 + \dfrac{\Delta V}{V}\right)} d\tau \\
\frac{\tilde{H}_\rho}{H_\rho} &= \frac{\tilde{p}}{p}\frac{\rho}{\tilde{\rho}}
\end{aligned}\right\} \tag{16}$$

Equations (12), (14), and (16) constitute the central results of this analysis. They can be used to estimate the accuracy to which an atmosphere can be defined from measurements of resultant acceleration. This is done as follows: First a model atmosphere is selected. Then, an accurate point-mass trajectory is computed with the selected model atmosphere and a given set of entry conditions. Finally, desired values for Δa_R, ΔV_E, $\Delta \sin \theta_E$, Δh_F, φ_{max}, C_D, and C_{D_O} are selected and equations (12), (14), and (16) are used to calculate $\tilde{\rho}$, $\tilde{p}$, $\tilde{H}_\rho$, and $\tilde{h}$.

Note that total trajectory time t_F plays an important role in this analysis so that the method of calculating the reference trajectory must be sufficiently precise to give reasonably accurate time histories of speed and altitude. Generally, the familiar analytic solution derived by Allen and Eggers (ref. 4) for steep entries with the assumption of zero gravity cannot be used. The reason is that in many situations the entry body will slow to terminal speed before impact with the planet surface and a considerable portion of the total trajectory time can be accumulated during this terminal descent. If gravity is neglected, the computed terminal speed can easily be in error by more than a factor of 2. Under these circumstances, values of time to traverse given altitude changes would be in error. An accurate trajectory can be calculated either by solving equations (9b) for the reference trajectory (e.g., ref. 5) or by integrating the exact equations numerically.

DISCUSSION

Results given by the present technique will be illustrated by showing the accuracy with which extreme model atmospheres of the planet Mars can be deduced from acceleration measurements. The models of the Mars atmosphere given in appendix C of reference 6 will be used. To check their accuracy the approximate analytic results will be compared with results derived by the numerical method described in reference 3. The latter results are essentially exact, since no approximations are made either to the governing equations or their solutions. Only the density variation with altitude will be considered since it is the quantity least accurately determined from acceleration measurements. Since, from aerodynamic considerations, the optimum entry body shape for this experiment is an aerodynamically stabilized sphere (ref. 3), most of the discussion will pertain to this shape. In particular, the accuracy with which atmosphere density as a function of altitude can be measured will be shown when errors in measured acceleration, entry speed, entry angle, and final altitude are assumed to occur both singly and combined in the most unfavorable fashion. Results will also be presented to indicate the effect of data sampling frequency on the description of density variation with altitude. Finally, some estimates will be made of the penalties incurred when entry bodies other than spheres are used. Except where otherwise noted, a value of the ballistic parameter at entry $(m/C_DA)_E$ of 0.25 slug/ft^2 will be assumed. Likewise, a typical entry speed of 26,000 ft/sec will be used. The coefficient of drag of the spherical shape will be assumed to be independent of Reynolds number but will be allowed to vary with flight speed in the manner shown in figure 1. The data given in figure 1 represent a mean fairing of sphere drag coefficient data for the range of Reynolds numbers appropriate to the entry conditions considered.

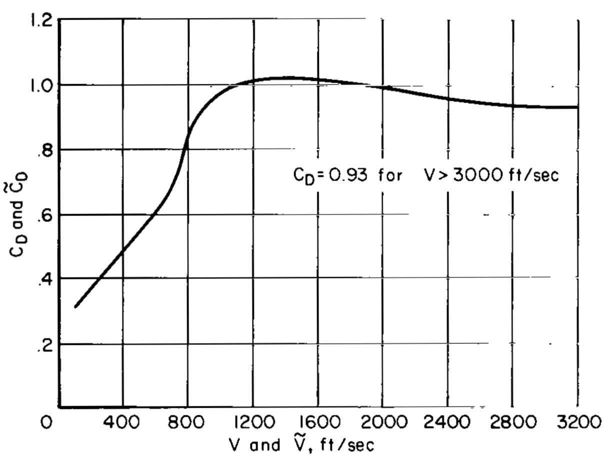


Figure 1.- Assumed variation of the drag coefficient with flight speed for a spherical body.

Results Obtained From Accelerations Measured Throughout the Entire Entry

Consider first the effects of entry angle on the accuracy with which the extreme model atmospheres for Mars can be determined by means of accelerometers with full-scale ranges chosen to match the specific entries. These are shown in figure 2 wherein biased errors in resultant acceleration equal to 0.1 percent of the maximum values experienced have been assumed. It is shown for

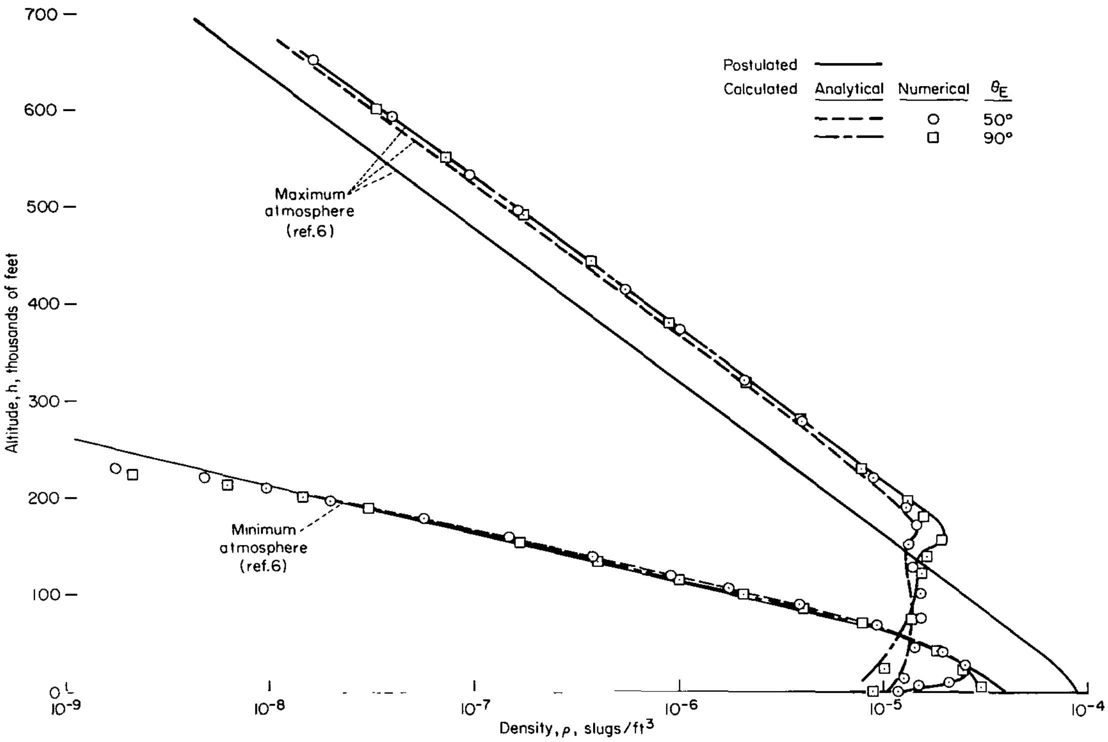


Figure 2.- Effect of entry path angle on the definition of extreme model atmosphere for Mars: spherical body; $(m/C_DA)_E = 0.25$ slug/ft^2; $V_E = 26{,}000$ ft/sec; biased acceleration error $\Delta a_R = -0.001\ (a_R)_{max}$.

these circumstances that entry angle affects the accuracy of atmosphere determination only slightly but that results are quite different, depending upon the model atmosphere encountered. The accuracy with which the maximum scale height atmosphere can be determined is poorer than for the minimum atmosphere, primarily because the flight time through the former is so much longer than through the latter. Errors are accumulated over a time period nearly six times longer in the maximum atmosphere than in the minimum atmosphere. Note that the method gives results in excellent agreement with those obtained numerically from the exact equations.

The results shown in figure 2 are idealized in the sense that the accelerometer ranges are matched to the chosen conditions while, in practice, it may not be known which atmosphere within the possible extremes will be encountered on a first mission into the Martian atmosphere. This means that the accelerometers will have to be designed to accommodate any magnitude of acceleration that can possibly be experienced. The maximum possible acceleration will occur during entry into the minimum scale height atmosphere with the steepest flight-path angle. It has a value of about 438 times the gravitational acceleration at the surface of Mars. For illustrative purposes, it will be assumed that the accelerometers must be designed to measure accelerations as large as 450 g_o with an accuracy of 0.1 percent of this value (0.45 g_o). The most unfavorable situation from the standpoint of atmosphere definition clearly would be that of designing for the maximum possible accelerations and actually experiencing the minimum values. The minimum values correspond to entering the maximum scale height atmosphere with the shallowest path angle. Therefore, emphasis will be placed on these conditions.

Figure 3 shows the additional penalty of designing for the largest accelerations in the minimum atmosphere and actually encountering the maximum atmosphere. The additional penalty in atmosphere definition is unacceptable. In order to gain insight into what steps might be taken to improve matters, this extreme case will be examined in more detail. This can be done with the aid of the results of figure 4. These results show how the error in atmosphere definition is divided among errors in measured accelerations, errors in speed due to integrating these acceleration errors, and errors in altitude due to integrating the errors in speed. It is shown that the atmosphere can be described quite well if speed and altitude are measured accurately by an independent method. The definition is equally good over a less extensive range of altitude if only the altitude is determined by an independent method. The altitude at which speed errors begin to increase rapidly corresponds to a speed of about 2000 ft/sec. The largest errors are shown to result from errors in altitude. Although the altitude errors are large over the entire trajectory, they are, in fact, accumulated largely over the low-speed part of the trajectory where speed errors are large. The high-altitude results are degraded by these errors only because the altitude at entry is not known a priori so that changes in altitude are, in effect, determined relative to the final altitude which is assumed to be zero at probe impact. These considerations suggest that either the accelerometer accuracy must be improved considerably or else different methods for measuring atmosphere properties must be used once the probe speed is reduced to low values. The improvements in atmosphere definition that can be expected to result from both of these techniques will be discussed next.

The effects of increasing accelerometer accuracy by factors of 2 and 10 are shown in figure 5, and the effects of terminating the accelerometer experiment at speeds ranging from that for impact at 378 ft/sec up to 1000 ft/sec are shown in figure 6. The results in figure 5 show that about an order of magnitude increase in accelerometer accuracy (to 0.01 percent of the maximum measurable) would be needed to provide good definition of the atmosphere at all altitudes. This accuracy is believed to be beyond that obtainable with either single or dual range accelerometer systems. The most attractive practical means for improving the definition of the atmosphere at altitudes where accelerations are large appears to be to accept data only over that range of altitudes for which the speed is greater than sonic speed. The results of reference 2 indicate that speed, altitude, and atmosphere properties can be determined well by direct measurement at subsonic speeds. For the remainder of this discussion, it will be assumed that the accelerometer experiment would be terminated at a speed of 750 ft/sec. Of course, errors in the quantities measured at low speed by the methods described in reference 2 will affect the accelerometer experiment because of its dependence on altitude at the final time - in this case, that corresponding to a flight speed of 750 ft/sec.

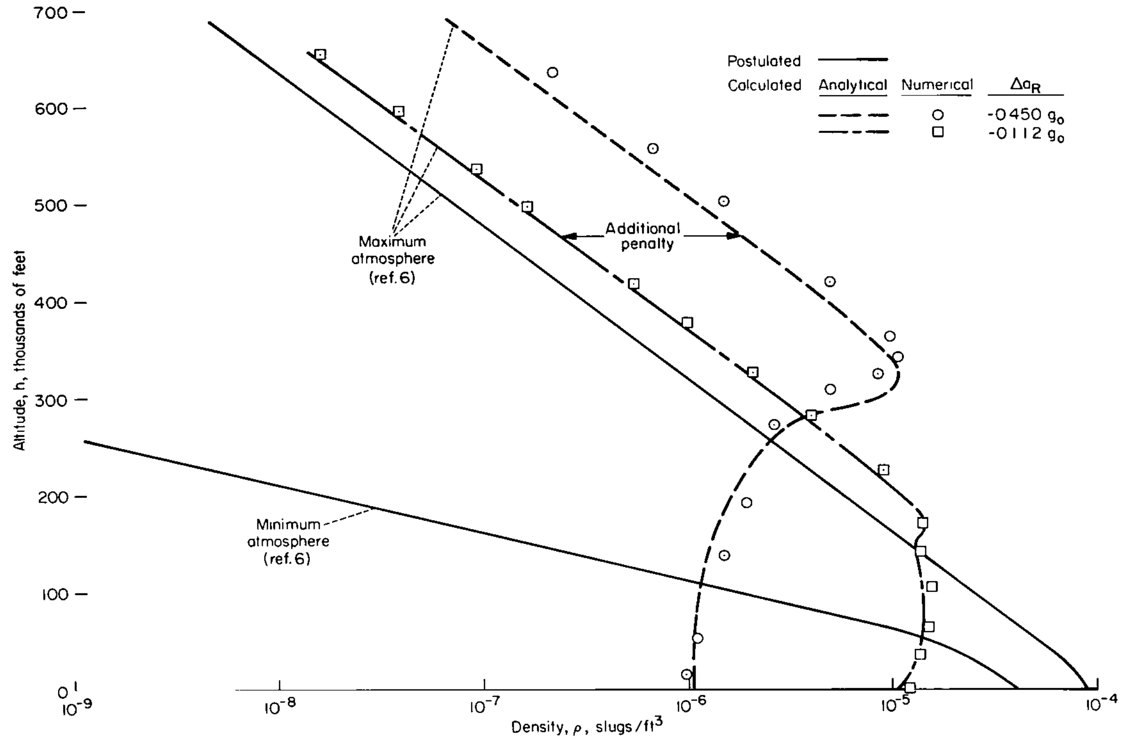


Figure 3.- Effect of magnitude of biased acceleration error on the definition of a model Mars atmosphere: spherical body; $(m/C_DA)_E$ = 0.25 slug/ft^2; V_E = 26,000 ft/sec; θ_E = 50°.

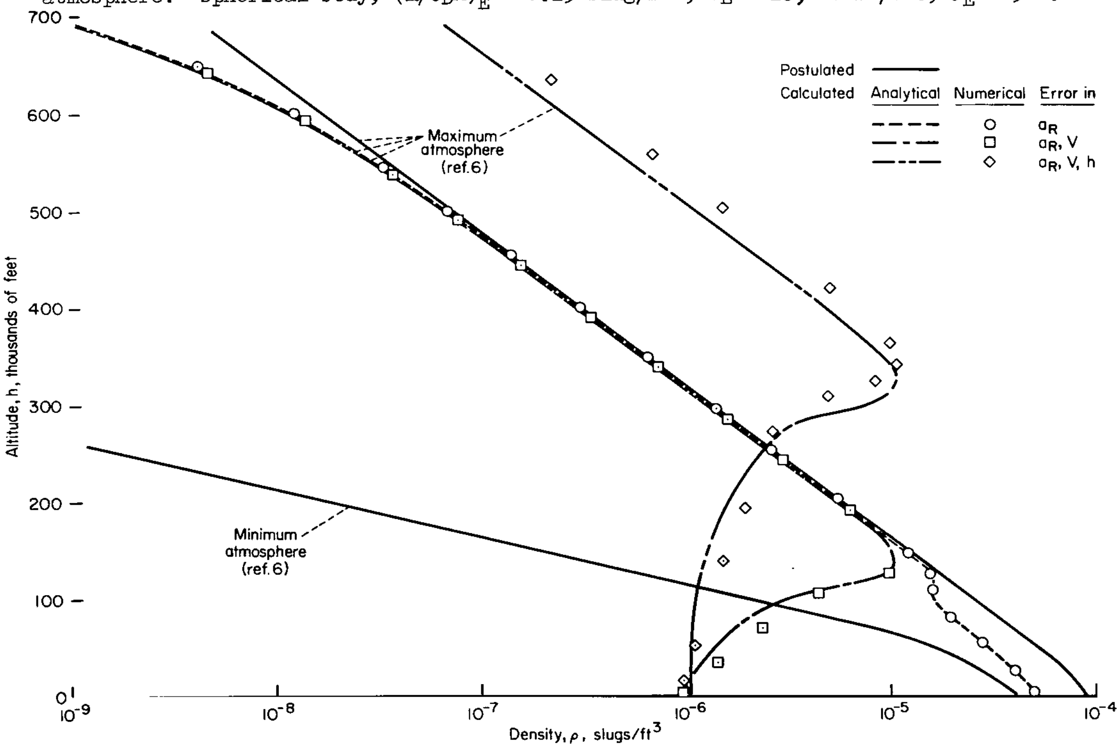


Figure 4.- Division of errors, in the definition of a model Mars atmosphere, between errors in resultant acceleration, speed and altitude: spherical body; $(m/C_DA)_E$ = 0.25 slug/ft^2; V_E = 26,000 ft/sec; θ_E = 50°; biased acceleration error Δa_R = -0.45 g_o.

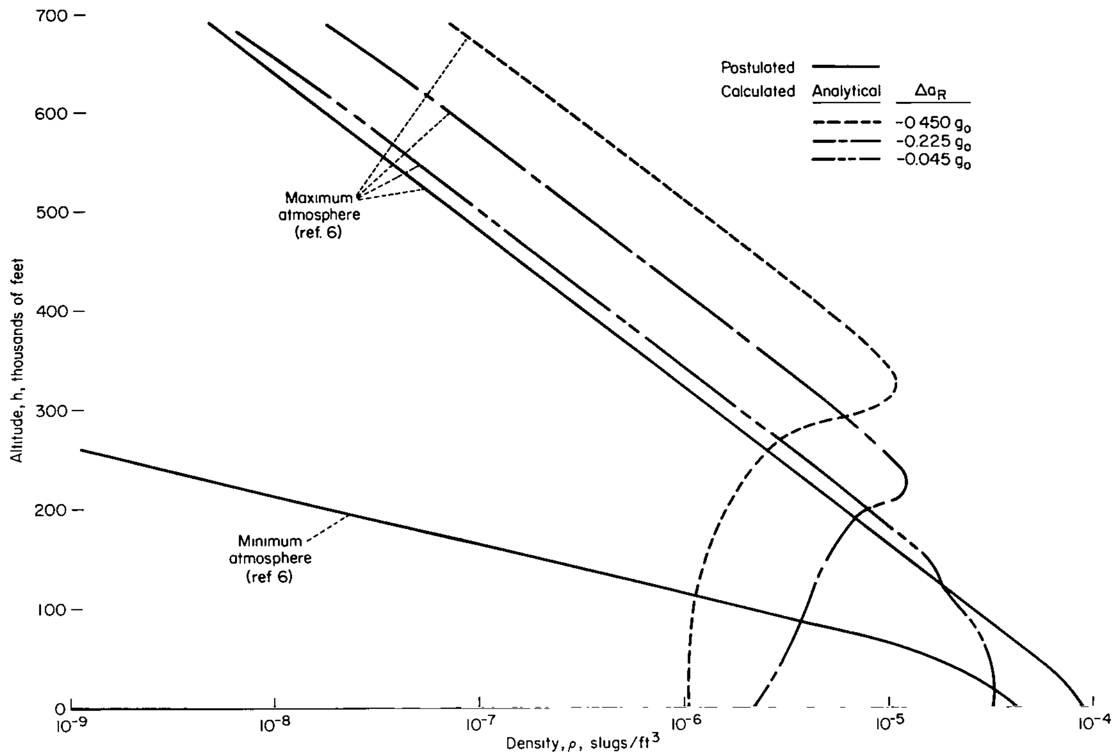


Figure 5.- Effect of accelerometer accuracy on the definition of a model Mars atmosphere: spherical body; $(m/C_DA)_E$ = 0.25 slug/ft^2; V_E = 26,000 ft/sec; θ_E = 50°.

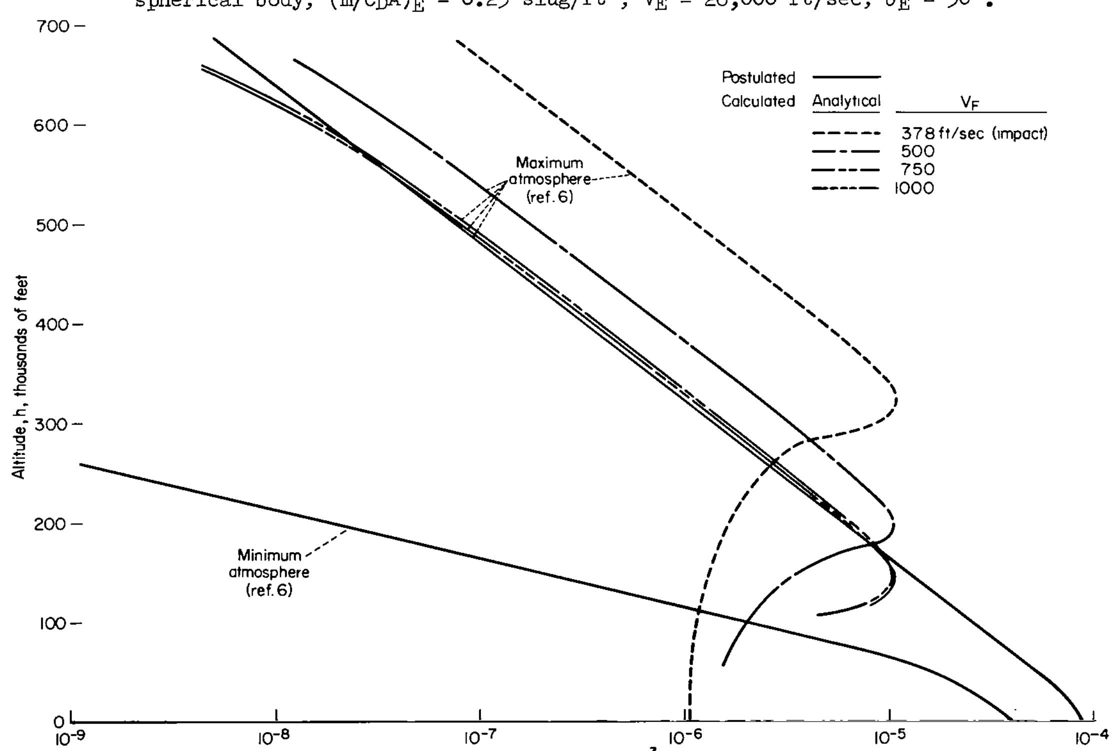


Figure 6.- Effect of speed at which accelerometer experiment is terminated on the definition of a model Mars atmosphere: spherical body; $(m/C_DA)_E$ = 0.25 slug/ft^2; V_E = 26,000 ft/sec; θ_E = 50°; biased acceleration error Δa_R = -0.45 g_o.

Effects of errors in the low speed measurements can be included in this study by the assumption of various errors in the final altitude.

Results Obtained From Accelerations Measured Only at Speeds Greater Than 750 Ft/Sec

First, attention will be directed at an examination of the individual and combined effects on atmosphere definition caused by less stringent accelerometer accuracies and errors in entry speed, entry angle, and altitude at the final speed. Then, consideration will be given to the effects of data sampling frequency. Finally, estimates will be made of the errors caused by the use of nonspherical entry bodies.

Effects of errors in measured acceleration, entry speed, entry angle, and final altitude.- The effects of reducing accelerometer accuracy by factors of 2 and 3 are illustrated in figure 7. Over most of the altitude range the effects vary linearly with the change in accuracy and are about the same magnitude for both positively and negatively biased errors. The comparison with results from numerical calculations is excellent throughout. The effects of errors in entry speed, entry angle, and final altitude are shown in figures 8, 9, and 10, respectively. In each case it was assumed that accelerations were known exactly, that is, $\Delta a_R \equiv 0$. It is seen that degradation of atmosphere definition resulting from reasonable values for each of these sources is not overwhelming, and in each case, the analytic method provides excellent predictions. It is noted that results of figure 10 show errors in altitude throughout the trajectory to be equal to the errors assumed to exist at the final speed. This means that the inaccuracy of the accelerometer experiment due to errors in various methods for measuring the atmosphere directly at low speed can be readily determined since altitude is the common link between the high and low speed methods.

The quality of atmosphere definition obtained when positive and negative errors in measured accelerations, entry speed, entry angle, and final altitude are combined in the most unfavorable manner is shown in figure 11. As expected, if the minimum scale height atmosphere were encountered, it would be defined more accurately than would the maximum scale height atmosphere should it be encountered instead. In either case, the present uncertainty of the Mars atmosphere would be reduced considerably by the measurements. Results of calculations not presented herein showed the data of figure 11, which are for values of speed and m/C_DA at entry of 26,000 ft/sec and 0.25 slug/ft^2, respectively, to be essentially the same for values of entry speed ranging from 22,000 to 28,000 ft/sec and values of m/C_DA at entry ranging from 0.20 to 0.35 slug/ft^2, provided the ranges for the accelerometers were adjusted for each combination of V_E and m/C_DA.

Effect of data sampling frequency.- The effects of data sampling frequency cannot be easily evaluated by analytic techniques. This possible source of degradation of the accelerometer measurements arises from the practical necessity of minimizing the amount of data transmitted from a probe to Earth. All of the results presented so far were based on the assumption that the acceleration measurements were described by a sufficient number of points

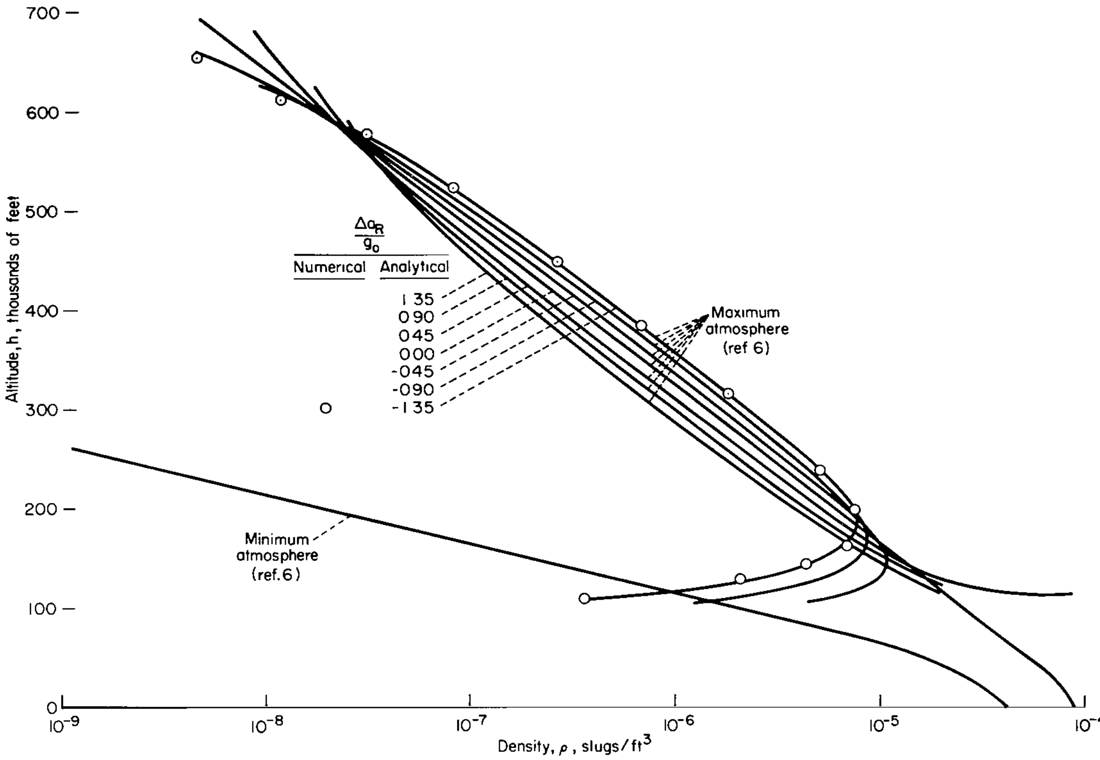


Figure 7.- Effect of accelerometer accuracy on the definition of a model Mars atmosphere: spherical body; $(m/C_DA)_E = 0.25$ slug/ft^2; $V_E = 26{,}000$ ft/sec; $\theta_E = 50^\circ$; $V_F = 750$ ft/sec.

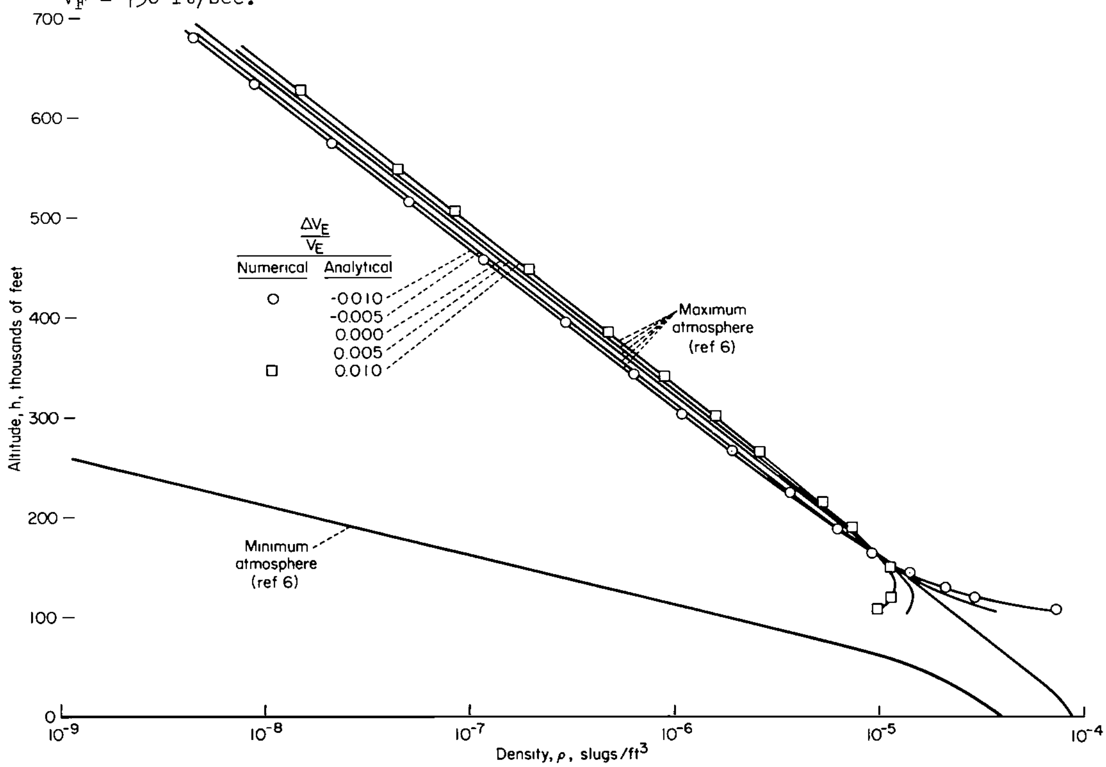


Figure 8.- Effect of errors in entry speed on the definition of a model Mars atmosphere: spherical body; $(m/C_DA)_E = 0.25$ slug/ft^2; $V_E = 26{,}000$ ft/sec; $\theta_E = 50^\circ$; $V_F = 750$ ft/sec.

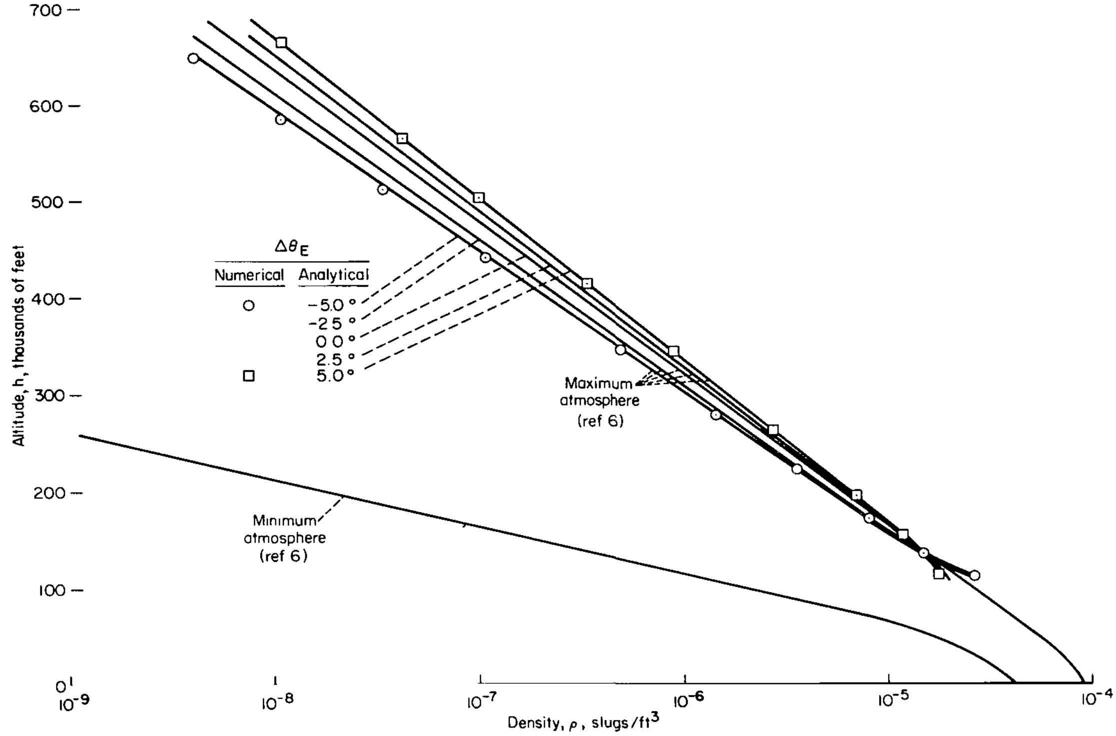


Figure 9.- Effect of errors in entry angle on the definition of a model Mars atmosphere: spherical body; $(m/C_DA)_E$ = 0.25 slug/ft^2; V_E = 26,000 ft/sec; θ_E = 50°; V_F = 750 ft/sec.

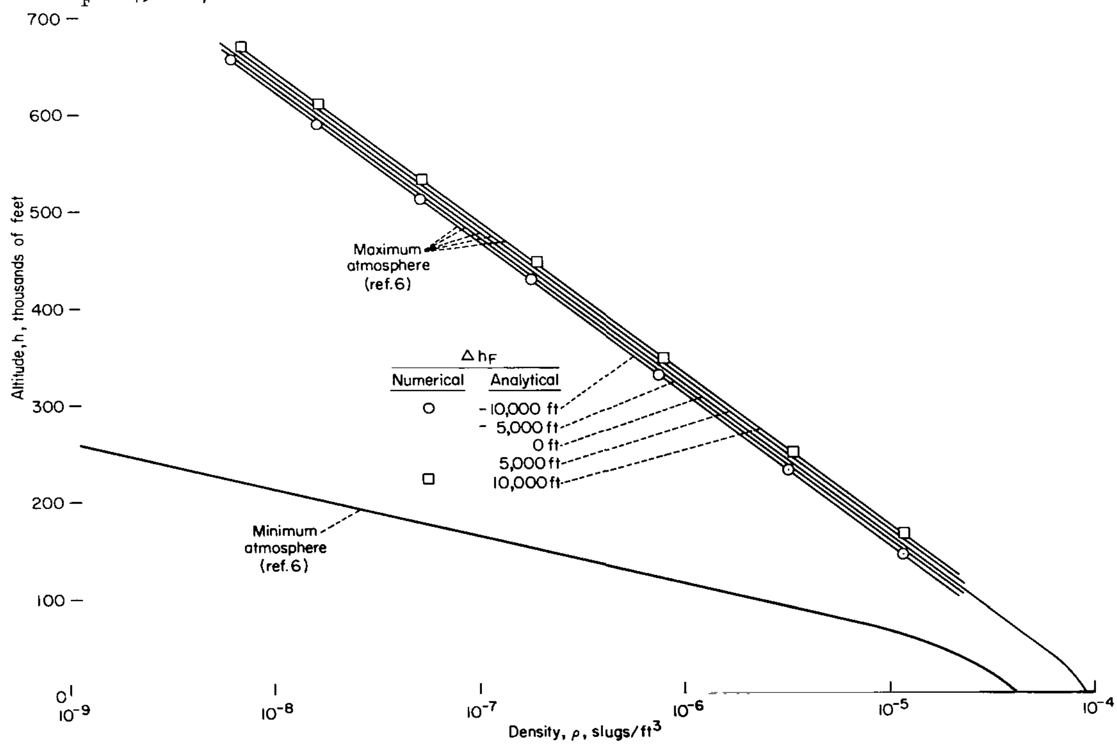


Figure 10.- Effect of errors in final altitude on the definition of a model Mars atmosphere: spherical body; $(m/C_DA)_E$ = 0.25 slug/ft^2; V_E = 26,000 ft/sec; θ_E = 50°; V_F = 750 ft/sec.

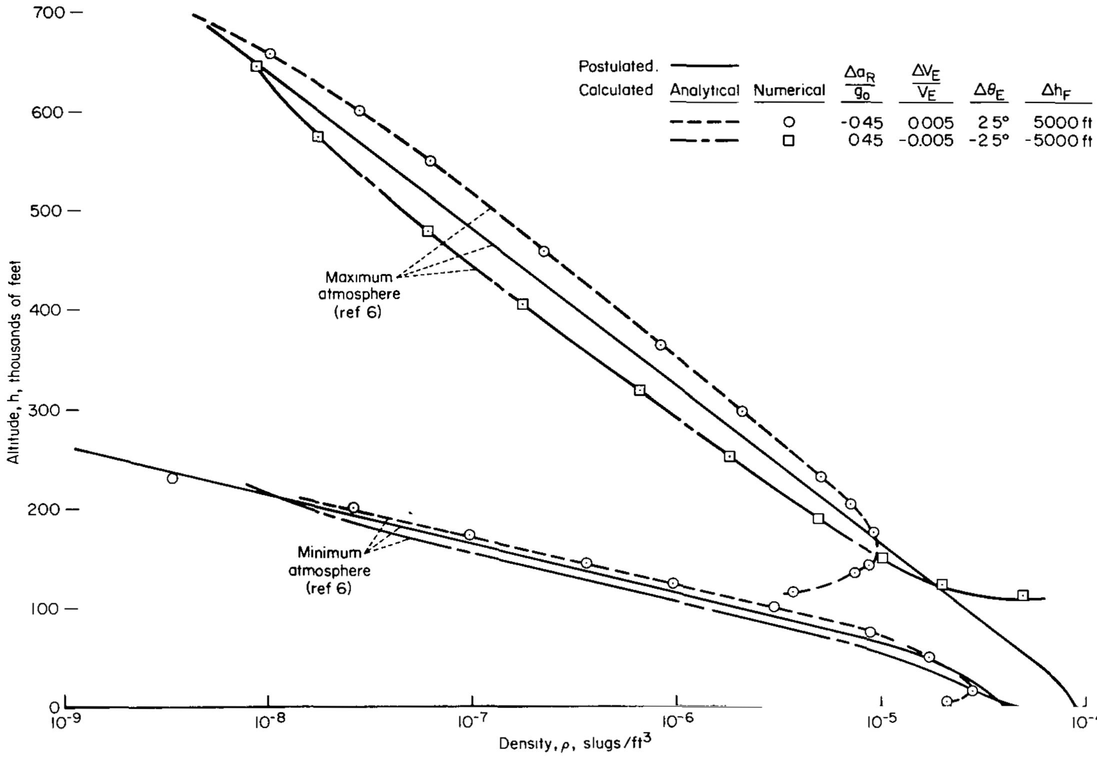


Figure 11.- Effect of errors in combination on the definition of extreme model atmosphere for Mars: spherical body; $(m/C_DA)_E = 0.25$ slug/ft^2; $V_E = 26{,}000$ ft/sec; $(V_F)_{min} = 750$ ft/sec; $\theta_E = 50^{\circ}$ and 90° for maximum and minimum atmospheres, respectively.

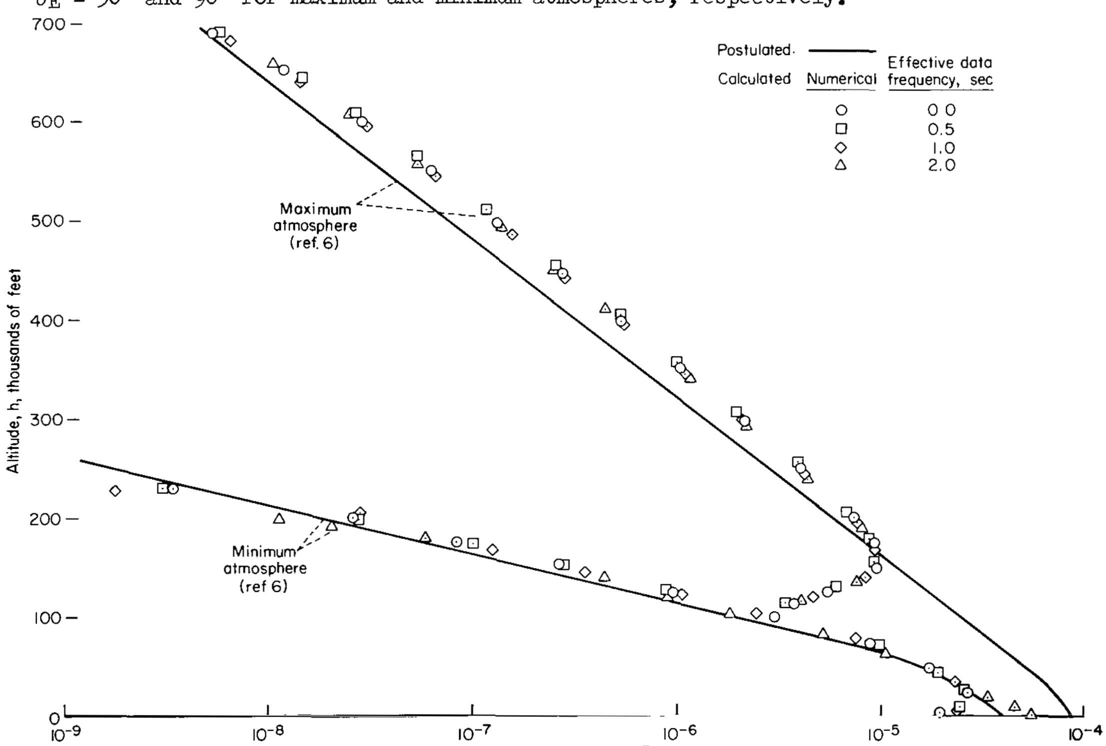


Figure 12.- Effect of time interval between measured accelerations on the definition of extreme model atmosphere for Mars: spherical body $(m/C_DA)_E = 0.25$ slug/ft^2; $V_E = 26{,}000$ ft/sec; $(V_F)_{min} = 750$ ft/sec; biased acceleration error $\Delta a_R = -0.45\ g_o$; $\theta_E = 50^{\circ}$ and 90° for maximum and minimum atmospheres, respectively.

to be effectively infinite. In other words, increasing the amount of data could not improve the accuracy. The effect of reducing the frequency of data sampling to allow for as much as 2 seconds of time between data points is shown in figure 12. These results were obtained by means of the numerical method of reference 3. Second-order interpolation on the logarithm of measured acceleration as a function of time was used. Note that these calculations also include a representative combination of errors from other sources. The magnitude of the sampling intervals considered has very little effect on the quality of atmosphere definition except at the very low density levels where the atmosphere is first sensed by the instruments and at very low altitudes in the minimum atmosphere case. These results are considered remarkable in view of the fact that the acceleration time history is defined by only 12 data points for the 90° entry into the minimum scale height atmosphere when the data sampling frequency is 2 seconds.

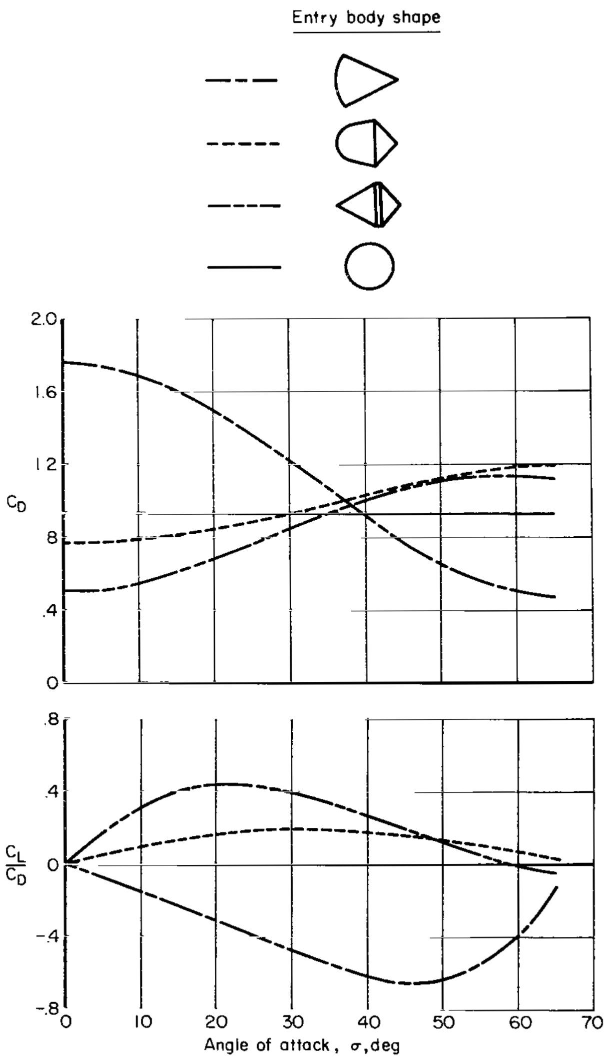


Figure 13.- Aerodynamic characteristics obtained from Newtonian impact theory for various entry body shapes.

Effects of using entry bodies other than a sphere.- Estimates have been made of the errors in atmosphere definition caused by the use of entry bodies other than spheres. These estimates were made for the shapes illustrated in figure 13. The aerodynamic characteristics presented in figure 13 were obtained from Newtonian impact theory. It is obvious from the data in figure 13 that conclusions regarding the relative merits of the various entry body shapes will depend strongly on the angle of attack selected for the comparison. In this discussion, a common angle of attack of 20° has been chosen; that is, the drag coefficient and the lift-to-drag ratio parameter φ_{max} were evaluated at this angle of attack for each configuration. These values were used in equations (12), (14), and (16) to obtain the results presented in figure 14. The aerodynamic characteristics were assumed invariant with Reynolds number and speed. No errors from any sources other than that related to unknown angle of attack were included in these calculations. The results of figure 14 show that the penalty in atmosphere definition associated with the angle of attack uncertainty makes two of the bodies definitely not suited for this application; however, the penalty for the body most closely resembling a sphere is not excessive. It will be recalled that all of the assumptions

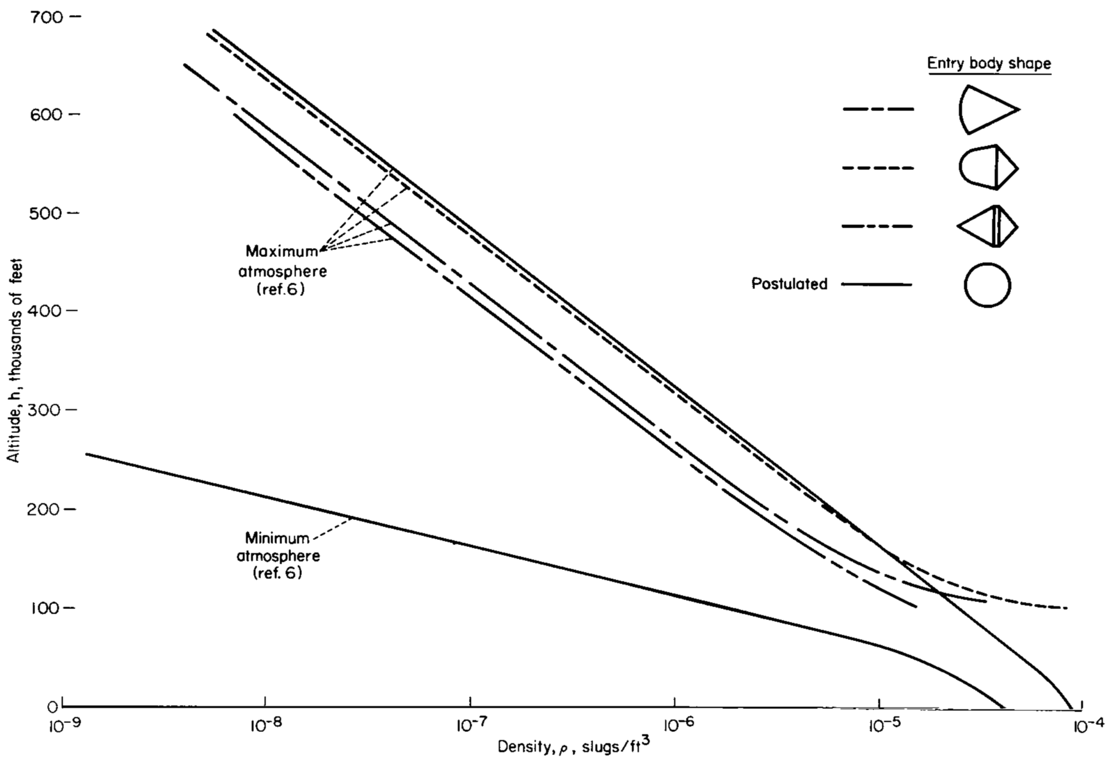


Figure 14.- Effect of entry body shape on the definition of a model Mars atmosphere: $(m/C_DA)_E = 0.25$ slug/ft^2; V_E = 26,000 ft/sec; θ_E = 50°; V_F = 750 ft/sec; maximum angle of attack assumed to be 20°.

made in the analysis, for deriving these results, were such as to make the effects of errors extreme. Actually, the errors due to using nonspherical configurations should not be as large as those shown in figure 14 for the same entry conditions. Also, the possibility exists of improving accuracy by correcting measurements for angle of attack. The body attitude can, in principle, be inferred from three-component accelerometer data and known aerodynamic characteristics. This possibility is discussed in more detail in references 2 and 3.

CONCLUDING REMARKS

An approximate method has been developed for estimating the errors in the density and pressure structure of a planetary atmosphere constructed from measurements of accelerations experienced by an entry vehicle. Among the sources of error considered both singly and in combination are accelerometer inaccuracies, uncertainties in entry speed, entry angle, final speed and altitude, aerodynamic characteristics, and frequency of data measurements. The effects of uncertainty of the attitude of entry bodies having nonzero lift-to-drag ratios were also considered.

The method was applied to the problem of estimating the accuracy with which the extremes of a range of postulated atmospheres for Mars could be defined by measurements of the accelerations experienced by a spherically shaped entry body. It was concluded that use of the accelerometer method for obtaining atmosphere structure should probably be confined to the portion of the atmosphere traversed at speeds greater than sonic speed. For this situation, it was shown that any of the atmospheres within the extremes postulated for Mars can be defined reasonably well even when errors from all sources are assumed to be combined in the most unfavorable manner. Selected portions of the results were compared with results obtained from more precise numerical procedures and the agreement was found to be excellent.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., April 13, 1965

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