DYNAMICAL BEHAVIOR OF MAGNETOSPHERE BOUNDARY FOLLOWING IMPACT BY DISCONTINUITY IN THE SOLAR WIND

John R. Spreiter and Audrey L. Summers

NASA, Ames Research Center
Moffett Field, California

29455

ABSTRACT

Time histories of the development of the magnetosphere boundary following impact of a sharp-fronted plasma cloud and of the compression of the magnetosphere following impact by a discontinuity in the solar wind are calculated. Results are presented for initially planar discontinuities having their normals either aligned with or inclined 30° to the wind direction. Although several minutes are generally required for a discontinuity in the solar wind to sweep past the geophysically significant portion of the magnetosphere, it is found that any individual element of the magnetosphere boundary usually completes the major part of its adjustment to the new conditions within a period of about a minute. Author

[redacted]

1. INTRODUCTION AND FORMULATION OF PROBLEM

Although the basic concepts fundamental to understanding how the geomagnetic field carves a hollow in a rapidly streaming solar plasma derive principally from a long series of pioneering studies of transient phenomena by Chapman and Ferraro (see CHAPMAN, 1963, for a recent review), most recent studies have been concerned with the corresponding steady-state problem (see BEARD, 1964, for a recent review). Since the latter problem is now becoming relatively well solved and the results generally agree with measurements made in space (see particularly NESS, SCEARCE, and SEEK, 1964), we return to two unsteady-state problems similar to those originally considered by Chapman and Ferraro over 30 years ago. We consider first that a huge isolated cloud of plasma advances toward and engulfs the earth, and, second, that a discontinuity in momentum flux density in an otherwise steady solar wind sweeps past the earth. The latter more realistic possibility may be associated either with a shock wave across which both the density and velocity are discontinuous, or with a contact surface across which the density, but not the velocity, changes.

We shall consider only the geomagnetic equatorial cross section of the hollow, and that the velocity vector $\underset{\sim}{V}$ of the undisturbed incident stream lies in this plane. We introduce rectangular Cartesian coordinates with the origin at the center of the earth, the xy plane lying in the geomagnetic equatorial plane, and the y axis directed upstream as illustrated in Fig. 1.

The magnetic field at the magnetosphere boundary will be denoted by $\underset{\sim}{B}_s$, and the angle between $\underset{\sim}{V}$ and the normal $\hat{n}$ to the boundary by ψ.

[redacted]

The dynamical balance of magnetic and fluid pressures on the two sides of the moving boundary provides the following boundary condition at the surface of the hollow (FERRARO, 1952, 1960)

$$B_s^2/8\pi = Kmn[(\underset{\sim}{V} - \underset{\sim}{v}) \cdot \hat{n}]^2 = Kmn[V \cos \psi - v_n]^2 \qquad (1)$$

where m and n are the mass and number density of ions in the incident stream, $\underset{\sim}{v}$ is the velocity of a segment of the boundary, and $v_n = \underset{\sim}{v} \cdot \hat{n} = v \cos \psi$ is its component normal to the boundary. Cosine ψ can moreover be expressed as follows:

$$\cos \psi = \partial x/(\partial x^2 + \partial y^2)^{1/2} \qquad (2)$$

where time is to be held constant in the evaluation of the partial derivatives. The coefficient K depends on the nature of the interaction between the particles of the solar wind and the boundary. Specular reflection, for which the appropriate value is $K = 2$, has been preferred historically, but recent developments (see, e.g., SPREITER and JONES, 1963) suggest that a hydromagnetic fluid model, for which $K = 1$, is to be favored.

Following FERRARO (1952), the unknown magnetic field B_s at the magnetosphere boundary will be approximated by twice the tangential component of the earth's magnetic dipole field. Thus,

$$B_s = 2B_o(a/r)^3 \qquad (3)$$

where $B_o = 0.312$ gauss is the equatorial value of the earth's surface magnetic field, $a = 6.37 \times 10^8$ cm is the radius of the earth, and $r = (x^2 + y^2)^{1/2}$ is the radial distance from the center of the earth.

Solving equations (1) and (3) for v_n/V and selecting the proper sign so that

$$\cos\psi = B_s/8\pi p_{st} \tag{4}$$

in the final steady state when $v_n = 0$ yields

$$\frac{v_n}{V} = \cos\psi - \frac{B_o}{(2\pi p_{st})^{1/2}}\left(\frac{a}{r}\right)^3 \tag{5}$$

where $p_{st} = KmnV^2$ represents the stagnation pressure at the apex of the hollow. Cartesian components of the normal velocity of a surface element are thus

$$\frac{\partial x}{\partial t} = v_n \sin\psi\ , \qquad \frac{\partial y}{\partial t} = -v_n \cos\psi \tag{6}$$

Given a curve or set of coordinates for the initial equatorial trace of the geomagnetic boundary, it is possible to integrate equation (6) numerically and determine the corresponding boundary coordinates for a slightly later time. Successive application of this process enables the calculation of the boundary coordinates for all later times.

The above equations may be expressed alternatively in dimensionless form by introduction of the distance ρ_o from the center of the earth to the apex of the steady-state hollow ($v_n = 0$, $\psi = 0$) as a unit of length, and the time τ_o required to travel a distance ρ_o at speed V as a unit of time; that is,

$$\rho_o = a(B_o{}^2/2\pi p_{st})^{1/6}\ , \qquad \tau_o = \rho_o/V \tag{7}$$

Thus, equations (5) and (6) can be rewritten as follows:

$$\frac{\partial\xi}{\partial\tau} = -\left(\frac{1}{\rho^3} - \cos\psi\right)\sin\psi\ , \qquad \frac{\partial\eta}{\partial\tau} = \left(\frac{1}{\rho^3} - \cos\psi\right)\cos\psi \tag{8}$$

where

$$\xi = x/\rho_o\ , \qquad \eta = y/\rho_o\ , \qquad \rho = r/\rho_o\ , \qquad \tau = t/\tau_o \tag{9}$$

2. INTERACTION WITH AN ISOLATED PLASMA CLOUD

Over 30 years ago, CHAPMAN and FERRARO (1931) described how an approximately parabolic hollow would be formed in an isolated and initially flat-faced plasma cloud as it approaches and passes beyond the earth. Although their sketches have been reproduced widely, and FERRARO (1952, 1960) has more recently integrated equation (6) with $\psi = 0$ (see also CHAPMAN, 1960, for a review), the time history of the motion of the remainder of the front of the plasma cloud has never been computed. We shall discuss this problem first because of its relative simplicity, conceptual value, and historical interest.

We consider that a uniform plasma cloud of density mn sweeps toward the earth with velocity V and that its leading surface is planar at time $t = 0$ when at a normal distance r_i from the center of the earth. The distance r_i is selected so as to be sufficiently large that the previous deformation of the boundary as it approaches r_i from infinity is small, but this is actually immaterial to the calculations since we are only concerned with the behavior of the plasma boundary at times subsequent to the initial time. The result may be presented either in convenient dimensional form or more universal dimensionless form. We have elected the first alternative, since the scaling laws are very simple, and have selected the following representative values for the pertinent variables: $V = 500$ km/sec $= 5\times10^7$ cm/sec; $n = 5$ protons/cm^3; $m = 1.67\times10^{-24}$ gm; and $K = 1$. Results are shown in Fig. 2 for the case in which the initially planar front face of the plasma cloud is parallel to the x axis at a distance $r_1 = 20$ earth radii. The corresponding results are shown in Fig. 3 for a case in which the initial planar face is inclined 30° to the x axis. The dashed line shows the final steady-state configuration of the boundary defined by the integral curve of equation (4) that passes through $\rho = 1$ at $x = 0$. The latter solution was indicated first in parametric form by FERRARO (1952), and evaluated explicitly in numerical form by BEARD (1960) and SPREITER and BRIGGS (1961).

The results in Figs. 2 and 3 are free of surprises and are quite consistent with the early qualitative descriptions of Chapman and Ferraro. Quantitative features of particular interest are the rapidity with which the boundary approaches its steady-state form as the plasma front sweeps by, the smallness of effects of obliquity of the plasma front, and the tendency to form waves that drift downstream along the boundary as it nears the steady-state configuration. The latter tendency is consistent with analytic results that can be derived for small scale waves on the magnetosphere boundary.

The results described above can be converted by a simple relabeling to those for other values for the pertinent variables by use of the following relations derived from equation (9)

$$r_2/r_1 = x_2/x_1 = y_2/y_1 = (p_{st_1}/p_{st_2})^{1/6}, \qquad t_2/t_1 = (p_{st_1}/p_{st_2})^{1/6}(V_1/V_2) \tag{10}$$

where subscript 1 refers to the values used to calculate the results shown in Figs. 2 and 3, and subscript 2 refers to any other set of values.

3. INTERACTION WITH A DISCONTINUITY IN MOMENTUM FLUX DENSITY IN THE SOLAR WIND

We consider next that a steady solar wind has existed for a sufficiently long time (of the order of 10 minutes on the basis of the preceding results) that the boundary between the geomagnetic field and the solar plasma is well established in the steady-state configuration associated with a value p_{st_a} for the stagnation pressure. We consider further that a discontinuity in momentum flux advances through or with the solar wind plasma with velocity V_d, and that the discontinuity surface is planar over lateral distances larger than the magnetosphere before it begins to interact with the magnetic field of the earth. Behind the discontinuity surface, conditions are considered to be steady and uniform, but are characterized by a larger stagnation pressure p_{st_b}. Ultimately, therefore, the magnetosphere boundary will assume a configuration geometrically similar to that possessed originally, but smaller in the ratio $(p_{st_a}/p_{st_b})^{1/6}$. We seek here to calculate the time history of the collapse of the magnetosphere from the initial to the final configuration.

The governing differential equations are the same as in the preceding case, but the initial conditions are different. Thus, the initial shape of the magnetosphere boundary is given by the solution of equation (4) with $p_{st} = p_{st_a}$. At an interval of time Δt after the discontinuity surface passes the magnetosphere nose, the most upstream portion of the magnetosphere extending over a distance $V_d \Delta t$ in the y direction will have been exposed to the greater pressure p_{st_b} behind the discontinuity surface and set into motion toward its final equilibrium position. The remainder of the magnetosphere boundary has not yet experienced the enhanced pressure, however, and remains fixed in its initial configuration. A general situation that may exist at any time after the discontinuity surface has passed the initial position of the magnetosphere nose is illustrated in Fig. 4. The shape of the magnetosphere boundary and the discontinuity surface at a small time interval later is calculated as follows. First, the part of the discontinuity surface exterior to the initial magnetosphere shape simply advances with velocity V_d and remains straight and parallel to its initial direction. The remainder of the discontinuity surface coincides at any moment with a portion of the magnetosphere boundary, and advances with velocity components given by equation (6) using the values V_d and p_{st_b} for V and p_{st}. The displacement of this portion of the magnetosphere boundary during a small time interval can then be determined numerically in the same manner as in the preceding case. Since each boundary element moves normal to itself in this calculation, there always remains small unconnected gaps separating the ends of the moving portion of the magnetosphere boundary and the straight portion of the discontinuity surface exterior to the magnetosphere. The necessary connections have been made with a parabola for the first time interval after impact of the discontinuity surface and the magnetosphere nose, and with straight lines for all subsequent times. The remainder of the magnetosphere boundary remains, of course, in its initial configuration until the arrival of the discontinuity surface.

Results are presented in Figs. 5 and 6 for two specific cases in which the initial planar discontinuity surface is inclined at 0° and 30° to the x axis. They are for a discontinuity surface which advances with a velocity

$V_d = 5\times10^7$ cm/sec and is characterized by a fourfold increase in p_{st} from $p_{st_a} = 2.09\times10^{-18}$ dynes/cm^2 to $p_{st_b} = 4p_{st_a}$. This value for p_{st_a} could represent, for instance, a flux of 5 protons/cm^3 traveling at 500 km/sec with $K = 1$, or any other combination of values for which the product $p_{st_a} = (KmnV^2)_a$ is the same. The initial and final steady-state solutions are indicated by dashed lines. As in the previous examples, the boundary is found to collapse quite rapidly to its final configuration. Although several minutes are required for the discontinuity surface to sweep past the forward part of the magnetosphere of greatest geophysical interest, each element of the boundary essentially completes its movement from initial to final position in less than a minute.

The following relations can be used to convert the results in Figs. 5 and 6 to those associated with other values of the stated parameters;

$$r_2/r_1 = \left(p_{st_1}/p_{st_2}\right)_b^{1/6}, \qquad t_2/t_1 = \left(p_{st_1}/p_{st_2}\right)_b^{1/6}\left(V_{d_1}/V_{d_2}\right) \tag{11}$$

where the notation with respect to subscripts 1 and 2 is the same as in the case of equation (10). It is required that the ratio $p_{st_b}/p_{st_a} = 4$ as stated above, however. Results for other values for this ratio could be computed in the same manner, but there is little reason to believe the results would differ substantially from those presented here.

REFERENCES

Beard D. B.	1960	J. Geophys. Res. 65, 3559-3568.
Beard D. B.	1964	Rev. Geophys. 2, 335-365.
Chapman S. and Ferraro V. C. A.	1931	Terr. Mag. and Atmos. Elec. 36, 171-186.
Chapman S.	1960	Rev. Mod. Phys. 32, 919-933.
Chapman S.	1963	"Solar Plasma, Geomagnetism and Aurora," in Geophysics, The Earth's Environment, ed. by C. DeWitt, J. Hieblot, and A. Lebeau, Gordon and Breach, New York.
Ferraro V. C. A.	1952	J. Geophys. Res. 57, 15-49.
Ferraro V. C. A.	1960	Rev. Mod. Phys. 32, 934-940.
Ness N. F., Scearce C. S. and Seek J. B.	1964	J. Geophys. Res. 69, 3531-3569.
Spreiter J. R. and Briggs B. R.	1961	NASA TR R-120.
Spreiter J. R. and Jones W. P.	1963	J. Geophys. Res. 68, 3555-3564.

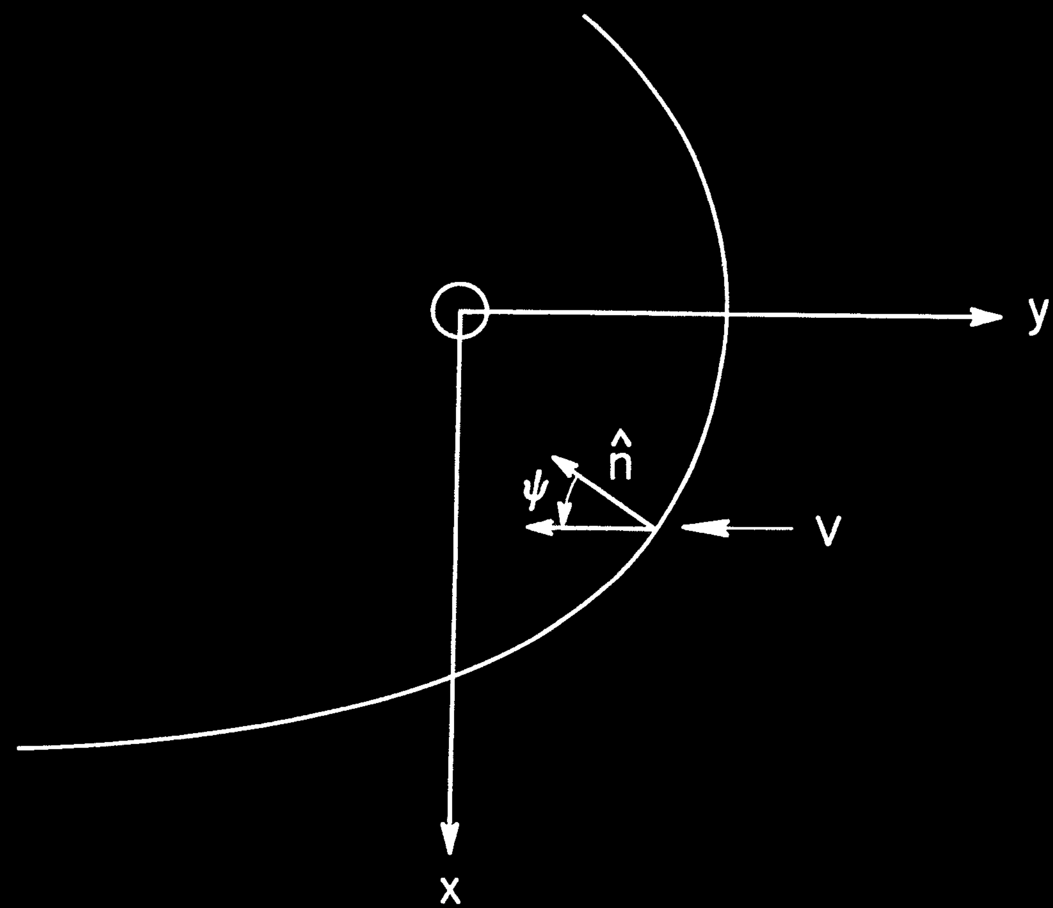
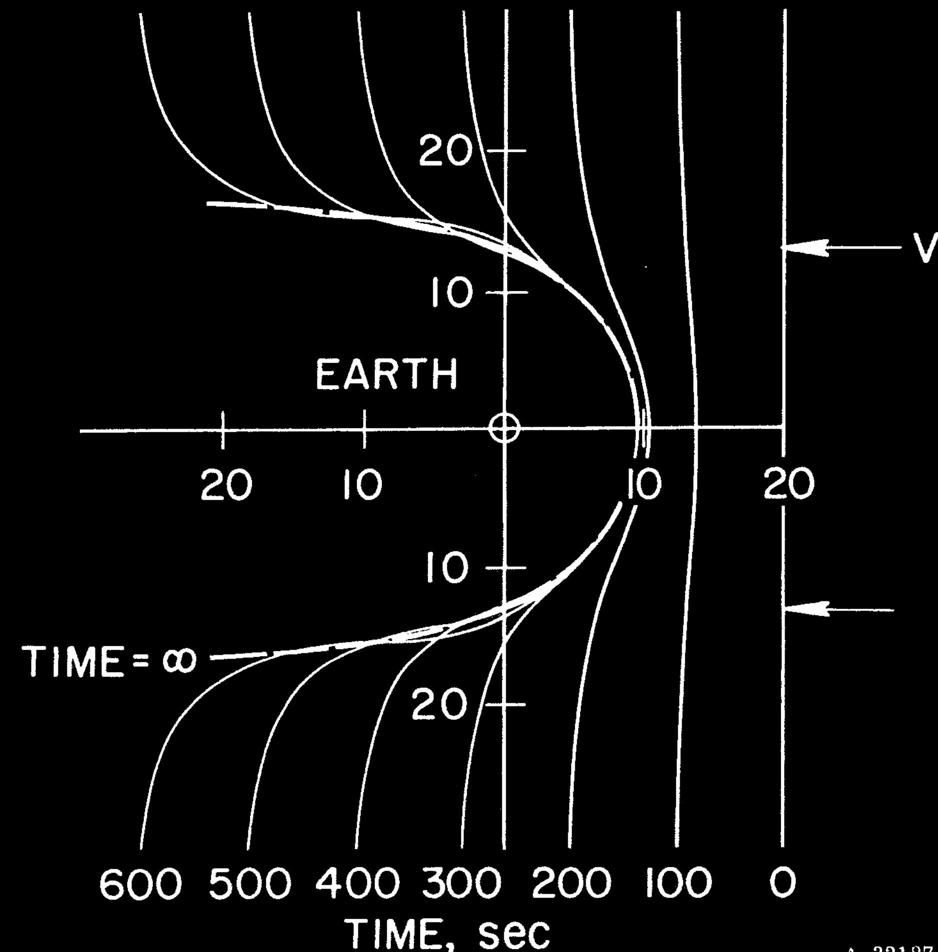


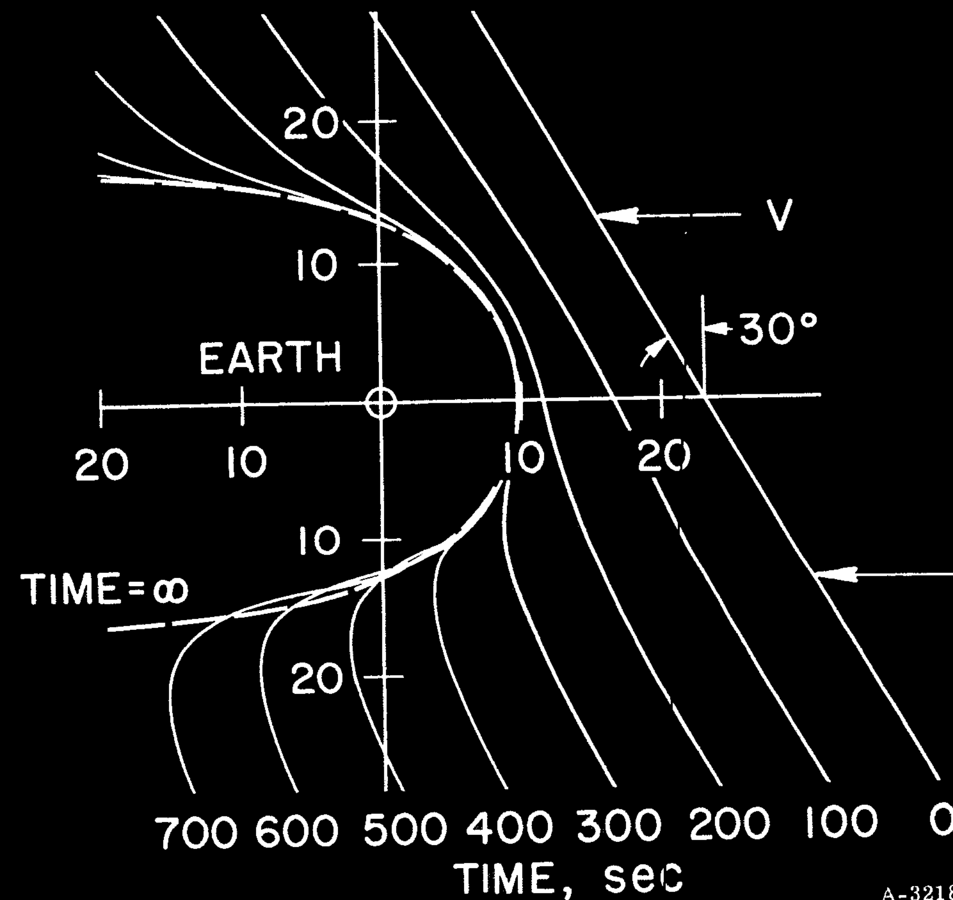
Fig. 1. View of coordinate system and equatorial trace of magnetosphere boundary.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
AMES RESEARCH CENTER, MOFFETT FIELD, CALIFORNIA



A-32187-1

Fig. 2.- Successive positions in equatorial plane of front of plasma cloud having a density of 5 protons/cm^3, velocity of 500 km/sec, and a planar face parallel the x axis when at 20 earth radii.



A-32187-2

Fig. 3.- Successive positions in equatorial plane of front of plasma cloud having a density of 5 protons/cm^3, velocity of 500 km/sec, and a planar face inclined 30° to the x axis when at 20 earth radii.

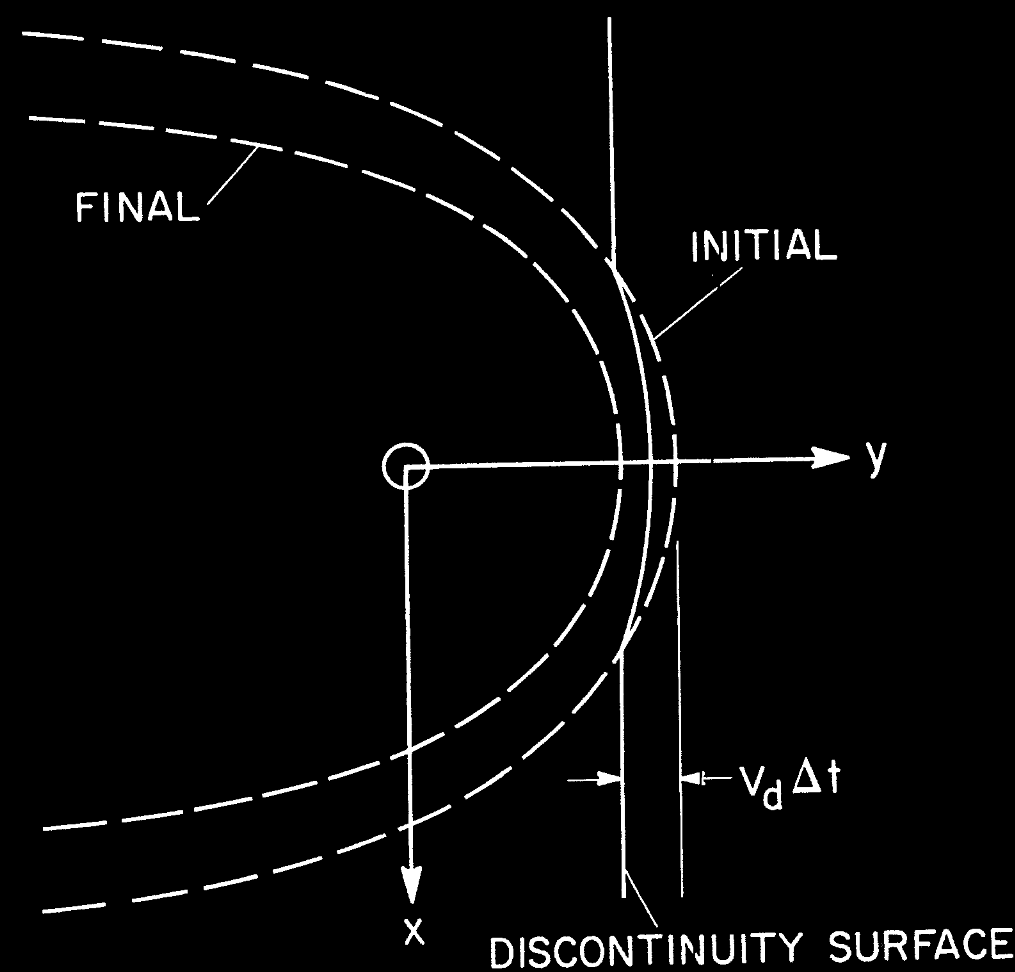
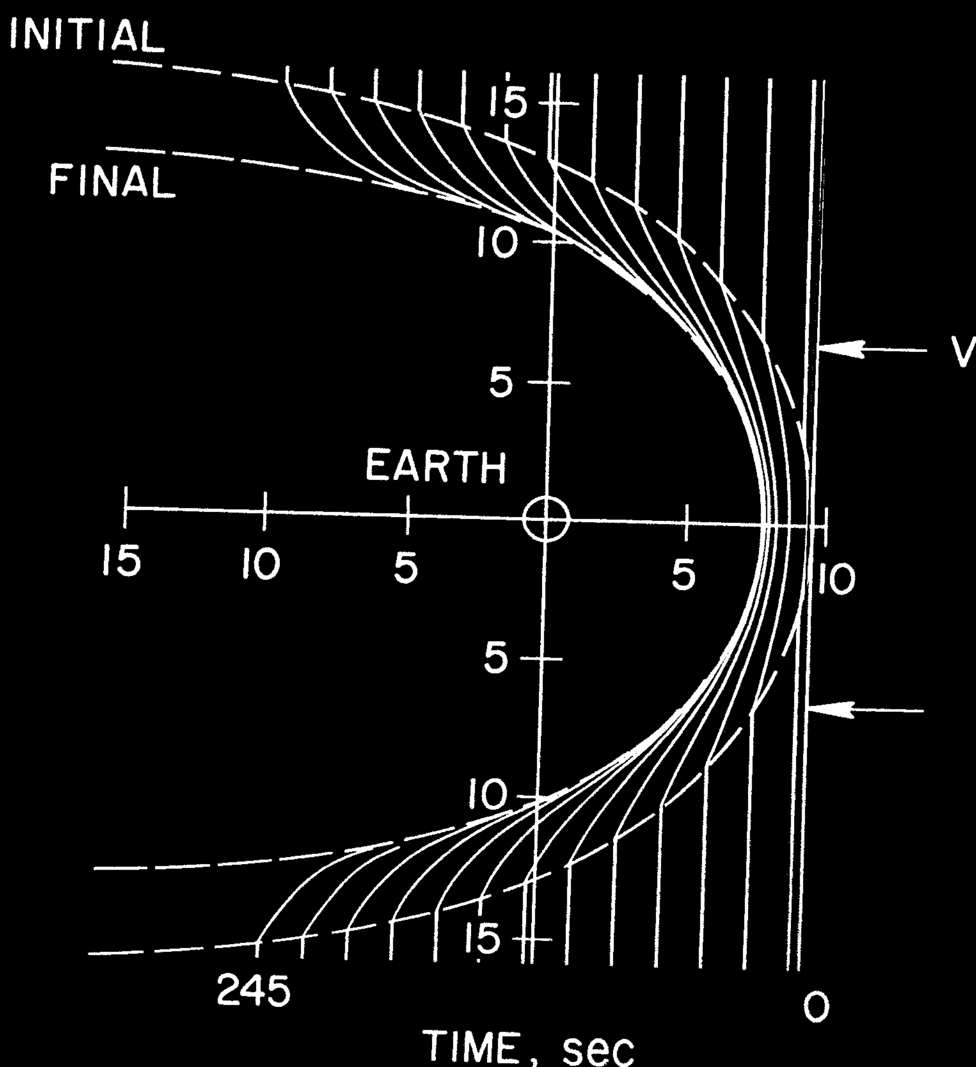
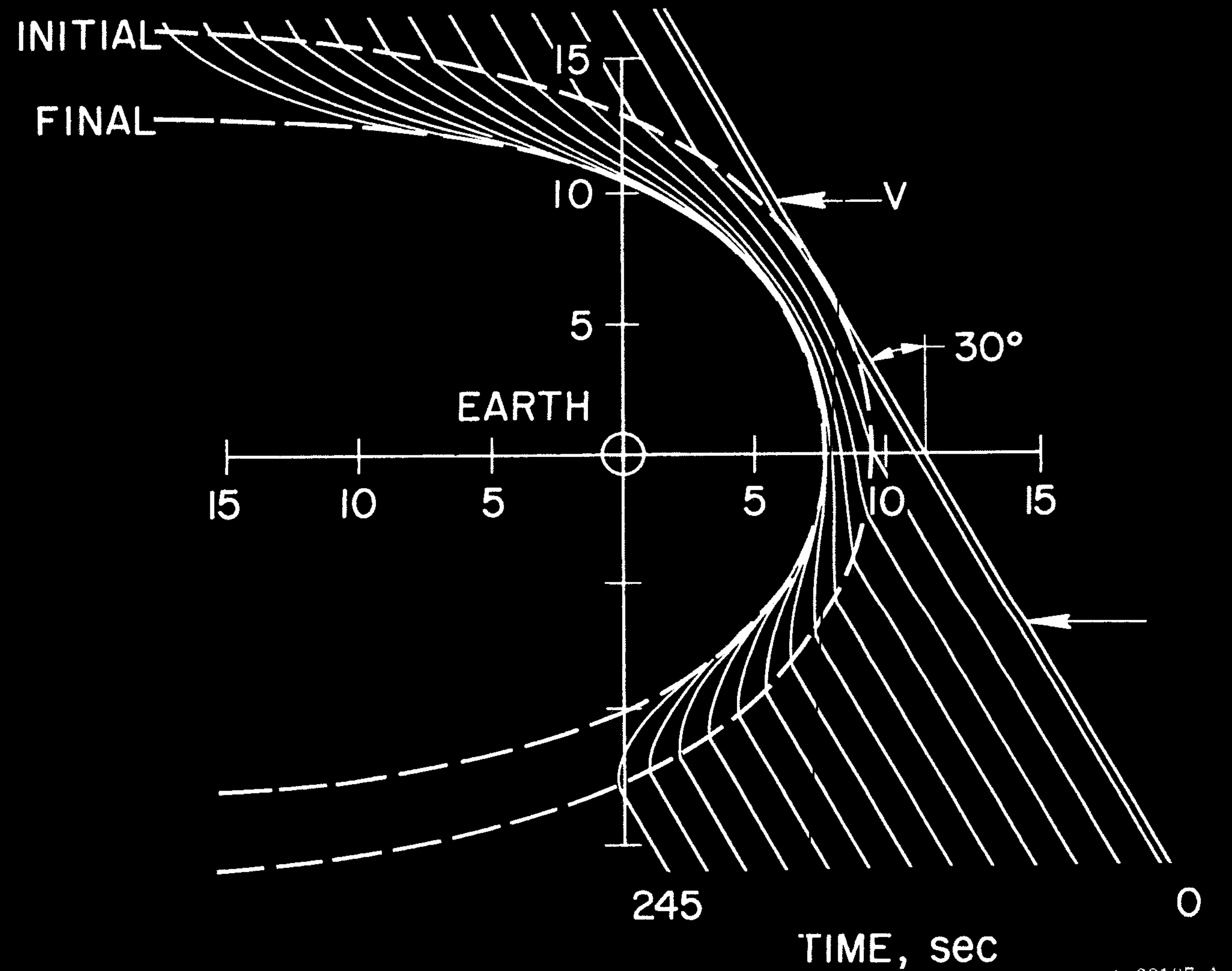


Fig. 4.- Illustration of magnetosphere boundary and discontinuity surface at time Δt after the latter passes the initial position of the magnetosphere nose.



A-32187-3

Fig. 9.- Successive positions in equatorial plane of discontinuity surface alined parallel to the x axis, traveling at 500 km/sec, and characterized by a fourfold increase in stagnation pressure from $p_{st_a} = 2.0 \times 10^{-13}$ dynes/cm^2 to $p_{st_b} = 4\,p_{st_a}$ before interaction with the magnetosphere.



A-32187-4

Fig. [illegible].- Successive positions in equatorial plane of discontinuity surface inclined 30° with the axis, traveling at 500 km/sec, and characterized by a fourfold increase in stagnation pressure from p_{st_a} [illegible] 10^{-10} dynes/cm^2 to $p_{st_b} = 4\, p_{st_a}$ before interaction with the magnetosphere.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
AMES RESEARCH CENTER, MOFFETT FIELD, CALIFORNIA