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Models of Uranus and Neptune

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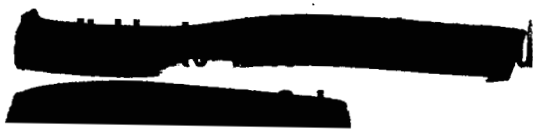
Abstract

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The equation of hydrostatic equilibrium and Clairaut's equation are integrated numerically to obtain mathematical models for solid bodies of planetary dimensions. Equations of state at 0° K for the more abundant materials within a solar mixture are estimated and mass-radius curves plotted for model planets composed of these materials. Sets of self-consistent models for Uranus and Neptune are computed upon the basis of a stated set of assumptions and the properties of these models are calculated and compared with available observational data. *Author*

Introduction

The composition and structure of the outer planets have been the subjects of numerous investigations since the calculation of a high pressure equation of state for metallic hydrogen by Kronig, De Boer, and Korringa [1946]. Wildt [1938] had previously indicated that a metallic phase of hydrogen should be of major importance regarding the composition of Jupiter and Saturn and that this equation of state presented the possibility of a quantitative approach to the problem. Detailed mathematical models have since been constructed for Jupiter and Saturn by various authors including DeMarcus [1951], Fesenkov and Masevich [1951], and Ramsey [1951]. DeMarcus [1958], in an exhaustive treatment of this



subject, concludes that these two planets are composed primarily of hydrogen in the metallic phase. He obtains values of 0.78 and 0.63 for the minimum weight fractions of hydrogen present within Jupiter and Saturn, respectively.

The higher mean densities of Uranus and Neptune, as shown in Table 1, imply that they must contain a much smaller proportion of hydrogen.

Table 1

Several previous attempts at the construction of models for Uranus and Neptune have been made, including the early work of Jeffreys [1923], Wildt [1947], and Brown [1950]. More recent efforts begin with Porter [1961] who computed 0° K models, with rotation, based upon the assumption of solid ammonium metal as the major constituent. He used an approximate equation of state for metallic ammonium due to Bernal and Massey [1954]. DeMarcus and Reynolds [1962] calculated equations of state at 0° K for a cosmic mixture of the nonhydrogen-helium elements and constructed nonrotating models of this material. Ramsey [1963] has obtained an equation of state for a cosmic mixture of CH_4 , H_2O , NH_3 , and Ne and has calculated four models for nonrotating masses composed of this mixture.

The relatively good agreement which has been reached within the literature regarding the composition of Jupiter and Saturn depends primarily upon the singular properties of hydrogen and the fact that hydrogen is the chief constituent of these planets. Quantitative calculations by DeMarcus and Reynolds [1962] have shown that the maximum weight fraction of hydrogen present within Uranus is about 0.23 and within

[REDACTED] and
[REDACTED]

Neptune about 0.14. The predominance of nonhydrogen material within Uranus and Neptune, as well as the relative lack of observational data, precludes the possibility of obtaining the type of well-defined solution to the compositional problem that has been developed for Jupiter and Saturn. Further assumptions are found to be necessary in the case of Uranus and Neptune.

It is the purpose of this investigation to develop, on the basis of a definite set of assumptions, a group of models of Uranus and Neptune which are consistent with the presently available physical data, and are compatible with current ideas regarding the development of the solar system. The difficulties and uncertainties are discussed and it is emphasized that, although the range of possible models may be restricted and bounds obtained upon the amounts of certain constituents, the current data are inadequate for the establishment of a unique model.

Primordial Composition and Subsequent Development of Uranus and Neptune

The assumption is made here that Uranus and Neptune were formed from a primitive solar nebula, having a so-called solar composition and which existed in low temperature chemical equilibrium. This assumption follows from the hypothesis of planetary formation by means of the accretion of material from a cold primitive nebula as developed by Spitzer, Urey, Hoyle, and others. The relative abundances of the constituent elements were taken from Aller [1961], with the exception of helium, whose abundance was modified slightly to conform with a 10:1 hydrogen to helium ratio by

number. Table 2 lists the relative abundances of the more abundant elements. The total mass of all the remaining elements combined amounts to only 1 per cent of the nonhydrogen-helium-neon portion of Aller's solar mixture and will be neglected here.

Table 2

Chemical equilibrium of this modified Aller mixture at the low temperatures characterizing the outer regions of the solar system will result in the silicon and magnesium being present in the form of oxides. The remaining oxygen as well as the carbon, nitrogen, and sulfur will exist in the form of hydrides. Iron and nickel are considered to remain in the uncombined state. The relative proportions of these elements and compounds are given by Table 3.

Table 3

The temperature of a small, rapidly rotating particle in equilibrium with the Sun and in a nearly circular solar orbit may be expressed by the relation

$$T = \frac{T_{\odot}}{\sqrt{2}} \left(\frac{R_{\odot}}{a} \right)^{1/2} (1 - A)^{1/4}$$

where

T_{\odot} effective temperature of the Sun

R_{\odot} radius of the Sun

a semimajor axis of the particle orbit

A albedo of particle

If we take the present temperature and radius of the Sun as representative values which should not have changed drastically over the period of planetary formation, we may obtain particle temperatures at the orbital distances of Uranus and Neptune. For particles which are blackbodies

(albedo = 0) the temperature is 64° K at Uranus and 51° K at Neptune. An albedo of 50 per cent will lower these values to 54° K and 43° K, respectively. A comparison of these figures with the data of Table 4 shows that the abundant materials which have a significant vapor pressure at room temperature are divided sharply into two groups on the basis of their melting points.

Hydrogen, helium, and neon remain gaseous at these temperatures while the remaining materials listed in Table 4 will exist predominantly in the solid phase. For purposes of calculation the hydrogen, helium, and neon are grouped together, in the proportions given by Table 3, to form a mixture labeled HHeNe. The nonvolatile materials are separated into two groups. The hydrides and argon are added, in cosmic proportions, to form a mixture designated here as Ice. This includes H_2O , CH_4 , NH_3 , H_2S , and Ar. The same procedure is employed for the oxides of silicon and magnesium and for iron-nickel and this mixture is called Rock.

Table 4

Calculation of Models

Two approximations may be made for large solid bodies which greatly simplify the calculation of mathematical models. First, the structural strength of the material comprising such bodies is greatly exceeded by the effect of the internal pressures at all depths below a negligibly small distance from the surface; hence, the equation of hydrostatic equilibrium may be used. Secondly, for solid bodies having the masses and dimensions characteristic of the outer planets and having central temperatures of

the order of a few thousand degrees, the effects of temperature will be completely overridden by pressure effects, and the temperature of the constituent materials may be considered to be effectively 0° K. The models presented in this paper are computed on the basis of 0° K equations of state and compared directly to the planets without allowance for temperature correction. There are several indications that the internal temperatures do not exceed a few thousand degrees but, at present, the evidence is not conclusive enough to rule out completely the possibility of temperature effects large enough to require consideration.

If the pressure-density relation for a 0° K gravitating fluid is known, the equation of hydrostatic equilibrium,

$$\frac{d^2P}{ds^2} - \frac{1}{\rho} \left(\frac{d\rho}{dP} \right) \left(\frac{dP}{ds} \right)^2 + \frac{2}{s} \left(\frac{dP}{ds} \right) + 4\pi G\rho^2 - 2\omega^2\rho = 0$$

where

P pressure

ρ density

s mean radius

G gravitational constant

ω angular velocity

which is given here to second order in the angular velocity, may be integrated from the center to the surface and the pressure and density distributions computed as a function of the mean radius within the model.

For a rotating planet with a known density distribution the ellipticity may be calculated. For bodies having the dimensions and angular velocities of Uranus and Neptune, the first-order equation

derived by Clairaut provides sufficient accuracy.

$$\frac{d^2\epsilon}{ds^2} + \left(\frac{8\pi\rho s^2}{m}\right) \frac{d\epsilon}{ds} + \left(\frac{8\pi\rho s}{m} - \frac{6}{s^2}\right) \epsilon = 0$$

where

$$\epsilon \text{ ellipticity} = \frac{(\text{equatorial radius} - \text{polar radius})}{(\text{equatorial radius})}$$

m mass contained within sphere of radius s

This system of equations was programmed for solution by digital computer and the numerical integration carried out by means of a variation of the Runge-Kutta technique.

Equations of State

Before the parameters of a model may be calculated, the equation of state for each constituent must be determined. Equations of state for 'cold' (0° K) hydrogen and helium are taken from DeMarcus [1959]. The equation of state of the HHeNe mixture may be expressed (assuming that the partial molar volumes of hydrogen, helium, and neon are additive) by the relation

$$\frac{1}{\rho_{\text{HHeNe}}} = \frac{w_{\text{H}}}{\rho_{\text{H}}} + \frac{w_{\text{He}}}{\rho_{\text{He}}} + \frac{w_{\text{Ne}}}{\rho_{\text{Ne}}}$$

For the proportions given in Table 3 the weight fractions are:

$w_{\text{H}} = 0.7105$, $w_{\text{He}} = 0.2822$, and $w_{\text{Ne}} = 0.0073$. Neon contributes less than 1 per cent to the density of the mixture and, hence, may be neglected in the calculation of the equation of state.

Since more than half of the nonvolatile portion of Aller's solar mixture is H_2O , the equation of state of H_2O is of prime interest.

Static pressure data are available from the work of Bridgman [1942] and a compilation of earlier work by Dorsey [1940] to about 50 kb. Walsh and Rice [1957] have published shock-wave data for water to 450 kb along a Hugoniot. Al'tshuler, Bakanova, and Trunin [1958] and Al'tshuler and Petrunin [1961] give a few pressure-density data points as high as 800 kb from shock-wave measurements. The shock-wave data are for H₂O in the liquid phase and, for the high pressures, at temperatures of the order of several thousand degrees Kelvin. For the program of model calculation outlined here these data must be corrected to correspond with the solid phase at 0° K.

Rice and Walsh [1957] have calculated isotherms at several temperatures from their Hugoniot data to 250 kb. Pistorius, et al. [1963] have determined the melting curve of ice VII to 200 kb. This curve was extrapolated to 250 kb and the density of liquid water obtained, along this fusion curve, by interpolation from the tables of Rice and Walsh. The density difference between liquid H₂O and ice VII is known from Bridgman's data to almost 40 kb. This density difference is decreasing and, on linear extrapolation, would become vanishingly small in the vicinity of 110 kb. The pressure-density relation for solid H₂O can thus be estimated along the melting curve by correcting for the effect of this phase change. This procedure gives the equation of state of solid H₂O along the fusion curve to 250 kb. The increase in uncertainty involved in extrapolating these corrections to higher pressures is partially counterbalanced by the fact that the density corrections to the Hugoniot data become smaller at the higher pressures. These corrections were applied to the experimental

data, giving values to 800 kb. (The density differences due to thermal expansion of the solid from 0° K to the fusion curve are small and may be neglected.) This temperature corrected curve for the solid was joined smoothly to a quantum mechanical equation of state for H₂O which should be valid at extremely high pressures. The interpolation is indicated in Figure 1. The high-pressure equation of state was constructed by a method suggested by Knopoff [1958]. The Thomas-Fermi-Dirac equation of state was employed for oxygen, calculated by the procedure of Metropolis and Reitz [1951], and the DeMarcus [1959] metallic equation of state was used for hydrogen. The two elements were mixed with a 2:1 number ratio, hydrogen to oxygen, under the assumption that the partial molar volumes are additive. This process of a smooth interpolation does not allow for the possibility of phase changes in the region of interpolation. Ramsey [1963] has proposed a large phase change for H₂O as the factor which determines the differing mean densities of Uranus and Neptune. At the present time, however, no evidence exists, either experimental or theoretical, to support this hypothesis.

Fig. 1

With the pressure-density relation for a material known, the equation of hydrostatic equilibrium may be integrated for a given central pressure and the mass and radius of the resulting model planet determined. This computation was made for various central pressures and a set of nonrotating models of H₂O. These models are represented on a mass-radius plot (Figure 2), along with similar curves for hydrogen and helium. Models of solid H₂O for Moon-sized objects with central pressures below 50 kb were constructed by Huauax [1951].

Fig. 2

The equations of state for CH_4 , NH_3 , and H_2S were calculated by means of similar procedures. The pressure-density relations for these compounds are less well determined than that for H_2O , however, since shock-wave data are not available in the intermediate pressure range. Low-temperature data are available from the work of Stewart [1959, 1960] up to 20 kb. These data were extrapolated to 100 kb by means of the Murnaghan-Birch equation of finite strain. High-pressure equations of state were constructed by combining the Thomas-Fermi-Dirac equations of state for carbon, nitrogen, and sulfur with the equation of state for 0°K hydrogen by means of the same process as outlined for H_2O . Smooth interpolations were then performed between the low pressure data and the high pressure equations of state. The equation of state of argon can be estimated in a like manner. 0°K equations of state for H_2O , CH_4 , and NH_3 are given in Table 5.

Only 8 per cent of the total mass of the Ice mixture is NH_3 while H_2S and Ar are even less abundant. The properties of the model planets are, consequently, not extremely sensitive to the equations of state of these substances. Fortunately the greatest amount of experimental data is available for H_2O , which is also the most abundant material.

For methane, however, a more difficult problem exists. No shock-wave data exist for intermediate pressure ranges and CH_4 forms more than one-fourth of the Ice mixture. The interpolation procedure outlined here is preferred, in spite of the uncertainties involved, over more rigorous appearing equations which, in addition to being based upon extremely insecure theoretical foundations, do not agree with the available high and low pressure information.

Table 5

The equation of state of Ice is then expressed by the relation

$$\frac{1}{\rho_{\text{Ice}}} = \frac{w_{\text{H}_2\text{O}}}{\rho_{\text{H}_2\text{O}}} + \frac{w_{\text{CH}_4}}{\rho_{\text{CH}_4}} + \frac{w_{\text{NH}_3}}{\rho_{\text{NH}_3}} + \frac{w_{\text{H}_2\text{S}}}{\rho_{\text{H}_2\text{S}}} + \frac{w_{\text{A}}}{\rho_{\text{A}}}$$

using the densities given by the individual equations of state and the weight fractions quoted in Table 6.

Bridgman's [1949] data for MgO to 30 kb was extrapolated to 100 kb by means of the Murnaghan-Birch equation and the resulting curve was joined smoothly at high pressures to a theoretical curve derived by Trubitsyn [1958].

Table 6

Static data for crystalline quartz to 100 kb are available from Bridgman [1948], and Wackerle [1962] has published shock-wave data to 700 kb. As was demonstrated by Knopoff and Uffen [1954], the high pressure Thomas-Fermi-Dirac relations for SiO₂ and MgO are very similar since the mean atomic weights are almost identical. Hence the SiO₂ experimental data were extrapolated to join the MgO equation of state at high pressures. The equations of state for iron and nickel were taken from the work of Al'tshuler, Bakanova, and Trunin [1962] who obtained shock-wave data to 9 Mb. Interpolation between these equations and the Thomas-Fermi-Dirac equations of state permits extension of the pressure-density relation to higher pressures.

The equation of state for Rock is then given by

$$\frac{1}{\rho_{\text{Rock}}} = \frac{w_{\text{SiO}_2}}{\rho_{\text{SiO}_2}} + \frac{w_{\text{MgO}}}{\rho_{\text{MgO}}} + \frac{w_{\text{Fe-Ni}}}{\rho_{\text{Fe-Ni}}}$$

where the densities are given by the individual equations of state and the weight fractions are 0.599 for SiO₂, 0.319 for MgO, and 0.082 for Fe-Ni.

Equations of state for HHeNe, Ice, and Rock are tabulated in Table 7 along with a mixture, in solar proportions, of Ice and Rock which is labeled Homall.

The approximate character of these equations of state is readily seen from the extrapolations and interpolations which must be performed to obtain them. These equations do provide reasonable and consistent estimates of the pressure-density relations and permit the calculation of models which approximate Uranus and Neptune.

The equations of state for each of the mixtures tabulated in Table 7 were used to plot mass-radius curves (Figure 3) for nonrotating bodies.

Model Planets

As may be seen from Figure 3, the actual compositions of Uranus and Neptune must lie somewhere between that of a pure HHeNe mixture and that of a planet composed entirely of nonvolatile materials.

Model planets, having the observational values of the masses and radii of Uranus and Neptune, may be constructed from the previously defined mixtures by a straightforward, but extremely nonlinear, process of mixing the various materials in the proper proportions.

Since the gravitational differentiation of light and heavy materials may produce different degrees of internal condensation within a planet, models were constructed to investigate the consequences of such a gravitational settling process. Models representing three different stages of this process were calculated and their density distributions

Table 7

Fig. 3

are displayed in Figures 4 and 5. The density is plotted as a function of s/R where R is the mean radius of the planet.

Figs. 4,5

The first is a completely homogeneous model (Model I) having Aller's proportions with respect to the nonvolatile materials and with sufficient HHeNe mixture added to provide the proper mass and radius. The second or two-shelled model (Model II) consists of a core of the Homall mixture with an outer layer of HHeNe. The final or three-shelled model (Model III) has an inner core of Rock, a mantle of Ice, and an outer layer of HHeNe. The moment of inertia, the ellipticity, and the pressure distributions were computed for sets of these models corresponding to Uranus and Neptune. Certain parameters calculated from these models are listed in Table 8.

Table 8

Apart from the masses and radii, which are constrained to fit the observed values, the relevant observational data are limited to the constant J (the visual ellipticities are not reliable) which is a measure of the departure of the external gravitational potential of the planet from spherical symmetry. The constant J appears in the equation

$$V = \frac{-GM}{r} \left[1 - \frac{2}{3} J \left(\frac{a^2}{r^2} \right) P_2(\cos \varphi) + \dots \right]$$

where

- V external gravitational potential
- G gravitational constant
- M mass of the planet
- a equatorial radius
- r distance from the center of mass of the planet
- φ colatitude
- P_2 Legendre polynomial of order 2

The constant J may be evaluated from a study of satellite motions and is available for Neptune, although not for Uranus. The values of J for the models may be obtained from the relation

$$J = \epsilon - \frac{3\omega^2}{8\pi G \bar{\rho}}$$

where $\bar{\rho}$ is mean density. These values are tabulated for Neptune and Uranus in Table 8. The observed value for Neptune is given as 0.0074 ± 0.0007 by Brouwer and Clemence [1961]. The uncertainty in the observational value of J rules out the possibility of obtaining a unique solution, even under the restrictive assumptions which have been made. The value of J for Model II, however, is within the quoted margin of error and is therefore consistent with the present data. The two-shelled model for Neptune is tabulated in Table 9. Although the value of J for Uranus is not currently available, the two-shelled model for Uranus is presented in Table 9 for purposes of comparison since the essential similarity in mass and radius would suggest a similar internal density distribution for Uranus and Neptune, at least as a first approximation.

Table 9

Summary

Models of Uranus and Neptune have been constructed on the basis of the following sets of principal assumptions and the resulting models compared with the available physical data.

General assumptions involved in the calculation of a model for a giant planet are:

- (a) The equation of hydrostatic equilibrium prevails throughout the planet.

- (b) The observed physical constants (as listed in Table 1) are correct.
- (c) The densities of the materials present within the planets are accurately represented by the given equations of state.
- (d) The partial molar volumes of those materials are additive.

Additional assumptions which were required to permit the calculation of models of Uranus and Neptune are:

- (a) These planets were formed from a primitive solar nebula, in low-temperature chemical equilibrium, whose relative molecular abundances are given by the modified Aller mixture of Table 3.
- (b) Differentiation of the planetary material from the primitive nebula proceeds only through loss of HHeNe.

The weight fractions of the constituent materials were calculated for Models I, II, and III of both Uranus and Neptune and the values for certain constituents listed in Table 10.

The difference in mean density of the two planets is, on the basis of the above assumptions, primarily dependent upon the relative amounts of HHeNe retained by the two bodies. A comparison of values obtained from the two shell models shows that the fraction of HHeNe retained by Uranus is 2.7 times the fraction retained by Neptune. The amounts of hydrogen present, within the two-shell Neptune model, of some 15 per cent and, within the range of Uranus models, of 18 to 22 per cent agree quite well with previous estimates. The internal density distribution preferred for Neptune, Model II, indicates a moderate degree of internal condensation with the formation of a large relatively homogeneous core.

Table 10

It is concluded that mathematical models can be constructed for Uranus and Neptune, on the basis of the stated set of assumptions, which are consistent with the presently available information regarding these planets. The models obtained for both planets are composed primarily of the Ice mixture with approximately one-half of their mass composed of H_2O .

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TABLE 1.- Physical Parameters of the Giant Planets:

Planet	Mass, ($\times 10^{27}$ gm)	Radius, ($\times 10^8$ cm)	Mean density, (gm/cm ³)	Period
Jupiter	1902	69.89	1.35	9 ^h 52 ^m
Saturn	569.4	57.53	.71	10 ^h 24.5 ^m
Uranus	86.63	2.37	1.55	10 ^h 49 ^m
Neptune	102.7	2.23	2.21	15 ^h 48 ^m

Sources: Masses and radii of Jupiter and Saturn, R. Wildt

[1961]; Period of Uranus, C. Allen [1963]; all other

data, D. Brouwer and G. Clemence [1961].

TABLE 2.- Elemental Abundances According to Aller [1961]

Element	Relative abundance by number	Atomic weight	Relative abundance by mass
H	1000	1.008	1008
He*	100	4.003	400
O	.891	16.00	14.3
Ne	.501	20.18	10.1
C	.398	12.01	4.78
N	.112	14.01	1.57
Si	.0316	28.09	.888
Mg	.0251	24.32	.610
S	.0223	32.07	.715
Ar	.00759	39.94	.303
Fe	.00372	55.85	.208
Ni	.00089	58.69	.052

*H/He number ratio of 10/1 employed to obtain He abundance.

TABLE 3.- Relative Abundances of Modified Aller Mixture.

Element or compound	Relative abundance by mass
H	1004
He	400
Ne	10.1
H ₂ O	14.3
CH ₄	6.39
NH ₃	1.91
H ₂ S	.760
Ar	.303
SiO ₂	1.90
MgO	1.01
Fe + Ni	.260

TABLE 4.- Melting Points for Relatively Volatile Materials at Low Pressures.

Element or compound	Melting point, (°K)
He	Liquid at 0° K
H	14.1
Ne	24.5
A	84
CH ₄	89
NH ₃	196
H ₂ S	190
H ₂ O	273

TABLE 5.- Estimated Equations of State (0° K)

Pressure (Mb)	H ₂ O	CH ₄	NH ₃
	Density (gm/cm ³)	Density (gm/cm ³)	Density (gm/cm ³)
0.0	0.917	0.507	0.815
.025	1.53*	.760	1.05
.05	1.63	.863	1.19
.10	1.75	1.01	1.36
.50	2.50	1.54	2.06
1.0	2.87	1.88	2.50
2.0	3.33	2.32	2.99
3.0	3.68	2.66	3.32
4.0	3.99	2.92	3.58
6.0	4.50	3.34	4.01
8.0	4.98	3.70	4.34
10.0	5.44	4.50	4.70
12.0	5.89	4.37	5.04
15.0	6.51	4.80	5.55
20.0	7.24	5.43	6.27

*Several phase changes below 0.025 Mb.

TABLE 6.- Weight Fractions of Ice Constituents

Constituent	Weight fraction
H ₂ O	0.605
CH ₄	.270
NH ₃	.080
H ₂ S	.032
Ar	.013

TABLE 7.- Estimated Equations of State (0° K)

Pressure, (Mb)	HHeNe	Ice	Rock	Homall
	Density, (gm/cm ³)	Density, (gm/cm ³)	Density, (gm/cm ³)	Density, (gm/cm ³)
0.0	0.105	0.750 *	3.03	0.828 *
.025	.258	1.17	3.16	1.27
.05	.299	1.28	3.24	1.39
.10	.345	1.44	3.38	1.55
.50	.541	2.12	4.60	2.28
1.0	.691	2.50	5.15	2.68
1.9	.900	2.94	5.93	3.14
1.9	1.202	2.94	5.93	3.14
2.0	1.22	2.97	6.01	3.18
3.0	1.40	3.33	6.69	3.56
4.0	1.56	3.63	7.27	3.87
6.0	1.83	4.11	8.20	4.39
8.0	2.06	4.54	8.99	4.84
10.0	2.26	4.96	9.78	5.29
12.0	2.48	5.36	10.5	5.71
15.0	2.69	5.91	11.5	6.28
20.0	3.10	6.62	12.8	7.04

*Low pressure phase transitions.

TABLE 8.- Physical Parameters Calculated From Models

Planet	Model	Central pressure (Mb)	Surface ellipticity	Moment of inertia, ($\times 10^{48}$ gm/cm ³)	J
Neptune	I	5.51	0.0196	17.3	0.0099
	II	7.45	.0165	14.8	.0068
	III	13.38	.0155	13.9	.0058
Uranus	I	3.13	.0584	16.4	.0283
	II	5.90	.0439	12.1	.0138
	III	10.28	.0423	11.5	.0122

TABLE 9.- Model II for Uranus and Neptune

s/R	Uranus		Neptune	
	Pressure (Mb)	Density (gm/cm ³)	Pressure (Mb)	Density (gm/cm ³)
0.	5.90	4.36	7.45	4.72
.05	5.87	4.35	7.40	4.70
.10	5.76	4.32	7.28	4.67
.15	5.57	4.27	7.10	4.64
.20	5.33	4.22	6.86	4.58
.25	5.02	4.15	6.54	4.51
.30	4.65	4.06	6.18	4.42
.35	4.23	3.94	5.71	4.31
.40	3.78	3.81	5.23	4.19
.45	3.28	3.66	4.69	4.04
.50	2.61	3.48	4.13	3.89
.55	2.29	3.29	3.55	3.72
.60	1.76	3.07	2.94	3.54
.65	1.23	2.82	2.37	3.32
.70	.776	2.53	1.80	3.09
.773	.207	1.83	---	---
.773	.207	.413	---	---
.80	.168	.390	.752	2.52
.85	.109	.352	.327	2.05
.883	---	---	.109	1.58
.883	---	---	.109	.351
.90	.061	.310	.083	.334
.95	.023	.255	.033	.273
1.00	0	.105	0	.105

TABLE 10.- Chemical Composition of Models

Planet,	Model	W_{Ice}	W_{Rock}	W_{HHeNe}	$W_{\text{H}_2\text{O}}$	W_{H}
Neptune	I	0.807	0.108	0.085	0.488	0.186
	II	.848	.114	.038	.513	.159
	III	.848	.114	.038	.513	.159
Uranus	I	.730	.098	.172	.441	.235
	II	.790	.106	.104	.478	.196
	III	.790	.106	.104	.478	.196

Figure Titles

- Fig. 1.- Estimated equation of state for H_2O .
- Fig. 2.- Mass vs. radius for nonrotating $0^\circ K$ bodies.
- Fig. 3.- Mass vs. radius for nonrotating $0^\circ K$ bodies.
- Fig. 4.- Density distributions for models of Uranus.
- Fig. 5.- Density distributions for models of Neptune.

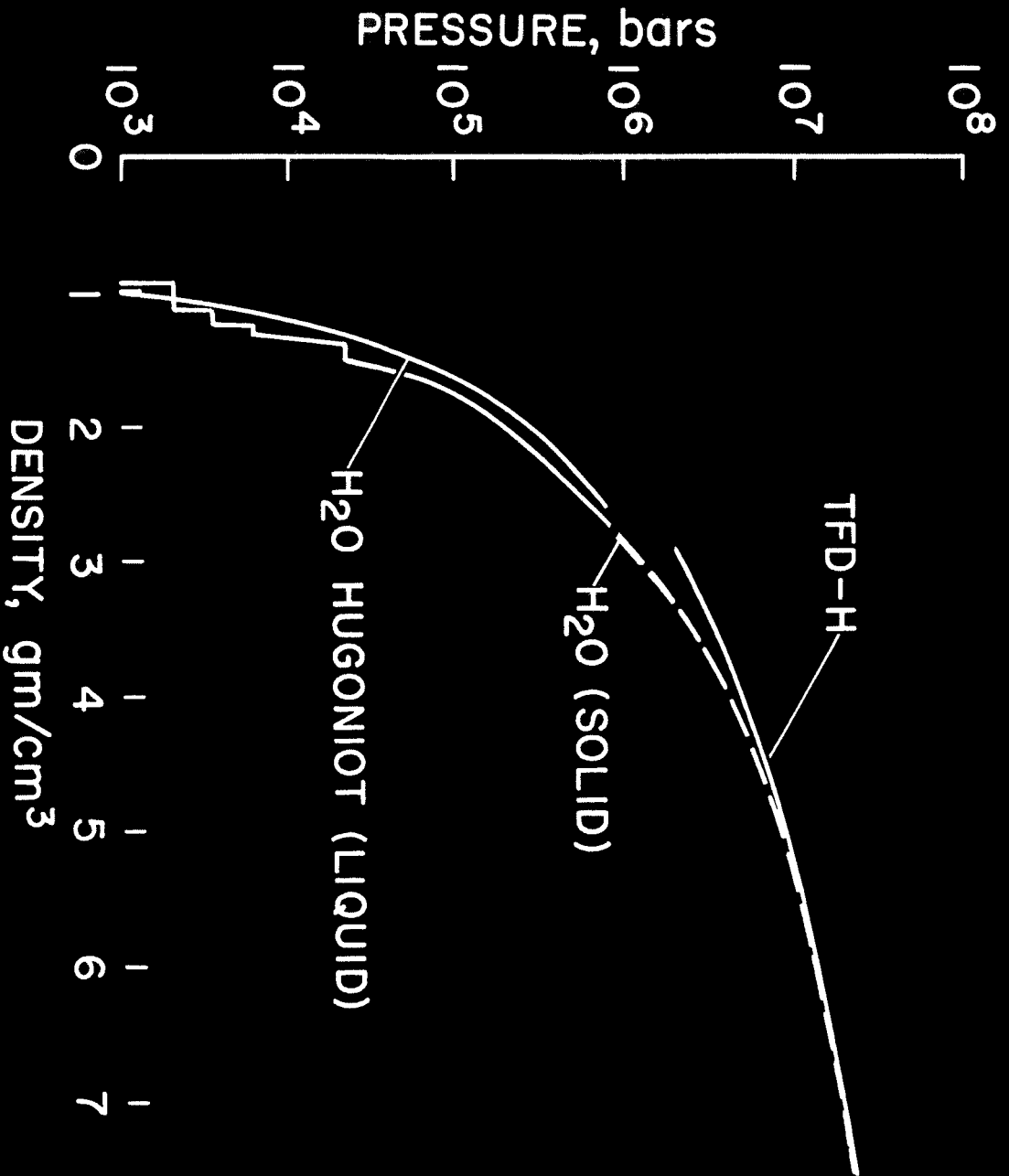


Fig. 1.- Estimated equation of state for H_2O .

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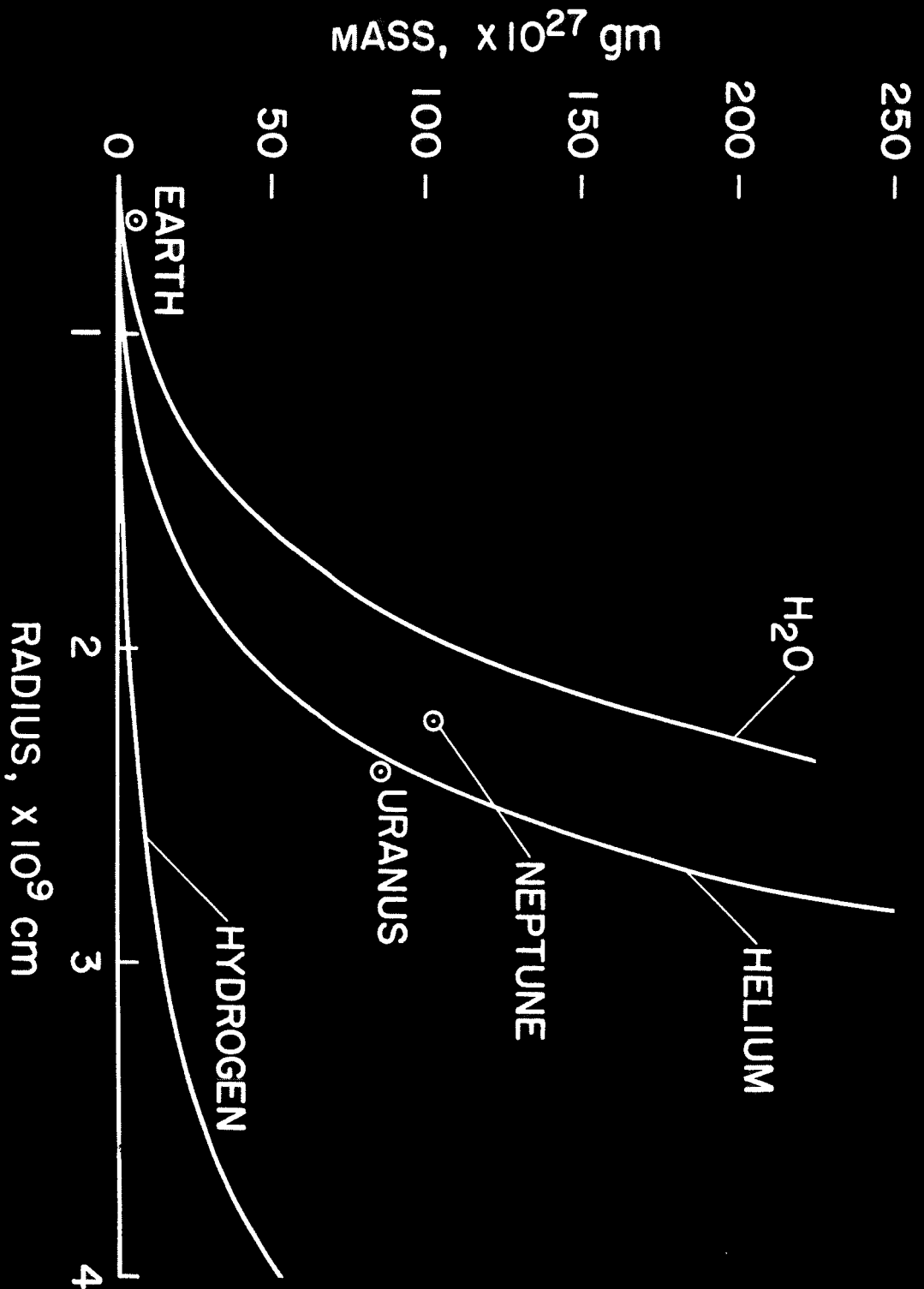


Fig. 2.- Mass vs. radius for nonrotating 0° K bodies.

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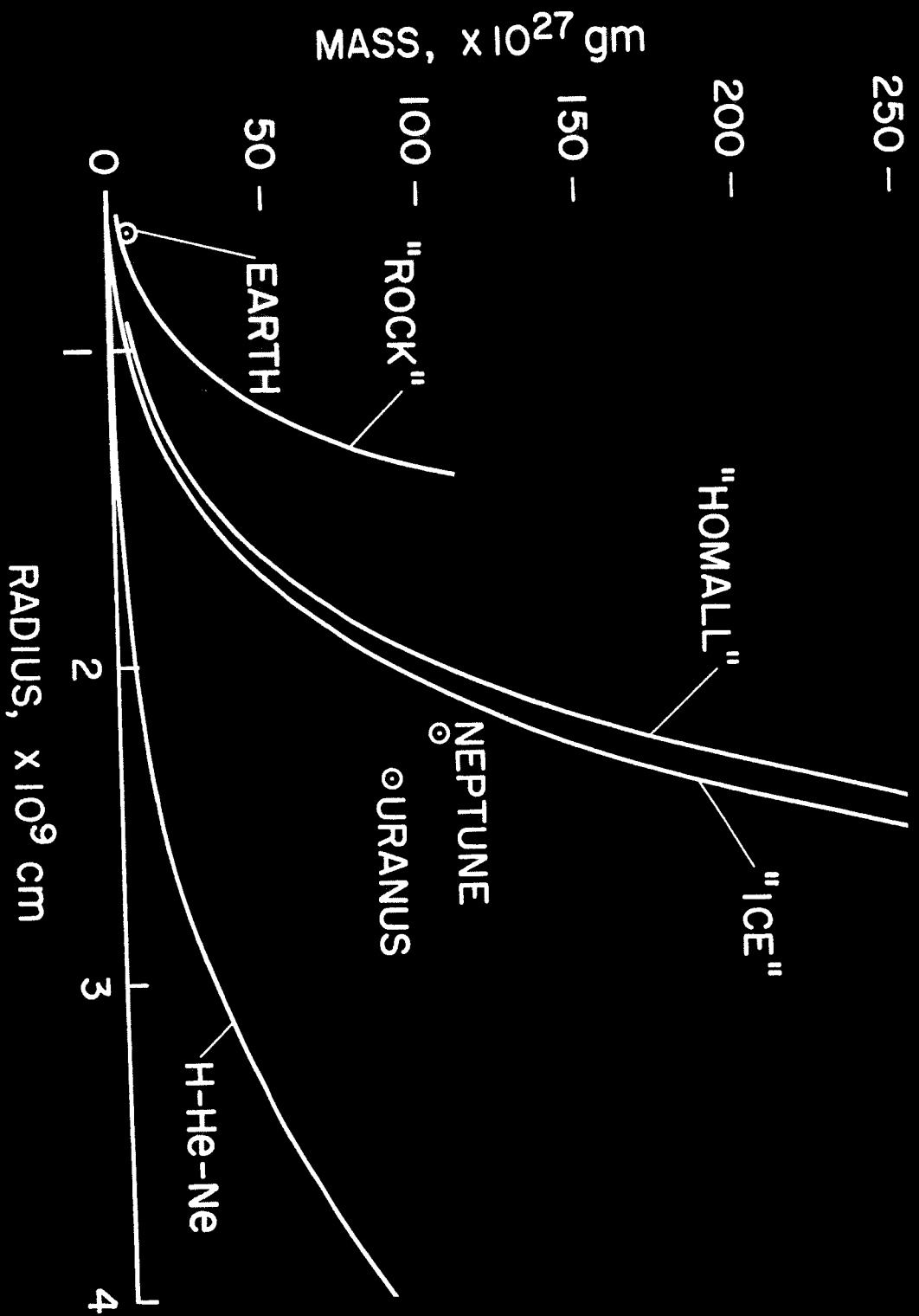


Fig. 3.- Mass vs. radius for nonrotating 0° K bodies.

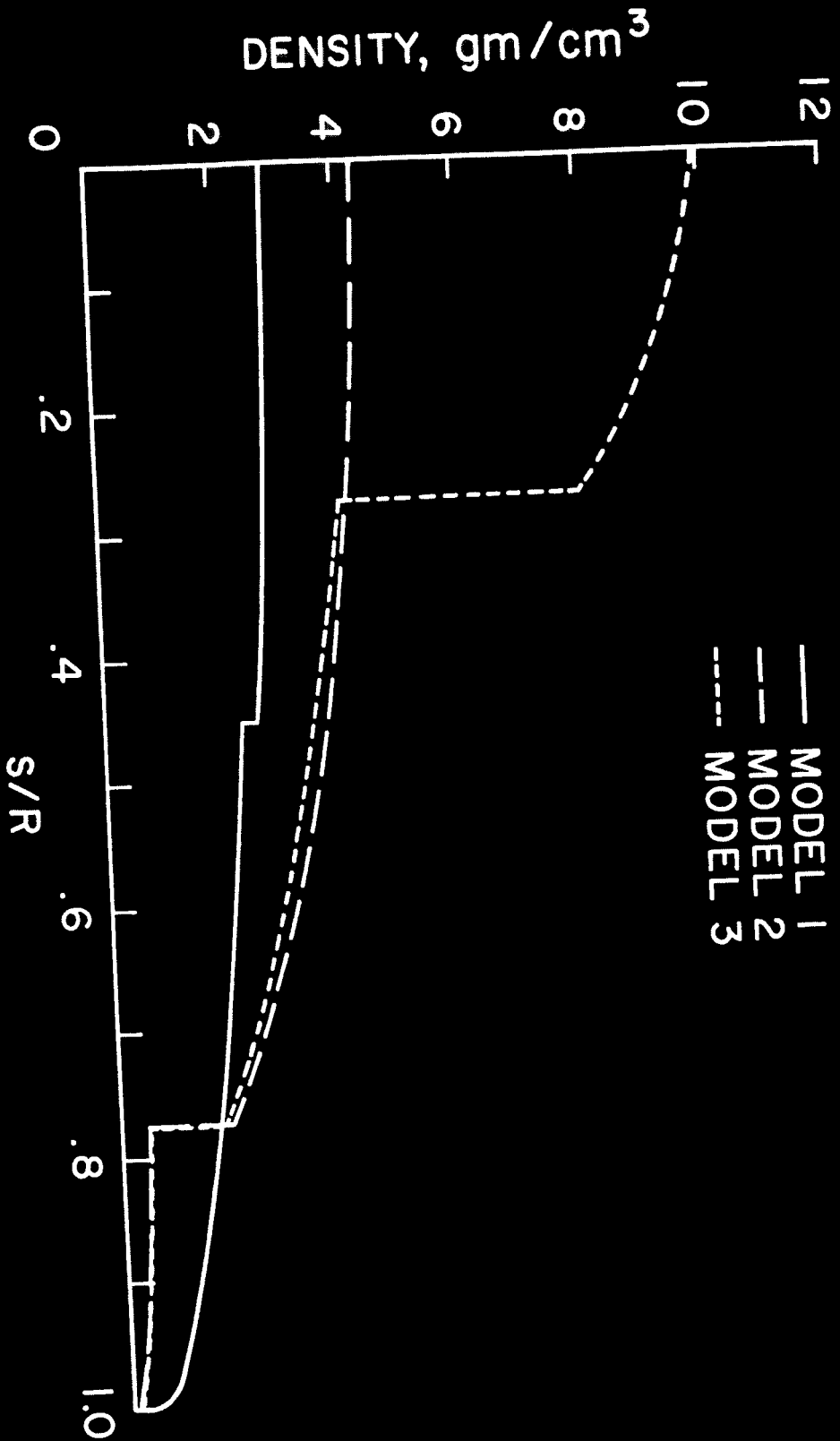


Fig. 4.- Density distributions for models of Uranus.

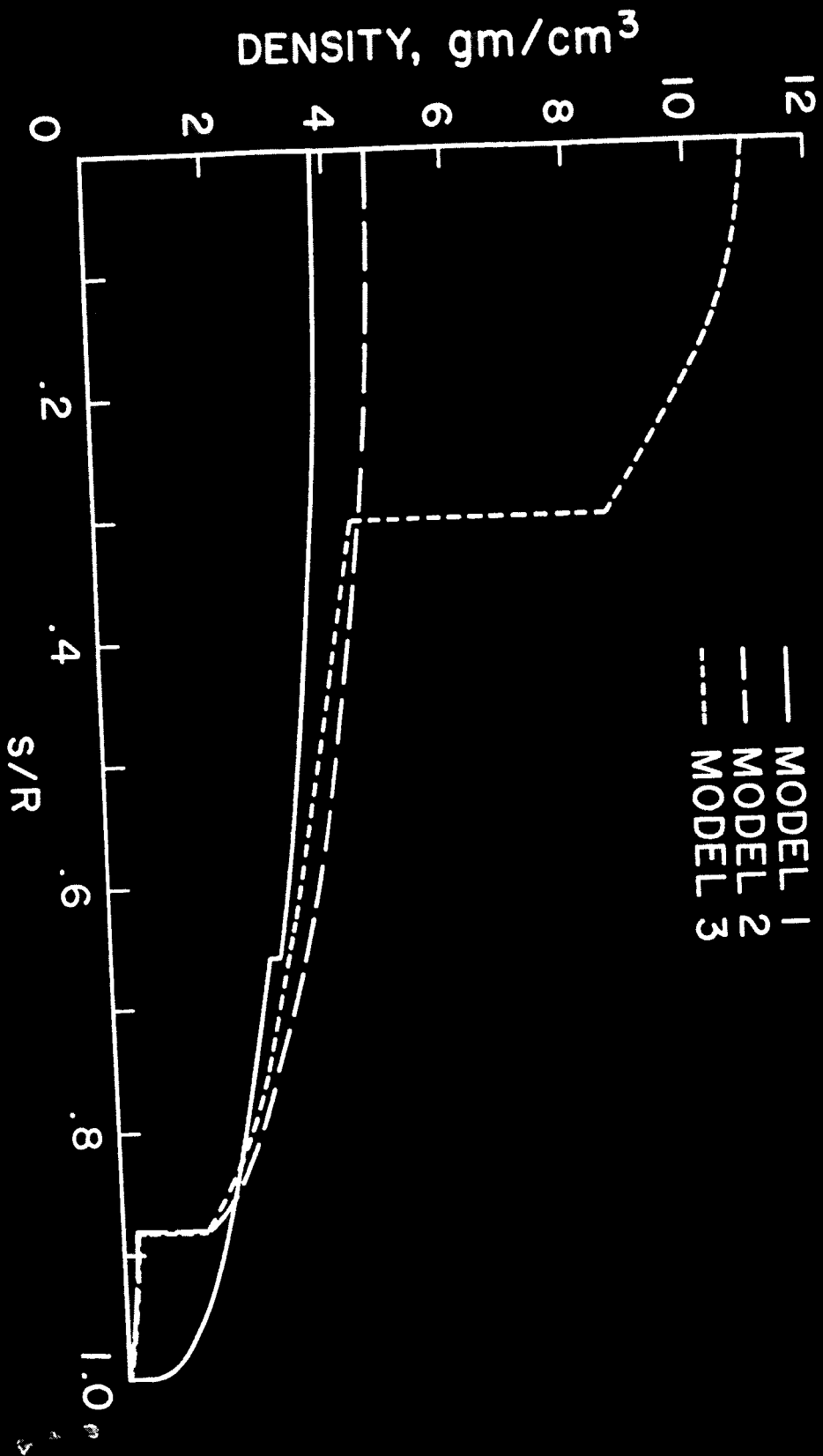


Fig. 5.- Density distributions for models of Neptune.

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