N65 29816

(ACCESSION NUMBER)

38

(PAGES)

(NASA CR OR TMX OR AD NUMBER)

(THRU)

1

(CODE)

~~8~~ 29

(CATEGORY)

FACILITY FORM 602

X-640-65-149

NASA TMX-55250

CALCULATIONS OF LONGITUDE DEPENDENCE OF GEOMAGNETICALLY TRAPPED ELECTRON FLUXES

PART I

THE GENERAL FOKKER-PLANCK EQUATION FOR ELECTRON DIFFUSION

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) 2.00

Microfiche (MF) .50

ff 653 July 65

BY

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APRIL 1965

NASA

GODDARD SPACE FLIGHT CENTER

GREENBELT, MARYLAND

To be presented at COSPAR, Buenos Aires, Argentina

May 13-19, 1965

X-640-65-149

CALCULATIONS OF LONGITUDE DEPENDENCE OF GEOMAGNETICALLY TRAPPED ELECTRON FLUXES

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THE GENERAL FOKKER-PLANCK EQUATION FOR ELECTRON DIFFUSION

by

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SUMMARY

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A general Fokker-Planck equation is deduced, which describes the distribution of geomagnetically trapped electrons as a function of longitude, time, energy and mirror point field intensity. A special variable for the longitudinal position of a particle is introduced.

The physical interpretation of this equation is analyzed for several special cases. In particular, it is found that the usual procedure of averaging over longitude in order to obtain a longitude-independent equation, is not valid for the description of electrons mirroring at low altitudes. In that case, the longitude dependence cannot be averaged out, and a four-dimensional equation must be used.

The coefficients representing longitudinal drift, ionization loss and multiple, screened Coulomb scattering in the general Fokker-Planck equation are deduced. An approximation is given for a simple model of the field and the atmosphere in the South American Anomaly. Qualitative conclusions are drawn for the mirror point "flow" along lines of force, for a stationary electron distribution drifting through the Anomaly. It is concluded that the region East of the Anomaly, initially depleted by precipitation, is replenished by electrons whose mirror points were situated in a narrow "window" in B, before passing through the Anomaly. A very limited extension of the atmosphere in the region of the Anomaly is responsible for this mechanism of replenishment. A diurnal effect for the electron fluxes in the replenished region is predicted, controlled by the diurnal variation of the atmosphere in the Anomaly.

Author

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CALCULATIONS OF LONGITUDE DEPENDENCE OF GEOMAGNETICALLY TRAPPED ELECTRON FLUXES

PART I

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INTRODUCTION

Time has come to look with more detail into the latitude dependence of geomagnetically trapped particle fluxes from the theoretical point of view. Until recently, experimental information on this subject was very scarce. Only after the discovery of the so-called South American Anomaly by Vernov, et al.[1] and the more detailed surveys with Discoverer satellites by Mann, Bloom and West[2], attention got more and more focussed on the experimental analysis of particle precipitation in the Anomaly, and the mechanism of subsequent replenishment of the depleted shell regions. In particular, Imhof and Smith[3], Paulikas and Freden[4] and Mihalov et al.[5] have made a careful study of artifically injected electron fluxes, measured by various satellites. On the other hand, Freden and Paulikas[6] and White[7] have analyzed proton fluxes at low altitudes in the Anomaly. Evidence for Bremsstrahlung X-rays from electrons precipitating into the Anomaly was recently found by Ghielmetti, et. al.[8]. It is, therefore, desirable to set up a theoretical description of the longitudinal behavior of trapped particles, and to test by comparison with experimental data, the various assumptions made about interaction processes governing particle diffusion.

Theoretical description of electron trapping, diffusion and precipitation was so far done only for configurations averaged over all longitudes[9,10,11,12,13]. In these papers, a time dependent Fokker-Planck equation was set up for the electron distribution function and used to follow the evolution in time of a given, initial electron flux.

In order to study the longitude dependence of the electron distribution on a given magnetic shell, it is necessary to derive a more general diffusion equation which contains an additional variable related to longitude. In this equation the coefficients, which depend upon the atmosphere, will be a function of longitude and local time of day in the Anomaly. One important by-product of the equation derivation will be the identification of an appropriate variable by which to describe the longitudinal dependence. We will call this variable X. It will be used, together with the particle energy E, the scalar magnetic field B at the mirror point, and the well-known McIlwain shell parameter L,[14] to describe the space in which particle densities change with time, t.

The main purpose of Part I is to set up such a general equation, to discuss its physical meaning, to compare it with the previously used longitude-averaged equation, and to draw some general, qualitative conclusions about longitude dependence of trapped electrons. Part II* will deal with atmosphere-field configurations to be used in this longitude dependent description. In Part III*, results of a numerical integration of the general equation will be presented.

Before setting up our equation, let us picture the problem in general terms. Consider the familiar B-L space, in which trapped radiation fluxes are usually described (Fig. 1). It can be shown that electrons whose mirror points are below 100 km cannot remain trapped for more than a few bounces. Thus, it is qualitatively useful to consider the 100 km level as the location of a sink. We shall return to this point in more detail later. At a given longitude the 100 km level can be displayed as a locus $B_c(L, X)$ in B-L space. Essentially no electrons will be found with mirror points above this curve, that is, for mirror point fields larger than $B_c(L, X)$. We have plotted two extreme loci of the 100 km level, corresponding to a longitude (that is, a value of X) right in the "center" of the South American Anomaly, and a longitude over the Pacific Ocean, respectively. From Fig. 1, one can see that for particles of a given energy, the 100 km curve, $B_c(L, X)$, "oscillates" with longitudinal drift frequency between these two extreme positions. As $B_c(L, X)$ lowers from its maximum, or "Pacific", position, the region in B-L space between the two extreme positions, called the "shadow region", is wiped clean of particles. As $B_c(L, X)$ rises from its minimum, or "Anomaly", position, the opportunity exists for atmospheric scattering, or any other nonadiabatic process, to repopulate this shadow region. This wiping clean and repopulation has been likened to the action of a windshield wiper.

Satellite observations do, in fact, find particles in the shadow region (Imhof and Smith[3]). This observation means that particles have enough non-adiabatic interaction during one drift around the earth to diffuse into the wiped-out or "shadow" region. Our main goal is to study this process of fast replenishment of the shadow region in the case of electrons interacting with the atmosphere by Coulomb scattering and energy loss.

*Will be published at a later date.

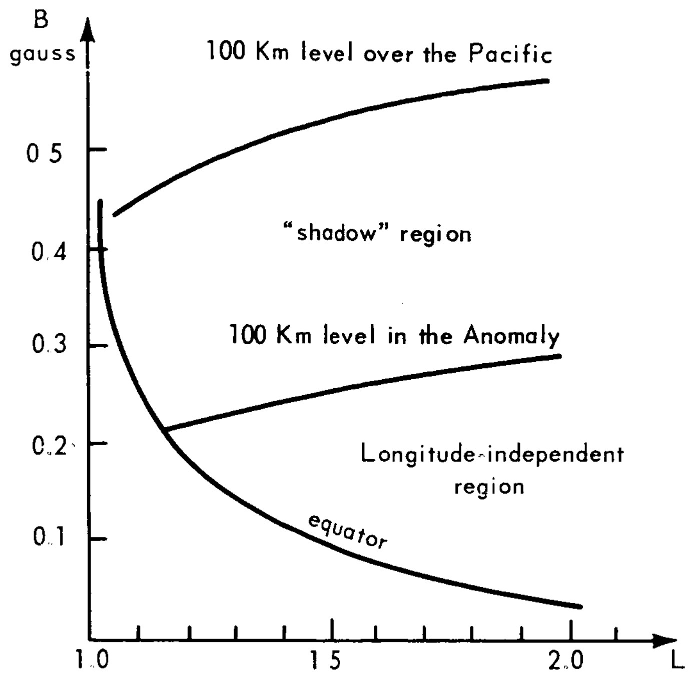

Figure 1–The "windshield wiper" effect in B-L space.

REPRESENTATION OF THE ELECTRON DISTRIBUTION AS A FUNCTION OF LONGITUDE

We shall describe the trapped electrons by the distribution function used by Welch, Kaufmann and Hess[10]. For a given L-shell we denote by

$$\delta N = U(B,\ E,\ \varphi,\ t)\ \delta\phi\ \delta B\ \delta E \tag{1}$$

the number of electrons contained at the time t in a tube of field lines of magnetic flux $\delta\phi$ situated at a longitude φ (for the time being, we shall use the geographic longitude of the magnetic equatorial point), with mirror points between B and $B + \delta B$, and with kinetic energies between E and $E+\delta E$. We shall neglect L-scattering; this is why we do not include explicitly L as a variable in U.

The distribution U is not directly measurable. It is, however, the physical magnitude which most appropriately describes trapped particles. Relations with experimentally accessible quantities are given in[10]. In particular, the number δn of particles contained in the volume element $\delta A \delta s$ of a tube at a point where the field is B', and which mirror in the interval dB at $B > B'$, (Fig. 2) is given by

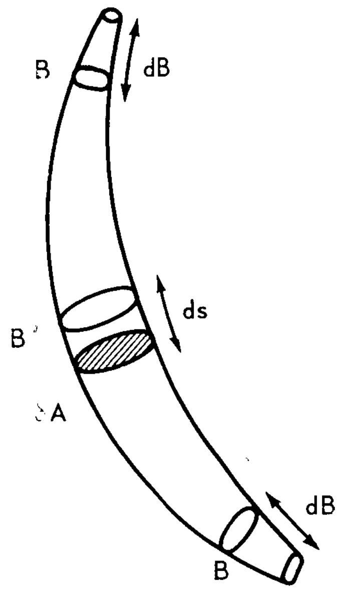

Figure 2

$$\delta n = U(B,\ E,\ \varphi,\ t)\ dB\ \delta E\ B\ \delta A$$

$$\frac{\delta s}{v_{||}(B',\ B,\ E)\ \tau_b\ (B,\ E,\ \varphi)} \qquad (2)$$

The ratio in (2) represents the probability of finding a particle of the population (1) in the volume element $\delta A \delta s$. $\tau_b(B,\ E,\ \varphi)$ is the half-period of bouncing of these particles, i.e., the time it takes to go from one mirror point to its conjugate. In the real geomagnetic field, it depends slightly on longitude. $v_{||}(B', B, E) = v(E)\sqrt{1 - B'/B}$ is the particle's velocity parallel to the field line, at the point B'. For a dipole field, the total path of a particle during a bounce period is $v\tau_b \approx 2LR$ (R = radius of earth). The total particle density per unit energy at a given point B' is therefore

$$n(B',\ E,\ \varphi,\ t) = \frac{\delta n}{\delta A\ \delta s\ \delta E} = \frac{B'}{v(E)} \int_{B'}^{B_c} \frac{U(B,\ E,\ \varphi,\ t)\ dB}{\tau_b(B, E, \varphi)\sqrt{1 - B'/B}} \approx \frac{B'}{2RL} \int_{B'}^{B_c} \frac{U dB}{\sqrt{1 - B'/B}} \qquad (3)$$

The integration is performed along the field line between the B value at the given point, and some upper cut-off B_c, beyond which all particles are absorbed in the atmosphere. The density (3) is related to the omnidirectional counting rate C of a spherical detector by

$$C = \int_{E_0}^{\infty} v n A\, dE \qquad (4)$$

where $A(E)$ is the effective cross sectional area of the counter for particles of energy E.

Let us now consider electrons of a given energy, mirroring at the same B, trapped between two neighboring shells labeled I and $I + \delta I$ (Fig. 3). I is the second adiabatic invariant[14] of these particles computed for field lines belonging to the inner shell:

$$I = \int_{-s(B)}^{s(B)} \frac{v_{\parallel}}{v} ds = \int_{-s(B)}^{s(B)} \frac{ds}{\sqrt{1 - \frac{B(s)}{B}}}$$

$I + \delta I$ is the value of the second invariant, taken along a line of force belonging to the outer shell, between mirror points with the same field intensity B. B_0 is the field at the equatorial point of a line of force (point of minimum B on a given shell); φ is the geographic longitude of this equatorial point. We call $\vec{\nabla}_0 I = \vec{n}\ \delta I/\delta y$ the gradient of I at the equatorial point of a magnetic shell (Figs. 3, 4). Northrop and Teller[15] have shown that the equatorial drift velocity, averaged over one bounce, is given by

$$\vec{u}_0 = \frac{m v}{e B_0^2 \tau_b} \vec{\nabla}_0 I \times \vec{B} = \frac{p}{e B_0} \frac{\nabla_0 I}{\tau_b} \vec{b} \tag{5}$$

where p is the particle's momentum, e the charge (<0 for electrons) and τ_b is the half-period of bouncing. For the different vectors, see Fig. 4.

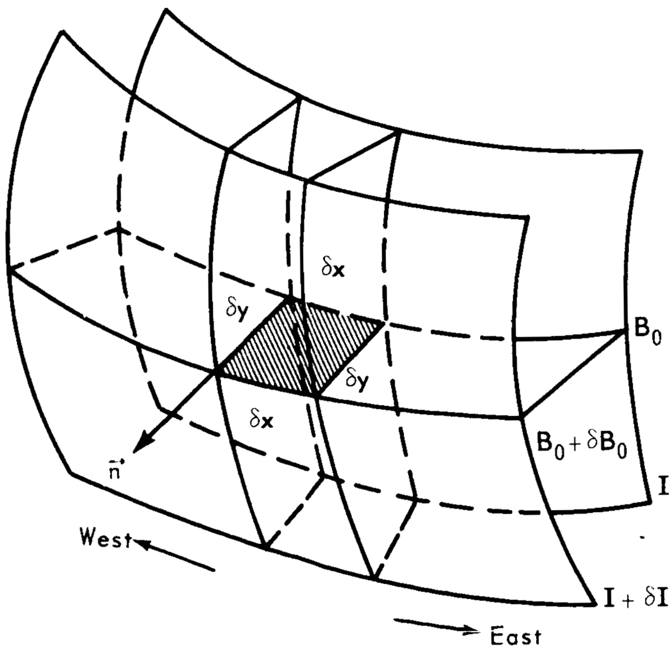

Figure 3–Shell geometry and elementary flux tube

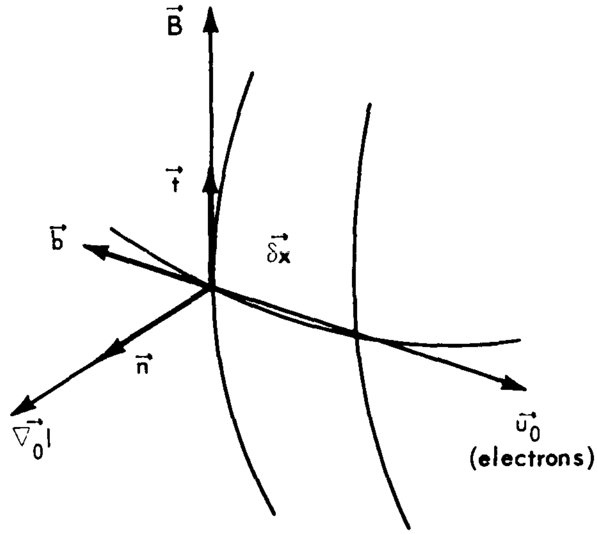

Figure 4—Unit Vectors, mean equatorial drift velocity and gradient of I

We denote with $\delta x = u_0 \delta t$ the distance between equatorial points of two neighboring field lines of a given shell; δx is then the element of arc of the shell's equatorial B_0-ring; $x = \int dx$ is the total arc length along the equator from a given initial point and can be used as a label to locate field lines on a given shell. Evidently

$$\delta x = \frac{\partial x}{\partial \varphi} \delta \varphi \tag{6}$$

and

$$x = \int_{\varphi_0}^{\varphi} \frac{\partial x}{\partial \varphi} d\varphi \tag{7}$$

where $\partial x/\partial \varphi$ is a field-geometric factor ($\partial x/\partial \varphi = 2\pi/360\, RL$ for a centered dipole), and φ the geographic longitude of the equatorial point. Let us take a tube of lines of force of equatorial cross section $\delta x\, \delta y$ and magnetic flux $B_0 \delta x \delta y$ (Fig. 3). By the definition of U (1), the number of electrons in this flux tube, which mirror between B and $B + \delta B$ and with kinetic energy between E and $E + \delta E$, is given by

$$\delta N = U B_0\, \delta x\, \delta y\, \delta B\, \delta E \tag{8}$$

We now want to follow the history of these particles, as a function of time. Before setting up the Boltzmann-type equation governing the distribution U, we have to introduce a convenient variable for the longitude, i.e. a convenient label for the line of force of a given magnetic shell, around which a particle is instantaneously spiralling. We cannot take φ or x (7) as suitable variables, because U is a distribution in flux, not in φ or x. And in the flux expression $B_0\,\delta x\,\delta y$ intervening in (8), not only δx, but also δy is a function of longitude, for the general geomagnetic field.

In order to find the correct longitudinal variable, we have to transform the flux into an expression $\delta\phi = B_0\,\delta X\,\delta Y$, in which δY – still related to the radial increment δy – is now longitude-independent, so that it may be ignored when following the particles during their longitudinal drift. In that case, X would be the correct longitudinal variable, its differential δX containing complete information about the longitude dependence of the flux $\delta\phi$ of a tube filled with particles, as they drift around the earth.

In order to find X and Y, we just have to determine the longitude dependence of δy, the equatorial distance between two neighboring magnetic shells. Notice that this dependence is a purely geometric one, independent of the particles' dynamic variables. Let us take McIlwain's definition of L,[14] which we write here:

$$\frac{R^3 L^3 B}{M} = F\left(\frac{I^3 B}{M}\right) \tag{9}$$

(R: radius of the Earth, L dimensionless, I in cm). We further consider the relation:

$$B_0 = \frac{M}{R^3 L^3} \tag{10}$$

(only valid within about a few percent, because L actually fluctuates along a line of force[14,16]). Combining (9) and (10), and differentiating, we have

$$\frac{\delta B_0}{B_0} = -\frac{3I^2}{R^3 L^3} F' \,\delta I \tag{11}$$

for a constant mirror field B. F' is the derivative of McIlwain's function with respect to its argument. Dividing by δy, we obtain, for absolute values

$$\frac{\delta I}{\delta y} = \frac{\frac{1}{B_0}\frac{\delta B_0}{\delta y}}{3/RL}\;\frac{R^2L^2}{I^2F'} \tag{12}$$

Notice that $1/B_0\ \delta B_0/\delta y \rightarrow 1/B_0\ \nabla_0 B$ as $\delta y \rightarrow 0$, where $\nabla_0 B$ is the gradient of $|B|$ taken at the equatorial point. For a current-free magnetic field, $1/B_0\ \nabla_\perp B$ is equal to the curvature of the line of force at the point where the normal gradient is taken; in our case, $1/B_0\ \nabla_0 B$ is therefore the curvature at the equatorial point. We shall introduce the dimensionless number

$$\kappa_0 = \frac{\frac{1}{B_0}\nabla_0 B}{3/RL} \tag{13}$$

which is the curvature of a line of force at the equatorial point, in units of the equatorial curvature $3/RL$ of the corresponding dipole line ($\kappa_0 = 1$ for all dipole shells). In the general case, κ_0 depends on longitude. From (12) and (13), we finally have:

$$\frac{\delta I}{\delta y} = \nabla_0 I = \frac{\kappa_0}{\lambda(B)} \tag{14}$$

where

$$\lambda(B) = \frac{I^2F'}{R^2L^2} \tag{15}$$

is a dimensionless function of the mirror point field intensity B. Notice that, according to (5) and (14), we have

$$u_0 = \frac{p}{eB_0}\,\frac{\kappa_0}{\tau_b\lambda} \tag{16}$$

and for the angular drift velocity, taking into account (6):

$$\dot{\varphi} = \frac{1}{\frac{\partial x}{\partial \varphi}} u_0 = \frac{1}{\frac{\partial x}{\partial \varphi}} \frac{p}{eB_0} \frac{\kappa_0}{\tau_b \lambda} \tag{17}$$

It is important to point out that $\lambda(B)$ (15) is a "pure dipole function", even in the case of a slightly distorted dipole as the real geomagnetic field (this is not true for τ_b). It is easy to verify that our function λ is identical with $1/6E$, where E is the function calculated by Hamlin et al.[17] (formula 22 in this reference). All this of course is valid only within a few percent (fluctuation of L on a magnetic shell). Notice finally that κ_0 contributes to the longitude dependence of the drift velocity. However, one must not forget that τ_b also depends on longitude in the real field. According to (14),

$$\delta y = \frac{\lambda \delta I}{\kappa_0} \tag{18}$$

In this relation, the only longitude dependent quantity is κ_0. We then introduce the variables Y and X in the following way:

$$\delta Y = \kappa_0 \, \delta y = \lambda \delta I \quad \therefore \quad Y = \lambda I \tag{19a}$$

$$\delta X = \frac{\delta X}{\kappa_0} \qquad \therefore \quad X = \int \frac{dx}{\kappa_0} = \int \frac{1}{\kappa_0} \frac{\partial x}{\partial \varphi} \, d\varphi \tag{19b}$$

X increases from West to East. Notice that labeling field lines by the coordinate X is equivalent to picking up lines of a shell with a "density" (number per unit equatorial ring arc length) inversely proportional to the curvature κ_0 (13). For a pure dipole field, or for the outer shells in the real field, $\kappa_0 = 1$, and X is identical with x (7). For low-L shells, κ_0 may differ from 1 as much as ±6% (See Part II). With the new coordinates (19a) and (19b), the flux $\delta\phi$ is

$$\delta\phi = B_0 \, \delta x \, \delta y = B_0 \, \delta X \, \delta Y$$

in which δY is now independent of longitude. Adopting X (19b) as the appropriate longitudinal variable, we can write the number of electrons in a flux tube as

$$\delta N = U(B, E, X, t)\ \delta B\, \delta E\, \delta X \cdot B_0\, \delta Y \tag{20}$$

As $B_0 \delta Y$ is a constant throughout the whole forthcoming discussion we shall drop it at once whenever we consider the number of particles in a flux tube.

Finally, it is important to remark that there is a more exact expression for the fundamental relation (14), which takes into account the fact that in the real geomagnetic field, L differs from $1/R\ (M/B_0)^{1/3}$ (10) by a small, longitude dependent amount $\Delta L = \Delta L(L, B, \varphi)$[16]. This relation is given by

$$\frac{\delta I}{\delta y} = \nabla_0 I = \frac{1}{\lambda(B)} \left[\kappa_0 - RL \frac{\partial}{\partial y}\left(\frac{\Delta L}{L}\right)\right]$$

However, the correction term $RL\ \partial/\partial y(\Delta L/L)$ is expected to be small as compared with κ_0, and the longitude variations of κ_0. On the other hand, the correction term implies the existence of a shell splitting as a function of the mirror point field intensity[16]. But in the whole treatment which follows, this splitting is being neglected, and all electrons, initially on a given line of force, are supposed to populate the same shell, regardless of their mirror points.

DERIVATION OF THE FOKKER-PLANCK EQUATION FOR LONGITUDE DEPENDENCE

We are now in condition to set up the general equation governing the distribution of trapped electrons. These electrons will undergo displacements in **B**, **E** and longitude, caused by three types of mutually independent interactions:

(1) A change in the mirror point field B, due to the <u>stochastic</u> process of multiple Coulomb scattering in the atmosphere;

(2) A change in kinetic energy E, due to ionization slowing-down in the atmosphere, considered here as a <u>non</u>-stochastic process (i.e. neglecting straggling);

(3) A change in longitude due to interaction with the static magnetic field (longitudinal drift), again a <u>non</u>-stochastic process.

All these processes are physically independent of each other, although all intervening parameters are in general functions of the three variables B, E and longitude.

Let us relate the distribution of electrons as it appears in (20), with the distribution of electrons at a slightly earlier time $t - \Delta t$, at a different longitudinal position. For the time being, we shall forget the non-stochastic character of E and X, and treat all variables as if they were of the same, stochastic nature. We write:

$$U(B, E, X, t) =$$

$$= \iiint U(B-\beta, E-\epsilon, X-\xi, t-\Delta t)\, \Pi(B-\beta, E-\epsilon, X-\xi, \beta, \epsilon, \xi, \Delta t)\, d\beta d\epsilon d\xi + Q(B, E, X, t)\, \Delta t \tag{21}$$

In this relation, the distribution of electrons which at the time t are at a position X, is linked to the distribution of those electrons which at an earlier time $t - \Delta t$ were at $X - \xi$, and which happened to scatter, slow down and drift the right amount in the interval Δt, in order to become part of the population described by the left hand of (21).

$\Pi d\beta\, d\epsilon\, d\xi$ is the a priori probability that these electrons have undergone just the right changes in mirror point field, energy and longitudinal position, in the time interval Δt.

The source term $Q\Delta t$ represents the contribution of electrons added by injection to the original bunch of particles, during Δt.

Expanding all intervening functions in Taylor series in β, ϵ, ξ and Δt, we obtain:

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial X}(U\langle\xi\rangle) = -\frac{\partial}{\partial E}(U\langle\epsilon\rangle) - \frac{\partial}{\partial B}(U\langle\beta\rangle) + \frac{1}{2}\frac{\partial^2}{\partial B^2}(U\langle\beta^2\rangle) +$$

$$+ Q + \text{higher order terms in } \xi^2, \epsilon^2, \epsilon\beta, \text{ etc.} \tag{22}$$

The brackets stand for the average per unit time of the enclosed variables:

$$\left\langle A \right\rangle = \frac{1}{\Delta t} \iiint A \, \Pi \, d\beta \, d\epsilon \, d\xi$$

We can now re-instate the non-stochastic character to E and X, by taking Π as a delta function in energy and longitude:

$$\Pi(B, E, X, \beta, \epsilon, \xi, \Delta t) = \Pi_0(B, E, X, \beta, \Delta t) \, \delta(\epsilon + \dot{\epsilon} \Delta t) \, \delta(\xi - \dot{\xi} \Delta t)$$

In this expression, $\dot{\epsilon}$, is the change of energy per unit time, due to ionization loss. Evidently,

$$\dot{\epsilon} = \frac{1}{\tau_b} \{\epsilon\} \tag{23}$$

where $\{\epsilon\}$ is the energy loss in one half-bounce (path from one mirror point to its conjugate). We have taken $\{\epsilon\}$ and $\dot{\epsilon}$ as positive quantities, and evidenced the energy decrease by the sign in the first delta function.

As to $\dot{\xi}$, the drift velocity in the X coordinate, we have, according to (19b), (16) and (17):

$$\dot{\xi} = \dot{\xi}(B, E, X) = \frac{dX}{dt} = \frac{u_0}{\kappa_0} = \frac{1}{\kappa_0} \frac{\partial x}{\partial \varphi} \dot{\varphi} = \frac{p}{eB_0} \frac{1}{\tau_b \lambda} \tag{24}$$

Π_0 is the probability for a change in the mirror point field due to Coulomb scattering, in the interval Δt. It is evidently given by

$$\Pi_0(B, E, X, \beta, \Delta t) = P(B, E, X, \beta) \frac{\Delta t}{\tau_b(B, E, X)} \tag{25}$$

where P is the probability for B-scattering during one half-bounce.

In all this we have implicitly assumed that scattering and slowing down are extremely small during one half-bounce of the electron. In other words, we suppose that $P \ll 1$ for $\beta \neq 0$ and approximately = 1 for $\beta = 0$ (this imposes a limiting condition to the applicability of our discussions when dealing with very high atmospheric densities).

Equation (22) finally becomes, up to the second order in β:

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial X}(U\dot{\xi}) = \frac{\partial}{\partial E}(U\dot{\epsilon}) - \frac{\partial}{\partial B}(U\langle\beta\rangle) + \frac{1}{2}\frac{\partial^2}{\partial B^2}(U\langle\beta^2\rangle) + Q \tag{26}$$

where

$$\langle\beta\rangle = \frac{1}{\tau_b}\{\beta\} \qquad \{\beta\} = \int \beta P \, d\beta$$

with

$$\langle\beta^2\rangle = \frac{1}{\tau_b}\{\beta^2\} \qquad \{\beta^2\} = \int \beta^2 P \, d\beta \tag{27}$$

The angular brackets $\langle\,\rangle$ are average changes per unit time; the curled brackets $\{\,\}$ represent average changes per half-bounce.

We can re-write the longitude-convection term as a function of ordinary geographic longitude, taking into account (24):

$$\frac{\partial}{\partial X}(U\dot{\xi}) = \frac{1}{\frac{1}{\kappa_0}\frac{\partial x}{\partial \varphi}}\frac{\partial}{\partial \varphi}\left(U\frac{1}{\kappa_0}\frac{\partial x}{\partial \varphi}\dot{\varphi}\right) = \frac{\kappa_0}{\frac{\partial x}{\partial \varphi}}\frac{p}{eB_0\lambda}\frac{\partial}{\partial \varphi}\left(\frac{1}{\tau_b}U\right) \tag{28}$$

Notice that in a pure, centered dipole, $\kappa_0 = 1$, $\partial x/\partial\varphi$ and $\dot{\varphi}$ constants, and the convection term would reduce to $\dot{\varphi}\,\partial U/\partial\varphi$.

Equation (26) is the most general Fokker-Planck equation describing the longitude and time dependence of a trapped particle flux, which scatters and slows down in the atmosphere.

The initial condition for equation (26) is the distribution U given at a certain longitude, for a given initial time. There is a "natural" boundary condition at the magnetic equator:

$$\frac{\partial U}{\partial B} = 0 \quad \text{for} \quad B = B_0 \tag{29}$$

At the ends of lines of force in the high atmospheric density region, there is no physical boundary condition for U as a function of B: as particles enter higher densities, they get slowed down by ionization and die away in energy space. Therefore, it would be incorrect to impose a condition of the type $U = 0$ for $B \gtrsim B_c$. Moreover, equation (26) is not valid at all near such fictious boundary (for derivatives and integrals could not be interchanged any more when passing from (21) to (22)). Of course, if one is only interested in electron fluxes in B regions which always remain high up, i.e. far away from high atmospheric densities, a boundary condition for U as a function of B may well be adopted for practical, computational reasons.

DISCUSSION OF THE GENERAL EQUATION

Let us now discuss equation (26) from the point of view of its physical meaning. First of all, in absence of scattering, slowing down and sources, the right hand of equation (26) is zero. We then have:

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial X}(U\dot{\xi}) = 0 \tag{30}$$

This equation tells us that a given initial distribution of electrons injected at a time t_0 at a line labeled X_0, will proceed drifting eastwards with speed $\dot{\xi}$, always keeping the electron distribution inversely proportional to the local drift speed $\dot{\xi}$:

$$U\dot{\xi} = \text{const} \tag{31}$$

As $\dot{\xi}$ is energy-dependent, equation (30) represents a "flight-time spectrometer effect" on the particles, after injection. Notice carefully that expression (31) means constancy along the drift path of a given bunch of particles. It is not a constancy in time.

If we now integrate equation (26) over one complete longitudinal cycle of X, for a fixed time t, the second term on the left vanishes. We obtain:

$$\frac{\partial}{\partial t}\oint U\,dX = \frac{\partial}{\partial E}\oint U\,\dot{\epsilon}\,dX - \frac{\partial}{\partial B}\oint U\left\langle\beta\right\rangle dX + \frac{1}{2}\frac{\partial^2}{\partial B^2}\oint U\left\langle\beta^2\right\rangle dX + \bar{Q} \tag{32}$$

Dividing by $\oint dX$ and calling

$$U_{AV} = \frac{\oint U\,dX}{\oint dX} = \frac{\oint U \frac{dx}{\kappa_0}}{\oint \frac{dx}{\kappa_0}} \tag{33a}$$

$$\dot{\epsilon}_{AV} = \frac{\oint \dot{\epsilon}\, U\,dX}{\oint U\,dX} \tag{33b}$$

$$\left.\begin{aligned} \left\langle \beta \right\rangle_{AV} &= \frac{\oint \left\langle \beta \right\rangle U\,dX}{\oint U\,dX} \\ \left\langle \beta^2 \right\rangle_{AV} &= \frac{\oint \left\langle \beta^2 \right\rangle U\,dX}{\oint U\,dX} \end{aligned}\right\} \tag{33c}$$

we obtain the longitude-independent equation

$$\frac{\partial U_{AV}}{\partial t} = \frac{\partial}{\partial E}\left(U_{AV}\,\dot{\epsilon}_{AV}\right) - \frac{\partial}{\partial B}\left(U_{AV}\left\langle \beta \right\rangle_{AV}\right) + \frac{1}{2}\frac{\partial^2}{\partial B^2}\left(U_{AV}\left\langle \beta^2 \right\rangle_{AV}\right) + \overline{Q} \tag{34}$$

This equation is formally equivalent to the time-dependent equation used by several authors[9,10,11,13]. Notice, however, the following remarks:

(1) The average distribution U_{AV} is computed integrating over the new variable X, which according to (33a), means that <u>U must be weighted at the different longitudes with the inverse of the equatorial curvature of the field lines.</u>

(2) The "coefficients" $\dot{\epsilon}_{AV}$, $\left\langle \beta \right\rangle_{AB}$ and $\left\langle \beta^2 \right\rangle_{AV}$ are not simple averages over the new coordinate, but <u>they are weighted with the distribution U itself. In other words, they are functionals of the unknown distribution.</u>

All this leads us to the conclusion that a longitude-average Fokker-Planck treatment of the problem of trapped particle diffusion is not valid, unless we restrict our description to only those particles which mirror high enough at all longitudes so that their distribution function can be expected a priori as very little longitude dependent. Equation (34) is certainly meaningless as a simple differential equation for that portion of B-L space, which descends below about 300 – 350 Km in the South American Anomaly. This is precisely the domain where the longitude-independent treatment has so far failed to give numerical results compatible with experimental measurements.

In order to "legalize" the longitude-independent description for such cases in which we know a priori that U will not depend strongly on X, we have to evaluate the coefficients (33) for a distribution U which has the smallest possible longitude dependence. This dependence is given precisely by (31), i.e. for the case of absence of interactions with the atmosphere. Introducing (31) in (33), and taking into account (24), we obtain "true" coefficients (independent of U):

$$\dot{\epsilon}_{AV} = \frac{\oint \dot{\epsilon} \frac{dX}{\dot{\xi}}}{\oint \frac{dX}{\dot{\xi}}} = \frac{\oint \dot{\epsilon} \frac{dx}{u_0}}{\oint \frac{dx}{u_0}} \tag{35}$$

$$\left\langle \beta \right\rangle_{AV} = \frac{\oint \left\langle \beta \right\rangle \frac{dx}{u_0}}{\oint \frac{dx}{u_0}} \qquad \left\langle \beta^2 \right\rangle_{AV} = \frac{\oint \left\langle \beta^2 \right\rangle \frac{dx}{u_0}}{\oint \frac{dx}{u_0}}$$

These expressions are formally identical with the longitude-average coefficients used by Hassitt[18], in which the atmospheric constituents (the only strongly longitude dependent variables actually contained in $\dot{\epsilon}$, $\langle \beta \rangle$ and $\langle \beta^2 \rangle$) are weighted inversely proportional to the equatorial drift velocity at each longitude.

We must, however, insist again that the use of a longitude-independent Fokker-Planck equation for the description of trapped electron diffusion, is not valid at all for those high B values, for which the atmospheric interactions in the region of the Anomaly will introduce a notable departure of U from the "collisionless" expression (31).

Let us now return to equation (26), for the steady state case $\partial U/\partial t = 0$. This is very nearly true for natural radiation belt electrons during quiet solar wind conditions, and even for artifically injected fluxes, provided we are interested in the longitude dependence of the electron distribution, at times after injection long compared to a typical longitudinal drift period. In that case, we can also neglect the contribution of the natural source term Q. Taking into account (24), (23) and (27), we introduce the distribution function

$$W = U\dot{\xi} = U \frac{p}{eB_0} \frac{1}{\tau_b \lambda} \tag{36}$$

and the coefficients

$$\bar{\dot{\epsilon}} = \frac{1}{\dot{\xi}} \dot{\epsilon} = \lambda \frac{eB_0}{p} \{\epsilon\}$$

$$\bar{\beta} = \frac{1}{\dot{\xi}} \langle \beta \rangle = \lambda \frac{eB_0}{p} \{\beta\} \tag{37}$$

$$\bar{\beta}^2 = \frac{1}{\dot{\xi}} \langle \beta^2 \rangle = \lambda \frac{eB_0}{p} \{\beta^2\}$$

which now represent changes per unit longitudinal coordinate X. Notice that the factor

$$\lambda \frac{eB_0}{p} = \frac{1}{\tau_b \dot{\xi}} = b(B, E) \tag{38}$$

physically represents the "bounce-density," i.e., the number of half-bounces per unit longitudinal coordinate, which a particle of mirror point field B and energy E makes at any position on the shell. Notice that this bounce density is independent of the longitudinal position X.

Using (36) and (37), the steady state form of (26) reads:

$$\frac{\partial W}{\partial X} = -\frac{\partial}{\partial B}(W\bar{\beta}) + \frac{1}{2}\frac{\partial^2}{\partial B^2}(W\overline{\beta^2}) + \frac{\partial}{\partial E}(W\bar{\dot{\epsilon}}) \tag{39}$$

This is the fundamental equation which describes the longitude dependence of a stationary, source free electron distribution, now represented by W (36). It will be the main subject for the rest of this work.

A useful concept for a qualitative analysis of equation (39) is that of the "mirror point flow in B-X space," i.e., the mean change of the mirror point field intensity of a particle, per unit longitudinal coordinate. Let us consider the following relation

$$\int_{B_0}^{B(X)} W\,dB = \text{const}, \tag{40}$$

where the integral is taken along a given field line between the equatorial point B_0 and a generic field point $B(X)$. We shall call the relation $B = B(X)$, which ensures the constancy of (40), "characteristic trajectory of a mirror point" (see Fig. 5). Notice carefully that, in general, $B = B(X)$ is not the real path of the

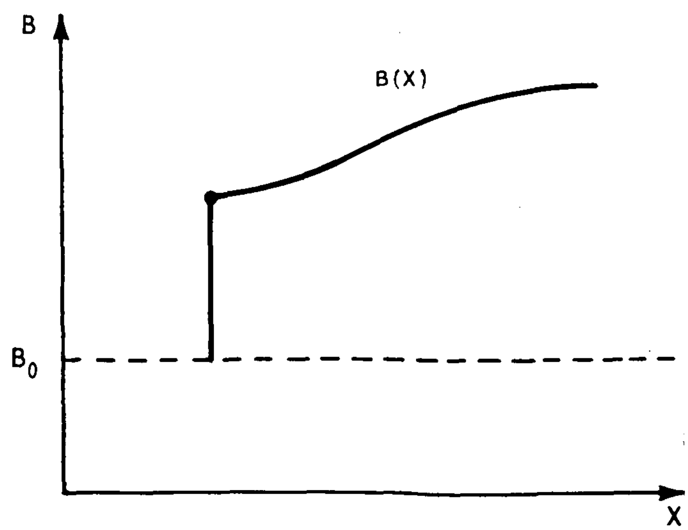

Figure 5

mirror point of a given electron, in B-X space. However, it does become the actual average path, if we neglect energy loss and also neglect the longitude dependence of τ_b. In that case, (40) is proportional to the number of particles mirroring between equator and B(X). But even if these conditions are not satisfied, the function $B = B(X)$ defined in (40) is a useful concept.

In absence of all interactions, $B(X) = \text{const.}$, i.e. the mirror point field intensities remain unchanged. We now turn on Coulomb scattering, but still neglect energy loss. We then introduce the quantity $v_m = dB(X)/dX$ as the "mirror point flow in B-X space," obtaining, with the use of (39) and (40):

$$v_m = \frac{dB(X)}{dX} = -\frac{1}{W}\int_{B_0}^{B(X)} \frac{\partial W}{\partial X}\, dB = \bar{\beta} - \frac{1}{2}\frac{\partial \overline{\beta^2}}{\partial B} - \frac{1}{2}\overline{\beta^2}\,\frac{\partial \ln W}{\partial B} \tag{41}$$

As defined in (41), v_m describes the average "flow" along the field lines, of mirror points of electrons of a given energy, as they drift in longitude. If we wish to add the energy loss mechanism, an additional term would appear in the right side of (41), and particle individuality would be lost.

Notice that there are three physically quite distinct contributions to the mirror point "flow." The first term of the right hand of (41) represents a steady increase of B, ($\bar{\beta}$ is positive), i.e. a steady lowering of mirror point altitude. This contribution comes from the first order, "streaming" term in B (39). In absence of dispersion ($\overline{\beta^2} = 0$), $\bar{\beta}$ is the only contribution to v_m; in that case, $B = B(X) = \int v_m dX$ is just the characteristic of the corresponding first order differential equation.

The second and third terms on the right hand side of (41) arise in the dispersion mechanism. Their presence in (41) clearly shows that dispersion also contributes to a steady, average "flow" of mirror points along the lines of force. These two terms are governed by the gradients (along field lines) of the coefficient $\overline{\beta^2}$, and of W (or U), respectively. The gradient of $\overline{\beta^2}$ is always positive, so that the second term in (41) always represents an upward motion of the particles's mirror points (back-scattering from the denser atmosphere). If, on the other hand $\partial W/\partial B < 0$, the third term represents a lowering of the mirror point altitude. If $W\,\overline{\beta^2} = \text{const.}$ along a line of force, the contribution from the two dispersion terms is zero: The number of mirror points thrown upwards by the gradient in $\overline{\beta^2}$, exactly compensates the number of mirror points streaming downwards due to the gradient of W. The only mirror point flow which remains, in this case, is that of the first order term: $v_m = \bar{\beta}$.

DETERMINATION OF THE COEFFICIENTS

In this section we shall obtain expressions for the various coefficients intervening in (39). According to (37), the principal quantities to be found are λ, $\{\epsilon\}$, $\{\beta\}$ and $\{\beta^2\}$.

With respect to the dimensionaless function λ, defined in (15), we recall that it is inversely proportional to a dipole-type function computed by Hamlin, et. al.[17] A good approximation of λ for the interval $1/4 < B_0/B \leq 1$ is given by

$$\lambda(B) \cong 0.384 + 0.128 \exp\left(-\frac{1}{1.37}\frac{B}{B_0}\right) \tag{42}$$

The energy loss per half-bounce $\{\epsilon\}$ is given by the integral

$$\{\epsilon\} = \int_{-s(B)}^{s(B)} \left|\frac{\partial E}{\partial s}\right| \frac{ds}{\sqrt{1-\frac{B'(s)}{B}}} \tag{43}$$

The origin of the field line arc length is taken at the geomagnetic equator. $ds/\sqrt{1 - B'(s)/B}$ is the element of trajectory of the electron at a point s where the field is B'. Using tables given by Berger and Seltzer,[19] we obtain the following very good approximation:

$$\left.\begin{aligned} &\left|\frac{\partial E}{\partial s}\right| = 10^{-20}\, N_{eff.ion.}\, h(E) \quad (kev/cm) \\ &\text{where} \\ &h(E) = \begin{cases} 2.61 + 349.0\, E^{-0.844} & \text{for } E < 470 \text{ kev} \\ 4.54 & \text{for } E \geq 470 \text{ kev} \end{cases} \end{aligned}\right\} \tag{44}$$

and

$$N_{eff.ion.} = N_{oxygen} + 0.881\, N_{nitrogen} + 0.273\, N_{helium} \tag{45}$$

is the "effective" atmospheric number density for ionization loss of electrons. As for the time being we are interested in low L shells only, we did not include the contribution from atmospheric ions and electrons.

In order to evaluate $\{\beta\}$, the average change of B per half-bounce, we first have to perform the average of collisons over the isotropic azimuthal distribution of scattering angles. Following Welch, Kaufmann and Hess[10] and slightly changing their notation, we obtain

$$\tilde{\beta} = B\left(\frac{B}{B'} - \frac{1}{2}\right)\sin^2\theta \tag{46}$$

$\tilde{\beta}$ is the change in mirror point field intensity B, when the electron's pitch angle scatters an amount θ at a field position B', averaged over all possible azimuthal angles of scattering. Now we have to find the average of $\sin^2\theta$ at the given field point B'. This will be given by

$$\overline{\sin^2\theta} = \sum_i N_i \eta_i \tag{47}$$

where

$$\eta_i = \int_0^{\pi/2} \sin^2\theta \frac{d\sigma_i}{d\theta}\, d\theta \tag{48}$$

is the average contribution from one atom of class i. N_i are the number densities of the different atmospheric constituents at the field point B'; $d\sigma_i/d\theta$ are the differential cross sections for screened Coulomb scattering, for each constituent. Using Molière's expression for $d\sigma_i/d\theta$,[20] and following closely Welch, Kaufmann, and Hess,[10] we obtain

$$\sum_i N_i\, \eta_i = 4 \times 10^{-22}\, N_{eff.scatt.} \frac{1 + 0.167 \ln \sqrt{\frac{E}{mc^2}\left(\frac{E}{mc^2} + 2\right)}}{\frac{E}{mc^2}\left(\frac{E}{mc^2} + 2\right)} (cm^{-1}) \tag{49}$$

where

$$N_{eff.scatt.} = N_{oxygen} + 0.774\, N_{nitrogen} + 0.058\, N_{helium} \tag{50}$$

is the "effective" atmospheric number density for screened Coulomb scattering of electrons. Again, the contribution of atmospheric ions and electrons is excluded.

We finally have to perform the integration along the path of a particle from one mirror point to its conjugate:

$$\{B\} = B \int_{-s(B)}^{s(B)} \left(\frac{B}{B'(s)} - \frac{1}{2}\right)\left(\sum_i N_i\, \eta_i\right) \frac{ds}{\sqrt{1 - \frac{B'(s)}{B}}} \tag{51}$$

Again following Welch, Kaufman and Hess,[10] we obtain $\tilde{\beta}^2$:

$$\tilde{\beta}^2 = 2B^2 \left(\frac{B}{B'} - 1\right) \sin^2 \theta \tag{52}$$

Therefore:

$$\{\beta^2\} = 2B^2 \int_{-s(B)}^{s(B)} \left(\frac{B}{B'(s)} - 1\right)\left(\sum_i N_i\, \eta_i\right) \frac{ds}{\sqrt{1 - \frac{B'(s)}{B}}} \tag{53}$$

If we now call:

$$S_1(B, X) = \int_{-s(B)}^{s(B)} \frac{B}{B'(s)} \frac{N_{eff\,scatt}(s, X)}{\sqrt{1 - B'(s)/B}}\, ds \quad (cm^{-2})$$

$$S_2(B, X) = \int_{-s(B)}^{s(B)} \frac{N_{eff\,scatt}(s, X)}{\sqrt{1 - B'(s)/B}}\, ds \quad (cm^{-2})$$

$$S_2'(B, X) = \int_{-s(B)}^{s(B)} \frac{N_{eff\,ion}(s, X)}{\sqrt{1 - B'(s)/B}}\, ds \quad (cm^{-2}) \tag{54}$$

$$K(B, E) = 4 \times 10^{-22}\, B \frac{1 + 0.167 \ln \sqrt{\frac{E}{mc^2}\left(\frac{E}{mc^2} + 2\right)}}{\frac{E}{mc^2}\left(\frac{E}{mc^2} + 2\right)} \quad (\text{gauss } cm^2)$$

$$C(E) = 10^{-20}\, h(E) \qquad (\text{kev } cm^2)$$

we can summarize expressions (51), (53) and (43) in the form

$$\{\beta\} = K\left(S_1 - \frac{1}{2} S_2\right)$$

$$\{\beta^2\} = 2BK\,(S_1 - S_2) \tag{55}$$

$$\{\epsilon\} = C\,S_2'$$

DISCUSSION OF ELECTRON DRIFT AND SCATTERING THROUGH A MODEL FOR THE ANOMALY

Before setting up numerical integrations for low altitude electrons, it is very helpful to discuss Equation (39) qualitatively, using a simple, approximate model of the atmosphere along field lines, in the region of the South American Anomaly, and where most of the scattering and energy loss occurs.

We shall restrict ourselves to the region between 250 and 750 Km altitude, and to longitudes within about ±30° of the "center" of the Anomaly (supposed at a longitude $\varphi_a = 318°E$). In the present discussion we shall neglect the energy loss term and suppose $\partial x/\partial\varphi$ = const. and $\kappa_0 = 1$. These latter suppositions lead to a proportionality between X and the longitude φ of the equatorial point of a field line (19b).

Using the Harris-Priester atmosphere[21] for low solar activity (S = 70), and Southern Hemisphere geomagnetic field B-L rings as calculated by Stassinopoulos,[22] we obtain a quite good fit for the effective number density (50) as a function of B and φ, of the type:

$$N_{eff\,scatt} = N_0 \exp\left(\frac{B - B_a}{\Delta B}\right) \exp\left(-\frac{|\varphi - \varphi_a|}{\Delta\varphi}\right) \tag{56}$$

Due to the supposed proportionality between φ and X, it is easy to convert this expression into a function of X. All this is valid only for very low L values.

For L = 1.25, we have:*

13 hs local time at the center of the Anomaly:

$N_0 = 10^{11}$ cm^{-3} $B_a = 0.243$ gauss $\Delta B = 5.25 \times 10^{-3}$ gauss $\Delta\varphi = 4.69°$

02 hs local time at the center:

$N_0 = 10^{11}$ cm^{-3} $B_a = 0.243$ gauss $\Delta B = 4.23 \times 10^{-3}$ gauss $\Delta\varphi = 3.78°$

For an expression of the form (56), with a "scale height" ΔB very small compared to B, the main contribution to the integrals (54) comes from a region close to that mirror point which lies in the high atmospheric density end of the line of force. We can therefore approximate:

$$S_1 \cong S_2 \cong N_0 \exp\left(\frac{B - B_a}{\Delta B}\right) \exp\left(- \frac{|\varphi - \varphi_a|}{\Delta\varphi}\right) \frac{\sqrt{\pi \frac{\Delta B}{B}}}{\frac{\partial B}{\partial s}} \tag{57}$$

and

$$S_1 - S_2 \cong S_1 \frac{\Delta B}{2B} \tag{58}$$

$\partial B/\partial s$ is the average gradient of B along the field line, in the region of interest. Taking into account (55), the following relation between the coefficients $\{\beta\}$ and $\{\beta^2\}$ holds:

$$\{\beta^2\} = 2\Delta B\,\{\beta\} \tag{59}$$

*For higher L shells ($L \gtrsim 1.8$), the approximation (56) becomes very crude. Qualitatively, as L increases, φ_a shifts to the East, and $\Delta\varphi$ increases considerably.

This is similar to a relation found by Walt and McDonald[12] for the coefficients of a time dependent (but longitude-independent) Fokker-Planck equation set up in terms of mirror point altitudes, in the region of high atmospheric densities. Relation (59) is expected to hold quite generally in the region of high densities, provided one inserts for ΔB the value $\Delta B = N_{eff}/|\partial N_{eff}/\partial B|$, where the derivation is taken along a line of force.

With (59) and the linear relationship between X and φ, Equation (39) becomes

$$\frac{\partial W}{\partial X} = \frac{1}{\frac{\partial X}{\partial \varphi}} \frac{\partial W}{\partial \varphi} = - \frac{\partial}{\partial B} (W\bar{\beta}) + \Delta B \frac{\partial^2}{\partial B^2} (W\bar{\beta}) \tag{60}$$

in which $\bar{\beta}$ is of the form

$$\bar{\beta} = K' \exp\left(\frac{B - B_a}{\Delta B}\right) \exp\left(- \frac{|\varphi - \varphi_a|}{\Delta\varphi}\right) \tag{61}$$

K' is obtained from (37), (54), (55) and (57), and is a slowly varying function of B (slow in comparison with exp $(B/\Delta B)$).

The mirror point flow (41) is now

$$v_m = \bar{\beta} - \Delta B \frac{\partial\bar{\beta}}{\partial B} - \Delta B\,\bar{\beta}\, \frac{\partial \ln W}{\partial B}$$

Taking into account (61) we can approximate:

$$\frac{\partial\bar{\beta}}{\partial B} \simeq \frac{\bar{\beta}}{\Delta B} \tag{62}$$

Therefore

$$v_m \cong - \Delta B\,\bar{\beta}\, \frac{\partial \ln W}{\partial B} \tag{63}$$

Physically, the cancellation of the first term with the second term in (41) means that the upstream of mirror points due to diffusion from the denser atmosphere ("backscattering" at lower altitudes) exactly compensates the downstream due to the first order term. What is left, is a mirror point streaming, entirely due to diffusion, and which may be upwards or downwards, according to the gradient of the actual electron distribution, along a line of force.

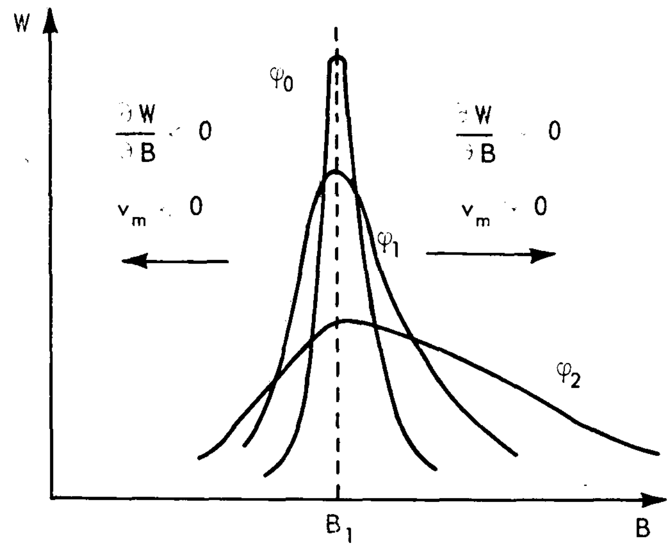

Figure 6–Qualitative description of the evolution of a mirror point distribution at very low altitudes.

In particular, if $\partial U/\partial B = 0$ for some value B_1, there will be no net flow of mirror points across this B value. An initial distribution of electrons at a longitude φ_0 like the one shown in Figure 6, will therefore keep the maximum at the same B_1, but will broaden towards both sides as the particles drift towards the east ($\varphi_2 > \varphi_1 > \varphi_0$). This smearing-out will be faster at the high-B side (towards lower altitudes), for $\bar{\beta}$ is much greater there (see expression (63)). A similar result was obtained numerically by MacDonald and Walt.[12]

If now W is of the form

$$W \sim \frac{1}{K' \exp\left(\frac{B - B_a}{\Delta B}\right)} \tag{64}$$

we have a solution of Equation (60), which represents a steady state in longitude. In this case, the mirror point flow (63) is approximately equal to $\overline{\beta}$:

$$v_m \cong -\Delta B \, \overline{\beta} \, \frac{\partial \ln W}{\partial B} \cong \overline{\beta} \tag{65}$$

It can be shown that such a steady state form (64) is attained very soon, provided the gradient of W along a line of force is negative. It is also easy to verify that (64) is an "equilibrium" configuration: Any small departure of W from (64) will cause mirror point flows which will tend to correct such a departure.

For all these reasons, it is extremely instructive to inspect in detail the "motion" of mirror points for such a longitude independent equilibrium state (64). Although, of course, all this is only a crude approximation, we expect all qualitative, physical features to be valid also in the real case. This qualitative analysis will then help us to set up and interpret the tedious numerical calculations (Part III).

We shall analyze the characteristic trajectories of mirror points $B = B(X)$ (40) throughout the South American Anomaly, described approximately by (56), for an equilibrium state electron distribution (64). First of all, we will do our discussion in terms of the more familiar longitude variable φ, rather than X, which in our description anyway is supposed proportional to the former. In order to obtain the characteristic trajectories in $B-\varphi$ space, we have to integrate (65), taking into account (61) and the relation (19b), which now reads

$$X = \frac{2\pi}{360} RL\varphi .$$

Starting from an initial longitude φ_0, well West of the center of the Anomaly, and for a shell $L = 1.25$, we obtain characteristic mirror point trajectories for 300 kev electrons, schematically represented in Figure 7. Remember that they correspond to a distribution of the type (64). In this figure, lines of constant altitude for the approximate expression (56) are also shown. Below 250 Km, the $N_{eff} = N_{eff}(B, \varphi)$ dependence is still nearly exponential in B and $|\varphi - \varphi_a|$, but with a smaller "scale height" ΔB. We can safely assume for this discussion, that particles getting below this level will be removed by precipitation. Above 750 Km, the scattering effect is extremely small during one drift through the region of interest.

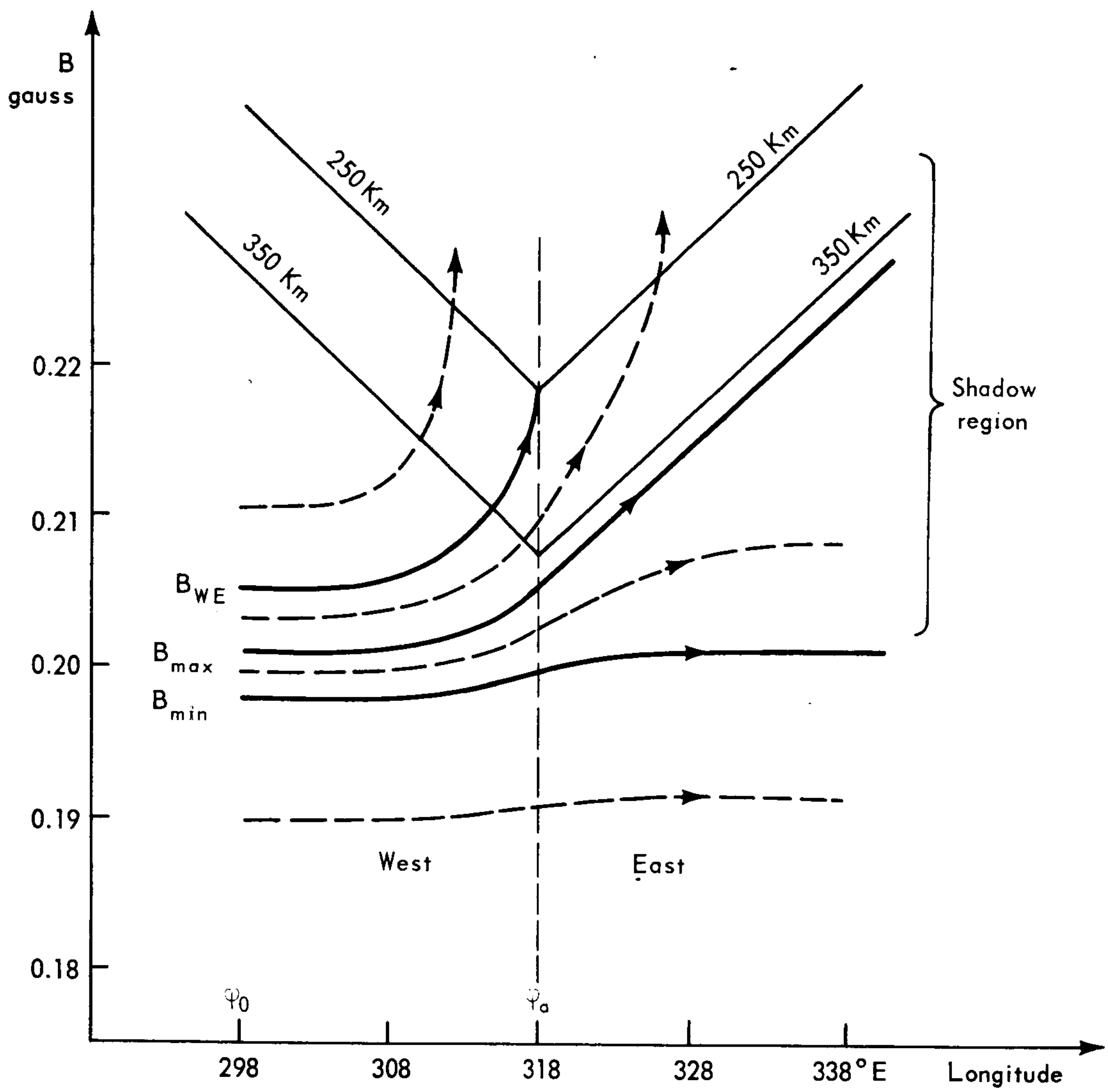

Figure 7–Characteristic mirror point trajectories in the region of the South American Anomaly (model (56)), for a steady state distribution of 300 Kev electrons of the form (64), and for $L = 1.25$ (13.00 LT in the Anomaly). Electrons whose mirror points are initially (i.e. West of the Anomaly) between B_{min} and B_{max}, populate the "shadow" region. Those between B_{max} and B_{WE} precipitate East of φ_a; those with $B > B_{EW}$ precipitate on the West side. Constant altitude lines for $h = 250$ and 350 Km are shown. Energy loss is neglected.

Notice in Figure 7 that there is a narrow "window" $B_{min} - B_{max}$, at the initial longitude φ_0 West of the Anomaly, which contains the mirror points of those electrons, which populate the high-B or "shadow" region, East of the Anomaly. After one more turn around the earth, all these electrons are precipitated into the dense layers. A certain portion of these ($B_{max} - B_{WE}$) will manage to get "around" the center of the Anomaly, and precipitate into its Eastern side. Electrons mirroring initially at higher altitudes than those corresponding to the "window," will lower slightly their mirror points during each passage through the Anomaly until they fall into the critical window. Please remember carefully that all this corresponds to an equilibrium distribution of the form (64), neglecting energy loss, and remember that the mirror point "trajectories" are average paths, not actual patterns for a given electron.

The most striking result of this analysis, is that if multiple Coulomb scattering is the only process responsible for the replenishment of the "shadow" region East of the Anomaly, then there is a very limited region of the atmosphere in the South American Anomaly (limited in both, longitude and altitude), which entirely governs this replenishing process. We therefore should expect that in the real case, electron fluxes in the "shadow" are strongly influenced by the instantaneous atmospheric structure in this critical region.

In particular, we may expect a diurnal variation of electron fluxes and electron precipitation, determined by the diurnal variation of the atmosphere within about ±5° of the center of the Anomaly, between about 250-400 Km altitude. We conclude, that it would be unrealistic, to carry out numerical calculations with a 24-hour averaged atmosphere. Rather, numerical integration should be performed for different local times at the Anomaly (Part II).

Notice finally, that according to this picture, replenishment occurs right at, or shortly "after," i.e. East, of the Anomaly. Diffusion into the "shadow" region far East of the Anomaly is negligible, for Coulomb scattering. We may conclude our discussion by pointing out that in the picture described above, there is a steady flow of particles out of the lower B region of a shell, towards higher B values, and from there, into the atmosphere at the Anomaly. In an equilibrium state, this loss must be exactly balanced by a continuous injection from a source mainly effective in the lower B region. In absence of such a source, i.e. in a non-equilibrium state, this flow and subsequent loss should determine the lifetime of the trapped radiation in question.

As a numerical example, we have calculated the position of the critical "window" in B (for the definition of B_{min} and B_{max} see Figure 7), for local times in the Anomaly as 1300 LT and 0200 LT. The values obtained for $L = 1.25$ are shown in Table 1. Although the B values differ only very little, remember that they are in a region where the distribution U is a very steep function of B.

Table 1

1300 LT	300 kev	625 kev
B_{min}	0.198 gauss	0.205 gauss
B_{max}	0.201	0.209
0200 LT		
B_{min}	0.207	0.213
B_{max}	0.210	0.216

Detailed quantitative results on electron fluxes can be obtained only by numerical integration of Equation (39). In particular, the energy loss term, neglected in the above discussion, has to be taken into account; this is particularly important if we analyze the behavior of electrons which dip below about 300 Km altitude in the Anomaly.

ACKNOWLEDGEMENTS

We are indebted to Drs. W. N. Hess, D. Stern, J. Herring and M. Liwshitz for valuable discussions and critical remarks.

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