RESONANCES OF THE EARTH-IONOSPHERE CAVITY OBSERVED

AT CAMBRIDGE, ENGLAND[1]

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[1]Some of the results reported in this paper were presented at the
Symposium on Ultra Low Frequency Electromagnetic Fields, Boulder,
Colorado, August 17-20, 1964.

Abstract

A program of observations of the natural vertical electric field in the atmosphere in the band 1 to 30 c/s is discussed. After rejection of unwanted signals, the ELF (extremely low frequency) signal is amplified, its waveform being preserved as a photographic record which can be digitized automatically. The power spectral density of the field is computed, and at all times shows statistically significant peaks at 8 and 14 c/s, with evidence of other peaks at 20 and 26 c/s. These are the resonant frequencies of modes of the electromagnetic field in the earth-ionosphere cavity, excited by radiation from lightning discharges. A comparison is made between such spectra and those observed elsewhere.

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The spectra are also compared with those calculated theoretically. The agreement is only fair for a sharply bounded, homogeneous, isotropic ionosphere concentric with the earth, but better for Galejs' model ionosphere with an exponential conductivity vs. height profile. From the derived propagation constant, the attenuation rate in the band 7 to 30 c/s is found to be approximately 0.25 db/Mm, i.e., 0.25 db/1000 km, and the phase velocity 0.77 c. The half width of the thunderstorm belt about an equator (with respect to a pole at Cambridge) is found to be $44^\circ \pm 12^\circ$, in agreement with thunderstorm statistics. Diurnal and seasonal variations of the power in the various modes are interpreted in terms of nodes of the field and the movement of thunderstorm centers. The fine structure of the spectrum is investigated. A typical measured field strength of 0.34 mV/m in a 1 c/s bandwidth at 8 c/s is shown to be reasonable.

The energy in the background spectrum, which overwhelms that in the cavity resonance spectrum below 5 c/s, may be due to nearby lightning activity, local electrostatic effects, or extraterrestrial phenomena.

"There is music in the spacing of the spheres," Pythagoras.

1. Introduction

Schumann [1952] suggested that modes of the electromagnetic field in the dielectric shell between the earth and ionosphere might be excited to resonance by radiation from lightning discharges. His theory indicated that the observed resonant frequencies of the cavity would be

$$f_n{}' = f_1{}'\sqrt{n(n + 1)/2} \ , \quad \text{with} \quad f_1{}' \lesssim c/\sqrt{2}\pi R = 10.6 \text{ c/s}, \quad (1)$$

in which n is an integer (n being 1 for the fundamental mode), c is the velocity of light, and R is the earth's radius. The first positive evidence for the existence of this resonant cavity was published by Balser and Wagner [1960]. At that time an observational program concerning natural radio noise below 40 c/s was initiated in Cambridge, England. A full description of the apparatus used, together with a discussion of some preliminary results and a brief review of related investigations, was presented by Rycroft and Wormell [1962]. The aim of this paper is to present a sample signal spectrum (having demonstrated how this is obtained experimentally), a mean spectrum of some 30 records, together with their interpretation in the light of current theories on earth-ionosphere cavity resonances.

Natural electromagnetic phenomena in the frequency range of interest have been reviewed recently by Kleimenova [1963].

2. Experiments

The antenna used to measure the natural vertical electric field consists of an insulated horizontal wire, of radius 0.15 cm and length 190 m, supported at an average height of 4.5 m above a meadow some 2 km from the center of Cambridge. The signal induced is fed to a series of filters that remove extraneous signals due to powerful local transmitters, oscillations of the antenna excited by wind, and the main power supplies. The signal in the band 1 to 30 c/s is amplified by a low noise, high gain feedback amplifier, the frequency response of which is shown in figure 1. A comparison of the response obtained when a low impedance signal generator feeds the amplifier (upper curve) with that obtained when a repre-sentation of the antenna is used (lower curve) reveals that it is difficult to optimize the coupling between antenna and amplifier. Examples of the extremely low frequency (ELF) waveform obtained when an oscilloscope spot is photographed by film moving at 7.06 cm/sec are given in figure 2. Above each trace appears the signal in the band 0.5 to 8 kc/s; only for the strong atmospherics at the left side of figure 2(b) does the ELF waveform appear correlated with impulses recorded on the kc/s channel. In general, there is no correlation between individual atmospherics and the ELF trace.

It is evident from figure 2 that there are no predominant frequency components in the signal waveform. In order to search for cavity resonance effects, the amplitude of the ELF trace has to be digitized at intervals of 1.18 mm (corresponding to 1/60 sec), from which the power spectral density of the record up to 30 c/s has to be computed. An automatic digitizer, constructed in the laboratory, is used. Its modus operandi is to scan the film record with the well focussed image of a spot of light, the scanning mechanism being a rotating plane mirror actuated by a linear cam, cyclically each 3.60 ±0.03 sec. While the spot scans between a mask at the edge of the film and the trace, light shines through the transparent part of the film on to a photomultiplier, during which time pulses of a standard frequency are counted by eight binary counters. When the spot reaches the opaque part of the film, counting is stopped. The count, representing the amplitude of the ELF waveform, is transferred to a tape punch. The counters are reset to zero before the next cycle commences. As the film moves continuously slowly past the scanning region, the amplitude of the radial electric field as a function of time, $E_r(t)$, modified by the transfer function of the recording equipment, is obtained on paper tape as desired. Each film is digitized at least twice to eliminate possible eccentricities and malfunctions of the digitizer. Records of different straight lines and sine waves of various frequencies are also digitized. The tapes are translated into the language of the EDSAC computer, either with or without a cubic correction for the nonlinearity of the cam.

3. Computations

The power spectral density function of the digitized waveform is computed as the Fourier transform of its autocovariance function [Swinnerton-Dyer, 1963]. A program, developed by Swinnerton-Dyer, removes the mean from the time series, which is then convolved with chosen prewhitening and smoothing functions, and computes the estimated autocorrelation and smoothed power spectral density. The output estimate of the power spectral density is compensated for the prewhitening factor.

It is important to consider the reliability of each estimate. Every record analyzed is at least $T = 10$ sec long, composed of $N \geq 600$ points at $1/60$ sec intervals. The power spectral density is required at $(M + 1)$ intervals from 0 to 30 c/s; with $M = 30$ the spectrum is found at $W = 1$ c/s intervals. The number of degrees of freedom of the system is defined as $k = N/M = 2TW$ and is always ≥ 20. Zukerman [1961] has shown that, for $k \geq 20$, and 80-percent confidence limits, the estimate of the power spectral density is correct to within $+2.0$ db or -1.5 db. A similar conclusion is reached by Swinnerton-Dyer [1963], who shows that the estimate may plausibly be supposed to behave like X^2 on k degrees of freedom, and by Balser and Wagner [1960], who find the standard deviation of random fluctuations.

The power spectral densities of two series of 600 random numbers generated by the computer are plotted in figure 3. They are independent of frequency, but have a statistical scatter due to the finite length of the sample of $\leq \pm 1.5$ db. A 1.5 db peak standing out above the background level is therefore statistically significant. The power spectral density of a sample record is plotted in figure 4, the points being marked by crosses; peaks are evident at 0, 8.7, 13.1, 20.1, and 26.0 c/s. To test whether these peaks may be spurious ones associated with the d-c peak, a numerical rejection filter at 0 c/s constructed by the computer is applied to the digitized waveform. The power spectral density of this series is computed and plotted with open circle points in figure 4. The higher frequency peaks occurring at resonant frequencies of modes of the field in the earth-ionosphere cavity are statistically significant.

From the test waveforms that are digitized, it is concluded that the noise introduced by the digitizer is typically 30 db below that in the signal. Power spectral densities computed with and without the correction for the nonlinearity of the cam differ only insignificantly.

POWER, db

FREQUENCY, c/s

n=1
n=2
n=3
n=4
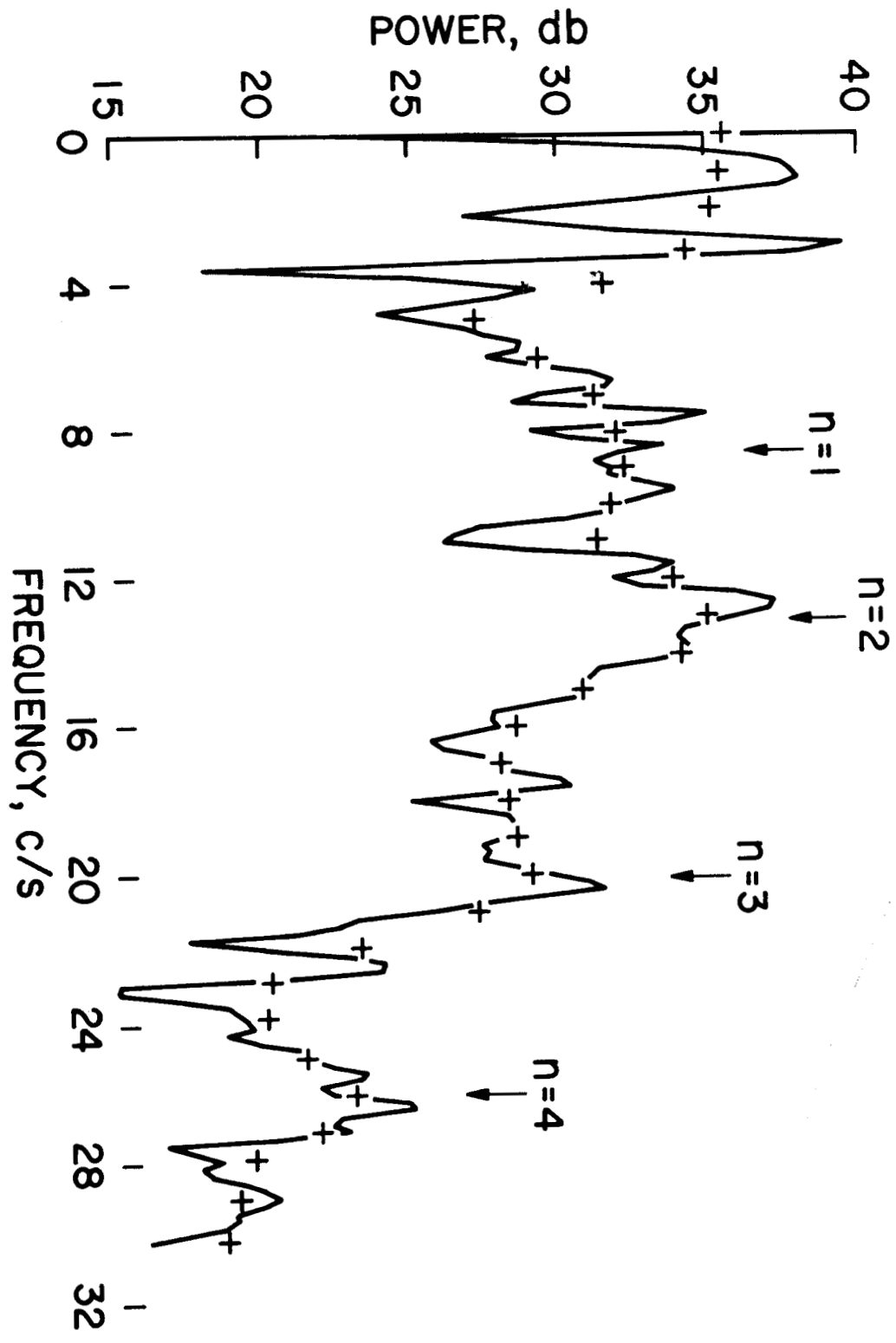
4. Spectra

The power spectral density of a sample of receiver noise obtained with the amplifier input grounded, which is typically >20 db below that of the ELF signal when new high-tension batteries are used in the amplifier, is subtracted from that of the signal. This spectrum is corrected for the receiver response (fig. 1) and is plotted in figure 5. The linear power scale emphasizes the peaks in the spectrum more than the logarithmic one. In fact, this second harmonic peak is the sharpest one, having the highest Q value, observed during the experiment. Peaks are noted at 8.5, 13.1, 20.2, and 26.2 c/s, which are identified as resonant frequencies of the earth-ionosphere cavity. Spectra are obtained for records taken randomly during meteorologically calm periods in June and December 1962, January, March, May, and July 1963. The first two resonant modes of the earth-ionosphere cavity are clearly shown in all records, together with evidence for the next two. From the best attempts at digitizing 31 records, the mean ELF spectrum is calculated and plotted in figure 6.

Knowing the amplifier gain, its frequency response, and the electrostatic characteristics of the antenna, the absolute power calibration shown in figures 5 and 6 is derived. This is checked having measured the signal induced by GBR transmissions. At 8 c/s the mean power spectral density is 0.117 $(mV/m)^2/c/s$, with a standard error of the mean of ±0.008 $(mV/m)^2/c/s$. This may be expressed as an equivalent amplitude of 0.342 ±0.012 mV/m in a 1 c/s bandwidth at 8 c/s. This value is in agreement with earlier quoted values [Large and Wormell, 1958; Balser and Wagner, 1962; Rycroft and Wormell, 1962]. It is equivalent to a Poynting flux of the order of 10^{-10} W/m^2/c/s.

The shapes of the spectra differ from those of sample spectra presented earlier [Rycroft and Wormell, 1962, figs. 8, 9, and 10] because at that time differences between the frequency response shapes (fig. 1) were ignored. The spectra presented here are similar to those of the magnetic field recorded in France by Gendrin and Stefant [1962] and Stefant [1963]. The main difference between these spectra and those presented by Balser and Wagner [1960] from observations made in Massachusetts is that the power spectral density at the minima between peaks is not independent of frequency, but decreases with increasing frequency. This point may be made in another way by noting that only when the background spectrum shown by the dashed line in figure 6 is subtracted from the ELF spectrum do these observations compare with those of Balser and Wagner [1960]. Although some of this background noise is introduced by the averaging process, it is probable that part represents a real noise source. This could be due to comparatively nearby lightning activity, energy propagated along a geomagnetic line of force, or extraterrestrial phenomena.

Below 5 c/s, the relatively immense power spectral density cannot be explained solely by energy from lightning and is ascribed to local electrostatic effects (as mentioned by Rycroft and Wormell [1962]), fluctuations of the fair weather field, or distant geo-electromagnetic phenomena. In figure 7, the power spectral density of the natural vertical electric field in a 1 c/s bandwidth at 1 c/s is plotted against the fair weather vertical electric field measured by a field mill at the time of the record. An association between the two quantities is apparent. At a certain value of the fair weather field, the power at 1 c/s tends to be larger the greater the wind force. This could be explained by the wind exciting harmonics of the oscillating antenna, or by clouds of space charge drifting past the antenna. Based on their observations of the natural magnetic field, Lokken, Shand, and Wright [1963] have asserted that lightning dominates the spectrum down to 1 or 2 c/s.

A few observations were made with an antenna consisting of three coils, each of 110,000 turns of 0.132 mm diameter wire which was enamel coated, mounted on a mu-metal rod 1 inch in diameter and 36 inches long. The induced signal was amplified and recorded as described in section 2; sample spectra showed the cavity resonance effect. On one occasion, at 21.00.13 G.M.T. on May 14, 1963, the waveform changed in 4 seconds from a typical cavity resonance waveform to a large amplitude (off-scale) wave at 8.0 c/s. This may be a man-made signal or an example of the ELF "noise storms" or "emissions" reported by Llanwyn Jones [1963].

Throughout the course of the experiments, the Mie scattering standing wave patterns required by Boyer's [1961] theory for the formation of the Van Allen trapped radiation belts were not detected.

5. Theoretical Considerations

In this section a review of the theories concerning earth-ionosphere cavity resonances, indicating their limitations, is attempted. Rationalized m.k.s.A. units are used.

The simplest model, used by Schumann [1952] and Wait [1960], is that of a perfectly conducting earth of radius R, surrounded by a thin dielectric shell of height h, h being $\ll$ R. This is surrounded by a concentric, sharply bounded, homogeneous, isotropic, good conducting model ionosphere of infinite extent.

A current I flowing vertically in the air near the ground forms, with its image in the ground, an electric dipole of infinitesimal length $\overrightarrow{ds}$. The dipole moment $\overrightarrow{M}$ is given by

$$\frac{\overrightarrow{dM}(\omega)}{dt} \exp(i\omega t) = I \, \overrightarrow{ds}(\omega) \, \exp(i\omega t). \tag{2}$$

Near a Hertzian dipole, for which ds $\ll$ λ, the free space wave equation for the Hertz vector must be satisfied. Near the source, where neither the ionosphere nor the earth's curvature is considered, the vertical electric field is given as a sum of electrostatic, induction, and radiation fields [Pierce, 1960]. It is obviously invalid to use the radiation field approximation of this equation in relation to cavity resonances.

A solution of the wave equation in spherical polar coordinates is the transverse magnetic (TM) wave with zero radial magnetic field. For example, in the air layer,

$$E_r(\omega,t) = -i\,\frac{\omega}{\sqrt{k^3 r^3}}\,\upsilon(\upsilon+1)P_\upsilon(\theta)\left[AH^{(1)}_{\upsilon+\frac{1}{2}}(kr) + BH^{(2)}_{\upsilon+\frac{1}{2}}(kr)\right]\exp(i\omega t) \tag{3}$$

in which $k = \omega/c$ is the free space wave number and $\upsilon+\frac{1}{2}$ is the order of the Hankel function. υ is determined by the boundary conditions. The Debye-Watson representation of the Hankel function, which is really a W.K.B. approximation to the radial part of the wave equation, is used. The approximations are only valid for $|\upsilon+\frac{1}{2}| > 1$ and $kr \gg 1$; the latter inequality is not met at cavity resonance frequencies. Hence Wait [1960] obtains a wave-guide mode condition in terms of Fresnel reflection coefficients. The sine of the complex angle of incidence is given by

$$S = \frac{\upsilon+\frac{1}{2}}{kR} \simeq \frac{\sqrt{\upsilon(\upsilon+1)}}{kR} \simeq 1 - \frac{i}{2khN_I} \simeq 1 - \frac{1}{2h}\sqrt{\frac{i^3}{\mu_0\omega\sigma_I}} \tag{4}$$

in which N_I is the refractive index of the ionosphere, which may be approximated in terms of the conductivity σ_I above 55 km (see, for example, Goldberg [1963]). Equation (4) may be rearranged, using $p = i\omega$, to give the modal equation

$$p^2 + \frac{1}{h\sqrt{\mu_0\sigma_I}}\,p^{3/2} + \upsilon(\upsilon+1)\,\frac{c^2}{R^2} \simeq 0. \tag{5}$$

Row [1962] derives the modal equation by another method.

Since only the zero-order waveguide mode is important at cavity resonator wavelengths, the resonator mode equation, of order n, is obtained,

$$E_r(\omega,t) = \frac{I\ ds(\omega)}{4\pi h R^2 \epsilon_0 \omega} \sum_{n=0}^{\infty} P_n(\cos\theta)\ \frac{\nu(\nu+1)(2n+1)}{n(n+1) - \nu(\nu+1)}\ \exp(i\omega t). \quad (6)$$

The nth resonance of the radial electric field occurs when

$$\frac{1}{h\omega}\ \frac{\nu(\nu+1)(2n+1)}{n(n+1) - \nu(\nu+1)}$$

is a maximum, ν being determined by (4) or (5).

For a perfectly conducting ionosphere, the resonant frequencies, f_n, are determined by (4):

$$\nu(\nu+1) = n(n+1) = k^2 R^2.$$

hence (1) is derived:

$$f_n = \frac{c}{2\pi R}\ \sqrt{n(n+1)}. \quad (7)$$

For an ionosphere of conductivity σ_I the resonant frequency is complex; thus the height of the model ionosphere is found to be frequency dependent which is physically untenable. Equation (5) may be solved under the approximation that

$$\left| \frac{1}{h\sqrt{\mu_o \sigma_I}} \, p^{1/2} \right| \ll |p|$$

as

$$p \simeq i\omega_n' - \Omega_n \tag{8}$$

where ω_n', the observed resonant frequency, is less than ω_n, calculated for a perfectly conducting ionosphere,

$$\omega_n' = \omega_n \left(1 - \frac{1}{h\sqrt{8\mu_o \sigma_I \omega_n}} \right), \tag{9}$$

and Ω_n is the amplitude damping coefficient,

$$\Omega_n = \frac{1}{h} \sqrt{\frac{\omega_n}{8\mu_o \sigma_I}} \tag{10}$$

This analysis, used by Gendrin and Stefant [1962] and Rycroft and Wormell [1962] to explain their observations, leads to a sharply bounded model ionosphere at about 76 km, where the conductivity is $\sim 5\times10^{-6}$ mho/m. It is not strictly valid because the approximation necessary to derive (8) is not met. The Q values of the resonances calculated from (10) are lower by a factor of 2 than those estimated as the ratio of the observed resonant frequency to the frequency difference at the half power points.

Besides fitting experimentally observed values of f_n' with theoretical ones, the theory based upon a particular model ionosphere must explain the observed Q values, of the order of 4, which tend to increase with increasing n. One more variable is required to describe the ionosphere. Llanwyn Jones [1963] and Chapman and Llanwyn Jones [1964] have achieved excellent agreement for a two-layer model ionosphere. Because the skin depth, $\sqrt{2/\mu_0\sigma\omega}$, is large at these extremely low frequencies the higher ionosphere regions play a significant part in determining the propagation of ELF electromagnetic energy.

Sharply bounded models do not consider the region where $\sigma/\epsilon_0\omega \approx 1$, typically near 50 km, where displacement currents and conduction currents are of comparable magnitude and where the W.K.B. approximation is not valid. Harris and Tanner [1962] consider this transition region to be as important as the upper region in determining ELF propagation. Galejs [1963] has reviewed his work on a model ionosphere, bounded at a height where $\sigma/\epsilon_0\omega = 1$, and having an exponential conductivity vs. height profile in agreement with observations. Maximizing E_r and using the condition that $\mathrm{Re}(S) \gg |\mathrm{Im}(S)|$, it is found that (compare (7))

$$f_n' = \frac{1}{\mathrm{Re}(S)} \frac{c}{2\pi R} \sqrt{n(n+1)}. \tag{11}$$

Wait [1964] obtains a more general form of this equation.

Cosmic ray induced ionization in the atmosphere contributes to the absorption of ELF radio energy; it is shown [Galejs, 1963] that

$$\frac{1}{Q} = -2\text{Re}(S)\text{Im}(S) \qquad (12)$$

From the mean spectrum presented in figure 6, the series for f_n' is found to be 7.8 ±0.2, 14.1 ±0.2, 20.0 ±0.2, and 26.0 ±0.4 c/s. These observations are compared with those of other workers in table 1 and with the series 0.7642 times (7). The observed fundamental resonant frequencies are all less than the calculated value in row 2, but f_4' is in all cases greater than the calculated value. Hence it is deduced that the Q value tends to increase with increasing n. This deduction is confirmed by estimating the Q values from the spectrum.

The real and imaginary parts of S at the four resonant frequencies are thus found. The propagation constant γ is related to S:

$$\gamma = ikS = \alpha + i\Phi. \qquad (13)$$

Hence the attenuation constant

$$\alpha = -k\ \text{Im}(S)\ \text{nepers/m} = -0.182\ f\ \text{Im}(S)\ \text{db/Mm} \qquad (14)$$

and the phase velocity is

$$\frac{\omega}{\Phi} = \frac{c}{\text{Re}(S)}. \qquad (15)$$

The attenuation rate is found to be approximately 0.25 db/Mm, but is only poorly determined because of the uncertainties in Q. The phase velocity is found to vary from 0.75 c at 8 c/s to 0.78 c at 30 c/s, with an estimated standard error of ±0.01 c.

It is important to consider the effect of the geomagnetic field which makes the ionosphere anisotropic. In the quasi-longitudinal approximation the attenuation is somewhat increased [Wait, 1962]. By considering a radial geomagnetic field and a multilayer stratified ionosphere above 85 km, Thompson [1963] has shown that energy would be lost from the high ionosphere, resulting in Q values decreased from ~16 to ~8, especially at night. Madden [private communication, 1964] has extended Thompson's analysis to include the lossy region below 85 km where displacement currents become important. In the quasi-transverse approximation, Wait [1962] and Galejs and Row [1964] have calculated differences between the attenuation rates for E to W and W to E propagation. The effect of ions on the attenuation constant at cavity resonance frequencies has been considered by Galejs [1963] and found to be small.

It has been assumed that the resonances are excited by vertical lightning discharges. The ratio of the far zone radial electric field excited by a vertical dipole to that excited by a horizontal dipole of the same moment (representing an intracloud or intercloud lightning flash) is $\gtrsim 10^5 \sqrt{\sigma_E}$ [Wait, 1960]. Typical values for the conductivity of the earth σ_E are shown in table 2, drawn from the paper of Watt, Mathews, and Maxwell [1963]. The effect of horizontal flashes is thus negligible at extremely low frequencies.

It may also be concluded from table 2 that little ELF energy is absorbed by the earth's surface and therefore that the Q values of the earth-ionosphere cavity are determined entirely by losses in the atmosphere and ionosphere.

6. Discussion

The detailed shape of ELF spectra may now be considered, bearing in mind the limitations in the theory mentioned in the previous section.

Equation (6) shows that the spectrum of the radial electric field depends on the spectrum of the exciting lightning discharge. Based on the statistics gathered by Williams [1959], Galejs [1963] has assumed that the return stroke excites the resonances. Pierce [1963] has contended that other processes in a lightning discharge contribute to the ELF noise. These ideas have been extended by Rycroft and Wormell [1964] who consider the spectrum generated by a "mean" lightning discharge. Accounting for the preponderance of cloud discharges, the result that the spectrum differs little from that of a median return stroke is rather surprising. The spectra computed by Galejs [1963] may thus be used with confidence.

The geographical distribution of sources needs to be considered. Galejs [1961, 1963] has calculated spectra for a uniform distribution of sources over the polar angle θ in the interval $90^\circ - \Delta \leq \theta \leq 90^\circ + \Delta$, θ being zero at the receiver. When the backround spectrum shown in figure 6 is subtracted from the observed spectrum the resulting cavity resonance spectrum is in good agreement with computed spectra. The ratio of the power at the fundamental $(n = 1)$ frequency to that at the second and third harmonics is taken from Galejs [1961] and plotted in figure 8. The experimental values of Balser and Wagner [1960] are plotted, as are the Cambridge observations, both with and without the background spectrum removed. The observations fit for a half width of the thunderstorm belt about an equator with respect to a pole at Cambridge Δ, of $44^\circ \pm 12^\circ$. This value is in agreement with that derived from the yearly average geographical distribution of thunderstorms as may be seen from figure 9. This map presents the world in an azimuthal equidistant projection centered on Cambridge; a radius vector is a great circle path on this projection. Also shown are loci of constant θ, for $\theta = 90^\circ$ where $P_1(\cos \theta) = \cos \theta = 0$, and for $\theta = 55^\circ$ and 125° where $P_2(\cos \theta) = 1/2 \, (3 \cos^2 \theta - 1) \simeq 0$. According to (6), the radial electric field is zero at these values of θ, for the fundamental and second-order resonator modes, respectively. Even if a source is present, its effect is not noted by a receiver situated at a certain angle θ with respect to the source.

It is well known that thunderstorms over tropical land masses occur preferentially during the local afternoon. These two factors which determine the intensity of the cavity resonance signals, termed "mode zeros" and "storm maxima," respectively, were discovered by Balser and Wagner [1962]. A specific example of the effect of these factors is shown in figure 4; presumably at this time thunderstorms were active over Brazil. The diurnal variation of the power spectral density in the fundamental mode, presented in figure 10, is also explained by these two factors. It should be noted that the $n = 1$ mode zero at $\theta = 90^{\circ}$ masks the Southeast Asia storm maximum effect.

The broad features of the ELF signal diurnal variation, that is the afternoon (G.M.T.) maximum and the nighttime (G.M.T.) minimum, have been noted by Holzer and Deal [1956] (see their fig. 1) and Rycroft and Wormell [1962] from records similar to their figure 7. The nighttime minimum is explained by the paucity of thunderstorms over the Pacific Ocean. The solid curve in figure 10 shows a fifth degree polynomial regression fitted to the points, with a dominant quadratic term. Although there is considerable deviation about the regression, the curve is significant at the 95-percent level. A search was made for seasonal variations in the spectra. The only noticeable feature is that the fundamental peak is relatively weaker than the higher harmonic peaks in winter. The explanation is that the thunderstorm centers over South America and Africa move to the South in winter.

A search was also made for diurnal variations in the resonant frequencies themselves. Complex variations have been reported by Balser and Wagner [1962] and Llanwyn Jones [1963]. The observations reported here are too few and too scattered to allow comparison with previous work. The mean resonant frequencies are 8.0 ±0.1, 14.0 ±0.2, 19.8 ±0.2, and 26.1 ±0.3 c/s, which differ slightly from those deduced from the mean spectrum in section 5.

Neither the power in the fundamental mode nor the resonant frequencies of the first two modes appeared to be correlated with the magnetic K_p index at the time of the record.

The power spectral densities of the longest signal records were computed from 0 to 30 c/s at 0.2 c/s intervals in order to investigate the fine structure of the cavity resonance spectrum. One such spectrum is shown in figure 11 and may be compared with those presented by Madden, et al. [1962] and Chapman and Llanwyn Jones [1964]. These spectra are all extremely jagged. One interpretation is that the radial ELF electric field given by (3) should contain an azimuthal dependence, in which case it would be proportional to

$$\sum_{m=-n}^{n} P_n^m(\cos\theta) \exp(im\varphi)$$

Since the nth order mode is not observed to split into precisely m = 2n + 1 peaks, it must be concluded that the "fine structure" is spurious, being brought about by statistical fluctuations due to the finite length of the sample.

As a final piece of evidence that the observed spectrum is the result of excitation of the earth-ionosphere cavity by lightning discharges, an order of magnitude calculation is performed to show that the observed field strength is reasonable. Inserting $n = 1$ for the fundamental resonance, (6) becomes

$$E_r(8 \text{ c/s}) = \frac{I \, ds(8 \text{ c/s})}{64\pi^2 hR^2 \epsilon_0} P_1(\cos \theta) \frac{3\nu(\nu + 1)}{2 - \nu(\nu + 1)}. \qquad (16)$$

For one "mean" lightning discharge, $I \, ds(8 \text{ c/s}) \simeq 1.4 \times 10^5$ m.k.s.A. units [Rycroft and Wormell, 1964]. It is important to know how many discharges play a part in determining the value of the field at any instant. To estimate this, an average value of 100 for the number of discharges that occur over the earth's surface per second is used, together with a value for the time constant of the cavity, $1/\Omega_1$, of $1/16.3$ sec [Rycroft and Wormell, 1962]. In this time there occur about $100/16.3 \simeq 6$ flashes, which tend to excite the cavity into resonance, but at random phase angles. Therefore, $\sqrt{6}$ times the 8 c/s component in the spectrum of one "mean" flash may be inserted for the quantity $I \, ds(8 \text{ c/s})$ in (16). Thunderstorms tend to occur in the range $\theta = 30°$ to $140°$, so a mean value for $P_1(\cos \theta)$ might be 1/2. Using Galejs' [1963] analysis,

$$\sqrt{\nu(\nu + 1)} \simeq \left(\nu + \frac{1}{2}\right) = kRS \simeq kR \, \text{Re}(S) = 1.74 , \qquad \text{for} \quad n = 1.$$

Taking a model ionosphere sharply bounded at $h = 76$ km, with $R = 6370$ km, E_r is found to be approximately 0.2 mV/m in a 1 c/s bandwidth at 8 c/s, which is in good agreement with the measured value of 0.34 ± 0.01 mV/m, or 0.26 mV/m having subtracted the background spectrum shown in figure 6.

7. Summary

In this paper experimentally obtained evidence has been presented which shows that resonant modes of the electromagnetic field in the dielectric cavity between the good conducting earth and ionosphere are observed. At all times when observations of the natural vertical electric field in the atmosphere could be made - at meteorologically calm periods - statistically significant resonances of the fundamental mode at 7.8 c/s and of the second harmonic at 14.1 c/s are detected. The next two higher harmonics, at 20.0 and 26.0 c/s, are often apparent. The standard error in these mean resonant frequencies, derived from 31 records, is typically ±0.2 c/s. The mean power in a 1 c/s bandwidth at 8 c/s is observed to be 0.117 ±0.008 $(mV/m)^2/c/s$. Spectra recorded by Gendrin and Stefant in France show striking similarities with those presented here, whereas those taken in Massachusetts by Balser and Wagner are somewhat different.

These resonances are excited by energy from tropical lightning discharges. Diurnal and seasonal variations of the power in the various modes are interpreted in terms of the movement of thunderstorm centers and modes of the field. The half width of a model thunderstorm belt, calculated for a model ionosphere with an exponential conductivity vs. height profile [Galejs, 1963], is in agreement with thunderstorm statistics. With this model ionosphere the attenuation rate in the band 7 to 30 c/s is found to be approximately 0.25 db/Mm, and the phase velocity 0.77 c.

For the longest records taken, the fine structure of the spectrum is investigated and found to be not statistically significant.

Below 5 c/s, the power spectral density of the electric field is relatively immense, and is mainly associated with wind borne local space charges or mechanical oscillations of the antenna in the fair weather electric field.

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Table 1. Calculated and observed resonant frequencies of the earth-ionosphere cavity.

Mode order, n	1	2	3	4
Resonant frequency, f_n, c/s from (7)	10.60	18.35	25.96	33.52
Resonant frequency, c/s, 0.7642 times row 1, see (11)	8.10	14.03	19.85	25.62
Observed f_n', c/s, June 27 and 28, 1960, Balser and Wagner (1960)	7.8	14.1	20.3	26.4
Observed f_n', c/s, mean 1961, 1962, Chapman and Llanwyn Jones (1964)	8.0 ±0.1	14.1 ±0.2	20.0 ±0.4	26.0 ±0.8
Observed f_n', c/s, July 13 and 14, 1962, Stefant (1963)	7.85 ±0.2	14.2 ±0.3	19.9 ±0.3	26.25 ±0.3
Observed f_n', c/s, mean spectrum 1962, 1963, observations at Cambridge, England	7.8 ±0.2	14.1 ±0.2	20.0 ±0.2	26.0 ±0.4

Table 2. Electrical properties of materials forming the earth's surface.

Material	Percentage of earth's surface covered	Average depth	σ_E, mho/m	Skin depth at 8 c/s
Sea water	71%	0-3 km	5	80 m
Basalt, ocean floor		3-12 km	10^{-6}	no effect
Rocks		0-35 km	10^{-2} to 10^{-6}	2 to 200 km
Mantle [Lahiri and Price, 1939]	24%			
		35-2900 km	10^{-5} to 10^{-6}	~100 km
Snow, ice	5%	0-1.5 km	10^{-6}	200 km

Figure Captions

Figure 1. Frequency response of the amplifying system.

Note that the response measured with a signal gen-
erator directly feeding the amplifier input differs
considerably from that measured when a representation of
the antenna is included.

Figure 2. Atmospherics and ELF traces.

Note that in general there is no correlation
between individual atmospherics and the amplitude of the
ELF waveform.

Figure 3. Spectra of two series of 600 random numbers, o with a
standard deviation of 40 about a mean of 130, and + with
a standard deviation of 25.

Figure 4. Spectrum of the digitized signal waveform observed at
16.18 G.M.T. on January 12, 1963.

The crosses show the spectrum as normally computed,
and the open circles that computed with a numerical rejec-
tion filter at 0 c/s. Statistically significant peaks
are observed at resonant frequencies of various modes
($n = 1, 2, 3,$ and 4) of the field in the earth-ionosphere
cavity.

Figure 5.   Spectrum of the natural vertical electric field observed

at 16.18 G.M.T. on January 12, 1963.

This is the spectrum presented in figure 4 corrected

for the receiver noise and frequency response. The power

ordinate is on a linear, rather than a logarithmic, scale.

Figure 6.   Mean ELF spectrum of the natural vertical electric field

observed in 1962 and 1963.

The similarity of this spectrum with those presented

by Gendrin and Stefant (1962) and Stefant (1963) may be

noted. Only when the background spectrum shown by the

dashed line is subtracted from the mean ELF spectrum do

these observations compare with those of Balser and

Wagner (1960). The calculations of Galejs (1963) may

be fitted to the observations of Balser and Wagner (1960).

The bars denote the standard error of the mean power at

each resonance, derived from 31 records.

Figure 7.   Dependence of the power spectral density of the vertical

electric field in a 1 c/s bandwidth at 1 c/s on the

strength of the wind and the magnitude of the fair

weather field.

Figure 8. Ratio of the power, G_1, in the fundamental (n = 1) mode

to that in the higher harmonic (n = 2 and 3) modes as a

function of Δ, the half width of the equatorial thunder-

storm belt.

The solid curve shows G_1/G_2, and the dashed curve

G_1/G_3, as computed by Galejs (1961). The open circle

points show these ratios observed by Balser and Wagner

(1960). For the Cambridge observations the crosses mark

the ratios derived from the mean ELF spectrum; the squares

show the ratios derived from the cavity resonance spectrum,

the spectrum obtained by subtracting the background spec-

trum from the mean ELF spectrum. The effect of the error

bars in figure 6 is also shown.

Figure 9. Map of the world in an azimuthal equidistant projection

centered on Cambridge, England.

The areas of greatest thunderstorm activity averaged

over a year are shown (Handbook of Geophysics, 1960).

Loci of constant θ, circles centered on Cambridge, cor-

responding to minima of the radial electric field in the

fundamental and second order modes, are also shown.

Figure 10. Diurnal variation of the power in the fundamental $(n = 1)$

mode. The crosses are for summer observations and the

open circle points for winter ones.

Both "mode zero" and "storm maxima" effects may be

noted. The solid curve, a fifth degree polynomial

regression, shows the trend of the diurnal variation.

Figure 11. Spectrum of the digitized signal waveform observed at

16.18 G.M.T. on January 12, 1963, computed at 0.2 c/s

intervals (solid line).

The crosses show the spectrum as normally computed,

at 1 c/s intervals, from figure 4. The "fine structure"

is not statistically significant.
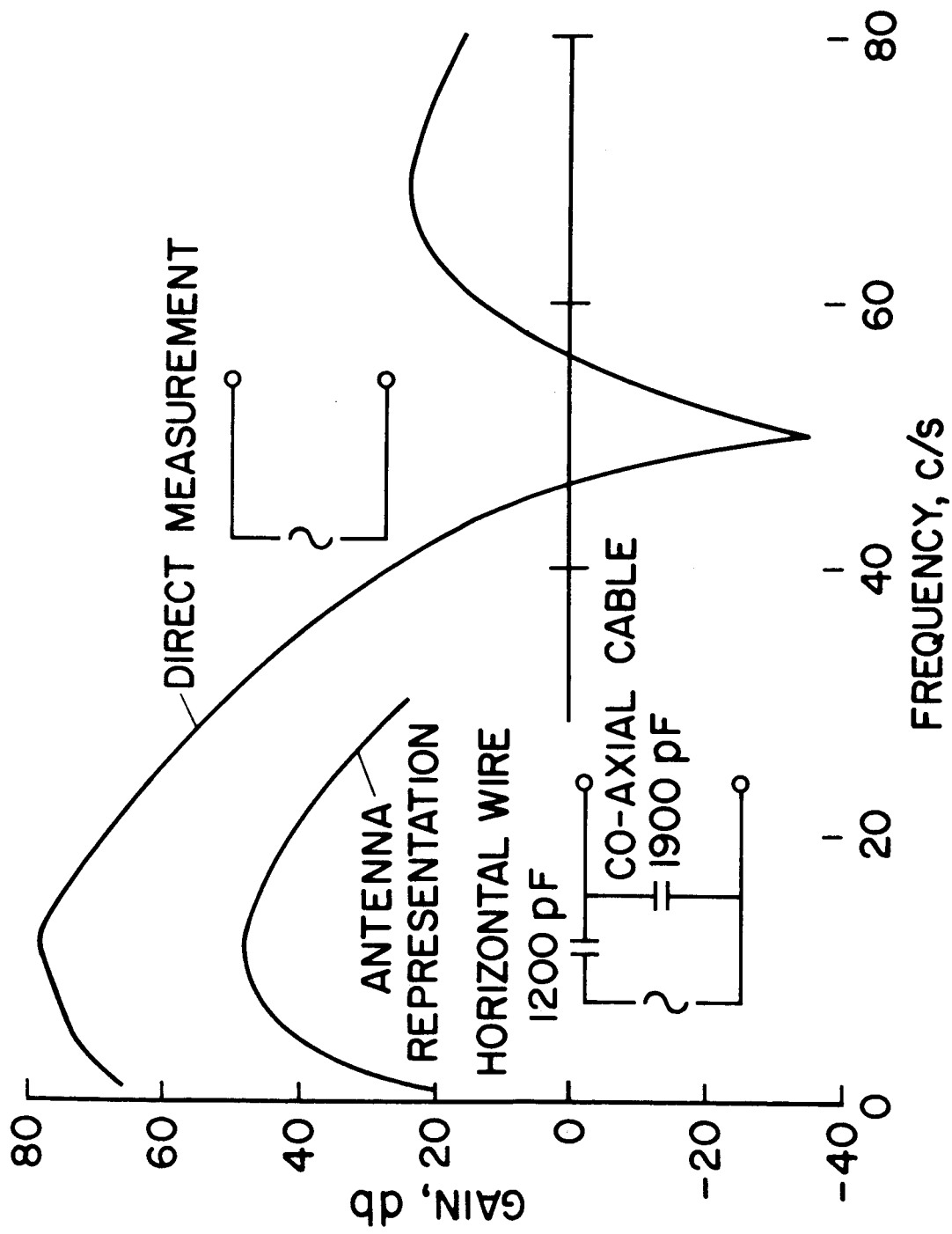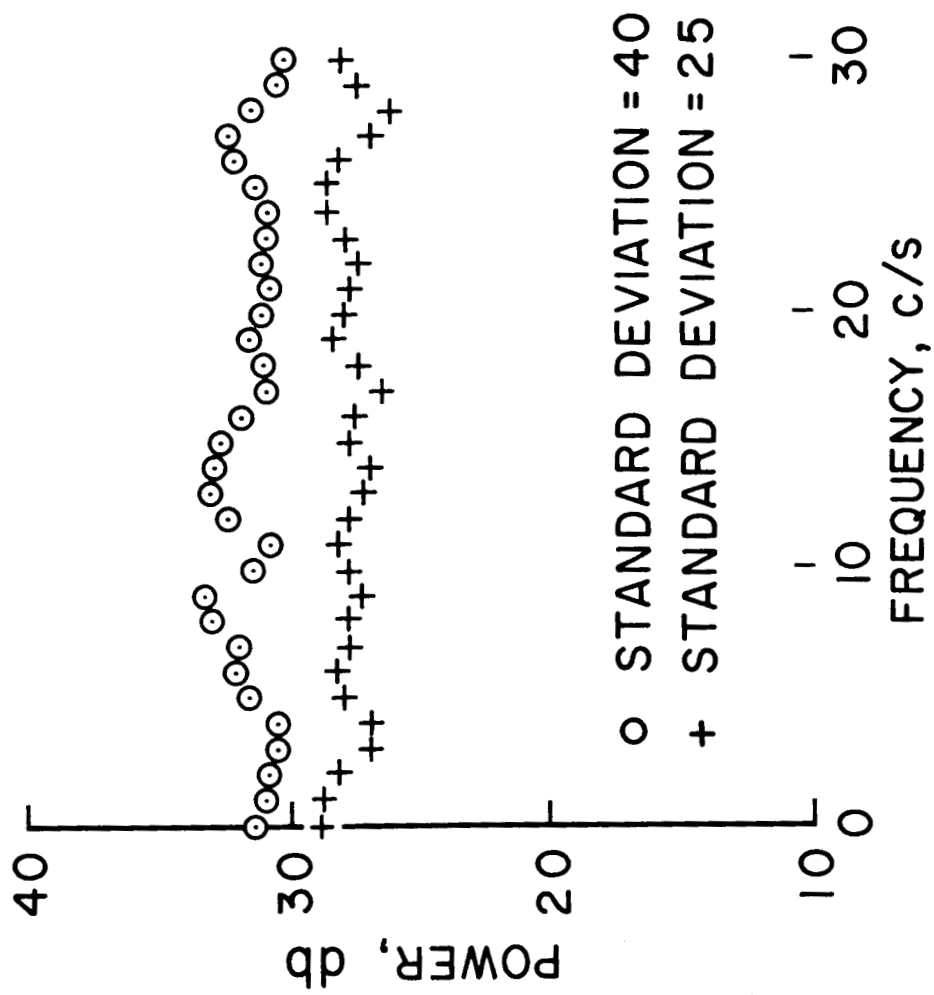
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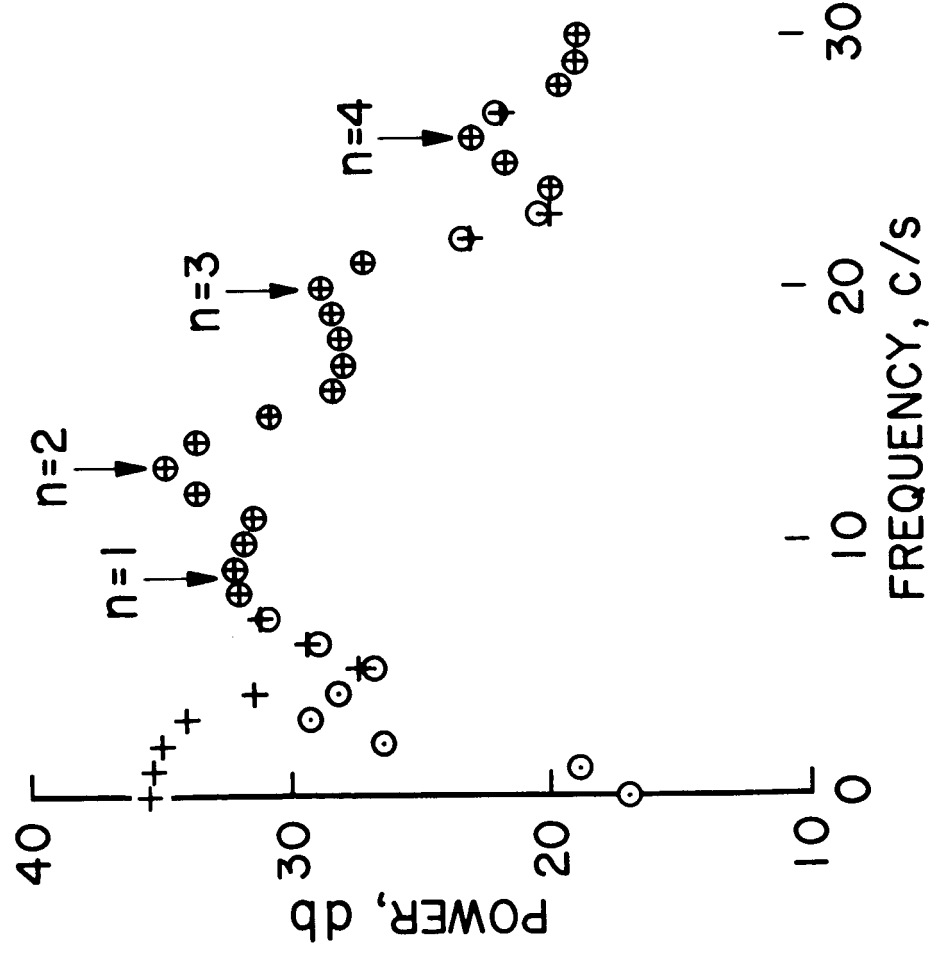
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JUNE 3, 1962 (c)

1 second

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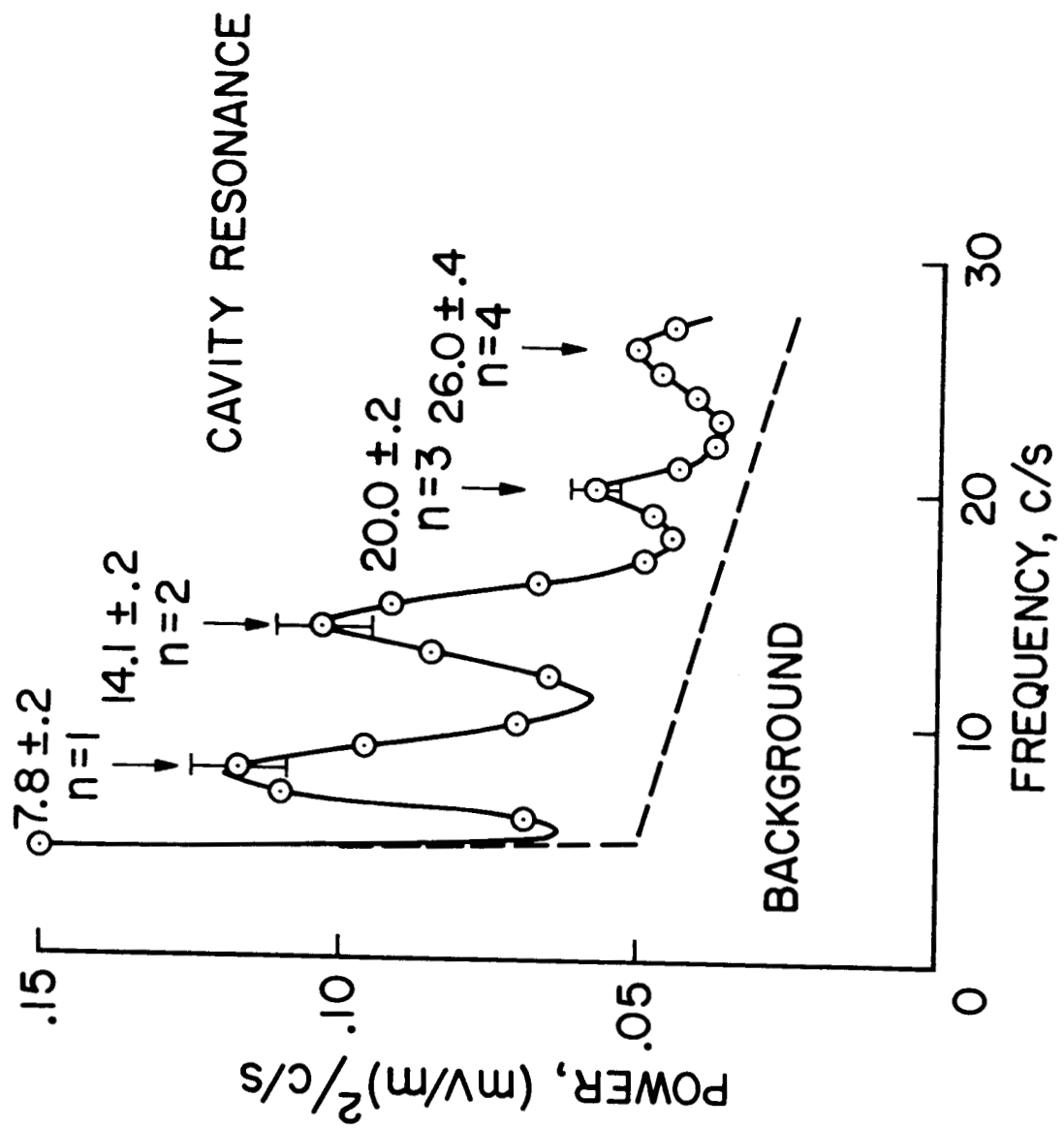
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FREQUENCY, c/s

POWER, db

3

$$f_n' = f_1' \sqrt{n(n+1)/2}$$

$$f_1' \simeq 0.75 \ c/\sqrt{2} \ \pi R_E$$

POWER, $(mv/m)^2/c/s$

.15

.10

.05

0

FREQUENCY, C/S

0   10   20   30

n=1

n=2

n=3

n=4

5

CAVITY RESONANCE

7.8 ±.2
n=1

14.1 ±.2
n=2

20.0 ±.2
n=3

26.0 ±.4
n=4

BACKGROUND

POWER, (mV/m)^2/c/s

.15

.10

.05

0

FREQUENCY, c/s

10

20

30

6

SPECTRAL PEAK POWER RATIO, G_1/G_n

Δ, deg
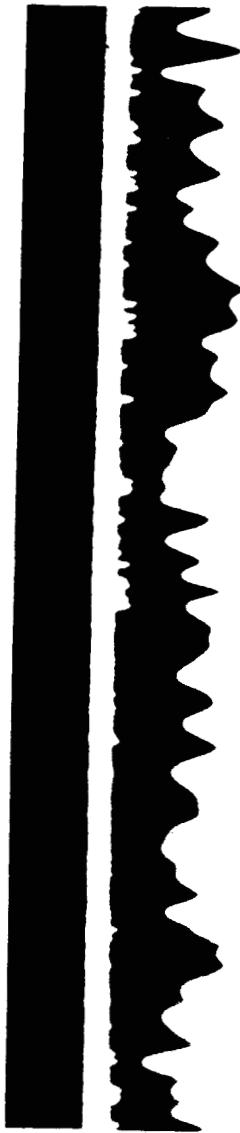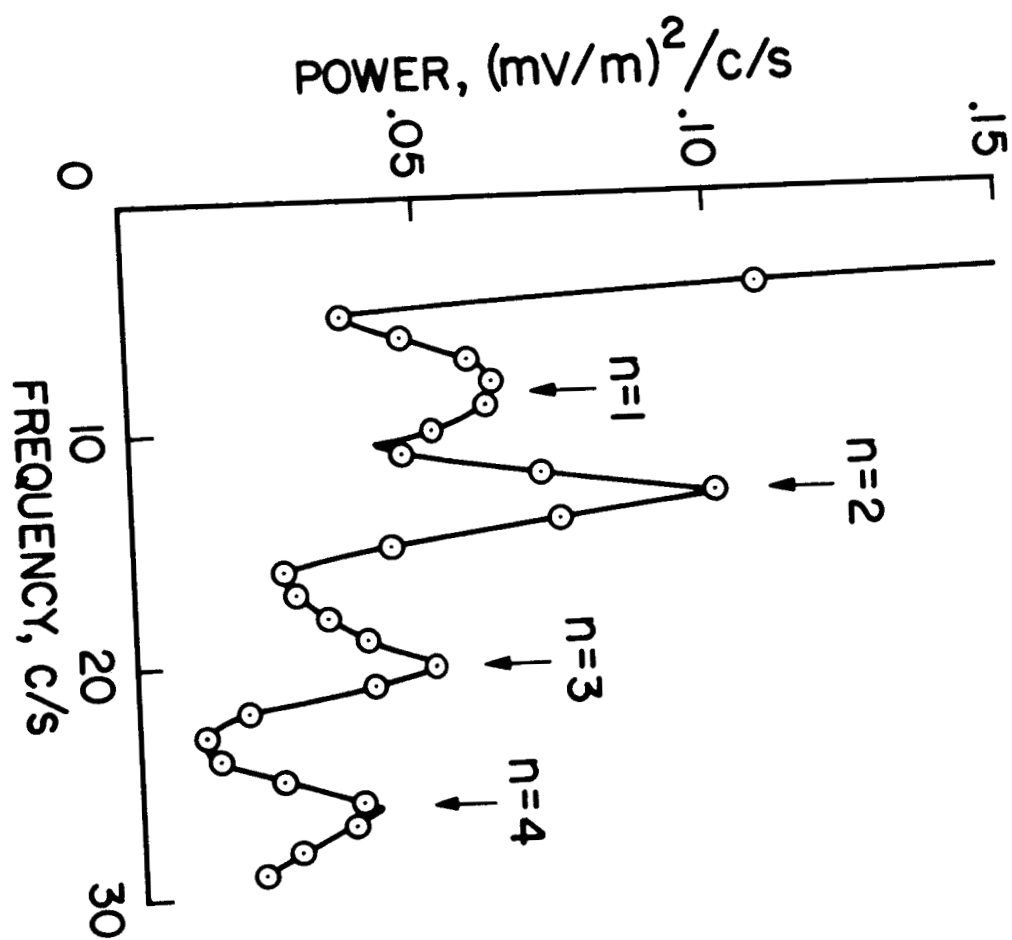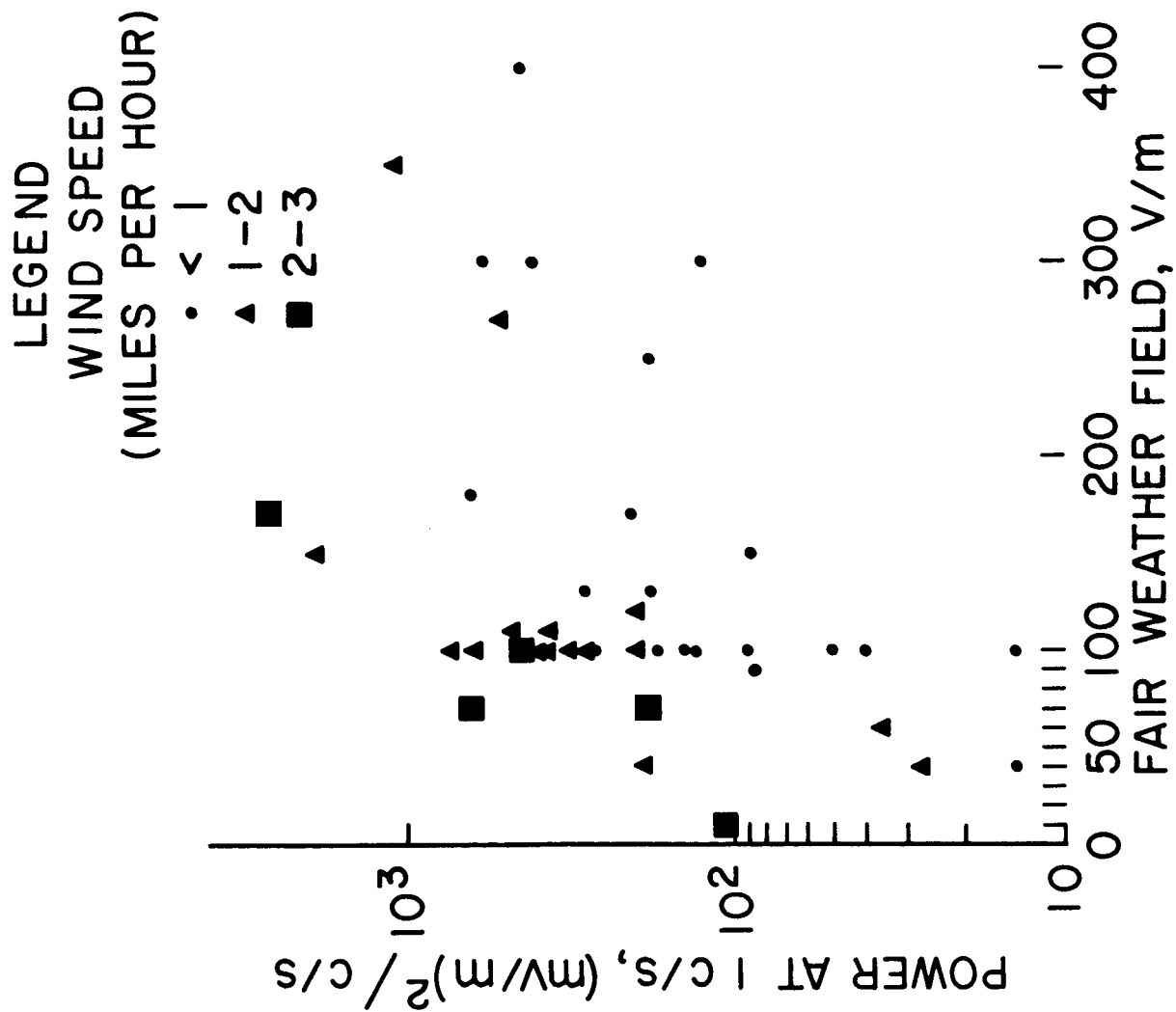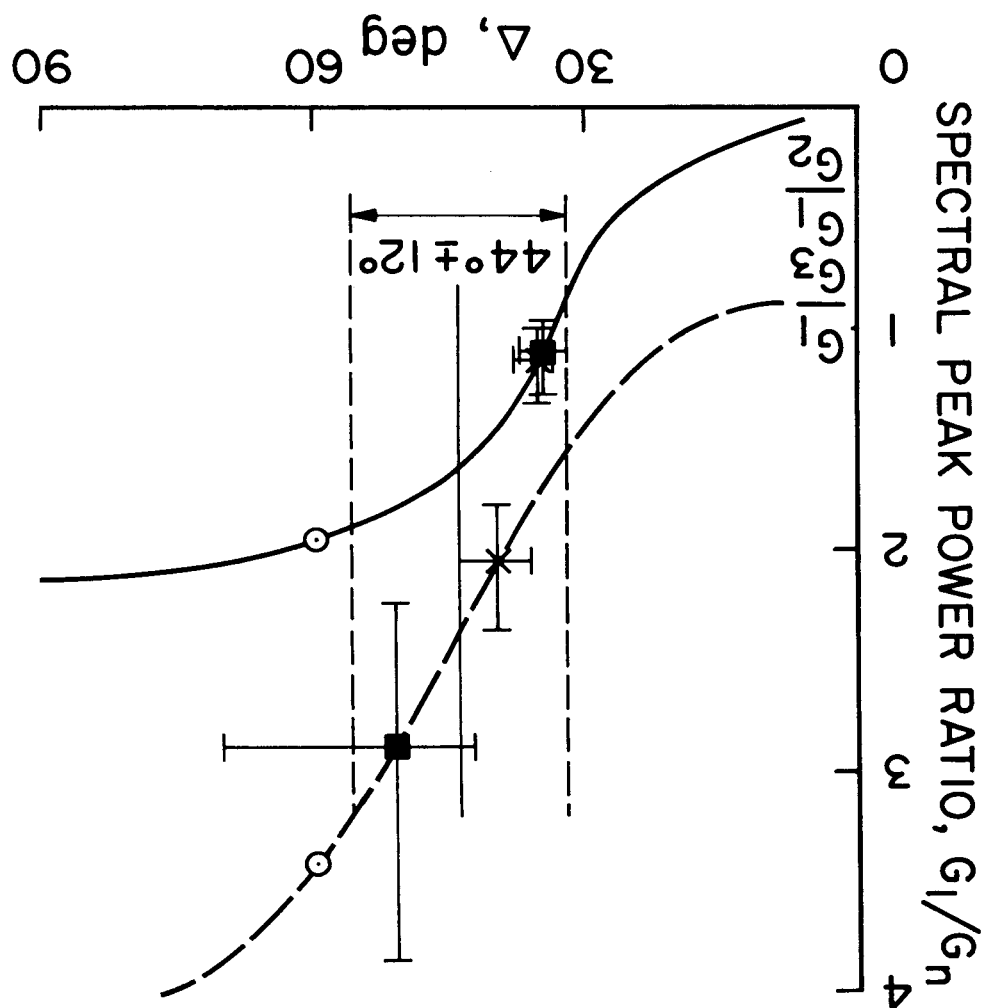
CAMBRIDGE:
× E.L.F. SPECTRUM
■ CAVITY RESONANCE-
BACKGROUND SPECTRUM
MASSACHUSETTS:
○ BALSER AND
WAGNER, 1960

$44° \pm 12°$

$\frac{G_1}{G_2}$ $\frac{G_1}{G_3}$ $\frac{G_1}{G_1}$

8
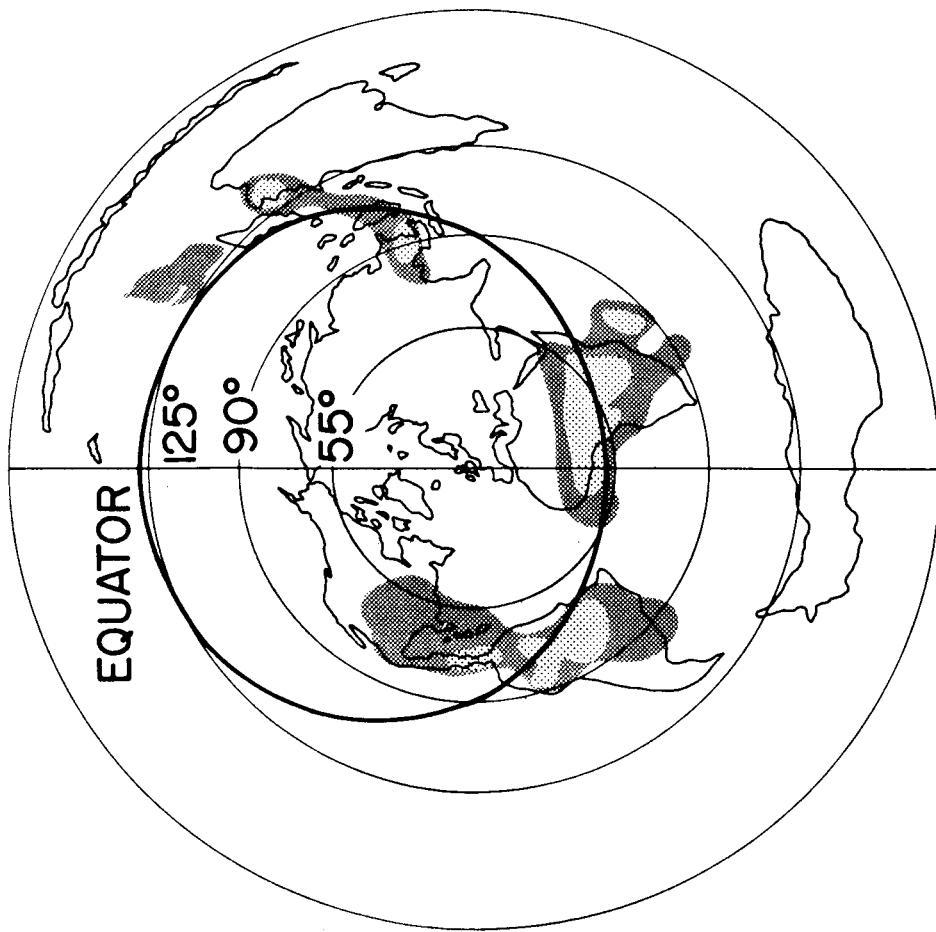
YEARLY AVERAGE OF NUMBER
OF THUNDERSTORM DAYS

GREATER THAN 100

50 TO 100

EQUATOR

125°
90°
55°