COHERENT FDM/FM TELEPHONE COMMUNICATION

Jean A. Develet, Jr.

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SPACE TECHNOLOGY LABORATORIES, INC.
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Prepared: Jean A. Develet, Jr.
Jean A. Develet, Jr., Head
Communication Theory Section

Approved: R C Booton Jr
R. C. Booton, Jr.
Study Director
Project Relay

SPACE TECHNOLOGY LABORATORIES, INC.
P. O. Box 95001
Los Angeles 45, California

ABSTRACT

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This paper analyzes telephone communication as applied to a satellite repeater system. In particular, emphasis is placed on a method of coherent reception which is important to our emerging communication satellite systems. This reception technique is not new to the field of space communications and telemetry; however, it is new to the field of common carrier telephony. As a consequence, for the class of signals utilized in common carrier telephony, an attempt is made to place on a quantitative footing the design of FDM/FM satellite communication systems. The interrelation among such quantities as sensitivity, bandwidth occupancy, and channel quality is presented for a simply realized second-order receiving system. In addition, the maximum sensitivity achievable with the optimum receiving system is shown. It is anticipated these two situations will bound the performance of the majority of systems.

Author

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I. INTRODUCTION

Telephone communication by means of active satellite repeaters is currently at hand. Our first spacecraft for this purpose will of necessity be limited in transmitter power output capability. For reliability reasons a considerable time may lapse before available spacecraft transmitter power becomes of little importance to the designer. Until such time, the communication system design must center around achieving the maximum information flow per watt of satellite power even at the expense of another precious commodity, bandwidth.

The most common modulation method utilized in ground microwave relays has the capability of trading bandwidth for transmitter power if this is desired. This modulation method consists of frequency modulating a carrier wave with a single-sideband frequency-division-multiplex (FDM) of telephone channels. Upon traversing the communication link from ground transmitter to spacecraft and thence to the ground receiver, corrupting noise can be considered as added for the most part in two places -- the satellite receiver and the ground receiver. Coherent demodulation techniques may then be applied to extract the desired signal from the noise with less required received signal power than standard FM discrimination.

This paper will be concerned only with the above mentioned sources of thermal noise and no attempt will be made to treat other sources of interference such as intermodulation noise due to link nonlinearities, direct adjacent channel crosstalk, co-channel interference, etc., each of which constitutes an extensive study in its own right. For more information on these topics the reader is referred to Reference 1.

Throughout this paper the standards of the International Radio Consultative Committee (CCIR) are adherred to in order to make the results useful to those familiar with these international guides to radio relay system design. In particular considerable reliance on Reference 2 and 3 was necessary.

II. TELEPHONE CHANNEL QUALITY

Prior to discussing receiving system sensitivity with coherent demodulation, it will be necessary to develop relationships among the various parameters of an FDM/FM system to determine the expected quality of an individual channel. Consider the satellite system of Figure 1. Regardless of which FM reception technique is utilized, standard discrimination or coherent demodulation with a frequency-following receiver, the same performance formulations hold above threshold. Thus, we may discuss this topic independently of receiver sensitivity, realizing that the extra sensitivity which may be provided by coherent demodulation will allow us to increase frequency deviations and obtain the same system performance at lower received signal levels.

Consider the voltage input to the system of Figure 1 to be that resulting from a composite of single-sideband channels extending from F_1 to F_2 c/s whose equivalent power-spectral density is shown in Figure 2. The CCIR has determined that a multichannel FDM signal can be represented during the busy hour by white Gaussian noise extending from F_1 to F_2 c/s where specific values of F_1 and F_2 depend on the channel arrangement.[4] The power level of this equivalent signal is given by the CCIR as:[5]

$$P_{eq} = -1 + 4 \log_{10} (N_c), \text{ dbm0} \quad 12 \leqslant N_c < 240$$

$$P_{eq} = -15 + 10 \log_{10} (N_c), \text{ dbm0} \quad 240 \leqslant N_c \qquad (1)$$

Where: N_c = number of channels in the system.

Dbm0 in Equation (1) is power in dbm referred to a point of zero relative level in the communication system. The zero relative level concept is convenient in that one may talk of absolute power with no ambiguity.

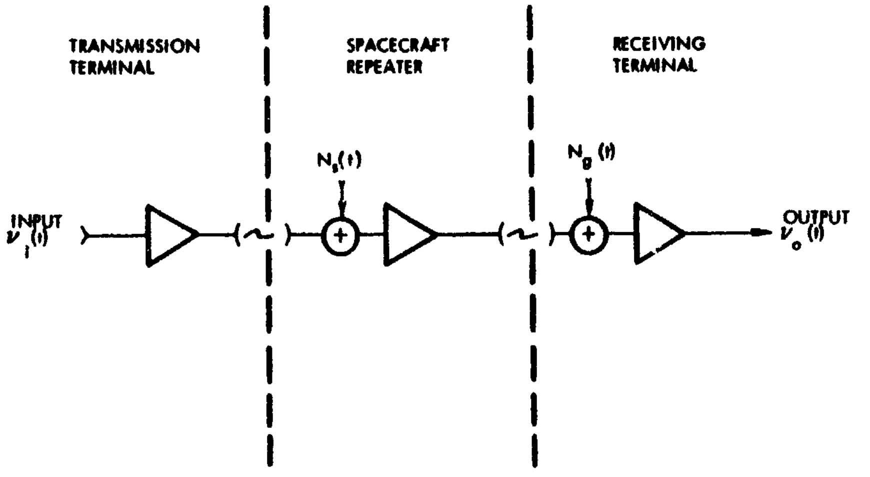

Figure 1. Active Communication Satellite System

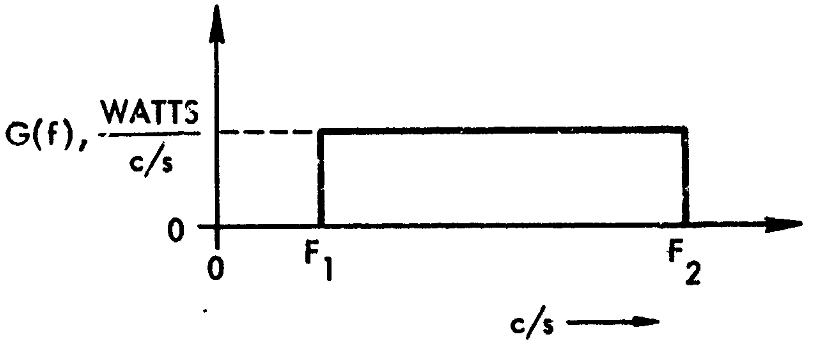

Figure 2. Equivalent Power-Spectral Density of the Multichannel Signal

As a direct result of Equation (1), a system design may now be carried out without reference to the intricate statistics of a number of actual talkers.

Since the signal described by Equation (1) must be passed / amplifiers and filters in the FM system, it is convenient to define a peak factor for the noise signal. In this paper the peak of the noise signal is defined as being 13 db above the rms value. This corresponds to a probability of overload less than 10^{-5} which the author has assumed adequate for satellite communication.

The signal and noise performance of the communication system may be found by considering the output of the receiver demodulator. Note that this point may not be at zero relative level. Let the signal power output at the receiver demodulator in Figure 1 be:

$$S_o = k\,F^2_{drms}\,,\ \text{mw} \tag{2}$$

where: S_o = Sinusoidal output power, milliwatts

k = Demodulator constant, $\text{mw}/\text{c/s}^2$

F_{drms} = RMS deviation due to an 800 c/s test tone of 0 dbm0 without pre-emphasis (CCIR Test Tone for Telephony Systems)

Familiar FM theory[6] gives the one-sided noise spectral density in the top channel of the radio system as:

$$\Phi_0 = k \left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g}\right] F_2^2\,,\quad \frac{\text{mw}}{\text{c/s}} \tag{3}$$

where: Φ_0 = Top channel <u>one-sided</u> spectral density, $\frac{\text{mw}}{\text{c/s}}$

k = Demodulator constant, $\text{mw}/\text{c/s}^2$

Φ_s, Φ_g = <u>One-sided</u> power spectral density of the additive spacecraft and ground receiver noise perturbations respectively, mw/c/s

S_s, S_g = Sinusoidal carrier power received at the spacecraft and ground receiver respectively, mw

F_2 = Highest channel frequency, c/s

The psophometrically weighted[7] noise power at the demodulator output in the top channel may be found by multiplying Equation (3) by the noise bandwidth of the psophometric weighting filter. Reference 3 gives this noise bandwidth as:

$$B = 3100 \times 10^{-0.25}, \text{ c/s} \tag{4}$$

Division of Equation (2) by the product of Equations (3) and (4) yields the test tone to noise power ratio in the top channel without pre-emphasis as follows:

$$\frac{S_o}{N_o} = \frac{10^{0.25} F_{drms}^2}{3100 \left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g}\right] F_2^2} \tag{5}$$

where: $\frac{S_o}{N_o}$ = Test tone to noise power ratio in the top channel of the satellite communication system.

Considering F_{drms} is caused by a 0 dbm0 test tone one may express the psophometrically weighted top channel noise power in picowatts[8] referred to zero relative level. It is easily shown:

$$N_{pw} = \frac{3.1 \left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g}\right] F_2^2 \times 10^{12}}{10^{0.25} F_{drms}^2}, \text{ pw(psoph)} \tag{6}$$

Since Equation (3) demonstrates that top channel performance is worse than the remaining channels, pre-emphasis is usually applied to equalize the noise in all the channels. If just thermal noise were present, 6 db/octave pre-emphasis would achieve equal noise in all channels. However, certain intermodulation products due to link non-linearities terminal noise, etc., prevent the use of this ideal pre-emphasis. Through study and practical experience with radio relay systems, CCIR has recommended a pre-emphasis curve.[9] This curve yields approximately 4 db improvement in top channel tone to noise power ratio instead of 4.8 db which is obtainable by use of 6 db/octave pre-emphasis. Recommended CCIR pre-emphasis and 6 db/octave pre-emphasis result in so nearly the same radio relay system performance that in Section IV of this paper the latter will be used because of its analytic simplicity.

Finally, Equation (6) may be written:

$$N_{pw} = \frac{3.1 \left\{ \frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g} \right\} F_2^2 \times 10^{12}}{10^{0.25}\ I\ F^2_{drms}} \quad , \text{ pw(psoph)} \qquad (7)$$

where: I = Numerical improvement achievable by use of pre-emphasis, e.g., $I = 3$ for 6 db/octave and $F_2 >> F_1$.

F_{drms} = Test tone deviation without pre-emphasis, c/s.

Equation (7) is the principal result of this section. It establishes the psophometrically weighted noise in any telephone channel (assuming pre-emphasis equalizes this quantity) versus the rms deviation of a 0 dbm0 800 c/s test tone, the spacecraft and ground receiver noise perturbations, the highest equivalent baseband frequency, and the received carrier powers at the spacecraft and ground receiver.

The value for N_{pw} in satellite communications has not yet been specified by the CCIR. For purposes of the performance curves in this paper, however, a nominal value of 10,000 pw (psoph) is chosen. This is consistent with present CCIR total noise objectives in a 2500 km link.[10] Enough latitude either side of nominal is displayed on the curves of Section V to provide for most eventualities.

III. RADIO FREQUENCY BANDWIDTH OCCUPANCY

The purpose of this section is to present formulations for estimating RF bandwidths necessary for communication with FDM/FM. Since the actual bandwidth utilized is a matter of considerable engineering judgment and depends on the number of repeaters in tandem, only a simplified bandwidth "occupancy" will be considered, given by the usual rule of thumb:

$$B_{rf} = 2 F_2 + F_{pp}, \text{ c/s} \tag{8}$$

where: B_{rf} = Bandwidth "occupancy" of the signal, c/s

F_2 = Highest equivalent baseband frequency, c/s

F_{pp} = Peak-peak deviation of the multichannel signal, c/s.

For more detailed treatment of bandwidths required for FDM/FM signals the reader is directed to the work of Medhurst.[11]

Referring back to the equivalent multichannel loading P_{eq} Equation (1) and the 13 db peak factor, one may calculate F_{p-p} in terms of F_{drms}, the deviation of the 0 dbm0 800 c/s test tone without pre-emphasis. A simple analysis yields:

$$F_{p-p} = 2 \; (10^{1.3} \; P_{eq})^{1/2} \; F_{drms}, \text{ c/s} \tag{9}$$

Equation (9) is not affected by pre-emphasis since a rule of pre-emphasis is to keep the rms value of the total frequency deviation the same with and without pre-emphasis.

Substitution of Equation (9) in Equation (8) yields the principal result of this section.

$$B_{rf} = 2 \left\{ F_2 + 10^{0.65} \; P_{eq}^{\;0.5} \; F_{drms} \right\}, \text{ c/s} \tag{10}$$

IV. COHERENT RECEIVER SENSITIVITY

A coherent receiver is one in which a "replica" of the received signal is generated locally to assist in more optimal demodulation of that signal.

There are many realizations of this type of reception. One realization is sometimes referred to as "FM feedback" reception. This technique was invented by J. G. Chaffee of Bell Telephone Laboratories.[12] Another realization is a modulation tracking phase-lock receiver. This reception technique is a variation of the type used to obtain horizontal synchronization in television receivers. Either realization of the receiver, and there are others such as exalted carrier techniques,[13] possess certain dynamic properties when driven by an FM or PM signal.

In addition, either realization possesses similar noise induced threshold properties. That is, coherent receivers cease to function when a "clean" local replica can no longer be generated. Dr. L. H. Enloe and C. L. Ruthroff of Bell Telephone Laboratories,[14,15] found that the FM feedback realization had a threshold close to the point where the rms value of the phase error due to <u>noise</u> alone between the local replica and the received signal became greater than 0.32 radian. C. R. Woods and E. M. Robinson of General Electric found a similar threshold point for the phase-lock receiver.[16] Their data indicates 0.354 radian maximum rms noise error is tolerable prior to onset of loss of lock.

The effect of <u>modulation</u> error manifests itself in a different manner in phase-lock reception than in "FM feedback" reception. In phase-lock reception of phase-encoded FDM/FM signals in the presence of additive white Gaussian noise, the sum total of the mean-square modulation plus noise error must remain small to prevent nonlinear operation of the loop phase detector. It will be assumed for purposes of this paper that phase-lock loop threshold occurs when the sum total mean-square error due to modulation and noise equals 1/8 radian^2. In "FM feedback" reception utilizing simple low-order transfer functions, Dr. L. H. Enloe has indicated lower thresholds are possible than the phase-lock receiver due to the fact larger modulation phase errors are tolerable prior to development of loop nonlinearities.

In the case of FDM/FM signals which can be represented by white Gaussian noise in Figure 2, Yovits and Jackson[17] have shown high-order actually infinite order transfer functions are necessary for optimal demodulators. The optimization performed was that of finding the transfer function which yields minimum total mean-square error. Study of the optimal transfer function derived by Yovits and Jackson shows that for the situation of high channel quality (large frequency deviations due to the signal), the modulation error becomes insignificant compared to the noise error. Thus for the special case of FDM/FM signals and use of the corresponding optimal transfer function at toll level qualities, threshold is determined primarily by noise error only. As a consequence of the similar noise induced threshold property, use of the Yovits and Jackson filter in either "FM feedback" or phase-lock reception will yield similar lower bounds on receiver sensitivity.

An "FM feedback" second-order receiver can be designed to be more sensitive than a second-order phase-lock receiver at high frequency deviations due to the larger allowable modulation error; however, by using more complex transfer functions to approach maximum sensitivity, one will find less and less difference between the two techniques until they both finally converge to the same performance with the optimal Yovits and Jackson filter.

This section will now treat two situations which will bound the performance of most coherent FDM/FM receivers designed for maximum sensitivity be they of the phase-lock configuration or the "FM feedback" configuration. First, the sensitivity of a receiver utilizing the optimal transfer function as derived by Yovits and Jackson will be treated followed by derivation of the sensitivity of a simple second-order transfer function utilized in a phase-lock configuration.

The latter case is most important in practice. The reason for the importance of the second-order loop is its simplicity. At the wide base-bandwidths necessary for 300, 600 and 1800 channels of telephony, it is difficult to realize much more than a second-order loop because of stability considerations.

A) The optimal receiver:

Consider the filter (or receiver) of Figure 3 as postulated by Yovits and Jackson.[18] The input to the receiver consists of the white Gaussian phase variable $\phi_m(t)$ proportional to the telephone multiplex signal input to the communication link.* $\phi_n(t)$ is the corrupting phase noise spectral density due to the ground receiver and spacecraft. Deleting proportionately constants which are identical for both signal and noise at any point in the system, it can be shown:

$$\Phi_n = \left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g}\right] , \quad \frac{\text{rad}^2}{\text{c/s}} \tag{11}$$

*6 db/octave pre-emphasis is assumed here for analytic simplicity. The resultant performance is within 1 db of an actual pre-emphasis schedule, however, the resulting threshold is within a fraction of a db.

where: Φ_n = One-sided phase noise spectral density due to spacecraft and ground receiver, $\text{rad}^2/\text{c/s}$.

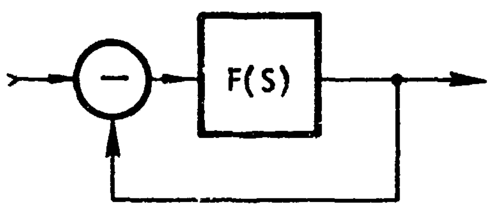

Figure 3. Yovits and Jackson Filter

By noting F_{drms} is the rms frequency deviation due to a 0 dbm0 800 c/s test tone inserted at a point where the deviation with and without pre-emphasis is the same, one may easily derive the one-sided phase spectral-density of the signal Φ_m . The result is:

$$\Phi_m = \frac{P_{eq}\, I\, F^2_{drms}}{(F_2 - F_1)\, F_2^2}\ , \quad \frac{\text{rad}^2}{\text{c/s}} \qquad F_1 \leqslant F \leqslant F_2 \quad \text{0 elsewhere} \tag{12}$$

where: P_{eq} is given by Equation (1)

$$I = \frac{3}{1 + \frac{F_1}{F_2} + \left(\frac{F_1}{F_2}\right)^2}$$ the improvement factor

attainable with 6 db/octave pre-emphasis.

Yovits and Jackson[19] have shown that no matter how complex the closed-loop transfer function in Figure 3, limited only by physical realizability, the minimum mean-square error $\overline{\epsilon(t)^2}$ for a white signal and noise spectral density is given by[20]

$$\overline{\epsilon(t)^2}_{min} = \Phi_n(F_2 - F_1) \log_\epsilon \left\{1 + \frac{\Phi_m}{\Phi_n}\right\} \tag{13}$$

Substitution of Equation (7), (11), (12), in Equation (13) yields the fundamental relation:

$$\epsilon^2_{min} = \left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g}\right] (F_2 - F_1) \log_\epsilon \left\{1 + \frac{3.1\ P_{eq} \times 10^{12}}{N_{pw} \cdot 10^{0.25}(F_2 - F_1)}\right\} \tag{14}$$

Equation (14) is fundamental in that given a channel quality N_{pw} and a maximum ϵ^2_{min} (1/8 rad^2) the maximum value of $\Phi_s/S_s + \Phi_g/S_g$ is determined. This is exactly the threshold relation desired.

Defining $\alpha = \frac{S_g\Phi_s}{S_s\Phi_g}$ as the fractional contribution of the ground to spacecraft link to over-all system noise and letting $\epsilon^2_{min} = 1/8$, one can obtain from Equation (14) the simple threshold criteria for the optimal loop:

$$S_g \geqslant 4 \left\{1 + \alpha\right\} \Phi_g B \tag{15}$$

where: $$B = 2(F_2 - F_1) \log_\epsilon \left\{1 + \frac{3.1\ P_{eq} \times 10^{12}}{N_{pw}\ 10^{0.25}(F_2 - F_i)}\right\},\ c/s \tag{16}$$

Thus the received signal-to-noise power ratio in a bandwidth B, c/s must be 6 db or more (depending on ground-spacecraft contribution) for proper demodulation.

Note that quantity B is not a strict noise bandwidth since it also includes the effects of modulation error. B is essentially a coefficient whose dimensions are c/s, which when multiplied by the two-sided phase noise spectral density input, yields the total mean-square loop error due to both noise and modulation.

In any particular receiver realization, one could compare the necessary received signal power S_g to that determined by Equation (15) and (16). This would give insight as to the efficiency of the design for FDM/FM signals.

Section V accomplishes this comparison for the second-order loop.

B) The Second-Order Receiver

This section will utilize the terminology of the phase-lock realization of this receiver since much documentation exists in this area.[21, 22, 23, 24]

Consider the block diagram of Figure 4. After Gruen[25], Martin[26], and others[27, 28] the loop was linearized and as can be seen is identical to the "filter" of Yovits and Jackson.

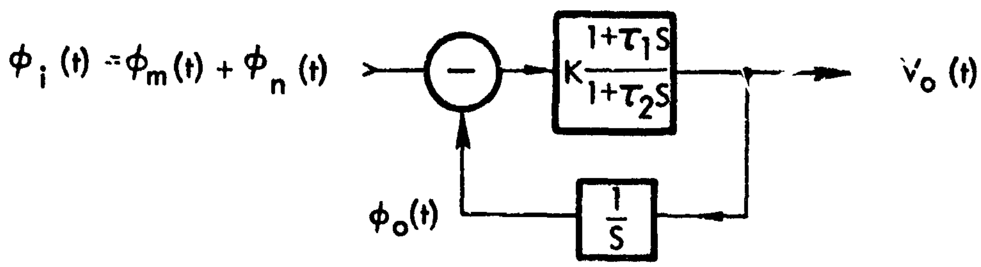

Figure 4. Second-Order Receiver

The definitions of $\phi_m(t)$ and $\phi_n(t)$ remain the same as those in SectionIV A. The phase detector in the phase-lock loop has been replaced by its linearized equivalent, the subtractor. The voltage controlled oscillator is a perfect integrator of baseband voltage to RF phase. The loop filter is a simply realized lead lag network. K represents the overall loop gain including all elements at the signal level of concern. After the work of Gruen[29] the following definitions are established.

$$\omega_n^2 = \frac{K}{\tau_2} \quad , \ (\text{rad/sec})^2$$

$$2\zeta\omega_n = \frac{1 + K\tau_1}{\tau_2} \quad , \ \text{rad/sec}$$

where: ω_n = Loop undamped natural frequency, rad/sec

ζ = Ratio of actual to critical loop damping

τ_1, τ_2 = Time constants of the lead lag compensation networks, seconds.

The loop transfer function as determined by Gruen[30] with the above definitions is the following:

$$\frac{\phi_o}{\phi_i} = \frac{\omega_n^2 \left\{ 1 + \left(\frac{2\zeta}{\omega_n} - \frac{1}{K} \right) s \right\}}{\omega_n^2 + 2\zeta\omega_n s + s^2} \qquad (17)$$

The error function which describes the faithfulness of tracking is easily established from the work of Yovits and Jackson[31] as follows:

$$\text{Noise error} \qquad\qquad \text{Modulation error}$$

$$\overline{\epsilon(t)^2} = \int_0^\infty \left|\frac{\phi_o}{\phi_i}\right|^2 \Phi_n \, df + \int_0^\infty \left|1 - \frac{\phi_o}{\phi_i}\right|^2 \Phi_m \, df; \ \text{rad}^2 \tag{18}$$

The first portion of Equation (18) has been evaluated by Gruen[32] and is repeated here as a slight modification of his Equation (36) in order to utilize our one-sided spectral densities. In addition, the following assumes small ω_n/K which is normally the case in practice. The first portion of (18) now becomes:

$$\overline{\epsilon_1^2} = \Phi_n \omega_n \left(\frac{1 + 4\zeta^2}{8\zeta}\right), \ \text{radians}^2 \tag{19}$$

For CCIR channel arrangements and the channel qualities normally encountered, the second half of Equation (18) can be approximated as follows to an excellent degree for $\zeta \cong 1/\sqrt{2}$ and $\omega_n/K \rightarrow 0$:*

$$\overline{\epsilon_2^2} \cong \frac{(2\pi)^4 \Phi_m}{\omega_n^4} \int_{F_1}^{F_2} f^4 \, df$$

or

$$\overline{\epsilon_2^2} \cong \frac{\Phi_m (2\pi)^4}{5\omega_n^4} \left[F_2^5 - F_1^5\right] \tag{20}$$

*These are reasonable values for most loop designs. Since Yovits and Jackson have shown the second-order loop is not optimum for FDM/FM signals, the author does not feel justified in using other than normally encountered values for ζ and ω_n/K.

Combining Equation (20) and (19) yields the total loop error.

$$\overline{\epsilon^2} = \Phi_n \omega_n \left\{ \frac{1 + 4\zeta^2}{8\zeta} \right\} + \frac{(2\pi)^4 \Phi_m (F_2^5 - F_1^5)}{5\ \omega_n^4}\ , \ \text{rad}^2 \qquad (21)$$

Substitution of Φ_n (Equation 11) and Φ_m (Equation 12) in Equation (21) finally gives:

$$\overline{\epsilon^2} = \left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g} \right] \omega_n \left(\frac{1 + 4\zeta^2}{8\zeta} \right)$$

$$+ \frac{(2\pi)^4\ P_{eq}\ I\ F_{drms}^2\ (F_2^5 - F_1^5)}{5(F_2 - F_1)\ F_2^2\ \omega_n^4}\ , \ \text{rad}^2 \qquad (22)$$

Utilizing Equation (7) we obtain:

$$\overline{\epsilon^2} = \left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g} \right] \left\{ \omega_n \left(\frac{1 + 4\zeta^2}{8\zeta} \right) + \frac{(2\pi)^4 P_{eq} (F_2^5 - F_1^5)\ 3.1 \times 10^{12}}{5\ N_{pw}\ 10^{0.25}\ \omega_n^4\ (F_2 - F_1)} \right\},$$

$$\text{rad}^2 \qquad (23)$$

Considering all other parameters fixed except ω_n, the natural frequency of the receiver loop, $\overline{\epsilon^2}$ may be minimized with respect to this quantity. The optimum ω_n becomes for a damping of $\zeta = 1/\sqrt{2}$:

$$\omega_{n\ opt} = 2 \left\{ \frac{3.1\ (2\pi)^4\ P_{eq}(F_2^5 - F_1^5) \times 10^{12}}{15\sqrt{2}\ 10^{0.25}\ N_{pw}(F_2 - F_1)} \right\}^{1/5}, \text{ rad/sec} \quad (24)$$

Substitution of Equation (24) back in Equation (23) yields the minimum error attainable contingent on the assumptions. One obtains for $\zeta = 1/\sqrt{2}$:

$$\overline{\epsilon^2} = \frac{1}{2}\left[\frac{\Phi_s}{S_s} + \frac{\Phi_g}{S_g}\right]\left[\frac{5}{4}\ B_N\right], \quad \text{rad}^2 \quad (25)$$

where: $B_N = \dfrac{3\ \omega_{n\ opt}}{2\sqrt{2}}$, c/s

B_N is the conventional definition of two-sided noise bandwidth in c/s for a second-order loop of damping $\zeta = 1/\sqrt{2}$.[33] The factor 5/4 takes into account the effects of modulation error. To be consistent with the results of Section IV, A, a quantity $B = 5/4\ B_N$, c/s will be defined as the noise coefficient, which when multiplied by the two-sided phase noise spectral density, will yield the total mean-square loop error due to the effects of both modulation and noise.

For a damping ratio of $\zeta = 1/\sqrt{2}$, it can also be shown $B_N = 3.24\ F_{3db}$, c/s where F_{3db} is the closed loop baseband 3 db bandwidth in c/s. Similarly, the newly defined noise coefficient, $B = 4.05\ F_{3db}$, c/s for the second-order loop with $\zeta = 1/\sqrt{2}$.

Applying the identical threshold criteria as in Section IV, A,

$\overline{\epsilon^2} \leq \frac{1}{8}$ rad^2, Equation (25) becomes:

$$S_g \geq 4(1+\alpha)\Phi_g B \qquad (26)$$

where: $B = \frac{5}{4} B_N$, c/s

Thus as in the optimum loop the received signal-to-noise power ratio in a bandwidth B, c/s must be 6 db or more (depending on ground-spacecraft contribution) for proper demodulation.

V. SUMMARY OF RESULTS

The previous sections have derived very important relationships for communication system design utilizing FDM/FM and coherent reception based on CCIR practice. Bounds on the performance expected by utilizing coherent reception have been established on the one hand by a second-order transfer function and on the other by the optimal transfer function derived by Yovits and Jackson.[34]

Table I summarizes the most important relationships under the key assumption that the desired quality of performance is achieved at threshold. In order to make the maximum use of spacecraft transmitter power and thereby allow every decibel of received power above threshold to represent true margin, that is both performance and threshold margin, this assumption should be made.

Table I. Formula Summary*

Quantity	Type of Demodulator	Formulation at Threshold
0 dbm0 Test Tone Deviation	Optimal and Second-Order	$F_{drms} = F_2 \left\{ \frac{3.1}{4 \times 10^{0.25} \, I \, N_{pw} \, B} \right\}^{\frac{1}{2}} 10^6$, c/s
Noise co-efficient	Optimal	$B = 2(F_2 - F_1) \log_\epsilon \left\{ 1 + \frac{3.1 \, P_{eq} \, 10^{12}}{N_{pw} \, 10^{0.25} (F_2 - F_1)} \right\}$, c/s
	Second-Order	$B = \frac{15}{4\sqrt{2}} \left\{ \frac{3.1 (2\pi)^4 P_{eq} (F_2^5 - F_1^5) \times 10^{12}}{15\sqrt{2} \; 10^{0.25} \, N_{pw} (F_2 - F_1)} \right\}^{1/5}$, c/s
Threshold Criteria	Optimal and Second-Order	$S_g \geqslant 4(1+\alpha)\, \Phi_g B$
Bandwidth Occupancy	Optimal and Second-Order	$B_{rf} = 2 \left\{ F_2 + 10^{0.65} \, P_{eq}^{\;0.5} \, F_{drms} \right\}$, c/s

*Note: These formulas apply for 6 db/octave pre-emphasis only. In addition for the second-order loop, the damping was taken to be $\zeta = 1/\sqrt{2}$ and $\omega_n / K \longrightarrow 0$.

Figures 5 through 12 plot the results of the formulations in the above table for the equivalent CCIR loading,[35] 6 db/octave pre-emphasis, CCIR channel arrangements, and of course the threshold assumption in which maximal use of spacecraft power is achieved.

One should note the large bandwidth occupancy required when one attempts to achieve full use of transmitter power. Presently these large bandwidths may be necessary; however, future satellite power development will allow the maximum bandwidths indicated in the figures to be reduced by increasing transmitter power and reducing deviations for the same net system performance. Of course in this situation the performance margin will always be less than the threshold margin.

Acknowledgement

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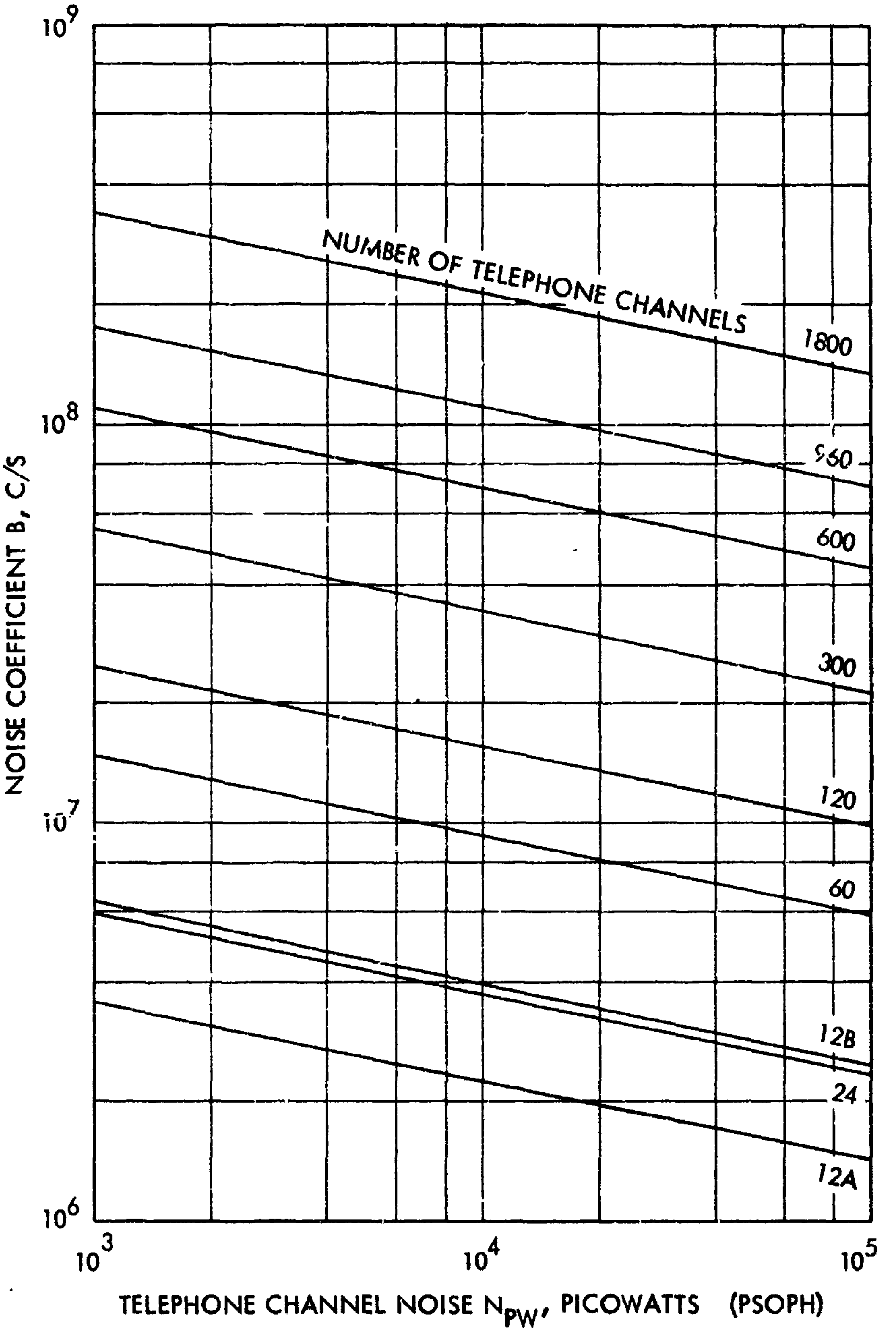

Figure 5. Noise Coefficient, Second-Order Loop

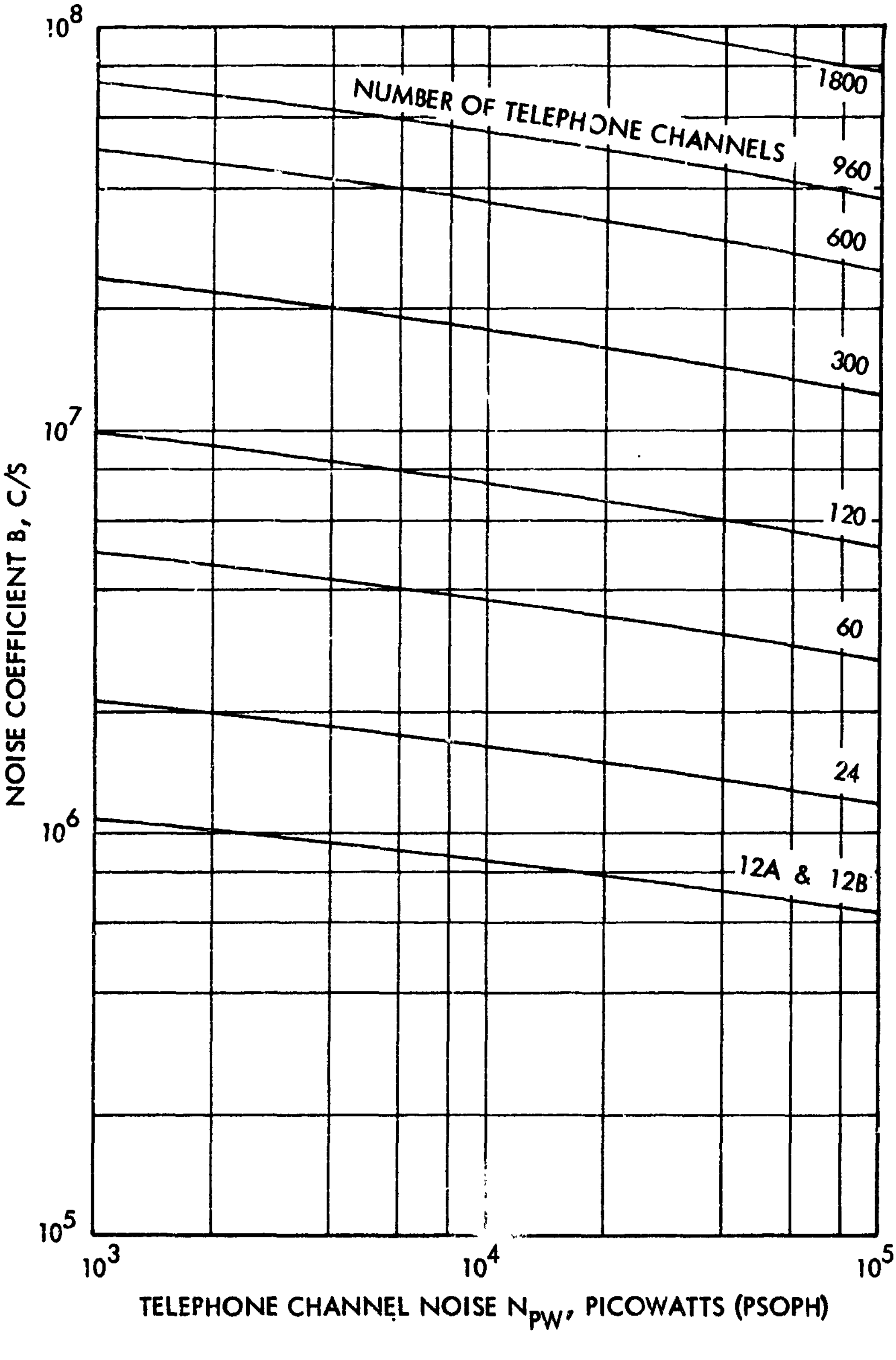

Figure 6. Noise Coefficient, Optimal Loop

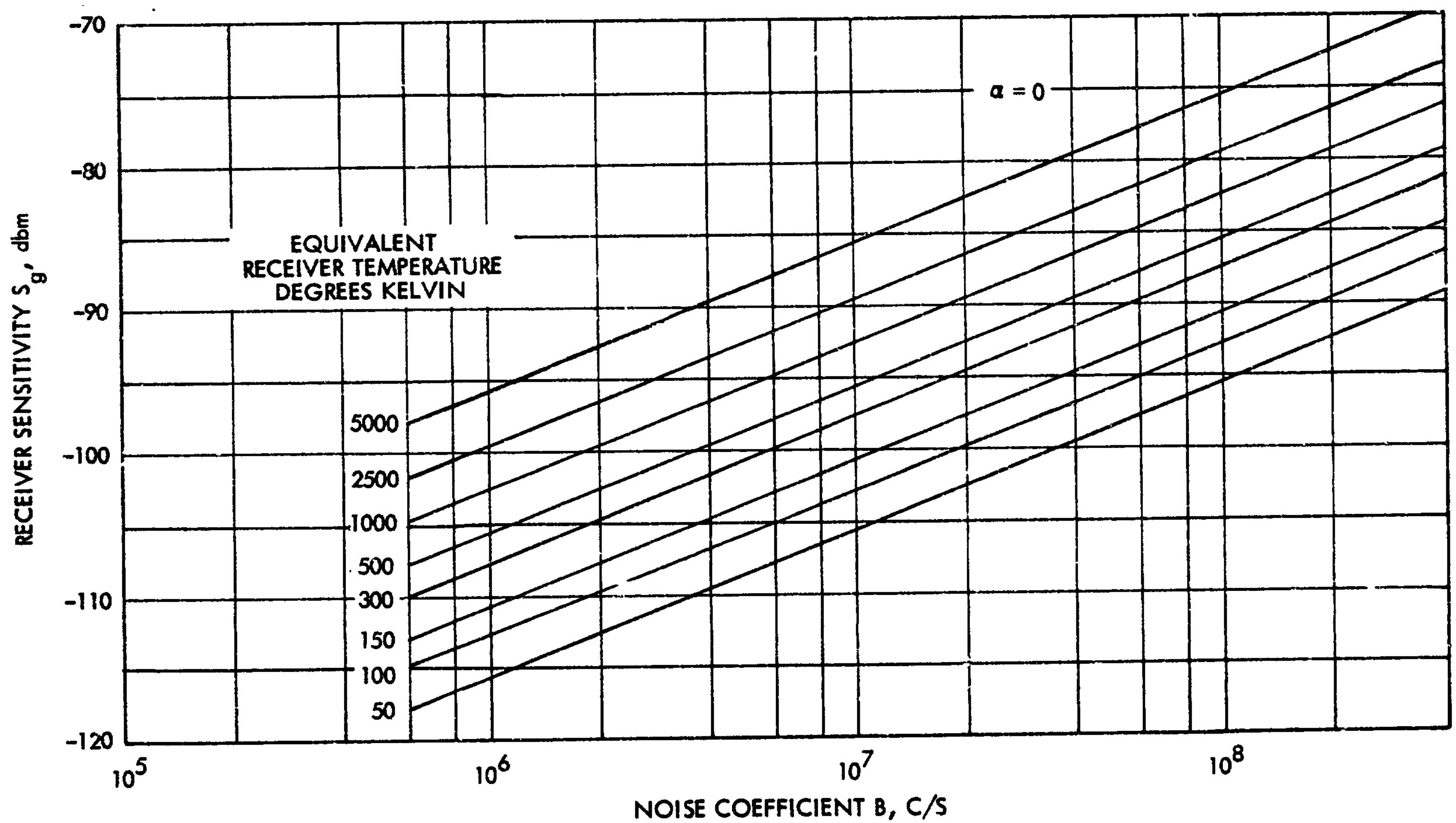

Figure 7. Receiver Sensitivity, dbm

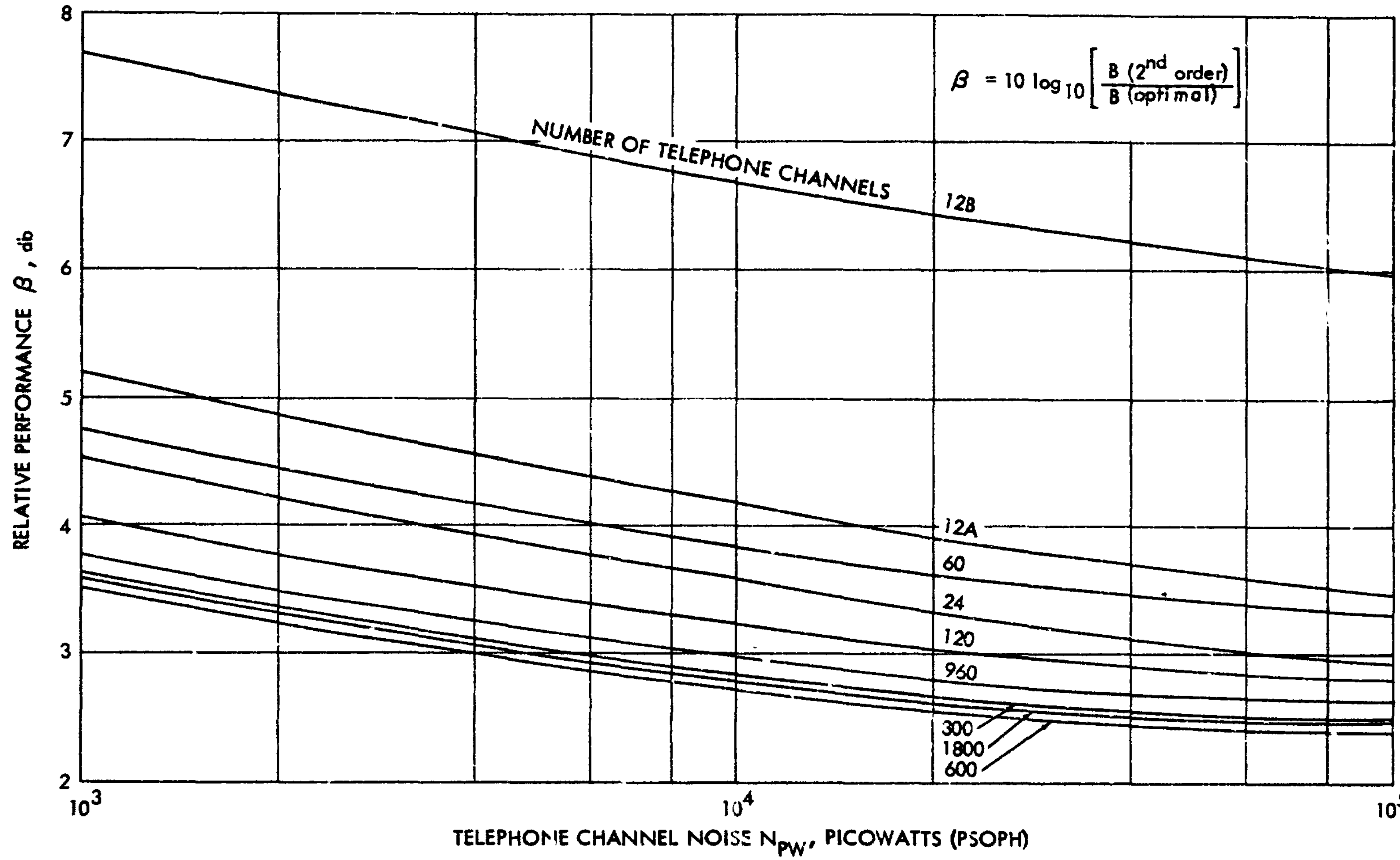

Figure 8. Sensitivity Comparison Second-Order versus Optimal Loop

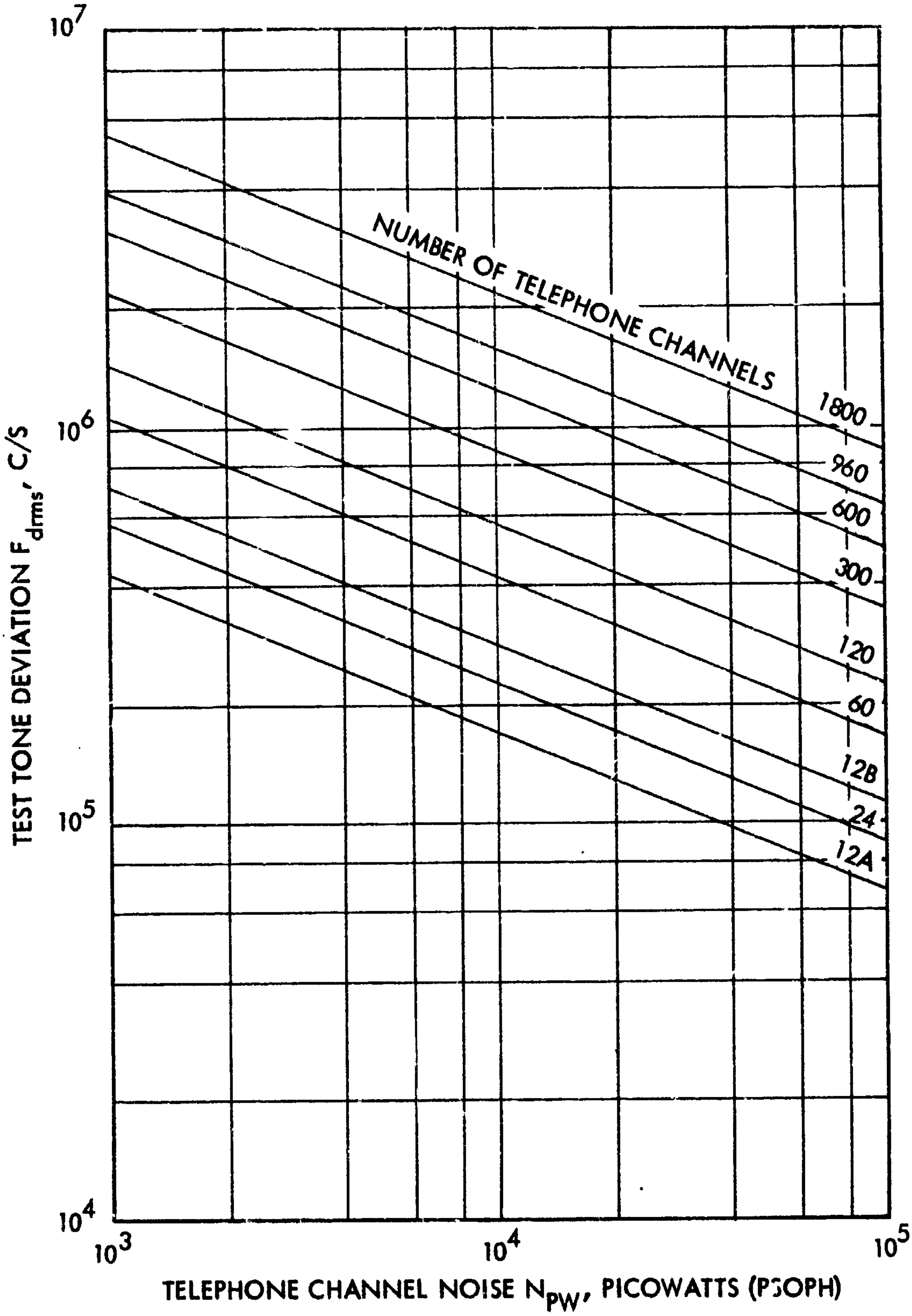

Figure 9. One Milliwatt 800 c/s Test Tone Deviation, Second-Order Loop

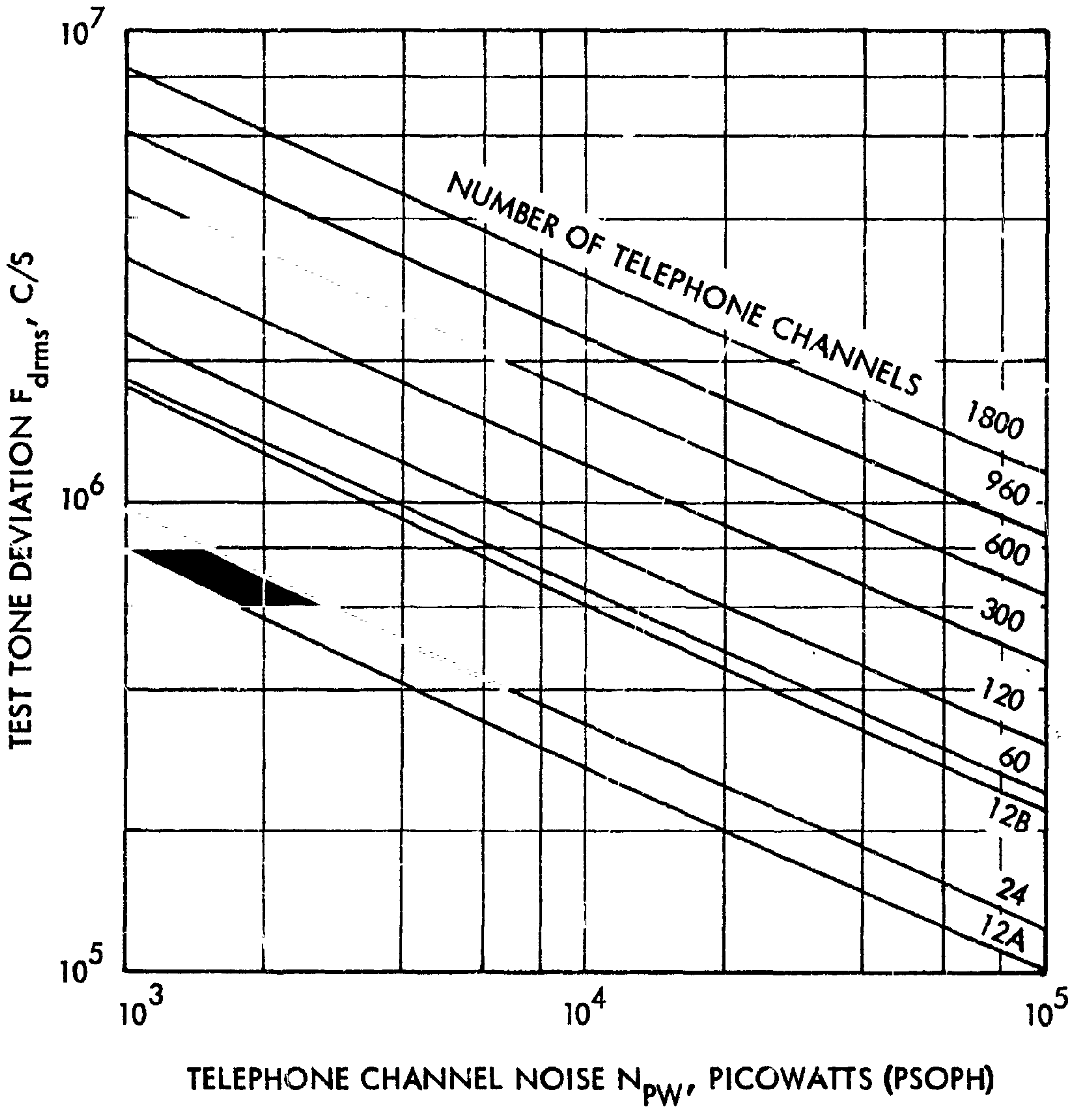

Figure 10. One Milliwatt 800 c/s Test Tone Deviation, Optimal Loop

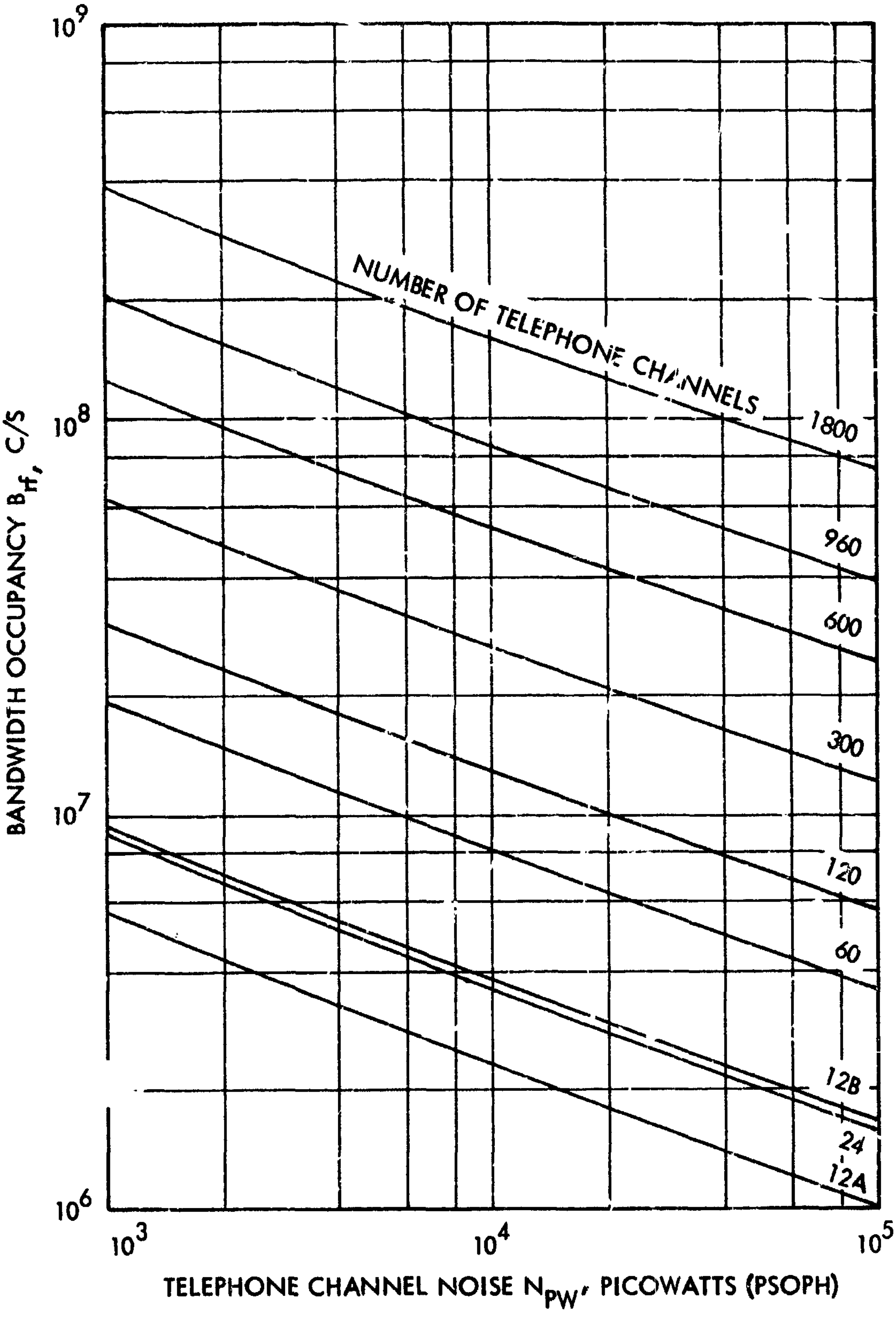

Figure 11. Radio Frequency Bandwidth Occupancy, Second-Order Loop

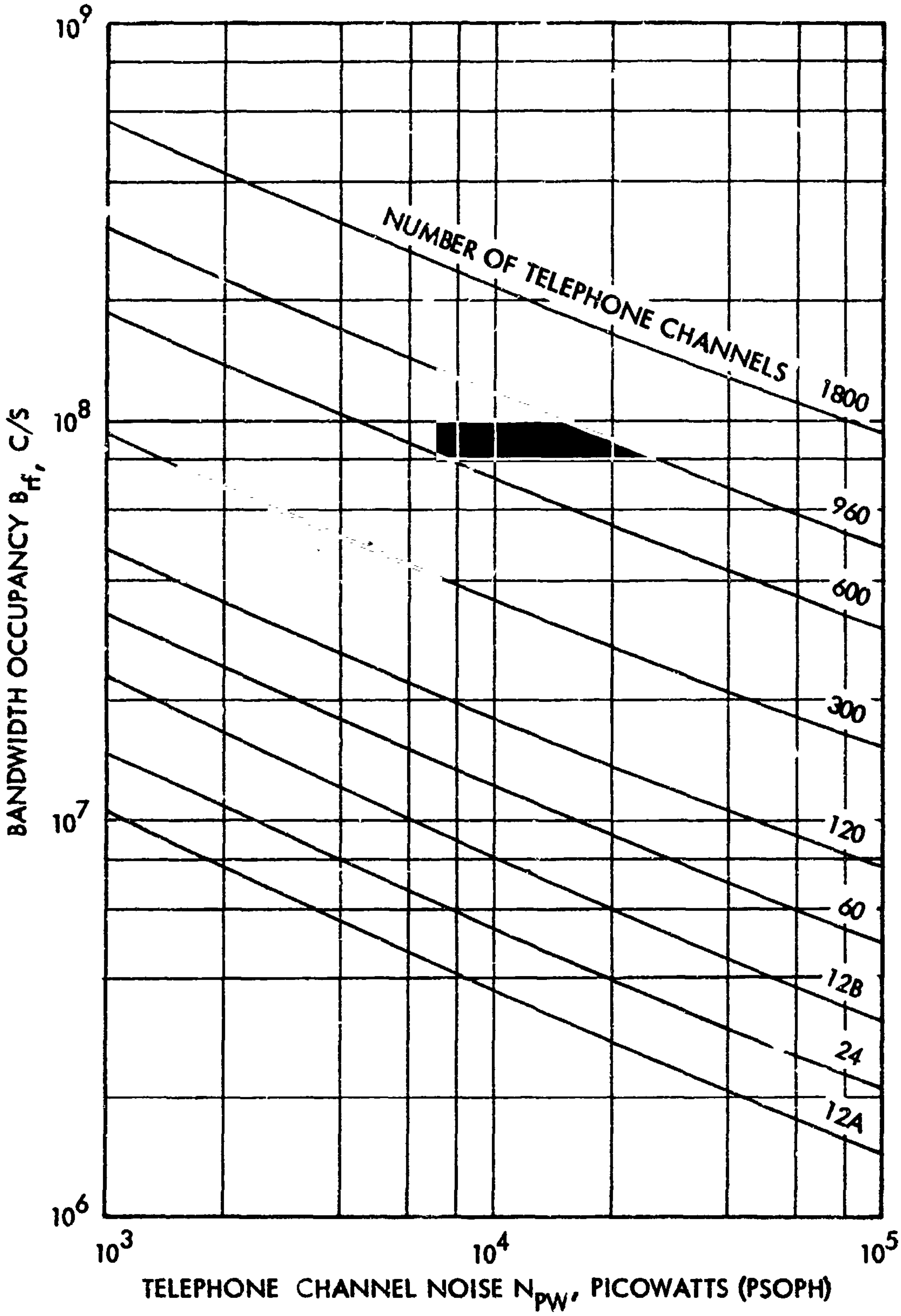

Figure 12. Radio Frequency Bandwidth Occupancy, Optimal Loop

FOOTNOTES

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FOOTNOTES (Cont'd.)

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