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Asymptotic Cones of Acceptance of Cosmic Ray Neutron Monitors in a Geomagnetic Field Distorted by the Solar Wind

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The concept of an asymptotic cone of acceptance of a cosmic ray neutron monitor has been widely used during the past few years for the study of spatial anisotropies in cosmic radiation in the interplanetary space [Rose and Lapointe, 1961; McCracken, 1962; Rao, et al., 1963; Lockwood and Razdan, 1963]. This cone is defined as the solid angle containing the asymptotic directions of approach of cosmic ray particles outside the influence of the geomagnetic field that significantly contribute to the counting of a ground detector [McCracken, 1959]. Knowing these cones at various stations, one can relate the intensity observed at a particular time to a direction in space outside the magnetosphere. So far, these cones have been calculated for the quiet geomagnetic field conditions for a simple dipole model of the earth [Lapointe and Rose, 1961] and for a high degree simulation of the geomagnetic field [McCracken, 1962 and McCracken, et al., 1962]. For neutron monitors at

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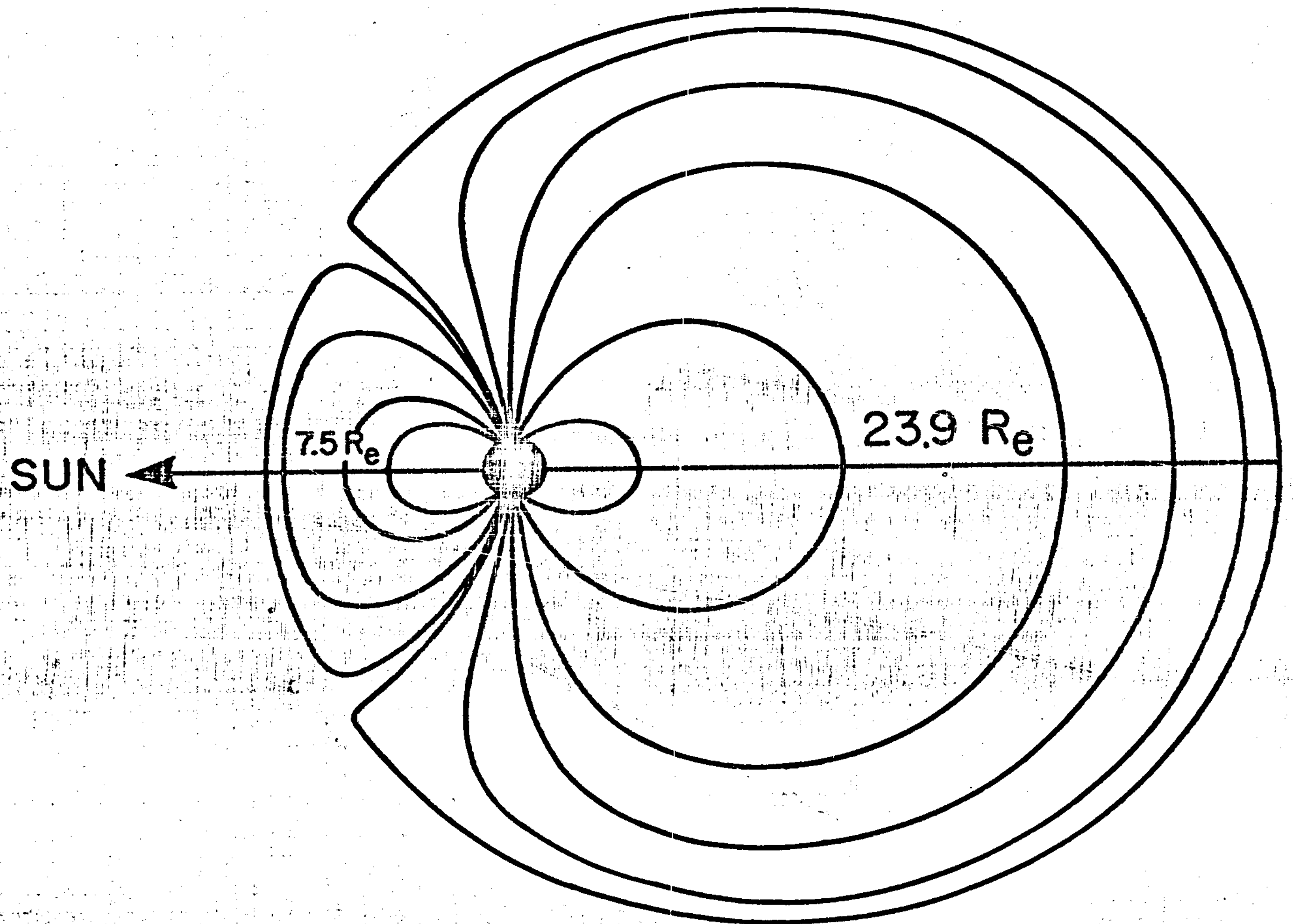
particular stations, differences of about 10° have been obtained in the effective directions of arrival in the latter case. However, during a period of geomagnetic disturbance, the solar wind appreciably deforms the geomagnetic field, confining it in a roughly ellipsoidal cavity with the magnetospheric boundary close to the earth on the sunward side and extended far out on the night side [Sonett, et al., 1960; Cahill and Amazeen, 1963; Heppner, et al., 1963]. The electromagnetic state of the interplanetary medium undergoes large changes at such time, leading to various types of observed variations in the cosmic ray intensity, such as Forbush decreases, enhanced diurnal variation, solar flare increases, etc. For the study of these variations and, consequently, the electromagnetic state of the interplanetary medium, it is important to know any change that might occur in the asymptotic cones of acceptance as a result of the distortion in the geomagnetic field. Thus cosmic ray trajectories were calculated numerically on an IBM 7094 computer, and the asymptotic cones for a distorted geomagnetic field were compared with those for a dipole field. The trajectory behavior near the geomagnetic cutoff for a dipole and the distorted geomagnetic field were also compared.

A deformed magnetosphere was represented by Hones' model [1963] where the distorted field is created by an image dipole of moment 28 times that of earth's equivalent dipole, placed at a distance of 28 earth radii along the noon meridian. The scalar potential due to these two parallel but unequal dipoles is given by

[redacted]



SUN
7.5 R_e
23.9 R_e

$$U = M \cos\theta \left\{ \frac{1}{r^2} + \frac{28r}{[(28R_e)^2 + r^2 - 2(28R_e)r \sin\theta \cos\varphi]^{3/2}} \right\} \quad (1)$$

where

M moment of earth's dipole (8.1×10^{25} gauss cm^3)

θ colatitude

φ azimuthal angle measured around earth's dipole from the noon meridian

r radial distance from the center of the earth

R_e mean radius of the earth

This leads to a magnetic field configuration in which the earth's field is completely confined in an approximately ellipsoidal cavity extending 7.56 R_e in the solar direction and 23.91 R_e in the night direction. A cross-sectional view of the deformed magnetosphere in the meridian plane is shown in Figure 1. It is found that an ellipse with major and minor axes of 15.736 R_e and 14.240 R_e fits the meridian plane trace fairly well. By then making a simplifying assumption that the boundary of the deformed magnetosphere is an ellipsoid of revolution, we arrive at the boundary equation for the ellipsoidal magnetosphere given by

Fig. 1

$$\frac{(r \sin\theta \cos\varphi + 8.176\,R_e)^2}{(15.736\,R_e)^2} + \frac{(r \cos\theta)^2}{(14.24\,R_e)^2} + \frac{(r \sin\theta \sin\varphi)^2}{(14.24\,R_e)^2} = 1 \quad (2)$$

It is clear from equation (1) that, by changing the strength of the image dipole and its distance from the center of the earth, we can obtain other types of ellipsoidal models for the magnetosphere where the position of the boundary on the solar side and the night side would be different. However, the above model which would approximately fit the experimental observations during a severe geomagnetic disturbance would be reasonable for the present calculations.

The asymptotic directions of cosmic ray particles outside the magnetosphere and arriving at a particular geomagnetic latitude were calculated by considering an equivalent problem of the motion of negative particles of the same rigidities moving in the reverse direction. The equation of motion

$$m \frac{d^2\vec{R}}{dt^2} = \frac{e}{c}\left(\frac{d\vec{R}}{dt} \times \vec{B}\right) \qquad (3)$$

where the symbols have their usual meaning, was solved in a spherical coordinate system by the Runge-Kutta-Blum method [Blum, 1957]. When the particles reached the boundary of the distorted magnetosphere, as given by equation (2), the trajectory direction was noted in terms of asymptotic geomagnetic latitude (Λ) and asymptotic geomagnetic longitude (ψ), measured eastward from station meridian. Asymptotic directions were calculated for various particle rigidities for the dipole field, and the distorted field at station azimuth angles of 0°, 90°, 180°, and 270°. For the dipole field the integration of the equation of motion was terminated at 25 earth radii, beyond which the geomagnetic deflection suffered by a particle is negligible.

In Table 1a, Λ, ψ values are presented for vertically incident particles near the cutoff at 50° geomagnetic latitude. Particle rigidities from 2.7 to 3.2 bv at steps of 0.1 bv are considered which mostly cover the penumbral region at this geomagnetic latitude (Störmer cutoff at $50^\circ = 2.54$ bv). In Table 1b, the Λ,ψ values are given only for a 3 bv particle arriving at 50° geomagnetic latitude along the vertical and zenith angles of 16° and 32° in the geomagnetic North, South, East, and West planes. A point which may be noted in these tables is that some particle trajectories in a distorted field, which are allowed in one longitude (i.e., $\varphi = 180^\circ$), may be prevented from entering in another. This indicates that during a geomagnetic disturbance there might occur changes in the allowed and forbidden cones in the penumbral region as a function of local time of the station. Recently, Shea, et al. [1964] have studied the trajectories in penumbral region at various geomagnetic latitudes at intervals of 0.01 bv, using a high degree simulation of the quiet geomagnetic field. For every forbidden trajectory, they have increased the Störmer cutoff at a particular station by 0.01 bv and arrived at what they call the effective geomagnetic cutoffs which give a better fit to the experimentally observed variations in the cosmic ray intensity as a function of geomagnetic latitude and longitude. From the results presented in Tables 1a and 1b, it appears then that during a geomagnetic disturbance these effective cutoffs would vary along different longitudinal planes, which would cause local time effects in especially low energy cosmic ray intensity observed at balloon heights or in low altitude satellites. No attempt has been made here to calculate these effective cutoffs, since for that purpose it would be appropriate to

Tables 1a and 1b

consider an accurate model of the distorted geomagnetic field where the higher harmonics of the surface field are also taken into account. It is also apparent from Tables 1a and 1b that except for the midnight meridian, there are more nonentrant particles in the distorted than in the dipole geomagnetic field. This result, though derived from a limited number of trajectories, is in agreement with the conclusion drawn by Akasofu, et al. [1963] that the limitation of the radial extent of the geomagnetic field due to the solar wind cannot produce reduction in the geomagnetic cutoffs.

To determine asymptotic acceptance cones, the asymptotic directions for particles of various rigidities were calculated for arrival at earth from the vertical and from the geomagnetic North, South, East, and West at zenith angles of 16° and 32°. The rigidity intervals from geomagnetic cutoff to an upper limit of 200 bv were chosen such that there were relatively small changes in asymptotic directions from one rigidity to the next. This is not strictly true near the station cutoff as discussed later. The asymptotic directions corresponding to each rigidity band and the arrival cone were given proper weighting for the cosmic ray primary rigidity spectrum taking into account the yield functions of neutrons at various atmospheric depths [Webber and Quenby, 1959], and the known zenithal response of neutron monitors. For further details of the method of calculations, see Lapointe and Rose [1961], McCracken [1962], and Rao, et al. [1963]. The asymptotic cones of acceptance of sea level neutron monitors have been calculated in a dipole and the distorted geomagnetic fields at three representative geomagnetic latitudes, 0°, 50°, and 70°, for azimuthal angles of 0°, 90°, 180°, and 270°. The results are presented in

Table 2 where an asymptotic cone of acceptance of a neutron monitor is represented by $\overline{\Lambda}$ and $\overline{\Psi}$, the weighted mean values of asymptotic geomagnetic latitudes and longitudes of particle trajectories, and $\sigma_{\overline{\Lambda}}$ and $\sigma_{\overline{\Psi}}$, their root-mean squares, which give an idea of the width of the acceptance cone. A maximum difference of about 6° is observed in these values at various geomagnetic latitudes when the acceptance cones in a dipole field are compared with those calculated in a distorted geomagnetic field at various azimuthal angles. These differences do not seem to vary in a definite manner with the geomagnetic latitude or longitude. We believe these are largely due to errors introduced in the calculation, particularly because of trajectories near the station cutoff. The asymptotic directions of particles change very rapidly there with as small a change in rigidity and the arrival direction as 0.1 bv and 1°, respectively. It is therefore difficult to choose a particle trajectory which would represent the average behavior of the whole rigidity band near the cutoff for various cones of arrival, unless rigidity bands and arrival cone widths of less than 0.1 bv and 1°, respectively, are considered. Because of the large amount of time consumed on the computer such fine intervals have not been considered here. Besides, the errors involved in the specific yield functions of cosmic ray particles and in our knowledge of the zenithal response of neutron monitors would also cause uncertainties in the calculated asymptotic cones of acceptance which would easily be of the order of 5°. It may therefore be concluded that during magnetically disturbed periods, the change in the geomagnetic

Table 2

field configuration due to solar wind does not by itself bring about any appreciable change in the asymptotic cones of acceptance of cosmic ray neutron monitors at sea-level stations beyond the possible errors involved in the method of calculations. A model calculation for a high altitude station (atmospheric depth 680 g/cm^2), at the geomagnetic equator, shows that though there is a shift in the $\overline{\Psi}$ values, the change from dipole to distortion field is negligible. However, during magnetically disturbed periods, there may be other effects such as a ring current around the earth, a change in the cosmic ray rigidity spectrum, or a change in station cutoffs, which might modify these acceptance cones. The former effects have been discussed by McCracken [1962], McCracken, et al. [1962], and Rao, et al. [1963]. The latter effect would again be negligible for small changes in station cutoffs for at least those neutron monitors below the atmospheric depth of 680 g/cm^2.

In the present calculations a highly distorted geomagnetic field was considered where one would expect to find maximum changes in the asymptotic cones of acceptance at various latitudes and longitudes. The negative results therefore imply that there would be no significant effects for less severely distorted geomagnetic fields. Further, a steep cosmic ray rigidity spectrum corresponding to a minimum solar active period of 1954 [Webber and Quenby, 1959] was used for giving proper weighting to asymptotic directions of various particle rigidities. This implies that for cosmic ray variations with flatter spectrums, such as Forbush decreases and the diurnal variation, the change in the asymptotic cones of acceptance due to geomagnetic field distortion would again be insignificant. However, for solar flare type variations in cosmic rays, asymptotic acceptance cones may undergo changes due to geomagnetic field distortion, since the spectrum for such variations is much steeper, thereby leading to a greater bias for asymptotic directions of low rigidity particles which undergo large changes. A model spectrum of the type R^{-5} (R = Rigidity) was used to calculate asymptotic cones of acceptance of neutron monitors at 680 g/cm^2 atmospheric depth at geomagnetic latitudes of 50° and 70° for the dipole and the distorted geomagnetic field along the noon meridian. The weighted mean values of asymptotic geomagnetic latitudes and longitudes presented in Table 3 show that while there are large changes in $\bar{\Psi}$ at $\lambda_m = 50^{\circ}$ the changes at $\lambda_m = 70^{\circ}$ occur mainly in $\bar{\Lambda}$. It should be recognized though that the errors associated with $\bar{\Psi}$ and $\bar{\Lambda}$ are also larger here because of more bias toward asymptotic directions of particles close to the geomagnetic cutoff. Extending these results to detectors at sea level and those at the top of the atmosphere, one would expect smaller changes in asymptotic cones of acceptance in the former case but much larger changes in the latter case. It therefore seems that when anisotropic effects of cosmic ray solar flare increases are studied by means of balloon borne detectors or low altitude satellites, it is important to take into account the geomagnetic field distortion in calculating asymptotic cones of acceptance. For each individual event, an appropriate rigidity spectrum and the geomagnetic field distortion would have to be considered. Besides, it would also be important to take into account the higher harmonics of the surface geomagnetic field which, together with solar wind distortion effects, may cause a larger change in the asymptotic acceptance cones at certain stations.

<Table 3

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TABLE 1.- Asymptotic geomagnetic latitudes and longitudes of particles in dipole and distorted field

(a) Vertically arriving cosmic rays near cutoff at 50° geomagnetic latitude

Rigidity	Dipole field	Distorted field			
		φ = 0°	90°	180°	270°
2.7 bv	Λ 92.3° Ψ 304.0°	No entry	95.6° 4379.1°	92.8° 525.8°	No entry
2.8 bv	Λ Ψ No entry	93.1° 918.4°	No entry	94.7° 821.7°	No entry
2.9 bv	Λ 89.0° Ψ 353.9°	100.2° 656.5°	80.0° 489.6°	84.6° 383.2°	99.2° 895.9°
3.0 bv	Λ 101.9° Ψ 254.6°	No entry	No entry	90.8° 383.5°	No entry
3.1 bv	Λ 73.1° Ψ 179.1°	102.3° 445.1°	102.8° 426.0°	91.1° 379.1°	86.8° 599.0°
3.2 bv	Λ 81.7° Ψ 151.8°	82.3° 229.7°	74.1° 177.0°	78.6° 152.1°	81.0° 509.7°

TABLE 1.- Asymptotic geomagnetic latitudes and longitudes of particles in dipole and distorted field - Concluded

(b) A 3 bv particle arriving at 50° geomagnetic latitude along the vertical and zenith angles of 16° and 32° in the geomagnetic North, South, East, and West planes

Rigidity	Arrival direction	Dipole field	Distorted field			
			φ = 0°	90°	180°	270°
3 bv	Vertical	Λ 101.9° ψ 254.6°	No entry	No entry	90.8° 383.5°	No entry
	16° N	Λ 99.4° ψ 402.6°	No entry	No entry	83.5° 271.6°	No entry
	32° N	Λ 76.3° ψ 168.6°	91.9° 267.5°	93.2° 234.4°	97.9° 275.5°	103.5° 382.1°
	16° E	Λ 88.4° ψ 1092.7°	90.4° 274.5°	89.0° 462.2°	No entry	98.8° 387.0°
	32° E	Λ ψ No entry	No entry	87.7° 620.2°	95.5° 445.8°	No entry
	16° S	Λ 78.7° ψ 209.0°	No entry	87.6° 408.8°	88.5° 362.2°	No entry
	32° S	Λ 75.2° ψ 193.2°	No entry	100.1° 528.5°	92.0° 290.8°	96.3° 635.2°
	16° W	Λ 73.3° ψ 174.0°	79.6° 721.0°	87.1° 564.0°	87.6° 244.0°	76.7° 346.2°
	32° W	Λ 77.6° ψ 156.0°	108.1° 287.9°	72.5° 190.8°	71.6° 167.2°	106.8° 293.1°

TABLE 2.- Mean asymptotic geomagnetic latitudes and longitudes ($\bar{\Lambda}$, $\bar{\Psi}$), and their root-mean squares ($\sigma_{\bar{\Lambda}}$, $\sigma_{\bar{\Psi}}$), of asymptotic cones of acceptance of cosmic ray neutron monitors at various geomagnetic latitudes and longitudes, for the cases of dipole and the distorted geomagnetic fields

Geomagnetic latitude		Dipole field	Distorted field			
			$\varphi = 0^\circ$	90°	180°	270°
0°	$\bar{\Lambda}$	0°	0.1°	0°	0°	0°
	$\bar{\Psi}$	70.2°	66.1°	70.2°	67.4°	63.8°
	$\sigma_{\bar{\Lambda}}$	10.0°	9.8°	10.5°	10.3°	10.5°
	$\sigma_{\bar{\Psi}}$	50.5°	53.1°	50.2°	47.8°	51.8°
50°	$\bar{\Lambda}$	6.9°	11.8°	7.4°	7.1°	9.9°
	$\bar{\Psi}$	49.3°	43.3°	49.6°	49.6°	42.5°
	$\sigma_{\bar{\Lambda}}$	22.3°	21.5°	22.3°	21.8°	26.1°
	$\sigma_{\bar{\Psi}}$	31.2°	34.6°	31.4°	36.5°	36.7°
70°	$\bar{\Lambda}$	47.0°	50.4°	48.6°	46.9°	48.6°
	$\bar{\Psi}$	24.3°	21.2°	25.1°	26.0°	22.4°
	$\sigma_{\bar{\Lambda}}$	11.6°	10.5°	11.3°	12.2°	11.3°
	$\sigma_{\bar{\Psi}}$	18.4°	19.0°	18.7°	18.4°	18.3°

TABLE 3.- Mean asymptotic latitudes and longitudes of neutron monitors at 680 g/cm^2 atmospheric depth for cosmic ray rigidity spectrum of the form R^{-5}

Geomagnetic latitude	Dipole field	Distorted field (Noon meridian)
50°	$\bar{\Lambda}$ -4.3°	-3.7°
	$\bar{\Psi}$ 111.8°	126.6°
70°	$\bar{\Lambda}$ 69.9°	58.2°
	$\bar{\Psi}$ 26.9°	20.3°

Figure Title

Fig. 1.- Meridian cross section of deformed magnetosphere [Hones, 1963].