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Local Resonances of a Plasma in a Magnetic Field

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Experiments with the Alouette topside sounder[1] gave some unexpected results which have been interpreted as the detection of resonances of the surrounding plasma. When excited at one of a discrete set of frequencies, the plasma continues to "ring" at that frequency. This may be termed a "local" resonance of the type detected in the resonance probe experiments of Takayama, Ikegami, and Miyasaki,[2] and in the experiments recently reported by Crawford, Kino, and Weiss.[3] Since the list of Alouette resonances includes harmonics of the electron cyclotron frequency, analysis of the Alouette problem may lead to an understanding of the anomalous emission[4] and absorption[5] at these frequencies observed in laboratory experiments.

Lockwood[6] suggests that the cyclotron harmonics are to be interpreted on single-electron theory, Johnston and Nuttal[7] believe this class of resonance to be a sheath phenomenon, and Walsh[8] presumes it to be a non-linear phenomenon. However, the observed resonances occur at frequencies for which plane waves in a homogeneous plasma have zero group velocity, indicating that the Alouette observations can be explained on the basis of the wave theory of a uniform plasma of nonzero temperature in a magnetic field. It is the purpose of this letter to show that this is the case.

For brevity, we here discuss only the electrostatic resonances. The theory of the electromagnetic resonances proceeds in identical fashion. With the notation

$$\varphi(\underset{\sim}{x},t) = \int\!\!\int d^3k\, d\omega e^{i(\underset{\sim}{k}\cdot\underset{\sim}{x} - \omega t)}\tilde{\varphi}(\underset{\sim}{k},\omega) \qquad (1)$$

and its inverse, we may relate the Fourier transforms of the electrostatic potential and an "applied" electric charge by

$$\tilde{\varphi} = 4\Pi F^{-1}\tilde{\rho} \qquad (2)$$

where F is the expression introduced in Eq. (103) of Ref. 9 (ch. 9). We consider a localized impulse excitation and calculate the asymptotic field at the point of excitation. This comes from waves with negligible Landau damping and near-zero group velocity. These waves have small wave vectors so that it is appropriate to expand F in powers of k. We find that the following expression includes the dominant terms contributing to the electrostatic resonances:

$$\begin{aligned} F = {} & k^2 - \frac{\Pi^2}{\omega^2} k^2 \cos^2\theta - \frac{\Pi^2}{\omega^2 - \Omega^2} k^2 \sin^2\theta \\ & - \frac{3}{2}\frac{\Pi^2}{\omega^4} v^2 k^4 \cos^4\theta + \frac{1}{2}\frac{\Pi^2}{\Omega^2\omega^2} v^2 k^4 \cos^2\theta \sin^2\theta \\ & + \frac{1}{2}\frac{\Pi^2 v^2 k^4 \sin^4\theta}{\Omega^2(\omega^2 - \Omega^2)} - \frac{1}{2}\frac{\Pi^2\omega^2(\omega^2 + 3\Omega^2)}{\omega^2(\omega^2 - \Omega^2)^3} v^2 k^4 \cos^2\theta \sin^2\theta \\ & - 4\Pi^2 \sum_{n=2}^{\infty} \frac{1}{2^{2n} n!} \frac{n^2\Omega^2}{(\omega^2 - n^2\Omega^2)\Omega^{2n}} v^{2n-2} k^{2n} \sin^{2n}\theta \end{aligned} \qquad (3)$$

Π and Ω are the electron plasma and gyro frequencies, respectively, $v = (2kT/m)^{1/2}$, and θ is the angle between the wave vector and the magnetic field.

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The resonance at the plasma frequency is associated with waves for which k and θ are small and $\omega^2 \approx \Pi^2$. The appropriate approximation to (3) is

$$F = \frac{k^2}{\Pi^2}\left(\omega^2 - \Pi^2 + \frac{\Pi^2\Omega^2}{\Omega^2 - \Pi^2}\,\theta^2 - \frac{3}{2}\,v^2k^2\right) \tag{4}$$

As a simple model of a pulsed transmitting antenna, we consider excitation by an infinitesimal dipole impulse. The resonance $\omega^2 = \Pi^2$ is excited by the component of the dipole parallel to the magnetic field, so that we may adopt this particular orientation:

$$\rho(\underset{\sim}{x},t) = \delta(x)\delta(y)\delta'(z)\delta(t) \tag{5}$$

and

$$\tilde{\rho}(\underset{\sim}{k},\omega) = i(2\Pi)^{-4}k\cos\theta \tag{6}$$

We may now evaluate the electric field at the origin from (1), (2), (4) and (6):

$$E_z(0,t) = 2^{-1}\Pi^{-2}\iiint d\omega\, dk\, d\theta k^4 \sin\theta \cos^2\theta e^{-i\omega t}F^{-1} \tag{7}$$

If we integrate first over frequency, using the contour indicated by causality, then over k and θ, we obtain

$$E_z(0,t) = \frac{4}{3^{3/2}\Pi^{1/2}}\,\frac{\Pi^{3/2}(\Omega^2 - \Pi^2)}{\Omega^2 v^3}\,t^{-5/2}\cos\left(\Pi t + \frac{3\Pi}{4}\right) \tag{8}$$

The upper hybrid resonance at $\omega^2 = \Pi^2 + \Omega^2$ may be evaluated in a similar way. The dipole should now be oriented normal to the magnetic field, for example, in the x direction. If $\theta = (\Pi/2) - \psi$, the appropriate approximation to F is

$$F = \frac{k^2}{\Pi^2}\left(\omega^2 - \Pi^2 - \Omega^2 + \frac{\Pi^2\Omega^2}{\Pi^2 + \Omega^2}\,\psi^2 + \frac{1}{2}\,v^2k^2\right) \tag{9}$$

which leads to

$$E_x(0,t) = 2^{1/2}\,\frac{\Pi(\Pi^2 + \Omega^2)}{\Omega v^3}\,t^{-2}\sin[(\Pi^2 + \Omega^2)^{1/2}t] \tag{10}$$

The expression (3) does not possess a zero near the gyro frequency, but it does have zeros at harmonics of the gyro frequency. At $\omega \approx n\Omega$, (3) may be approximated by

$$F = \left[1 - \frac{\Pi^2}{\Omega^2}\frac{n^2 - \cos^2\theta}{n^2(n^2 - 1)}\right]k^2 - \frac{n}{2^{2n-2}(n - 1)!}\frac{\Pi^2 V^{2n-2}k^{2n}\sin^{2n}\theta}{\Omega^{2n-2}(\omega^2 - n^2\Omega^2)} \qquad (11)$$

The approximation of an antenna by an infinitesimal dipole is invalid for $n = 2,3,4$, since the integral proves to be divergent. The response of the plasma to a line dipole does not have this peculiarity. Another simple model, more closely related to observations made with a long antenna, is that of excitation by a line charge. If one makes the choice

$$\rho(\underset{\sim}{x},t) = \delta(x)\delta(y)\delta(t) \qquad (12)$$

so that

$$\rho(\underset{\sim}{k},\omega) = (2\Pi)^{-3}\delta(k_z) \qquad (13)$$

and allows for arbitrary orientation of the line charge to the magnetic field by writing

$$\underset{\sim}{B} = (B\sin\Theta, 0, B\cos\Theta) \qquad (14)$$

the electric potential at the line charge may be evaluated by means of equations (1), (2), (11), and (13). The integrals are elementary and lead to the result

$$\varphi(0,t) = -2(n - 1)^{-1}\left(1 - \frac{\Pi^2}{\Omega^2}\frac{1}{n^2 - 1}\right)^{-1/2}\left[1 - \frac{\Pi^2}{\Omega^2}\frac{n^2 - \sin^2\Theta}{n^2(n^2 - 1)}\right]^{-1/2} t^{-1}\cos n\Omega t \qquad (15)$$

It is clear from this formula that the "strength" of the resonance falls off only slowly with n.

From published figures of the antenna dimensions and impedance, of the power and pulse length of the transmitter, and of the sensitivity of the receiver,[10] the equivalent dipole impulse is estimated to be about 10^2 esu and the detectable field strength about 2×10^{-12} esu. These estimates indicate that the plasma resonance would be observable for about 10 sec and the other electrostatic resonances even longer. The resonances are in fact observable for only 1 to 10 msec. This rapid damping cannot be explained by electron collisions, but it seems that it can be explained by the velocity of the satellite relative to the plasma, about 10^6 cm sec^{-1}, since the vehicle moves about 20 gyro-radii or 30 Debye lengths per millisecond.

It seems, therefore, that the finite satellite velocity must be included in any extension of the theory here outlined. Not only would this modification lead to more rapid damping of the plasma response, but it would also yield a zero of the dielectric coefficient F near the electron gyro frequency which would then explain the resonance observed at this frequency.

It is possible that the resonances detected by emission and absorption measurements[3,4] are related to the resonances discussed in this letter by coupling of the electrostatic and electromagnetic modes through density and current inhomogeneities. This mechanism has previously been advanced as the explanation of the observed emission spectra of Type II solar radio bursts.[11]

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