

FLIGHT DYNAMICS AND CONTROL ANALYSIS
OF THE CENTAUR VEHICLE
(ATLAS/CENTAUR AC-5)

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SUMMARY

This report presents the stability and control analyses for Centaur stage flight of the Atlas/Centaur AC-5 configuration. The analyses include all flight phases from Atlas/Centaur separation through drifting (unstabilized) flight of the Centaur vehicle in earth orbit.

Stability and control of the AC-5 configuration during Centaur stage flight is provided by the Centaur vehicleborne flight control system which corrects vehicle deviations from the reference attitude provided by the Centaur inertial guidance system. Separate control channels in pitch, yaw, and roll admit control signals to the autopilot from the control system sensors (rate gyros and inertial platform). These signals affect engine gimbaling or attitude control jet firing to provide control forces throughout both powered and coast flight phases.

The analyses were performed to verify the ability of the flight control system to adequately stabilize:

- a. The short period oscillations about the vehicle center of gravity during powered phase flight (including parasitic modes of oscillation, i.e., propellant sloshing and elastic vehicle bending).
- b. The longer period oscillations resulting from use of an integral term in the steering equation during powered phase flight.
- c. The attitude excursions during coast phase flight.

In addition, the various transient conditions occurring during Centaur stage flight are analyzed. These include the Atlas/Centaur separation and the Centaur ignition, steering enable, cutoff transients, and Centaur/payload separation.

Stability during powered phase flight was analyzed using the root-locus technique with actuator nonlinearities represented by describing functions. The results of a time-varying analog simulation of AC-5 were not available at the time this report was written. However, a "quick look" at simulation results shows agreement with the analytical results, as expected. Coast phase behavior was analyzed using analytical techniques as well as an analog computer simulation. Both direct analytical techniques and simulations were used in the analyses of vehicle transient behavior.

The major additions to this report over preceding reports are the discussion of payload separation and the updating of preceding analyses to reflect the AC-5 Centaur vehicle configuration. Revisions and additions have been made to material previously presented for purposes of clarification.

The results of these studies show that the flight control system provides adequate stability and control of the AC-5 configuration throughout all phases of Centaur stage flight. Analysis of the spacecraft separation system verifies its adequacy for vehicles AC-5 and on, using presently contemplated spacecraft parameters. However, a tighter (than present spacecraft design envelopes allow) restriction on the allowable Surveyor center-of-gravity location is necessary to meet dynamic constraints. This envelope is described in Section 3.5.

January 1965

FOREWORD

This report presents the results of the dynamic stability and control system analyses performed to verify the integrity of the Centaur vehicle flight control system and the Atlas/Centaur and Centaur/Spacecraft separation systems for the AC-5 mission. These analyses were performed without benefit of AC-4 flight test results and do not include the effects of any changes that may be made to the AC-5 configuration as a consequence of these results. The studies were conducted and the results published under Contract NAS3-3232, Redirection of the Centaur Program, in accordance with General Dynamics/Astronautics Report 55-00207D, Vehicle Dynamics Reports (Item 61).

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SECTION 1

INTRODUCTION

The Atlas/Centaur AC-5 vehicle is the fifth high energy space vehicle in the Centaur research and development program. This report covers the control dynamic analysis of Centaur stage flight from Atlas SECO/VECO to unstabilized flight on a simulated lunar impact trajectory. Atlas stage analysis is covered in a separate report (Reference 1), and only a summary of the results is included herein for reference purposes.

1.1 MISSION DESCRIPTION. Atlas 156D/Centaur 6C will be launched from Complex 36A, Eastern Test Range (ETR), on a simulated lunar direct ascent trajectory.

1.1.1 Flight Test Objectives. Test objectives of the AC-5 mission include, but are not limited to, the following:

- a. Demonstrating the spacecraft separation system and the capability of Centaur to perform the retro-maneuver.
- b. Demonstrating the performance of the Centaur propellant utilization system.
- c. Demonstrating the launch-on-time capability of the Atlas/Centaur vehicle.
- d. Demonstrating the operation of the 165,000-pound thrust Atlas booster engines.
- e. Obtaining data on the closed-loop operation and performance of the Centaur guidance system.
- f. Obtaining data on the inflight environment of the spacecraft.

1.1.2 Mission Parameters. The AC-5 vehicle will be used in a simulated launch-on-time experiment with the launch time biased so that an intentional lunar miss will occur. The AC-5 vehicle after liftoff rolls to a launch point azimuth between 90 and 111 degrees. The vehicle will pitch downrange and inject the Centaur vehicle into a lunar transfer trajectory with Centaur MECO at an altitude of 90 nautical miles. After a short coast phase, the Surveyor dynamic model is separated, following which the Centaur vehicle executes a retromaneuver. The trajectory and guidance design is such that if launch occurred at the proper time, and within the launch window, lunar impact of the Surveyor dynamic model would result.

1.2 STRUCTURAL DESCRIPTION. The AC-5 vehicle consists of Atlas LV-3C space booster 156D (EID 55-0502-6), Centaur space vehicle 6C (EID 55-0501-6), and a Surveyor Dynamic Model (SD-1). Figure 1.2-1 is a cutaway view of the Centaur vehicle configuration as mated to the interstage adapter at launch. A general description of both stages follows.

1.2.1 Atlas Stage. The LV-3C is a modified D-series Atlas space booster. The major structural modifications are the substitution of a 10-foot diameter cylindrical tank section for the Atlas liquid oxygen tapered tank section and the attachment of an interstage adapter.

The tank configuration consists of two cylindrical tanks separated by an intermediate bulkhead (no insulation). The Atlas liquid oxygen tank contains a baffle cage consisting of 11 ring baffles 10 to 11 inches apart.

The Atlas and Centaur stages are joined by an interstage adapter permanently attached to the first stage. The Centaur vehicle is released from the interstage adapter by a flexible linear-shaped charge attached around the outer periphery of the adapter just below the adapter attachment points. Eight retrorockets at the base of the Atlas fuel tank provide the force to back the Atlas away from Centaur.

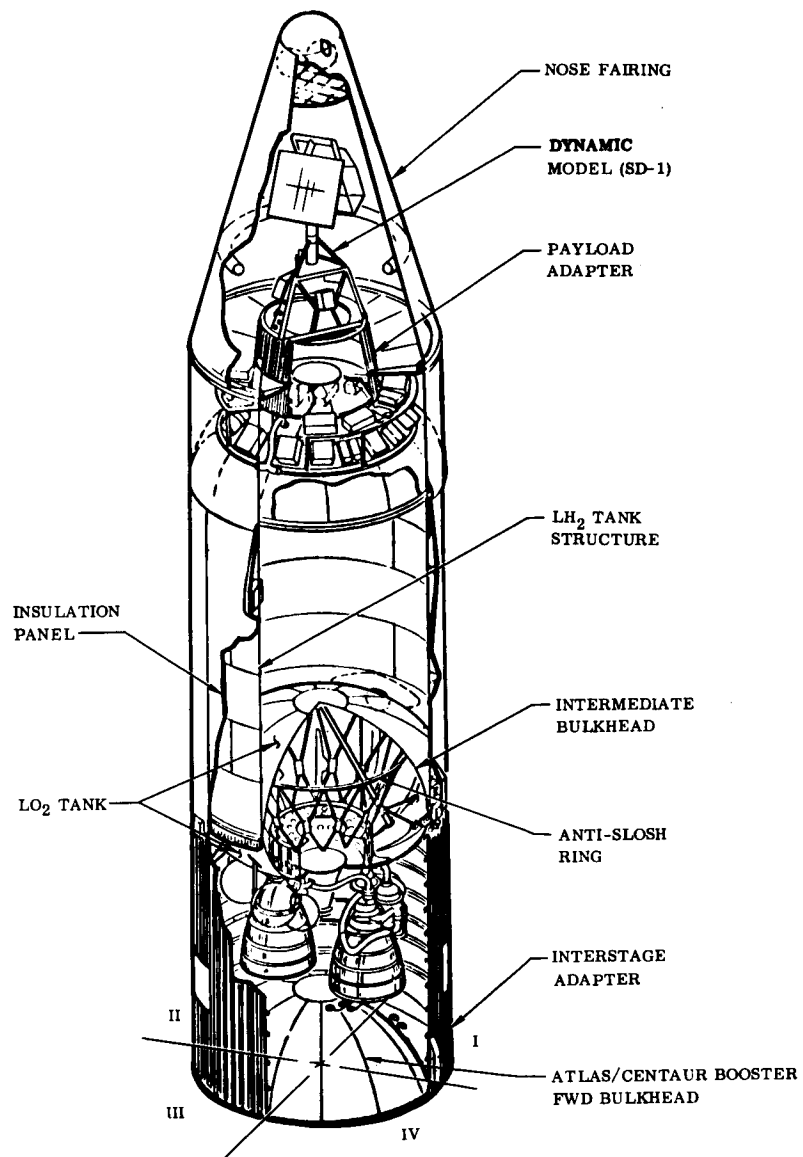


Figure 1.2-1. 6-C (AC-5) Centaur Vehicle Structural Configuration

The Atlas utilizes a Rocketdyne MA-5 propulsion system, which burns a liquid oxygen/RP-1 propellant combination. The system consists of two booster thrust chambers, each rated at 165,000 pounds nominal thrust at sea level; a single sustainer thrust chamber of 57,000 pounds nominally rated sea level thrust; and two vernier thrust chambers, each having a nominally rated sea level thrust of 1,000 pounds. The two booster engines used baffled injectors. All engines employ hypergolic ignition. The booster engine thrust level represents an uprating from previous Atlas/Centaur vehicles.

During the booster phase of Atlas stage flight, the two booster engines are gimballed to provide pitch, yaw, and roll control. The sustainer engine is nulled and the verniers are active in roll. After jettisoning the booster package, which consists of the two booster engines and the associated hardware, the sustainer engine is gimballed to provide pitch and yaw control, and the verniers provide roll control. During booster package jettison, the verniers are active in pitch and yaw, as well as roll.

1.2.2 Centaur Stage. The 6-C Centaur vehicle is a 10-foot diameter, balloon-type, pressure stabilized structure. The liquid oxygen and liquid hydrogen tanks are separated by a structural and insulation bulkhead. The structure is made of 301 CRES of 0.016-inch nominal skin gauge. A lightweight baffle is carried within the oxidizer tank to suppress propellant sloshing. A dynamic model is carried as payload, and is separated from the vehicle. These are the major structural differences between the 6-C and the 4-C (AC-4) Centaur vehicles.

The vehicle utilizes two Pratt & Whitney RL 10A-3 main engines, burning liquid oxygen and liquid hydrogen and having a nominally rated thrust (at altitude) of 15,000 pounds each. These engines are gimballed to provide pitch, yaw, and roll control during Centaur powered flight. During coast phase flight, attitude control is provided by a series of six hydrogen peroxide (monopropellant) jets. Figure C-1, in Appendix C, illustrates the jet location. Two 3-pound jets, radially directed, provide control in pitch; and four 1.5-pound jets, tangentially directed, provide yaw and roll control. These comprise the coast phase attitude holding system. Figure 1.2-2 shows the Centaur vehicle coordinate system used in the description of vehicle motion in this report.

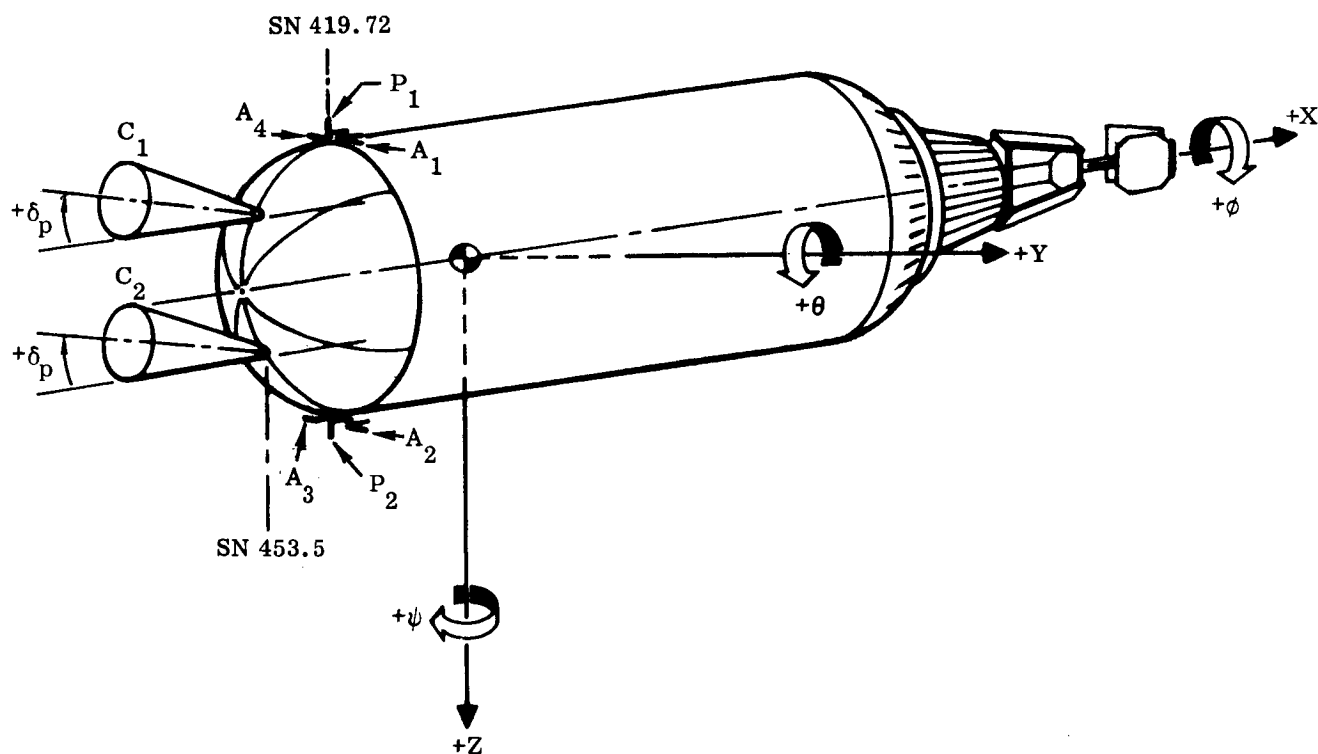
1.3 FLIGHT DESCRIPTION. The Atlas/Centaur AC-5 will fly a trajectory similar to that of the operational, lunar direct ascent trajectories.

1.3.1 Atlas Stage. The Atlas/Centaur rises vertically for the first 15 seconds after liftoff, which is defined as the time ($t = 0$) of 2 inches of vertical motion off the launcher. From 4 seconds to 12 seconds of flight, the vehicle executes the roll program designed to orient the vehicle pitch plane to the proper azimuth as determined by the launch date. From 15 seconds after liftoff to approximately 145 seconds, the Atlas/Centaur executes the predetermined pitch program. Booster phase flight is terminated by a booster staging discrete issued by Centaur guidance, with backup provided by two staging accelerometers.

Eight seconds after BECO, the Centaur guidance signals are admitted to the Atlas stage autopilot, with the guidance system operating in a closed-loop configuration throughout sustainer phase flight. The Centaur insulation panels and the nose fairing are jettisoned during sustainer phase flight. SECO/VECO is initiated by propellant depletion, which also initiates the Centaur electromechanical timer. During the next 2 seconds, the Atlas programmer energizes the electrical disconnect, fires the flexible linear shaped charge (FLSC), and fires the eight retrorockets to back Atlas away from Centaur.

Atlas flight sequencing is given in Table 1.3-1.

1.3.2 Centaur Stage. The hydraulic recirculating pump is activated 1/2 second after SECO/VECO, causing the Pratt & Whitney thrust chambers to move toward null. Main engine start (MES) is initiated 8 seconds later by the Centaur programmer. The total time of the separation sequence is 8.5 seconds, as compared with 9.6 seconds for AC-4. This represents the operational sequence.



NOTES

1. THE ORIGIN OF THE COORDINATE AXES (X, Y, Z) IS ON THE VEHICLE GEOMETRIC CENTERLINE AT THE LONGITUDINAL CENTER OF GRAVITY LOCATION.
2. THE CONTROL CONVENTION SIGNS ARE SUCH THAT A POSITIVE DISPLACEMENT OF THE ENGINE (δ_p , δ_y , δ_r) WILL RESULT IN A POSITIVE DISPLACEMENT OF THE MISSILE (θ , ψ , ϕ).
3. THE REFERENCE POINT FOR LONGITUDINAL STATION NUMBERS (SN) IS 162 INCHES FORWARD OF THE FORWARD BULKHEAD OF THE CENTAUR VEHICLE.

Figure 1.2-2. Centaur Vehicle Coordinate System

Table 1.3-1. Atlas Stage Flight Sequencing*

EVENT	START (sec)	END (sec)
Guidance to Flight Mode	A - 8.0 (Nominal)	---
Unnull Atlas Displacement Gyros	Engine Start A - 3.37 (Nominal)	---
Programmer Starts Subroutine A (A = 0)	2 inches of Rise	B + 0
Unnull Atlas Servo Integrators	8 inches of Rise	B + 1.0
Activate Autopilot Control (Booster P, Y, R; Vernier R)	42 inches of Rise	---
Roll Program	A + 4	A + 12
Pitch Program	A + 15	---
LO ₂ Tank Pressure Change	A + 20	---
Filter Response Change (P, Y)	A + 40	---
Open LH ₂ Vent Valve Command	A + 70	A + 71
Displacement Gyro Gain to Low (P, Y)	A + 110	---
Rate Gyro Gain to Low (P, Y)	A + 110	B + 0
Enable Staging Sequence	A + 135	B + 0
Staging (STG)	Staging Discrete	C + 0
Programmer Starts Subroutine B (B = 0)		
Booster Engine Cutoff (BECO)	B + 0	---
Close LH ₂ Vent Valve Command	B + 0	B + 1.0
Activate Sustainer Control (P, Y)	B + 0	B + 3.0
Activate Vernier Control (P, Y)	B + 0	B + 6.7
Rate Gyro Gain to High (P, Y)	B + 0	---
Deactivate Booster Control (P, Y, R)	B + 1.0	---
Null Integrators (P, Y)	B + 1.0	B + 6.7
Deactivate Sustainer Control (P, Y)	B + 3.0	B + 3.7

* As of 8 December 1964, (Reference 2).

Table 1.3-1. Atlas Stage Flight Sequencing*, Contd

EVENT	START (sec)	END (sec)
Jettison Booster Package	B + 3.1	---
Bias Verniers to 40 deg (Y)	B + 6.7	---
Deactivate Vernier Control (P, Y)	B + 6.7	---
Open LH ₂ Vent Valve Command	B + 7	B + 8
Admit Guidance Steering (P, Y)	B + 8	---
Disable Displacement Gyros (P, Y)	B + 8	---
Jettison Insulation Panels	B + 30	---
Unlatch Nose Fairings	B + 66.5	---
Fire Thruster Bottles	B + 67	---
Start Centaur Boost Pumps	B + 77	---
Enable Separation Sequence	B + 88	C + 0
Separation Sequence	Propellant Depletion	---
Programmer Starts Subroutine C (C = 0)		
Sustainer Engine Cutoff (SECO)	C + 0	---
Vernier Engine Cutoff (VECO)	C + 0	---
Start Centaur Electromechanical Timer	C + 0	---
Energize Electrical Disconnect	C + 0.1	---
Fire Shaped Charge	C + 1.9	---
Fire Retrorockets	C + 2.0	---
Reset Programmer	C + 30	---

* As of 8 December 1964, (Reference 2).

The Centaur engines fire for approximately 7 minutes, the exact time of cutoff being determined by guidance. During this time, the vehicle's attitude reference is supplied by the Centaur guidance system. The guidance vector is nulled at MES and re-admitted 4 seconds later after the transients produced by main engine ignition have passed. At this time (MES + 4) the vehicle undergoes a pitch transient, the size of which largely depends upon the change in vehicle attitude between Atlas SECO/VECO and MES + 4. The change in the orientation of the reference vector over this time interval is essentially zero.

Following main engine cutoff (MECO), the hydrogen peroxide attitude control system is activated and the vehicle rates produced by engine shutdown are reduced to within the threshold levels of the system. The dynamic model is then separated, following which a guidance vector is admitted to the autopilot, causing the vehicle to turn around so it points back along the flight path. A prestart sequence is initiated, causing the residual propellants to be vented through the engine bells and cooldown valves, producing a small amount of thrust. This thrust backs the Centaur away from the dynamic model (the retromaneuver). The controlled portion of flight ends some time later when the tank vents are enabled and the attitude control system is shut down.

The Centaur flight sequencing is given in Table 1.3-2.

Table 1.3-2. Centaur Stage Flight Sequencing *

EVENT	START (sec)	END (sec)
Booster Staging	Guidance Discrete	---
Start Boost Pumps	Atlas Discrete	MECO
Start Programmer	C + 0	---
Close LO ₂ Vent Valve Signal	C + 0.2	C + 0.5
Close LH ₂ Vent Valve Signal	C + 0.2	C + 0.5
Pressurize LO ₂ Tank	C + 0.2	C + 1.0
Pressurize LH ₂ Tank	C + 0.2	C + 1.0
Start Hydraulic Recirculating Pump	C + 0.5	C + 12.5
Propellant Utilization Segmented Signal	C + 0.5	C + 3.5
Open LH ₂ Boost Pump Accelerating Valve	C + 1.0	C + 12.5
Propellant Utilization Null Signal	C + 3.5	C + 98.5
Prestart	C + 3.5	MECO
Steering Reference to Centaur ϕ A	C + 3.5	---
Start Main Engines (MES)	C + 8.5	MECO
Energize Igniters	C + 8.5	C + 12.5
Unnull Main Engine Integrators	C + 8.5	C + 468.4
Low Rate Gain	C + 8.5	MECO
Admit Guidance for Steering	C + 12.5	MECO
Propellant Utilization Segmented Signal	C + 333.5	C + 450.5
Enable Main Engine Cutoff	C + 403.5	C + 450.5
Main Engine Cutoff (MECO)	Guidance Discrete	---
Main Engine Cutoff Backup	C + 450.5	---
High Rate Gain	MECO	---

* As of 8 December 1964, (Reference 2).

Table 1.3-2. Centaur Stage Flight Sequencing *, Contd

EVENT	START (sec)	END (sec)
Inhibit Guidance	MECO	C + 515.5
Activate H ₂ O ₂ Attitude Control System	MECO	---
Low Displacement Gain	MECO	---
Enable Open LH ₂ Vent Valve Signal	C + 450.5	C + 451
Propellant Utilization Null	C + 450.5	---
Safe Surveyor Destruct	C + 451	---
Pre-Separation Arming	C + 468.5	C + 499.5
Extend Landing Gear ¹	C + 468.5	C + 468.6
Null Integrators	C + 468.5	---
Unlock Omni Antenna ¹	C + 479	C + 479.1
Turn on High Power Transmitter ¹	C + 499.5	C + 499.6
Spacecraft Electrical Disconnect	C + 505	---
Switch Guidance/Spacecraft Telemetry Channels ¹	C + 505	---
Separate Spacecraft	C + 510.5	---
Admit Guidance for Attitude Control (Begin Turnaround Maneuver)	C + 515.5	---
Prestart (Begin Retrothrust)	C + 715.5	---
Calibrate Telemetry	C + 1091.5	C + 1094.5
Open LO ₂ Vent Valve	C + 1615.5	---
Power Off	C + 1616.5	---

Notes: * As of 8 December 1964, (Reference 2).

¹ Autopilot signal only.

SECTION 2

FLIGHT CONTROL SYSTEM DESCRIPTION

The Atlas/Centaur flight control system provides the reference attitude (or rate) and maintains the vehicle at this attitude (or rate). Throughout the powered flight of both stages, thrust-vector control maintains vehicle attitude. This is also true of the coast phase if the usual definition of thrust-vector control is extended to include the control provided by fixed jets in an on-off mode of operation.

The AC-5 flight vehicle has closed-loop guidance during sustainer phase of Atlas stage flight and throughout Centaur stage flight. In addition, the guidance system provides the booster staging discrete and starts subroutine B in the Atlas programmer. The Centaur electromechanical timer is initiated by the Atlas programmer. With these three exceptions, the flight control systems of the Atlas and Centaur stages are entirely independent. A brief description of the flight control systems follows; more detailed information is presented in References 3, 4, 5, 6, and 7.

2.1 ATLAS STAGE. Thrust vector control of the booster, sustainer, and vernier engines during Atlas stage flight is provided by electrohydraulic actuators that gimbal the thrust chambers. The actuators are in turn controlled by the servoamplifier outputs. Signals from rate and displacement gyros are summed in a displacement gyro package whose output drives the servoamplifier. The sequencing of the whole system, including pitch program and gain changes, is provided by the Atlas programmer. These components constitute the Atlas stage flight control system and are described individually in more detail below.

2.1.1 Programmer Package and Sequencing. The Atlas programmer provides a time base and a preset switching sequence for subsystems in the vehicle. The autopilot gain changes and the roll and pitch programs are control functions of the programmer. In addition, discrettes that cause panel and nose fairing jettison, sustainer engine cutoff, separation sequence, and initiation of the Centaur programmer sequencing are provided.

Certain other discrettes are provided by separate systems. The autopilot is activated and the Atlas programmer started as a function of vehicle rise from the launcher. The booster staging discrete is provided by Centaur guidance which also starts subroutine B (see Table 1.3-1) in the Atlas programmer. Atlas SECO/VECO and initiation of subroutine C is provided by the propellant depletion discrete. In addition, the other discrettes are provided as backup signals to the programmer. For example, backup accelerometers ensure that the axial loading due to g-level prior to BECO does not exceed allowable limits.

2.1.2 Displacement Gyro Package. The Atlas displacement gyro package is mounted at approximately Station 990 in the B-1 pod. This package contains the displacement gyros, gyro torquer amplifiers, and gyro signal amplifiers for all three channels (pitch, yaw, and roll). The signal amplifiers sum the rate and displacement gyro signals or the rate gyro and Centaur guidance signals, and have provisions for changing autopilot gains during flight.

2.1.3 Rate Gyro Package. The Atlas rate gyro package is mounted at Station 600 on the side of the Atlas liquid oxygen tank. This location was chosen for reasons of flight control system stability; i.e., it is just forward of the first bending mode antinode during the booster phase of Atlas flight. Rate gyros for pitch, yaw, and roll channels are included in the forward rate group. A Centaur-type (overdamped) rate gyro is used in the roll channel to eliminate the need for roll channel filtering.

2.1.4 Filter-Servoamplifier Package. This package contains the autopilot filters, which stabilize the flight control system; the autopilot integrators; and the servoamplifiers which provide control signals to the booster, sustainer, and vernier engine actuators. Provisions are made for changes in the integrator and/or filter characteristics during flight.

The autopilot integrator ensures that the error signal has a steady state value of zero, thus ensuring a zero steady state attitude error to the vehicle. In effect, the integrator assumes the output voltage to point the net thrust vector through the vehicle center of gravity under steady state conditions. Thus, the effects of thrust misalignment, rigging errors, and lateral offsets of the vehicle center-of-gravity on the vehicle attitude are removed.

The servoamplifier receives signals from the integrator, autopilot-stabilization filter, and actuator position feedback transducer. It produces a signal to drive the servovalve on the hydraulic actuator.

2.1.5 Electrohydraulic Actuators. The actuators that gimbal each of the five Atlas thrust chambers consist of three major components: the actuator cylinder, a servovalve, and a feedback transducer. Signals from the servoamplifier command the servovalve to open, generating a hydraulic flow into one side of the actuator cylinder and out the other. This flow also depends upon the load pressure required to cause the actuator piston to move with respect to the case (which causes engine motion). The motion is measured by the feedback transducer, which generates an electrical signal that is fed back to the servoamplifier. Thus, the servoamplifier input signals (less the transducer signal) correspond to a commanded engine position.

The actuators that gimbal the booster and sustainer engines employ flow-control servovalves with leakage orifices in the actuator pistons. These leakage orifices provide damping to the servoloop.

2.2 CENTAUR STAGE. Thrust-vector control of the two main engines during Centaur stage flight is provided by electrohydraulic actuators which gimbal the thrust chambers. The actuator is in turn driven by signals from the servoamplifier package. This package also provides the hydrogen peroxide jet on-off signals during coast phase. Position signals from the inertial platform resolver chain are summed with rate gyro signals to produce the autopilot error signal that drives the servoamplifier. The input to the resolver chain is provided in the form of guidance originated vectors. The sequencing of the overall system is provided by a timer. These components constitute the flight control system and are described in more detail below. Figure 2.2-1 illustrates the location of the autopilot and guidance system components. The block diagram of the autopilot is given in Figure 2.2-2.

2.2.1 Electromechanical Timer. The Centaur electromechanical timer has a function similar to the Atlas programmer in that it provides a preset switching sequence. In addition, it accepts signals from the Atlas programmer and the guidance computer, and by relay logic performs switching functions on the Centaur vehicle. The guidance vectors are fed through the electromechanical timer, which acts as a junction box, to the resolver chain. The timer provides the command signal to start and stop the main engines and attitude control jets. It switches the autopilot configuration at the appropriate times during the powered and coast phases of flight. The sequencer in the electromechanical timer is started by a signal received at Atlas propellant depletion from the Atlas programmer.

2.2.2 Guidance Set. The Centaur guidance system will be flown closed loop on AC-5. The system provides certain discretized and steering commands to the autopilot.

2.2.2.1 Guidance Platform. The guidance platform maintains an inertial reference for the measurement of vehicle accelerations in inertial coordinates. The resolver chain, mounted on the gimbals of the platform, transforms the position reference vector (in inertial coordinates) to vehicle coordinates. Physically, three guidance computer voltages are impressed on the resolver chain, and two are read out as the components of the reference vector along the pitch and yaw axes of the vehicle. The third voltage is shorted to ground through a loading resistance.

Mounted on the platform is a set of three pulse-rebalance accelerometers. These measure the accelerations of the vehicle in inertial space. The acceleration measurements are used by the computer to determine the location, velocity, and acceleration of the vehicle, as well as the steering command.

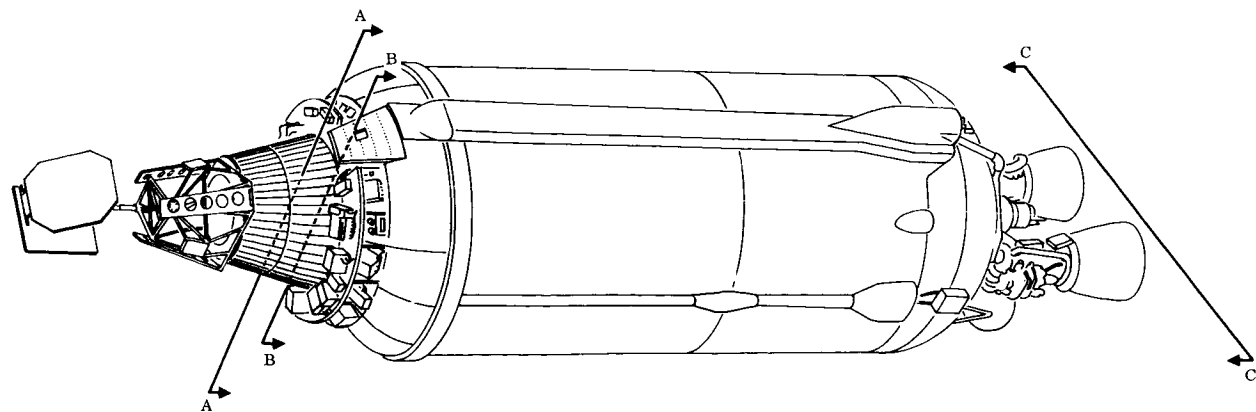
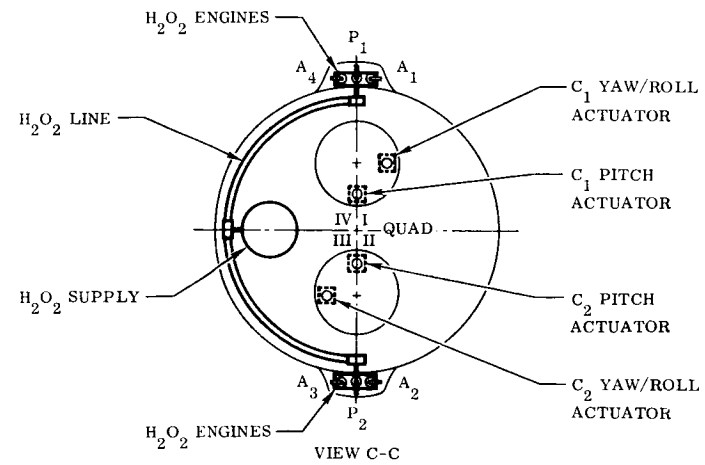
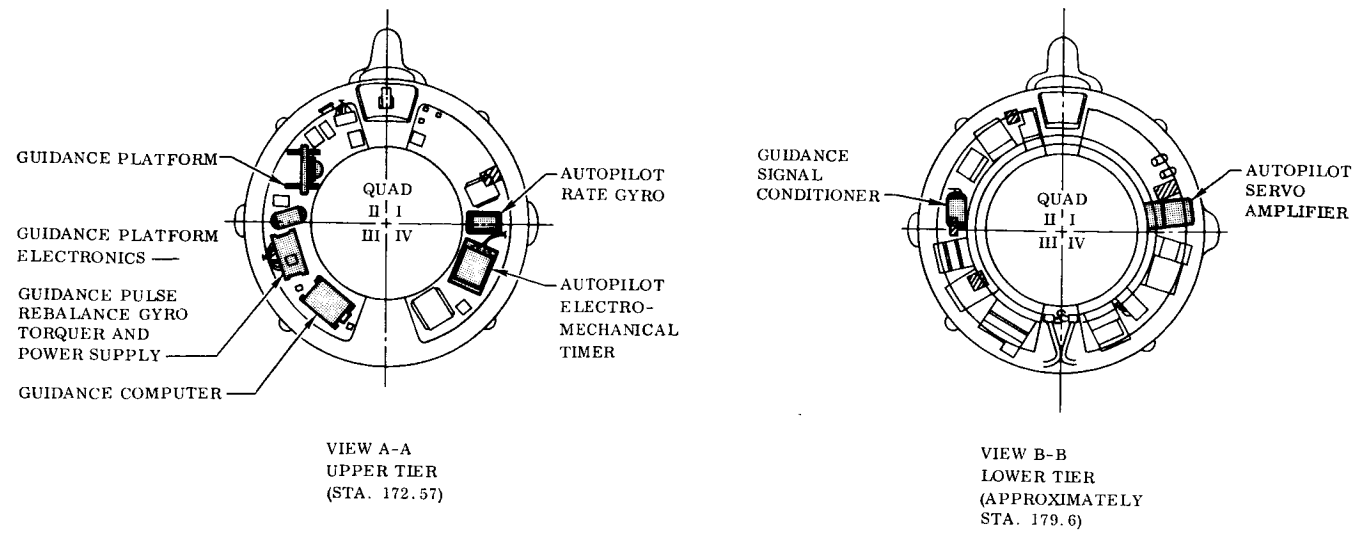
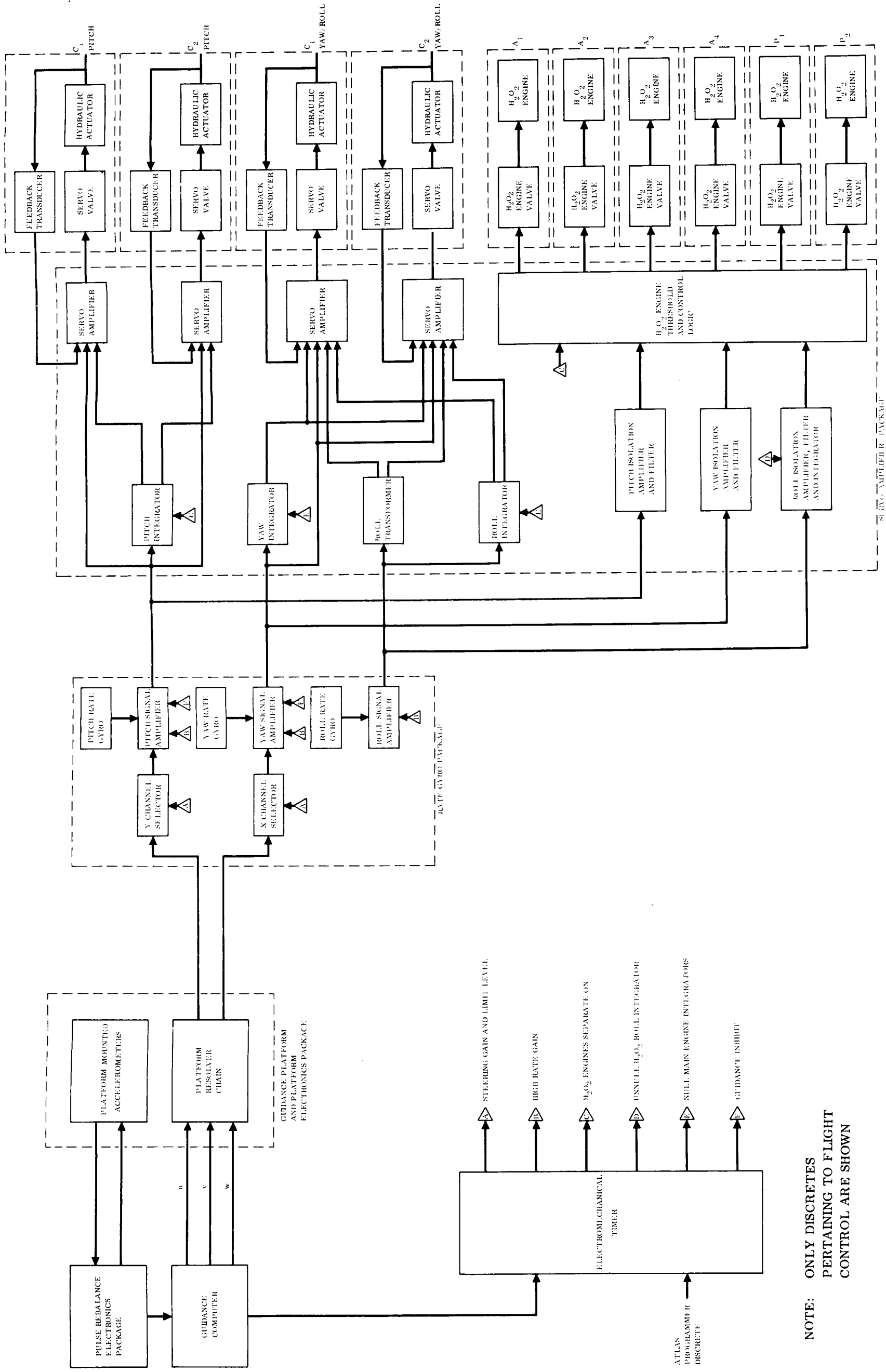


Figure 2.2-1. Location of Centaur Autopilot and Guidance System Components



NOTE: ONLY DISCRETES PERTAINING TO FLIGHT CONTROL ARE SHOWN

Figure 2.2-2. Centaur Autopilot Block Diagram

Three gyros are also mounted on the platform to detect platform rotation. Signals from these gyros control the platform gimbal positions.

2.2.2.2 Platform Electronics Package. This package provides the switching for caging and aligning platform gimbals and also amplification and isolation of the signals transmitted through the resolver chain. The platform-mounted gyro error signals are used to produce the platform gimbal-torquing signals.

2.2.2.3 Pulse Rebalance Electronics Package. This package provides signals to rebalance (null) the platform-mounted accelerometers. These signals are supplied to the computer as a measure of the incremental inertial velocities. The package is driven by the accelerometer error signals.

2.2.2.4 Guidance Computer. The guidance computer uses the pulse rebalance signals to compute the attitude reference direction. In addition, gyro compensation signals are fed through the pulse rebalance electronics package to the platform gyro torquers, and discretes are fed to the electromechanical timer.

2.2.3 Rate Gyro Package. The rate gyro package is mounted on the forward equipment shelf, together with the inertial platform, at approximately Station 173. The rate gyros for pitch, yaw, and roll channels are of the nominally overdamped Centaur type whose characteristics are:

- a. Natural frequency ~ 15 to 23 cps.
- b. Damping ratio ~ 0.77 to 2.98 (nondimensional).

These characteristics result in a nominal, equivalent first-order lag time constant of $\tau_R = 0.03$ second, although testing of 12 Centaur rate gyros before the flight of AC-2 indicated a better value to be $\tau = 0.025$ second.

2.2.3.1 Channel Selector. The channel selector inputs are the two resolver chain output voltages corresponding to the components of the reference attitude vector along the pitch and yaw axes. These form the positive yaw and negative pitch rate commands within the channel selector. Hence the output of the channel selector corresponds to a position error. The channel selector also limits the error signal, thus limiting the commanded rate.

2.2.3.2 Signal Amplifier. The rate gyro and channel selector signals are summed in the signal amplifier to produce the autopilot error signals. Autopilot gain changes are made within the signal amplifier and channel selector.

2.2.4 Servoamplifier Package. The Centaur servoamplifier package has essentially the same function as the corresponding Atlas package. The autopilot error signal is integrated by the autopilot integrator within the package. The integrator output, the error signal, and the engine feedback transducer signals are summed in the servoamplifier to produce the servovalve command signal. No filtering is required for autopilot stability because of the large lag already existing within the engine-gimbaling servo and the rate gyro.

Additionally, the autopilot error signals are used to generate the on and off commands to the hydrogen peroxide jets used for attitude and rate control during coast phase flight. This circuitry contains a built-in hysteresis and filter to mitigate the effects of signal noise.

2.2.5 Electrohydraulic Actuators. The actuators that gimbal each of the two Pratt & Whitney RL10A-3 engines, for pitch, yaw, and roll control during powered flight, consist of three major components as do the Atlas stage actuators: actuator cylinder, servovalve, and feedback transducer. Operation is the same. Signals from the servoamplifier command the servovalve to open, generating a flow into one side of the actuator cylinder and out the other. This flow is dependent upon load pressure opposing motion as well as valve opening. The motion of the actuator piston with respect to the case (and hence engine

motion) is measured by the feedback transducer, and the resulting signal is fed back to the servoamplifier input. The sum of the servoamplifier inputs, less the transducer signal, constitutes the commanded engine position.

The actuators used on the Centaur employ derivative load pressure feedback servovalves, and have no leakage orifices in the actuator pistons. This configuration gives a relatively large stiffness to the static loads such as caused by the engine center-of-gravity offset. These servovalves are described more fully in Appendix B. Briefly, they employ within them a hydraulic feedback that causes the valve to act as a flow-control valve at low frequencies and as a pressure-feedback valve at high frequencies. This valve action, coupled with the low actuator loop gain greatly attenuates engine motion at high frequencies but without the hydraulic power consumption associated with a leakage orifice. The major parameters of the electrohydraulic engine-actuator gimbal system are listed in Table E-2.

2.2.6 Hydrogen Peroxide Jets. The Centaur vehicle has six hydrogen peroxide jets for attitude holding during coast phase flight. Signals from the servoamplifier package energize solenoids. These in turn open the valves to release hydrogen peroxide into the jet combustion chambers. Here a catalyst causes decomposition of the hydrogen peroxide. This results in steam and oxygen which produce thrust by exhausting out the jet nozzle. Switching logic to determine which jet fires, and when, is incorporated in the servoamplifier package. The location of these jets is shown in Figure C-1. The jets themselves have certain response characteristics that influence the behavior of the vehicle during coast phase flight. These characteristics are depicted in Figures C-4 and C-5. They include delay in energizing the solenoid, opening the valve, etc., and are taken from the results of recent tests.

2.2.7 Autopilot Parameters. The overall flight control system has a configuration similar to that of Centaur 4C (AC-4), with one exception: the electronic programmer used on AC-4 is replaced by an electromechanical timer. The sequence is given in Table 1.3-2.

Table 2.2-1. Centaur Stage End-to-End Autopilot Gains (See Figure B2-4; equations in Appendix B2).

END-TO-END GAIN	PITCH AND YAW	ROLL	TOLERANCE (Percent)
A. POWERED PHASE FLIGHT			
$k_p K_A$ (position) (δ/θ)	0.0 MES TO MES + 4 0.24 MES + 4 TO MECO	0.0	± 15
$K_A K_R$ (rate) ($\delta/\dot{\theta}$)	0.32	0.11	± 12
$K_A K_R K_I$ (integral-rate) ($\delta/\ddot{\theta}$)	0.25	0.25	± 30
B. COAST PHASE FLIGHT			
$k_p K_A$ (position) (δ/θ)	0.15	0.0	---
$K_A K_R$ (rate) ($\delta/\dot{\theta}$)	1.25	0.43	---
$K_A K_R K_I$ (integral-rate) ($\delta/\ddot{\theta}$)	0.98*	0.98*	---

*Until nulled by a command from the programmer (nulled prior to MES).

2.2.7.1 Autopilot Gains. Between MES and MECO, the Centaur autopilot operates in its powered phase configuration. The end-to-end gains given in Table 2.2-1, Part A, assume an 11.05-volt absolute value for the reference vector provided by the programmer. Note that the integral gain is specified by an end-to-end integral-rate gain, because of the absence of a roll position reference indicated by $k_p = 0$ in the roll channel. Roll attitude is maintained during powered phase flight by the autopilot integrator.

2.2.7.2 Coast Phase Parameters. Prior to MES and after MECO, the autopilot is in a coast phase configuration. The characteristics and locations of the attitude control jets are described in Section 2.2.6. The autopilot parameters of interest here are: the error signal thresholds above which one or more attitude control jets will be commanded to fire; the roll integrator characteristics; the filtering of the error signal; the hysteresis in the command circuitry; and the composition of the error signal (ratio of position-to-rate information).

The autopilot is switched to a coast phase configuration by the Centaur electromechanical timer or by guidance. This involves electrical gain changes within the channel selector and gyro signal amplifiers, as well as nulling of the autopilot integrators. The operation of the coast phase attitude control is outlined in Appendix C.

The coast phase autopilot design parameters are summarized in Table 2.2-2, assuming a position reference vector magnitude of 11.05 volts as input to the resolver chain.

Table 2.2-2. Coast Phase Autopilot Parameters (See Appendix C).

CHANNEL	PITCH	YAW	ROLL
Rate Threshold, $T_{c 1.5, 3}$ (deg/sec)	0.2	0.2	0.2
Integrator Gain, K_I (sec ⁻¹)	-	-	0.12
Integrator Limit (deg)	-	-	16.1
Integrator Limit (deg/sec)	-	-	1.95*
Rate Hysteresis, h (deg/sec)	0.02	0.02	0.012
Filter Time Constant, τ (sec)	0.1	0.1	0.00825
"Rate Gain", K'_R (sec)	8.0	8.0	8.25
Channel Selector Limit, η_c (deg/sec)	1.6	1.6	-
*Approximate			

It should be noted that the main thrust chambers will be gimbaled during coast phase flight if there is hydraulic power available. This is because the servoamplifiers are still active and are receiving signals from the servovalves. This is of consequence prior to MES, when the recirculation pump is active, and after MECO, while the hydraulic power and engine thrust are decaying to zero. The end-to-end gains that apply during this portion of flight are given in Table 2.2-1, Part B.

SECTION 3

FLIGHT DYNAMICS ANALYSIS

The results presented in this section concern the AC-5 vehicle flight dynamics and stability characteristics from liftoff to uncontrolled flight in a simulated lunar impact trajectory. A brief description of the Atlas-stage flight dynamics is followed by a discussion of the following five phases of Centaur flight:

- a. Atlas/Centaur separation.
- b. Main engine ignition.
- c. Powered-phase flight.
- d. Coast-phase flight.
- e. Spacecraft separation.

All phases are discussed in sufficient detail to demonstrate the design integrity of the AC-5 flight control system configuration.

3.1 ATLAS STAGE FLIGHT. A complete analysis of the AC-5 configuration during Atlas-stage flight was unavailable at the time this report was being written. The following discussion is therefore based upon extrapolation from the AC-4 analysis. This procedure is justified since the two major changes, uprated booster engines and reduced autopilot gains during booster phase, compensate for one another, resulting in stability properties which are more-or-less unchanged from the AC-4 configuration. The booster phase will be somewhat shorter, and the sustainer phase somewhat longer due to the use of the uprated engines and propellant depletion cutoff at SECO/VECO.

3.1.1 Launch Phase. Two major areas associated with the Atlas launch phase are of concern: launch drift and longitudinal oscillations following liftoff.

3.1.1.1 Launch Drift. For the first 15 seconds of flight, the Atlas/Centaur is programmed to rise vertically. A true vertical ascent would maintain initial clearances between the vehicle and the umbilical tower. However, because of such factors as ground winds, vehicle initial misalignment in the launcher, thrust vector misalignments, launcher release forces, and gyro null errors, the vehicle will drift laterally with respect to the launch pad as it rises. Since this drift is uncontrolled, it is necessary to examine the clearances between the vehicle and the umbilical tower structure, including the umbilical booms, as the vehicle rises.

A "worst case" analysis of the AC-4 launch drift was undertaken, making use of a detailed analog simulation. Three critical clearances exist other than the initial clearances of the vehicle from the umbilical booms. These clearances are between the booster engine skirt and the lower umbilical boom, the upper umbilical boom, and the umbilical-tower hydrogen vent stack. These points of closest approach are approximately 8.9, 3.4, and 5.4 feet at 4.9, 5.3 and 5.8 seconds after liftoff, assuming the worst case conditions where the vehicle drifts toward the umbilical tower under the assumed additive effects of the parameters producing drift. It should be noted that the AC-5 configuration with its higher thrust will rise faster, giving the vehicle less time for lateral drift. This will result in even greater minimum lateral clearances than indicated here; therefore, there is no risk of collision between the vertically rising Atlas/Centaur and the umbilical booms and/or tower structure.

3.1.1.2 Longitudinal Oscillations Following Liftoff. The Atlas D-series launcher mechanism produces forces of sufficient magnitude to excite a structural longitudinal oscillation in the vicinity of six cycles/second. This oscillation, occurring immediately after vehicle liftoff, has been evidenced in numerous Atlas flight records and is known to be caused by coupling between the Atlas LO₂-tank helium-pressurization

system and the elastic structure of the vehicle. Extensive analysis has been made of this phenomenon in connection with other programs. Using the techniques developed for these programs, the AC-5 configuration is being analyzed for the possibility of longitudinal oscillations after liftoff. The mathematical model includes a spring mass approximation to the vehicle longitudinal dynamics, the ullage-volume thermodynamics (pressure-temperature-volume relationship), the pressure sensing line dynamics, the pneumatic regulator characteristics, and the helium supply duct dynamics. The effects of the Hadley pneumatic regulator nonlinearity (hysteresis and flow limiting) are included in the analysis. This analysis is not expected to show behavior which differs greatly from that of AC-4. A maximum possible limit cycle in sensed ullage pressure (Atlas LO₂ tank) with initial value of 2.2 psi peak-to-peak and ceasing by 35 seconds of flight time was predicted for AC-4. The precise behavior depends on the launcher release forces, which at present are only qualitatively known.

3.1.2 Booster Phase. The booster phase of flight begins with autopilot activation at launcher release (42 inches of rise) and ends with booster engine cutoff (BECO). The primary flight control system objective is to maintain adequate stability in the presence of parasitic modes of oscillation while traversing the denser portion of the earth's atmosphere. The high control system gains (required for adequate rigid body stability in the presence of high aerodynamic loading) aggravate the stability of flexible vehicle bending modes which have their lowest frequencies during this phase of flight. Propellant sloshing becomes significant just prior to BECO where the bending moment capability of the vehicle is lowest (high axial acceleration).

A complete stability analysis of a flexible-bodied vehicle, in which all of the interactions affecting stability are considered simultaneously, is not practical. Fortunately, the various degrees of freedom of the launch vehicle are, to a large extent, dynamically uncoupled. Thus, stability of the rigid body control (pitch, yaw, and roll) modes and propellant sloshing modes are analyzed independently of the higher frequency elastic bending modes.

3.1.2.1 Rigid-Body/Propellant Sloshing Stability. The widely used pendulum analogy is used to duplicate the forces and moments associated with propellant sloshing. Sloshing in the Centaur LH₂ tank is ignored because of negligible influence during Atlas-stage flight. Both digital (root locus) and analog (time varying simulation) techniques are used to study the stability qualities of the resulting mathematical model of the vehicle dynamics. The nonlinear character of the engine-positioning servo response results in the prediction of limit cycle behavior.

These studies indicate the rigid-body limit cycle to be dominant throughout most of booster phase and with maximum amplitude, $\delta \leq 0.05$ degree, at maximum dynamic pressure. The system is relatively insensitive to autopilot parameter variations within their tolerance bands, with the exception of booster engine gimbal friction. Analysis of flight data from several Atlas /Agena flights indicate gimbal friction values that would produce the limit cycle amplitude predicted here. However, larger values, resulting in larger limit cycle amplitudes, are possible although not expected.

Past Atlas/Centaur flight data correlates well with the predicted results; the effects of propellant sloshing are observable on the flight traces just prior to BECO. The rigid body/propellant sloshing behavior of the present configuration of AC-5 is expected to be quite similar to that of AC-4 because of the similarity in end-to-end flight control system gains (including the effect of the increased thrust level). It is presently felt that a reduction in autopilot gain during the latter part of booster phase should largely eliminate the pre-BECO sloshing.

3.1.2.2 Elastic Vehicle Bending Stability. The coupling of the flexible bodied vehicle bending modes with the flight control system is analyzed by root locus techniques. The mathematical model consists of a spring-mass representation of the elastic vehicle dynamics from which mode shapes and frequencies are obtained. The high frequency terms in the engine positioning servo dynamics are included together with the rate gyro dynamics in the system equations. The analysis is carried out at several discrete flight times.

The usual philosophy in synthesis of the control system parameters is to phase stabilize the first bending mode, relying on filter and engine position servo attenuation to gain stabilize the higher modes. In the absence of autopilot filtering, the forward rate gyro location used on Atlas/Centaur results in a first mode locus departing into the right half plane. Filter lag is used to rotate the locus clockwise into the left half plane. Then, as flight time increases, the locus continues to rotate, further increasing the stability, as the modal frequency and the filter and servo lags increase. Thus the most critical time is at launch where the AC-4 analysis shows a first mode phase margin of 55 degrees for an assumed engine limit cycle amplitude of 0.1 degree zero-to-peak. After the filter change at 40 seconds of flight time, the margin is 33 degrees for $\delta = 0.1$ degree, and increases thereafter. The higher modes are gain stabilized by factors of 2 (6 db) or greater, independent of phase. The AC-5 analysis is not expected to show any great change in the bending stability properties because of the great similarity in configuration between AC-4 and AC-5.

3.1.3 Booster Staging Phase. The AC-5 configuration has essentially the same staging sequence as the standard SLV-3 boosters. Centaur guidance initiates staging with a staging discrete when the axial acceleration of the vehicle exceeds approximately 5.7 g's. The booster engines are shut down (BECO) and centered, and the sustainer and vernier engines are activated in pitch and yaw. Shortly thereafter, the sustainer engine is centered, and the booster package jettisoned; the sustainer engine then resumes pitch and yaw control, and verniers deactivate in pitch and yaw.

The sequence produces two distinct transients which are greatest in the vehicle's yaw plane. These occur at STG (BECO) and at STG + 3.1 seconds (Booster Package Jettison). The first of these is due primarily to moments caused by the booster engines' thrust decay and the difference between the engine positions before and after BECO. An analysis of the AC-5 configuration yields an expected transient yaw rate at BECO of 0.19 ± 0.24 deg/sec (3σ) with lesser rates in pitch. This transient is smaller than the usual D-series vehicles because of the greater moment of inertia of the Atlas/Centaur configuration. The vernier engines contribute very little to the reduction of these rates, for the same reason.

The booster-package separation forces produce the major portion of the moment which causes the second transient. Booster package jettison analysis, similar to the BECO transient analysis (based on past flight experience), gives 1.25 ± 0.95 deg/sec (3σ) rates in yaw and lesser rates in pitch. Again, these rates are smaller than other D-series vehicles having similar staging sequences because of the greater inertial properties of Atlas/Centaur.

3.1.4 Sustainer Phase. Sustainer phase flight is that portion of flight after reactivation of sustainer engine gimbaling at STG + 3.7 seconds until sustainer and vernier engine cutoff (SECO/VECO). This portion of flight is characterized by relatively mild demands on the flight control system. The bending frequencies sharply increase after jettison of the booster package, resulting in gain stabilization of all bending modes. At the same time, the propellant sloshing frequencies drop, and the reduced system lag at these frequencies improves the sloshing stability. The speed of response of the autopilot is not critical, and the autopilot gains are reduced accordingly. As during booster phase, the rigid body limit cycle is predominant at an engine deflection amplitude on the order of 0.04 degree zero-to-peak.

3.1.4.1 Guidance/Autopilot Coupling. Activation of Centaur guidance at STG + 8 seconds replaces the position gyros with the Centaur platform as a position reference. Two questionable influences occur concerning vehicle dynamic stability: the influence of the inertial platform dynamics, and the influence of the sample-and-hold nature of the guidance commands.

Analysis has shown (Appendix B4) that the departure of the platform transfer function from the ideal (frequency-independent) position reference assumed in the analysis is significant for frequencies corresponding to the third and higher mode frequencies during sustainer phase. Since the attenuation of the engine positioning servo and the autopilot stabilization filter at these frequencies is so great, the practical influence of the platform dynamics on the modal stability is nil; and, in fact, use of the platform as a position reference has no influence on the system behavior at these frequencies.

The same techniques used to analyze the Centaur stage vehicle are used in the analysis of the sample-and-hold influence on the Atlas/Centaur sustainer-phase flight control system. The low frequency "guidance mode" is produced by the integral term in the steering equation. This term is not admitted until approximately 15 seconds after steering enable, at which time the transient caused by steering enable has passed. The influence of this term on the vehicle dynamic behavior should be barely perceptible on the flight traces. In any case, the magnitude of the gain associated with this term, $k_1' = 0.25$, is such as to result in a "guidance mode" frequency of 0.42 rad/sec and damping of 0.5, assuming the current compute cycle length, Δt , of 1.25 seconds.

3.1.4.2 Insulation Panel and Nose Fairing Jettison. At STG + 30 seconds, the insulation panels are jettisoned. The nose fairing is jettisoned at STG + 67 seconds. Analysis has shown that the worst input to the vehicle autopilot at panel jettison is equivalent to a step rate change of 0.04 deg/sec in pitch or yaw. This corresponds to one panel coming off, i.e., and asymmetric condition. The AC-3 flight data showed no discernible rigid-body transient at panel jettison.

Nose fairing jettison is initiated by the firing of thruster bottles mounted near the tip of each nose fairing half. The far forward location of the nose fairing and its increased mass mean a greater vehicle disturbance if only one nose fairing half should come off (worst case condition from the control standpoint). This, in turn, can result in a larger rigid body transient. A simplified analysis shows the peak engine response to be less than 0.16 degree for this condition, well within control capability margins. It should be noted in addition that the jettison of the AC-3 nose fairing halves resulted in a 0.07 degree, 0.7 cps disturbance on the sustainer engine traces. It can therefore be concluded that no control difficulties exist as a result of insulation panel or nose fairing jettison.

3.1.4.3 Simultaneous SECO/VECO. At propellant depletion, the sustainer and vernier engines are shut down and the separation sequence begins. Of interest at this time are the residual vehicle rates after shutdown. A simplified analysis based on guidance-commanded rates and command signal levels when the electrical disconnect is fired yields an approximately -0.21 ± 0.42 deg/sec rate in pitch and 0 ± 0.42 deg/sec in yaw. A second approach, based upon the flight data from six Atlas/Mercury flights, yields ± 0.58 deg/sec in pitch, ± 0.66 deg/sec in yaw, and ± 0.49 deg/sec in roll. All of these rates are within the allowable levels necessary for successful Atlas/Centaur separation, and it may be concluded that simultaneous SECO/VECO creates no difficulties with the separation sequence which follows.

3.2 ATLAS/CENTAUR SEPARATION PHASE. The separation sequence for the AC-5 and all succeeding vehicles differs only slightly from that of AC-4. The Atlas retrorockets are fired two seconds after SECO/VECO (2.6 seconds for AC-4) and the Centaur coast-phase attitude-control system is not activated, since positive control of the Centaur stage is not necessary during the separation sequence. The analysis which follows was made by summarizing past analyses performed in support of the separation system design.

3.2.1 Design Criteria. The philosophic basis of the present separation system design is fundamentally one of minimizing the vehicle performance lost (due to gravity losses) between SECO/VECO and MES while reducing the probability of collision between the two stages to acceptable (near zero) levels. This dictates a fast (as opposed to the earlier slow) separation sequence to minimize the time spent at essentially zero thrust. At the same time, the exit plane of the engine bells must clear the forward edge of the adapter before the prestart sequence begins, which is approximately nine feet of relative separation. This is to prevent an explosive concentration of oxygen and hydrogen collecting within the interstage-adapter section. In addition, the two stages require a relative separation of about 27 feet at MES to prevent undue heating of the Centaur aft bulkhead because of flame impingement on the expended Atlas sustainer stage.

3.2.2 Design Considerations. The major design problem is to minimize the probability of collision while the Centaur engines are still within the adapter. Speed of separation is dictated only by the thrust level and number of retrorockets used.

3.2.2.1 Guide Rails (Latch Straps). The guide rails (latch straps on AC-5 and earlier vehicles) serve to prevent collision between the forward edge of the interstage adapter for the first few inches of longitudinal motion. In effect, they serve to damp out any relative angular motion between the two stages produced by the forces and disturbances acting during the first few inches of motion. Beyond this point they are unnecessary since the relative clearance becomes much greater.

3.2.2.2 Location of Retrorockets. The ideal location of the retrorockets is one that will minimize the relative lateral motion between the forward edge of the interstage adapter and the Centaur vehicle. This requires that the net thrust vector pass through the center of percussion of the sustainer section. The obvious location of the retrorockets would be forward of this center of percussion with the thrust vector directed through it. However, this does not account for uncertainties in the thrust vector angle; small angular deviations can produce large lateral motions at the forward edge.

The retrorockets could be circumferentially distributed about the center of percussion such that the longitudinal components of the retro thrust produce no pitching or yawing moments about the center of gravity. The lateral components, since they act through the center of percussion, produce no lateral motion at the adapter forward edge.

The optimum location is actually somewhere between the two extremes as shown in Reference 8 and depends on the uncertainties in the retrorocket thrust-vector angle and thrust-versus-time history. Since these uncertainties are difficult to evaluate with precision, the safest location is at the center of percussion.

The optimum circumferential location is likewise a function of the uncertainties of the retrorocket performance and also a function of the residual rates of the Atlas/Centaur prior to separation. The residual rates are small (less than ± 1 deg/sec at SECO/VECO). (See Reference 9.) Their influence on the choice of circumferential location is likewise small, and therefore ignored. (See Reference 8.) With the longitudinal location at the center of percussion, the ideally selected circumferential locations would prevent pitching and yawing moments about the center of gravity.

3.2.2.3 Number of Retrorockets. The number of retrorockets chosen is a function of the impulse per retrorocket. However, as brought out in Reference 10, the more retrorockets, the better; the failure of one of a large number would produce a smaller effect on the separation dynamics.

3.2.2.4 Additional Considerations. For a given impulse from all retrorockets, the preference is for a high thrust over a short period of time. This is because a great part of the impulse will be expended while the angular disturbances due to retro thrust are restrained by the guide rails.

The time interval between SECO/VECO and the firing of the shaped charge and subsequently the retrorockets, should be sufficient to allow the sustainer engine thrust to decay to near zero. This time interval can be shortened, but there is a greater risk of collision.

3.2.3 Mathematical Model. The analysis is confined to the time subsequent to the firing of the eight retrorockets. The two stages are treated as rigid bodies having the following forces acting on them:

- a. Atlas retrorockets.
- b. Atlas sustainer and vernier engine thrust decays.
- c. Guide rail (latch strap) forces.

The remaining forces due to the interstage electrical and Centaur LO_2 -tank boiloff valve disconnects are insignificant by comparison (Reference 8). The Centaur boost-pump exhaust forces are likewise insignificant, being less than 10 pounds apiece. The guide rail (latch strap) forces are important only because they restrain relative lateral motion between the two stages for the first twelve inches or so of longitudinal motion.

The separation analysis discussed in this section is based on the equations derived in Appendix A. The basic data pertinent to the separation system is given in Appendix E.

3.2.4 Separation Distance. Since the forces acting on the Centaur stage produce such small accelerations, they are neglected in the calculation of the longitudinal separation distance. The only significant forces acting are those of the retrorockets and the sustainer and vernier-engine thrust decays. These forces (3σ) are given in Figure 3.2-1 and result in the displacement versus time relationship given in Figure 3.2-2 for the case of one retrorocket failure.

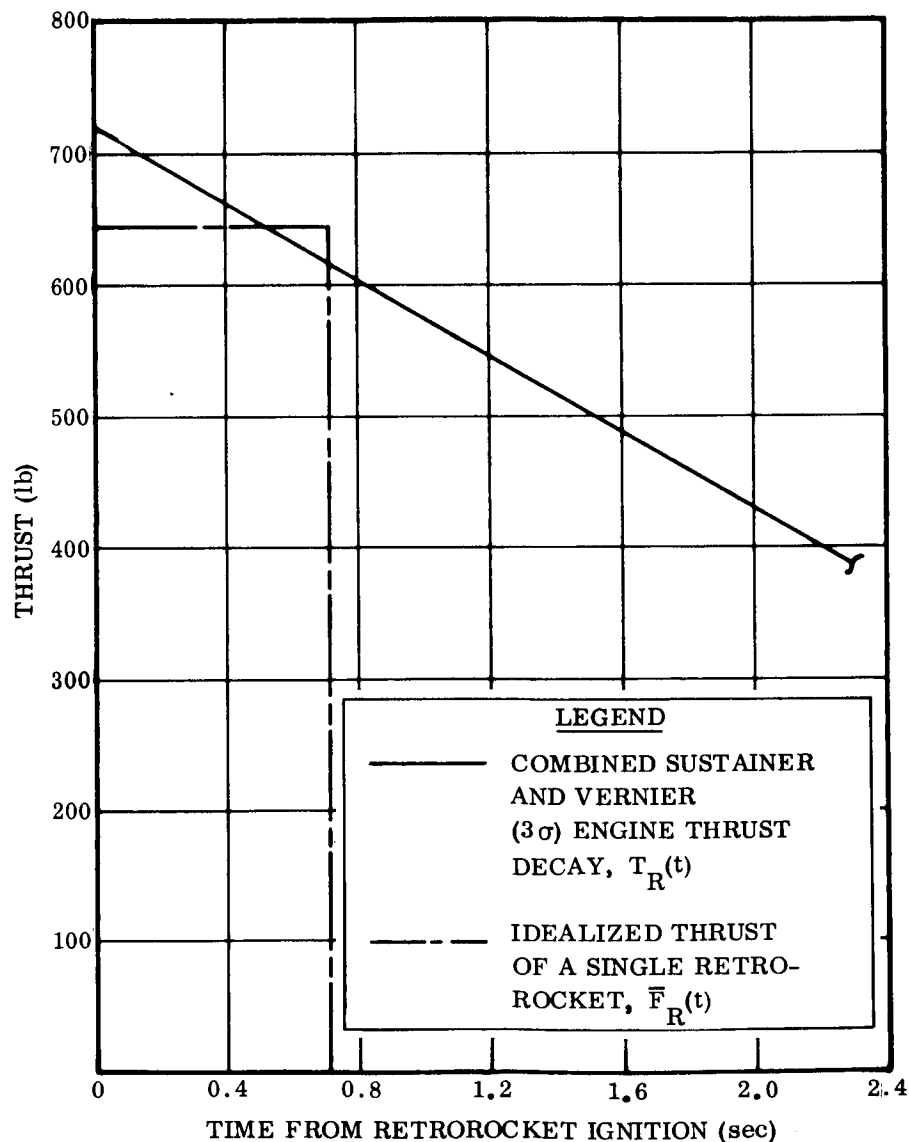


Figure 3.2-1. Sustainer and Vernier Thrust Decay and Idealized Retrorocket Thrust Characteristics

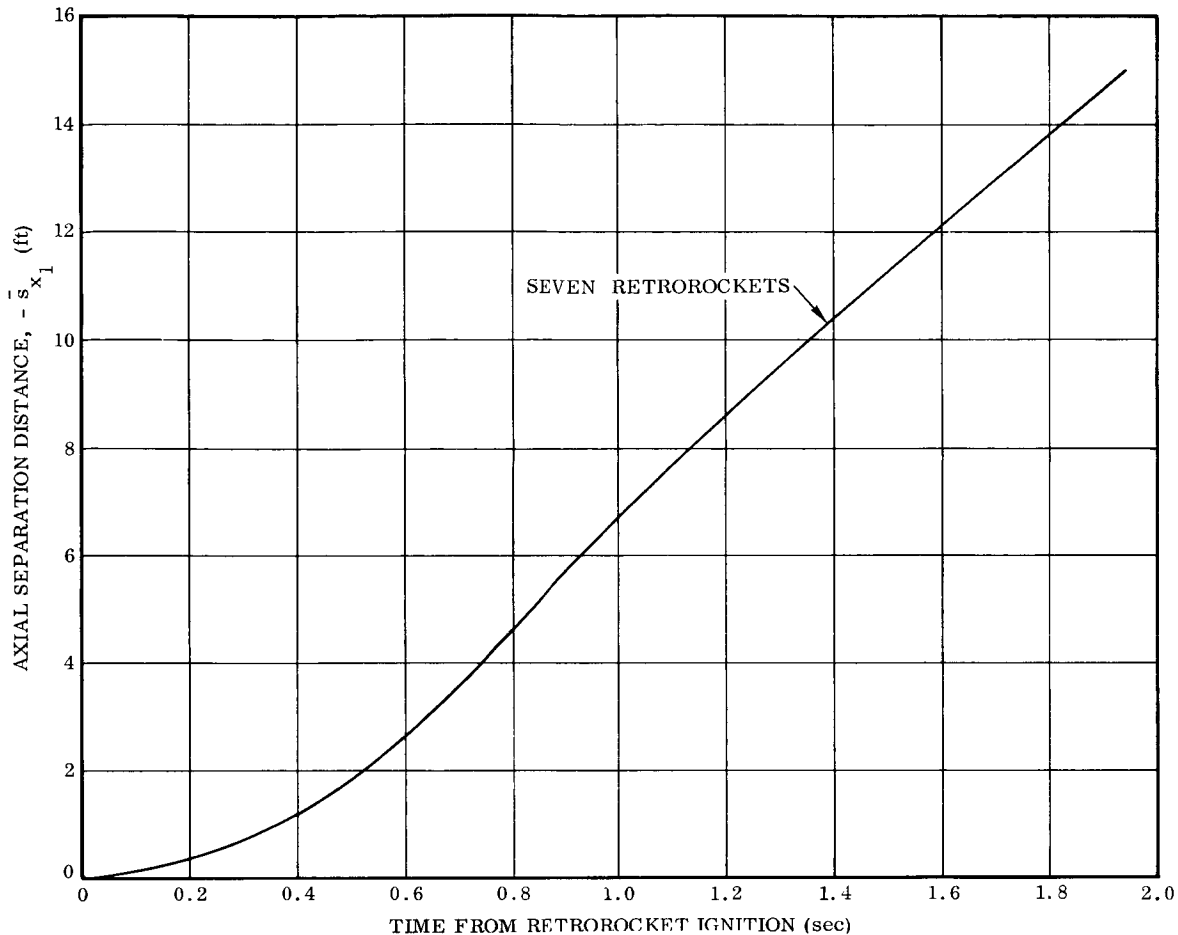


Figure 3.2-2. Nominal Separation Distance versus Time Assuming One Retrorocket Failure

The guide rails are cleared in 0.375 second, and 9 feet of separation is achieved in 1.25 seconds after retrorocket ignition. There is more than 11 feet of separation at prestart and 40 feet at MES. This satisfies the separation distance criteria.

3.2.5 Probability of Collision. The probability of collision between the interstage adapter and the Centaur main-engine bells or associated hardware can be determined by using Equation A-36 from Appendix A:

$$P(E) \leq q \sum_{k=1}^n P(E|E_k) + \frac{n}{2}(n-1)q^2 \quad (3.2-1)$$

Of the eight single retrorocket failure possibilities, the failure of the one nearest the center of gravity, No. 8, will result in the greatest torque about the sustainer-stage center of gravity (Figure E-1) and hence the largest lateral displacement of the interstage-adapter forward edge. Equation 3.2-1 can be simplified considerably by using the following conservative approximation:

$$P(E) \leq nq P(E|E_8) + \frac{n}{2}(n-1)q^2 \quad (3.2-2)$$

The probability of collision relative to the hypothesis that retrorocket No. 8 has failed is

$$P(E|E_8) \equiv P(|S_{z_1}| > C|E_8) \quad (3.2-3)$$

This probability can be calculated by considering S_{Z1} as being composed of two components. The first is the nominal or mean value of S_{Z1} , $\bar{S}_{Z1}$, relative to the hypothesis that retrorocket No. 8 has failed. The second component is the dispersion in S_{Z1} with respect to $\bar{S}_{Z1}$ relative to the same hypothesis. The mean displacement $\bar{S}_{Z1}$ is evaluated from Equation A-18. This equation is

$$\bar{S}_{Z1} = -\frac{m_2^L R}{I_2} \bar{\chi} \frac{1}{n} \sum_{i=1}^n (\sin \phi_i + \frac{Z}{R}) \quad (3.2-4)$$

where

$$\bar{\chi} = -S_{x1} + \frac{1}{m_2} \left[\int_0^{\bar{t}_x} (\bar{t}_x - \tau) T_R(\tau) d\tau - n \int_0^{\bar{t}_0} (\bar{t}_x - \tau) \bar{F}_R(\tau) d\tau \right] \quad (3.2-5)$$

The upper limits in the integrals of (3.2-5), $\bar{t}_0$ and $\bar{t}_x$, are the nominal times required to clear the guide rails (effective latch strap length) and achieve x feet of axial separation respectively. The nominal axial separation distance versus time relationship is determined by using Equation A-16. This relationship is given by

$$\bar{S}_{x1} = \frac{1}{m_2} \int_0^t (t - \tau) [T_R(\tau) - n \bar{F}_R(\tau)] d\tau. \quad (3.2-6)$$

and is represented by the plot of Figure 3.2-2.

Assuming the effective length of the guide rails (latch straps) to be one foot, $\bar{t}_0$ is 0.375 second and $\bar{t}_9$ (the time to achieve 9 feet of axial separation) is 1.25 seconds. Using the basic data given in Appendix E,

$$\left. \begin{aligned} \frac{m_2^L R}{I_2} &= 0.530 \\ \frac{1}{n} \sum_{\substack{i=1 \\ i \neq 8}}^n (\sin \phi_i + \frac{Z}{R}) &= -0.206 \end{aligned} \right\} \quad (3.2-7)$$

the parameter $\bar{\chi} (-S_{x1} = 9.0)$, upon numerically evaluating (3.2-5), is

$$\bar{\chi} (-S_{x1} = 9.0) = 4.05 \quad (3.2-8)$$

Therefore, relative to the hypothesis that retrorocket No. 8 has failed, the nominal lateral displacement of the forward end of the interstage adapter with respect to the Centaur clearance envelope is

$$\begin{aligned} \bar{S}_{Z1} &= -(0.530)(4.05)(-0.206) \\ &= 0.442 \text{ ft (5.30 inches)} \end{aligned} \quad (3.2-9)$$

From Equation A-20, the standard deviation of lateral displacement is given by

$$\sigma_{S_{z_1}} = \sqrt{\left[\sum_{i=1}^n \left(\frac{\partial S_{z_1}}{\partial \xi_i} \right)^2 \right] \sigma_{\xi}^2 + \left(\frac{\partial S_{z_1}}{\partial \omega_o} \right)^2 \sigma_{\omega_o}^2} \quad (3.2-10)$$

The standard deviation of the retrorocket thrust attenuation parameter, ξ_i , has been calculated from test results to be 0.057 at a confidence level of 95 percent. The standard deviation of residual turning rate, ω_o , from Atlas/Mercury flight tests, is estimated to be approximately 0.50 deg/sec. Using Equations A-28 and A-29 from Appendix A, (3.2-10) yields

$$\sigma_{S_{z_1}} = 0.772 \text{ inch} \quad (3.2-11)$$

The available clearance, c , as the adapter passes the aft end of the Centaur main engine bells ($-S_{x1} = 9.0$ ft), is 10.8 inches. Therefore, in addition to the failure of retrorocket No. 8, it would require a 7σ dispersion in rocket thrusts and residual turning rate in order to have a collision.

3.2.6 System Adequacy. In view of the preceding results, the first term in Equation 3.2-2 can be neglected for any reasonably acceptable retrorocket reliability. The overall probability of collision is therefore approximately:

$$P(E) \cong \frac{n}{2} (n-1) g^2 \quad (3.2-12)$$

Assuming the retrorockets to be 99 percent reliable, the overall probability of collision would be about 0.003. This figure is regarded as adequate.

3.3 POWERED FLIGHT PHASE. Centaur powered flight, for purposes of this report, begins with the main engine ignition transient and ends with main engine cutoff. The AC-5 flight is a one-burn mission with the main engines firing for approximately 412 seconds.

The flight control system maintains control of the vehicle in pitch, yaw, and roll. The vehicle attitude reference in the pitch and yaw planes is obtained from the Centaur guidance computer via the resolver chain. The roll attitude reference is obtained by integrating a signal proportional to the vehicle roll rate within the autopilot.

The basic control problem is concerned with the stability of the rigid body motions. Sufficient gimbal capability is built into the system to enable the engines to be trimmed (Reference 11), i.e., the net thrust vector can be pointed through the vehicle center of gravity at all flight times. However, the major problem of flight control is that of ensuring stability of the system in the presence of the principal modes of oscillation, i.e., guidance, propellant sloshing, and body bending. A complete analysis of the flexible bodied vehicle including all system parasitics would be needlessly complicated. The wide separation between the low frequency dynamics, i.e., guidance, propellant sloshing and rigid body modes, and the higher frequency elastic body bending modes permits separate treatment of each phase of the problem, since there is negligible coupling between the two.

3.3.1 Ignition Phase. Reference 12 documents a study to determine the nature and magnitude of transient attitude excursions of Centaur during and immediately following engine ignition. Also considered was the extent to which these excursions depend upon: (1) uneven ignition of the two engines, (2) operation of the engine servohydraulic power supplies, (3) initial thrust chamber positions, and (4) rigging errors

and thrust misalignments. The AC-2 configuration was recently analyzed to update the Reference 12 study. The results, with a discussion of system operation as applied to the AC-5 configuration, are given below.

3.3.1.1 Autopilot. The ignition transient period is defined as that portion of powered flight between MES and Steering Enable (MES + 4). The initial conditions at MES result from events occurring before and during the separation sequence. (The autopilot is in a coast phase configuration with its integrators nulled until MES.) When the hydraulic system recirculating pump is activated 0.5 second after SECO/VECO, the main engines will begin to gimbal in response to the rate signals existing within the autopilot. Since these signals are at low levels, the engines will move toward a centered, or nulled position.

At MES, the autopilot is changed to the powered-phase configuration, and the integrators are unnulled. The thrust of each main engine increases from zero to approximately 15,000 pounds in less than two seconds. The main hydraulic supply pump is driven by the propellant turbopump. The hydraulic supply pressure for the four engine actuators rises to approximately 1100 psi simultaneously with the turbopump speed and the thrust buildup. Because the reference vector is removed at MES, the Centaur vehicle has no position reference until MES + 4 seconds. As a result of the transient condition and/or the conditions immediately preceding MES, the vehicle may be oriented several degrees from the desired attitude when steering is enabled. Another transient will be produced, but under the more easily analyzed conditions of full thrust and full power. This transient is discussed in Subsection 3.3-2.

3.3.1.2 Hydraulic System. At 8 seconds prior to MES, the recirculation pump is activated, permitting coast phase gimbaling of the engines. In the absence of any significant position error, the engines will move to within a few tenths of a degree of null, thus minimizing the vehicle attitude excursions and attitude rates during the transient following MES.

MES marks the beginning of main hydraulic pump operation. As the pressure on the high pressure side of the system rises, the bootstrap piston displaces, proportionately increasing the reservoir pressure. When the system supply pressure reaches and exceeds approximately 600 psi, the accumulator piston is displaced, permitting fluid flow into the accumulator. The pressure rises still further until a pressure of approximately 1020 psid is established between the high pressure side and the reservoir. The relief valve opens, permitting the fluid to flow back to the reservoir. During this supply pressure buildup, an attitude excursion of the Centaur vehicle takes place, causing the appropriate servovalves to activate and permitting high pressure flow to the actuators. When the pump has reached its steady-state flow rate, the supply pressure seeks some steady-state level, normally approximately 1100 psi, with the net flow into or out of the high pressure side equal to zero. The time required to reach this steady level depends on the flow rate versus time history of the pump, the severity of the vehicle attitude excursion, and hence the flow into the pitch and yaw/roll actuators.

3.3.1.3 Vehicle Dynamics. For purposes of this study, the operation of the propulsion system is summarized by the pump speed rise curve shown in Figure 3.3-1 and the envelope of thrust buildup curves shown in Figure 3.3-2. The former is taken from recent acceptance tests by the engine manufacturers and the latter from the engine specification (Reference 13). It is assumed that the actual thrust buildup curve lies within the limits of Figure 3.3-2. The difference in the thrust rise between one engine and the other constitutes the major parameter variation in the study. This difference, in terms of differential impulse, ΔI , is

$$\Delta I = \int_0^2 (T_1 - T_2) dt \quad (3.3-1)$$

In addition, there can be a steady-state thrust difference once the engines have reached full thrust. These two parameters, thrust and pump speed buildup, constitute major variables in the parameter study of the system and are discussed below in connection with overall system behavior.

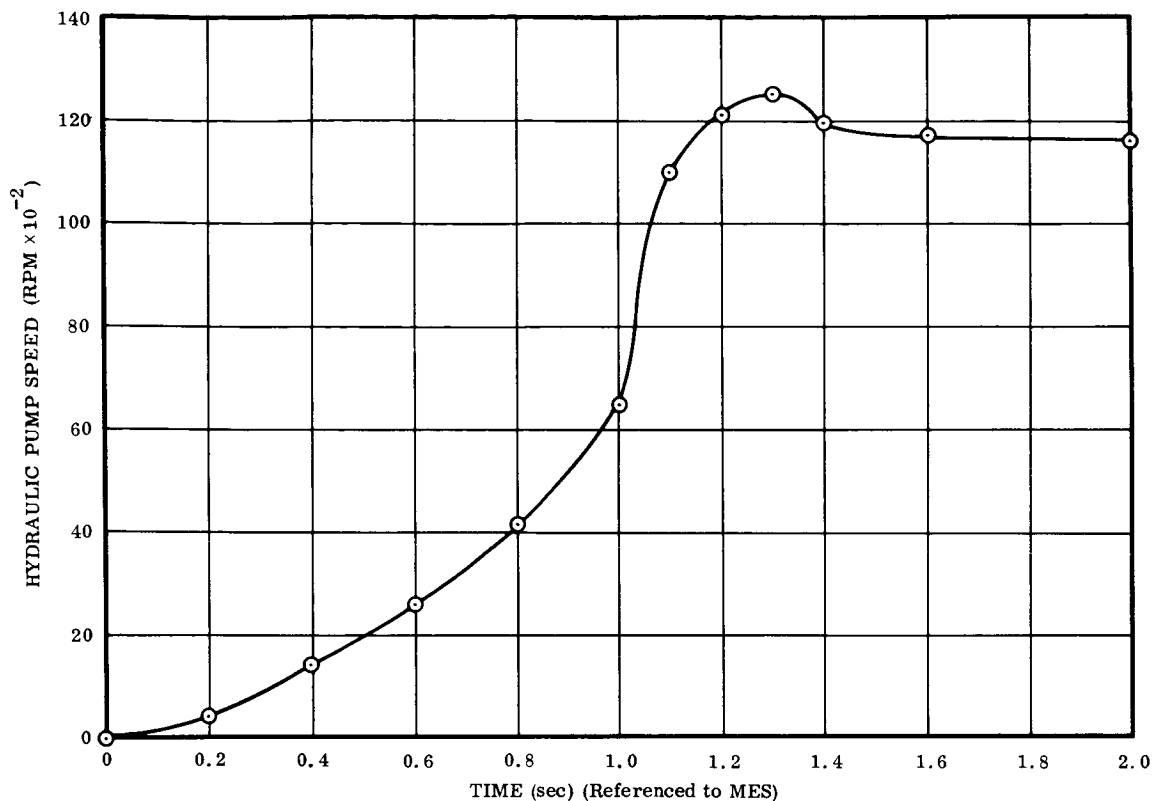


Figure 3.3-1. Typical Hydraulic Pump Speed Starting Transient for Centaur Engines

The mathematical model used to describe the system behavior in the study was that of a rigid vehicle with two pendulums to duplicate the effect of propellant slosh, and two gimbaled masses representing the main engines. The earlier studies (Reference 12) indicated that only pitch and roll plane dynamics need be considered because the yaw transient is negligible. Engine thrust is the major force acting on the system. The conditions having the greatest influence on the system are the initial rates and the position of the thrust chambers prior to main engine start. The model is the same as that used in the stability studies of powered flight except that the main engine dynamics are included in greater detail. Since the time period analyzed encompasses four seconds of flight, the dynamic properties of the vehicle are assumed to be constant with the values which apply at MES for the AC-2 vehicle.

The major engine parameters, other than their inertial properties (Appendix E) are those of gimbal block friction, effective thrust misalignment, and spring rates of the supporting structure. Main engine gimbal block friction was assumed to consist of two components, coulomb and viscous friction. It was further assumed that the coulomb component had to be overcome by the application of, at minimum, 20 ft-lb of torque to the engine about the gimbal block for zero engine thrust, and 200 ft-lb for full engine thrust. Application of torque would be a linear function of thrust levels. The viscous, or engine velocity dependent component, was assumed to be constant, independent of the thrust level, and equal to five ft-lb/deg/sec. The thrust vector of each engine was assumed to be misaligned in pitch by 0.54 degree, in yaw by 0.36 degree, and in such a direction to cause the greatest attitude excursion. This misalignment includes rigging error, vehicle center-of-gravity uncertainty, as well as actual misalignment of the thrust vector with respect to the thrust chamber centerline. The figures are not unrealistically large.

The thrust vector was assumed to pass to one side of the intersection of the pitch and yaw/roll gimbal axes by 0.01 foot in such a way to oppose the corrective motion of the engines in response to the ignition transient. The spring rates assumed for the actuators and their supporting structure are as quoted in

- NOTE: 1. AT 200,000 FEET ALTITUDE
STANDARD PROPELLANT INLET
CONDITIONS AND THRUST CHAMBER
INITIAL METAL TEMPERATURE OF
540° R
2. ZERO SECONDS TAKEN AS TIME OF MES
3. TAKEN FROM REFERENCE 13

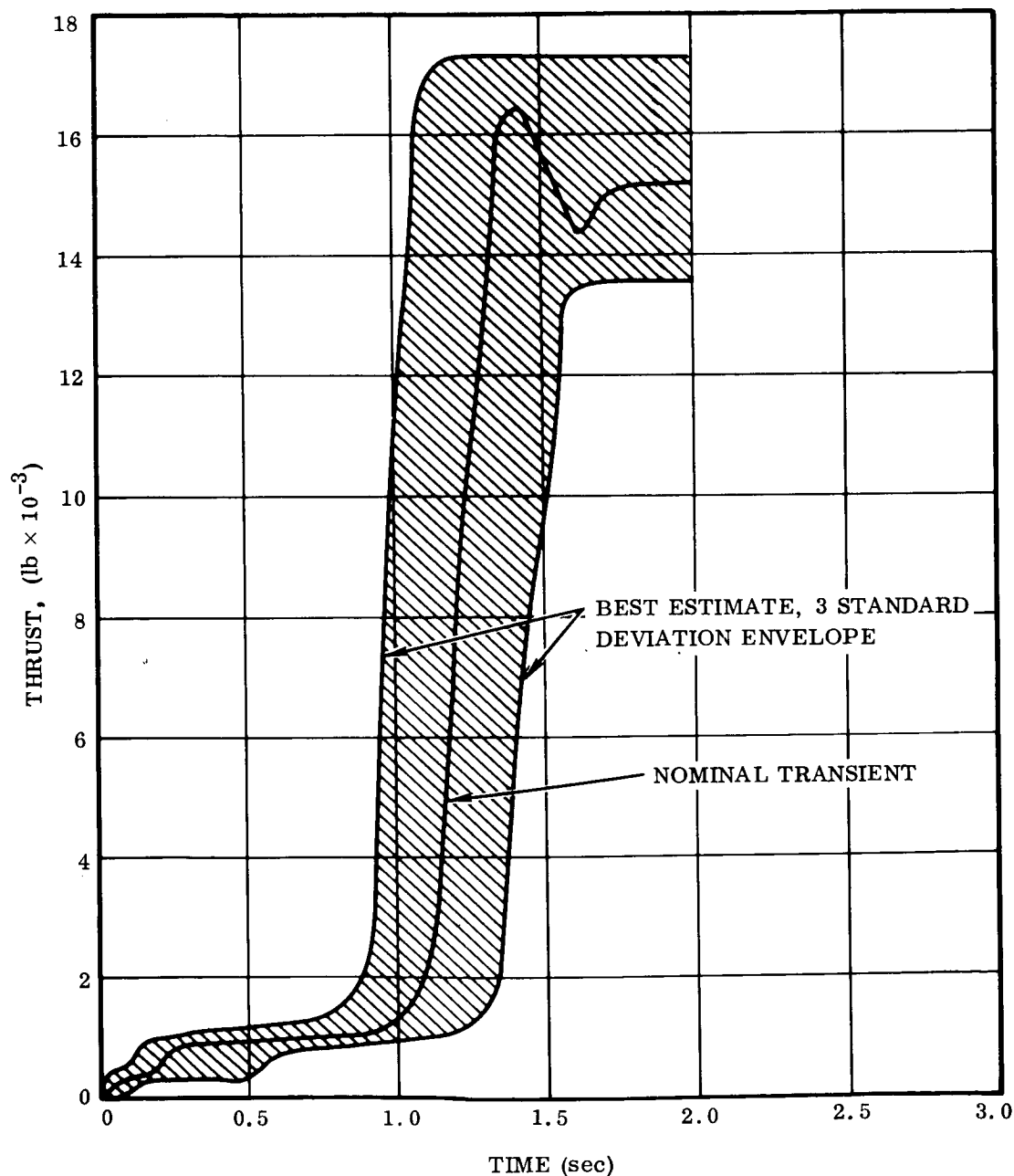


Figure 3.3-2. Estimated Thrust Starting Transient Envelope for Centaur Engines

Appendix E. There is an additional effective spring rate of the gimbal block in the longitudinal direction. This is so high, as determined from static loading tests, as to be negligible in its effects on the study.

The initial conditions at MES assumed for the study were zero initial vehicle rates and engine positions, and slosh pendulum angles of five degrees in a direction to maximize the attitude transient. In the actual case, the engines at MES are oriented to reduce the attitude error (with respect to the reference vector). Likewise the actual vehicle rate tends to reduce the attitude error. Thus the assumptions above are conservative in that they tend to increase the attitude error at steering enable. In addition, the hydraulic system supply pressure is assumed to be zero at MES, even though the recirculation pump provides a small supply pressure of approximately 90 psi.

The engine differential impulse is assumed equivalent to taking a typical thrust versus time curve as determined from engine firing (Figure 3.3-3) and delaying it in time with respect to an identical curve on the other engine. The steady-state thrust level difference is assumed equivalent to attenuating the thrust rise curve of one thrust chamber with respect to the other. In all cases, both thrust rise curves fit within the shaded area of Figure 3.3-2.

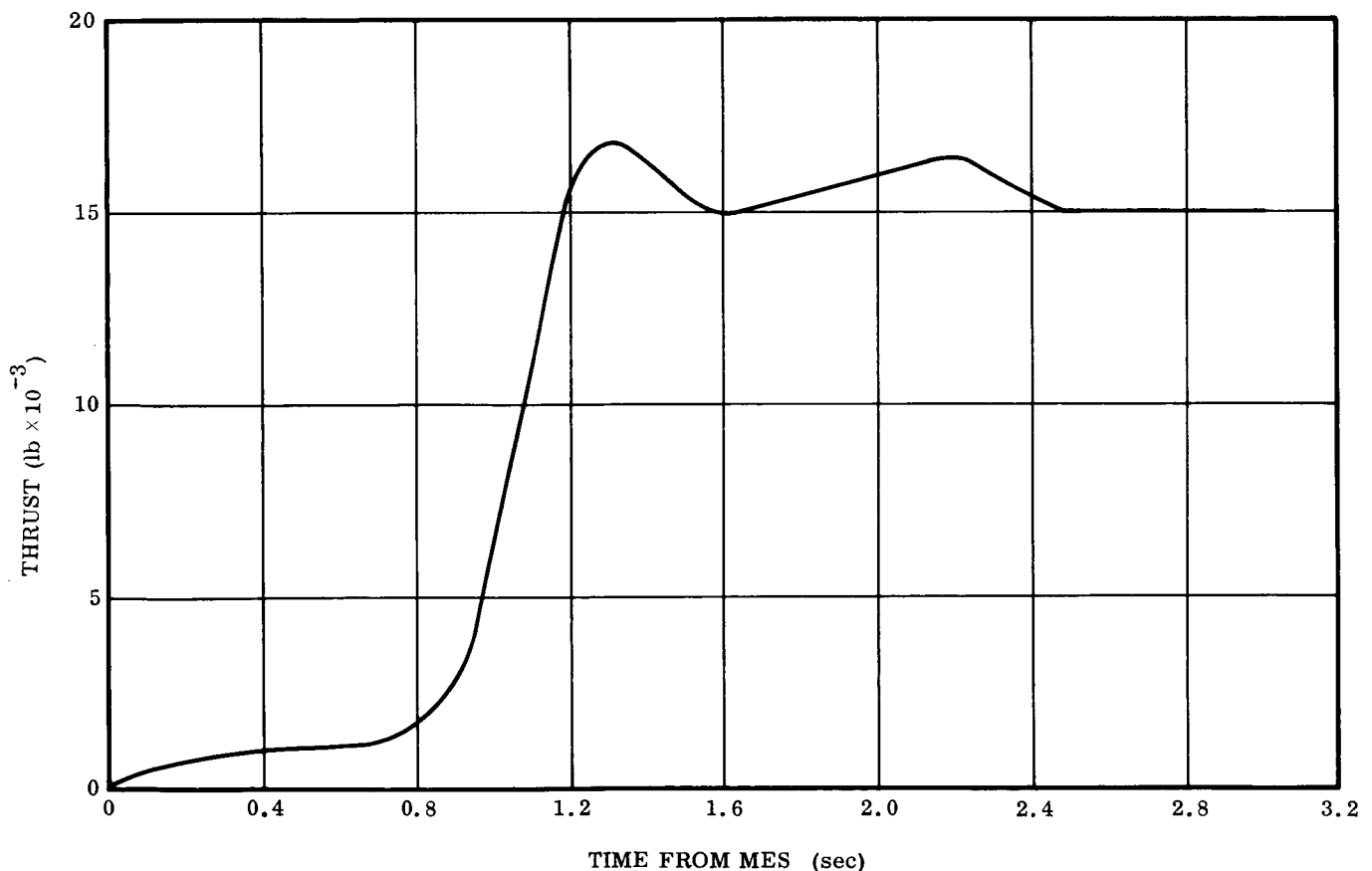


Figure 3.3-3. Thrust Rise versus Time Used in Analog Computer Simulation

The vehicle dynamic behavior is as follows. At MES, the autopilot programmer issues a discrete command to activate the turbopumps on the two main engines. The hydraulic supply pressures and engine thrusts increase as the speed of the turbopumps increase. Differences in the response times of the two engines produce unavoidable differences in the rates of thrust buildup, measured in terms of differential impulse. The differential impulse and the initial misalignment of the thrust vectors with respect to trim conditions cause the vehicle to begin rotating. The vehicle rates are sensed by the Centaur rate gyros

and commands are sent to the engine actuators to gimbal the two main engines to reduce these rates. The amplitude and duration of this command is dependent upon the severity of the disturbance. Depending upon the command, the time rate of change of the hydraulic fluid supply pressure may decrease and possibly even become temporarily negative. Coupling of the pitch and roll actuators on an engine through their common hydraulic supply causes the flow available to an actuator to decrease if the other actuator is in a high demand condition. It is expected that this coupling is of importance only for relatively large ignition transients and/or for markedly low values of hydraulic supply pressure. The latter condition can exist if, for example, the hydraulic pump efficiency is low due to absorbed gases in the hydraulic fluid. The thrust and the supply pressure reach more or less steady-state conditions by MES + 3. Thereafter the vehicle flight control system continues to respond to the conditions caused by the transient. The equations of motion of the overall system are derived in Reference 12 and in Appendix B.

3.3.1.4 System Criteria. To determine the adequacy of the Centaur stage flight control system in the ignition transient condition, it is desirable to have some criteria by which to judge the system. Unfortunately, no criteria have been formally established, and one must rely on good design practice.

The most obvious consideration is that of performance. The impulse expended during the transient condition must not result in fuel depletion before the desired guidance objectives are reached. No upper limit has been established for the allowable impulse perpendicular to the desired flight path, although 50,000 lb-sec was determined acceptable (Reference 14) for the AC-3 configuration. In actuality, the orientation of the vehicle at MES can be other than that specified by the guidance system and performance could be lost over and above that lost due to the ignition transient.

Lacking formal criteria, the approach used was to require the autopilot to regain control by the time of steering enable. This is achieved when the thrust chambers come off their stops. The major flight control system parameter affecting this situation is total gimbal angle capability. The larger this is, the larger the moment which can be balanced by gimbaling the thrust chamber. For the 1C (AC-3) and following vehicles, the gimbaling capability was increased to ± 3.0 degrees to ensure control capability in the presence of engine alignment and vehicle center-of-gravity uncertainties.

The ignition transient condition imposes conditions on other portions of the system. For example, the inertial platform must be capable of sustaining the vehicle rates imposed on it by the vehicle. Reference 5 specifies that the platform should be capable of sustaining a body rate of the form

$$\dot{\theta}, \dot{\psi}, \text{ or } \dot{\theta} = K_1 e^{-K_2 t} \sin K_3 t \quad (3.3-2)$$

where

$$K_1 = 30 \text{ deg/sec}$$

$$K_2 = 0.36 \text{ sec}^{-1}$$

$$K_3 = 3.2 \text{ rad/sec}$$

This represents a rigid body transient, such as would be experienced at MES, having a peak rate of approximately 25 deg/sec.

Another consideration is that of loads. In this area, the loads on the aft bulkhead are largely a function of the thrust buildup with the control system contributing a small additional increment. The present bulkhead is capable of sustaining control loads well in excess of those predicted by the following analysis. It should be noted that the major control system load will come from the engine hitting the actuator stops, a

shock loading determined by knowing the engine velocity and mass and the spring rate of the actuator support structure. This loading is a design consideration on the bulkhead and actuator support structure.

3.3.1.5 System Analysis. An analog computer simulation of the vehicle and flight control system as defined by the set of equations derived in Appendix B was set up. The major parameter in the transient at engine ignition is the differential impulse during the first two seconds after MES.

The results showed that the vehicle was able to recover with an impulse as great as 8,000 lb-sec (the largest impulse investigated) but required the engines to be driven to their stops for a period of six seconds. For any impulse a little greater than 4,000 lb-sec, the engines will drive to their stops in the recovery maneuver.

Another variable was the delay between the beginning of thrust buildup and beginning of hydraulic pump flow rise. At the lower impulses there was almost no noticeable difference in the transient as the delay was increased from zero to 0.3 second. At 8,000 lb-sec the only noticeable difference was an increase in maximum attitude excursion of one degree, and a decrease in the rate at which the thrust chambers drove to their stops. A maximum steady-state thrust differential of 2,200 pounds required 1.85 degrees of engine deflection to zero the body pitching moment. The maximum deflection available was 2.5 degrees for the AC-2 configuration. Note that these figures are well in excess of those reasonably expected and those observed on the flights of AC-2 and AC-3.

A nominal case for impulse and pump flow delay was set up and runs were made to determine: 1) the effect of engine thrust misalignment in pitch and yaw, and 2) the initial condition on the oxidizer and fuel pendulums.

A nominal impulse of 2,000 lb-sec, a steady-state thrust differential of 550 pounds, and no delay in pump flow were simulated. The combination of thrust misalignment and initial pendulum angles giving greatest attitude transient during engine ignition was determined. This was used as the nominal case. Using the nominal parameters, analog computer runs were made for the various combinations of differential impulse and steady-state thrust differential listed in Table 3.3-1. For each of these combinations, hydraulic supply pump speed buildup was delayed 0.0, 0.15, and 0.3 second (with respect to the nominal buildup). It should be noted that the ratio of differential impulse to steady-state thrust differential is held constant.

Table 3.3-1. Impulse and Thrust Combinations Used in Computer Simulations

DIFFERENTIAL IMPULSE (lb-sec)	STEADY-STATE DIFFERENTIAL THRUST (lb)
0	0
2000	550 (nominal)
4000	1100
6000	1650
8000	2200

Figure 3.3-4 illustrates some of the important variables as a function of time for the nominal case as taken from the analog simulation. Note that, 1) steering enable at MES + 4 seconds is not simulated, and 2) the so-called nominal parameters for differential impulse and steady-state thrust difference are easily double those actually encountered on the AC-2 and AC-3 flights.

Because of the importance of differential impulse, the variations in angular displacements and rates were plotted as functions of ΔI defined as the integral of the thrust difference between main engine start (MES)

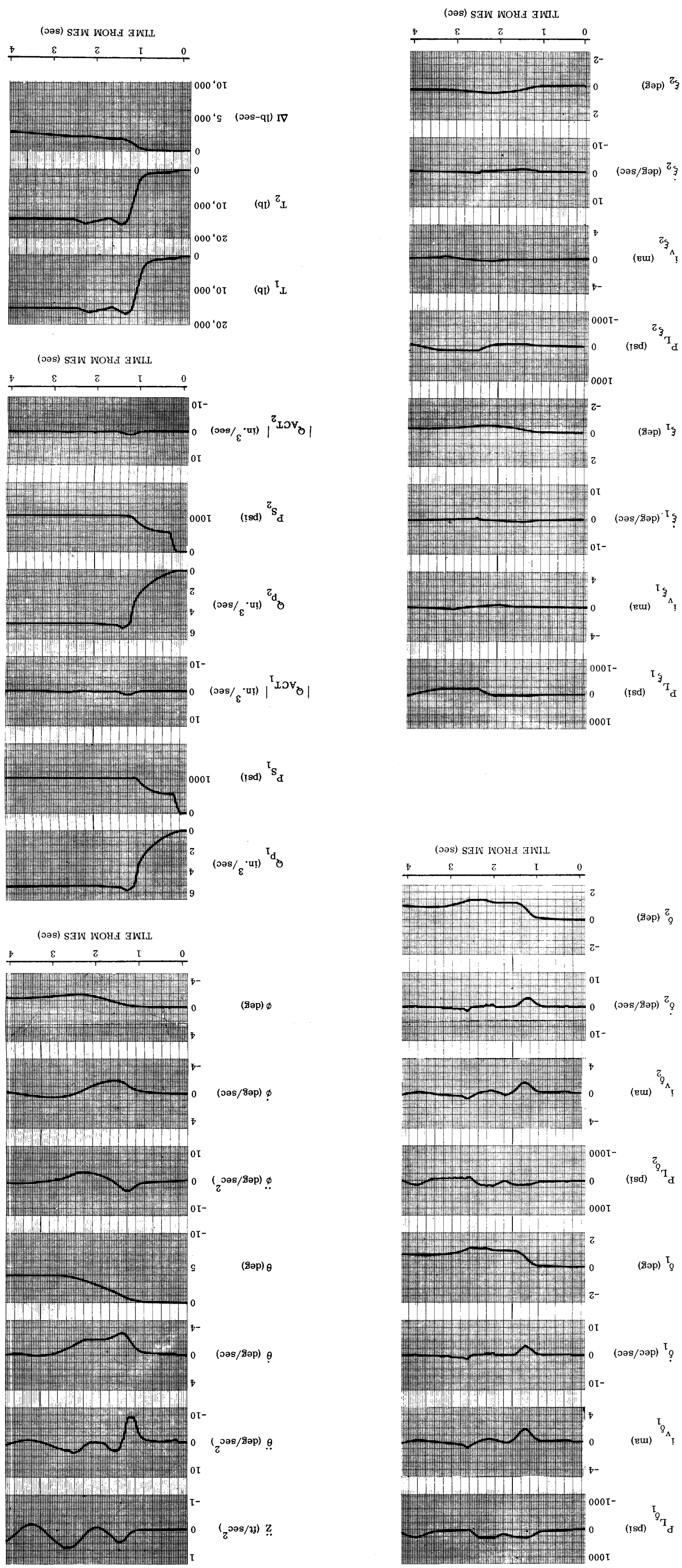


Figure 3.3-4. Simulated Nominal AC-2 Ignition Transient Response

and MES plus two seconds (Equation 3.3-1). Figure 3.3-5 is a plot of the maximum value of pitch attitude and pitch rate occurring during the ignition transient. The pitch rate shows a gradual increase in slope as the differential impulse is increased, reaching a value of 12.4 deg/sec at 8000 lb-sec. The pitch attitude, on the other hand, shows a marked increase in slope after 4000 lb-sec. This is caused by the fact that the engines are at their maximum deflection as shown in Figure 3.3-6. This figure also shows the length of time the engines are at their maximum deflection. The longer they are deflected, the greater the attitude error. It is seen from Figure 3.3-6 that a differential impulse greater than about 6000 lb-sec causes the engines to remain on their stops until steering enable and beyond. This does not mean loss of mission; the transient is more severe than should be reasonably expected. The maximum engine velocities are shown in Figure 3.3-7. The maximum velocities increase as the impulse is increased, then begin to level out as the maximum velocity for the actuator is approached. The difference in rates between $\dot{\delta}_1 \text{ max}$ and $\dot{\delta}_2 \text{ max}$ is a result of the delay in supply pressure buildup in the C2 hydraulic system. Figure 3.3-8 shows the maximum load pressure as a function of differential impulse. Until the actuator goes to the end of its travel the load is low, about 340 psi. After hitting the limits the pressure goes to the full value between the supply and return pressure, this being 1100 psi. This pressure is not transmitted to the structure. In yaw, the load pressure remains at 200 psi for almost all conditions, increasing 10 to 20 psi as the impulse is increased. The maximum roll rate and roll attitude were both small under all conditions. The thrust misalignment in roll had been set to give a maximum roll rate. The rate experienced at engine ignition on the AC-2 flight was considerably less than that shown for the nominal case of 2000 lb-sec differential impulse in the simulation. See Figure 3.3-9.

The yaw actuator rates and deflections shown in Figure 3.3-10 were likewise small. These rates include the effects of the roll channel only, the yaw channel being omitted from the simulation.

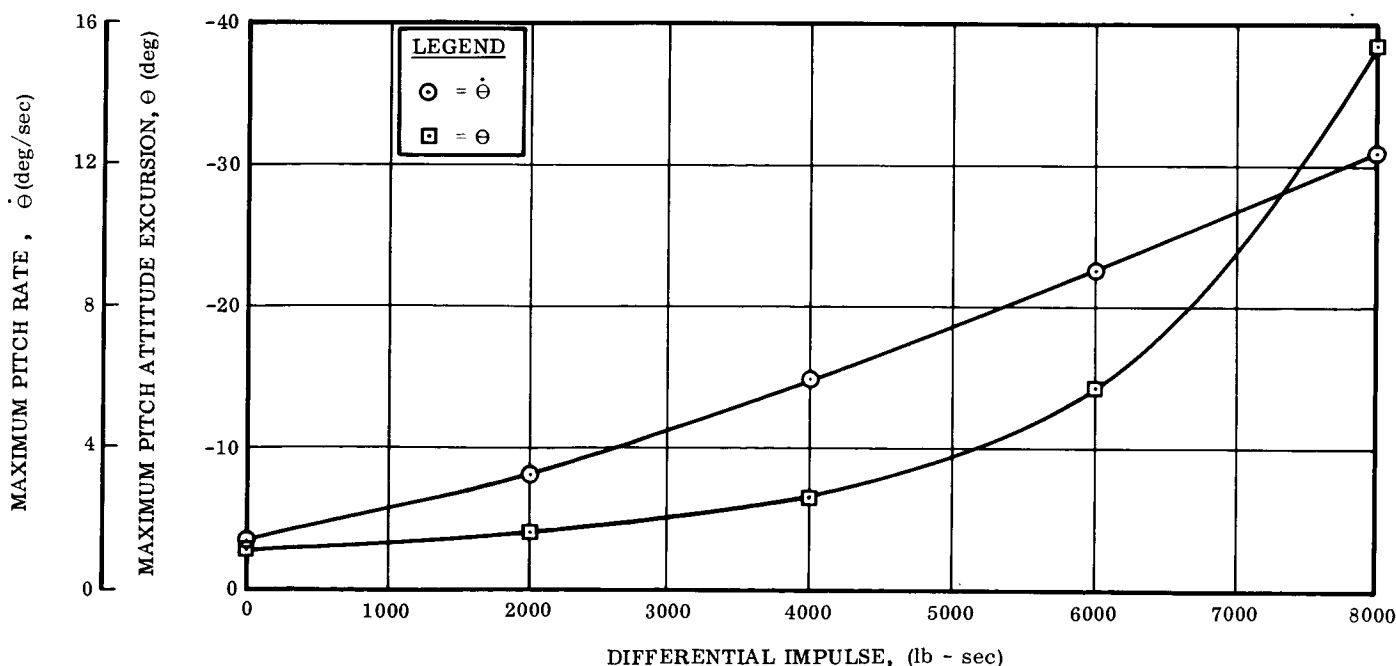


Figure 3.3-5. Maximum Pitch Attitude and Attitude Rate Excursions versus Differential Impulse

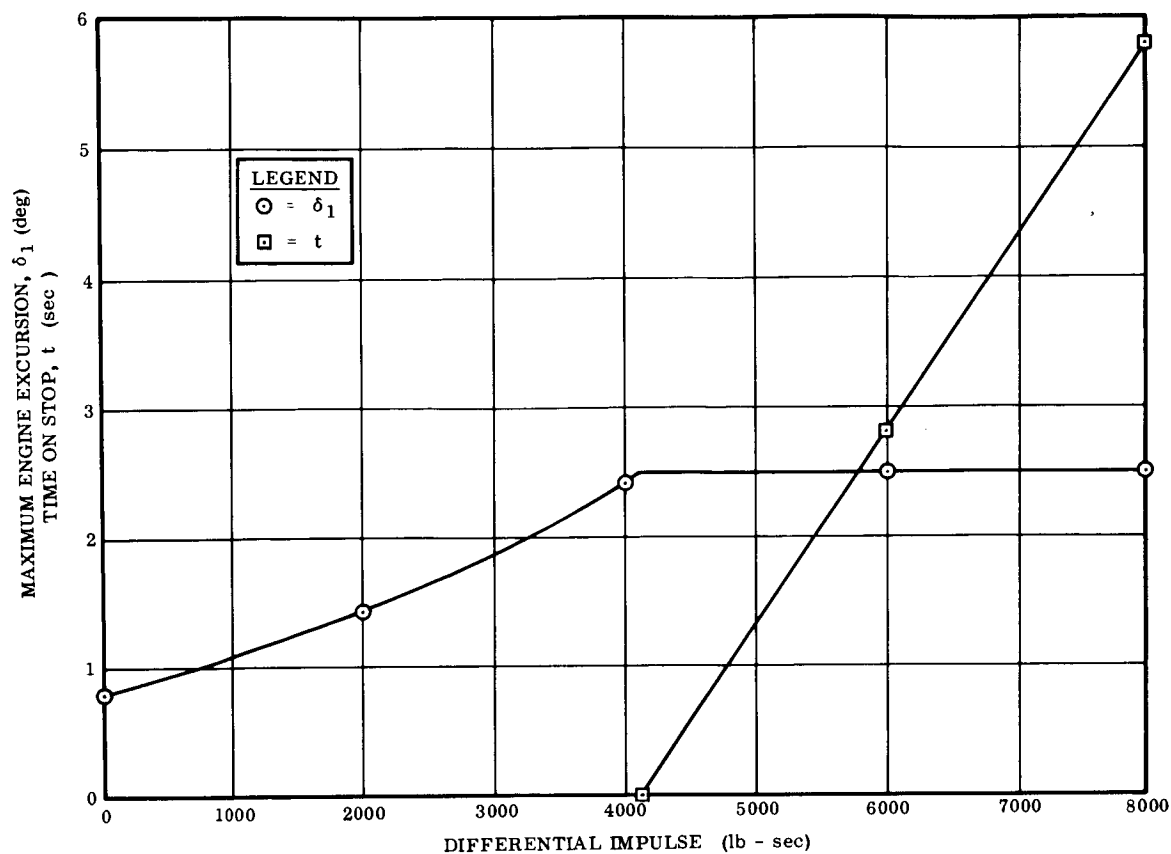


Figure 3.3-6. Maximum C-1 Engine Pitch Excursion and Time on Stop versus Differential Impulse

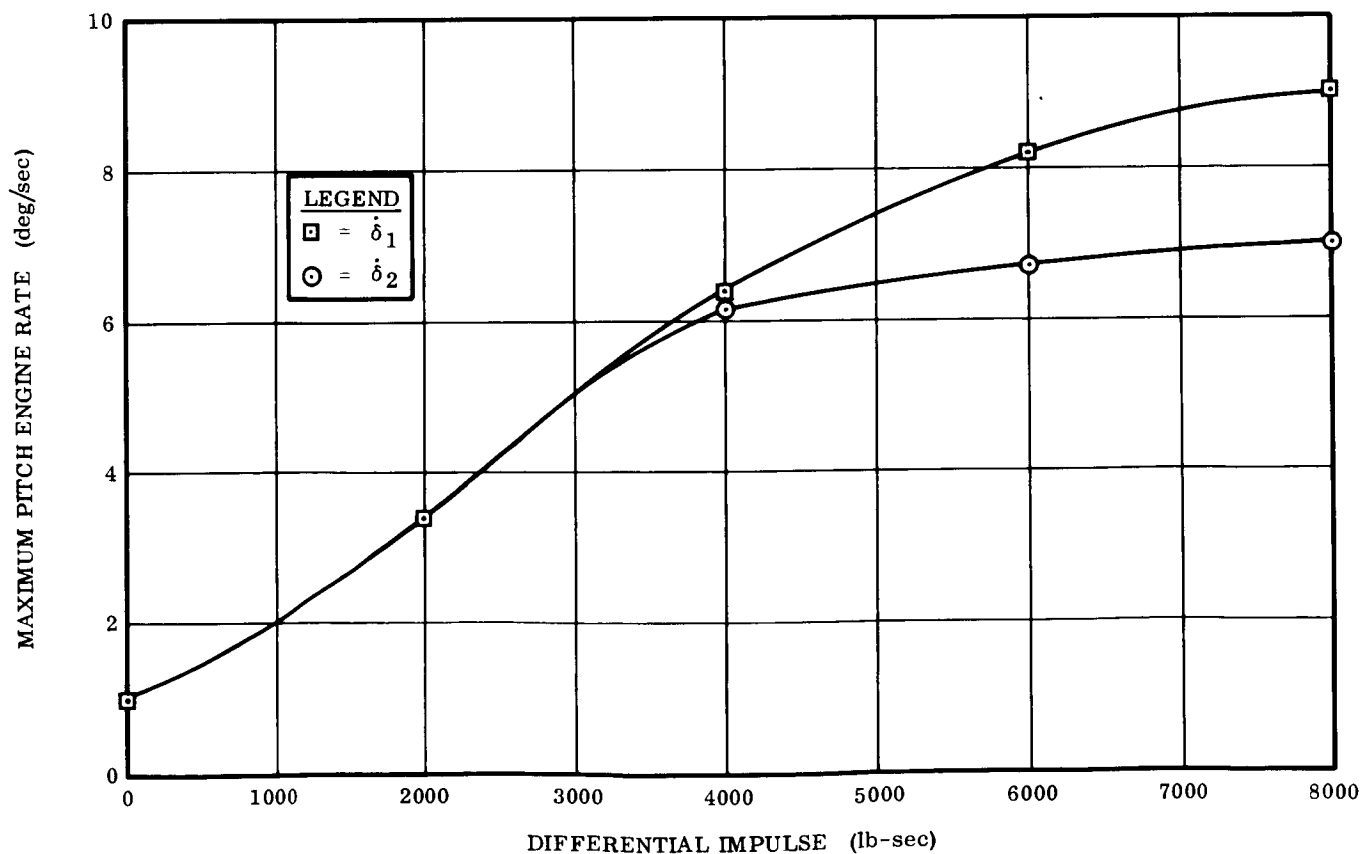


Figure 3.3-7. Maximum Engine Pitch Gimbaling Rates versus Differential Impulse

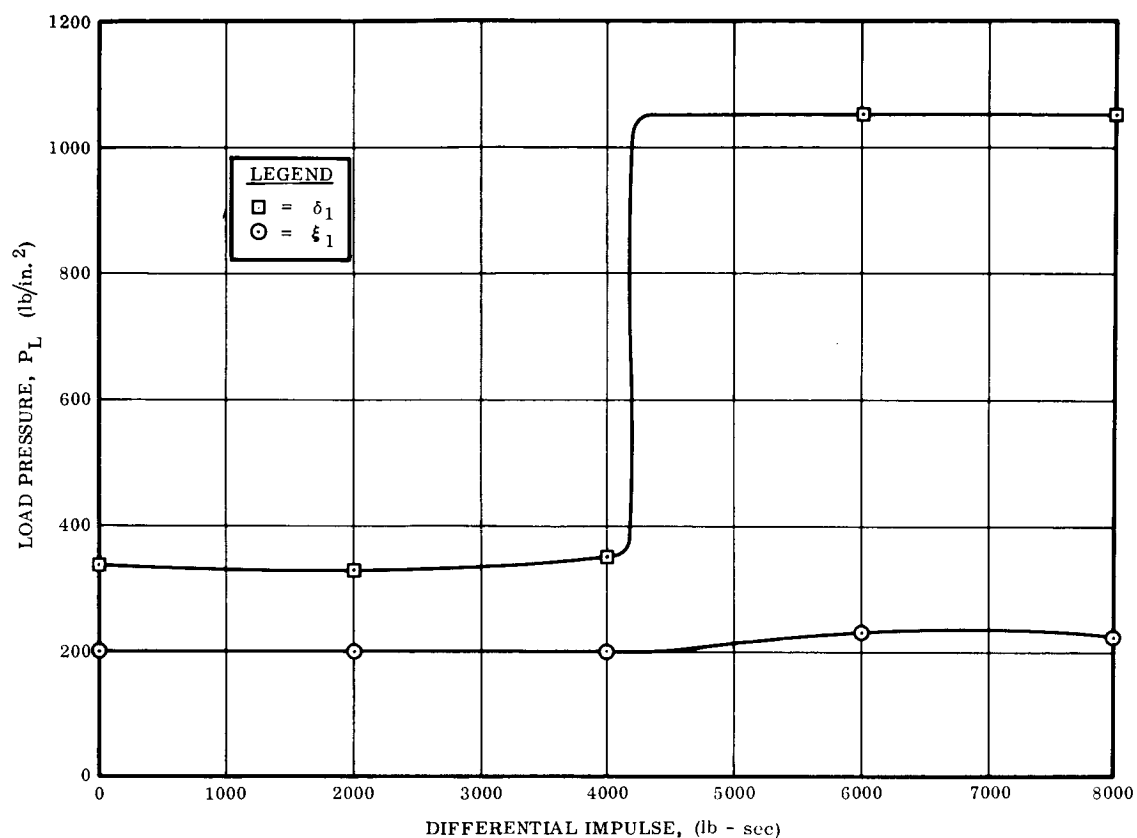


Figure 3.3-8. Maximum Load Pressure in C-1 Engine Actuators versus Differential Impulse

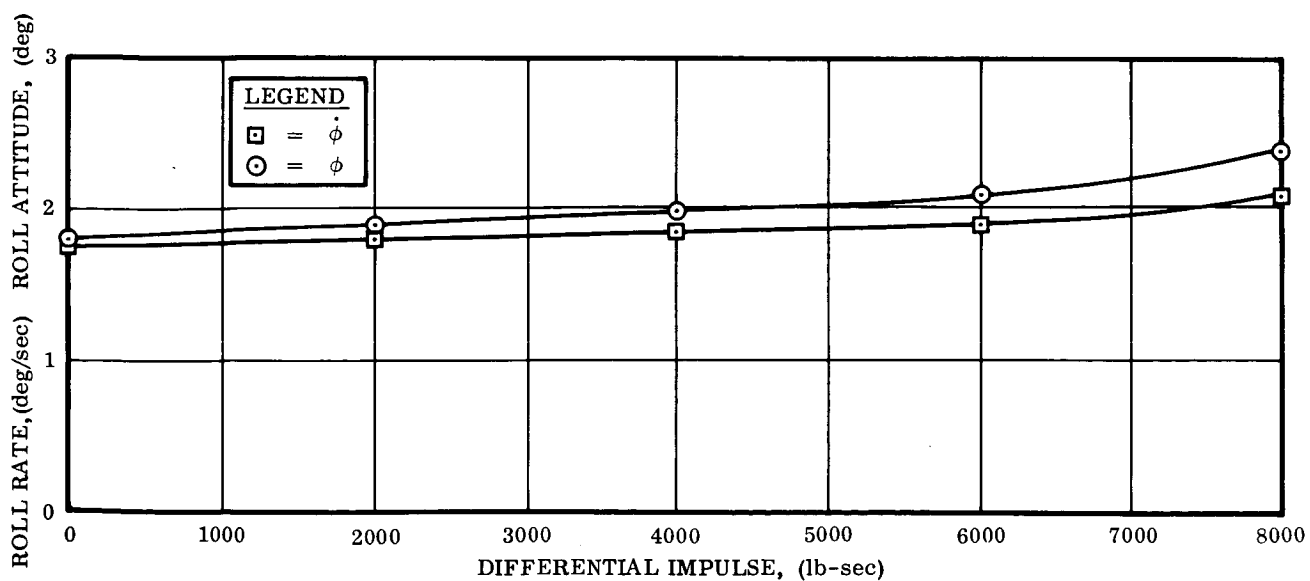


Figure 3.3-9. Maximum Roll Attitude and Roll Rate Excursions versus Differential Impulse

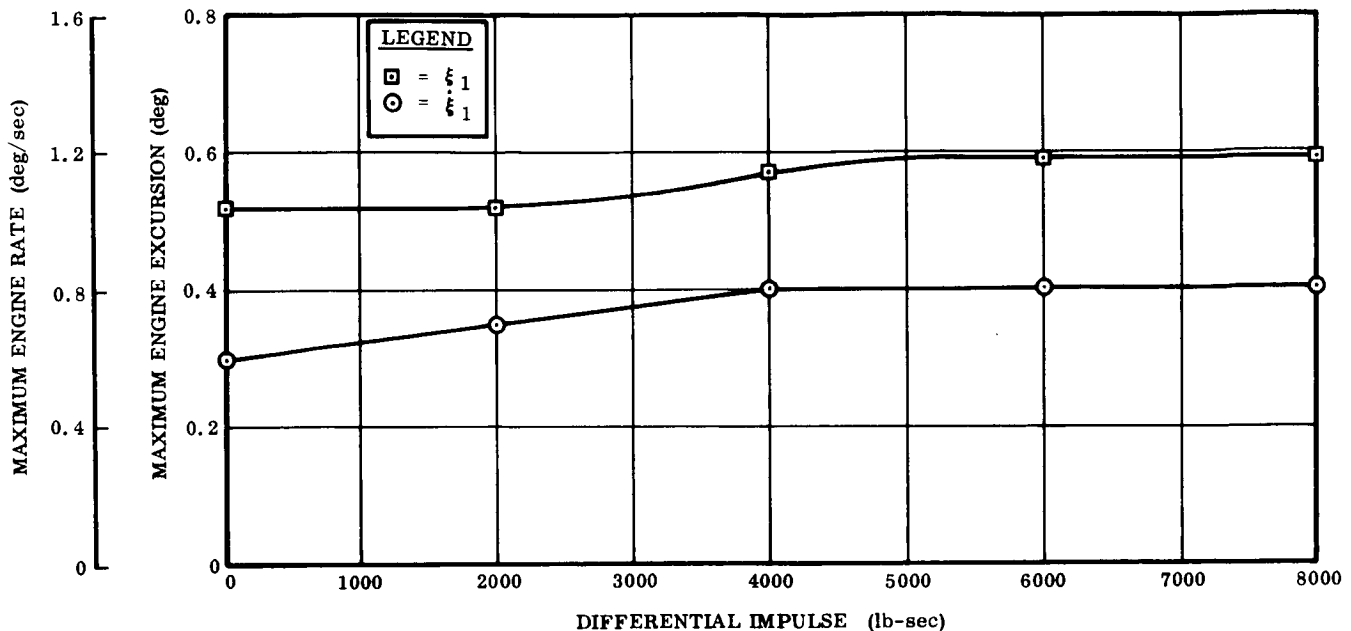


Figure 3.3-10. Maximum C-1 Engine Yaw Gimbal Angles and Gimbaling Rates versus Differential Impulse

The total impulse between MES and MES + 4 seconds, as measured in the pitch plane at MES is given by:

$$\Delta I_z = \int_0^4 \frac{1}{57.3} \left[T_1 (\theta - \delta_1 - \nabla_{\delta_1}) + T_2 (\theta - \delta_2 - \nabla_{\delta_2}) \right] dt \quad (3.3-3)$$

This quantity, (ΔI_z) , is assumed to be a measure of the impulse expended perpendicular to the desired attitude and is shown in Figure 3.3-11 as a function of ΔI . As expected, this impulse increases with increasing differential impulse. Note that it does not become large until the differential impulse is such as to cause the engines to gimbal to their stops, and that in all cases, it is less than that previously judged tolerable (50,000 lb-sec). This must be interpreted with caution, because the referenced 50,000 lb-sec figure includes the impulse that would be used in a recovery maneuver.

3.3.1.6 Conclusions and System Adequacy. The preceding analysis shows that the Centaur flight control system can cope with extremely large differential impulses or, to put it in another way, with large variations in engine behavior at ignition. For impulses six times as large as seen on the AC-2 and AC-3 flights, the vehicle will recover control before steering enable, and well within the allowable impulse loss. All parameters in the analysis were set in such a way as to produce the most detrimental effect. The analysis is strictly applicable only to the AC-2 configuration. However, an examination of Table 3.3-2 shows the AC-2 results to be indicative of AC-5 behavior. The control capability for very large differential impulses, where the engines have gimballed to their stops, is greater than for AC-2 because of the increased gimbal angle capability. The roll transient will be somewhat more severe because of the smaller roll moment of inertia on AC-5. In sum, the response at MES is basically similar to that shown here. The wide range of differential impulse shown to be within system capabilities indicates considerable margin in the AC-5 flight control system for controlling uneven engine ignition.

3.3.2 Steering Enable Transients. Four seconds after MES, guidance signals are admitted to the autopilot. At the instant of steering enable the autopilot will suddenly sense and respond to a large position error. The transient caused by steering enable is a function of the commanded attitude change and the

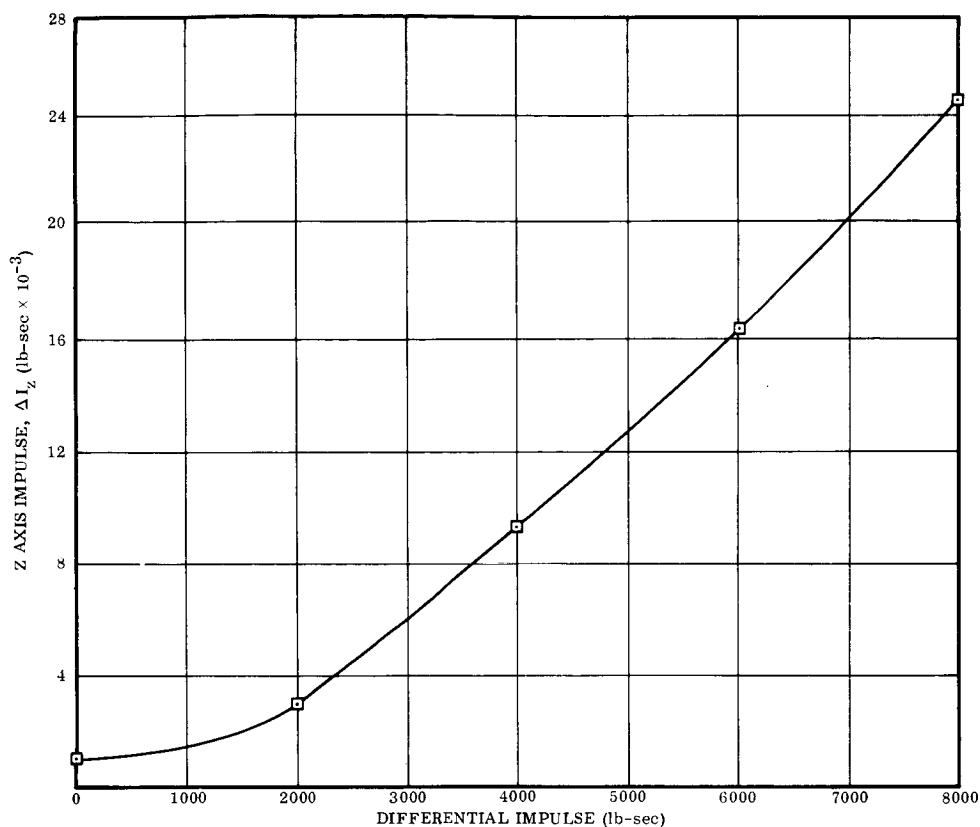


Figure 3.3-11. Total Perpendicular Impulse at MES + 4 versus Differential Impulse

Table 3.3-2. Comparison of Major Differences Affecting Ignition Transient Response of the AC-2 and AC-5 Centaur Vehicles at MES

QUANTITY	AC-2 MES	AC-5 MES
$\ddot{x}$ (ft/sec ²)	26.1	27.42
M_r (slugs)	919	834.2
M_T (slugs)	1,150	1,091
I_{yy_r} (slug-ft ²)	56,200	56,224
I_{xx_r} (slug-ft ²)	3,800	2,105.6
μ_δ (sec ⁻²)	4.17	4.18

channel selector limits. The magnitude of the command is equal to the angle between the vehicle roll axis and the reference attitude vector in the pitch (and yaw) planes. Roll plane is not of concern since the only roll attitude reference is that provided by the roll autopilot integrator. Since the channel selector limiting characteristics are well defined, the magnitude of the attitude command becomes the independent variable in any attempt to estimate the steering enable transients. For the AC-5 vehicle this command is primarily the result of the main engine ignition transient.

The steering enable transients of the AC-5 flight will be significantly less severe than those of the AC-2 and AC-3 flights. This is the result of 1) a smaller change in the reference attitude vector at MES + 4 relative to SECO/VECO (0.4 degree as compared to approximately 20 degrees for AC-2 and AC-3), and 2) AC-5 vehicle commanded rates are limited to 1.6 deg/sec (nominal), the result of a new channel selector limiter (Reference 15) compared to approximately 8 deg/sec for the AC-2 and AC-3 configurations. The characteristics of the new channel selector are shown in Figure 3.3-12. Thus the demands on the AC-5 flight control and hydraulic systems at steering enable will be minimal.

Reference 16 estimates the maximum (3σ) attitude dispersions at MES + 4 to be:

$$\text{pitch} \sim -0.4 \pm 8.04$$

$$\text{yaw} \sim \pm 7.44$$

The steady-state bias results from the steering vector change at MES + 4 relative to SECO/VECO. It should be borne in mind that there are many factors for which there is no detailed knowledge available. The above estimates are based on the following assumptions:

- a. The residual rates at SECO/VECO, used to calculate the attitude dispersions at MES, have been derived from Mercury flight data. This data is analogous to the conditions on Atlas/Centaur because both have simultaneous SECO/VECO. It is assumed that these rates remain constant from SECO/VECO to MES; i.e., it is assumed that a "clean" separation will occur.
- b. Attitude dispersions as the result of the main engine ignition transient assume a maximum differential impulse of 6,000 lb-sec. Saturation effects of the autopilot integrator and engine trim angle dispersions have been included. Autopilot errors arising from integrator drift have been neglected.

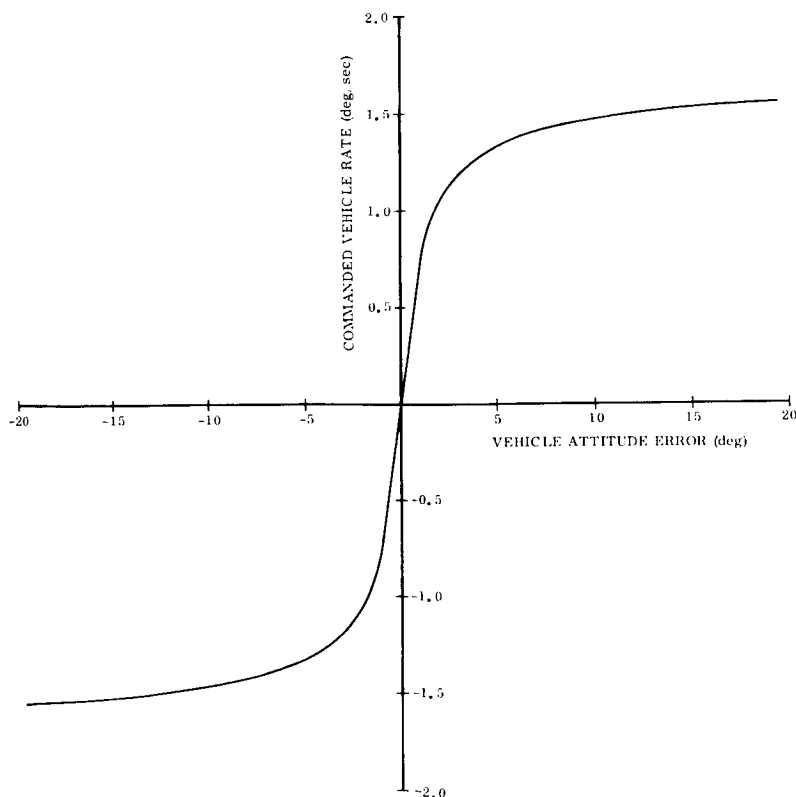


Figure 3.3-12. Channel Selector Limiting Characteristics

Figure 3.3-13 illustrates the effects of various step commands on peak vehicle angular accelerations, rates, and engine deflections as the result of the channel selector limiter. The data for this figure has been obtained from a single-plane analog computer simulation. The mathematical model assumes rigid body dynamics only and includes a nonlinear actuator (see Appendix B2.4.2.4).

3.3.3 Rigid-Body/Propellant-Sloshing Stability. Normally, two methods are used to analyze the propellant-sloshing/rigid-body stability, time-slice root loci and time-varying analog simulations. The root loci provide valuable insight regarding the stability characteristics of the system. The nonlinearities of the engine actuator system are represented by a simplified describing function. The disadvantage that prevents exclusive use of this technique is that nonlinear, amplitude-dependent propellant slosh damping cannot be accurately represented in the linear fixed-time analysis. Therefore, the usual procedure is to use the root loci for synthesis of the control system, then to use the time-varying analog simulation (which contains the effects of nonlinear propellant slosh damping) to obtain quantitative limit cycle predictions.

Preliminary analysis of the AC-5 configuration showed that its stability characteristics are very similar to those of its predecessors, the AC-2, AC-3, and AC-4 vehicles. Due to weight differences among the configurations, slightly different sloshing mode frequencies are evidenced at corresponding propellant levels. However, the general stability characteristics are the same. The most critical time is that at which the rigid-body control mode and the Centaur-oxidizer sloshing-mode frequencies approach each other. Since this occurs late in flight, the slosh-mode damping by the thrust barrel in the oxidizer tank is more than sufficient to prevent instability.

Previous correlation of root loci predicted frequencies with the analog computer and flight test data for AC-2 (Reference 17) has shown the mathematical model used to be reasonably accurate. Further, experience with the Centaur analog computer simulation has shown the time-varying and time-slice limit

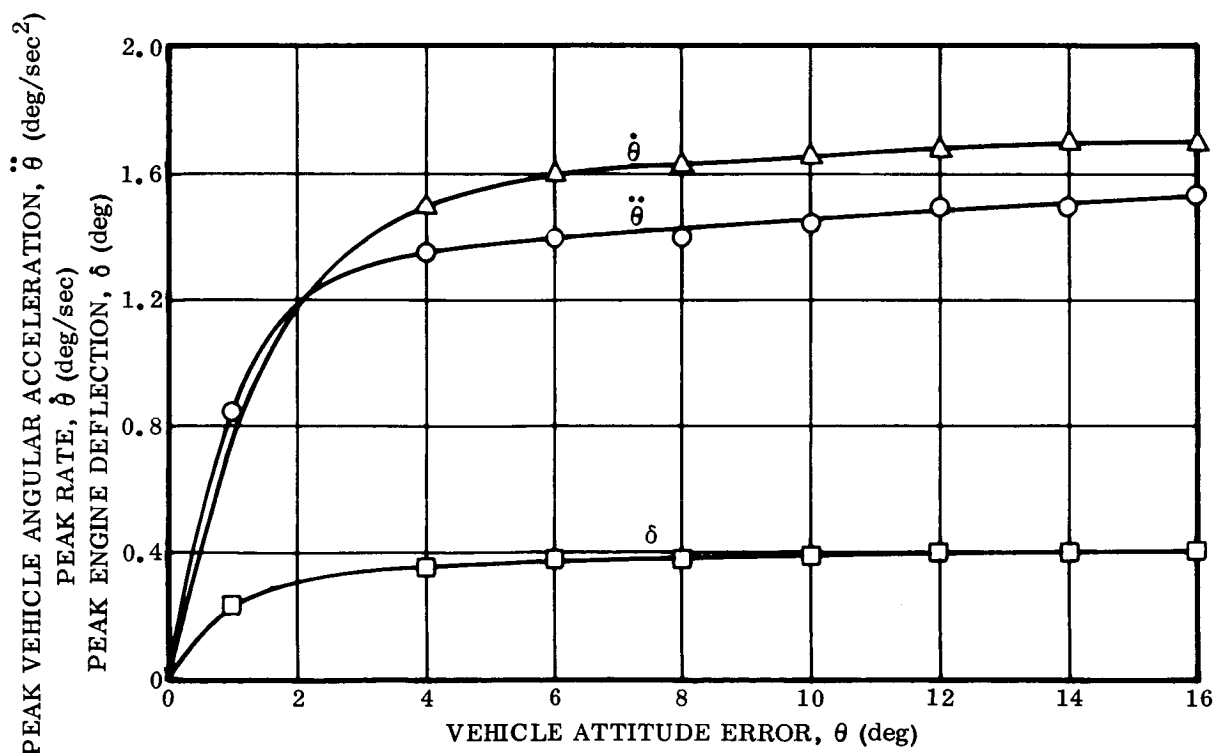


Figure 3.3-13. Steering Enable Transient at MES + 4

cycle amplitudes to be virtually the same. This is due to the very slowly varying nature of the coefficients of the differential equations relative to the frequencies of the dynamic oscillations. Consideration of the above and the root loci for AC-5 was the reason a time-varying analog computer simulation for AC-5 was not performed.

3.3.3.1 Mathematical Model. The analysis presented here is based on the pendulum analogy to duplicate the forces and moments produced by sloshing propellants. Only the fundamental sloshing mode in each propellant stage is considered. The vehicle body is assumed rigid. The dynamics of the overdamped Centaur rate gyros have been approximated by a single lag. The analysis is the same for the pitch and yaw planes, since the vehicle inertial properties in these two planes are essentially identical. The major difference is in the engine actuator system characteristics, as illustrated in the low frequency engine servoactuator approximation shown in Figures 3.3-14 and 3.3-15. Roll plane analysis considers the rigid body only, as the vehicle is assumed to rotate about its propellants.

3.3.3.2 Stability Criteria. No uniform set of stability criteria exists for a nonlinear system such as the Centaur stage flight control system. The usual concepts of settling time, response time, etc., have no meaning in a system whose response is nonlinear with respect to control input amplitude. For example, the settling time, as usually defined, may well be infinite if the limit cycle amplitude is of the same order of magnitude as the command.

In a situation like this, flexible approach must be used in synthesizing a system. A rigid criterion could unnecessarily penalize a system design because of cost or unreliability. Basically, then, the problem is to keep the control loads to a minimum, while maintaining the proper attitude, and using as simple a design as possible. Also in considering loads, one is interested not only in their magnitude, but also their history; for example, propellant duct flexing should be kept to a minimum in order to reduce fatigue.

In an evaluation of the existing Centaur flight control system, if the control loads are well within hydraulic system capability, the limit cycles are stable, proper attitude is maintained, and commanded engine excursions do not exceed gimbaling capability (except during special situations like the ignition transient condition), then the system is judged to be adequate.

3.3.3.3 Stability Analysis. Figures 3.3-16 through 3.3-20 depict the stability analysis and show the root loci at five flight times during powered flight. These loci together with the low frequency engine servoactuator approximation, Figures 3.3-14 and 3.3-15, can be used to predict possible engine limit cycles. It can be seen that there is a possibility of three limit cycles throughout most of powered flight. The engine gimbal angle will oscillate about its trim condition with an amplitude and frequency that correspond to the particular conditions prevailing, such as amplitude of sloshing propellants, external disturbances, and the degree of coupling between the sloshing and rigid-body dynamics. An examination of the root loci indicate that all possible limit cycles are self-exciting, i.e., instability is indicated for low values of K'_C , hence low amplitudes.

Since the roll moment of inertia has been assumed to be independent of the changing liquid oxygen and liquid hydrogen masses, the roll plane stability characteristics remain the same throughout Centaur powered flight. Figure 3.3-21 presents a typical roll plane root locus.

The pitch plane root loci predict stable limit cycles throughout powered flight. The most critical flight time is encountered about 40 seconds prior to MECO. Figure 3.3-19 shows that the liquid oxygen tank slosh dipole has flipped to a potentially unstable configuration (pole below the zero), a situation similar to that encountered in the analysis of the AC-3 vehicle. There exists, however, one significant difference: the rigid body frequency of the AC-5 configuration is lower than that of AC-3. This decreases coupling between the liquid oxygen sloshing mode and the rigid body control mode, hence the engine response is decreased. The lower frequency has resulted from an increase in weight and moment of inertia of the

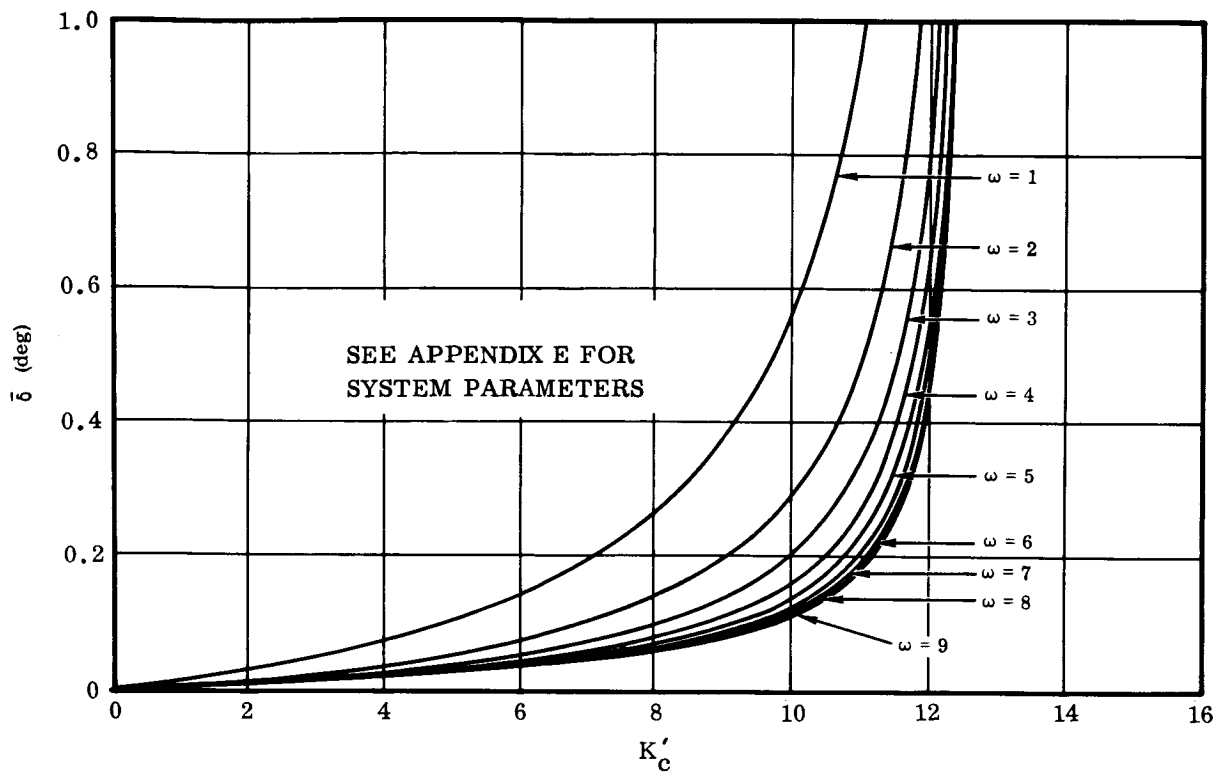


Figure 3.3-14. Low Frequency Engine-Servoactuator Approximation, $K'_C/(s+K'_C)$ (Pitch Actuator)

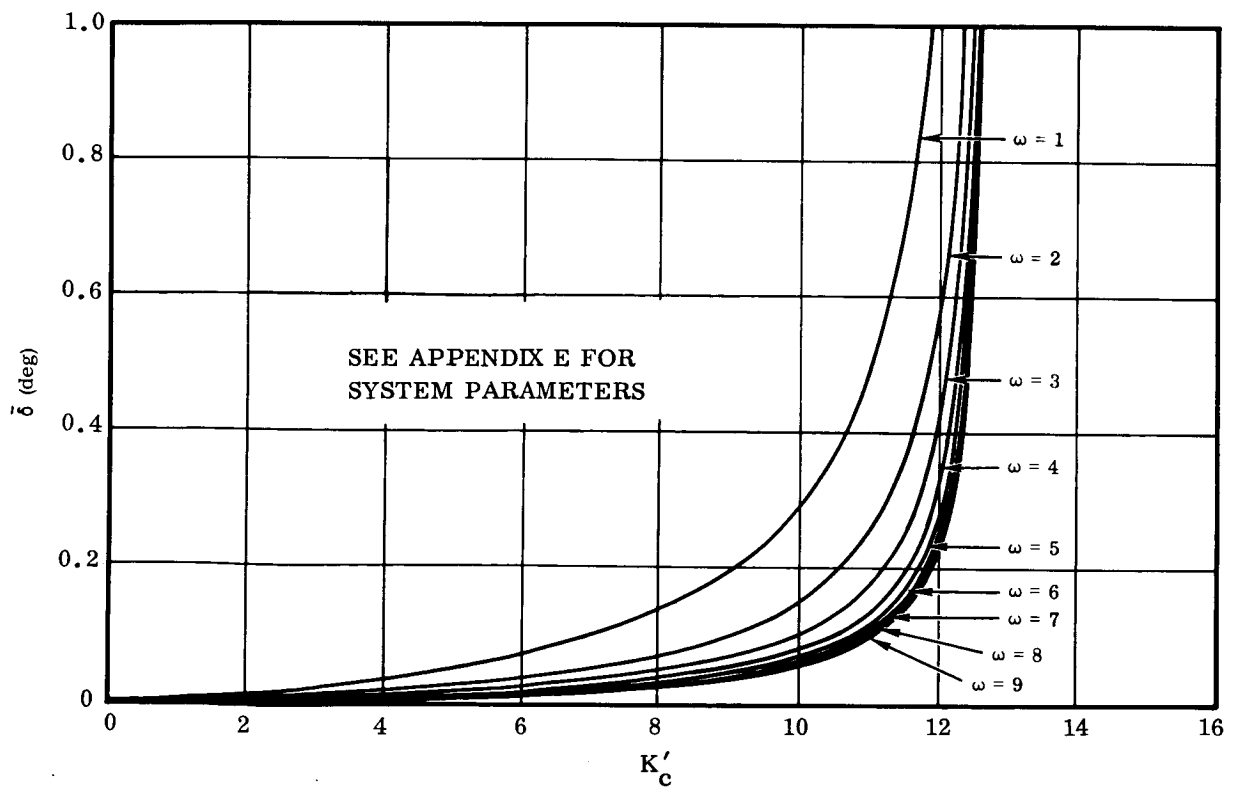


Figure 3.3-15. Low Frequency Engine-Servoactuator Approximation, $K'_C/(s+K'_C)$ (Yaw/Roll Actuator)

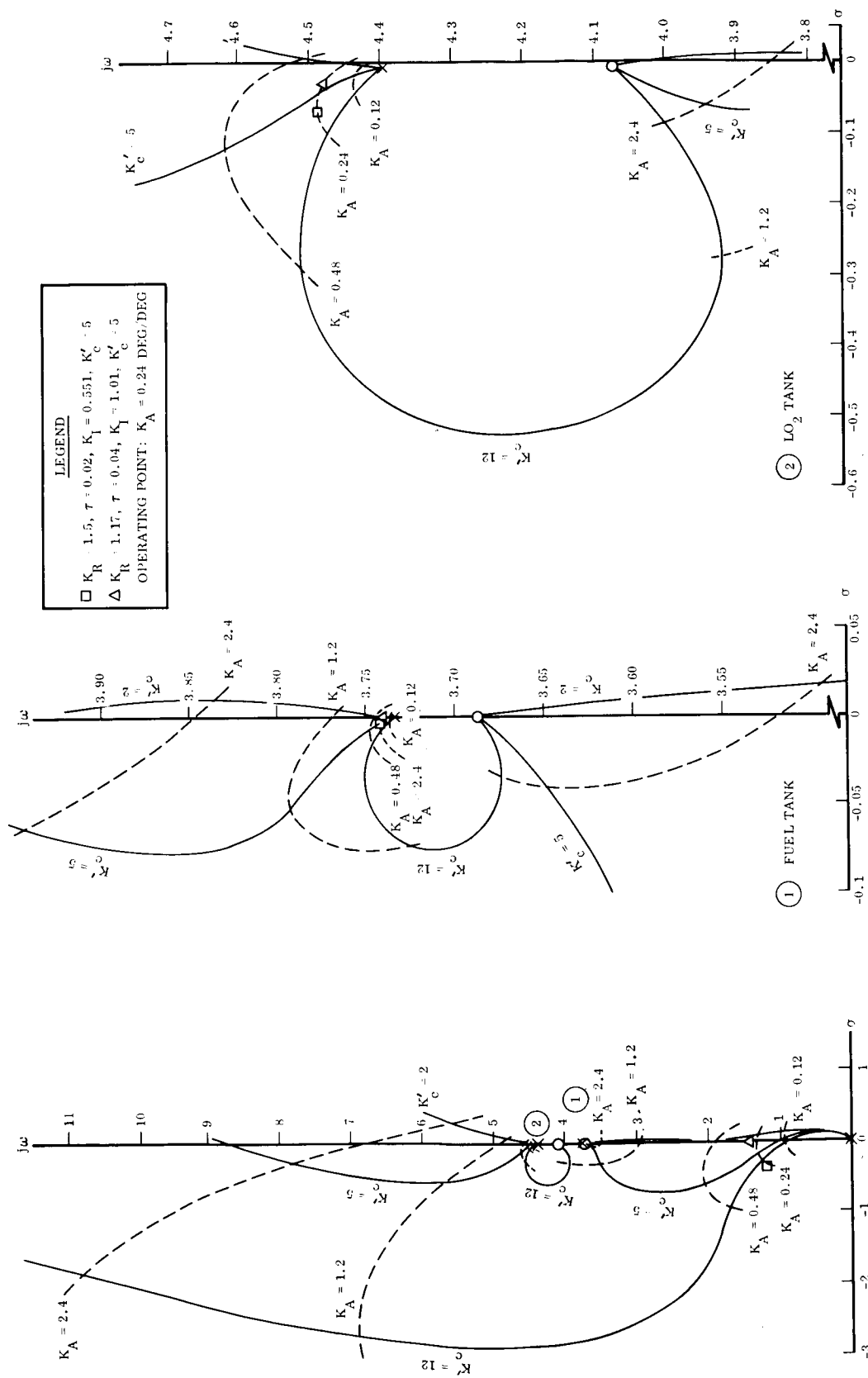


Figure 3.3-16. AC-5 Centaur Stage Rigid Body/Propellant Sloshing Stability at MES + 4 (Steering Enable)

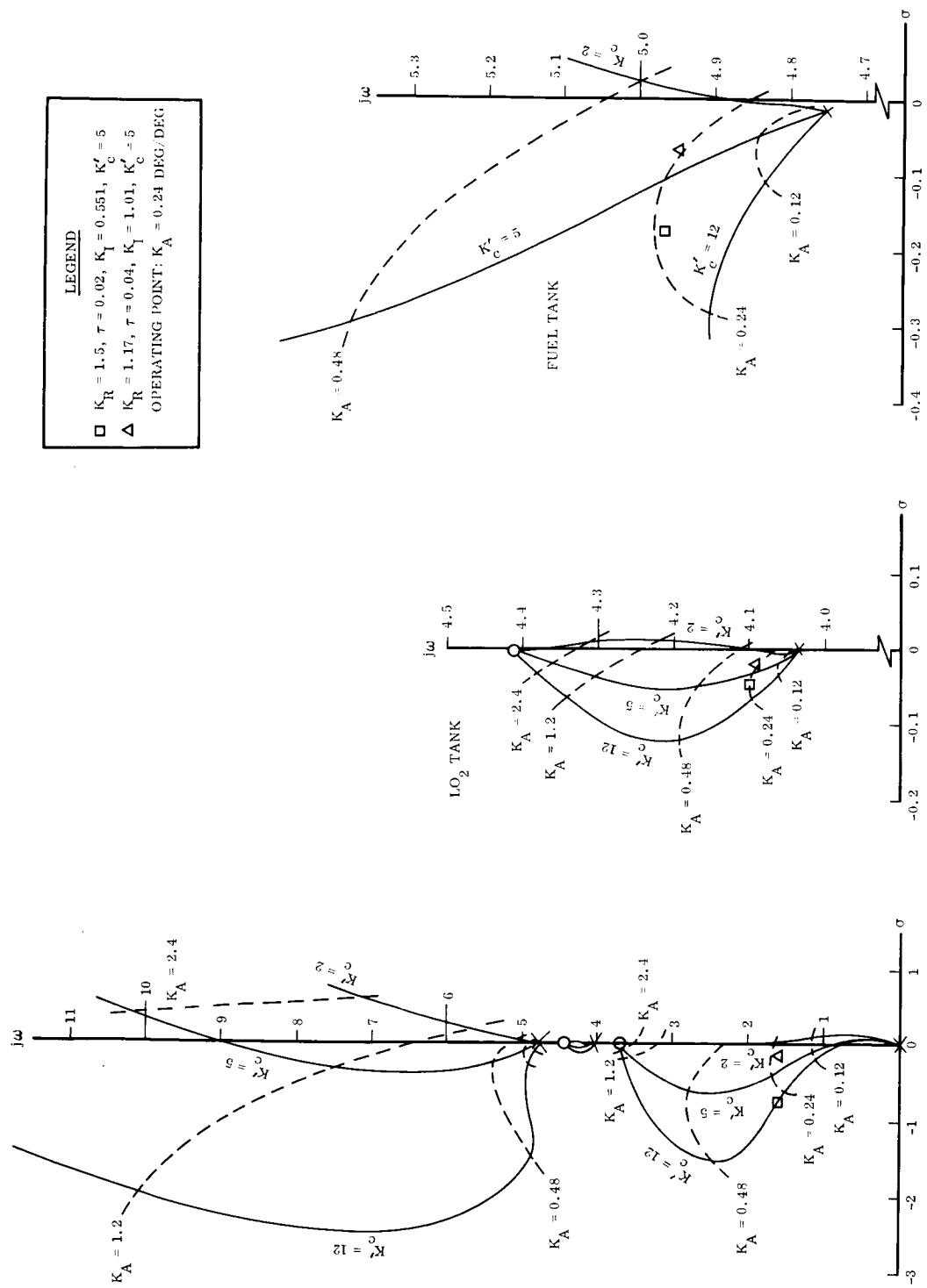


Figure 3.3-17. AC-5 Centaur Stage Rigid Body/Propellant Sloshing Stability at MES + 210

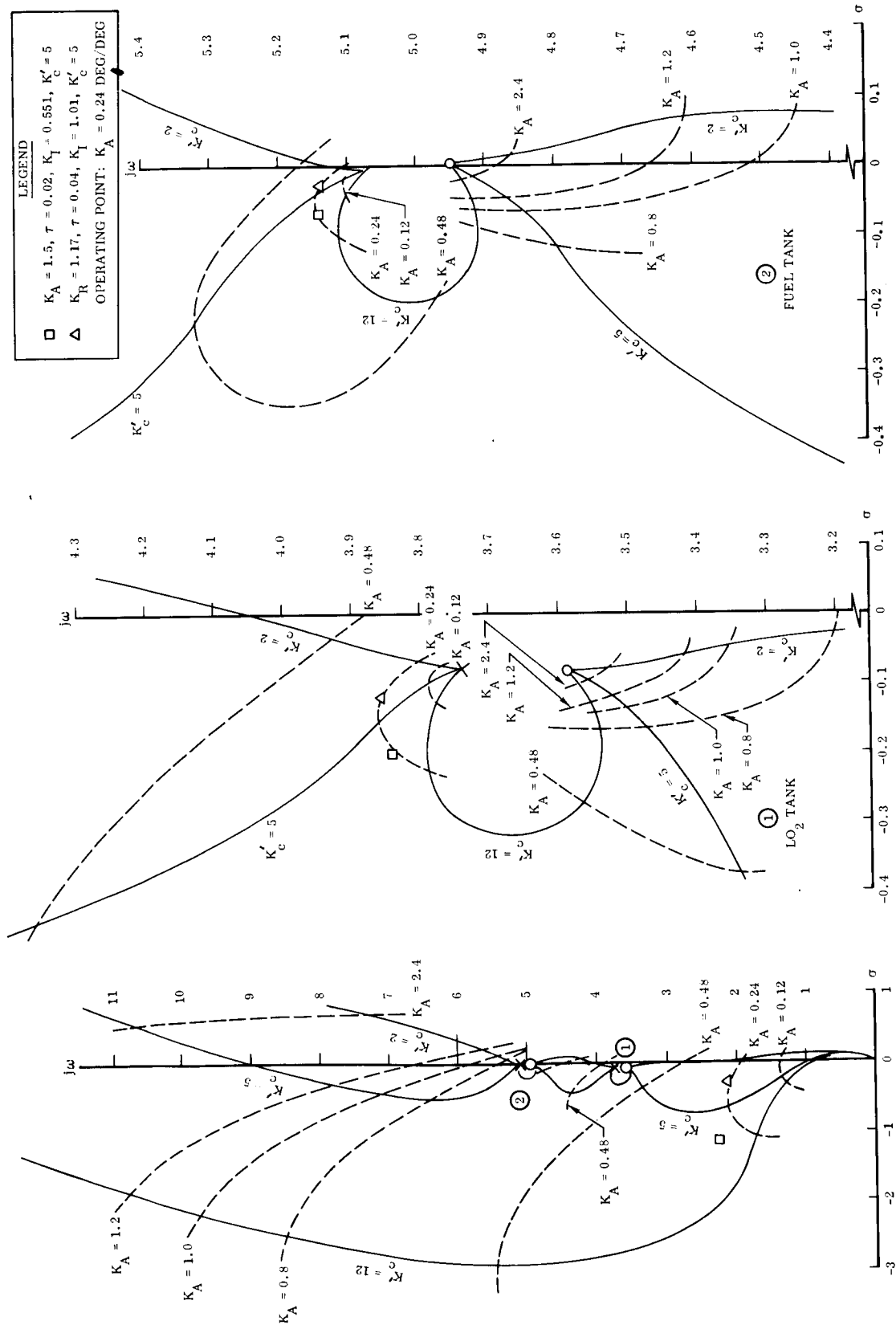


Figure 3.3-18. AC-5 Centaur Stage Rigid Body/Propellant Sloshing Stability at MES + 310

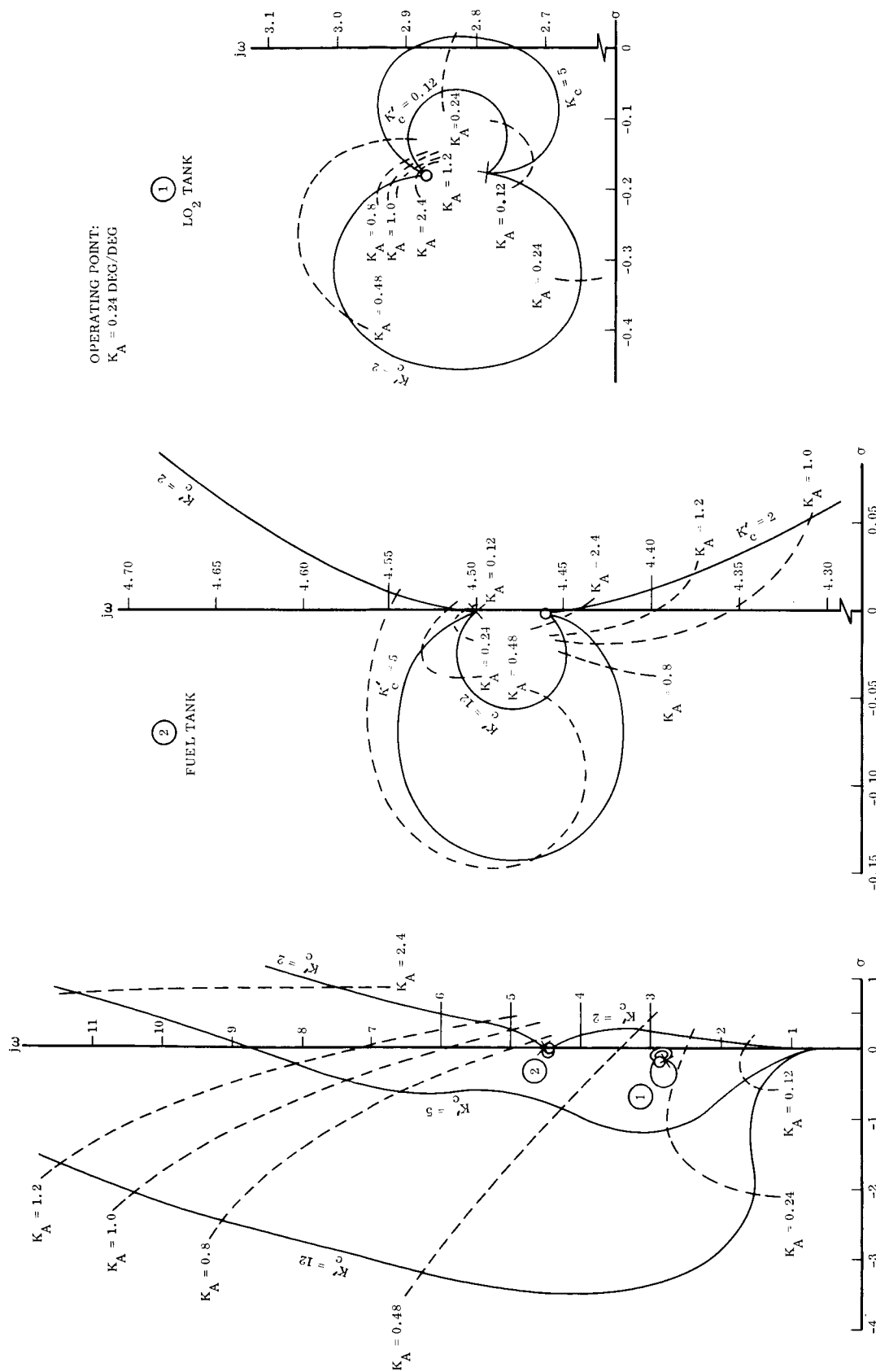


Figure 3.3-19. AC-5 Centaur Stage Rigid Body/Propellant Sloshing Stability at MES + 370

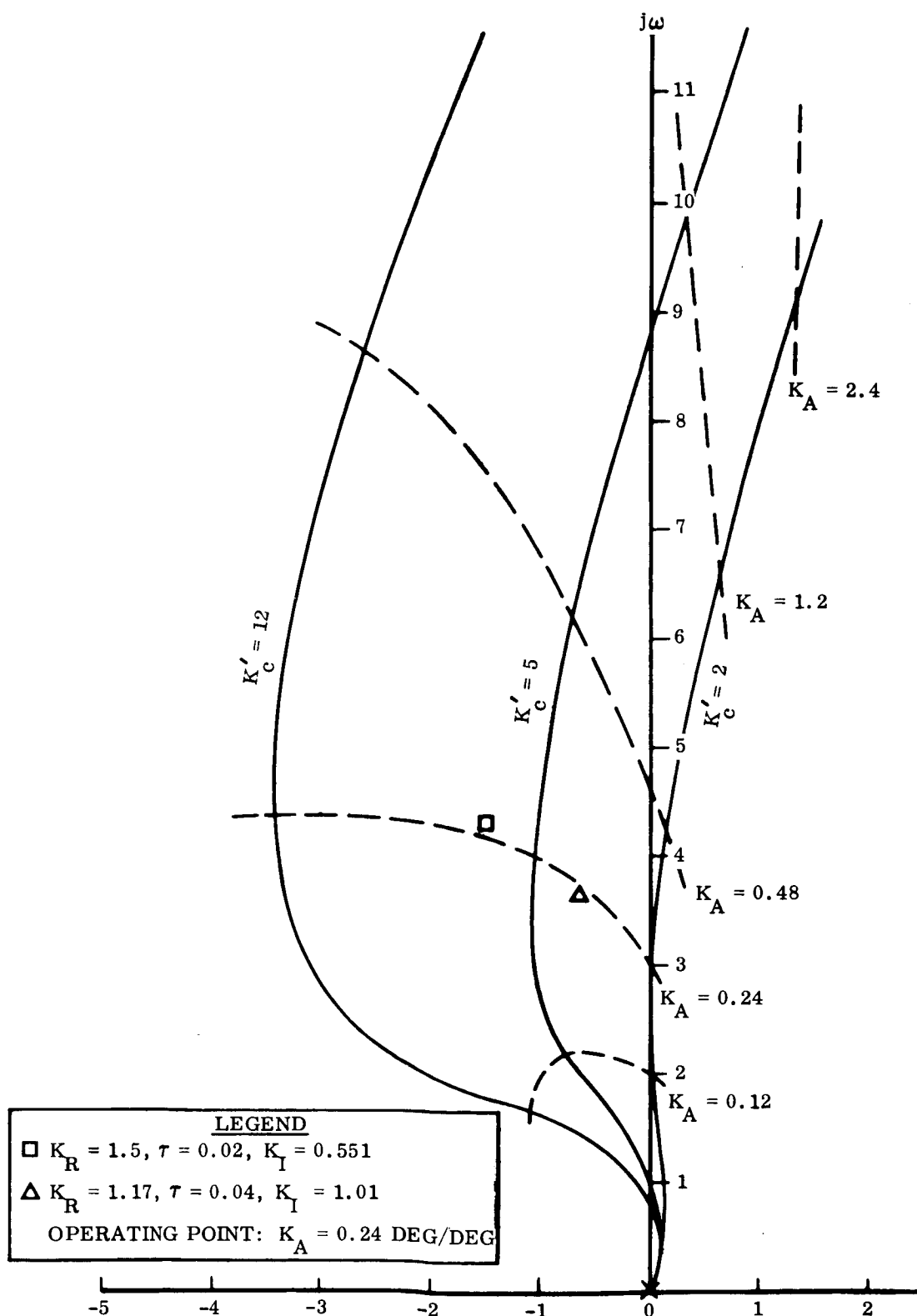


Figure 3.3-20. AC-5 Centaur Stage Rigid Body/Propellant Sloshing Stability at MECO

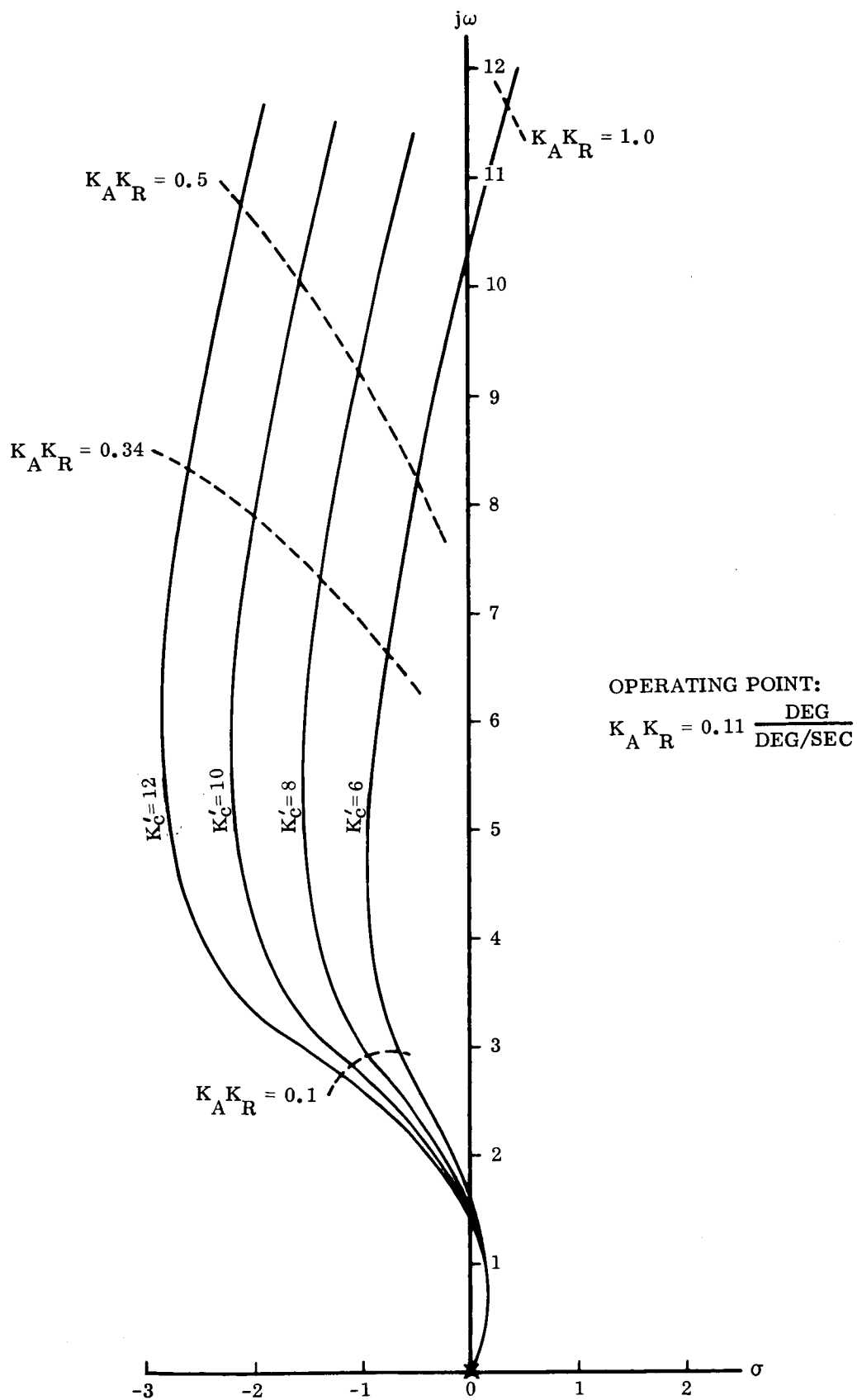


Figure 3.3-21. AC-4 Centaur Stage Rigid Body Roll Plane Stability

AC-5 vehicle over that of the AC-3 configuration. Figure 3.3-19 together with the pitch low frequency engine servomotor curve, Figure 3.3-14, predicts an engine limit cycle amplitude of 0.05 degree.

This predicted limit cycle amplitude is very conservative since it is based on linear wall wiping damping only. The fluid level at this time is approximately three inches below the top of the thrust barrel. At this fluid level it is not unreasonable to expect a large amount of nonlinear amplitude-dependent damping as the result of the thrust barrel and baffle support structures. Under these circumstances, the limit cycle amplitude is likely to be much smaller than predicted. The analysis performed uses damping data from Reference 18. This data is based on experimental slosh test results for the Centaur oxidizer tank.

Additional points are shown on the loci showing the effect of autopilot phase lead and phase lag extremes that may exist for $K_C' = 5$, and nominal K_A with K_I , K_R , and τ taking extreme values from their tolerance band as set forth in Reference 2. Although the extreme phase lag condition noticeably deteriorates all dynamic responses, the system remains adequately damped and stable.

3.3.3.4 Stability Summary. In summarizing stability, the root loci predict small rigid body limit cycles with still smaller slosh limit cycles throughout powered flight. Because of the frequency separation between the liquid oxygen sloshing and rigid body modes, it is expected that the slosh limit cycle will not couple with the rigid body control mode. Thus it is expected that the rigid body mode will be the primary driving force of the flight control system throughout powered flight.

Figures 3.3-22 and 3.3-23 present a comparison of the root locus predicted limit cycle frequencies and amplitudes of the AC-3, AC-4, and AC-5 configurations. Note that although the AC-4 powered flight phase consists of two portions separated by a coast phase, the vehicle weight and inertia data do not vary significantly during coast; hence these figures have been plotted versus main engine burn time. In general, the AC-5 limit cycle amplitudes are too small to be accurately determined from the root loci. Liquid oxygen sloshing limit cycles have not been predicted near MECO because the corresponding loci, Figures 3.3-19 and 3.3-20, show that the liquid oxygen tank is well damped.

Figure 3.3-22 shows that the AC-5 slosh and rigid body frequencies in general fall between those of AC-3 and AC-4, nearly matching those of AC-4. The AC-5 payload and dry vehicle weights are lighter than those of AC-4, resulting in slightly higher rigid body and sloshing frequencies from the decrease in vehicle inertia and increase in axial acceleration respectively (for comparable tanking levels). However, the AC-5 is not nearly as light as the AC-3. A comparison of the predicted limit cycle amplitudes (Figure 3.3-23) shows the predicted slosh limit cycle amplitudes of AC-3, AC-4, and AC-5 are approximately the same. The AC-5 rigid body limit cycle amplitude is close to that of AC-4.

The Centaur hydraulic system's ability to sustain a limit cycle at MECO is shown in Figure 3.3-24. The upper curve represents the maximum engine deflection which can be sustained as a function of assumed limit cycle frequency, while the lower curve illustrates the maximum amplitudes and frequencies predicted by root locus analysis for the dominant rigid body mode throughout powered flight. It is apparent that the predicted amplitudes fall well below the hydraulic system capability for all observed frequencies. Note that these predicted amplitudes do not include those expected during transient conditions, i.e., MES, steering enable, or MECO. The upper curve is based on the assumption of a sinusoidal limit cycle and parameters used for AC-5 just prior to MECO. This represents a worst-case condition since inertia forces restraining the motion of the engines are largest when the vehicle inertias are smallest and accelerations greatest. In addition, worst-case conditions have been assumed for thrust misalignment, coulomb and viscous friction, and duct restraints. See Appendix B2.4.2.5 for the mathematical description.

3.3.4 Autopilot/Guidance Loop Stability. This section is concerned with the stability of the autopilot/guidance loop. Centaur obtains its position reference through the guidance system. The usual steering equation for reference attitude steering is modified by addition of an integral term which corrects for

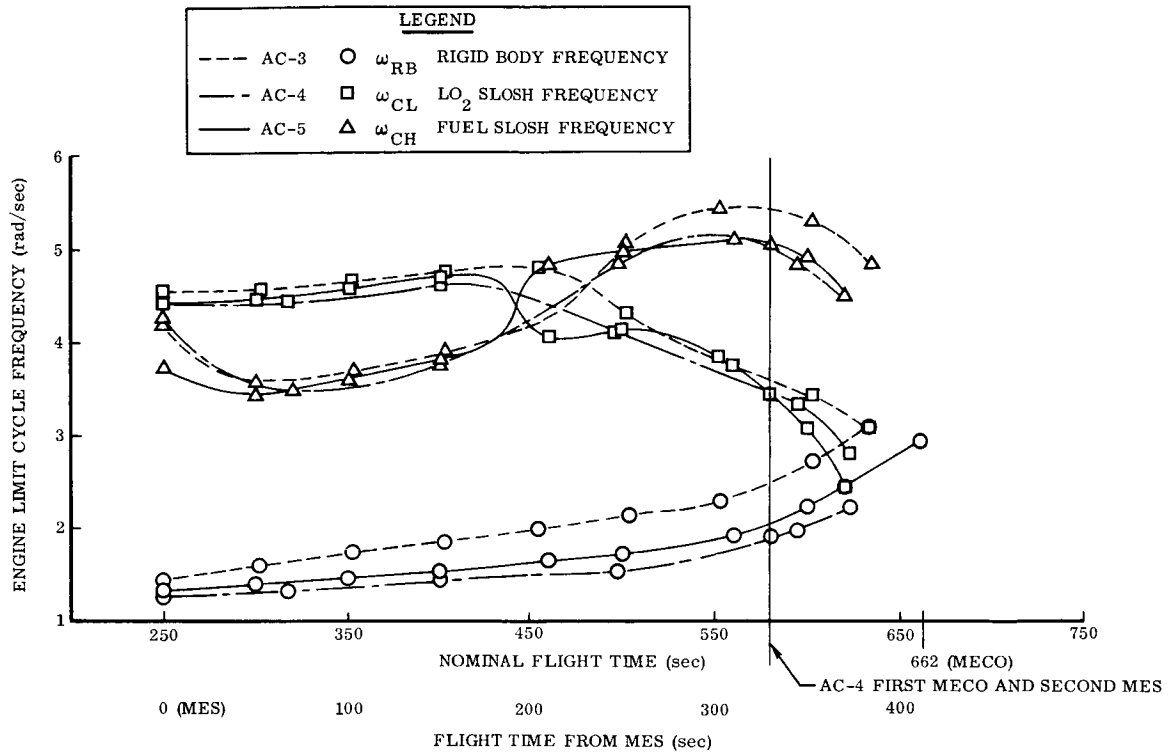


Figure 3.3-22. Root Locus Predicted Engine Limit Cycle Frequencies

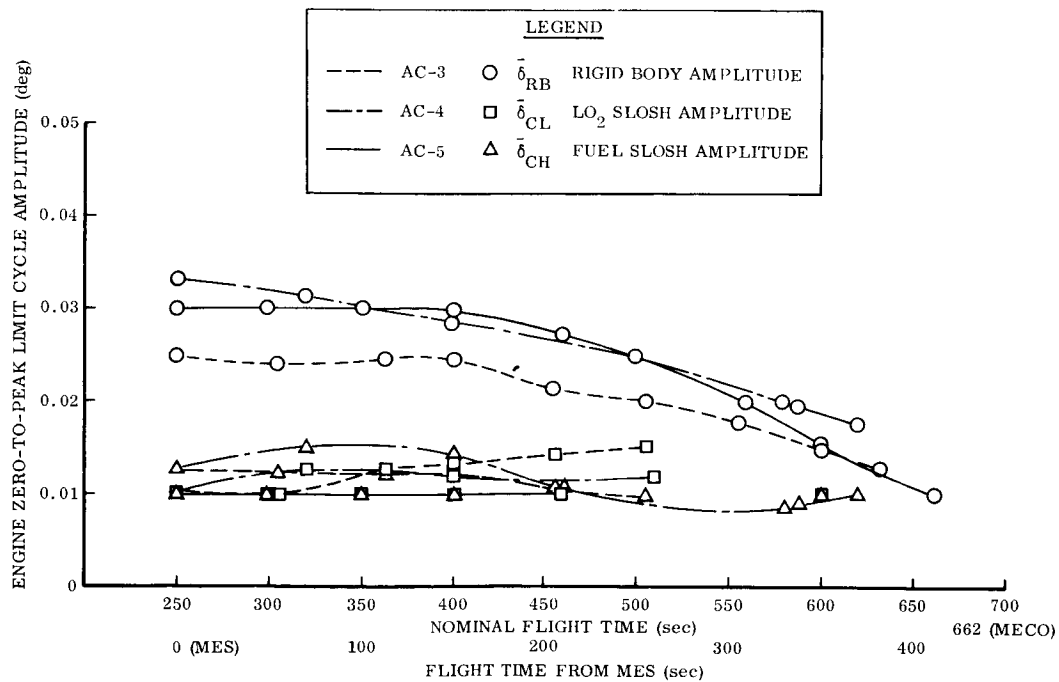


Figure 3.3-23. Root Locus Predicted Engine Limit Cycle Amplitudes

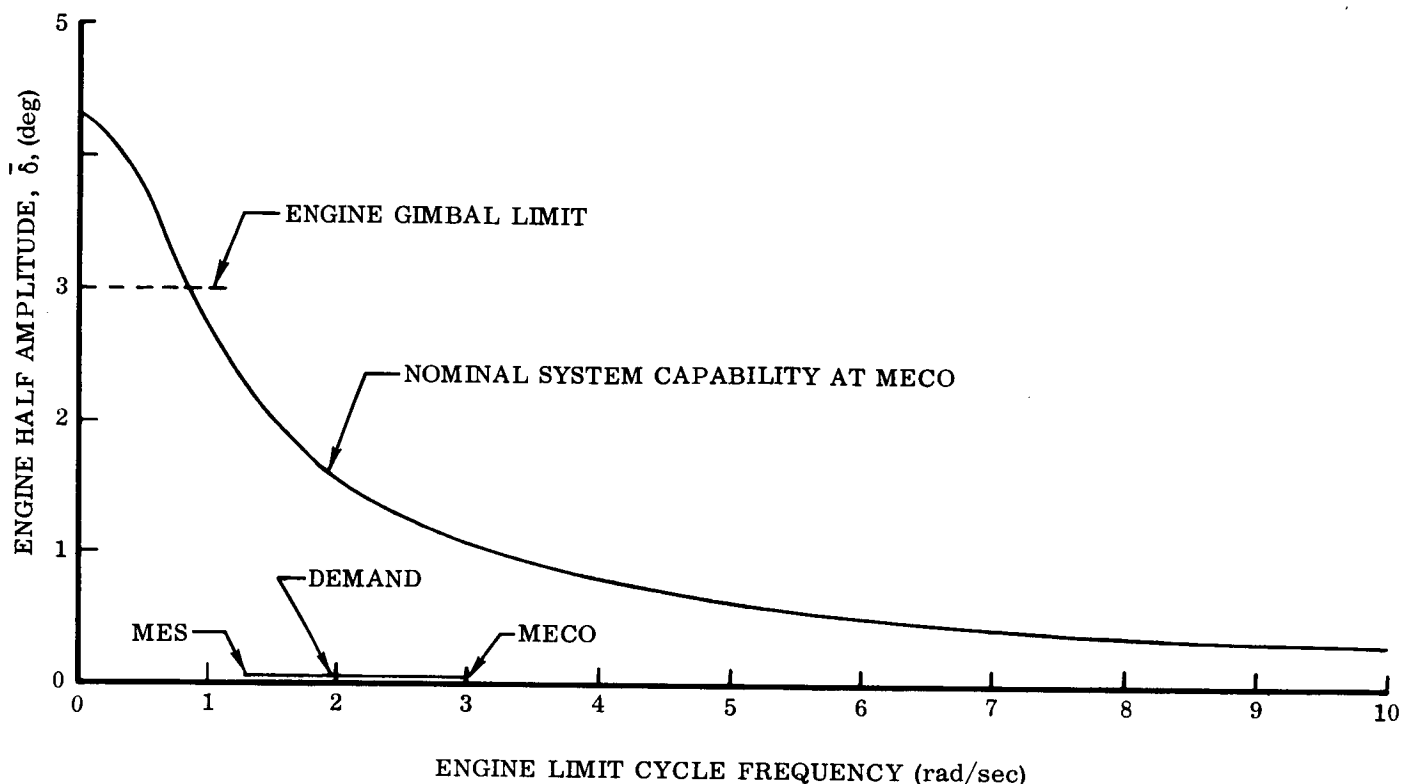


Figure 3.3-24. Comparison of Hydraulic System Limit Cycle Capability at MECO with Predicted Demand

center-of-gravity offset and similar errors, so that the thrust vector, rather than the vehicle centerline, is aligned with the reference attitude vector. This vector is computed by the on-board digital computer. Guidance signals are applied to the autopilot in a discrete rather than a continuous fashion. Under these conditions, the system is described by a combination of linear difference and differential equations. Development of the mathematical model representing the feedback dynamics of the guidance loop is presented in Appendix B4.

Autopilot/guidance loop stability for the AC-5 configuration has been analyzed by the time-fixed root locus technique. The addition of the guidance loop to the autopilot causes considerable complexity to the root locus analysis. Since signals are not applied to the autopilot in a continuous fashion, the conventional Laplace transform method of analysis becomes unduly cumbersome. The z -transform method of analysis is used instead, and root loci illustrating autopilot/guidance loop stability are plotted in the z -plane.

Perturbations of the reference attitude vector, $\bar{f}$, with vehicle short-period motions are neglected (see Appendix B4). As in the conventional root locus analysis, nonlinearities of the engine servoactuator are approximated by a describing function. The mathematical model used to represent the vehicle dynamics is the same used for the rigid-body/propellant-sloshing stability analysis (see Section 3.3.3.1). Because of the large separation between the flexible vehicle bending mode frequencies and the guidance system frequencies, the vehicle bending modes do not couple with the guidance system. Vehicle bending is therefore neglected in this analysis.

Several bases exist on which a stability criterion can be predicted. The usual criterion, in reference to the flight control system, is to maintain "adequate" control and to minimize possible engine limit cycle amplitudes. It is also desirable to keep the speed of response high and the settling time at a minimum while minimizing overshoot, thus maximizing performance. For application to the integral guidance control loop, the requirements are similar. The criterion used here is that the major root must be 0.3 damped.

Figures 3.3-25 through 3.3-27 are z -plane root loci illustrating autopilot/guidance loop stability during the powered flight phase. Roll plane stability is not analyzed, since it is not influenced by the guidance loop. The poles of the z -loci (varying k_I') are the closed-loop roots of the system excluding the guidance loop with nominal autopilot gains. These loci have been generated for a $K_C' = 5$ only. In general the slosh and rigid body poles lie inside the unit circle. At MES + 370, however, an LO₂ sloshing limit cycle can exist as seen from the corresponding continuous root locus. Thus the z -locus at this time apparently shows an LO₂ sloshing instability (roots outside the unit circle), with the K_C' of 5 whereas this is only a low-amplitude limit cycle. The loci show that at the operating gain ($k_I' = 0.25$), the guidance mode frequency remains approximately constant at 0.32 rad/sec.

The root locus analysis shows that vehicle dynamic stability is not deteriorated significantly by the guidance loop. Adequate vehicle stability with the guidance loop closed has been obtained by selection of the proper loop gain. The existence of only small coupling between the guidance system and the vehicle dynamics is illustrated by:

- a. The closed-loop rigid body and propellant sloshing poles are close to the open-loop poles.
- b. The propellant slosh dipoles are small.
- c. The rigid body pole is well damped, even when substantial coupling exists.

These results are not surprising since there is approximately a five-to-one separation between the lowest vehicle dynamic frequency and that of the guidance mode. This frequency separation, and thus the lack of coupling between the guidance mode and vehicle dynamics, is evidenced by the fact that the guidance mode root does not move significantly throughout flight. These results show that at the operating gain the guidance loop does not couple with the vehicle dynamics and that the system is adequately stabilized.

3.3.5 Elastic Vehicle Bending Stability. Fundamentally, the phenomenon of elastic vehicle/autopilot coupling is caused by the feedback of "parasitic" signals arising from vehicle elasticity. The feedback sensors, in this case the inertial guidance platform and rate gyros, are mounted on the vehicle structure and sense missile body bending in addition to rigid body attitude changes. These modal contributions provide a gimbaling command to the engine thrust chambers. Since the thrust vector movement, in turn, excites the body bending modes, a closed loop is formed which can lead to a divergent oscillation at the bending frequency, if excessive gains and incorrect phasing are present. The problem is further aggravated because the damping associated with these modes is very low.

This coupling phenomenon is usually of paramount importance during the booster phase of flight where the conditions are such that the vehicle is most susceptible to coupling of this nature. These conditions include:

- a. A large slenderness ratio, i.e., length-to-width (diameter) ratio.
- b. Amplification of feedback signals proportional to vehicle elastic deformations by the high autopilot gains needed for control of the vehicle through the high aerodynamic pressure region of flight.
- c. Large engine thrust forces.
- d. Low elastic mode frequencies.
- e. The presence of high velocity wind gusts.

However, during Centaur powered flight, the reverse is true:

- a. The slenderness ratio is low.
- b. The autopilot gains are low.

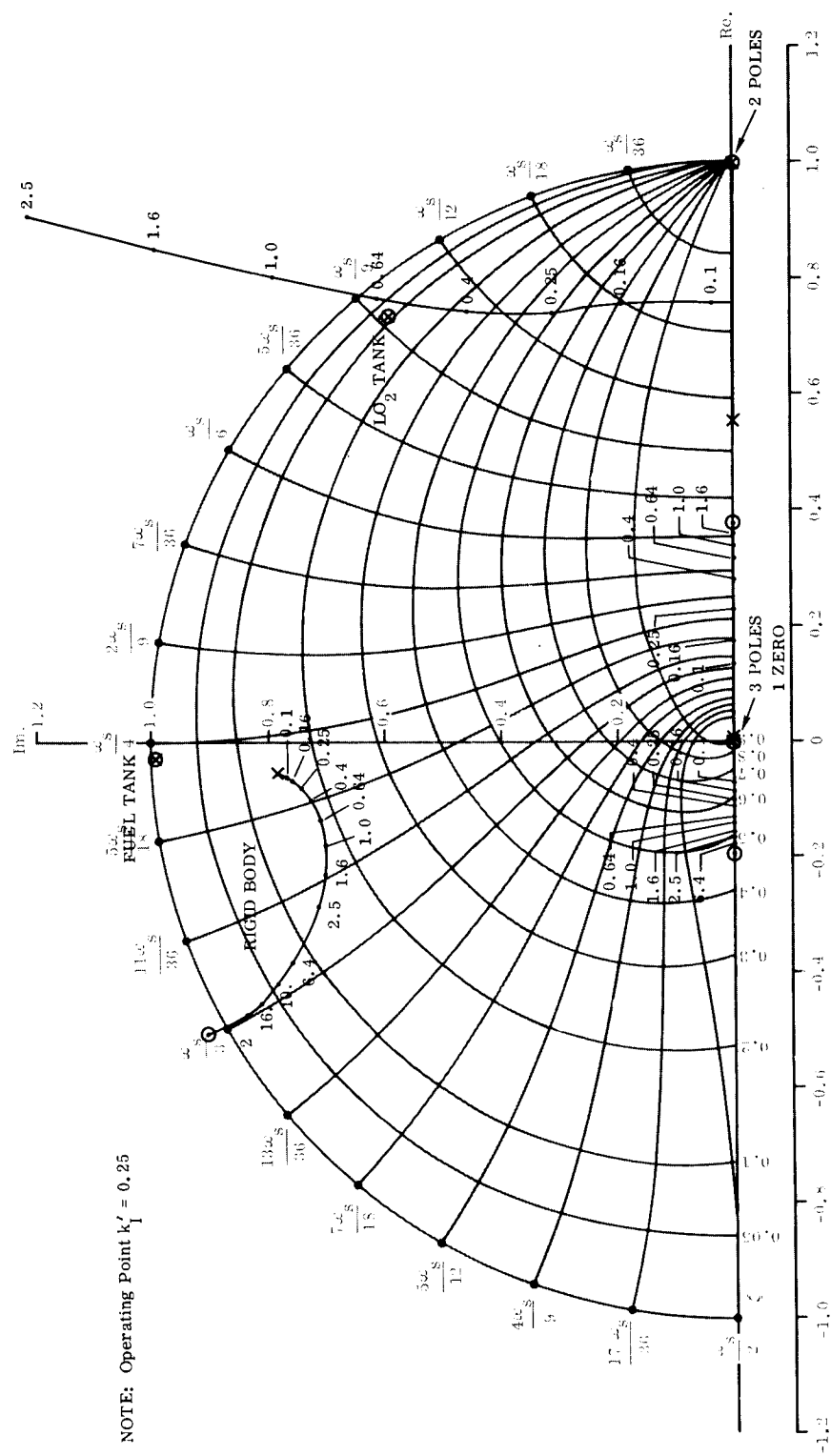
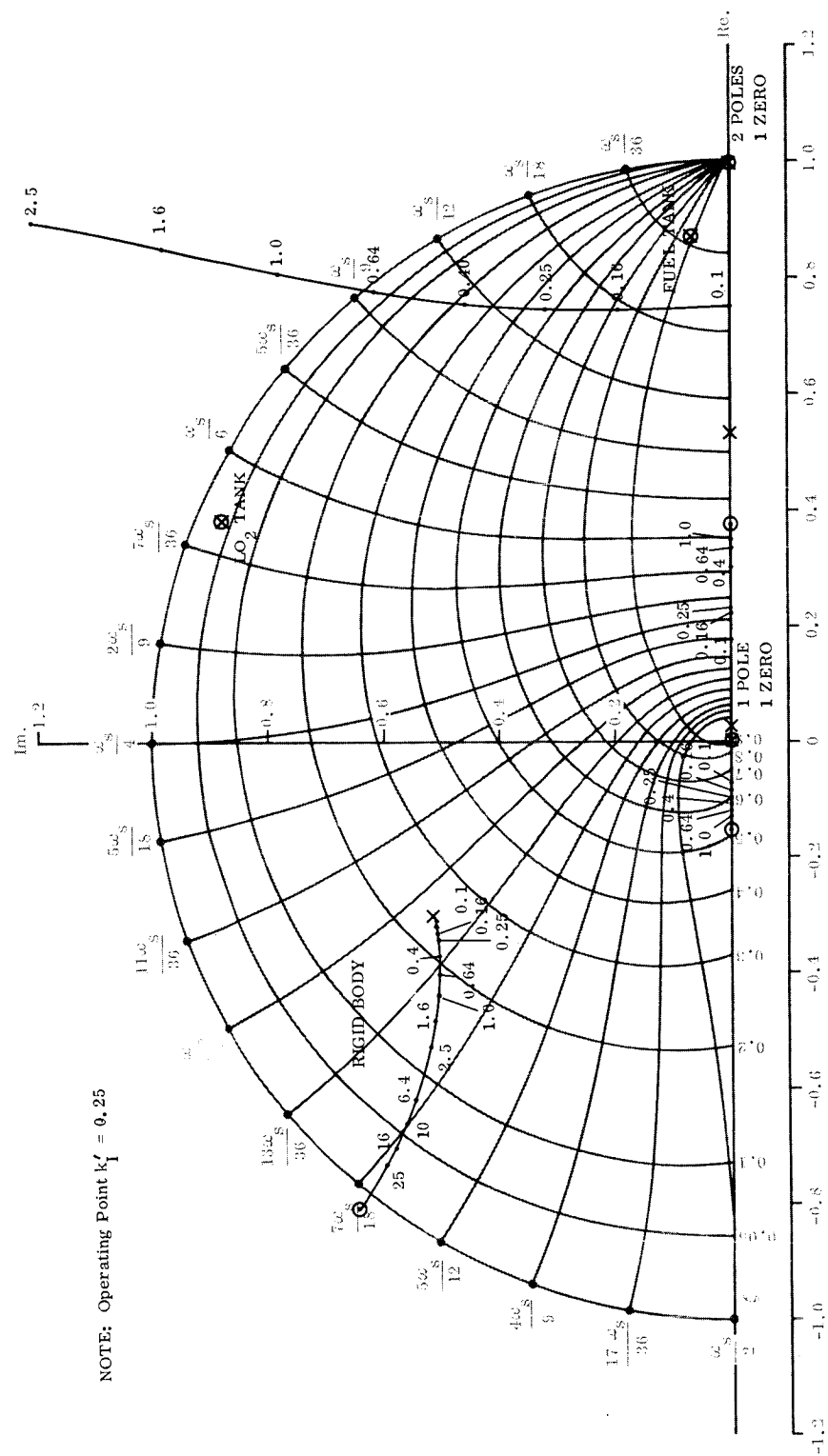


Figure 3.3-25. AC-5 Centaur Stage Autopilot/Guidance Loop Stability at MES + 4



NOTE: Operating Point $k_1' = 0.25$

Figure 3.3-26. AC-5 Centaur Stage Autopilot/Guidance Loop Stability at MES + 210

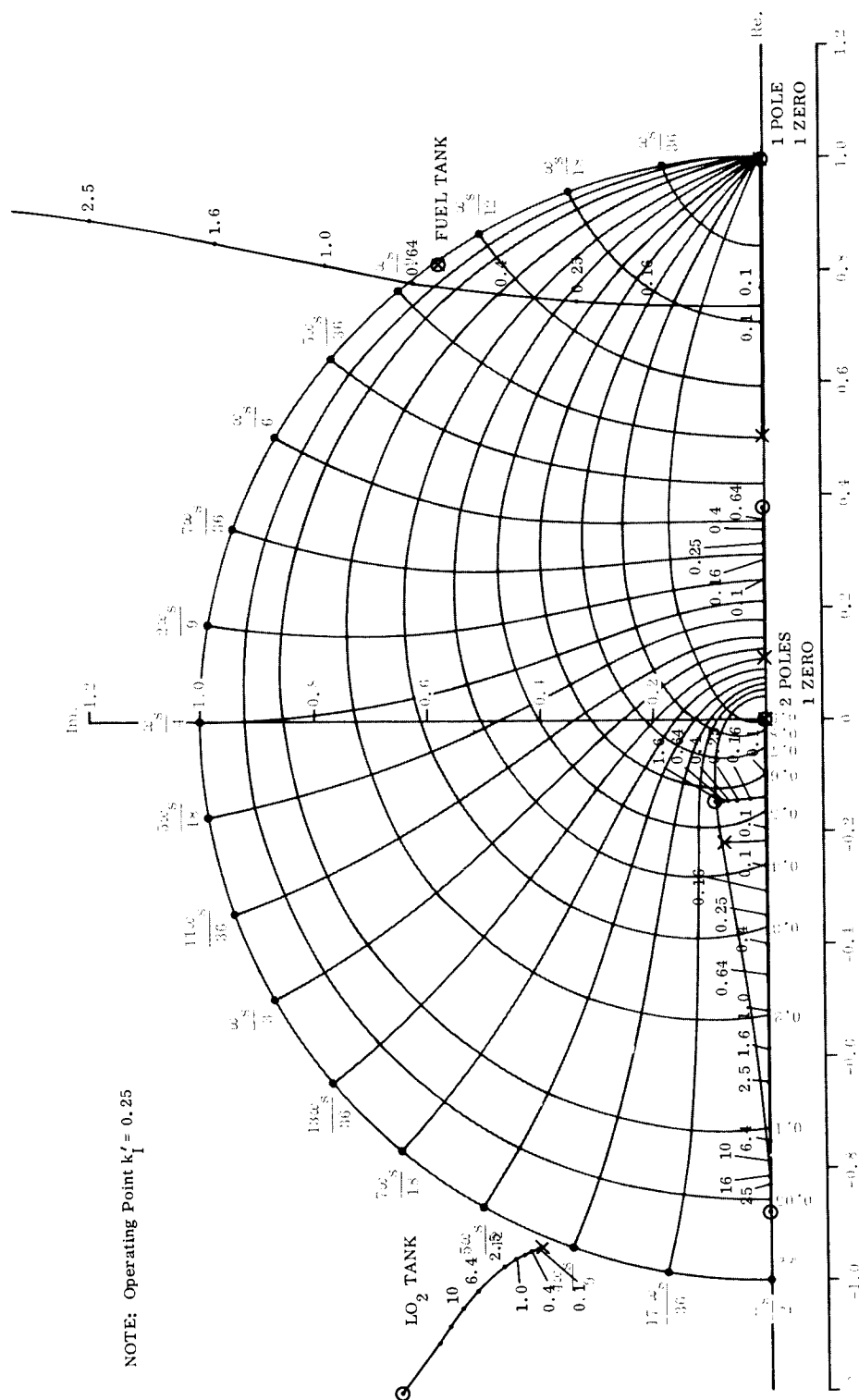


Figure 3.3-27. AC-5 Centaur Stage Autopilot/Guidance Loop Stability at MES + 370

- c. The engine thrust forces are low.
- d. The modal frequencies are high.
- e. There are no disturbing aerodynamic forces to contend with.

In short, conditions during upper stage flight are such that flexible vehicle/autopilot coupling is not a problem. This is demonstrated in the following analysis.

The bending frequencies associated with the AC-5 configuration are higher than those of the AC-4 configuration. This is primarily the result of the differences between the Surveyor mass model carried by the AC-4 vehicle and the Surveyor dynamic model carried by the AC-5 vehicle. A separation of greater than ten to one exists between the rigid body and first elastic bending mode frequency. This together with the fact that the Centaur type (overdamped) rate gyro has the filtering characteristic of a 4 to 6 cps filter, prevents significant coupling of the elastic bending modes with the autopilot.

Figure B3-1 in Appendix B illustrates the lumped parameter spring-mass model used in the analysis. A simplified block diagram, Figure 3.3-28, illustrates the effect of elastic vehicle bending on the feedback dynamics of the flight control system. The feedback provided by the guidance loop has been omitted in this analysis since its frequency is far outside the range of interest, being lower than the rigid body and slosh frequencies by a factor of five. In addition, filtering within the guidance loop, provided by the zero-order hold, is such that signals at the bending frequencies are greatly attenuated (see Figure B4-2). Thus the above simplification is valid. The dynamics of the platform have also been neglected based on analysis presented in Appendix B3.

The bending stability is analyzed entirely by the time slice root locus technique. The analysis includes the effects of the nonlinear electrohydraulic actuator by means of a describing function. As the engine amplitude (δ) changes, the operating point shifts due to changes in the actuator characteristics. In order to duplicate more precisely the effects of the rate gyro at bending frequencies, its dynamics are described by a second-order equation. The forces and moments produced by propellant sloshing have not been included in this analysis since the response of the sloshing masses at the relatively high bending frequencies is negligible. Damping for all bending modes is based on a very conservative modal damping ratio of one-half of one percent. The development of the bending stability equations is presented in Appendix B3.

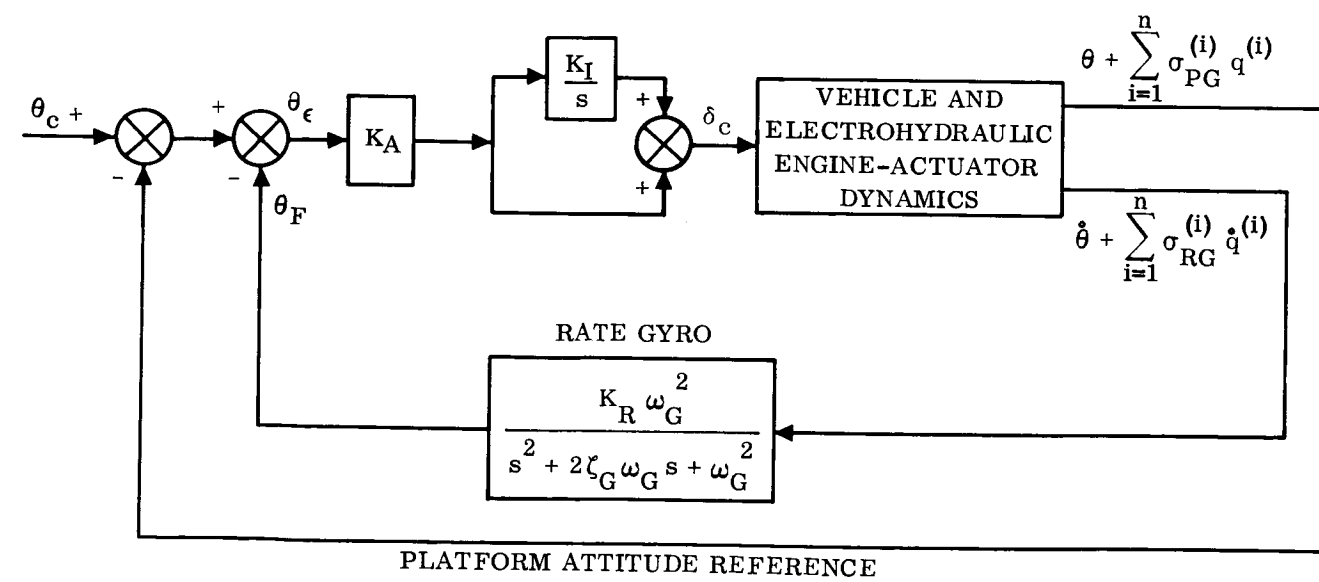
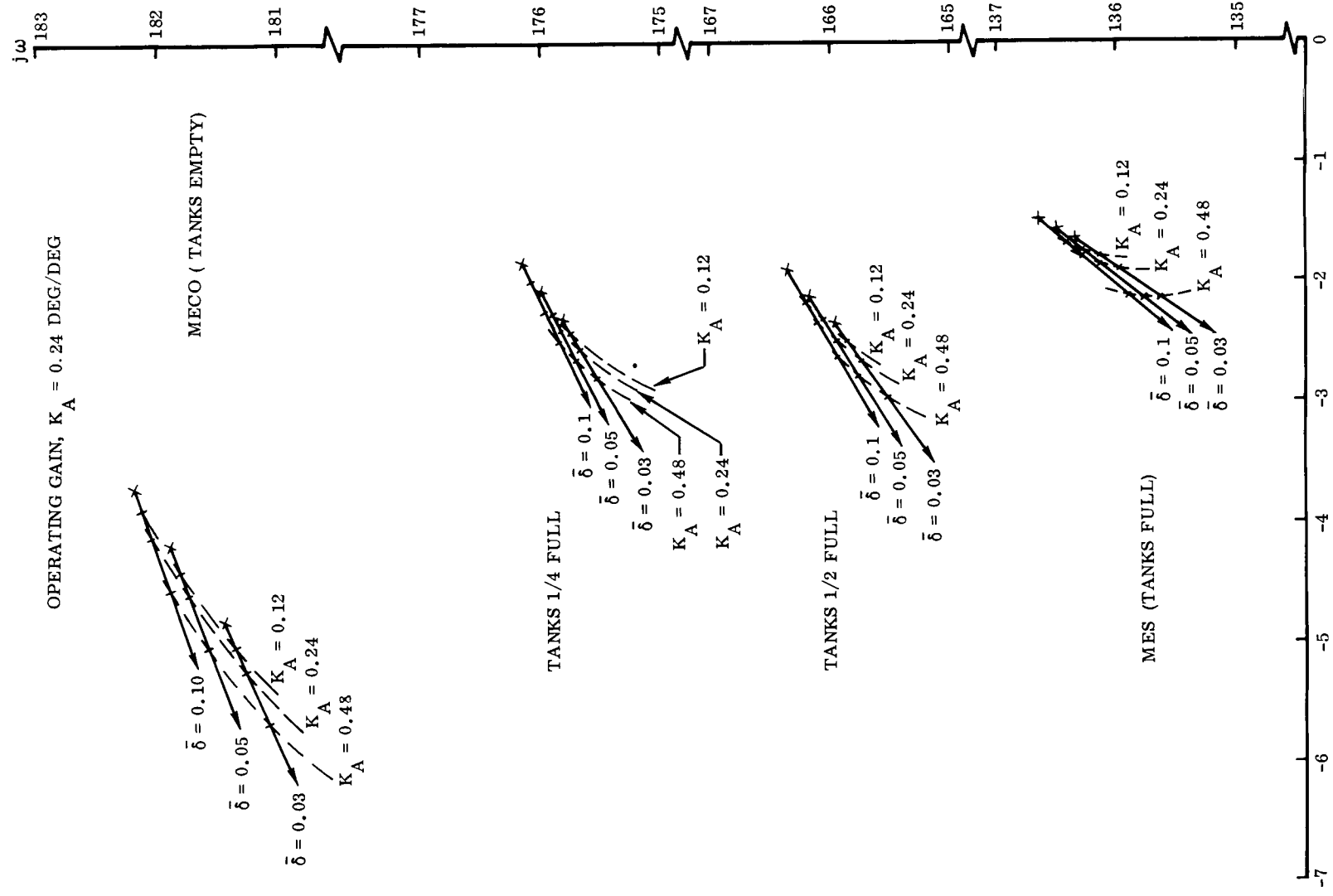
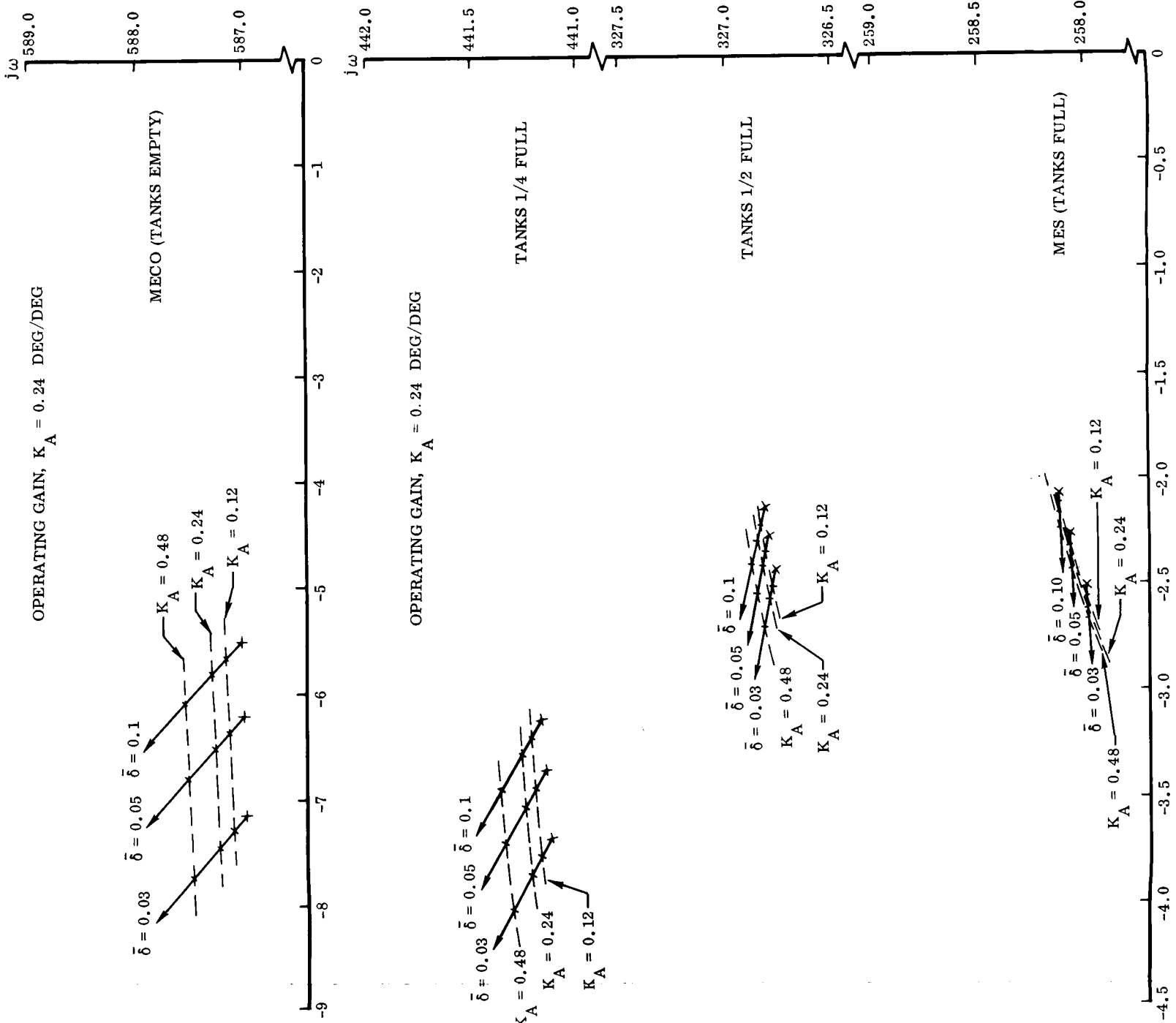


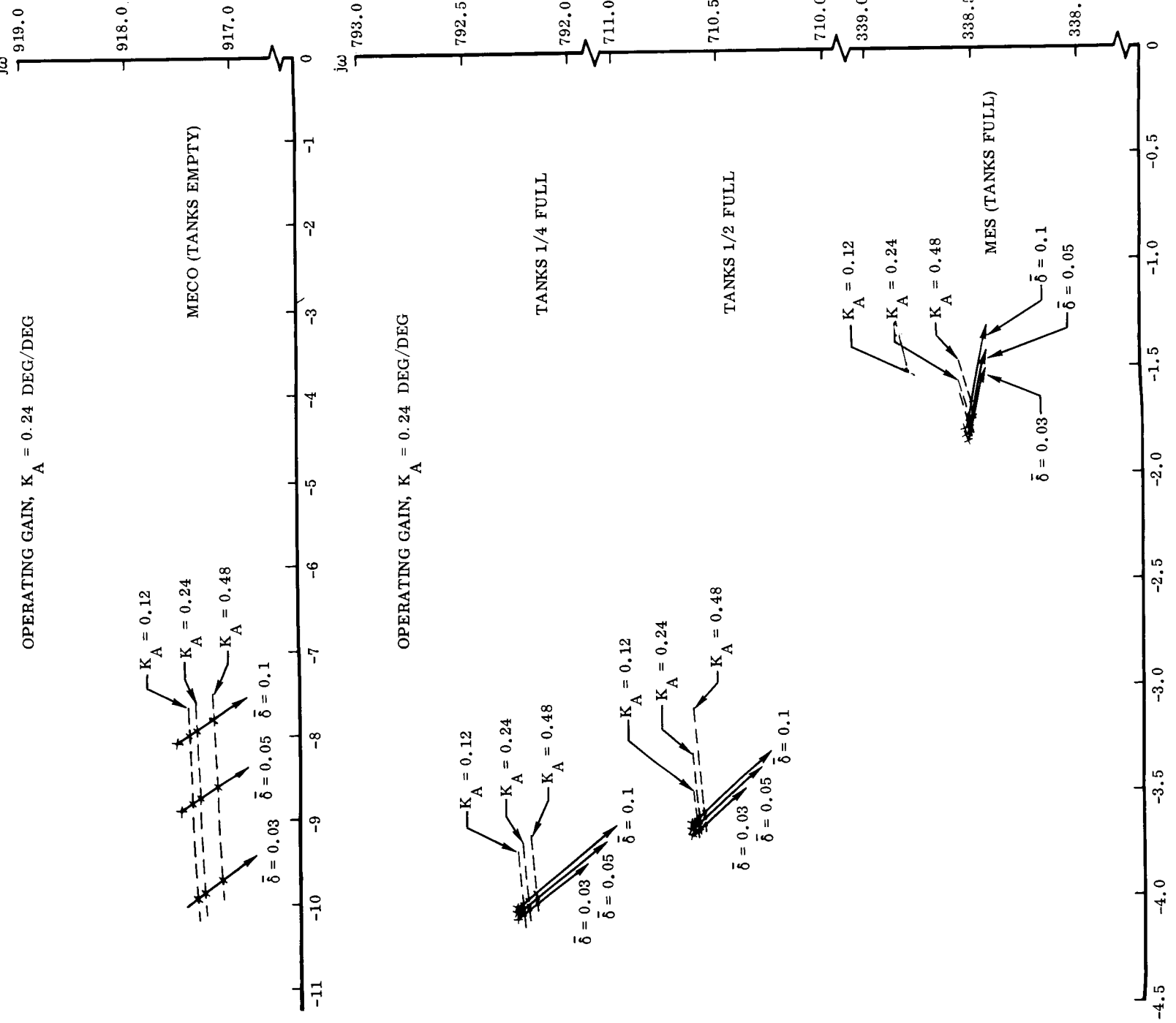
Figure 3.3-28. Block Diagram of the Flight Control System (Pitch or Yaw) Showing Structural Feedback



(a). First Elastic Mode Stability



(b). Second Elastic Mode Stability



(c). Third Elastic Mode Stability

The results of the bending stability analysis for the first three elastic modes at flight times corresponding to MES, propellant tanks half full, propellant tanks one-quarter full, and MECO are shown in Figure 3.3-29. This figure shows that the elastic bending modes are so stable as to preclude the need for any formal stability criteria. Figure 3.3-29(a) shows that the first mode is gain and phase stable; i.e., at the operating gain the mode is stable at any phase, and at the departure angle shown the mode is stable for any gain. Figure 3.3-29(b) shows that the second mode is also gain and phase stable. Figure 3.3-29(c) shows that the third mode is gain stable.

3.3.6 Main Engine Cutoff Transients. The Centaur vehicle prior to main engine cutoff is in a limit-cycle condition with the engine and the vehicle oscillating at very low amplitudes about their trim positions at approximately three radians per second. When the programmer issues the discrete commanding MECO, the autopilot changes to a coast phase configuration. This means that the rate and position gains are changed (Table 2.2-1). The integrators, however, are left in an unnullled condition. The thrust begins its rapid decay to zero after a few tenths of a millisecond delay, and the hydraulic supply pressure drops to zero as the pump speed drops off. In addition, the thrust structure becomes unloaded, and the deflections due to thrust forces are removed. The delay in nulling the integrator reduces the resulting transient by eliminating the step engine command which has a magnitude proportional to the integrator voltage prior to MECO. The combination of all these factors occurring within a very short interval of time (less than half a second) will impart rates to the vehicle which must be nulled by the coast phase attitude control system. An estimate of the expected values of these rates is needed to determine the maximum expected reorientation times for the attitude control system.

In the analysis which follows, it should be borne in mind that there are many factors for which there is no detailed knowledge available. The resulting analysis is therefore based on approximations made to arrive at an estimate for the transient rates. These assumptions and approximations are as follows:

- a. Integrator Nulling. The integrators in all three channels are assumed to remain at a fixed voltage until after the thrust decays to zero. This fixed value is that required to trim the vehicle just prior to MECO.
- b. Engine Shutdown Impulse. The estimated shutdown thrust decay envelope as supplied by the manufacturer is shown in Figure 3.3-30. The shutdown impulse per engine is taken as 1308 ± 250 pound-seconds.
- c. Engine Response. Unloading of the thrust structure occurs concurrently with thrust decay and the thrust vector will move through an angle proportional to the deflection of the aft bulkhead. The deflection is assumed equal to the thrust structure loading correction angle used in aligning the engine prior to launch. This gives rise to yaw and roll transients primarily; the pitch engine deflections approximately cancel each other. The deflection is assumed to occur instantaneously at MECO.
- d. Guidance Commanded Rates. The steering equation used during the powered phase is assumed to result in maximum vehicle rates at MECO of ± 0.5 deg/sec in pitch or yaw.
- e. Contingencies. This factor is included to account for uncertainties in the cutoff transients not considered in the mathematical model, and is taken as ± 0.25 deg/sec.

The rates resulting from these factors are summarized as follows:

First MECO	
Pitch (deg/sec)	± 1.55
Yaw (deg/sec)	-0.206 ± 0.56
Roll (deg/sec)	-1.53 ± 0.25

The pertinent basic data for the calculation is listed in Table E-7. The calculations are summarized in Table 3.3-3. Because of the crudeness of the assumptions and approximations made in this analysis, a

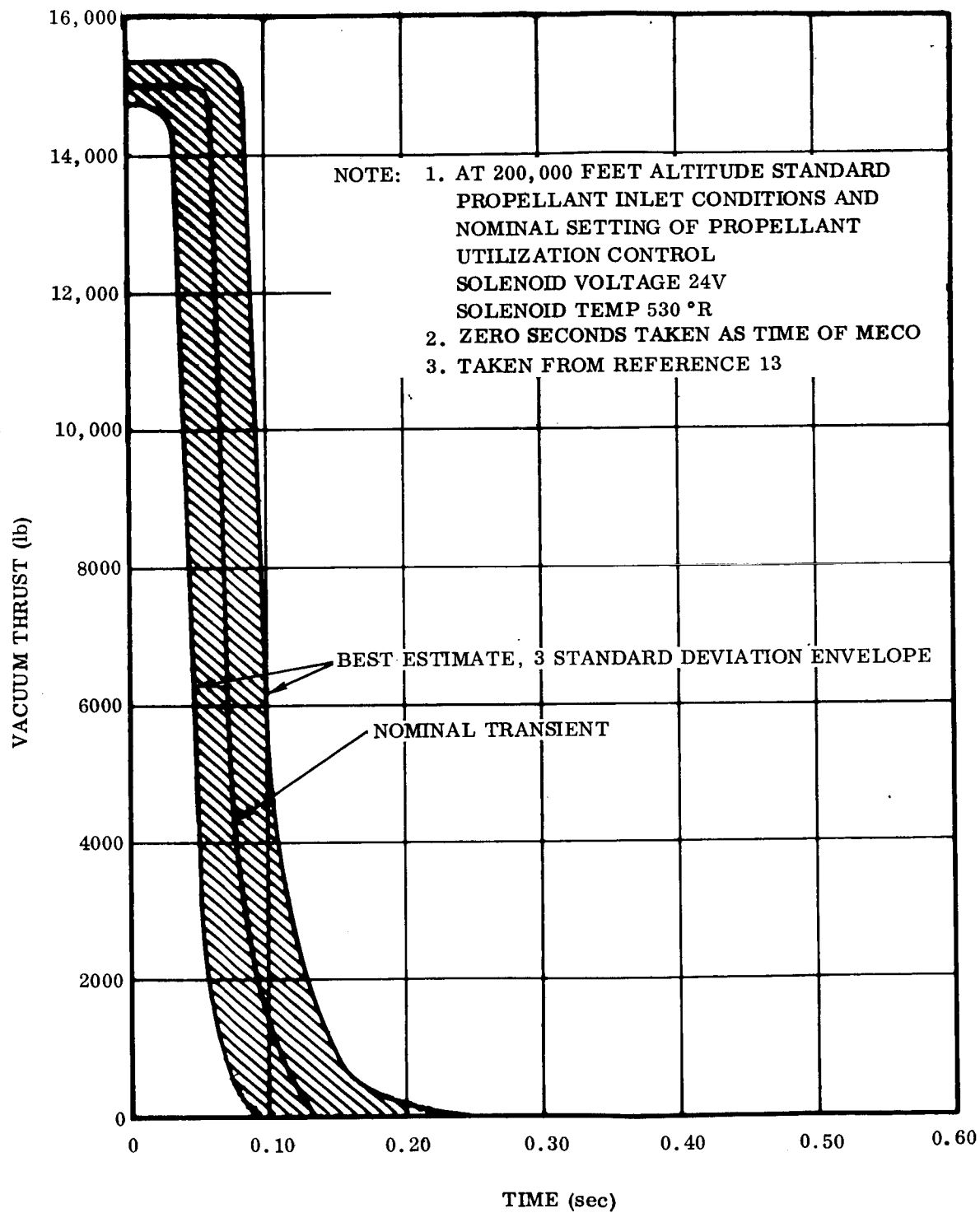


Figure 3.3-30. Estimated Shutdown Thrust Decay Envelope versus Time, No Programmer Delay

great deal of confidence cannot be placed in the results. However, not enough flight data exists to improve upon this model at this time.

Table 3.3-3. Summary of Uncertainties (3σ) in Main Engine Cutoff Transients

DEFINITION		FIRST MECO		
		Pitch	Yaw	Roll
Guidance Commanded Rates	(deg/sec)	± 0.50	± 0.50	0
Rate Due to Differential Impulse				
$= 57.3 \times \ell_z \times 250.5 \times \sqrt{2/I_{yyr}}$	(deg/sec)	± 1.45	0	0
Contingencies (Estimated 3σ)	(deg/sec)	± 0.25	± 0.25	± 0.25
Total MECO Transient Dispersion (3σ)	(deg/sec)	± 1.55	± 0.56	± 0.25
Rate Bias Due to Thrust Structure Unloading				
$= I_{yyr} \times 1308 \times (\delta_1 + \delta_2)$	(deg/sec)	0	+0.109	-1.38

3.4 COAST PHASE FLIGHT. The Centaur coast phase begins at Main Engine Cutoff (MECO) and ends with completion of the retromaneuver approximately 20 minutes later. The first minute or so of the coast phase is spent in nulling the rates caused by the MECO transient to within the rate thresholds of the attitude control system. Following this, the spacecraft dynamic model is separated in a test of the separation system. (See Section 3.5.) The Centaur vehicle then executes a re-orientation or turn around maneuver, pointing its nose back along the flight path. A prestart sequence is then initiated, producing a small amount of thrust, and the Centaur vehicle backs away from the spacecraft. Approximately 15 minutes later the attitude control system is turned off, leaving the Centaur vehicle free to drift in unstabilized flight.

3.4.1 Method of Analysis. Both analytical and simulation techniques were utilized to determine the vehicle behavior during the coast phase. In the absence of a satisfactory model describing the behavior of the propellants in near-zero gravitational conditions, the residual propellants that will remain in the vehicle were assumed to be rigid and in the aft end of each tank. This approximation is more or less academic with AC-5 since the on-board residuals will be very small. A simplified single-plane simulation was performed for this analysis. This was done on the assumption that, for the time involved, the dynamics in each plane could be uncoupled without significantly affecting the results. Disturbing torques were also neglected in the simulation, since their effect on the duty cycle could be handled better analytically, particularly with the variances and variation in modes of firing the attitude engines. Appendix C gives the equations defining the mathematical model in detail and the attitude control engine thrust versus time characteristics.

Statistical methods have been utilized wherever possible to determine dispersions; i.e., whenever statistical significance of data could be assumed it was used. The disturbing torques and control torques, for example, are functions of several assumed random and independent variables. These variables include the magnitudes and directions of the forces, (direct thrust and/or impingement) and their application points. The parameters, having stated nominal values and maximum allowable variations, were treated as statistical quantities. The nominal values of the torques were calculated and the variances found by using the technique outlined in Appendix C (Section C4).

3.4.2 System Criteria. The fundamental criterion to be applied to the AC-5 configuration during the coast phase is that it successfully perform the intended maneuver. To accomplish this, the vehicle rates must be within the attitude control system thresholds when the separate-spacecraft command is given, and the vehicle attitude at MECO. Secondly, the vehicle must be at least 336 kilometers away from the spacecraft dynamic model 5 hours after it is separated, which is the intended purpose of the retromaneuver.

For AC-5, one of the objectives is to gain data on the retromaneuver thrust - thus determining whether or not the distance-from-spacecraft versus time objective is accomplished. The best data will be obtained when the vehicle is within ± 10 degrees of the commanded attitude when retrothrust begins. This was, therefore, adopted as a criterion on the re-orientation maneuver.

To ensure that the system has an adequate margin for controlling the disturbance torques acting on the vehicle during the coast phase, a forty-percent criterion is adopted for external torques. The system is judged adequate if the maximum (3σ) disturbance torques due to retrothrust and/or venting are less than 40 percent of the control torque capability. A greater disturbance torque than this will result in a substantial departure from the intended behavior due to the change in resulting acceleration available for controlling the vehicle.

3.4.3 System Behavior. The change in the inertial properties of the AC-5 configuration relative to AC-4 have resulted in larger control torques and accelerations as shown in Tables 3.4-1 and 3.4-2. The assumed rates following MECO are 3σ maximum as derived in Section 3.3. The vehicle behavior is depicted in Figures 3.4-1 through 3.4-3. These phase plane trajectories show that, with the conditions assumed, the system response is adequate for the stated criteria. The higher control accelerations reduce the initial rates faster than on AC-4. The rates about all three axes are less than ± 0.2 deg/sec at the time the spacecraft dynamic model is separated. The phase plane plots are referenced from MECO and the MECO time was arbitrarily chosen to be 9 seconds before the electromechanical timer discrete for MECO backup. The guidance system gives the MECO discrete command, and since the electro-mechanical timer does not have the capability of restarting at this time, there is a period of 47 seconds within which MECO may occur before a backup signal is given by the timer. (See sequence Table 1.3-1 and Figure 3.4-4.) The only effect this uncertainty has on the phase plane plots (Figures 3.4-1 through 3.4-3) is to increase or decrease the time spent within the rate threshold in pitch and yaw or in the limit cycle condition in the roll plane.

The legs will not actually extend on the spacecraft dynamic model, but it will be separated from the Centaur vehicle to demonstrate the separation mechanism (see Section 3.5). Five seconds after the separation, a guidance vector is admitted which initiates the re-orientation maneuver. As can be seen in Figure 3.4-1, even if MECO occurred 38 seconds earlier than shown (which would be earliest that it could possibly occur) the attitude of the Centaur vehicle would still be within ± 5 degrees of that commanded when retrothrust begins.

The major disturbing torques are those due to LH_2 tank vent and to retrothrust. Disturbing torques due to cross coupling when the hydrogen-peroxide jets are firing (in response to various combinations of

Table 3.4-1. Summary of Coast Phase External Disturbing Torques (3σ)

DEFINITION	PITCH	YAW	ROLL
MECO Transient (deg/sec)	± 1.55	-0.2 ± 0.56	-1.5 ± 0.25
LH_2 Tank Vent (in.-lb)	42.4 ± 10.8	43.6 ± 10.9	15.3 ± 3.9
Retrothrust (in.-lb)	-49.6 ± 96	36.3 ± 75	2.56 ± 19

Table 3.4-2. Summary of Coast Phase Control Torque and Accelerations (3σ)

PLANE	BEFORE PAYLOAD SEPARATION (in. -lb)/(deg/sec ²)	AFTER PAYLOAD SEPARATION (in. -lb)/(deg/sec ²)
Pitch	$404.2 \pm 21.2/0.066 \pm 0.0035$	$261.4 \pm 18/0.07 \pm 0.0048$
Yaw	$406.4 \pm 15.8/0.067 \pm 0.0025$	$262.8 \pm 13.7/0.071 \pm 0.0037$
*	$203.2 \pm 25.3/0.033 \pm 0.0042$	$131.4 \pm 18.7/0.035 \pm 0.005$
Roll	$190 \pm 5.1/0.43 \pm .01$	$190 \pm 5.1/0.47 \pm 0.013$
*	$95 \pm 10.4/0.22 \pm 0.024$	$95 \pm 10.4/0.23 \pm 0.026$

* Minimum torque when the control system responds to yaw and roll commands simultaneously; the control logic permits only one "A" engine to fire.

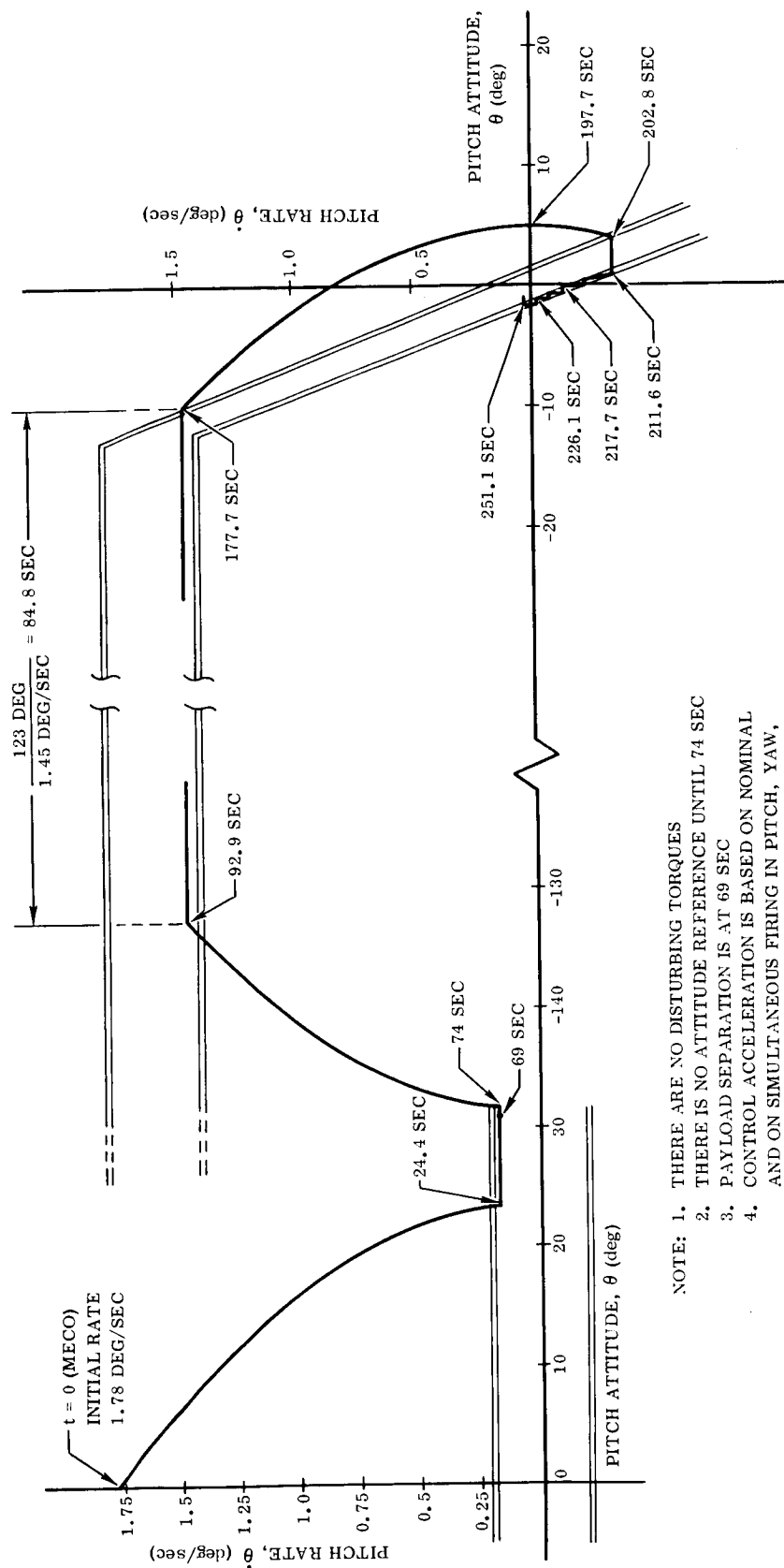
commands in pitch, yaw, and roll) are less than 1.5 percent of the nominal control torques and less than 10 percent of the 3σ dispersion. Therefore, these cross-coupling torques were ignored. The LH₂ tank vent torques are based on preliminary calculations and test data. A zero-thrust plenum chamber type vent similar to that on AC-4 and operating as intended was assumed. Initial test results indicate a steady-state thrust imbalance on the order of 0.3 pound and a transient condition with a peak thrust imbalance of 1.74 pounds when venting commences. The disturbing torques in pitch and roll resulting from the vent transient last for a period of about 7.5 seconds and are momentarily uncontrollable (greater than the control torque). The resulting attitude loss is insignificant.

The duty cycles during the limit cycle condition are given in Table 3.4-3. These are based on the control and disturbance torques given in Tables 3.4-1 and 3.4-2.

Table 3.4-3. Coast Phase Duty Cycles

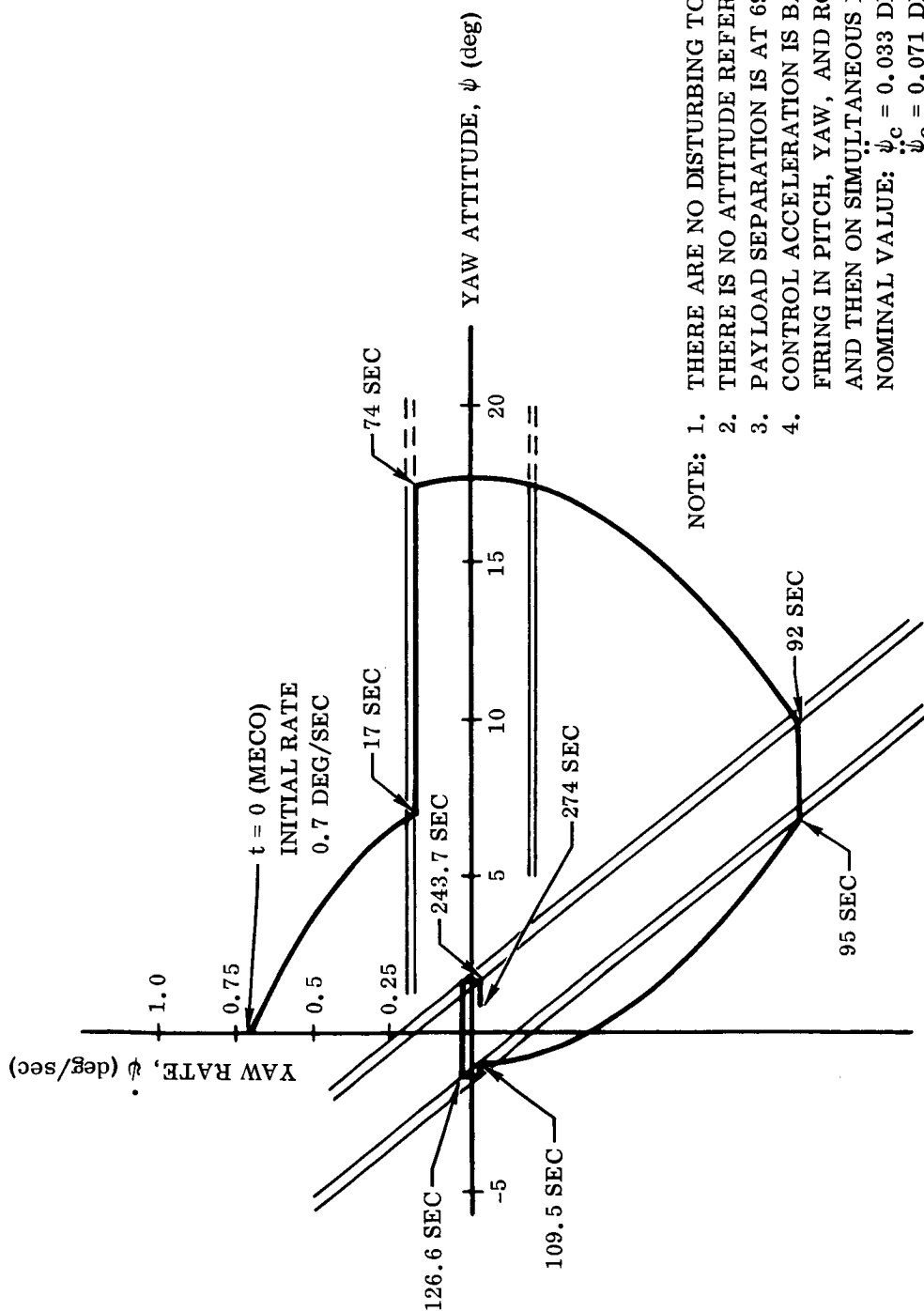
PLANE	DISTURBING TORQUE	DUTY CYCLE	THRUST (lb)
Pitch	Maximum Nose Up	0.38	3
	Maximum Nose Down	0.436	3
	Nominal (Nose Down)	0.03	3
Yaw	Maximum Nose Right	0.63	3
	Maximum Nose Right	1.26 *	1.5
	Maximum Nose Left	0.023	3
	Maximum Nose Left	0.046 *	1.5
	Nominal (Nose Right)	0.3	3
	Nominal (Nose Right)	0.6 *	1.5
Roll	Maximum CW Roll	0.195	3
	Maximum CW Roll	0.39 *	1.5
	Maximum CCW Roll	0.0074	3
	Maximum CCW Roll	0.0037 *	1.5
	Nominal (CW Roll)	0.09	3
	Nominal (CW Roll)	0.18 *	1.5

* Logic allows only one "A" engine firing when correcting for yaw and roll simultaneously.



- NOTE: 1. THERE ARE NO DISTURBING TORQUES
 2. THERE IS NO ATTITUDE REFERENCE UNTIL 74 SEC
 3. PAYLOAD SEPARATION IS AT 69 SEC
 4. CONTROL ACCELERATION IS BASED ON NOMINAL
 AND ON SIMULTANEOUS FIRING IN PITCH, YAW,
 AND ROLL
 $\ddot{\theta}_c = 0.067 \text{ DEG/SEC}^2 \text{ FOR } 0 < t < 69$
 $\ddot{\theta}_c = 0.071 \text{ DEG/SEC}^2 \text{ FOR } 69 < t$
 5. MECO AS SHOWN HERE IS ASSUMED TO OCCUR 9
 SECONDS PRIOR TO MECO BACKUP DISCRETE
 (C + 450.5) AS SHOWN IN TABLE 1.3-2

Figure 3.4-1. AC-5 Pitch-Phase Plane



NOTE: 1. THERE ARE NO DISTURBING TORQUES

2. THERE IS NO ATTITUDE REFERENCE UNTIL 74 SEC

3. PAYLOAD SEPARATION IS AT 69 SEC

4. CONTROL ACCELERATION IS BASED ON SIMULTANEOUS FIRING IN PITCH, YAW, AND ROLL UNTIL ABOUT 69 SEC, AND THEN ON SIMULTANEOUS PITCH AND YAW FIRING.

NOMINAL VALUE: $\ddot{\psi}_c = 0.033 \text{ DEG/SEC}^2$ FOR $0 < t < 68$

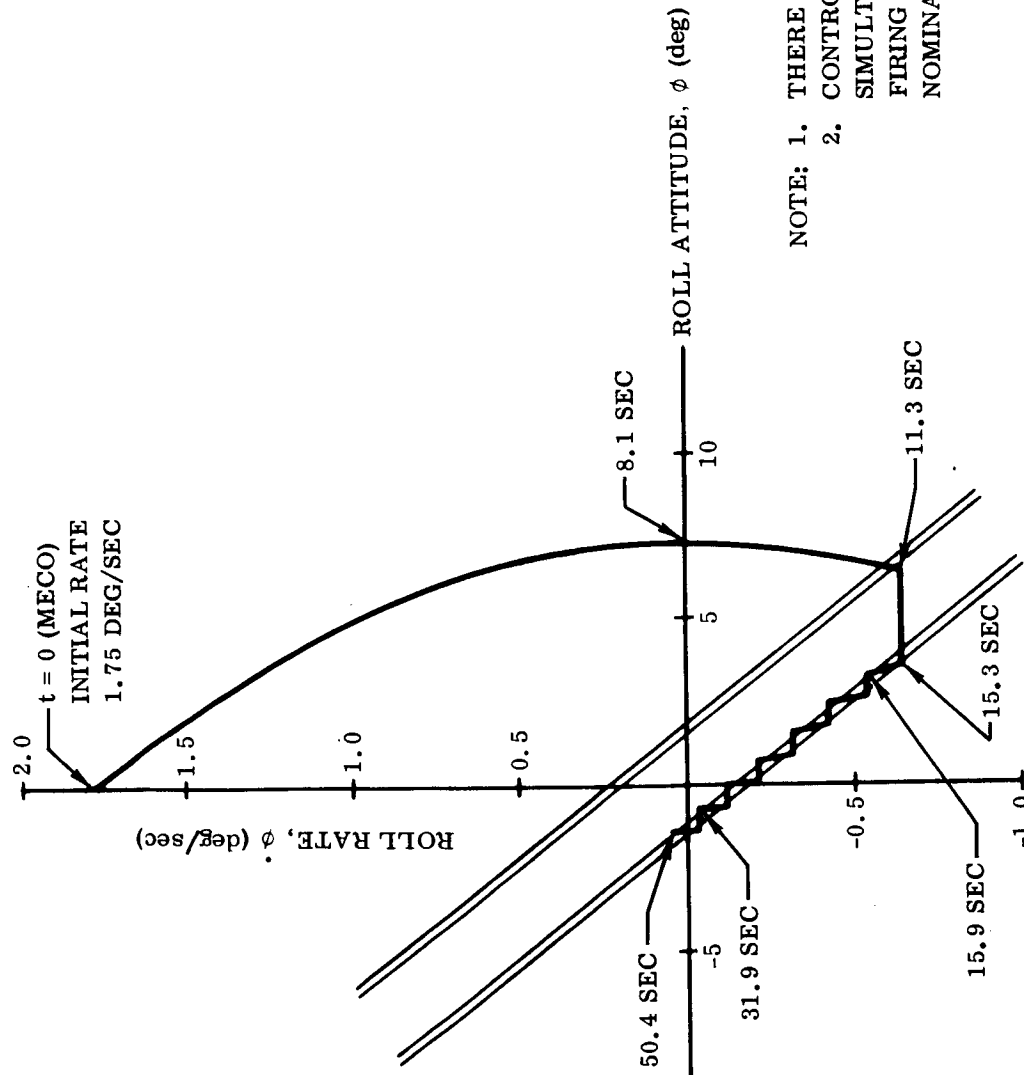
$\ddot{\psi}_c = 0.071 \text{ DEG/SEC}^2$ FOR $69 < t$

5. MECO AS SHOWN HERE IS ASSUMED TO OCCUR 9

SECONDS PRIOR TO MECO BACKUP DISCRETE

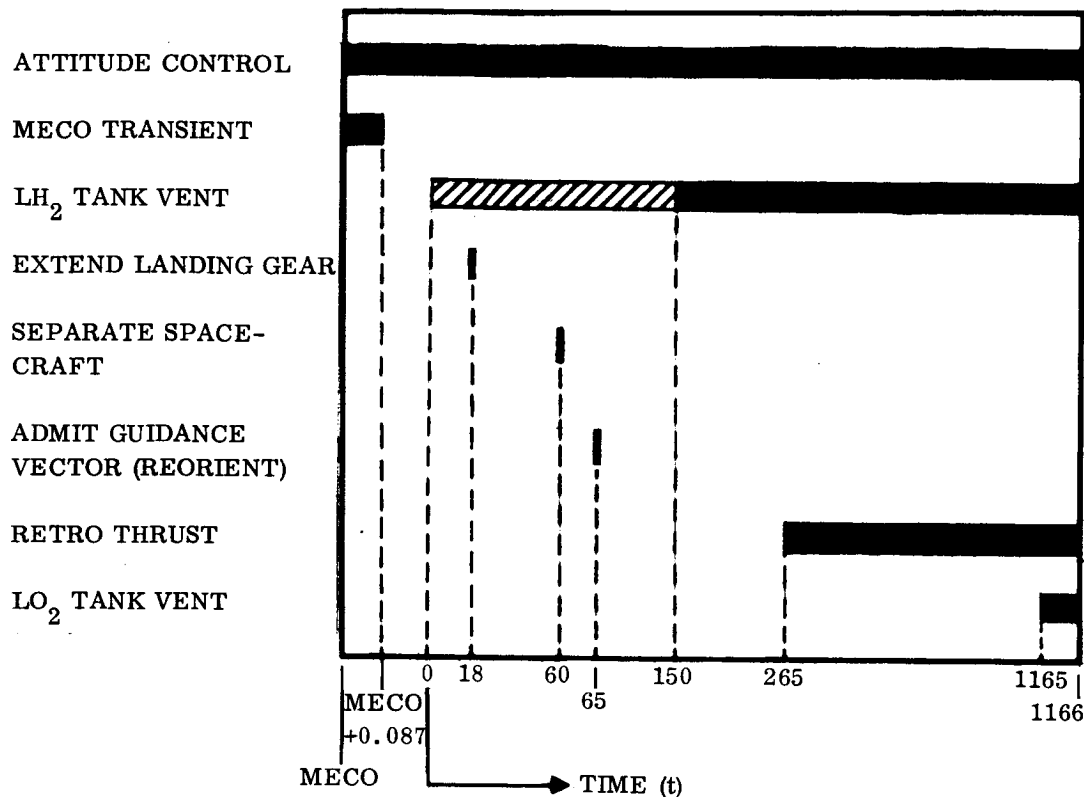
(C + 450.5) AS SHOWN IN TABLE 1.3-2

Figure 3.4-2. AC-5 Yaw-Phase Plane



NOTE: 1. THERE ARE NO DISTURBING TORQUES
 2. CONTROL ACCELERATION IS BASED ON
 SIMULTANEOUS PITCH, YAW, AND ROLL
 FIRING
 NOMINAL VALUE: $\ddot{\phi}_c = .22 \text{ DEG/SEC}^2$ $0 < t < 69$

Figure 3.4-3. AC-5 Roll-Phase Plane



- NOTE: 1. TIME SCALE NOT TO SCALE
 2. MECO CAN OCCUR ANYWHERE FROM $t = -47$ SEC TO $t = 0$ SEC
 3. TIME ZERO IS THE TIMER MECO BACKUP DISCRETE

Figure 3.4-4. External Torque and Event Sequence, AC-5 Coast Phase

3.4.4 System Adequacy. The attitude control system is capable of accomplishing the mission objectives with the criteria stated in Paragraph 3.4.2. As the duty cycles indicate, the 40 percent criterion for the disturbing torque and control torque relationship is sometimes violated. This occurs only under 3σ maximum conditions and does not significantly affect the system adequacy.

3.5 CENTAUR/SPACECRAFT SEPARATION. A major objective of the AC-5 mission is to demonstrate a successful separation of the payload from the Centaur vehicle. A dynamic model, SD-1, is used to simulate the spacecraft and is intended to duplicate the dynamic behavior of the actual Surveyor. However, because of payload capability constraints on AC-5, the mass of the dynamic model is only two thirds that of the Surveyor. This is judged sufficient to demonstrate the separation mechanism and to approximate the motion of the actual spacecraft.

For purposes of comparison, additional analysis was performed based upon Surveyor A-21 and Centaur 3D, (the AC-7 mission). The pertinent basic data for the AC-5 and AC-7 configurations are given in Appendix E. The data used in an earlier separation analysis (Reference 19) is also given for comparison.

The dynamic model does not have the spacecraft legs unfolded as does the Surveyor at separation. This does not appreciably affect the separation dynamics.

This could affect longitudinal CG & will affect mass moments of inertia. The effect of I's is to increase and hence lower rotation rates etc. - longitudinal CG could be adversely affected if moved forward.

3.5.1 Method of Analysis. The equations of motion were derived (Appendix D), and certain simplifications made. These simplifications resulted in an essentially planar analysis. It has been shown, Reference 19, that angular displacements do not appreciably affect the longitudinal motion, and that for the small angular displacements assumed, certain second order coupling terms could be neglected. Additionally, it was assumed that the roll angle, all products of inertia, and transverse velocities in the Y and Z directions could be neglected. Dispersions in the parameters contributing to the mechanics of the separation were treated statistically and variances determined for the principle parameters involved in the criteria for a successful separation.

The results of a separation system validation test (Reference 20) are not directly comparable with the results given here because of the test conditions. The primary objectives of this test were only to demonstrate the separation mechanism. Air bearings were used to support the simulated spacecraft, while the Centaur portion was held fixed. The longitudinal separation velocity resulting from the test is therefore somewhat higher than would be the actual case. Rotational velocities are similarly affected.

3.5.2 Separation System Criteria. The criteria for successful operation of the spacecraft separation system are as follows (Reference 21):

- a. Incremental velocity. The velocity imparted to the spacecraft will be considered as a portion of the injection velocity. The uncertainty in the separation velocity is considered a part of the injection errors. No specific limits or criteria have been set on this parameter, consequently the nominal separation velocity and its 3σ dispersion are given without comment.
- b. Angular rate. The spacecraft rotational velocity at separation shall not exceed a total angular vector sum of 3 degrees/second, including the Centaur residual angular rate at initiation of spacecraft separation.
- c. Collision. The separation system shall function to preclude damage to the spacecraft through collision.

3.5.3 Separation Velocity. The longitudinal separation velocity of the spacecraft with respect to the reference frame is:

- a. AC-5 (6C/SD-1) $0.988 \pm 0.017 (3\sigma)$ ft/sec.
- b. AC-7 (3D/A-21) $0.701 \pm 0.013 (3\sigma)$ ft/sec.

These give the contribution to the injection velocity and its errors. In the Separation Validation Test (Reference 20), the average separation velocity for the 2110-pound Surveyor Test Vehicle (T-1) plus the 210-pound air bearing was 0.766 ft/sec.

For collision considerations, the relative separation velocities between the spacecraft and the Centaur vehicle are important. These are:

- c. AC-5 (6C/SD-1) $1.27 \pm 0.017 (3\sigma)$ ft/sec.
- d. AC-7 (3D/A-21) $1.07 \pm 0.013 (3\sigma)$ ft/sec.

In determining the collision clearance, a slightly conservative figure (1 ft/sec) was used for the relative separation velocity.

3.5.4 Angular Rates. The analysis revealed that the major effect on the angular rates of the spacecraft resulted from the lateral center-of-mass offset. In fact, the equations of motion may be reduced, without significant (less than 5 percent in this case) effect on the results, to simplify the lateral center-of-mass

offset and its coefficient c_1 or d_1 for the appropriate plane (see Appendix D). Since this coefficient remains approximately constant for the Surveyor design configurations (Reference 22), it affords a convenient way to determine the nominal planar angular rate for a given lateral center of mass offset. (Figures 3.5-1 and 3.5-2). The nominal and 3σ uncertainties in the planar angular rates for the two configurations considered are given in Table 3.5-1.

Table 3.5-1. Spacecraft Planar Angular Rates

SPACECRAFT	PITCH RATE (deg/sec)	YAW RATE (deg/sec)
Dynamic Model SD-1	$+0.70 \pm 1.5 (3\sigma)$	$0.00 \pm 1.5 (3\sigma)$
Surveyor A-21	$+0.133 \pm 1.38 (3\sigma)$	$0.147 \pm 1.38 (3\sigma)$

From the Separation Test (Reference 20) the average rates imparted to the Surveyor Test Vehicle were 0.184 deg/sec in Yaw and 0.06 deg/sec in pitch.

The results in Reference 19 are somewhat different from those given here. This is primarily because of differences in the variances of two major parameters (see Table E-11). The $\sigma_{\dot{\psi}_{1(0)}}^2$ and $\sigma_{\dot{\theta}_{1(0)}}^2$ have changed because of changes in the attitude control system. The present system without the 50-lb vernier engines is in a rate control mode rather than a limit cycle condition at spacecraft separation. This results in an approximately discrete distribution which has modes just under the positive or negative rate threshold, depending on the direction of the initial rate. The variance of this discrete distribution has little significance by itself, but is valid in determining the variance of the spacecraft angular velocity. The mean of this residual rate $\dot{\psi}_{(0)}$ or $\dot{\theta}_{(0)}$ was considered to be zero in determining the spacecraft angular velocity and 3σ variance. From simulation results, the value of $\dot{\psi}_{(0)}$ and $\dot{\theta}_{(0)}$ was found to be ± 0.15 deg/sec. The other major parameter change was that of $\sigma_{\frac{2}{y}}$ and $\sigma_{\frac{2}{z}}$. The lateral center-of-mass offset uncertainty of the spacecraft has been determined to be ± 1 inch rather than the earlier value of ± 0.5 inch. This value is assumed to be the 3σ deviation of a normal distribution.

The vector sum angular rates are (see Equation D-48, Appendix D):

- AC-5 (6C/SD-1) $0.7 \pm 1.72 (3\sigma)$ deg/sec.
- AC-7 (3D/A-21) $0.2 \pm 1.58 (3\sigma)$ deg/sec.

3.5.5 Separation Clearance. The principal concern in separation clearance is the clearance distance between the bell of the spacecraft retromotor and the payload adapter on the Centaur vehicle. Figure 3.5-3 shows the general arrangement of the interface for a Surveyor spacecraft. (Principal dimensions are given in Appendix E.) The clearance is a function of the relative spacecraft-to-Centaur longitudinal velocity; the relative angular velocity (a function of lateral center-of-gravity offset); and the spacecraft longitudinal center of gravity.

For a given relative longitudinal velocity of 1 ft/sec and two given angular velocities, the variation in clearance (at the top of the payload adapter) with spacecraft longitudinal center of gravity is shown in Figure 3.5-4. This figure is for the Surveyor (A-21) configuration. The AC-5 configuration having the SD-1 with a retromotor simulator has no clearance problem. (See shaded area on Figure 3.5-3.) For the Surveyor (A-21), the predicted clearance is 3.2 inches. This is based on a relative longitudinal velocity of 1 ft/sec, a vector sum angular rate of 1.78 (3σ maximum), and a longitudinal center-of-gravity location at Station Number 112.87.

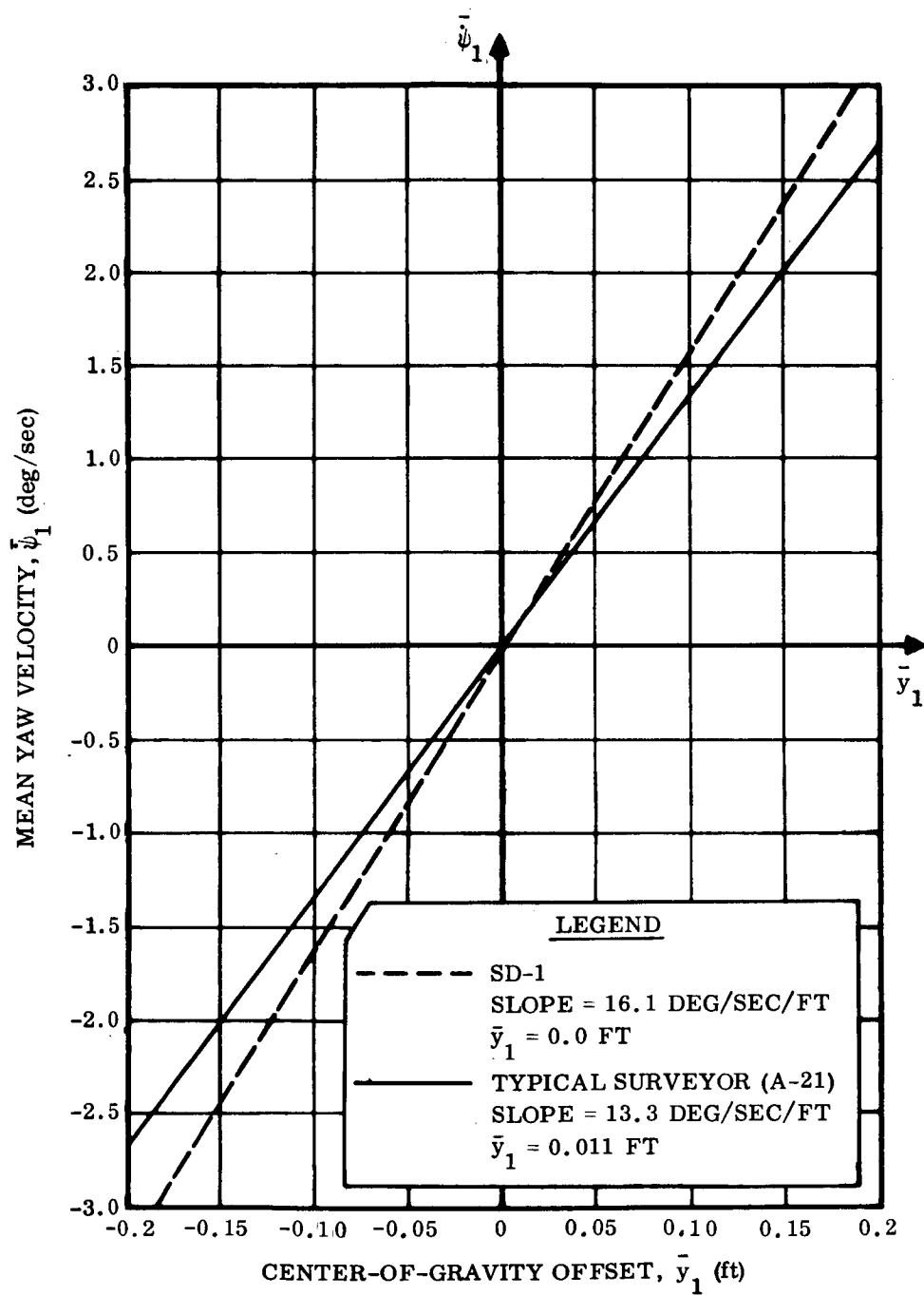


Figure 3.5-1. Mean Yaw Rates versus Spacecraft Lateral Center-of-Gravity Offsets

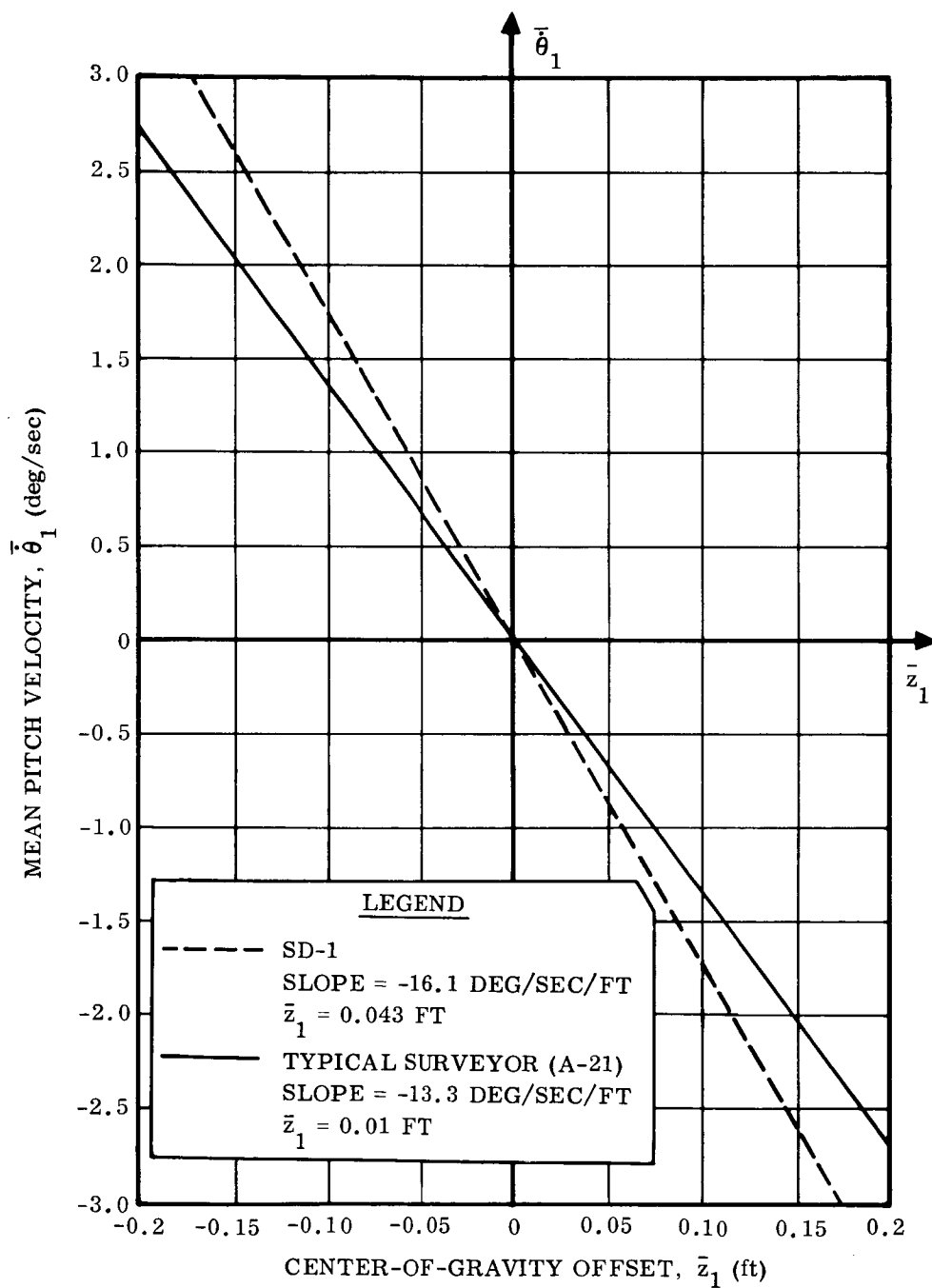
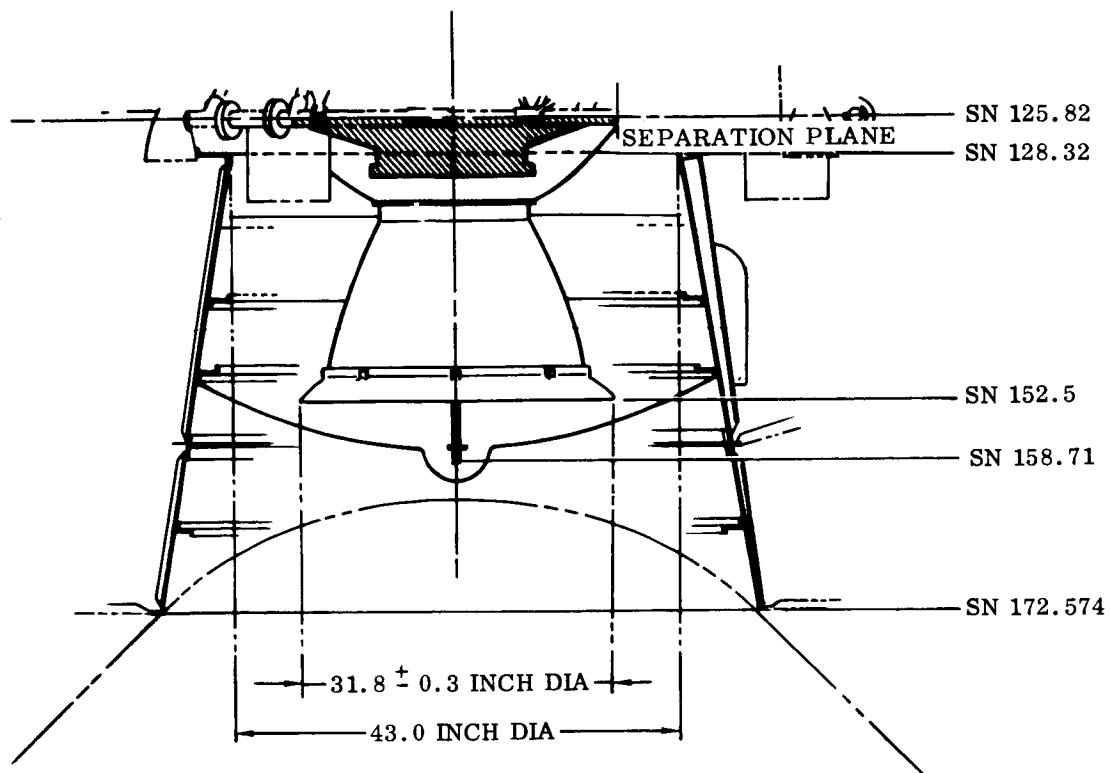


Figure 3.5-2. Mean Pitch Rates versus Spacecraft Lateral Center-of-Gravity Offsets



NOTE: 1. SHADED AREA REPRESENTS THE
RETROMOTOR SIMULATOR ON THE
DYNAMIC MODEL SD-1 ON 6D (AC-5)
2. SN INDICATES STATION NUMBER

Figure 3.5-3. Principal Spacecraft Separation Clearances

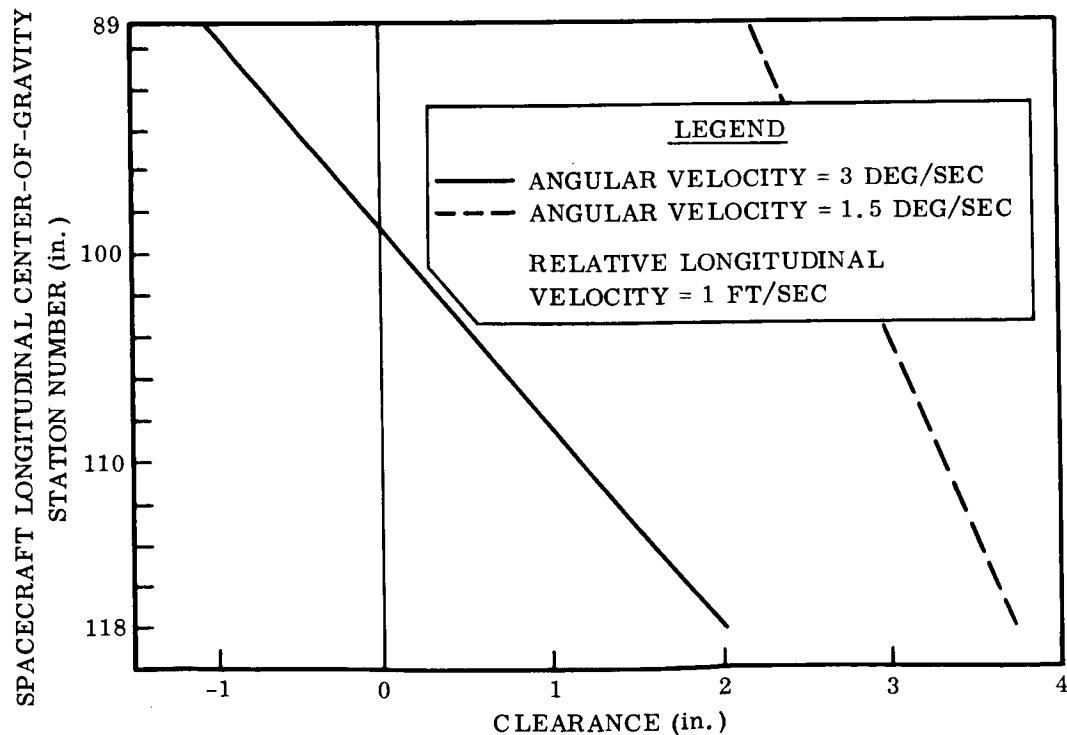


Figure 3.5-4. Surveyor Spacecraft Separation Clearance

NOTES:

1. SHADED AREA INDICATES CENTER-OF-GRAVITY LOCATION, NOMINAL (DOT) PLUS DISPERSIONS, FOR A TYPICAL SURVEYOR (BASED ON A-21)
2. SOLID LINES INDICATE ABSOLUTE LIMITS, I.E., NOMINAL PLUS DISPERSIONS OF CENTER OF GRAVITY
3. DASHED LINES INDICATE NOMINAL CENTER-OF-GRAVITY LIMIT CONSIDERING PRESENT DISPERSIONS

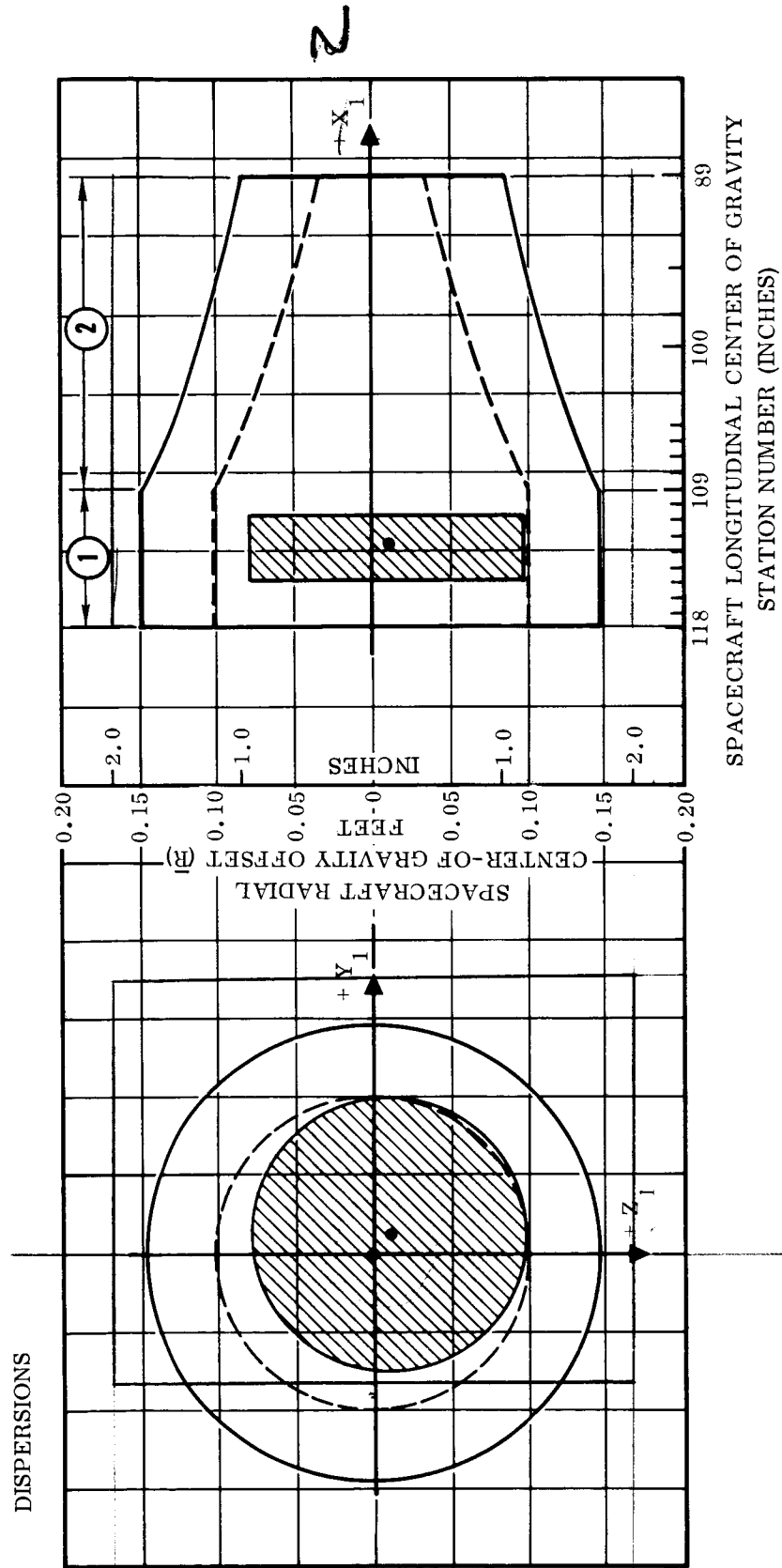
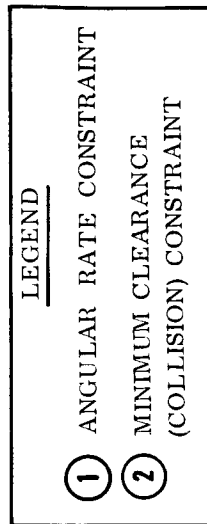


Figure 3.5-5. Proposed Spacecraft Center-of-Gravity Envelope

CG ENVELOPE
 IN EXISTING CASE
 CG ENVELOPE
 IN EXISTING CASE

3.5.6 System Adequacy. The AC-5 configuration (6C/SD-1) is adequate with respect to all the separation criteria as stated in Section 3.5.2.

The Typical Surveyor/Centaur, AC-7, operational configuration (3D/A-21), is adequate with respect to the separation criteria as stated in Section 3.5.2. However, this adequacy cannot be assured under the presently allowed design criteria envelopes for lateral and longitudinal center of gravity of the spacecraft as stated in References 21 and 23.

A proposed envelope for the spacecraft lateral center-of-gravity offset is shown in Figure 3.5-5. This proposed envelope for design criteria preserves the present longitudinal center-of-gravity limits and the maximum angular velocity of 3 deg/sec (vector sum) criteria. The following assumptions were made in deriving this envelope:

- a. The equations used are shown in D4.1.
- b. A minimum allowable clearance of 1 inch between the Surveyor retromotor bell (with antenna cover) and the top of the payload adapter is arbitrarily assumed.
- c. The relative longitudinal velocity is 1 ft/sec.
- d. The variances in angular rates $\sigma_{\dot{\psi}_1(t)}^2$ and $\sigma_{\dot{\theta}_1(t)}^2$ are assumed to be equal.
- e. The coefficients c_1 and d_1 , Equations D-28 and D-29, are assumed to be equal.

SECTION 4
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APPENDIX A
ATLAS/CENTAUR SEPARATION EQUATIONS

LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
$\bar{F}_R(t)$	Nominal thrust vs. time characteristic for a single retrorocket $\bar{F}_R(t) = \begin{cases} 644 \text{ lbs: } 0 < t < 0.71 \text{ sec} \\ 0 \text{ lbs: } t > 0.71 \text{ sec} \end{cases}$	lb
$\bar{F}_2$	Force acting on Atlas booster	lb
I_2	Moment of inertia of booster vehicle about a transverse axis through center-of-gravity	slug-ft ²
$\bar{L}_2$	Torque about Atlas booster center of gravity	ft-lb
$\bar{l}_2$	Vector from center-of-gravity of booster vehicle to forward end of interstage adapter in x_1, y_1, z_1 reference frame	ft
m_2	Mass of booster vehicle	slugs
n	Number of retrorockets	N. D.
$P(A)$	Probability of the event A	N. D.
$P(A B)$	Conditional probability of the event A relative to the hypothesis that the event B has occurred	N. D.
p	Reliability of a single retrorocket	N. D.
q	Unreliability of a single retrorocket	N. D.
R	Distance from the centerline of booster vehicle to center of retrorocket exit nozzle	ft
$\bar{r}_2$	Vector from separation plane to center of gravity of booster vehicle	ft
$\bar{S}$	Vector from separation plane to forward edge of interstage adaptor	ft
S_{x_1}	Relative axial separation distance	ft
S_{z_1}	Lateral clearance used (z component as seen by an observer whose reference frame is x_1, y_1, z_1)	ft
$T_R(t)$	Residual "tail-off" thrust of sustainer and vernier engines vs. time	lb
t	Time	sec
i, j, k, r	Summation indices	N. D.
x_1, y_1, z_1	Centaur body-fixed reference system	N. D.
x_2, y_2, z_2	Atlas body-fixed reference system	N. D.
θ_{21}	Relative pitch angle between Atlas and Centaur vehicles	deg

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
ξ_i	Attenuation factor for magnitude of thrust of i^{th} retrorocket	N. D.
σ_{ξ}	Standard deviation of ξ_i ($\sigma_{\xi} = \sigma_{\xi_i} = 0.057$)	N. D.
τ	Dummy variable used for time	sec
ϕ_i	Angle defining angular location of i^{th} retrorocket	deg
$\bar{\omega}_0$	Initial angular velocity of Atlas/Centaur vehicle at instant of stage separation	rad/sec
$\bar{\omega}_2$	Angular velocity of booster vehicle	rad/sec

APPENDIX A

ATLAS/CENTAUR SEPARATION EQUATIONS

A.1 INTRODUCTION. The analysis presented in this appendix is concerned with the separation dynamics of the Centaur vehicle from its Atlas booster. The derivation of the clearance equations has been simplified by considering only planar motion of rigid bodies and then retaining only the dominant first-order terms. The adequacy of these approximations has been proved previously by extensive manual calculations and an "exact" analog computer simulation (Reference A-1).

A.2 MATHEMATICAL MODEL. Figure A-1 shows the essential two-body geometrical relationship required to define stage separation.

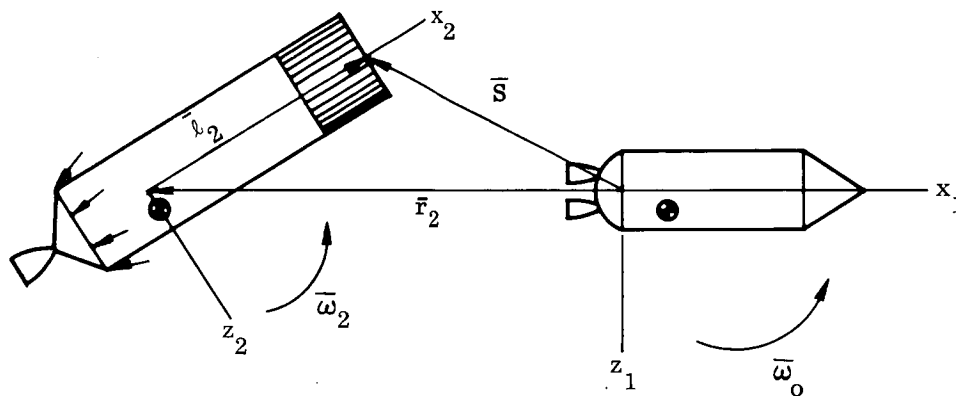


Figure A-1. Atlas/Centaur Separation Geometry

As shown in Figure A-1, two sets of coordinate axes are used. The first set—the x_1, y_1, z_1 coordinate frame—is rigidly attached to the Centaur vehicle (body fixed axes) and has its origin on the geometric centerline at the station number corresponding to the separation plane, Station 413. The second set—the x_2, y_2, z_2 coordinate frame—is rigidly attached to the Atlas booster, its origin being located on the geometric centerline at the station number corresponding to the longitudinal center of gravity of the booster vehicle.

Since we are only concerned here with approximately the first two seconds of time following the detonation of the FLSC (flexible linear-shaped charge), the only expected forces and moments that are acting on the Centaur vehicle are those due to the boost pump exhausts. Furthermore, since the thrust of these exhausts is so small (less than 10 lb apiece), the thrust acceleration of the Centaur vehicle is negligible. In particular, we may assume that the point corresponding to the origin of the x_1, y_1, z_1 reference frame is moving with constant velocity. The x_1, y_1, z_1 reference frame itself is rotating in space at the rate ω_0 , the initial angular velocity of the combined Atlas/Centaur vehicle at the initiation of stage separation.

A.3 DEVELOPMENT OF SEPARATION CLEARANCE EQUATIONS. From a clearance standpoint, we are interested in the motion of the Atlas interstage adapter with respect to the Centaur clearance envelope. In other words, we require the vector $\bar{S}$ as "seen" by the observer in the rotating x_1, y_1, z_1 reference

frame. From elementary mechanics involving rotating coordinate systems, one has

$$\ddot{\bar{\mathbf{S}}} = \frac{d^2 \bar{\mathbf{S}}}{dt^2} - 2 \bar{\boldsymbol{\omega}}_o \times \frac{d\bar{\mathbf{S}}}{dt} \quad (\text{A-1})$$

where the $\dot{}$ (dot) denotes rate of change with respect to time as seen by the observer in the x_1, y_1, z_1 system and $\frac{d}{dt}$ denotes rate of change with respect to time as seen by an inertial observer. Now, since $\bar{\mathbf{S}} = \bar{\mathbf{r}}_2 + \bar{\boldsymbol{\ell}}_2$, (A-1) can also be written as

$$\ddot{\bar{\mathbf{S}}} = \frac{d^2 \bar{\mathbf{r}}_2}{dt^2} + \frac{d\bar{\boldsymbol{\omega}}_2}{dt} \times \bar{\boldsymbol{\ell}}_2 - 2 \bar{\boldsymbol{\omega}}_o \times \frac{d\bar{\mathbf{S}}}{dt} \quad (\text{A-2})$$

where we have again neglected terms involving the products of angular velocities. The first two terms on the right side of (A-2) can be expressed by Newton's Laws of Motion as

$$\left. \begin{aligned} \frac{d^2 \bar{\mathbf{r}}_2}{dt^2} &= \frac{\bar{\mathbf{F}}_2}{m_2} \\ \frac{d\bar{\boldsymbol{\omega}}_2}{dt} \times \bar{\boldsymbol{\ell}}_2 &= \frac{\bar{\mathbf{L}}_2}{I_2} \times \bar{\boldsymbol{\ell}}_2 \end{aligned} \right\} \quad (\text{A-3})$$

and

where $\bar{\mathbf{F}}_2$ is the force acting on the Atlas booster and $\bar{\mathbf{L}}_2$ is the torque about its center of gravity. Equation A-2 can now be rewritten as

$$\ddot{\bar{\mathbf{S}}} = \frac{\bar{\mathbf{F}}_2}{m_2} + \frac{\bar{\mathbf{L}}_2}{I_2} \times \bar{\boldsymbol{\ell}}_2 - 2 \bar{\boldsymbol{\omega}}_o \times \frac{d\bar{\mathbf{S}}}{dt} \quad (\text{A-4})$$

The spacial derivative $\frac{d\bar{\mathbf{S}}}{dt}$ can be expressed equivalently as

$$\frac{d\bar{\mathbf{S}}}{dt} = \dot{\bar{\mathbf{S}}} + \bar{\boldsymbol{\omega}}_o \times \bar{\mathbf{S}} \quad (\text{A-5})$$

Now, substituting this expression for $\frac{d\bar{\mathbf{S}}}{dt}$ into (A-4) and neglecting the small term $\bar{\boldsymbol{\omega}}_o \times (\bar{\boldsymbol{\omega}}_o \times \bar{\mathbf{S}})$, we have

$$\ddot{\bar{\mathbf{S}}} = \frac{\bar{\mathbf{F}}_2}{m_2} + \frac{\bar{\mathbf{L}}_2}{I_2} \times \bar{\boldsymbol{\ell}}_2 - 2 \bar{\boldsymbol{\omega}}_o \times \dot{\bar{\mathbf{S}}} \quad (\text{A-6})$$

Before Equation A-6 can be integrated, it is necessary to resolve all vectors into their components in the x_1, y_1, z_1 reference frame. This yields the following two scalar equations:

$$\ddot{S}_{z_1} = \begin{cases} 0; t < t_o \\ \frac{F_{2z}}{m_2} - \frac{L_{2y}}{I_2} \ell_{2z} + 2\omega_o \dot{S}_{x_1} \left[\frac{F_{2x}}{m_2} \theta_{21} \right]; t > t_o \end{cases} \quad (\text{A-7})$$

$$\ddot{S}_{x_1} = \frac{F_{2x}}{m_2} \left[-\frac{L_{2y}}{I_2} \theta_{21} - 2\omega_0 \dot{S}_{z_1} \right] \quad (A-8)$$

Here, t_0 is the effective time that the latch straps or guide rails provide lateral restraint. θ_{21} is the relative pitch angle between the two vehicles. That is, $\theta_{21} = \iint \frac{L_{2y}}{I_2} dt_1 dt_2 \approx \frac{1}{2} \frac{L_{2y}}{I_2} t^2$. The effects of the terms enclosed by the dashed lines in (A-7) and (A-8) are negligible in the present application. They will therefore be dropped in the remaining analysis.

The retrorockets are mounted on the booster's waveguide fairing at Station 1133. Since the center of percussion (with respect to the forward end of the interstage adapter) is at Station 1090, the transverse components of the retrorocket forces will result in a clearance loss of less than an inch at nine feet of axial separation. These lateral components will therefore be neglected in the following analysis since their effect is so small.

The significant forces and moments appearing in (A-7) and (A-8) above are given by

$$F_{2x} = T_R - \sum_{i=1}^n F_{R_i} \quad (A-9)$$

$$F_{2z} = 0 \quad (A-10)$$

$$L_{2y} = \sum_{i=1}^n (R \sin \phi_i + \bar{z}) F_{R_i} \quad (A-11)$$

where T_R is the combined residual thrust of the sustainer and vernier engines. It will be assumed here that the thrust vs. time characteristic of each retrorocket is functionally the same. Thus, one may write

$$F_{R_i}(t) = \xi_i \bar{F}_R(t) \quad (A-12)$$

where $\bar{F}_R(t)$ represents the nominal thrust vs. time for the particular type of rocket being used. ξ_i is an attenuation factor which accounts for individual thrust dispersions. Thus, the ξ_i ($i = 1, 2, \dots, n$) are independent random variables having unity for mean and σ_ξ^2 for variance. Substituting from Equations A-9 through A-12, Equations A-7 and A-8 become

$$\ddot{S}_{z_1}(t) = -\frac{\ell_2 R}{I_2} \left[\sum_{i=1}^n \left(\sin \phi_i + \frac{\bar{z}}{R} \right) \xi_i \right] \bar{F}_R(t) + 2\omega_0 \dot{S}_{x_1}(t); t > t_0 \quad (A-13)$$

$$\ddot{S}_{x_1}(t) = \frac{1}{m_2} \left[T_R(t) - \left(\sum_{i=1}^n \xi_i \right) \bar{F}_R(t) \right] \quad (A-14)$$

Equations A-13 and A-14 can be integrated directly to give

$$S_{z_1}(t) = -\frac{\ell_2 R}{I_2} \left[\sum_{i=1}^n \left(\sin \varphi_i + \frac{\bar{z}}{R} \right) \xi_i \right] \int_{t_0}^t (t - \tau) \bar{F}_R(\tau) d\tau + 2 \left[\int_{t_0}^t (t - \tau) \dot{S}_{x_1}(\tau) d\tau \right] \omega_0 \quad ; t > t_0 \quad (A-15)$$

$$S_{x_1}(t) = \frac{1}{m_2} \int_0^t (t - \tau) \left[T_R(\tau) - \left(\sum_{i=1}^n \xi_i \right) \bar{F}_R(\tau) \right] d\tau \quad (A-16)$$

Now, Equation A-16 can be used to eliminate $\int_{t_0}^t (t - \tau) \bar{F}_R(\tau) d\tau$ and $\dot{S}_{x_1}(\tau)$ from Equation A-15. Thus, one obtains the result

$$S_{z_1} = -\frac{m_2 \ell_2 R}{I_2} \left\{ -S_{x_1} + \frac{1}{m_2} \left[\int_0^t (t - \tau) T_R(\tau) d\tau - \left(\sum_{i=1}^n \xi_i \right) \int_0^{t_0} (t - \tau) \bar{F}_R(\tau) d\tau \right] \right\} \frac{\sum_{i=1}^n \left(\sin \varphi_i + \frac{\bar{z}}{R} \right) \xi_i}{\sum_{i=1}^n \xi_i} + \frac{1}{m_2} \left\{ (t - t_0)^2 \int_0^{t_0} \left[T_R(\tau) - \left(\sum_{i=1}^n \xi_i \right) \bar{F}_R(\tau) \right] d\tau + \int_{t_0}^t (t - \tau)^2 \left[T_R(\tau) - \left(\sum_{i=1}^n \xi_i \right) \bar{F}_R(\tau) \right] d\tau \right\} \omega_0 \quad (A-17)$$

The random variable S_{z_1} in Equation A-17 is here considered to be a function only of the independent random variables ξ_i ($i = 1, 2, \dots, n$) and ω_0 . The displacement S_{x_1} is treated as a specified parameter. t_0 and t are both dependent upon the ξ_i ($i = 1, 2, \dots, n$). For instance, t_0 is obtained by substituting the effective guide rail length for the left side of Equation 16 and solving the resulting integral equation for t . This value of t is t_0 and obviously depends on the ξ_i . In the same manner, the value of t which corresponds to any specified value of S_{x_1} can be found. Unless both $T_R(\tau)$ and $\bar{F}_R(\tau)$ are extremely simple functions, a graphical integration will usually afford the most efficient solution.

The mean value of the lateral displacement at the forward end of the interstage adapter is readily obtained from Equation A-17 as

$$\bar{S}_{z_1} = -\frac{m_2 \ell_2 R}{I_2} \left\{ -S_{x_1} + \frac{1}{m_2} \left[\int_0^{\bar{t}_x} (\bar{t}_x - \tau) T_R(\tau) d\tau - n \int_0^{\bar{t}_0} (\bar{t}_x - \tau) \bar{F}_R(\tau) d\tau \right] \right\} \frac{1}{n} \sum_{i=1}^n (\sin \varphi_i + \frac{\bar{z}}{R}) \quad (A-18)$$

where the mean residual turning rate $\bar{\omega}_0$ has been neglected. The symbol $\bar{t}_x$ used above and in the following denotes the nominal time required to achieve x feet of axial separation. Likewise, $\bar{t}_0$ denotes the nominal time required for the booster vehicle to clear the end of the guide rails. The word "nominal" as used here actually means nominal relative to a given hypothesis.

Optimum design considerations require that the mean lateral displacement S_{z_1} be zero. This requirement could be satisfied by biasing (judiciously spacing) the retrorockets such that

$$\frac{1}{n} \sum_{i=1}^n \sin \varphi_i = -\frac{\bar{z}}{R} \quad (A-19)$$

although, in the case of the Atlas/Centaur, physical considerations prevent this.

The effects due to dispersions in retrorocket thrust and residual turning rate are now evaluated statistically. As noted earlier, the lateral displacement, S_{z_1} given by Equation A-17, is a function of the retrorocket thrust magnitudes and the residual turning rate; i. e.,

$$S_{z_1} = S_{z_1}(\xi_1, \xi_2, \dots, \xi_n, \omega_0)$$

Now, the variance of S_{z_1} is given by

$$\sigma_{S_{z_1}}^2 = \sum_{i=1}^n \left(\frac{\partial S_{z_1}}{\partial \xi_i} \right)^2 \sigma_{\xi_i}^2 + \left(\frac{\partial S_{z_1}}{\partial \omega_0} \right)^2 \sigma_{\omega_0}^2 + [\dots \text{higher-order terms} \dots] \quad (A-20)$$

where the partial derivatives are evaluated at $\xi_i = \bar{\xi}_i = 1$ ($i = 1, 2, \dots, n$) and $\omega_0 = \bar{\omega}_0$. The higher-order terms can be shown to be negligible in the present application. Furthermore, since $\sigma_{\xi_i}^2 = \sigma_{\xi}^2$ ($i = 1, 2, \dots, n$), Equation A-20 can be simplified to read

$$\sigma_{S_{z_1}}^2 = \left[\sum_{i=1}^n \left(\frac{\partial S_{z_1}}{\partial \xi_i} \right)^2 \right] \sigma_{\xi}^2 + \left(\frac{\partial S_{z_1}}{\partial \omega_0} \right)^2 \sigma_{\omega_0}^2 \quad (A-21)$$

Referring to Equation A-17, it will be found convenient to introduce an auxiliary variable χ defined as follows:

$$\chi = -s_{x_1} + \frac{1}{m_2} \left[\int_0^t (t - \tau) T_R(\tau) d\tau - \left(\sum_{i=1}^n \xi_i \right) \int_0^t (t - \tau) \bar{F}_R(\tau) d\tau \right] \quad (A-22)$$

The partial derivatives, $\frac{\partial S}{\partial \xi_j}$ ($j = 1, 2, \dots, n$), are now given by

$$\frac{\partial S_{z_1}}{\partial \xi_j} = -\frac{m_2 \ell_2 R}{I_2} \left[\frac{1}{n} \bar{\chi} \left(\sin \varphi_j + \frac{\bar{z}}{R} \right) - \frac{1}{n^2} \left(\bar{\chi} - n \frac{\partial \chi}{\partial \xi_j} \right) \sum_{i=1}^n \left(\sin \varphi_i + \frac{\bar{z}}{R} \right) \right] \quad (A-23)$$

where

$$\bar{\chi} = -s_{x_1} + \frac{1}{m_2} \left[\int_0^{\bar{t}_x} (\bar{t}_x - \tau) T_R(\tau) d\tau - n \int_0^{\bar{t}_o} (\bar{t}_x - \tau) \bar{F}_R(\tau) d\tau \right] \quad (A-24)$$

and

$$\begin{aligned} \frac{\partial \chi}{\partial \xi_j} = \frac{1}{m_2} & \left[\frac{\partial \bar{t}_x}{\partial \xi_j} \int_0^{\bar{t}_x} T_R(\tau) d\tau - \int_0^{\bar{t}_o} (\bar{t}_x - \tau) \bar{F}_R(\tau) d\tau \right. \\ & \left. - n \frac{\partial \bar{t}_x}{\partial \xi_j} \int_0^{\bar{t}_o} \bar{F}_R(\tau) d\tau - n (\bar{t}_x - \bar{t}_o) \bar{F}_R(\bar{t}_o) \frac{\partial \bar{t}_o}{\partial \xi_j} \right] \end{aligned} \quad (A-25)$$

The partial derivatives, $\frac{\partial \bar{t}_x}{\partial \xi_j}$ and $\frac{\partial \bar{t}_o}{\partial \xi_j}$, can be obtained by differentiating (A-16). Thus, one obtains the

following results:

$$\frac{\partial \bar{t}_x}{\partial \xi_j} = - \frac{\int_0^{\bar{t}_x} (\bar{t}_x - \tau) \bar{F}_R(\tau) d\tau}{\int_0^{\bar{t}_x} [n \bar{F}_R(\tau) - T_R(\tau)] d\tau} \quad (A-26)$$

$$\frac{\partial t_o}{\partial \xi_j} = - \frac{\int_0^{\bar{t}_o} (\bar{t}_o - \tau) \bar{F}_R(\tau) d\tau}{\int_0^{\bar{t}_o} [n \bar{F}_R(\tau) - T_R(\tau)] d\tau} \quad (A-27)$$

The coefficient of σ_{ξ}^2 in Equation A-21 is

$$\begin{aligned} \sum_{i=1}^n \left(\frac{\partial S_{z_1}}{\partial \xi_i} \right)^2 &= \left(\frac{m_2 \ell_2 R}{I_2} \right)^2 \frac{\bar{X}^2}{n^2} \left\{ \sum_{i=1}^n \sin^2 \varphi_i - \frac{1}{n} \left[1 - n^2 \frac{\left(\frac{\partial X}{\partial \xi} \right)^2}{\bar{X}^2} \right] \left(\sum_{i=1}^n \sin \varphi_i \right)^2 \right. \\ &\quad \left. + 2 \frac{\bar{Z}}{R} n^2 \frac{\left(\frac{\partial X}{\partial \xi} \right)^2}{\bar{X}^2} \left[\sum_{i=1}^n \sin \varphi_i + \frac{n}{2} \frac{\bar{Z}}{R} \right] \right\} \quad (A-28) \end{aligned}$$

The coefficient of $\sigma_{\omega_o}^2$ in Equation A-21 is

$$\begin{aligned} \left(\frac{\partial S_{z_1}}{\partial \omega_o} \right)^2 &= \left(\frac{1}{m_2} \right)^2 \left\{ (\bar{t}_x - \bar{t}_o)^2 \int_0^{\bar{t}_o} [T_R(\tau) - n \bar{F}_2(\tau)] d\tau \right. \\ &\quad \left. + \int_{\bar{t}_o}^{\bar{t}_x} (\bar{t}_x - \tau)^2 [T_R(\tau) - n \bar{F}_R(\tau)] d\tau \right\}^2 \quad (A-29) \end{aligned}$$

A.4 STATISTICAL ANALYSIS OF COLLISION. The Centaur clearance envelope consists of the frustum of a cone having a half angle of $5^\circ 45'$. The radial clearance thus varies linearly from zero at Station 413 to 10.8 inches at the aft end of the main engine bells at Station 521. The trajectory of the forward edge of the interstage adapter with respect to the clearance envelope (see Equation A-17) is such that one need consider only the clearance used (the random variable S_{z_1}) at Station 521.

If c denotes the available clearance, the probability of collision is then equal to the probability that $|S_{z_1}|$ is greater than c . Let E represent the event, $|S_{z_1}| > c$. Then,

$$P(E) = P(|S_{z_1}| > c) \quad (A-30)$$

Now, consider the finite set of mutually exclusive events $E_0, E_1, \dots, E_m$.

E_0 = the event, no retrorockets fail

E_1 = the event, retrorocket No. 1 fails

E_2 = the event, retrorocket No. 2 fails

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E_n = the event, retrorocket No. n fails

E_{n+1} = the event, retrorockets No. 1 and No. 2 fail

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E_m = the event, all retrorockets fail

The total number of events in this set is $m + 1$. This number is

$$m + 1 = \sum_{r=0}^n \frac{n!}{r! (n-r)!} \quad (\text{A-31})$$

The event E , ($S_{Z_1} > c$), obviously can only occur in conjunction with one of events in the set $E_0, E_1, \dots, E_m$. Stated mathematically, the previous statement becomes

$$E = \sum_{k=0}^m E_k E$$

Now, since the events $E_k E$ ($k = 0, 1, \dots, m$) are mutually exclusive, one has

$$P(E) = \sum_{k=0}^m P(E_k E)$$

or using the relationship, $P(E_k E) = P(E_k)P(E|E_k)$, for conditional probabilities, one finally obtains

$$P(E) = \sum_{k=0}^m P(E_k)P(E|E_k) \quad (\text{A-32})$$

Here, the standard notation $P(E|E_k)$ is used to denote the probability of the event E relative to the hypothesis that the event E_k has occurred. Since $P(E|E_k) \leq 1$ for all k , it follows from Equation A-32 that

$$P(E) \leq \sum_{k=0}^n P(E_k)P(E|E_k) + \sum_{k=n+1}^n P(E_k) \quad (A-33)$$

$$\begin{aligned} \text{now, } \sum_{k=n+1}^n P(E_k) &= \sum_{r=2}^n \frac{n!}{r!(n-r)!} p^{n-r} q^r \\ &= \frac{n}{2} (n-1) p^{n-2} q^2 + \dots \end{aligned} \quad (A-34)$$

where p is the reliability of a single rocket and $q = 1 - p$ is the unreliability (probability of failure) of a single rocket. For normally accepted reliabilities, say, $p > 0.99$, only the first term in Equation A-34 is of any practical significance. Also, since $p^{n-2} \leq 1$, Equation A-33 can now be rewritten as

$$P(E) \leq \sum_{k=0}^n P(E_k)P(E|E_k) + \frac{n}{2} (n-1) q^2 \quad (A-35)$$

Now the probability of collision relative to the hypothesis that no retrorockets fail is $P(E|E_0)$. This probability will usually be so small as to be negligible. Also, since $P(E_1) = P(E_2) = \dots = P(E_n) = q p^{n-1} \approx q$, Equation 35 can be further simplified to read

$$P(E) \leq q \sum_{k=1}^n P(E|E_k) + \frac{n}{2} (n-1) q^2 \quad (A-36)$$

REFERENCES

- A-1 Backus, F. I., Greiner, H. G., and Huxtable, A. J., Atlas/Centaur Separation Study, AWW63-001, 1 May 1963.

APPENDIX B
POWERED PHASE FLIGHT STABILITY
EQUATIONS

This appendix is divided into four major sections representing the intermediate, high, and low frequency ranges used in the description of the powered phase flight behavior. The major sections are:

	<u>Page</u>
a. Section B1, Ignition Phase Equations	B1-3
b. Section B2, Rigid Body/Propellant Sloshing Stability Equations	B2-1
c. Section B3, Elastic Vehicle Bending Stability Equations	B3-1
d. Section B4, Guidance/Autopilot Stability Equations	B4-1

SECTION B1
IGNITION PHASE EQUATIONS

LIST OF SYMBOLS FOR SECTION B1

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A	Actuating piston area	ft ²
A _A	Accumulator piston area	ft ²
A _{BS}	"Bootstrap" piston area	ft ²
A _f	Feedback piston area	ft ²
A _{RA}	Reservoir piston area	ft ²
A _s	Sensing piston area	ft ²
A _{RH}	A _{RA} + A _{BS}	ft ²
A _V	Valve spool area	ft ²
B	Bulk modulus of hydraulic fluid	lb/ft ²
C _B	Coulomb friction coefficient	ft-lb
C _V	Viscous friction coefficient	$\frac{\text{ft-lb}}{\text{deg/sec}}$
C _f	Discharge coefficient for washout orifice	$\frac{\text{ft}^3/\text{sec}}{\sqrt{\text{lb/ft}^2}}$
dV	Volume change due to fluid compressibility in hydraulic power supply	ft ³
F _S	Initial reservoir spring force	lb
I _{yy_r}	Reduced moment of inertia about the pitch axis	slug-ft ²
I _{xx_r}	Reduced moment of inertia about the roll axis	slug-ft ²
I _{xy_r}	Reduced product of inertia about the pitch-roll axis	slug-ft ²
I _R	Engine "wet" moment of gimballed mass about gimbal point	slug-ft ²
i _V	Servo valve current	ma
i _V '	Equivalent servo valve current	ma
K	Power of adiabatic compression	N. D.
K _a	Servoamplifier gain	ma/volt
K _A K _{R_θ}	End-to-end rate gain, pitch	$\frac{\text{deg}}{\text{deg/sec}}$

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
$K_A K_R$	End-to-end rate gain, roll	$\frac{\text{deg}}{\text{deg/sec}}$
K_f	Feedback spring constant	lb/ft^2
K_i	Torque motor gain	lb/ma
K_{I_θ}	Pitch autopilot integrator gain	sec^{-1}
K_{I_ϕ}	Roll autopilot integrator gain	sec^{-1}
K_m	Spring constant of actuator support structure	lb/ft
K_s	DPF valve pressure feedback gain	$\frac{\text{ma}}{\text{lb/ft}^2}$
K_t	Feedback transducer gain	volts/deg
K_V	Servovalve discharge coefficient	$\frac{\text{ft}^3/\text{sec}}{\text{ma} \sqrt{\text{lb/ft}^2}}$
K_1	Valve spool spring constant	lb/ft
K_2	Sensing piston spring constant	lb/ft
$L_{P_{CL}}, L_{P_{CH}}$	Liquid oxygen and liquid hydrogen pendulum lengths	ft
ℓ_c	Rocket engine control moment arm	ft
ℓ_{CL}, ℓ_{CH}	Liquid oxygen and liquid hydrogen pendulum moment arms	ft
ℓ_R	Distance between rocket engine center of gravity and gimbal point	ft
ℓ_z	Rocket engine roll control moment arm	ft
M_r	Reduced mass of vehicle (without sloshing mass)	slugs
$M_{P_{CL}}, M_{P_{CH}}$	Liquid oxygen and liquid hydrogen pendulum masses	slugs
M_R	Rocket engine gimballed mass, one engine, "wet"	slugs
M_T	Total mass of vehicle	slugs
η	Duct restraint moment (fixed)	ft-lb
η_x	Duct restraint moment coefficient per unit axial acceleration	$\frac{\text{ft-lb}}{\text{ft/sec}^2}$
η_δ or ξ	Duct restraint moment coefficient per unit deflection	ft-lb/deg

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
N	Engine hydraulic pump speed	rev/sec
P_A	Pressure on lefthand side of valve spool	lb/ft ²
P_{AC}	Pressure in Region d of accumulator (see Figure B-6)	lb/ft ²
P_B	Pressure on righthand side of valve spool (see Figure B-6)	lb/ft ²
P_C	Relief valve cracking pressure	lb/ft ²
P_{FF}	Relief valve pressure drop at full flow	lb/ft ²
P_L	Dynamic load pressure (pressure drop across actuating piston)	lb/ft ²
P_{RA}	Pressure, Region a of reservoir (see Figure B-6)	lb/ft ²
P_R	Hydraulic system reservoir pressure (return pressure from servovalve)	lb/ft ²
P_S	Hydraulic system supply pressure to servovalve	lb/ft ²
$P_{S(ref)}$	Reference supply pressure	lb/ft ²
ΔP_f	Pressure drop across sensing piston in dpf valve	lb/ft ²
Q_{ACC}	Flow from accumulator	ft ³ /sec
Q_{ACT}	Actuator flow	ft ³ /sec
Q_{BS}	Flow into bootstrap region	ft ³ /sec
Q_{FF}	Relief valve full flow	ft ³ /sec
Q_p	Pump flow	ft ³ /sec
Q/rev	Volume per pump revolution	ft ³ /rev
Q_T	Total flow into high pressure side of hydraulic power supply	ft ³ /sec
Q_V	Relief valve flow into reservoir	ft ³ /sec
R	Engine actuator moment arm	ft
s	Laplace operator	sec ⁻¹
T_c	Control Engine Thrust	lb
T_i	Temperature of hydraulic system at launch	°K
T_o	Temperature of hydraulic system at MES	°K

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
V_A	Total volume of hydraulic fluid in one actuator cylinder	ft ³
V_{lines}	Volume of lines in hydraulic system on high pressure side	ft ³
V_T	Total volume, high-pressure side	ft ³
V_{AC}	Volume, Region d of accumulator (see Figure B-6)	ft ³
V_{RA}	Volume, Region a of reservoir (see Figure B-6)	ft ³
$\ddot{x}$	Vehicle axial acceleration	ft/sec ²
x_A	Accumulator piston displacement	ft
x_f	Isolation piston displacement	ft
x_m	Actuator piston displacement	ft
x_{res}	Reservoir piston displacement	ft
x_s	Sensing piston displacement	ft
x_V	Valve spool displacement	ft
$\ddot{z}$	Vehicle normal acceleration (plunging)	ft/sec ²
Δ	Angular offset of the rocket engine center -of-gravity from the engine centerline	deg
δ	Rocket engine gimbal angle, pitch	deg
δ_c	Commanded engine angle	deg
∇	Thrust misalignment	deg
ϵ	Thrust offset	ft
ζ_{CL}, ζ_{CH}	Liquid oxygen and liquid hydrogen slosh damping ratio	N. D.
ζ_V	Pump efficiency	N. D.
θ	Vehicle pitch angle	deg
ξ	Rocket engine gimbal angle, yaw/roll	deg
τ_R	Equivalent rate gyro time constant	sec

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
ϕ	Vehicle roll angle	deg
ϕ_{CL}, ϕ_{CH}	Liquid oxygen and liquid hydrogen pendulum angles	deg
ω_{CL}, ω_{CH}	Liquid oxygen and liquid hydrogen slosh frequencies	rad/sec

Subscripts

1, 2	Refers to C-1 or C-2 engine system
c	Refers to command
f	Refers to feedback
δ	Refers to engine in pitch
ξ	Refers to engine in yaw/roll

B1.2 VEHICLE DYNAMICS. The mathematical model used to describe the behavior of the Centaur vehicle following engine ignition is that of a rigid body with two pendulums (representing effects of propellant sloshing), and two gimbaled masses (representing the thrust chambers). Only perturbational motion in the pitch and roll planes is considered. This results in some error for large vehicle excursions; however, for the limited purposes of the analysis it was considered tolerable.

The vehicle center-of-gravity offset was assumed to be zero, without loss of generality. The actuator lengths are preset so that the nominal thrust vector of both engines taken together will point through the center of gravity of the vehicle at ignition in the presence of changes in the actuator geometry due to temperature changes and thrust loading. In Figures B1-1 through B1-3 the line labeled "vehicle centerline" is actually parallel to a line passing through the vehicle c.g. and the vehicle centerline at Station 453.5.

[illegible]

B1-11

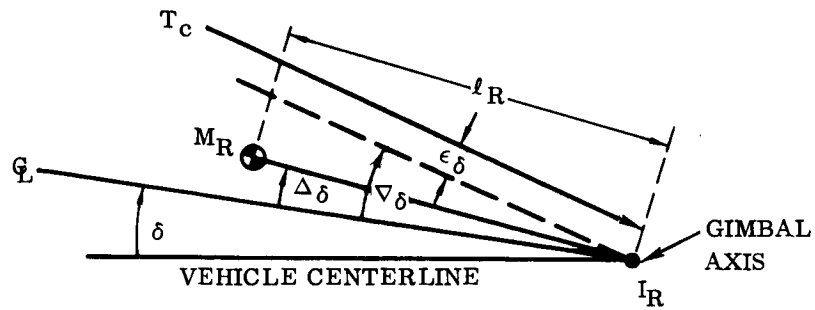


Figure B1-2. Engine Coordinate System, Pitch Plane

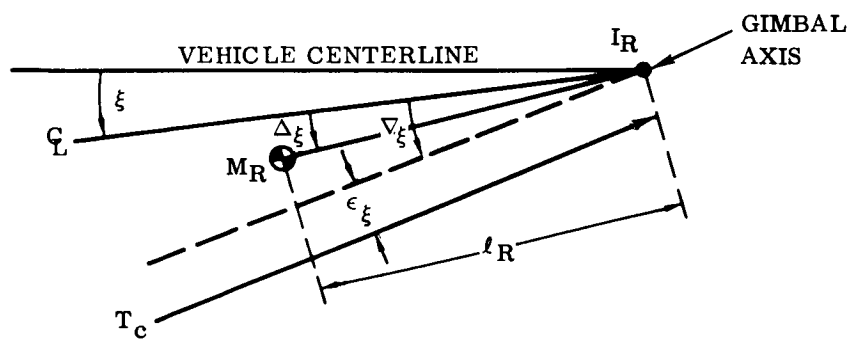


Figure B1-3. Engine Coordinate System, Yaw Plane

The vehicle dynamics equations are as follows:

a. Angular Acceleration, Pitch Plane:

$$\begin{aligned}
 I_{yy_r} \ddot{\theta} = & I_{xy_r} \ddot{\phi} - M_{P_{CL}} \ell_{CL} \ddot{x}_{CL} \phi_{CL} - M_{P_{CH}} \ell_{CH} \ddot{x}_{CH} \phi_{CH} \\
 & + T_{c_1} \ell_c (\delta_1 + \nabla_{\delta_1}) + T_{c_2} \ell_c (\delta_2 + \nabla_{\delta_2}) \\
 & + (I_R + M_R \ell_R \ell_c) (\ddot{\delta}_1 + \ddot{\delta}_2) + M_R \ell_R \ddot{x} (\delta_1 + \Delta_{\delta_1}) \\
 & + M_R \ell_R \ddot{x} (\delta_2 + \Delta_{\delta_2}) - 57.3 T_{c_1} (\ell_z + \epsilon_1) + 57.3 T_{c_2} (\ell_z + \epsilon_2)
 \end{aligned} \tag{B1-1}$$

b. Angular Acceleration, Roll Plane:

$$I_{xx_r} \ddot{\phi} = I_{yx_r} \ddot{\theta} - T_{c_1} \ell_z (\xi_1 + \nabla_{\xi_1}) + T_{c_2} \ell_z (\xi_2 + \nabla_{\xi_2}) - M_R \ell_R \ell_z (\ddot{\xi}_1 + \ddot{\xi}_2) \tag{B1-2}$$

c. Lateral Acceleration, Pitch Plane:

$$\begin{aligned}
 57.3 M_r \ddot{z} = & M_{P_{CL}} \ddot{x}_{CL} \phi_{CL} + M_{P_{CH}} \ddot{x}_{CH} \phi_{CH} + T_{c_1} (\delta_1 + \nabla_{\delta_1}) + T_{c_2} (\delta_2 + \nabla_{\delta_2}) \\
 & + M_R \ell_R (\ddot{\delta}_1 + \ddot{\delta}_2)
 \end{aligned} \tag{B1-3}$$

d. Axial Acceleration:

$$M_T \ddot{x} = T_{c_1} + T_{c_2} \tag{B1-4}$$

e. Propellant Sloshing, Pitch Plane:

$$L_{P_{CL}} \ddot{\phi}_{CL} + 2 \zeta_{CL} \omega_{CL} L_{P_{CL}} \dot{\phi}_{CL} + \ddot{x}_{CL} \phi_{CL} = (\ell_{CL} - L_{P_{CL}}) \ddot{\theta} - 57.3 \ddot{z} \tag{B1-5}$$

$$L_{P_{CH}} \ddot{\phi}_{CH} + 2 \zeta_{CH} \omega_{CH} L_{P_{CH}} \dot{\phi}_{CH} + \ddot{x}_{CH} \phi_{CH} = (\ell_{CH} - L_{P_{CH}}) \ddot{\theta} - 57.3 \ddot{z} \tag{B1-6}$$

f. Engine Gimbal Torques:

$$\begin{aligned}
 \frac{1}{57.3} I_R \ddot{\delta}_1 + C_V \dot{\delta}_1 + C_B \frac{\dot{\delta}_1}{|\dot{\delta}_1|} + \frac{M_R \ell_R \ddot{x}}{57.3} (\delta_1 + \Delta_{\delta_1}) + m_{\delta_1} \delta_1 + m_{\delta_1} \\
 + m_{\ddot{x}_{\delta_1}} \ddot{x} = M_R \ell_R \ddot{z} + \frac{1}{57.3} (I_R + M_R \ell_R \ell_c) \ddot{\theta} + T_{c_1} \epsilon_{\delta_1} + A R P_{L_{\delta_1}}
 \end{aligned} \tag{B1-7}$$

$$\begin{aligned}
& \frac{1}{57.3} I_R \ddot{\delta}_2 + C_V \dot{\delta}_2 + C_B \left[\frac{\dot{\delta}_2}{\delta_2} \right] + \frac{M_R \ell_R \ddot{x}}{57.3} (\delta_2 + \Delta_{\delta_2}) + m_{\delta_2} \delta_2 + m_{\delta_2} \\
& + m_{\ddot{x}_{\delta_2}} \ddot{x} = M_R \ell_R \ddot{z} + \frac{1}{57.3} (I_R + M_R \ell_R \ell_c) \ddot{\theta} + T_{c_2} \epsilon_{\delta_2} + A R P_{L_{\delta_2}}
\end{aligned} \tag{B1-8}$$

$$\begin{aligned}
& \frac{1}{57.3} I_R \ddot{\xi}_1 + C_V \dot{\xi}_1 + C_B \left[\frac{\dot{\xi}_1}{\xi_1} \right] + \frac{M_R \ell_R \ddot{x}}{57.3} (\xi_1 + \Delta_{\xi_1}) + m_{\xi_1} \xi_1 + m_{\xi_1} \\
& + m_{\ddot{x}_{\xi_1}} \ddot{x} = - \frac{1}{57.3} M_R \ell_R \ell_z \ddot{\phi} + T_{c_1} \epsilon_{\xi_1} + A R P_{L_{\xi_1}}
\end{aligned} \tag{B1-9}$$

$$\begin{aligned}
& \frac{1}{57.3} I_R \ddot{\xi}_2 + C_V \dot{\xi}_2 + C_B \left[\frac{\dot{\xi}_2}{\xi_2} \right] + \frac{M_R \ell_R \ddot{x}}{57.3} (\xi_2 + \Delta_{\xi_2}) + m_{\xi_2} \xi_2 + m_{\xi_2} \\
& + m_{\ddot{x}_{\xi_2}} \ddot{x} = \frac{1}{57.3} M_R \ell_R \ell_z \ddot{\phi} + T_{c_2} \epsilon_{\xi_2} + A R P_{L_{\xi_2}}
\end{aligned} \tag{B1-10}$$

B1.3 HYDRAULIC ACTUATOR. Figure B1.4 is a schematic diagram of the engine support and actuation mechanism. Both sides of the actuator are considered separately, using the law of hydraulic compressibility to equate flow and pressure on each side of the actuator piston and the pressure differential across the piston face.

The linear law of hydraulic compressibility is given by

$$\Delta P = \frac{B \Delta V}{V} \tag{B1-11}$$

where

ΔP = change in pressure within the cavity (lb/ft²)

B = bulk modulus of the hydraulic fluid (lb/ft²)

V = volume of the cavity (ft³)

ΔV = volume of fluid to be added to the cavity to cause a pressure rise of ΔP (ft³)

Differentiating Equation B1-11 gives

$$\begin{aligned}
\dot{P} &= \frac{B}{V} (\Delta \dot{V}) - \frac{B}{V^2} (\Delta V) \dot{V} \\
&= \frac{B}{V} Q_c - \frac{B}{V^2} (\Delta V) \dot{V}
\end{aligned} \tag{B1-12}$$

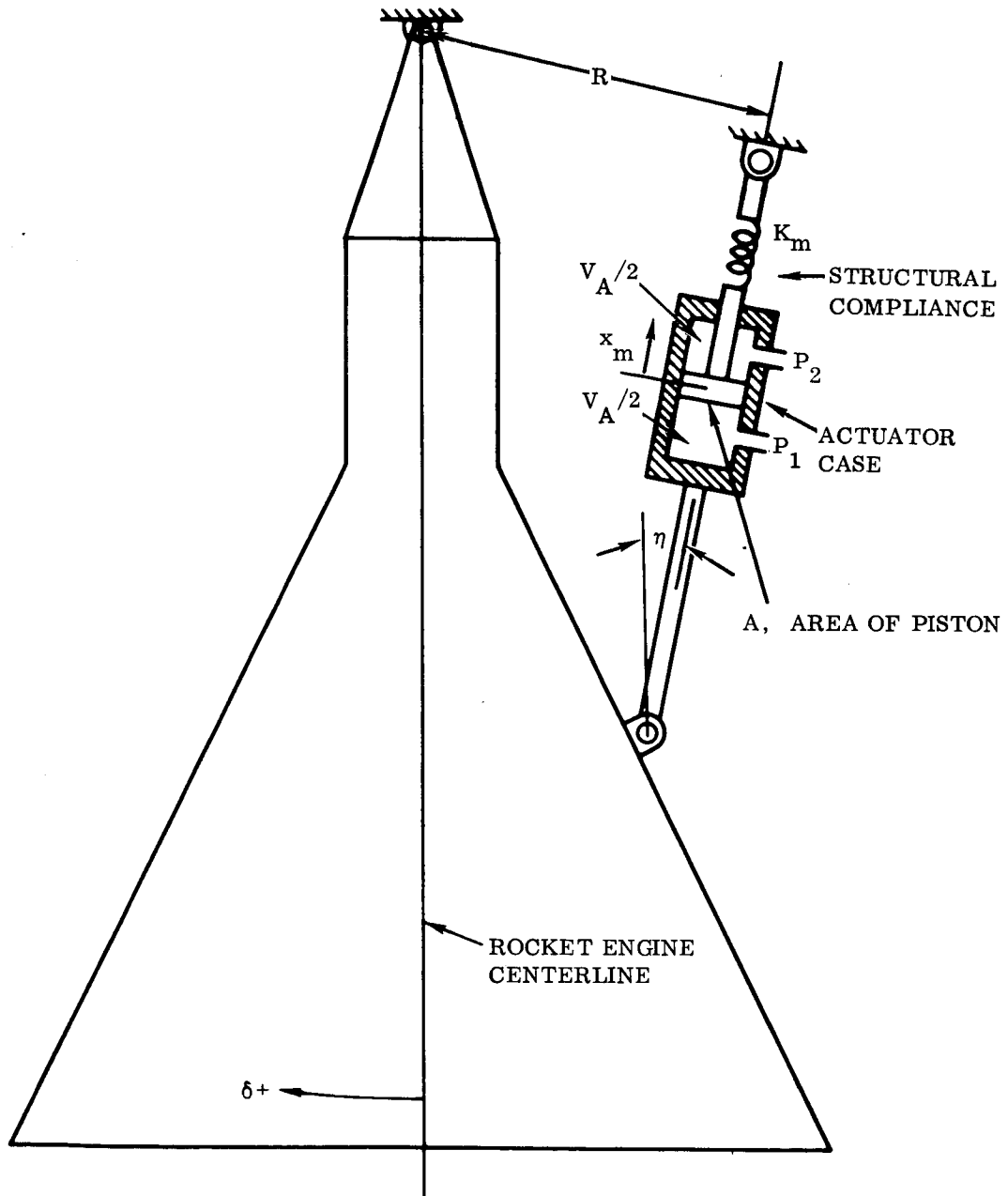


Figure B1-4. Engine Position Servo, Schematic Diagram

where Q_c is the compressibility flow, in ft^3/sec . Since the bulk modulus of the hydraulic fluid is of the order of $4 \times 10^7 \text{ lb/in.}^2$, the increase of hydraulic fluid, ΔV , accompanying a rise in the pressure within the cavity, is of the order of 10^{-2} times the volume of the cavity. The rate of change of the volume, V , due in this case to motion of the actuator piston, is of the same order of magnitude as the flow, Q_c . Therefore the second term on the right-hand side of Equation B1-12 is of second order and may be omitted, giving

$$\dot{P} = \frac{B}{V} Q_c \quad (\text{B1-13})$$

The volume, V_1 , on one side of the actuator piston may be considered to consist of a fixed volume, V_1' , plus a displacement volume, dV_1 , due to the motion of the piston with respect to the actuator case.

$$V_1 = V_1' + dV_1 \text{ (for side 1)} \quad (\text{B1-13a})$$

$$V_2 = V_2' - dV_2 \text{ (for side 2)} \quad (\text{B1-13b})$$

It is assumed that the null, or center position of the actuator piston, divides the volume of the actuator case, V_A , into two equal parts,

$$V_1' = V_2' = \frac{V_A}{2} \quad (\text{B1-13c})$$

Obviously the displacement value, dV_1 , is equal to the negative of that on the other side of the piston, dV_2 . Therefore

$$dV_1 = dV_2 \quad (\text{B1-13d})$$

Equation (B1-13) can be restated by two equations pertinent to the two sides of the actuator piston

$$Q_1 = \frac{V_1' + dV_1}{B} \dot{P}_1 \quad (\text{B1-14a})$$

$$Q_2 = \frac{V_2' + dV_2}{B} \dot{P}_2 \quad (\text{B1-14b})$$

The two flows, Q_1 and Q_2 , into the actuator case on each side of the actuator piston consist of two parts, one due to compressibility and the other due to piston displacement.

$$Q_1 = Q_{\text{ACT}1} + Q_{p1} \quad (\text{B1-14c})$$

$$Q_2 = Q_{\text{ACT}2} + Q_{p2} \quad (\text{B1-14d})$$

Q_{ACT} equals valve flow in cubic feet per second.

Q_p results from three movements: 1) movement of the case with piston fixed due to angular movement of the engine, $\dot{\delta}$; 2) movement of the piston with case fixed due to compliance of the actuator rod and support structure, $\dot{x}_m$; and 3) movement of the piston due to leakage flow around the piston. Piston movement due to leakage flow is assumed negligible, and the resulting equations for Q_p are

$$Q_{p_1} = -\frac{AR\dot{\delta}}{57.3} - A\dot{x}_m \quad (B1-14e)$$

$$Q_{p_2} = -Q_{p_1} = \frac{AR\dot{\delta}}{57.3} + A\dot{x}_m \quad (B1-14f)$$

where A is the area of actuator piston face in square feet and R is the moment arm of the actuator about the engine gimbal point in feet.

Equations B-14a and B-14b may be written as:

$$\frac{(V_1' + dV_1)}{B} \dot{P}_1 = -\frac{AR\dot{\delta}}{57.3} - A\dot{x}_m + Q_{ACT_1} \quad (B1-15a)$$

$$\frac{(V_2' + dV_2)}{B} \dot{P}_2 = +\frac{AR\dot{\delta}}{57.3} + A\dot{x}_m + Q_{ACT_2} \quad (B1-15b)$$

The valve design is such that the orifice areas of both inlet and exit are equal. Assuming small displacements of the actuator piston, $dV \ll V$, the valve flows, Q_{ACT_1} and $-Q_{ACT_2}$, will be equal. Subtracting Equation B-15b from B-15a gives:

$$\frac{V}{2B} (\dot{P}_1 - \dot{P}_2) = -2\frac{AR\dot{\delta}}{57.3} - 2A\dot{x}_m + 2Q_{ACT} \quad (B1-16)$$

The load the actuator can support is a function of the pressure differential across the actuator piston face. Therefore this pressure differential is defined as load pressure, P_L ,

$$P_L = P_1 - P_2 \quad (B1-17)$$

Neglecting the inertia force of the mass of the actuator when compared with the elastic forces (spring constant of K_m , lb/ft) of the support structure, the displacement of the actuator piston is given by:

$$x_m = \frac{AP_L}{K_m} \quad (B1-17a)$$

Differentiating the formula,

$$\dot{x}_m = \frac{A}{K_m} \dot{P}_L \quad (B1-18)$$

Substituting Equations B1-17 and B1-18 into B1-16 and transposing terms.

$$\left(\frac{V_A}{4B} + \frac{A^2}{K_m}\right) \dot{P}_L = -\frac{AR\dot{\delta}}{57.3} + Q_{ACT} \quad (B1-19)$$

This equation gives load pressure as a function of engine motion and valve flow. With the proper subscripting, the equations become:

$$\left(\frac{V_A}{4B} + \frac{A^2}{K_m}\right) \dot{P}_{L_{\delta_1}} = -\frac{AR}{57.3} \dot{\delta}_1 + Q_{ACT_{\delta_1}} \quad (B1-20)$$

$$\left(\frac{V_A}{4B} + \frac{A^2}{K_m}\right) \dot{P}_{L_{\delta_L}} = -\frac{AR}{57.3} \dot{\delta}_2 + Q_{ACT_{\delta_2}} \quad (B1-21)$$

$$\left(\frac{V_A}{4B} + \frac{A^2}{K_m}\right) \dot{P}_{L_{\xi_1}} = -\frac{AR}{57.3} \dot{\xi}_1 + Q_{ACT_{\xi_1}} \quad (B1-22)$$

$$\left(\frac{V_A}{4B} + \frac{A^2}{K_m}\right) \dot{P}_{L_{\xi_2}} = -\frac{AR}{57.3} \dot{\xi}_2 + Q_{ACT_{\xi_2}} \quad (B1-23)$$

B1.4 ELECTROHYDRAULIC SERVOVALVE

B1.4.1 Valve Flow. The servovalve used in the Centaur engine servo system is a dynamic pressure feedback flow control valve as shown in Figure B1-5. This valve is used to yield a highly damped servo for transient and dynamic conditions and yet maintain a high static stiffness in the face of large static loads with negligible error signal. The valve flow, Q_{ACT} , is a function of the control current, i_V , load pressure, P_L , supply pressure, P_S , and reservoir pressure, P_R . The valve flow, Q_{ACT} , is assumed proportional to the linear displacement of the valve spool, x_V , and the square root of the pressure drop across the valve, P_V .

$$Q_{ACT} = K_o x_V \frac{P_V}{|P_V|} \sqrt{|P_V|} \quad (B1-24)$$

where K_o is the constant of proportionality.

The pressure drop across the valve is composed of two parts: one due to a pressure drop on the supply side of the actuator piston and the other due to the pressure drop of the reservoir side of the piston. These two drops were assumed equal because of the symmetry of the valve orifices.

$$\frac{P_V}{2} = P_S - P_1 = P_2 - P_R; x_V > 0 \quad (B1-25a)$$

$$\frac{P_V}{2} = P_S - P_2 = P_1 - P_R; x_V < 0 \quad (B1-25b)$$

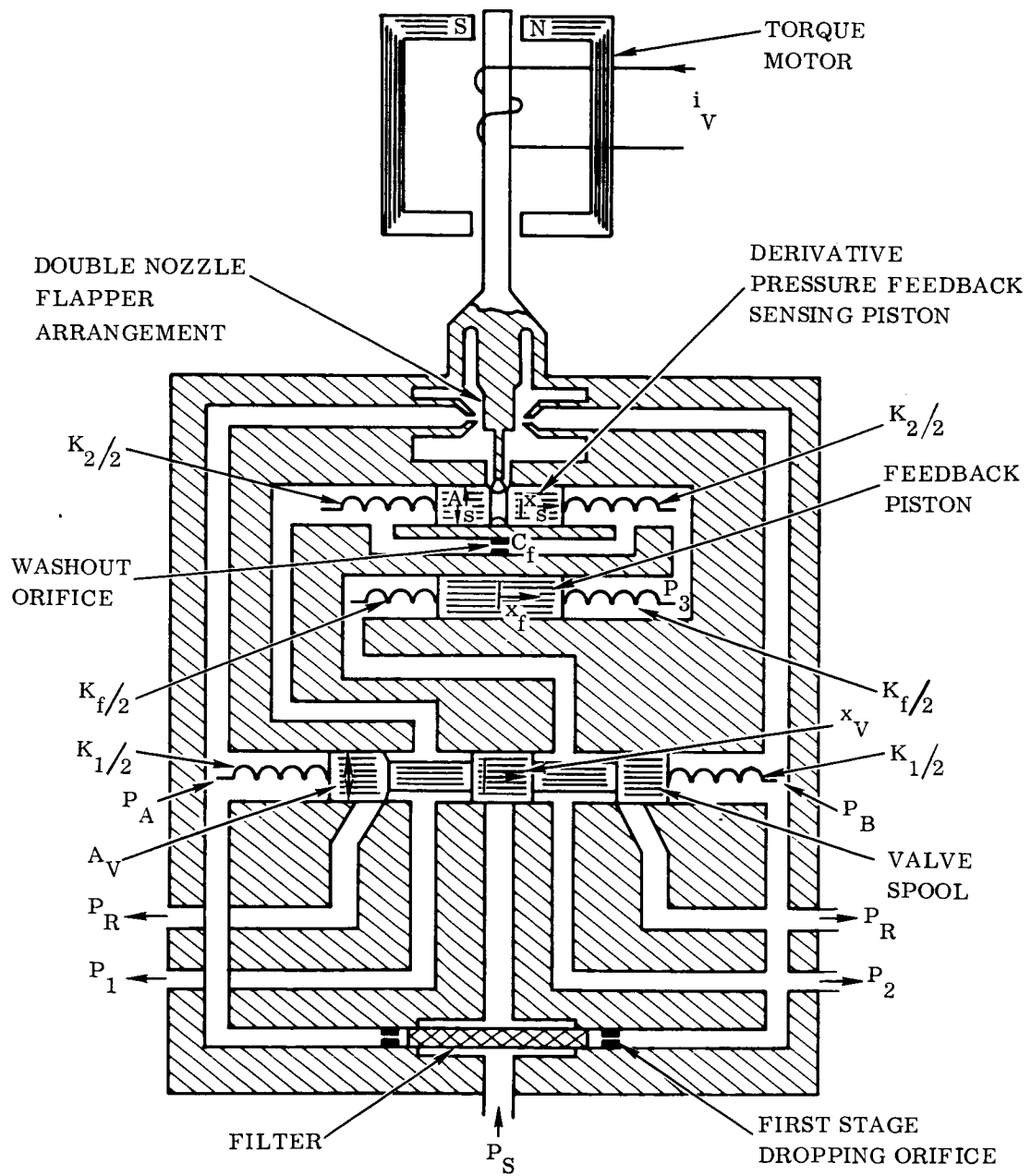


Figure B1-5. Servovalve Schematic

These equations may be combined for the pressure drop across the valve.

$$P_V = P_S - P_R - \frac{x_V}{|x_V|} P_L \quad (B1-26)$$

where x_V is the displacement of the valve spool from the center position. Summing the forces on the valve spool,

$$K_1 x_V = A_V (P_A - P_B) \quad (B1-27)$$

where

K_1 = spring constant of the springs locating the valve spool

A_V = area of spool cross section

P_A = pressure exerted on left hand end of valve spool

P_B = pressure exerted on right hand end of valve spool

Summing forces on the flapper due to the valve current, i_V , and the displacement of the sensing piston, x_s ,

$$\Sigma \text{Flapper Forces} = K_1 i_V - K_2 x_s \quad (B1-28)$$

The pressure differential, $P_A - P_B$, across the valve spool depends directly on the extent to which the flapper restricts the flow through the nozzles. The flapper position, in turn, depends on the net force acting on the flapper. The available supply pressure dictates how far the flapper will be required to move to cause a given pressure differential, $P_A - P_B$. These considerations may be stated by:

$$(P_A - P_B) = \frac{P_S}{P_{S(\text{ref})}} K_q \Sigma \text{Flapper Forces} \quad (B1-29)$$

where

K_q = proportionality constant

$P_{S(\text{ref})}$ = reference supply pressure

Substituting Equation B1-28 into B1-29 and rearranging,

$$(P_A - P_B) = \frac{P_S}{P_{S(\text{ref})}} K_q K_1 \left[i_V - \frac{K_2}{K_1} x_s \right] \quad (B1-30)$$

But the displacement, x_s , of the sensing piston is proportional to the differential pressure acting on it.

$$x_s = \frac{A_s}{K_2} (P_1 - P_3) \quad (B1-31)$$

Substituting into B1-30,

$$(P_A - P_B) = \frac{P_S}{P_{S(\text{ref})}} K_q K_i \left[i_V - K_s (P_1 - P_3) \right] \quad (B1-32)$$

where

$$K_s = \frac{A_s}{K_i} \text{ in ma/psf}$$

This equation may be substituted into Equation B1-27.

$$x_V = \frac{A_v}{K_1} K_q K_i \frac{P_S}{P_{S(\text{ref})}} \left[i_V - K_s \Delta P_f \right] \quad (B1-33)$$

where

$$\Delta P_f = P_1 - P_3 = \text{pressure drop across sensing piston}$$

It is convenient to define an equivalent valve current by

$$i_V' = i_V - K_s \Delta P_f \quad (B1-34)$$

Then, substituting Equations B1-26, B1-33, and B1-34 into B1-24 yields, for the valve flow,

$$Q_{\text{ACT}} = \frac{P_S}{P_{S(\text{ref})}} K_V i_V' \frac{P_S - P_R - \frac{i_V'}{i_V} P_L}{\left| P_S - P_R - \frac{i_V'}{i_V} P_L \right|} \sqrt{\left| P_S - P_R - \frac{i_V'}{i_V} P_L \right|} \quad (B1-35)$$

where K_V , the valve gain at full supply pressure in $\frac{\text{in.}^3/\text{sec}}{\text{ma}\sqrt{\text{lb/in.}^2}}$, is given by:

$$K_V = K_o \frac{A_v}{K_1} K_q K_i \quad (B1-35a)$$

Applying the proper subscripting to Equation B1-34, the equivalent value currents for the four Centaur actuators are obtained.

$$i'_{V_{\delta_1}} = i_{V_{\delta_1}} - K_s \Delta P_{f_{\delta_1}} \quad (\text{B1-36})$$

$$i'_{V_{\delta_2}} = i_{V_{\delta_2}} - K_s \Delta P_{f_{\delta_2}} \quad (\text{B1-37})$$

$$i'_{V_{\xi_1}} = i_{V_{\xi_1}} - K_s \Delta P_{f_{\xi_1}} \quad (B1-38)$$

$$i'_{V_{\xi_2}} = i_{V_{\xi_2}} - K_s \Delta P_{f_{\xi_2}} \quad (\text{B1-39})$$

Similarly, the four valve flows are obtained.

$$Q_{ACT_{\delta_1}} = \frac{P_{S_1}}{P_{S_{(ref)}}} K_V i'_{V_{\delta_1}} \frac{P_{S_1} - P_{R_1} - \left| \frac{i'_{V_{\delta_1}}}{i'_{V_{\delta_1}}} \right| P_{L_{\delta_1}}}{P_{S_1} - P_{R_1} - \left| \frac{i'_{V_{\delta_1}}}{i'_{V_{\delta_1}}} \right| P_{L_{\delta_1}}} \sqrt{P_{S_1} - P_{R_1} - \left| \frac{i'_{V_{\delta_1}}}{i'_{V_{\delta_1}}} \right| P_{L_{\delta_1}}} \quad (B1-40)$$

$$Q_{ACT_{\delta_2}} = \frac{P_{S_2}}{P_{S_{(ref)}}} K_V i'_{V_{\delta_2}} \sqrt{\frac{P_{S_2} - P_{R_2} - \frac{i'_{V_{\delta_2}}}{P_{L_{\delta_2}}} P_{L_{\delta_2}}}{P_{S_2} - P_{R_2} - \frac{i'_{V_{\delta_2}}}{P_{L_{\delta_2}}} P_{L_{\delta_2}}}} \quad (B1-41)$$

$$Q_{ACT_{\xi_1}} = \frac{P_{S_1}}{P_{S_{(ref)}}} K_V \frac{i'_{V_{\xi_1}}}{i'_{V_{\xi_1}}} \frac{P_{S_1} - P_{R_1} - \frac{i'_{V_{\xi_1}}}{i'_{V_{\xi_1}}} P_{L_{\xi_1}}}{P_{S_1} - P_{R_1} - \frac{i'_{V_{\xi_1}}}{i'_{V_{\xi_1}}} P_{L_{\xi_1}}} \sqrt{\frac{P_{S_1} - P_{R_1} - \frac{i'_{V_{\xi_1}}}{i'_{V_{\xi_1}}} P_{L_{\xi_1}}}{P_{S_1} - P_{R_1} - \frac{i'_{V_{\xi_1}}}{i'_{V_{\xi_1}}} P_{L_{\xi_1}}}} \quad (B1-42)$$

$$Q_{ACT \xi_2} = \frac{P_{S_2}}{P_{S(ref)}} K_V i_{V \xi_2} \frac{P_{S_2} - P_{R_2} - \frac{i_{V \xi_2}'}{i_{V \xi_2}} P_{L \xi_2}}{\left| \frac{P_{S_2} - P_{R_2} - \frac{i_{V \xi_2}'}{i_{V \xi_2}} P_{L \xi_2}}{\frac{i_{V \xi_2}'}{i_{V \xi_2}}} \right|} \sqrt{\left| \frac{P_{S_2} - P_{R_2} - \frac{i_{V \xi_2}'}{i_{V \xi_2}} P_{L \xi_2}}{\frac{i_{V \xi_2}'}{i_{V \xi_2}}} \right|} \quad (B1-43)$$

B1.4.2 DPF Network. Summing the flows around the hydraulic feedback loop,

$$A_s \dot{x}_s + C_f \frac{P_1 - P_3}{|P_1 - P_3|} \sqrt{|P_1 - P_3|} + A_f \dot{x}_f = 0 \quad (B1-44)$$

where

A_s = area of sensing piston (ft²)

x_s = displacement of sensing piston (ft)

C_f = orifice flow coefficient $\frac{\text{ft}^4}{\sqrt{\text{lb-sec}^2}}$

A_f = area of feedback piston (ft²)

x_f = displacement of feedback piston (in.)

Summing forces on the feedback piston,

$$K_f x_f = A_f (P_2 - P_3) \quad (B1-45)$$

where K_f is the spring constant of the springs restraining the motion of the feedback piston. Substituting Equations B1-31 and B1-45 into B1-44,

$$\frac{A_s^2}{K_2} (\dot{P}_1 - \dot{P}_3) + C_f \frac{P_1 - P_3}{|P_1 - P_3|} \sqrt{|P_1 - P_3|} + \frac{A_f^2}{K_f} (\dot{P}_2 - \dot{P}_3) = 0 \quad (B1-46)$$

But

$$P_1 - P_2 = P_L \quad (B1-46a)$$

and

$$P_1 - P_3 = \Delta P_f \quad (B1-46b)$$

Therefore

$$\frac{A_f^2}{K_f} \dot{P}_L = \frac{A_s^2}{K_2} \dot{\Delta P}_f + C_f \frac{\Delta P_f}{|\Delta P_f|} \sqrt{|\Delta P_f|} + \frac{A_f^2}{K_f} \dot{\Delta P}_f \quad (B1-47)$$

In the Centaur servovalve $A_f = A_s$, and $K_2 = K_f$. This makes further simplification possible.

$$\dot{\Delta P}_f = \frac{1}{2} \left[\dot{P}_L - \frac{C_f K_f}{A_f^2} \frac{\Delta P_f}{|\Delta P_f|} \sqrt{|\Delta P_f|} \right] \quad (B1-48)$$

The proper subscripting yields the four equations for the four DPF networks.

$$\dot{\Delta P}_{f_{\delta_1}} = \frac{1}{2} \left[\dot{P}_{L_{\delta_1}} - \frac{C_f K_f}{A_f^2} \frac{\Delta P_{f_{\delta_1}}}{|\Delta P_{f_{\delta_1}}|} \sqrt{|\Delta P_{f_{\delta_1}}|} \right] \quad (B1-49)$$

$$\dot{\Delta P}_{f_{\delta_2}} = \frac{1}{2} \left[\dot{P}_{L_{\delta_2}} - \frac{C_f K_f}{A_f^2} \frac{\Delta P_{f_{\delta_2}}}{|\Delta P_{f_{\delta_2}}|} \sqrt{|\Delta P_{f_{\delta_2}}|} \right] \quad (B1-50)$$

$$\dot{\Delta P}_{f_{\xi_1}} = \frac{1}{2} \left[\dot{P}_{L_{\xi_1}} - \frac{C_f K_f}{A_f^2} \frac{\Delta P_{f_{\xi_1}}}{|\Delta P_{f_{\xi_1}}|} \sqrt{|\Delta P_{f_{\xi_1}}|} \right] \quad (B1-51)$$

$$\dot{\Delta P}_{f_{\xi_2}} = \frac{1}{2} \left[\dot{P}_{L_{\xi_2}} - \frac{C_f K_f}{A_f^2} \frac{\Delta P_{f_{\xi_2}}}{|\Delta P_{f_{\xi_2}}|} \sqrt{|\Delta P_{f_{\xi_2}}|} \right] \quad (B1-52)$$

B1.4.3 Valve Current. The valve current is dependent upon the gain of the servoamplifier, K_a , the feedback transducer gain, K_t , the command signal, δ_c , and the feedback signal, δ_f , giving:

$$i_V = K_a K_t (\delta_c - \delta_f) \quad (B1-53)$$

The four valve currents are:

$$i_{V_{\delta_1}} = K_a K_t (\delta_{c_1} - \delta_{f_1}) \quad (B1-53a)$$

$$i_{V_{\delta_2}} = K_a K_t (\delta_{c_2} - \delta_{f_2}) \quad (B1-53b)$$

$$i_{V_{\xi_1}} = K_a K_t (\xi_{c_1} - \xi_{f_1}) \quad (B1-53c)$$

$$i_{V_{\xi_2}} = K_a K_t (\xi_{c_2} - \xi_{f_2}) \quad (B1-53d)$$

B1.5 FEEDBACK TRANSDUCER. The engine feedback signal, δ_f , depends not only on engine position but also upon the actuator support structure compliance. Therefore

$$\delta_f = \delta + \frac{57.3 A}{K_m R} P_L \quad (B1-54)$$

The four engine feedback signals become

$$\delta_{f_1} = \delta_1 + \frac{57.3 A}{K_m R} P_{L_{\delta_1}} \quad (B1-55)$$

$$\delta_{f_2} = \delta_2 + \frac{57.3 A}{K_m R} P_{L_{\delta_2}} \quad (B1-56)$$

$$\xi_{f_1} = \xi_1 + \frac{57.3 A}{K_m R} P_{L_{\xi_1}} \quad (B1-57)$$

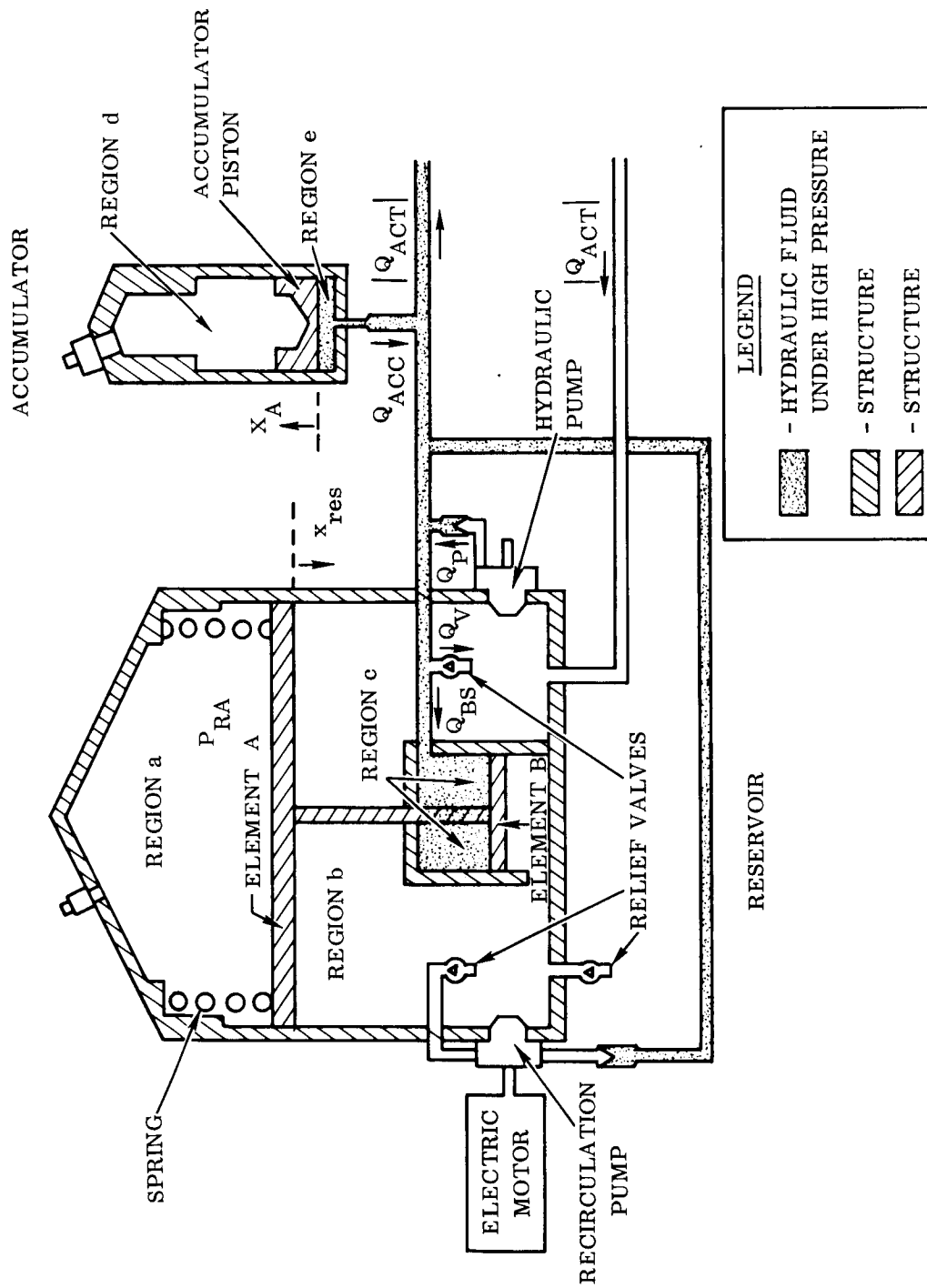
$$\xi_{f_2} = \xi_2 + \frac{57.3 A}{K_m R} P_{L_{\xi_2}} \quad (B1-58)$$

B1.6 AUTOPILOT EQUATIONS. The autopilot equations are taken directly from Appendix B2, with the proper subscripting.

$$\delta_{c_1} = \delta_{c_2} = - \frac{K_A K_R \theta}{\tau_R s + 1} \left(1 + \frac{K_I \theta}{S} \right) \dot{\theta} \quad (B1-59)$$

$$\xi_{c_1} = - \xi_{c_2} = \frac{K_A K_R \phi}{\tau_R s + 1} \left(1 + \frac{K_I \phi}{S} \right) \dot{\phi} \quad (B1-60)$$

B1.7 HYDRAULIC POWER SUPPLY. Figure B1-6 is the schematic diagram of the Centaur hydraulic power supply. The equations governing behavior of the hydraulic power supply during the ignition transient are derived in the following sections.



B1.7.1 Pump Flow. Fluid is supplied to the high-pressure side of the system by a fixed-vane rotary pump which displaces constant volume of fluid per revolution. The pump output flow is given by

$$Q_p = \zeta_V \left(\frac{Q}{\text{rev}} \right) N \quad (\text{B1-61})$$

Subscripting for the two power supplies,

$$Q_{p_1} = \zeta_V \left(\frac{Q}{\text{rev}} \right) N_1 \quad (\text{B1-62})$$

$$Q_{p_2} = \zeta_V \left(\frac{Q}{\text{rev}} \right) N_2 \quad (\text{B1-63})$$

B1.7.2 Relief Valve Flow. The relief valve maintains the supply pressure within set limits while meeting the normal flow requirements of the system. It is normally open to three-quarters of full-flow position. Since the valve opening depends on supply pressure, flow through the valve is maintained equal to pump output. When there is a flow demand on the system, the flow is maintained by the relief valves closing in response to a supply pressure drop.

Excluding higher-order effects and relief valve nonlinearities, the pressure excursions usually encountered from no-flow to full-flow may be 5 to 25 percent of the supply pressure, depending upon orifice geometry and the spring rate of the relief valve poppet assembly. Also, depending upon the relief valve, the reseal pressure may be as low as 50 percent of the cracking (opening) pressure. The reseal pressure of the relief valve placed between the high-pressure supply and the reservoir is greater than 95 percent of the cracking pressure. The difference between the reseal pressure and the cracking pressure is less than 10 percent of the difference between the cracking pressure and full-flow pressure. It is assumed that the relief valve operates in a linear fashion about the three-quarters-full-flow point between cracking pressure and full-flow pressure. The flow is given by

$$Q_V = Q_{FF} \frac{P_S - P_C}{P_{FF} - P_C} ; P_S \geq P_C \quad (\text{B1-64})$$

$$= 0 ; P_S < P_C$$

Subscripting for the two power supplies,

$$Q_{V_1} = Q_{FF} \frac{P_{S_1} - P_C}{P_{FF} - P_C} ; P_{S_1} \geq P_C \quad (\text{B1-65})$$

$$= 0 ; P_{S_1} < P_C$$

$$Q_{V_2} = Q_{FF} \frac{P_{S_2} - P_C}{P_{FF} - P_C} ; P_{S_2} \geq P_C \quad (\text{B1-66})$$

$$= 0 ; P_{S_2} < P_C$$

This flow represents that from the high-pressure side of the system into the reservoir through the relief valve.

B1.7.3 Actuator Flow. The flow demand by the two actuators represents a flow out of the high-pressure side of the system

$$|Q_{ACT_1}| = |Q_{ACT_{\delta_1}}| + |Q_{ACT_{\xi_1}}| \quad (B1-67)$$

$$|Q_{ACT_2}| = |Q_{ACT_{\delta_2}}| + |Q_{ACT_{\xi_2}}| \quad (B1-68)$$

B1.7.4 Bootstrap Flow. The flow into the bootstrap region (Region c) depends directly on motion of the bootstrap piston. The rate at which this piston moves can be equated to the sum of the flows into the low-pressure side (Region b) of the reservoir.

$$\dot{x}_{res_1} = \frac{1}{A_{RH}} \left[Q_{p_1} - Q_{V_1} - |Q_{ACT_1}| \right] \quad (B1-69)$$

$$\dot{x}_{res_2} = \frac{1}{A_{RH}} \left[Q_{p_2} - Q_{V_2} - |Q_{ACT_2}| \right] \quad (B1-70)$$

The bootstrap flows are then given by:

$$Q_{BS_1} = A_{BS} \dot{x}_{res_1} \quad (B1-71)$$

$$Q_{BS_2} = A_{BS} \dot{x}_{res_2} \quad (B1-72)$$

B1.7.5 Accumulator Flow. The flow out of the accumulator, Q_{ACC} , depends directly upon motion of the accumulator piston.

$$Q_{ACC_1} = -A_A \dot{x}_A ; 0 \leq x_A \leq 0.1375 \text{ ft} \quad (B1-73)$$

$$= 0 ; x_A > 0.1375 \text{ ft}$$

$$Q_{ACC_2} = -A_A \dot{x}_A ; 0 \leq x_A \leq 0.1375 \text{ ft} \quad (B1-74)$$

$$= 0 ; x_A > 0.1375 \text{ ft}$$

This, in turn, depends on the precharge inside Region d of the accumulator. Assuming adiabatic expansion during operation,

$$P_{AC} V_{AC}^K = \frac{P_{AC_1} T_o}{T_1} V_{AC_1}^K \quad (B1-75)$$

where V_{AC} is the volume of Region d, P_{AC_1} is the initial pressure (at launch) in Region d, and V_{A_1} is the initial volume in Region d. Here a constant-volume process is assumed to relate the pressure at launch (subscript i) to that at main engine start (subscript o).

The volume on the gas side of the accumulator is given by

$$V_{AC} = V_{AC_1} - A_A x_A \quad (B1-76)$$

Substituting Equation B1-76 into B1-75,

$$P_{AC} (V_{AC_1} - A_A x_A)^K = \frac{P_{AC_1} T_o}{T_i} V_{AC_1}^K \quad (B1-77)$$

Raising to the $\frac{1}{K}$ power,

$$P_{AC}^{\frac{1}{K}} (V_{AC_1} - A_A x_A) = \left(\frac{P_{AC_1} T_o}{T_i} \right)^{\frac{1}{K}} V_{AC_1} \quad (B1-78)$$

Rewriting in terms of x_A ,

$$x_A = \frac{V_{AC_1}}{A_A} \left[1 - \frac{\left(\frac{P_{AC_1} T_o}{T_i} \right)^{\frac{1}{K}}}{P_S^{\frac{1}{K}}} \right] \quad (B1-79)$$

since $P_S = P_{AC}$ to balance pressures. Differentiating Equation B1-79 and subscripting,

$$\dot{x}_{A_1} = \frac{1}{K} \frac{V_{AC_1}}{A_A} \left(\frac{P_{AC_1} T_o}{T_i} \right)^{\frac{1}{K}} P_{S_1}^{-(1+\frac{1}{K})} \dot{P}_{S_1} \quad (B1-80)$$

$$\dot{x}_{A_2} = \frac{1}{K} \frac{V_{AC_1}}{A_A} \left(\frac{P_{AC_1} T_o}{T_i} \right)^{\frac{1}{K}} P_{S_2}^{-(1+\frac{1}{K})} \dot{P}_{S_2} \quad (B1-81)$$

B1.7.6 Total Flow. The total flow into the high-pressure side at the system may be written

$$Q_T = Q_P - Q_V - |Q_{ACT}| - Q_{BS} + Q_{ACC} \quad (B1-82)$$

Substituting for the bootstrap flow and subscripting,

$$Q_{T_1} = \left(1 - \frac{A_{BS}}{A_{RH}}\right) \left[Q_{P_1} - Q_{V_1} - |Q_{ACT_1}| \right] + Q_{ACC_1} \quad (B1-83)$$

$$Q_{T_2} = \left(1 - \frac{A_{BS}}{A_{RH}}\right) \left[Q_{P_2} - Q_{V_2} - |Q_{ACT_2}| \right] + Q_{ACC_2} \quad (B1-84)$$

B1.7.7 Reservoir Pressure. The pressure on the gas side of element A, Region a, is referred to as P_{RA} and related to the launch condition by the following equation of state.

$$P_{RA_o} = \frac{P_{RA_i} T_o}{T_i} \quad (B1-85)$$

where the subscripts o and i refer to main engine start and launch respectively. During the powered phase of flight the gas is assumed to follow the adiabatic expansion relationship given by

$$P_{RA} V_{RA}^K = \frac{P_{RA_i} T_o}{T_i} V_{RA_i}^K \quad (B1-86)$$

This can be rewritten

$$P_{RA} = \frac{P_{RA_i} T_o}{T_i} \left(\frac{V_{RA_i}}{V_{RA}} \right)^K \quad (B1-87)$$

But substituting

$$V_{RA} = V_{RA_i} + A_{RA} x_{res} \quad (B1-88)$$

into Equation B1-87 yields

$$P_{RA} = \frac{P_{RA_i} T_o}{T_i} \left(\frac{1}{1 + \frac{A_{RA} x_{res}}{V_{RA_i}}} \right)^K \quad (B1-89)$$

The pressure in the low-fluid-pressure side of the system, P_R , is given by

$$P_R = \frac{1}{A_{RH}} \left[P_S A_{BS} + F_S + K_{res} x_{res} + P_{RA} A_{RA} \right] \quad (B1-90)$$

Since the incremental volume caused by the displacement, x_{res} , is small when compared with V_{RA_i} , P_{RA} is almost a constant. Likewise $K_{res} x_{res} \ll F_S$, and the reservoir pressure can be written as

$$P_R = \frac{A_{BS}}{A_{RH}} P_S + \left(1 - \frac{A_{BS}}{A_{RH}}\right) P_{R_o} \quad (B1-91)$$

In this analysis P_{R_o} , the reservoir pressure at main engine start, was assumed zero, yielding for the two hydraulic power supplies

$$P_{R_1} = \frac{A_{BS}}{A_{RH}} P_{S_1} \quad (B1-92)$$

$$P_{R_2} = \frac{A_{BS}}{A_{RH}} P_{S_2} \quad (B1-93)$$

B1.7.8 Supply Pressure. The supply pressure is a function of the volume of the high-pressure side of the system:

$$P_S = \frac{B}{V_T} dV + P_{R_o} \quad (B1-94)$$

Differentiating Equation B-94,

$$\dot{P}_S = \frac{B}{V_T} Q_T - \frac{B dV}{V_T^2} \dot{V}_T \quad (B1-95)$$

where

$$dV = \int Q_T dt \quad (B1-96)$$

$$V_T = V_{lines} + A_A x_A + V_{BS_o} + A_{BS} x_{res} \quad (B1-97)$$

$$\dot{V}_T = A_A \dot{x}_A + A_{BS} \dot{x}_{res} \quad (B1-98)$$

This can be rewritten in terms of the bootstrap and accumulator flows.

$$\dot{V}_T = -Q_{ACC} + \frac{A_{BS}}{A_{RH}} \left(Q_P - Q_V - |Q_{ACT}| \right) \quad (B1-99)$$

Rewriting Equations B1-95, B1-96, B1-97, and B1-99 with the proper subscripting,

$$P_{S_1} = \int \dot{P}_{S_1} dt \quad (B1-100)$$

$$\dot{P}_{S_1} = \frac{B}{V_{T_1}} Q_{T_1} - \frac{B}{V_{T_1}^2} \dot{V}_{T_1} \int Q_{T_1} dt \quad (B1-101)$$

$$V_{T_1} = V_{T_o} + \int \dot{V}_{T_1} dt \quad (B1-102)$$

$$\dot{V}_{T_1} = -Q_{ACC_1} + \frac{A_{BS}}{A_{RH}} \left(Q_{P_1} - Q_{V_1} - |Q_{ACT_1}| \right) \quad (B1-103)$$

$$P_{S_2} = \int \dot{P}_{S_2} dt \quad (B1-104)$$

$$\dot{P}_{S_2} = \frac{B}{V_{T_2}} Q_{T_2} - \frac{B}{V_{T_2}^2} \dot{V}_{T_2} \int Q_{T_2} dt \quad (B1-105)$$

$$V_{T_2} = V_{T_o} + \int \dot{V}_{T_2} dt \quad (B1-106)$$

$$\dot{V}_{T_2} = -Q_{ACC_2} + \frac{A_{BS}}{A_{RH}} \left(Q_{P_2} - Q_{V_2} - |Q_{ACT_2}| \right) \quad (B1-107)$$

SECTION B2
RIGID BODY/PROPELLANT SLOSHING
STABILITY EQUATIONS

LIST OF SYMBOLS FOR SECTION B2

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A	Actuating piston area	ft ²
A _f	Feedback piston area	ft ²
B	Bulk modulus of hydraulic fluid	lb/ft ²
C _B	Coulomb friction coefficient	ft-lb
$\bar{C}_F$	Equivalent admittance for gimbal friction	$\frac{\text{ft-lb}}{\text{deg/sec}}$
C _f	Discharge coefficient for washout orifice	$\frac{(\text{ft}^3/\text{sec})}{\sqrt{\text{lb/ft}^2}}$
$\bar{C}_f$	Equivalent admittance for C _f	$\frac{\text{ft}^3/\text{sec}}{\text{lb/ft}^2}$
C _L	Discharge coefficient for leakage bypass orifice	$\frac{(\text{ft}^3/\text{sec})}{\sqrt{\text{lb/ft}^2}}$
$\bar{C}_L$	Equivalent admittance for C _L	$\frac{\text{ft}^3/\text{sec}}{\text{lb/ft}^2}$
C _V	Viscous friction coefficient	$\frac{\text{ft-lb}}{\text{deg/sec}}$
C ₁	Nonlinear slosh damping proportionality constant	deg ⁻¹
$\bar{f}$	Attitude reference vector	ft/sec ²
I _R	Engine "wet" moment of inertia of gimballed mass about the gimbal point	slug-ft ²
I _{xx_r} , I _{yy_r} , I _{zz_r}	Reduced moment of inertia about the roll axes, pitch axes, and yaw axes respectively (See Reference C-1)	slug-ft ²
i _V	Servo valve current	ma
i' _V	Equivalent servo valve current	ma
K _A	End-to-end position gain	$\frac{\text{deg } \delta}{\text{deg } \theta}$
K _a	Servoamplifier gain	ma/volt
K _A K _R	End-to-end rate gain	$\frac{\text{deg}}{\text{deg/sec}}$
K _C	Actuator no-load open-loop velocity gain	$\frac{\text{deg/sec}}{\text{deg}}$
K' _C	Simplified low-frequency approximation of the open-loop nonlinear actuator velocity gain under load	$\frac{\text{deg/sec}}{\text{deg}}$
K _f	Feedback spring constant	lb/ft ²
K _I	Autopilot integrator gain	sec ⁻¹

LIST OF SYMBOLS FOR SECTION B2, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
K_m	Spring constant of actuator support structure	lb/ft
K_R	Rate gyro gain	$\frac{\text{deg}}{\text{deg/sec}}$
K_s	DPF valve pressure feedback gain	$\frac{\text{ma}}{\text{lb/ft}^2}$
K_t	Feedback transducer gain	volt/deg
K_V	Servo valve discharge coefficient	$\frac{(\text{ft}^3/\text{sec})}{\text{ma}\sqrt{\text{lb/ft}^2}}$
$L_{P_{CL}}, L_{P_{CH}}$	Liquid oxygen and liquid hydrogen pendulum lengths	ft
l_{CL}, l_{CH}	Liquid oxygen and liquid hydrogen pendulum moment arms	ft
l_c	Rocket engine control moment arm	ft
l_p	Distance between vehicle center-of-gravity and inertial guidance platform	ft
l_R	Distance between rocket engine center-of-gravity and gimbal point	ft
l_z	Rocket engine roll control moment arm	ft
M_r	Reduced mass of vehicle (without sloshing mass)	slugs
$M_{P_{CL}}, M_{P_{CH}}$	Liquid oxygen and liquid hydrogen pendulum masses	slugs
M_R	Rocket engine gimballed mass, one engine "wet"	slugs
M_T	Total mass of vehicle	slugs
P_L	Dynamic load pressure (pressure drop across actuating piston)	lb/ft ²
$\bar{P}_L$	Equivalent admittance for P_L	lb/ft ²
P_R	Hydraulic system reservoir pressure (return pressure from servovalve)	lb/ft ²
P_S	Hydraulic systems supply pressure to servovalve	lb/ft ²
Q_{ACT}	Actuator flow	ft ³ /sec
R	Engine actuator moment arm	ft
s	Laplace operator	sec ⁻¹
T_c	Control engine thrust	lb
T_L	Load torque feedback to actuating piston	ft-lb
t	Time	sec

LIST OF SYMBOLS FOR SECTION B2, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
V_A	Total volume of hydraulic fluid in one actuator cylinder	ft ³
x, y, z	Vehicle body axis coordinate system (roll, pitch, and yaw)	
$\ddot{x}$	Vehicle axial acceleration	ft/sec ²
$\ddot{z}$	Vehicle normal acceleration (plunging)	ft/sec ²
ΔP_f	Pressure drop across sensing piston in dpf valve	lb/ft ²
$\overline{\Delta P_f}$	Equivalent admittance for ΔP_f	lb/ft ²
$\Delta \delta$	Angular offset of the rocket engine center-of-gravity from engine center line	deg
δ	Rocket engine gimbal angle	deg
$\bar{\delta}$	Sinusoidal engine gimbal angle limit cycle amplitude from null to peak	deg
δ_c	Commanded engine angle	deg
δ_f	Engine feedback angle	deg
δ_ϕ	Engine roll deflection	deg
ϵ	Thrust offset	ft
ζ_{CL}, ζ_{CH}	Liquid oxygen and liquid hydrogen slosh damping ratios	N. D.
θ	Rigid body vehicle pitch angle	deg
θ_c	Commanded vehicle pitch angle	deg
θ_{CI}^*	Integral guidance control feedback signal	deg-sec
θ_F	Rate gyro feedback signal	deg
θ_p	Position reference feedback signal	deg
θ_ϵ	Autopilot error signal	deg
π	3.1416	N. D.
τ_f	Effective dpf network time constant	sec
τ_R	Equivalent lag of rate gyro	sec
ϕ	Vehicle roll angle	deg
ϕ_{CL}, ϕ_{CH}	Liquid oxygen and liquid hydrogen pendulum angles	deg
ω	Frequency	rad/sec
ω_{CL}, ω_{CH}	Liquid oxygen and liquid hydrogen slosh frequencies	rad/sec

SECTION B2

RIGID BODY/PROPELLANT SLOSHING

STABILITY EQUATIONS

B2.1 INTRODUCTION. This appendix describes the equations used in the rigid body/propellant sloshing stability analysis. The equations are all basically linear, with modifications made to account for non-linearities associated with propellant slosh damping and the electrohydraulic actuator-gimbaled engine system.

B2.2 VEHICLE DYNAMICS. The mathematical model used to describe the behavior of the Centaur vehicle for stability analyses may be summarized as follows:

- a. Five perturbational degrees of vehicle freedom (two translational, three rotational) from a nominal trajectory are assumed, a so-called "fly-the-wire" model. The vehicle is restrained to fly a nominal trajectory in a predetermined fashion; all mass and inertial properties and the axial acceleration are predetermined functions of time.
- b. The widely used pendulum analogy is used to duplicate the forces and moments associated with the first mode of propellant sloshing. The second and higher modes are ignored. Both theoretical and empirical means are used to obtain the sloshing parameters (References B-1, B-2, B-3, B-4, and B-5).
- c. The inertial forces due to gimbaling the thrust chambers are neglected in root locus analyses, and a transfer function between commanded and actual thrust vector directions is used. This approximation is valid in the frequency range being studied. These forces are not neglected in time varying analyses.

The above three assumptions define the basic mathematical model of the vehicle dynamics. To shorten the analysis, the following additional simplifications have been made: one plane at a time is considered, with cross coupling due to cross moments of inertia, center-of-gravity offset, etc., giving only second-order effects. This is valid for the virtually cylindrically symmetric Centaur. Roll plane analysis ignores sloshing, with the vehicle assumed to rotate about its propellants. Pitch and yaw plane dynamics are identical except for actuator response, which is reflected in the K'_c , ω , $\bar{\delta}$ curves presented in Section 3.3.3.1. Hence, only the pitch and roll plane equations are given.

Figure B2-1 illustrates the pitch plane coordinate system for the mathematical model.

B2.2.1 Pitch Plane Vehicle Equations of Motion

- a. Angular Acceleration:

$$I_{yy_r} \ddot{\theta} = T_c \ell_c \delta - M_{P_{CL}} \ell_{CL} \ddot{x} \phi_{CL} - M_{P_{CH}} \ell_{CH} \ddot{x} \phi_{CH} \quad (B2-1)$$

- b. Lateral Acceleration:

$$57.3 M_r \ddot{z} = T_c \delta + M_{P_{CL}} \ddot{x} \phi_{CL} + M_{P_{CH}} \ddot{x} \phi_{CH} \quad (B2-2)$$

- c. Axial Acceleration:

$$M_T \ddot{x} = T_c \quad (B2-3)$$

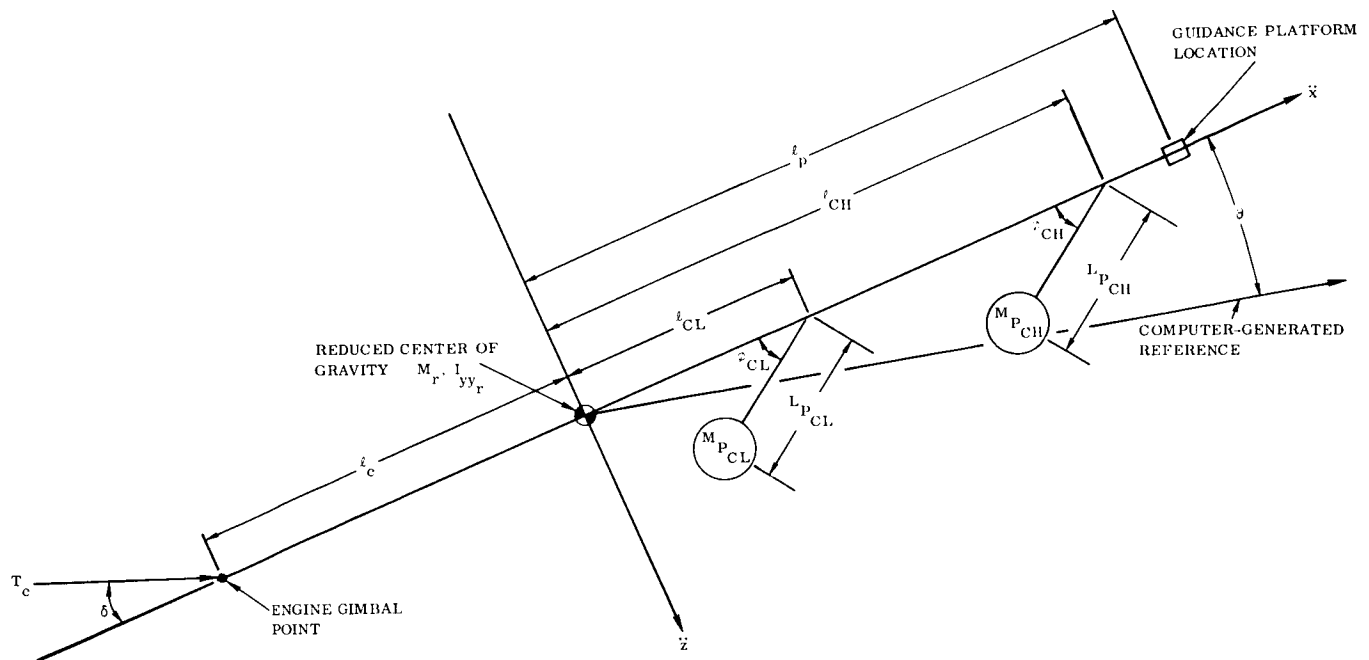


Figure B2-1. Centaur Vehicle Pitch Plane Parameters

d. Propellant Sloshing:

$$L_{P_{CL}} \ddot{\phi}_{CL} + 2\zeta_{CL} \omega_{CL} L_{P_{CL}} \dot{\phi}_{CL} + \ddot{x} \phi_{CL} = (\ell_{CL} - L_{P_{CL}}) \ddot{\theta} - 57.3 \ddot{z} \quad (B2-4)$$

$$L_{P_{CH}} \ddot{\phi}_{CH} + 2\zeta_{CH} \omega_{CH} L_{P_{CH}} \dot{\phi}_{CH} + \ddot{x} \phi_{CH} = (\ell_{CH} - L_{P_{CH}}) \ddot{\theta} - 57.3 \ddot{z} \quad (B2-5)$$

B2.2.2 Roll Plane Vehicle Equation of Motion

a. Angular Acceleration:

$$I_{xx_r} \ddot{\phi} = T_c \ell_z \delta \quad (B2-6)$$

B2.3 PROPELLANT SLOSH DAMPING. The propellant slosh damping ratios, ζ_{CL} and ζ_{CH} , in the preceding equations are functions of the tank geometry and internal structure as well as the amplitude of sloshing motion. In Centaur stability studies the damping in any given tank at any given fluid level is treated as having two parts: a small linear (with slosh amplitude) portion called inherent damping and a larger, amplitude dependent portion called structural slosh damping. The former is due to three terms: the free surface, side wall wiping, and bottom scrubbing damping (Reference B-1). The latter is due to baffles and/or other internal tank structures. Figure B2-2 illustrates the internal tank structures of both the liquid hydrogen and oxygen tanks contributing to damping.

B2.3.1 Linear Damping. The linear portion of the propellant slosh damping included in the root locus analysis presented in this report has the values shown in Appendix E.

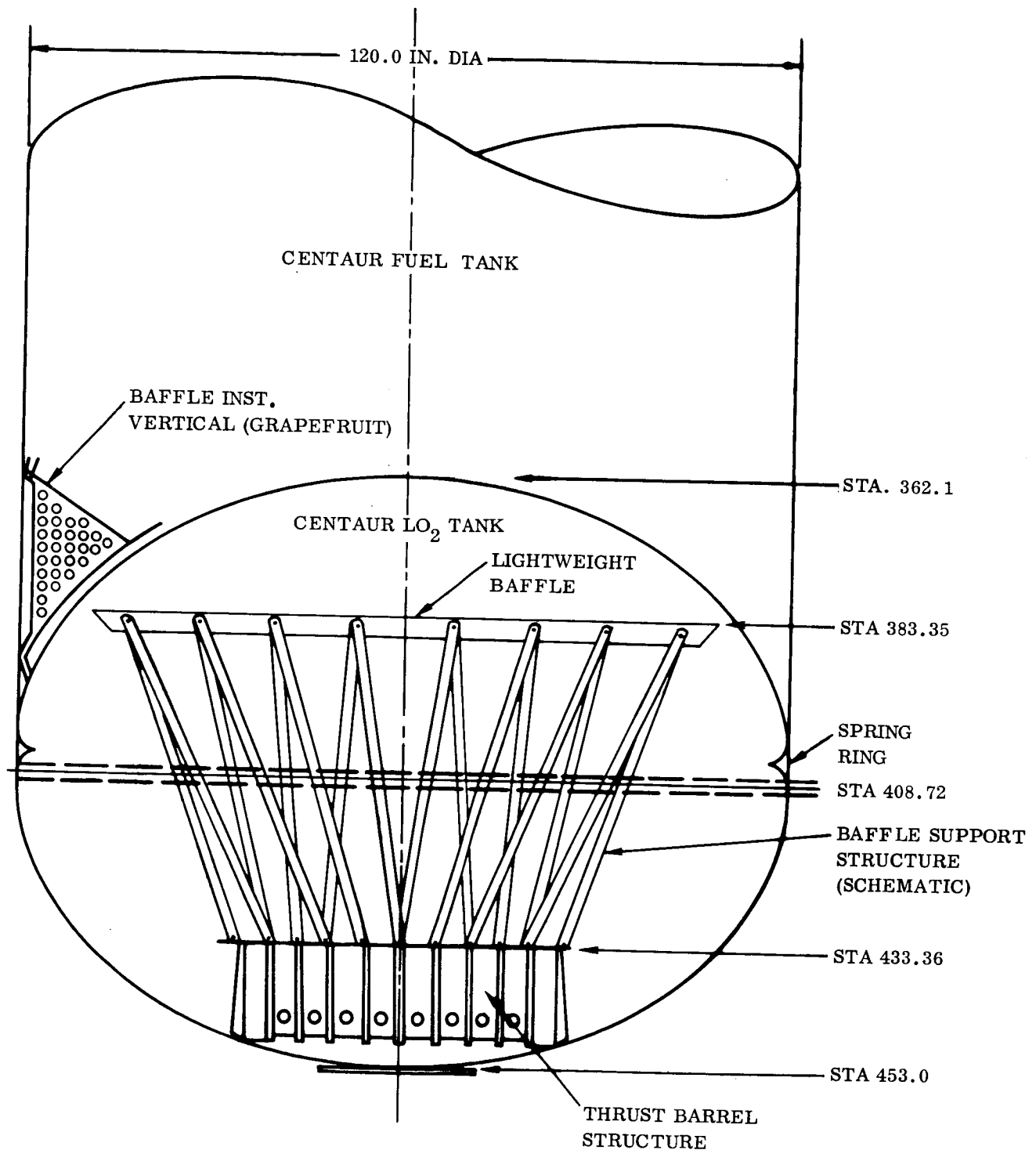


Figure B2-2. Structures Providing Damping in the Propellant Tanks

B2.3.2 Amplitude Dependent Damping. The damping provided by the baffle, spring ring, baffle support and thrust barrel structures in the Centaur liquid oxygen tank is assumed to be proportional to the sloshing amplitude. In general the amplitude dependent damping is only included in time varying analysis and not in root locus analysis. In past analog computer studies, only that damping provided by the thrust barrel has been considered, the damping provided by the other structures being ignored because the damping is not necessary for stability of sloshing.

It is assumed that the nonlinear damping is proportional to the sloshing amplitude:

$$(\zeta)_{N.L.} \propto \phi_{SLOSH} \quad (B2-7)$$

For time varying studies on the analog computer, a means of mechanizing the determination of $\bar{\phi}$ is needed. This is done by first assuming

$$\bar{\phi} = \frac{\dot{\phi}}{\omega} = \frac{\pi}{2\omega} |\dot{\phi}|_{AVG} \quad (B2-8)$$

where $|\dot{\phi}|_{AVG}$ is the average value of the instantaneous absolute value of the pendulum angular rate, in deg/sec. The solution is mechanized by finding the instantaneous absolute value and filtering through a simple lag having a time constant of one second.

$$|\dot{\phi}|_{AVG} \cong \frac{|\dot{\phi}|}{s+1} \quad (B2-9)$$

B2.3.2.1 Liquid Oxygen Tank Thrust Barrel. In time varying sloshing stability studies, the nonlinear damping provided by the thrust barrel is mechanized according to the following equation:

$$2(\zeta_{CL})_{NL} \omega_{CL} \dot{\phi}_{CL} = C_1 \frac{|\dot{\phi}_{CL}|}{s+1} \dot{\phi}_{CL} \quad (B2-10)$$

Hence

$$C_1 = \pi \frac{(\zeta_{CL})_{NL}}{\bar{\phi}_{CL}} \quad (B2-11)$$

A plot of C_1 , the nonlinear damping proportionality constant, versus station number is shown in Figure B2-3. This data has been obtained from results of slosh testing of full scale tanks, Reference B-6.

B2.4 FLIGHT CONTROL SYSTEM. For purposes of rigid body/propellant sloshing stability analysis, the flight control system of the Centaur vehicle during any powered flight phase may be represented by the block diagram of Figure B2-4, or by minor modifications to it. An autopilot error signal, generated by subtracting a signal proportional to the vehicle rate from the position error, is used to generate engine commands. It is integrated to ensure that the steady-state error signal, and hence the position error, is zero in the presence of thrust misalignments, center-of-gravity offsets, etc.

The rate signal is obtained from rate gyros which have significant dynamic effects. Centaur type rate gyros are overdamped, resulting in significant lag within this frequency range. The lag is assumed equivalent to a single time constant, τ_R .

The position error signal is obtained from the guidance platform resolver chain. The platform dynamics are negligible for purposes of rigid body/propellant sloshing stability analysis. The position reference

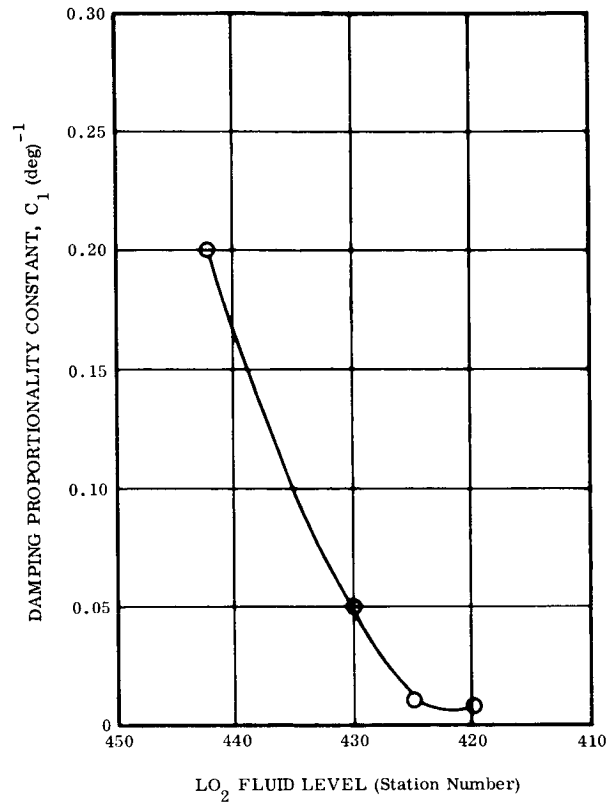


Figure B2-3. Nonlinear Damping Proportionality Constant versus Fluid Level

is provided by the guidance computer via the programmer. Figure B2-4 includes a simplified sample-and-hold representation of the effect of the guidance computer on the vehicle dynamics. The derivation of this model is discussed in Section B4. In root locus analysis, the effect of the sample-and-hold is ignored; i. e., $k_I' = 0$; and the stability of the rest of the system is analyzed using the usual techniques. Analog studies, however, do include the effects of the sample-and-hold.

B2.4.1 Autopilot Error Signals

a. Pitch and Yaw (Steering Enabled)

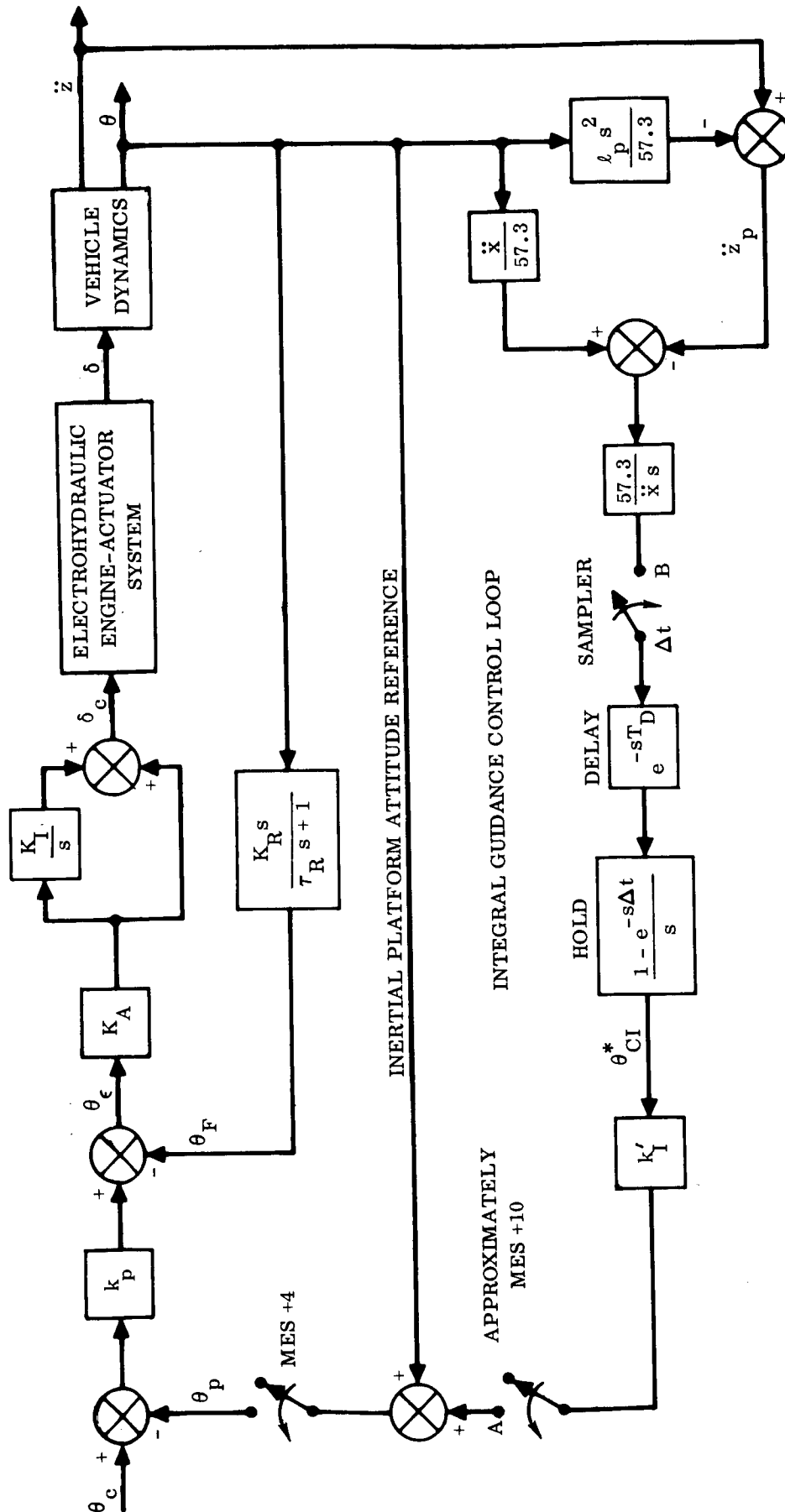
$$\theta_{\epsilon} = -k_p \theta_p - \frac{K_R s}{\tau_R s + 1} \theta \quad (B2-12)$$

b. Roll; Pitch and Yaw (Prior to Steering Enable)

$$\theta_{\epsilon} = -\frac{K_R s}{\tau_R s + 1} \theta \quad (B2-13)$$

c. Autopilot Engine Command Signal (Pitch, Yaw and Roll)

$$\delta_c = K_A \left(1 + \frac{K_I}{s} \right) \theta_{\epsilon} \quad (B2-14)$$



NOTE: THE GAIN, k_p , IS DIMENSIONLESS; IT IS ALWAYS EQUAL EITHER TO ONE OR ZERO, DEPENDING UPON WHETHER OR NOT THERE IS AN AUTOPILOT ATTITUDE REFERENCE.

Figure B2-4. Powered Phase Autopilot Block Diagram

B2.4.2 Electrohydraulic Actuator-Engine System. The engine position servo system is the major source of nonlinearity in the flight control system. Depending upon the analysis being performed, one of several approximations to this behavior is made. This section outlines the approximations.

The equations describing the servo system have been developed elsewhere (References B-1 and B-7 and Appendix B1) and are repeated here for convenience. It is assumed that the system described by these equations is undergoing perturbational motions about a central position and that leakage around the actuator piston and a dynamic pressure feedback network in the servo valve are included.

B2.4.2.1 System Equations

a. Engine Gimbal Torques:

$$\frac{I_R}{57.3} \ddot{\delta} = -C_V \dot{\delta} - C_B \frac{\delta}{|\dot{\delta}|} - T_L(t) + ARP_L \quad (B2-15)$$

b. Actuator Piston Load Pressure:

$$\left(\frac{V_A}{4B} + \frac{A^2}{K_m} \right) \dot{P}_L = -\frac{AR}{57.3} \dot{\delta} - C_L \frac{P_L}{|P_L|} \sqrt{|P_L|} + Q_{ACT} \quad (B2-16)$$

c. Servovalve Flow:

$$Q_{ACT} = K_V i_V' \frac{P_S - P_R - \frac{i_V'}{|i_V'|} P_L}{\left| P_S - P_R - \frac{i_V'}{|i_V'|} P_L \right|} \sqrt{\left| P_S - P_R - \frac{i_V'}{|i_V'|} P_L \right|} \quad (B2-17)$$

d. Equivalent Servovalve Current:

$$i_V' = i_V - K_S \Delta P_f \quad (B2-18)$$

e. Servovalve Current (Output of Servoamplifier):

$$i_V = K_a K_t (\delta_c - \delta_f) \quad (B2-19)$$

f. Engine Position Feedback:

$$\delta_f = \delta + \frac{57.3 A}{K_m R} P_L \quad (B2-20)$$

g. Dynamic Pressure Feedback:

$$\Delta \dot{P}_f = \frac{1}{2} \left[\dot{P}_L - \frac{C_f K_f}{A_f^2} \frac{\Delta P_f}{|\Delta P_f|} \sqrt{|\Delta P_f|} \right] \quad (B2-21)$$

B2.4.2.2 System-Describing Function. In these equations, flow through an orifice is assumed proportional to the square root of the pressure drop across the orifice. The equations are now linearized by defining "equivalent admittances."

$$\frac{I_R}{57.3} \ddot{\delta} = -\bar{C}_F \dot{\delta} - T_L(t) + AR P_L \quad (B2-22)$$

$$\left(\frac{V_A}{4B} + \frac{A^2}{K_m} \right) \dot{P}_L = -\frac{AR}{57.3} \dot{\delta} - \bar{C}_L P_L + Q_{ACT} \quad (B2-23)$$

$$\Delta \dot{P}_f = \frac{1}{2} \left[\dot{P}_L - \frac{K_f}{A_f} \bar{C}_f \Delta P_f \right] \quad (B2-24)$$

where the barred variables are the equivalent admittances for gimbal friction, C_L and C_f ; and are both amplitude- and frequency-dependent. The values for the barred variables are defined in terms of describing functions; i.e., in terms of the ratio between the fundamental component of the output to the assumed sinusoidal input. Reference B-7 derives the following relationships:

$$\bar{C}_F = C_V + \frac{4C_B}{\pi \omega \bar{\delta}} \quad (B2-25)$$

$$\bar{C}_L = \frac{1.11 C_L}{\sqrt{\bar{P}_L}} \quad (B2-26)$$

$$\bar{C}_f = \frac{1.11 C_f}{\sqrt{\Delta \bar{P}_f}} \quad (B2-27)$$

where $\bar{\delta}$, $\bar{P}_L$, and $\Delta \bar{P}_f$ are the maximum sinusoidal amplitudes of the engine motion, load pressure, and feedback pressure (across the sensing piston) at an assumed amplitude, $\bar{\delta}$, and frequency, ω . The pressure amplitudes are given by:

$$\bar{P}_L = \frac{\bar{\delta}}{57.3 AR} \sqrt{(I_R \omega^2)^2 + (57.3 \bar{C}_F \omega)^2} \quad (B2-28)$$

$$\Delta \bar{P}_f = \frac{1}{2} \left[\sqrt{\bar{P}_L^2 + \frac{1}{(4a_f^2 \omega^2)^2}} - \frac{1}{4a_f^2 \omega^2} \right] \quad (B2-29)$$

where

$$a_f = \frac{A_f^2}{1.11 C_f K_f} \quad (B2-30)$$

In addition to these linearizations, the influence of load pressure on the valve gain is assumed negligible. Equation B2-17 then becomes:

$$Q_{ACT} = K_V i_V' \sqrt{P_S - P_R} \quad (B2-31)$$

The equations can now be represented by the block diagram of Figure B2-5. The general expression for engine position, $\delta(s)$, in terms of the commanded position, $\delta_c(s)$, and the load torques, $T_L(s)$, becomes:

$$\delta(s) = \frac{K_c \omega_c^2 (2\tau_f s + 1) \delta_c(s) - \frac{57.3}{I_R} (2\tau_f s^2 + K_1 s + K_o) T_L(s)}{2\tau_f s^4 + N_3 s^3 + N_2 s^2 + N_1 s + K_c \omega_c^2} \quad (B2-32)$$

where

$$\tau_f = \frac{A_f^2}{\bar{C}_f K_f} \quad (B2-33)$$

$$\omega_c^2 = \frac{(AR)^2}{I_R \left(\frac{V_A}{4B} + \frac{A^2}{K_m} \right)} \quad (B2-34)$$

$$K_c = \frac{57.3 K_V K_a K_t \sqrt{P_S - P_R}}{AR} \quad (B2-35)$$

$$K_1 = \left\{ 1 + \frac{2\tau_f \left[\bar{C}_L + K_c \left(\frac{A^2}{K_m} + \frac{K_s AR}{2 \times 57.3 K_a K_t} \right) \right]}{\frac{V_A}{4B} + \frac{A^2}{K_m}} \right\} \quad (B2-36)$$

$$K_o = \frac{\bar{C}_L + \frac{A^2}{K_m} K_c}{\frac{V_A}{4B} + \frac{A^2}{K_m}} \quad (B2-37)$$

$$N_3 = K_1 + \frac{2 \times 57.3 \tau_f \bar{C}_F}{I_R} \quad (B2-38)$$

$$N_2 = 2\tau_f \omega_c^2 \left(1 + \frac{\bar{C}_F K_s K_c}{2 K_a K_t AR} \right) + \frac{57.3 \bar{C}_F}{I_R} + K_o \left(1 + \frac{2 \times 57.3 \tau_f \bar{C}_F}{I_R} \right) \quad (B2-39)$$

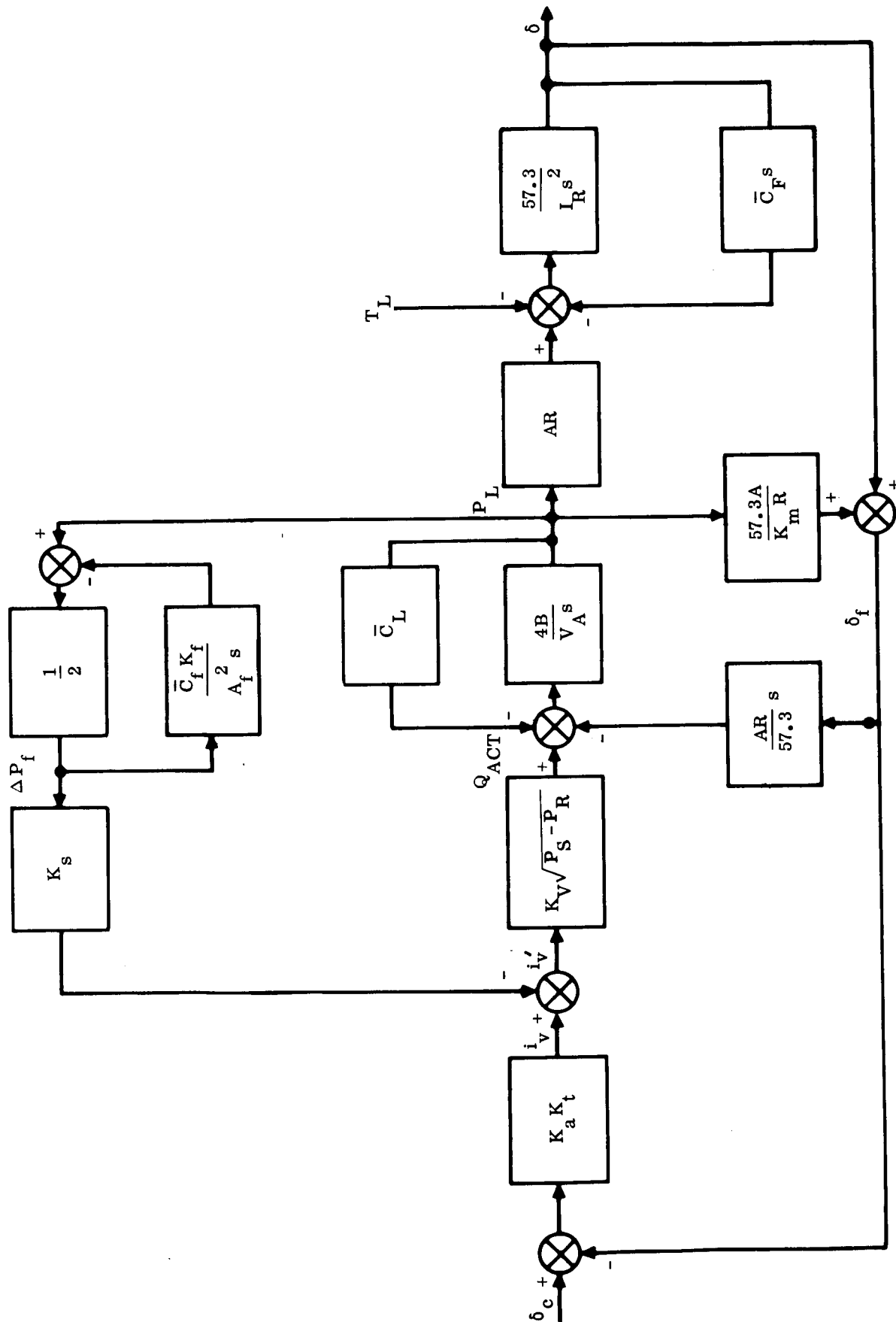


Figure B2-5. Block Diagram of Electrohydraulic Engine Position Servo

$$N_1 = \frac{57.3 \bar{C}_F K_o}{I_R} + \omega_c^2 (1 + 2 \tau_f K_c) \quad (B2-40)$$

The Centaur actuators do not have a leakage orifice ($\bar{C}_L = 0$); under this condition Equations B2-36 and B2-37 can be simplified to

$$K_1 = \left\{ 1 + \frac{2 \tau_f K_c \left(\frac{A^2}{K_m} + \frac{K_s AR}{2 \times 57.3 K_a K_t} \right)}{\left(\frac{V A}{4 B} + \frac{A^2}{K_m} \right)} \right\} \quad (B2-41)$$

$$K_o = \frac{\frac{A^2}{K_m} K_c}{\frac{V A}{4 B} + \frac{A^2}{K_m}} \quad (B2-42)$$

These expressions are strictly valid for $s = j\omega$, but are approximately valid in the region of the $j\omega$ axis. Thus they can be used to predict limit cycles over the entire frequency spectrum of interest.

Figure B2-6 illustrates the gain and phase characteristics of the Centaur actuator describing function.

B2.4.2.3 Low-Frequency Approximation; Root-Locus Studies. For purposes of studying rigid body/propellant sloshing stability using root-locus techniques, it is desirable to use a simple transfer function of the form (Reference B-8):

$$\frac{\delta}{\delta_c}(s) = \frac{K_c'}{s + K_c'} \quad (B2-43)$$

where K_c' is a function of both the amplitude, $\bar{\delta}$, and the frequency, ω , of engine motion. Root-loci can be plotted for several values of equivalent actuator lag frequency, K_c' , and interpreted in terms of possible limit cycles.

It is assumed that the load disturbance torques, $T_L(t)$, are of minor importance in the frequency range of interest and can be omitted. Then the phases of the transfer functions in Equations B2-32 and B2-43 are equated for various engine amplitudes and frequencies. This results in an unavoidable but small (at low frequencies) error in gain. The results, $K_c'(\omega, \bar{\delta})$, are plotted as in Figures 3.3-14 and 3.3-15 in the main body of the report. It should be noted that these curves are for particular values of actuator parameters and will change as these parameters change. For example, gimbal friction varies over a wide range, and the predicted limit cycle amplitudes will vary accordingly.

B2.4.2.4 Low Frequency Approximation; Analog Studies. Implicit in the preceding discussion was the validity of the describing function method in analyzing the actuator nonlinearities. This kind of analysis assumes both sinusoidal excitation and an essentially sinusoidal output (as far as its effects on the succeeding elements in the system are concerned). At very low frequencies and amplitudes this approximation begins to lose validity, primarily because of the coulomb friction in the gimbal bearing. The gimbal friction creates a dead zone in the engine motion. In analog studies a so-called dead-zone approximation is used in analyzing system behavior.

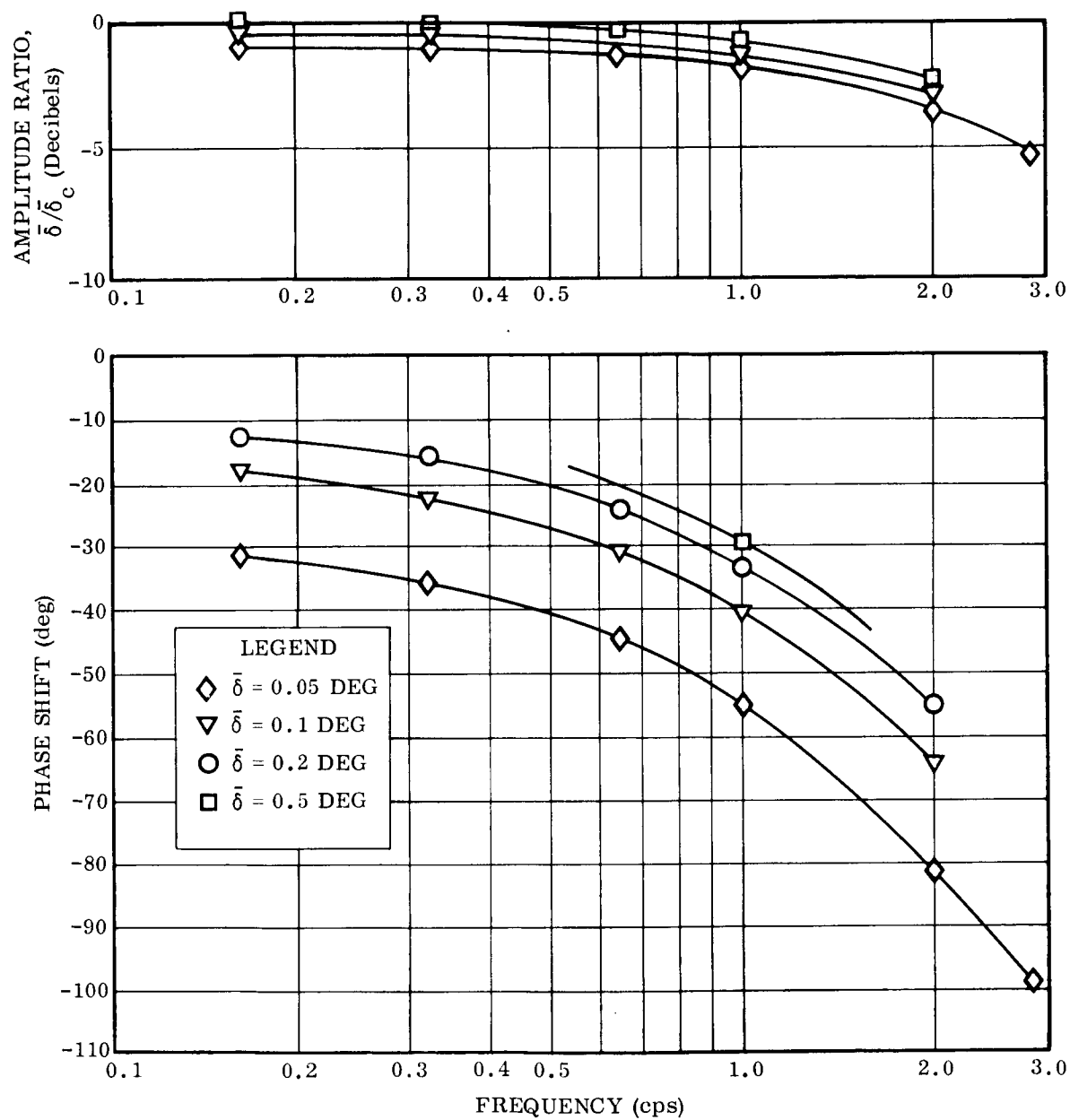


Figure B2-6. Describing Function of Engine Actuator

The dead zone arises from the Coulomb friction in the gimbal bearing. A consideration of the friction forces acting will make this evident. Consider static friction. Until the breakaway friction torque is exceeded, motion will not take place. Once motion has taken place, both Coulomb and viscous friction forces will be present. This static friction causes the dead zone, because an output motion is not produced in the presence of a command until the maximum static friction force is exceeded. The dead zone is the angular command required such that motion is impending.

The system equations given at the beginning of Section B2.4.2 may be combined to yield:

$$\frac{57.3}{AR} \left(\frac{VA}{4B} + \frac{A^2}{K_m} \right) \dot{P}_L + \dot{\delta} + \frac{57.3 C_L}{AR} \frac{P_L}{|P_L|} \sqrt{|P_L|} = K_c \left\{ \delta_c - \delta - \left[\frac{57.3 A}{K_m R} P_L + \frac{K_s}{2 K_a K_t} \frac{\dot{P}_L}{s + \frac{C_f K_t}{2 A_f^2 \sqrt{|\Delta P_f|}}} \right] \right\} \quad (B2-44)$$

where the influence of load pressure on valve gain is assumed negligible. It is further assumed that in the frequency range of interest, the washout orifice around the sensing piston in the dpf network is completely effective; i.e., there is no feedback ($K_s = 0$).

With C_L and K_s equaling zero, for the Centaur stage actuators, Equation B2-44 may be solved for P_L , yielding:

$$P_L = \frac{1}{\frac{57.3}{AR} \left(\frac{VA}{4B} + \frac{A^2}{K_m} \right)} \frac{K_c \delta_c - (s + K_c) \delta}{s + \frac{K_c}{1 + \frac{VA K_m}{4BA^2}}} \quad (B2-45)$$

The equation of motion of the thrust chamber is given by:

$$ARP_L = \frac{I_R}{57.3} \ddot{\delta} + C_V \dot{\delta} + C_B \frac{\dot{\delta}}{|\dot{\delta}|} + T_L \quad (B2-46)$$

where T_L represents the inertial load torques acting on the engine;

$$T_L = M_R \ell_R \ddot{x} - \frac{\delta}{57.3} - \frac{1}{57.3} (I_R + M_R \ell_R \ell_c) \ddot{\theta} - M_R \ell_R \ddot{z} \quad (B2-47)$$

Terms representing duct restraints, engine center-of-gravity offsets, etc., are omitted in these equations. Equations B2-45, B2-46, and B2-47 are simulated for analog studies of the Centaur stage vehicle. The Coulomb friction is simulated by feedback of engine velocity, $\dot{\delta}$, through a high gain limited amplifier, and thus produces a rate dead zone.

B2.4.2.5 Gimbaling Capability. The hydraulic system adequacy is indicated by comparing the gimbaling demand with gimbaling capability. The equations describing the vehicle and engine dynamics are given by References B-9 and B-10. The capability is determined as follows:

$$\begin{aligned} \text{ARP}_L = & \frac{I_R}{57.3} \ddot{\delta} + C_V \dot{\delta} + C_B \frac{\dot{\delta}}{|\dot{\delta}|} + \text{ARP}_{L(\text{ACT})} \frac{\dot{\delta}}{|\dot{\delta}|} + \frac{M_R \ell_R}{57.3} \ddot{x} \\ & - M_R \ell_R \ddot{z} - \frac{1}{57.3} (I_R + M_R \ell_R \ell_c) \ddot{\theta} + \frac{T_c}{2} \epsilon + \mathfrak{m} \end{aligned} \quad (\text{B2-48})$$

$$I_{yy_r} \ddot{\theta} = T_c \ell_c \delta + 2 (M_R \ell_R \ell_c + I_R) \ddot{\delta} + 2 M_R \ell_R \ddot{x} \delta + 57.3 T_c \epsilon \quad (\text{B2-49})$$

$$57.3 M_r \ddot{z} = T_c \delta + 2 M_R \ell_R \ddot{\delta} \quad (\text{B2-50})$$

$$M_T \ddot{x} = T_c \quad (\text{B2-51})$$

where

$$P_{L(\text{ACT})} = \text{Load pressure required to balance coulomb friction in actuator} = 15 \text{ lb/in.}^2$$

$$\mathfrak{m} = \text{Total duct restraint moment acting on engine} = 135 \text{ ft-lb.}$$

In these equations the forces and moments produced by propellant sloshing have been neglected. Since the engine angle, δ , is the excursion from trim conditions, terms for center-of-gravity offset and thrust differential do not appear.

If $\ddot{\theta}$ and $\ddot{z}$ are eliminated from the above equations, the following equation defining instantaneous load pressure in terms of engine motion applies.

$$\begin{aligned} \text{ARP}_L = & \frac{1}{57.3} \left\{ I_R - 4 \frac{I_R}{I_{yy_r}} M_R \ell_R \ell_c - 2 \frac{I_R^2}{I_{yy_r}} - 2 \frac{(M_R \ell_R \ell_c)^2}{I_{yy_r}} - \frac{2(M_R \ell_R)^2}{M_r} \right\} \ddot{\delta} \\ & + C_V \dot{\delta} + \left(C_B + \text{ARP}_{L(\text{ACT})} \right) \frac{\dot{\delta}}{|\dot{\delta}|} + \frac{1}{57.3} \left\{ M_R \ell_R \ddot{x} - \frac{I_R T_c \ell_c}{I_{yy_r}} \right. \\ & \left. - 2 \frac{I_R}{I_{yy_r}} M_R \ell_R \ddot{x} - \frac{M_R \ell_R \ell_c}{I_{yy_r}} T_c \ell_c - 2 \frac{M_R^2 \ell_R^2}{I_{yy_r}} \ell_c \ddot{x} - \frac{M_R \ell_R T_c}{M_r} \right\} \delta \\ & + \left\{ M_R \ell_R \ddot{x} \frac{\Delta \delta}{57.3} - \frac{I_R}{I_{yy_r}} T_c \epsilon - \frac{M_R \ell_R \ell_c}{I_{yy_r}} T_c \epsilon + \frac{T_c}{2} \epsilon + \mathfrak{m} \right\} \end{aligned} \quad (\text{B2-52})$$

This equation is more simply stated:

$$\text{ARP}_L = A_1 \ddot{\delta} + A_2 \dot{\delta} + \left[C_B + \text{ARP}_{L(\text{ACT})} \right] \frac{\dot{\delta}}{|\dot{\delta}|} + A_3 \delta + A_4 \quad (\text{B2-53})$$

Except for those defined above, the parameters given in Appendix E are used to evaluate the terms in the equation. Table B2-1 lists values for gimbal torque coefficients.

The available torque is conservatively assumed to be:

$$T_{\text{AVAIL}} = 1620 - 142 |\dot{\delta} \text{ (in deg)}| \text{ ft-lb} \quad (\text{B2-54})$$

where the engines are assumed to gimbal in both planes simultaneously. This represents a conservative approximation of the hydraulic power available to produce actuator motion, since the effects of the accumulator are neglected. If $\delta(t)$ is assumed sinusoidal, i. e., $\delta(t) = \bar{\delta} \cos \omega t$ then T_{AVAIL} as a function of time is given by:

$$T_{\text{AVAIL}} = 1620 - 142 \omega \bar{\delta} |\sin \omega t| \quad (\text{B2-55})$$

The required torque is given by the right-hand side of Equation B2-53:

$$T_{\text{REQ}} = (A_3 - \omega^2 A_1) \bar{\delta} \cos \omega t + A_4 + \left[C_B + \text{ARP}_{L(\text{ACT})} \right] \frac{\dot{\delta}}{|\dot{\delta}|} \quad (\text{B2-56})$$

where the phase and amplitude changes due to the $A_2 \dot{\delta}$ term (viscous friction) are small and have been neglected. The critical point in the limit cycle, where the demand on the hydraulic power supply is greatest, is where the required torque just equals the available torque. Using this fact, the following relation between limit cycle amplitude and frequency is obtained:

$$\bar{\delta} \sqrt{(141.6 \omega)^2 + (A_3 - \omega^2 A_1)^2} = 1620 - A_4 - (C_B + \text{ARP}_{L(\text{ACT})}) \quad (\text{B2-57})$$

In Figure 3.3-24, the curve representing hydraulic system capability is a plot of Equation B2-57. The key assumption in this analysis is that the sinusoidal engine motion. This assumption is most valid where large demands, as in this derivation, are made on the system. The nonlinear terms, chiefly coulomb friction, form a relatively small part of the overall torques.

Table B2-1. Evaluation of Gimbal Torque Coefficients

COEFFICIENT	VALUE
$A_1 \left(\frac{\text{ft-lb}}{\text{rad/sec}^2} \right)$	1.4
$A_3 \text{ (ft-lb/rad)}$	110
$A_4 \text{ (ft-lb)}$	914
$C_B + \text{ARP}_{L(\text{ACT})} \text{ (ft-lb)}$	230

SECTION B3

ELASTIC VEHICLE BENDING STABILITY EQUATIONS

LIST OF SYMBOLS FOR SECTION B3

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A	Actuating piston area	ft ²
A _f	Feedback piston area	ft ²
B	Bulk modulus of hydraulic fluid	lb/ft ²
C _B	Coulomb friction coefficient	ft-lb
$\bar{C}_F$	Equivalent admittance for gimbal friction	$\frac{\text{ft-lb}}{\text{deg/sec}}$
C _f	Discharge coefficient for washout orifice	$\frac{(\text{ft}^3/\text{sec})}{\sqrt{\text{lb/ft}^2}}$
$\bar{C}_f$	Equivalent admittance for C _f	$\frac{\text{ft}^3/\text{sec}}{\text{lb/ft}^2}$
C _V	Viscous friction coefficient	$\frac{\text{ft-lb}}{\text{deg/sec}}$
D	Platform gyro damping	dyne-cm-sec
f _m	Inertial guidance platform gimbal viscous friction coefficient	$\frac{\text{in./oz}}{\text{rad/sec}}$
G(s)	Platform servo stabilization electronic transfer function	N. D.
H _r	Angular momentum of platform gyro motor	$\frac{\text{gm cm}^2}{\text{sec}}$
I _r	Vehicle moment of inertia about pitch axis (I _{yy_r})	slug-ft ²
I _R	Engine "wet" moment of inertia of gimballed mass about gimbal axis	slug-ft ²
I _{R_o}	Engine "wet" moment of inertia of gimballed mass about engine center-of-gravity	slug-ft ²
J	Platform gimbal moment of inertia	in.-oz-sec ²
K _A	End-to-end position gain	$\frac{\text{deg } \delta}{\text{deg } \theta}$
K _a K _t	Product of servoamplifier gain and feedback transducer gain	ma/deg
K _C	Actuator no-load open-loop velocity gain	$\frac{\text{deg/sec}}{\text{deg}}$
$\bar{K}_C$	Actuator effective (at load) open loop velocity gain	$\frac{\text{deg/sec}}{\text{deg}}$
K _f	Feedback spring constant	lb/ft ²
K _I	Autopilot integrator gain	sec ⁻¹
K _m	Spring constant of actuator support structure	lb/ft
K _R	Autopilot rate gyro gain	$\frac{\text{deg}}{\text{deg/sec}}$
K _V	Servovalve discharge coefficient	$\frac{\text{ft}^3/\text{sec}}{\text{ma } \sqrt{\text{lb/ft}^2}}$
K ₃	Inertial guidance platform servo stabilization loop gain	in.-oz/rad

LIST OF SYMBOLS FOR SECTION B3, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
l_c	Rocket engine control moment arm	ft
l_R	Distance between rocket engine gimballed mass center-of-gravity and gimbal axis	ft
M_R	Rocket engine gimballed mass, one engine, "wet"	slugs
$m^{(i)}$	Generalized mass, i^{th} mode	slugs
M. U.	Platform gimbal mass unbalance parameter	$\frac{\text{in.-oz}}{\text{ft/sec}}$
$\bar{P}_L$	Amplitude of actuator load pressure	lb/ft ²
P_R	Hydraulic system reservoir pressure (return pressure from servovalve)	lb/ft ²
P_S	Hydraulic supply pressure to servovalve	lb/ft ²
$q^{(i)}$	Generalized coordinate, i^{th} mode	ft
R	Engine actuator moment arm	ft
T_c	Control engine thrust	lb
T_D	Total platform gimbal disturbance torque ($T_f + T_{MU}$)	in.-oz
T_f	Platform disturbance torque due to gimbal friction	in.-oz
T_{GR}	Platform gyro reaction torque	in.-oz
T_L	Load torque feedback to actuating piston	ft-lb
T_m	Platform gimbal servo stabilization torque	in.-oz
T_{MU}	Platform disturbance torque due to gimbal mass unbalance	in.-oz
V_A	Total volume of hydraulic fluid in one actuator cylinder	ft ³
$\beta_{XT}^{(i)}$	Normalized slope-coupling term, i^{th} mode ($\beta_{XT}^{(i)} = \sigma_{XT}^{(i)} - \sigma_{XE}^{(i)}$)	deg/ft
ΔP_f	Pressure drop across sensing piston in dpf valve	lb/ft ²
$\overline{\Delta P_f}$	Equivalent admittance for ΔP_f	lb/ft ²
δ	Engine displacement with respect to the elastic slope at the gimbal point	deg
$\bar{\delta}$	Sinusoidal engine gimbal angle limit cycle amplitude from null to peak	deg
δ_c	Commanded engine angle	deg
δ_{ik}	Kronecker delta	N. D.
$\zeta^{(i)}$	Damping ratio, i^{th} mode	N. D.
ζ_G	Rate gyro damping ratio	N. D.

LIST OF SYMBOLS FOR SECTION B3, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
ζ_P	Inertial guidance platform case damping ratio	N. D.
θ	Rigid body vehicle pitch angle	deg
θ_c	Commanded vehicle pitch angle	deg
θ_F	Rate gyro feedback signal	deg
θ_{GP}	Platform gyro rotation about its output axis	rad
θ_{mu}	Incremental position feedback signal error due to gimbal mass unbalance and shock mount transmissibility	deg
θ_P	Position reference feedback signal	deg
θ_P'	Ideal position reference feedback signal modified by platform case rocking dynamics	deg
θ_e	Autopilot error signal	deg
π	3.1416	N. D.
$\sigma_{PG}^{(i)}$	Normalized slope of i^{th} mode at position reference location	deg/ft
$\sigma_{RG}^{(i)}$	Normalized slope of i^{th} mode at rate gyro location	deg/ft
$\sigma_{XE}^{(i)}$	Normalized slope of i^{th} mode at engine center of gravity	deg/ft
$\sigma_{XP}^{(i)}$	Normalized slope of i^{th} mode at inertial guidance platform location	deg/ft
$\sigma_{XT}^{(i)}$	Normalized slope of i^{th} mode at engine gimbal point	deg/ft
τ_f	Effective dpf network time constant	sec
τ_G	Inertial Guidance platform gyro time constant	sec
$\phi_{XP}^{(i)}$	Normalized displacement at inertial guidance platform location	ft/ft
$\phi_{XT}^{(i)}$	Normalized displacement of i^{th} mode at engine gimbal point	ft/ft
ω	Frequency	rad/sec
$\omega^{(i)}$	Natural frequency, i^{th} mode	rad/sec
ω_c	Undamped natural frequency of open-loop electrohydraulic position servo	rad/sec
ω_G	Rate gyro natural frequency	rad/sec
ω_P	Inertial guidance platform case natural frequency	rad/sec

LIST OF SYMBOLS FOR SECTION B3, Contd

Symbol	Definition	Units
Parameters used for intermediate calculations in general equations:		
a_f	Defined in Equation B3-27	
α	Defined in Equation B3-31	
ϕ	Defined in Equation B3-32	
ϕ_{IV}	Defined in Equation B3-34	
ϕ_L	Defined in Equation B3-33	

SECTION B3

ELASTIC VEHICLE BENDING STABILITY EQUATIONS

B3.1 INTRODUCTION. The equations of motion derived in this section are based on the Rayleigh-Ritz method (Reference B-11) of employing assumed structural mode shapes. The particular set of assumed modes used herein were derived for normal (orthogonal) modes, calculated with all elastically mounted rocket engines included, by utilizing all of the normal mode function, $\phi^{(i)}(X)$, except that portion pertaining to the rocket engines. Hence, these new modes are no longer orthogonal, being both inertially- and elastically-coupled as will be seen in the following derivation.

The flexible vehicle bending stability equations which follow are developed assuming a position reference having a transfer function independent of frequency. Since both the Atlas sustainer and Centaur flight phases use the Centaur inertial guidance platform to maintain a position reference rather than a position gyro, this assumption is not strictly valid. The effect of the inertial platform dynamics on the flexible vehicle bending stability is discussed in Section B3.4.

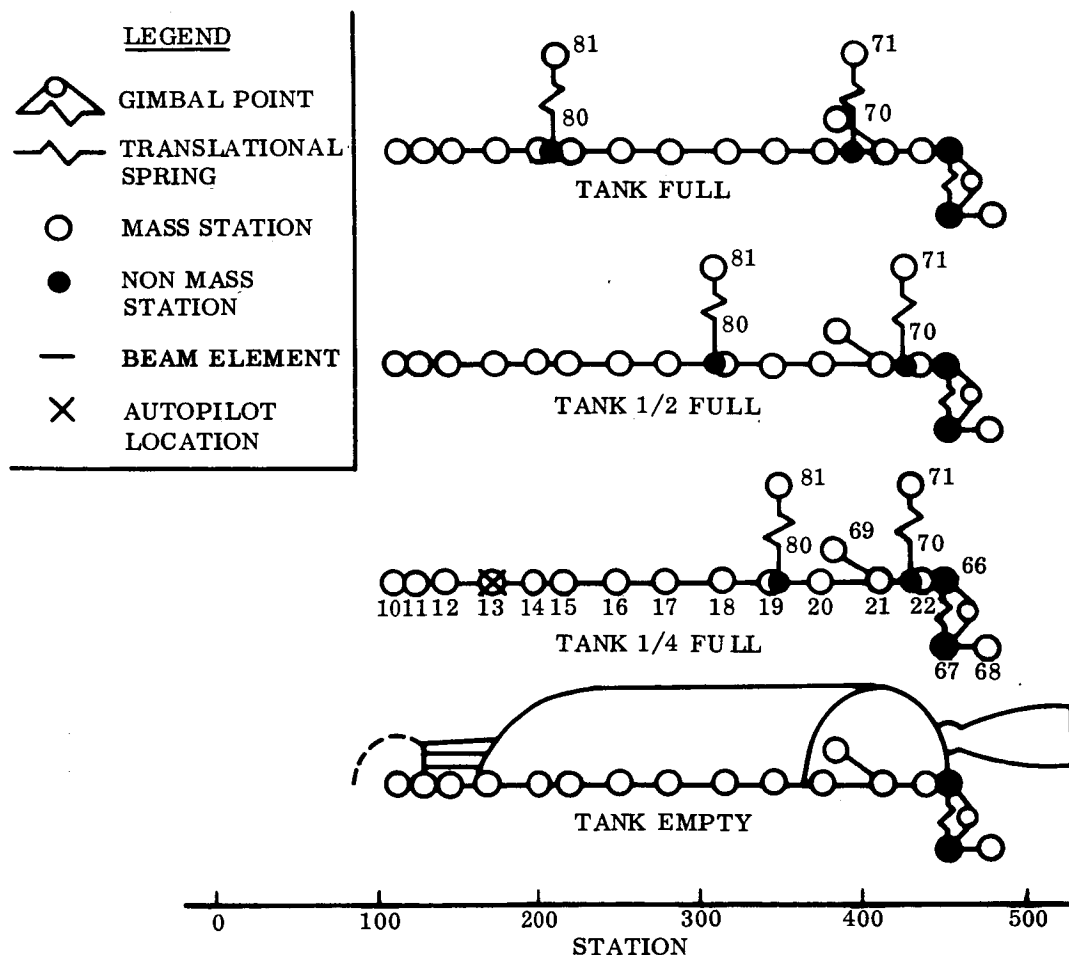


Figure B3-1. AC-5 Spring Mass Model

B3.2 ANALYTICAL DEVELOPMENT OF THE ELASTIC EQUATIONS OF MOTION. The following assumptions were made:

- a. Lateral and angular displacements are small.
- b. Aerodynamic and propellant sloshing forces are neglected.

B3.2.1 Lagrange's Equation. The general form of Lagrange's equation is:

$$\frac{d}{dt} \left(\frac{\partial T_V}{\partial \dot{q}^{(k)}} \right) - \frac{\partial T_V}{\partial q^{(k)}} + \frac{\partial U_V}{\partial q^{(k)}} = Q^{(k)} \quad (B3-1)$$

Where T_V and U_V are the kinetic and potential energies, respectively, of the elastic vehicle excluding engines, and $q^{(k)}$ is the k^{th} generalized coordinate. $Q^{(k)}$ is the generalized force for the k^{th} generalized coordinate and is defined by

$$Q^{(k)} \triangleq \frac{\partial W_e}{\partial q^{(k)}} \quad (B3-2)$$

where W_e is the work done by the externally applied forces.

For small lateral and angular displacements the kinetic energy is a positive quadratic function of the velocities (Reference B-11), and Lagrange's equation reduces to:

$$\frac{d}{dt} \left(\frac{\partial T_V}{\partial \dot{q}^{(k)}} \right) + \frac{\partial U_V}{\partial q^{(k)}} = Q^{(k)} \quad (B3-3)$$

Now let

$$T_V = T - T_E \quad (B3-4)$$

$$U_V = U - U_E \quad (B3-5)$$

where T and U are the kinetic and potential energy, respectively, of the vehicle with engines, and T_E and U_E are the kinetic and potential energies respectively, of the engines.

Substituting Equations B3-4 and B3-5 into B3-3,

$$\frac{d}{dt} \frac{\partial}{\partial \dot{q}^{(k)}} (T - T_E) + \frac{\partial}{\partial q^{(k)}} (U - U_E) = Q^{(k)} \quad (B3-6)$$

Now

$$T = \frac{1}{2} \sum_i m^{(i)} (\dot{q}^{(i)})^2 \quad (B3-7)$$

$$U = \frac{1}{2} \sum_i m^{(i)} (\omega^{(i)} q^{(i)})^2 \quad (B3-8)$$

where $m^{(i)}$ and $\omega^{(i)}$ are the generalized mass and natural frequency, respectively, of the i^{th} normal mode (including all gimbaled engines).

Using Equations B3-6, B3-7, and B3-8,

$$m^{(k)} \ddot{q}^{(k)} - \frac{d}{dt} \frac{\partial T_E}{\partial \dot{q}^{(k)}} + m^{(k)} (\omega^{(k)})^2 q^{(k)} - \frac{\partial U_E}{\partial q^{(k)}} = Q^{(k)} \quad (B3-9)$$

To simplify the expressions for the T_E , U_E , and $Q^{(k)}$ terms above, we assume only one engine and express all angles in radians. Modal parameters are those obtained from the assumed modes. Refer to Figure B3-2 for sign convention and symbology.

NOTE: $K_{\alpha} = K_m R^2$; EQUIVALENT TORSIONAL SPRING

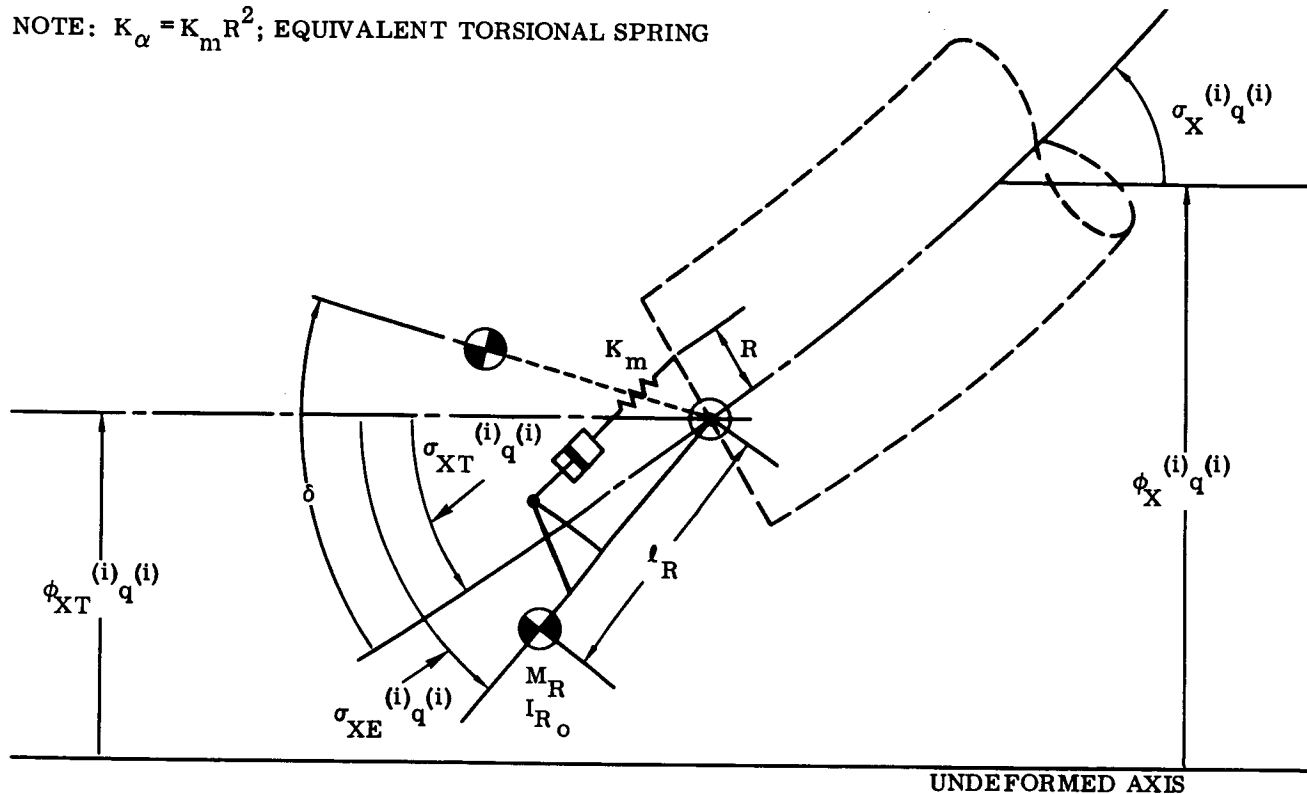


Figure B3-2. Vehicle Bending Schematic

B3.2.2 Kinetic Energy of the Engine, T_E ($i = 1, 2, \dots, k, \dots, n$)

$$T_E = \frac{1}{2} M_R \left\{ \sum_{i=1}^n \left[\phi_{XT}^{(i)} + \ell_R (\beta_{XT}^{(i)} - \sigma_{XT}^{(i)}) \right] \dot{q}^{(i)} \right\}^2 + \frac{1}{2} I_{Ro} \left\{ \sum_{i=1}^n (\beta_{XT}^{(i)} - \sigma_{XT}^{(i)}) \dot{q}^{(i)} \right\}^2 \quad (B3-10)$$

$$\frac{d}{dt} \left(\frac{\partial T_E}{\partial \dot{q}^{(k)}} \right) = \sum_{i=1}^n \left\{ M_R (\phi_{XT}^{(k)} - \ell_R \sigma_{XT}^{(k)}) \phi_{XT}^{(i)} + M_R \ell_R \beta_{XT}^{(k)} \phi_{XT}^{(i)} - I_R \beta_{XT}^{(k)} \sigma_{XT}^{(i)} \right. \\ \left. - (M_R \ell_R \phi_{XT}^{(k)} - I_R \sigma_{XT}^{(k)}) \sigma_{XT}^{(i)} + \left[M_R \ell_R \phi_{XT}^{(k)} + I_R (\beta_{XT}^{(k)} - \sigma_{XT}^{(k)}) \right] \beta_{XT}^{(i)} \right\} \ddot{q}^{(i)} \quad (B3-11)$$

B3.2.3 Potential Energy of the Engine, U_E ($i = 1, 2, \dots, k, \dots, n$)

$$U_E = \frac{1}{2} K_\alpha \left(\sum_{i=1}^n \beta_{XT}^{(i)} q^{(i)} \right)^2 \quad (B3-12)$$

$$\frac{\partial U_E}{\partial q^{(k)}} = K_\alpha \beta_{XT}^{(k)} \sum_{i=1}^n \beta_{XT}^{(i)} q^{(i)} \quad (B3-13)$$

Let

$$K_\alpha = K_m R^2; \quad (B3-14)$$

$$\frac{\partial U_E}{\partial q^{(k)}} = K_m R^2 \beta_{XT}^{(k)} \sum_{i=1}^n \beta_{XT}^{(i)} q^{(i)} \quad (B3-15)$$

B3.2.4 Virtual Work Done By Externally Applied Forces Due to Virtual Displacements $\Delta q^{(k)}$ ($k = 1, 2, \dots, n$).

$$\begin{aligned} \Delta W_e &= \sum_{k=1}^n \frac{\partial W_e}{\partial q^{(k)}} \Delta q^{(k)} \\ &= \sum_{k=1}^n \left[-M_R \ell_R \ddot{\phi}_{XT}^{(k)} + M_R \sum_{i=1}^n \left(\ell_R \sigma_{XT}^{(i)} - \phi_{XT}^{(i)} \right) \ddot{q}^{(i)} \phi_{XT}^{(k)} + I_R \ddot{\sigma}_{XT}^{(k)} \right. \\ &\quad \left. + M_R \ell_R \sum_{i=1}^n \phi_{XT}^{(i)} \ddot{q}^{(i)} \sigma_{XT}^{(k)} - I_R \sum_{i=1}^n \sigma_{XT}^{(i)} \ddot{q}^{(i)} \sigma_{XT}^{(k)} - T_c \delta \phi_{XT}^{(k)} \right] \Delta q^{(k)} \end{aligned} \quad (B3-16)$$

where Δ denotes a virtual increment.

B3.2.5 Generalized Forces in the k^{th} Mode, $Q^{(k)}$ ($i = 1, 2, \dots, k, \dots, n$). From Equation B3-2,

$$\begin{aligned} Q^{(k)} &\triangleq \frac{\partial W_e}{\partial q^{(k)}} \\ Q^{(k)} &= -\phi_{XT}^{(k)} \left[T_c \delta + \left(M_R \ell_R - I_R \frac{\sigma_{XT}^{(k)}}{\phi_{XT}^{(k)}} \right) \ddot{\phi} \right] + \sum_{i=1}^n \left[M_R \ell_R \sigma_{XT}^{(k)} \phi_{XT}^{(i)} - M_R \phi_{XT}^{(k)} \phi_{XT}^{(i)} \right. \\ &\quad \left. + M_R \ell_R \phi_{XT}^{(k)} \sigma_{XT}^{(i)} - I_R \sigma_{XT}^{(k)} \sigma_{XT}^{(i)} \right] \ddot{q}^{(i)} \end{aligned} \quad (B3-17)$$

After substituting Equations B3-11, B3-14, and B3-16 into B3-9, rearranging terms, accounting for both engines, and using proper units, the final elastic equation is given by Equation B3-18 in the section that follows.

B3.3 SUMMARY OF THE CONTROL SYSTEM ELASTIC VEHICLE EQUATIONS OF MOTION (Reference B-12)

a. Elastic Modes: Equation for k^{th} mode ($i = 1, 2, \dots, n$), $n \leq 5$

$$\begin{aligned} \sum_{i=1}^n \left(\left\{ \delta_{ik} - \frac{2}{57.3m^{(k)}} \left[M_R \ell_R \phi_{XT}^{(k)} + \frac{I_R}{57.3} (\beta_{XT}^{(k)} - \sigma_{XT}^{(k)}) \right] \beta_{XT}^{(i)} \right. \right. \\ \left. \left. - \frac{2 M_R \ell_R}{57.3m^{(k)}} \beta_{XT}^{(k)} \phi_{XT}^{(i)} + \frac{2 I_R}{m^{(k)} (57.3)^2} \beta_{XT}^{(k)} \sigma_{XT}^{(i)} \right\} \ddot{q}^{(i)} + 2 \zeta^{(k)} \omega^{(k)} \delta_{ik} \dot{q}^{(i)} \right. \\ \left. + \left[(\omega^{(k)})^2 \delta_{ik} - \frac{2 K_m R^2 \beta_{XT}^{(k)} \beta_{XT}^{(i)}}{m^{(k)} (57.3)^2} \right] q^{(i)} \right) + \frac{T_c \phi_{XT}^{(k)}}{57.3m^{(k)}} \delta \\ + \frac{2}{57.3m^{(k)}} \left(M_R \ell_R \phi_{XT}^{(k)} - \frac{I_R \sigma_{XT}^{(k)}}{57.3} \right) \ddot{\delta} = 0 \end{aligned} \quad (B3-18)$$

b. Rigid Body: ($i = 1, 2, \dots, n$), $n \leq 5$

$$\begin{aligned} \frac{I_o \ddot{\theta}}{57.3} - \frac{T_c \ell_c}{57.3} \delta - \frac{2}{57.3} (M_R \ell_R \ell_c + I_R) \ddot{\delta} + \left\{ \frac{1}{57.3} \sum_{i=1}^n \left[2 \left(I_R \right. \right. \right. \\ \left. \left. \left. + M_R \ell_R \ell_c \right) \beta_{XT}^{(i)} \right] \right\} \ddot{q}^{(i)} + \sum_{i=1}^n \left[T_c \left(\phi_{XT}^{(i)} + \frac{\ell_c}{57.3} \sigma_{XT}^{(i)} \right) \right] q^{(i)} = 0 \end{aligned} \quad (B3-19)$$

c. Position Reference and Rate Gyro: ($i = 1, 2, \dots, n$), $n \leq 5$

$$\begin{aligned} \ddot{\theta}_F + 2 \zeta_G \omega_G \dot{\theta}_F + \omega_G^2 \theta_F - K_R \omega_G^2 \dot{\theta} \\ - \sum_{i=1}^n (K_R \omega_G^2 \sigma_{RG}^{(i)}) \dot{q}^{(i)} = 0 \end{aligned} \quad (B3-20)$$

d. Autopilot Error Signal: ($i = 1, 2, \dots, n$), $n \leq 5$

$$\theta_\epsilon - \theta_F - \left[\theta + \sum_{i=1}^n \sigma_{PG}^{(i)} q^{(i)} \right] = 0 \quad (B3-21)$$

e. Command to Servoactuators:

$$\dot{\delta}_c - K_A \theta_\epsilon - K_A K_I \theta_\epsilon = 0 \quad (B3-22)$$

f. Nonlinear Actuators Describing Function:

$$\begin{aligned}
& 2\tau_f \ddot{\delta} + \left\{ 1 + \frac{2 \times 57.3}{I_R} \tau_f \bar{C}_F + \frac{\tau_f}{\frac{V_A}{4B} + \frac{A^2}{K_m}} \left[\left(\frac{A^2}{K_m} + \frac{AR K_s}{2 \times 57.3 K_a K_t} \right) \bar{K}_c \right] \right\} \ddot{\delta} \\
& + \left\{ 2\tau_f \left(1 + \frac{\bar{C}_F K_s}{2 K_a K_t AR} \bar{K}_c \right) \omega_c^2 + \frac{57.3}{I_R} \bar{C}_F + \left(\frac{\frac{A^2}{K_m} \bar{K}_c}{\frac{V_A}{4B} + \frac{A^2}{K_m}} \right) \left(1 + \frac{2 \times 57.3}{I_R} \tau_f \bar{C}_F \right) \right\} \ddot{\delta} \\
& + \left\{ \left(\frac{\frac{A^2}{K_m} \bar{K}_c}{\frac{V_A}{4B} + \frac{A^2}{K_m}} \right) \frac{57.3}{I_R} \bar{C}_F + (1 + 2\tau_f \bar{K}_c) \omega_c^2 \right\} \dot{\delta} + \omega_c^2 \bar{K}_c \delta - \omega_c^2 \bar{K}_c 2\tau_f \dot{\delta}_c \\
& - \omega_c^2 \bar{K}_c \delta_c + \frac{2 \times 57.3}{I_R} \tau_f \ddot{T}_L + \frac{57.3}{I_R} \left\{ 1 + \frac{2\tau_f}{\frac{V_A}{4B} + \frac{A^2}{K_m}} \left[\left(\frac{A^2}{K_m} + \frac{AR K_s}{2 \times 57.3 K_a K_t} \right) \bar{K}_c \right] \right\} \dot{T}_L \\
& + \frac{57.3}{I_R} \left\{ \frac{\frac{A^2}{K_m} \bar{K}_c}{\frac{V_A}{4B} + \frac{A^2}{K_m}} \right\} T_L = 0
\end{aligned} \tag{B3-23}$$

g. General Equations:

$$\tau_f = \frac{A_f^2}{\bar{C}_f K_f} \tag{B3-24}$$

$$\bar{C}_f = \frac{1.11 C_f}{\sqrt{\Delta P_f}} \tag{B3-25}$$

$$\Delta P_f = \frac{1}{2} \left\{ \sqrt{\bar{P}_L^2 + \frac{1}{(4a_f^2 \omega^2)^2}} - \frac{1}{4a_f^2 \omega^2} \right\} \tag{B3-26}$$

$$a_f = \frac{A_f^2}{1.11 C_f K_f} \tag{B3-27}$$

$$\bar{P}_L = \frac{\bar{\delta}}{AR} \sqrt{\left(\frac{I_R}{57.3} \omega^2\right)^2 + (\bar{C}_F \omega)^2} \quad (B3-28)$$

$$\bar{C}_F = C_V + \frac{4 C_B}{\pi \omega \bar{\delta}} \quad (B3-29)$$

$$\bar{K}_c = \alpha K_c \quad (B3-30)$$

$$\alpha = 1 + \frac{4}{3\pi} \left(\frac{\bar{P}_L}{P_R - P_S} \right) \cos \phi \quad (B3-31)$$

$$\phi = \phi_{IV} - \phi_L \quad (B3-32)$$

$$\phi_L = -\tan^{-1} \left(\frac{\bar{C}_F}{\frac{I_R}{57.3} \omega} \right) \quad (B3-33)$$

$$\phi_{IV} = \tan^{-1} \left[\frac{\omega^2 - \omega_c^2}{\left(\frac{I_R}{57.3} \bar{C}_F \right) \omega} \right] \quad (B3-34)$$

$$K_c = \frac{57.3}{AR} \cdot K_a K_t K_V \sqrt{P_S - P_R} \quad (B3-35)$$

$$\omega_c^2 = \frac{A^2 R^2}{I_R \left(\frac{V_A}{4B} + \frac{A^2}{K_m} \right)} \quad (B3-36)$$

$$T_L = \frac{1}{57.3} (I_R + M_R \ell_R \ell_c) \ddot{\theta} + \sum_{i=1}^n \left(M_R \ell_R \phi_{XT}^{(i)} - \frac{I_R}{57.3} \sigma_{XT}^{(i)} \right) \ddot{q}^{(i)} \quad (B3-37)$$

B3.4 ANALYSIS OF PLATFORM DYNAMICS. The Centaur inertial platform is used as the vehicle attitude reference for the flight control system during Atlas sustainer phase and throughout Centaur stage flight. The Centaur programmer provides a reference attitude vector (generated by the programmer or by the guidance computer) in the form of three voltages which represent the three components of the reference attitude vector in inertial (platform stable element) coordinates. These voltages are impressed on the resolver chain of the platform whose output is a similar set of three voltages which represent the attitude reference vector in vehicle coordinates. The voltage corresponding to that component of the reference vector along the positive pitch axis represents a positive (yaw right) rate command; along the positive yaw axis, a negative (pitch down) rate command. The voltage representing the component along the positive roll axis is shorted to ground through a loading resistance. The rate commands represent position errors about the pitch and yaw axes with respect to the reference attitude, and thus are equivalent to displacement gyro outputs. It is the purpose of this subsection to show that the transfer function relating the platform position error signal to the angular displacement of the case mounting with respect to the reference vector

is independent of frequency, for purposes of stability analyses, and also independent of the translational accelerations applied to the case. Moreover, it will be shown that the vehicle dynamics do not affect the stability of the platform servo loops used to stabilize the inertial element. This results in the simplified flight control system block diagram shown in Figure B3-4, which neglects platform dynamics.

B3.4.1 Mathematical Model. A more complete single-plane block diagram is shown in Figure B3-3, where the vehicle and platform angles are defined in Figure B3-4. The mathematical model includes:

- a. Flexible vehicle dynamics.
- b. Rocket engine position servo dynamics.
- c. Single axis (pitch) inertial platform servo loop dynamics.
- d. Platform gimbal friction.
- e. Platform mass unbalance.
- f. Platform shock mount dynamics.

The platform servo stabilization electronics represented by $G(s)$ in Figure B3-3 is a complex transfer function (Reference B-13) in the case of all four of the platform servo stabilization loops. Because of the similarity between them, this analysis considers only the pitch, or third gimbal stabilization loop, and is representative of yaw and roll. The method of analysis consists of establishing the transfer functions pertinent to each of the feedback loops in Figure B3-3, and evaluating the effects of the different parameters on the overall system stability. A more detailed description of the analysis is contained in Reference B-14.

B3.4.1.1 Servo Loop Dynamics $\left(\frac{\phi_P}{T_D}\right)$. The single axis platform stabilization dynamics are given within the dashed line box of Figure B3-3. The root locus for this loop is given in Figure B3-5. The two roots (A and B) represent the closed loop oscillatory roots of the platform dynamics at the operating gain, K_3 . The gain can vary between rather wide limits such that the natural frequency of A varies from 130 to 195 radians/second. The change in B for this gain variation is of little consequence because of this root's relatively large damping.

B3.4.1.2 Shock Mount Dynamics. The platform case mounting is a cantilever arrangement, four shock mounts secure the inertial platform to the mount. The transmissibility of the mounts and the transfer function used to simulate the mount characteristics are illustrated in Figure B3-6. All four of the isolators are dynamically matched to within 0.25 cps, and have a natural frequency of approximately 30 cps (Figure B3-5).

From the shock mount load-deflection envelopes, the critical platform case rocking frequencies (nearest to the platform servo loop frequencies and vehicle bending frequencies) are estimated to be:

Pitch	38 cps
Yaw	48 cps

Cables fastened to the platform case will alter the frequencies somewhat. This analysis assumes the critical condition whereby the platform rocking frequency in pitch is essentially equal to the frequency for which the shock mount transmissibility is a maximum (Figure B3-6), approximately 28 cps. The damping associated with this rocking motion is conservatively assumed to be the same as that obtained for the shock mount transmissibility, $\zeta = 0.17$.

B3.4.2 Platform/Vehicle Coupling. The coupling between the platform gimbal servo loops and the vehicle dynamics arises from three sources:

- a. Platform gimbal friction.
- b. Platform rocking dynamics.
- c. Platform gimbal mass unbalance.

Figure B3-7 illustrates two rearrangements of the block diagram of Figure B3-3 which allows the effect of the various parameters to be examined separately. The platform gimbal servo loop dynamics are represented in these figures by the transfer function $\left(\frac{\phi_P}{T_D}\right)$.

B3.4.2.1 Platform Gimbal Friction. Platform case angular velocities relative to the platform gimbal result in friction torques being applied to the platform gimbals. The root locus for the gimbal friction loop is illustrated in Figure B3-8. As shown, the platform's two oscillatory roots are unaffected by the specified maximum allowable gimbal friction ($f_m = 0.066 \frac{\text{in.-oz}}{\text{rad/sec}}$). The transfer function between the platform case attitude, θ_2 , and the inertial platform gimbal attitude, θ_0 , as read by the resolver is unity as far as the effects of gimbal friction are concerned. Only when the friction is increased to an unrealistically high value does the root locus show any noticeable change in the stability characteristics of the platform. Therefore it is neglected, as shown in Figure B3-7b, for the remainder of this analysis.

B3.4.2.2 Platform Rocking Dynamics and Mass Unbalance. Figures B3-9 and B3-10 present straight line bode plot comparisons of the three remaining feedback inputs shown in Figure B3-7b for the Centaur phase and Atlas sustainer phase of flight, respectively.

The rate feedback transfer function (loop 2) considers a typical Centaur rate gyro characteristic ($\omega_G = 107 \text{ rad/sec}$, $\zeta_G = 1.83$) or Atlas rate gyro ($\omega_G = 157 \text{ rad/sec}$, $\zeta_G = 0.75$), whichever the case may be. The remaining two loops (3 and 4), which together represent the platform position feedback, include the platform case rocking dynamics ($\omega_P = 173.3 \text{ rad/sec}$, $\zeta_P = 0.17$) and the destabilizing gimbal mass unbalance effects. Representative elastic vehicle slopes (σ_{XP}) and deflections (ϕ_{XP}), for vehicle bending frequencies in the critical platform frequency range of 100 to 300 rad/sec, were used to determine the relative magnitudes of these three feedback signals.

B3.4.2.2.1 Centaur Phase. A comparison of the autopilot control signals, of Figure B3-9, shows that the feedback signals during Centaur phase flight consist chiefly of a large rate signal with the signals resulting from the platform position feedback contributing only a small portion to the total feedback signal. The position feedback signal remains less than 10 percent of the corresponding rate signal, even at its resonant frequency. Furthermore, this represents a "worst case" condition since the damping associated with the platform case rocking motion is conservatively assumed to be 0.17. The feedback signal resulting from platform mass unbalance remains well below both the position and rate feedback signals even when amplified by the shock mount transmissibility and platform stabilization dynamics at frequencies near 170 rad/sec. In short, the Centaur flexible vehicle bending stability is expected to have little or no dependence on the platform position feedback.

Figure B3-11 shows a root locus which illustrates the closed-loop bending stability and the effect of the platform position feedback. For this analysis the elastic vehicle modal frequency was assumed to be the same as both the platform stabilization loop and platform shock mount resonant frequencies. In addition, a conservative engine half amplitude of 0.1 deg was assumed.

The results show that the platform dynamics do not affect the vehicle bending stability significantly even under the assumed ideal coupling conditions. For nominal loop parameters the platform rocking dynamics show the only noticeable effect on vehicle elastic stability. This is a result of the conservative damping ratio assumed for the platform rocking case motion.

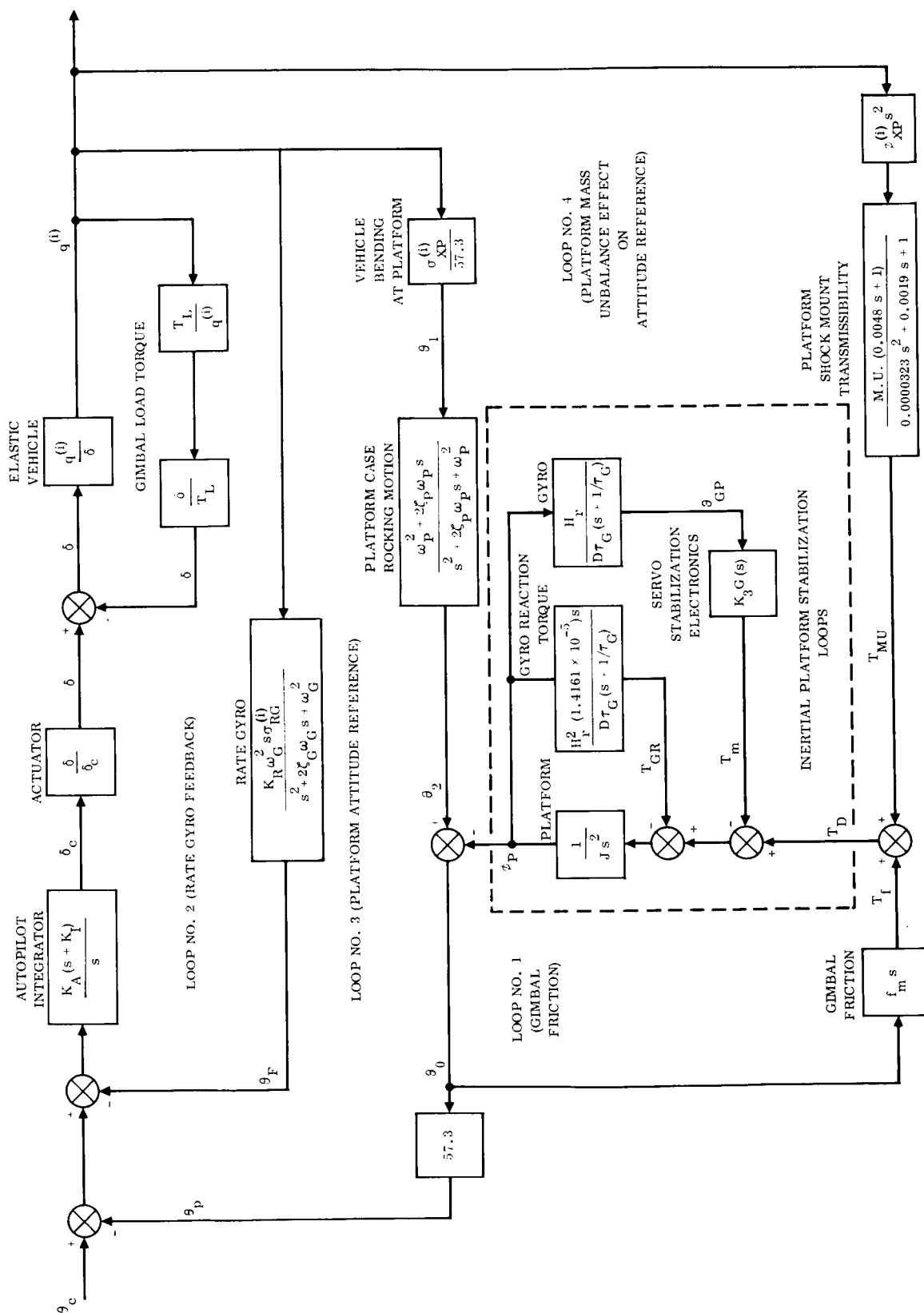


Figure B3-3. Single Plane Block Diagram for (Flexible) Vehicle/Autopilot Control Loop and Inertial Platform Servo Control Loop

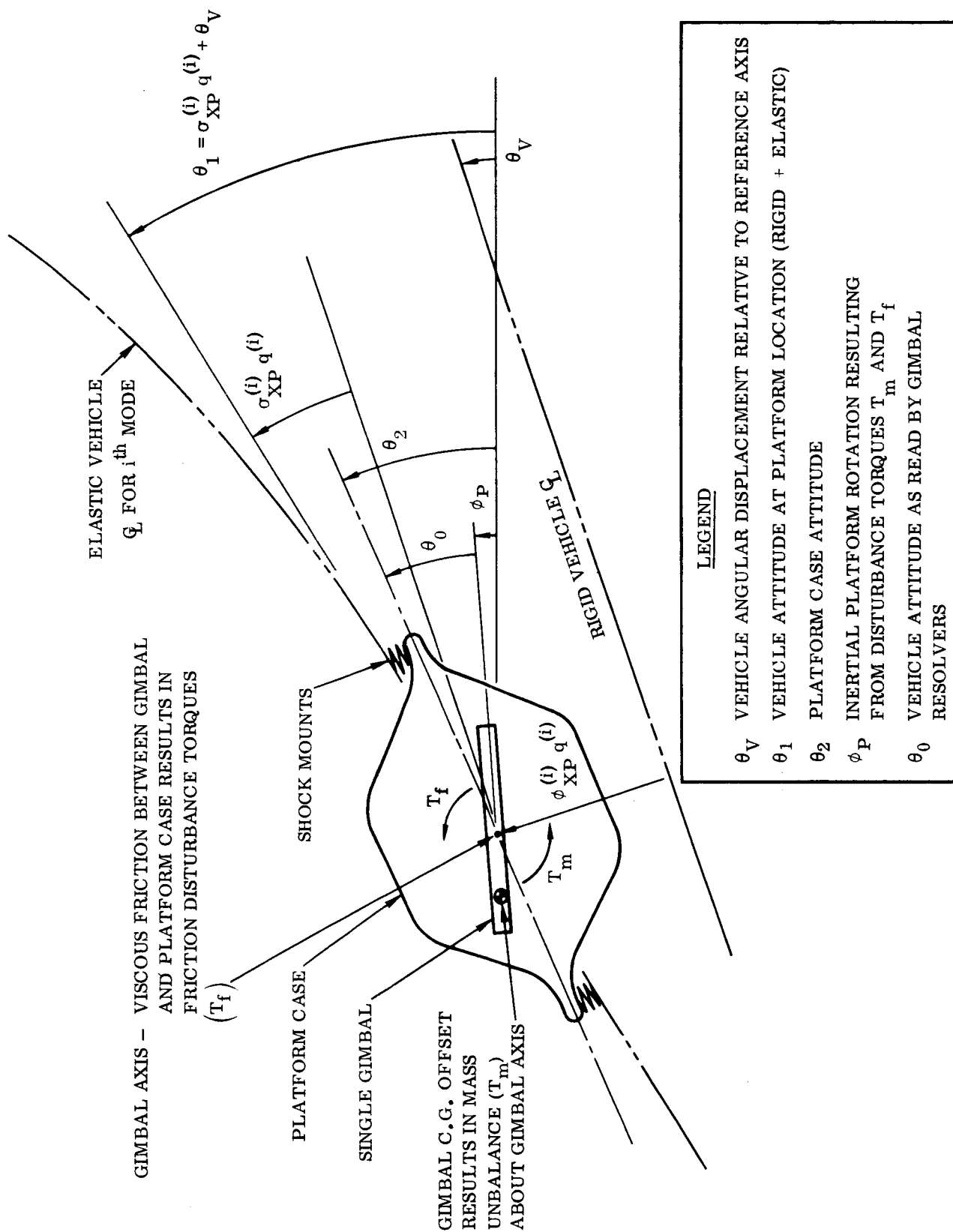


Figure B3-4. Definition of Vehicle and Platform Angles

NOTES: 1. PLATFORM/CYRO REACTION TORQUE CLOSED-LOOP TRANSFER FUNCTION

$$\frac{\dot{\phi}_P}{T} = \frac{s - \frac{1}{\tau_G}}{J s \left(s^2 - \frac{1}{\tau_G} s - \frac{1.4161 \times 10^{-5} H_F}{D J \tau_G} \right)} = \frac{s - 166.7}{3.29 s (s - 2.618)(s - 164.05)}$$

2. PLATFORM/SERVO STABILIZATION OPEN-LOOP TRANSFER FUNCTION

$$(GH)_{\text{OPEN}} = \frac{H K_3 4.863 \cdot 10^{-9} (s + 166.7)(s - 12.62)(s + 69.64)(s - 63.13)(s + 3704.1)}{D \tau_G J 5.9146 \cdot 10^{-17} (s + 2.618)(s + 164.05)(s + 6.08)(s - 171.82)(s - 714.3)(s + 3508.8)(s + 1476.7 \pm j2069.3)(s + 166.7)}$$

3. PLATFORM/SERVO STABILIZATION CLOSED-LOOP TRANSFER FUNCTION
FOR $K_3 = 7500 \text{ IN. - OZ/RAD}$

$$\frac{\dot{\phi}_P}{T_D} = \frac{(s + 166.7)^2 (s + 6.08)(s + 171.82)(s + 714.3)(s + 3508.8)(s + 1476.7 \pm j2069.3)}{3.29 (s + 12.476)(s + 60.285 \pm j37.105)(s + 61.76 \pm j173.06)(s + 802.4)(s + 1476.7)(s + 166.7)(s + 166.7)(s + 3508.7)}$$

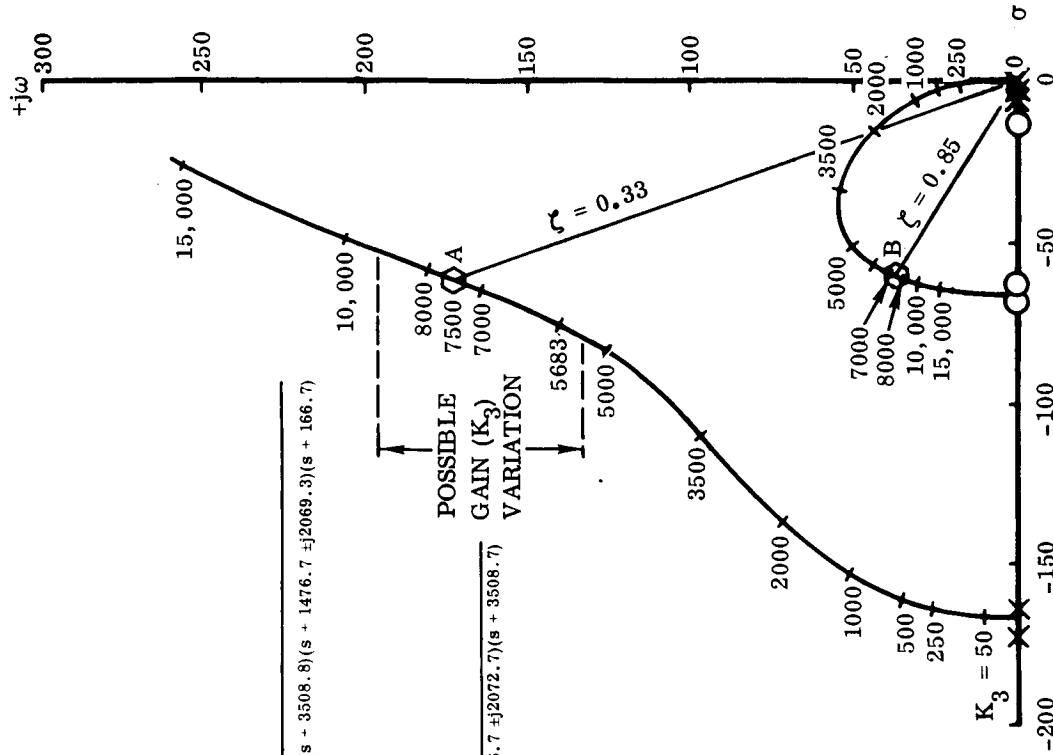


Figure B3-5. Stability of the Inertial Platform Stabilization Loops

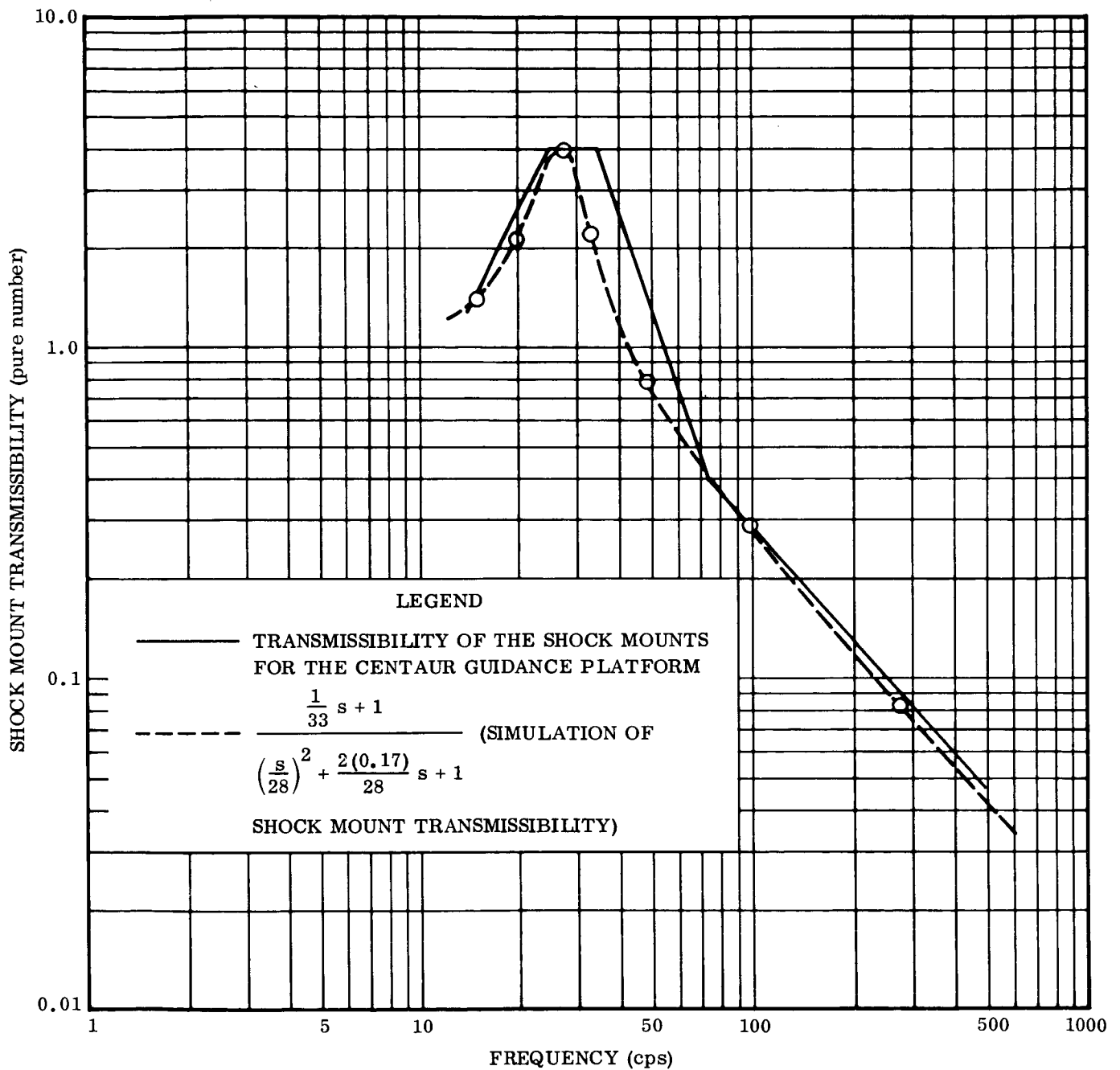
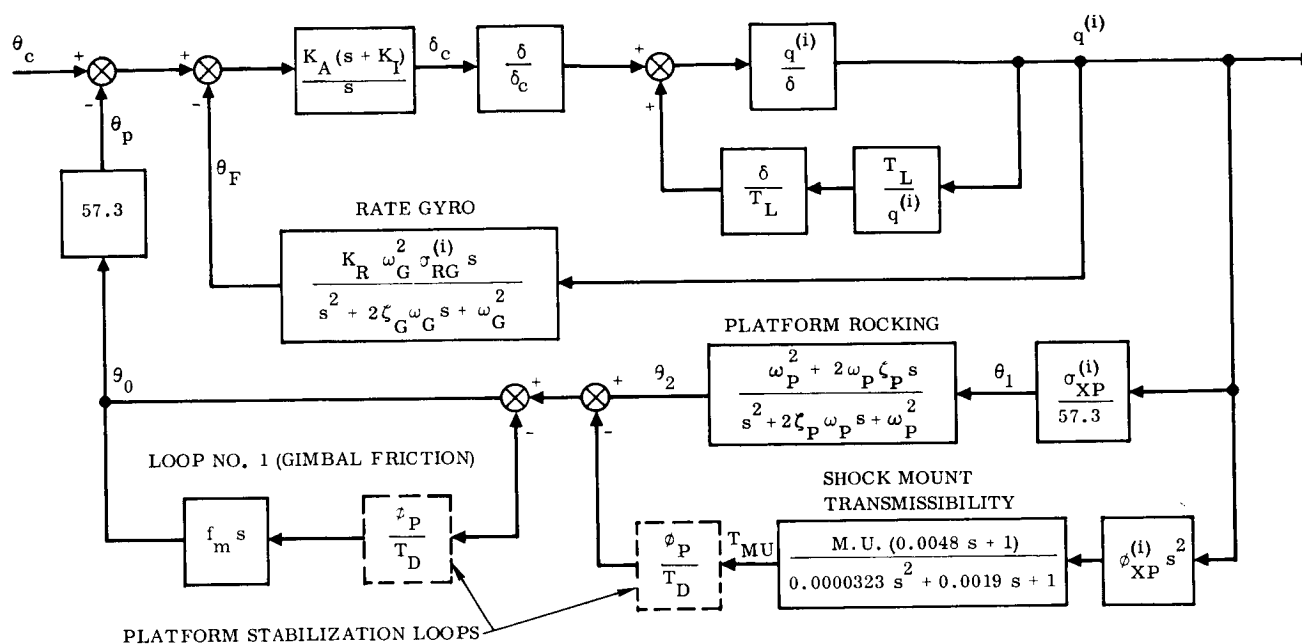
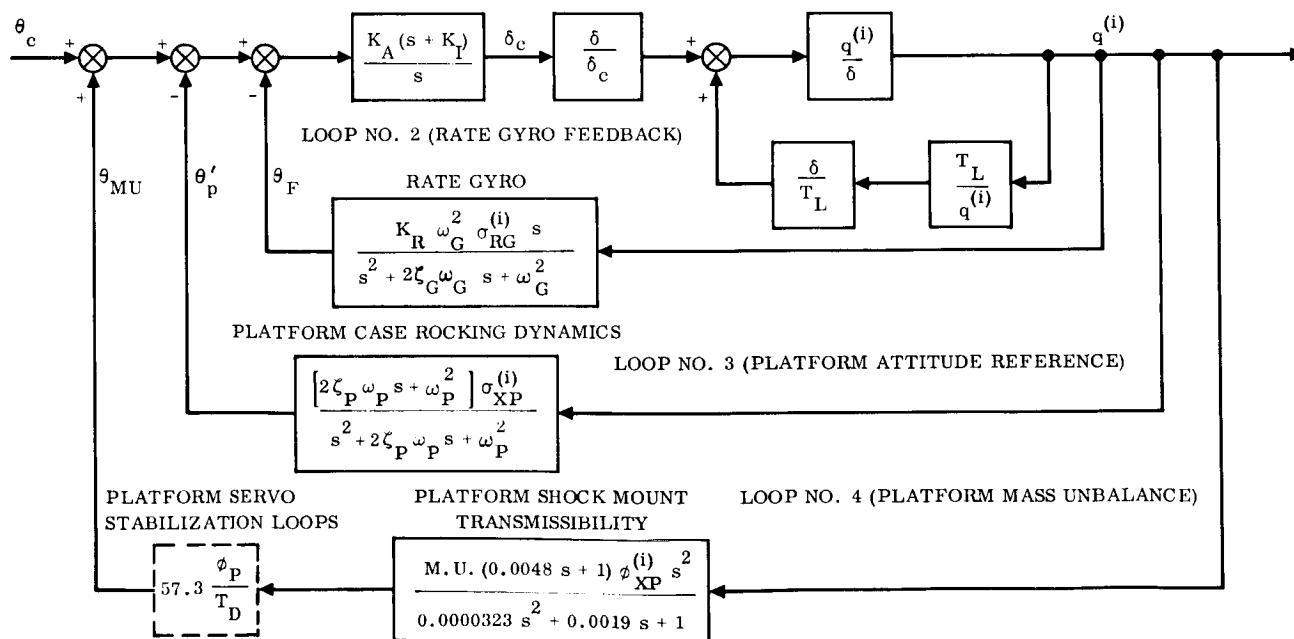


Figure B3-6. Transmissibility Limit of the Inertial Guidance Platform Mounting Isolators



a. Modified Block Diagram Shown in Figure B3-3



b. Simplified Block Diagram; No Gimbal Friction

Figure B3-7. Flexible Vehicle, Autopilot, and Inertial Platform Block Diagrams

NOTES: INERTIAL PLATFORM STABILIZATION/GIMBAL FRICTION CLOSED-LOOP TRANSFER FUNCTION:

1. FOR $K_3 = 7500$ IN.-OZ./RAD

$$\frac{\theta_0}{\theta_2} = \frac{(s + 166.7)(s + 3508.7)(s + 12.476)(s + 60.285 \pm j37.105)(s + 61.76 \pm j173.06)(s + 1476.7 \pm j2072.7)(s - 802.4)}{(s - 166.7)(s + 3508.7)(s - 12.476)(s + 60.285 \pm j37.105)(s + 61.76 \pm j173.06)(s - 1476.7 \pm j2072.7)(s + 802.4) - \frac{f_m}{j} [s(s + 171.82)(s + 166.7)^2 (s + 6.09)(s + 714.3)(s + 3508.8)(s + 1476.7 \pm j2069.3)]}$$

2. FOR $K_3 = 7500$ IN.-OZ./RAD AND $f_m = 0.066$ IN.-OZ./RAD/SEC

$$\frac{\theta_0}{\theta_2} = \frac{(s + 166.7)(s + 3508.7)(s + 12.476)(s + 60.285 \pm j37.105)(s + 61.76 \pm j173.06)(s - 1476.7 \pm j2072.7)(s - 802.4)}{(s - 166.7)(s + 3508.7)(s + 12.476)(s + 60.285 \pm j37.097)(s + 61.77 \pm j173.07)(s + 1476.67 \pm j2072.7)(s - 802.4)} = 1.0$$

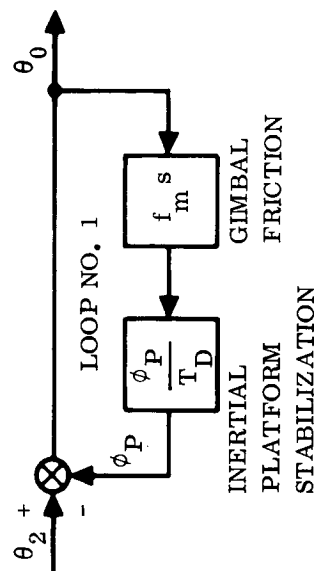
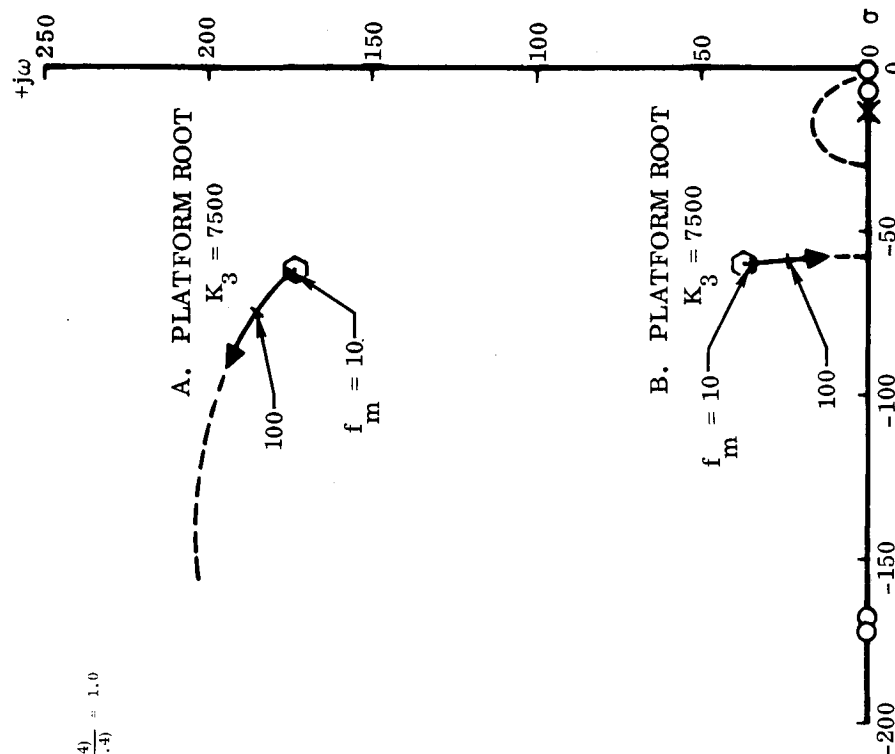


Figure B3-8. Stability of the Platform Gimbal Friction Loop

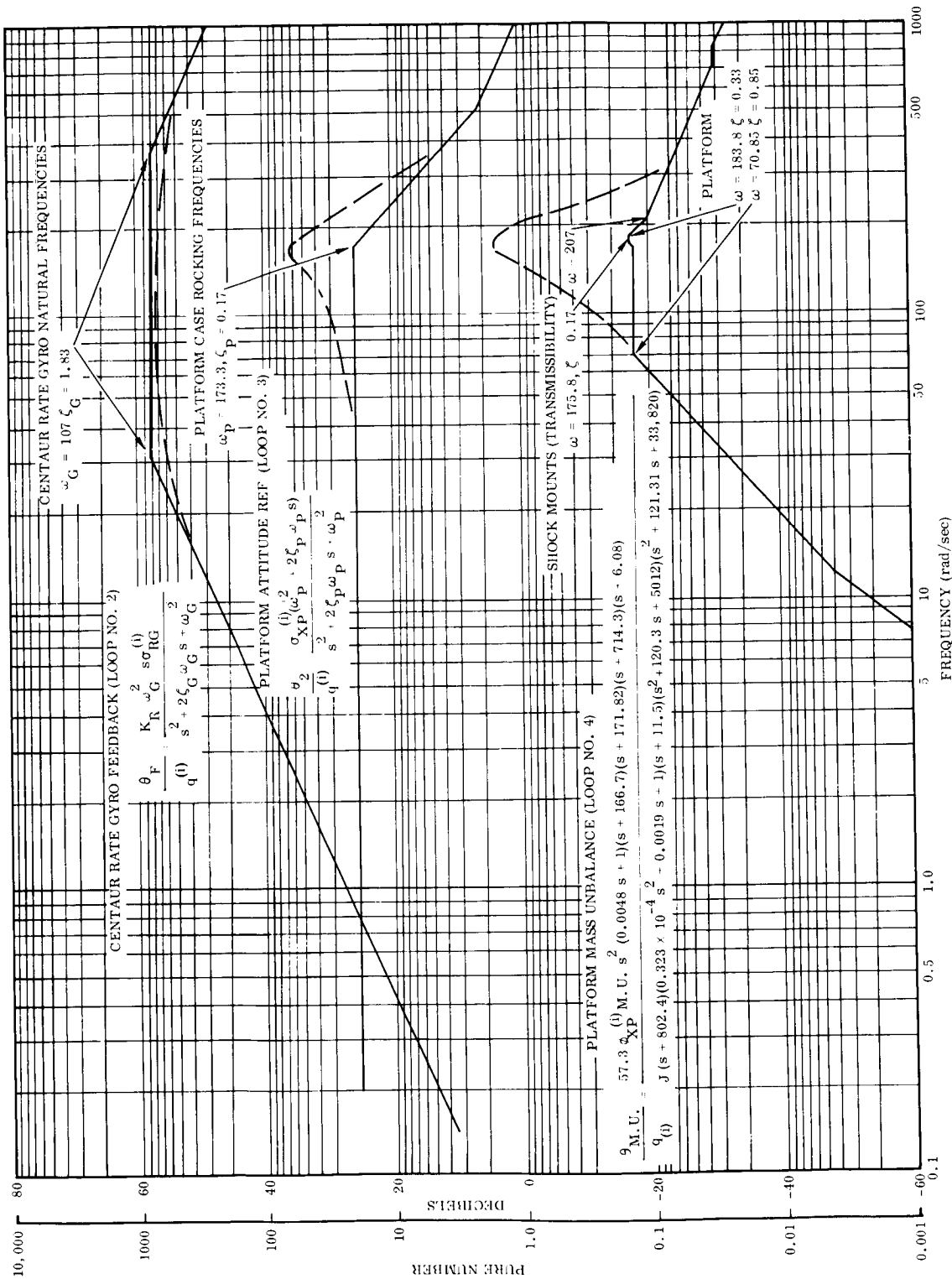


Figure B3-9. Bode Plot Comparison of the Rate, Position, and Mass Unbalance Feedback Signals for the Centaur Vehicle

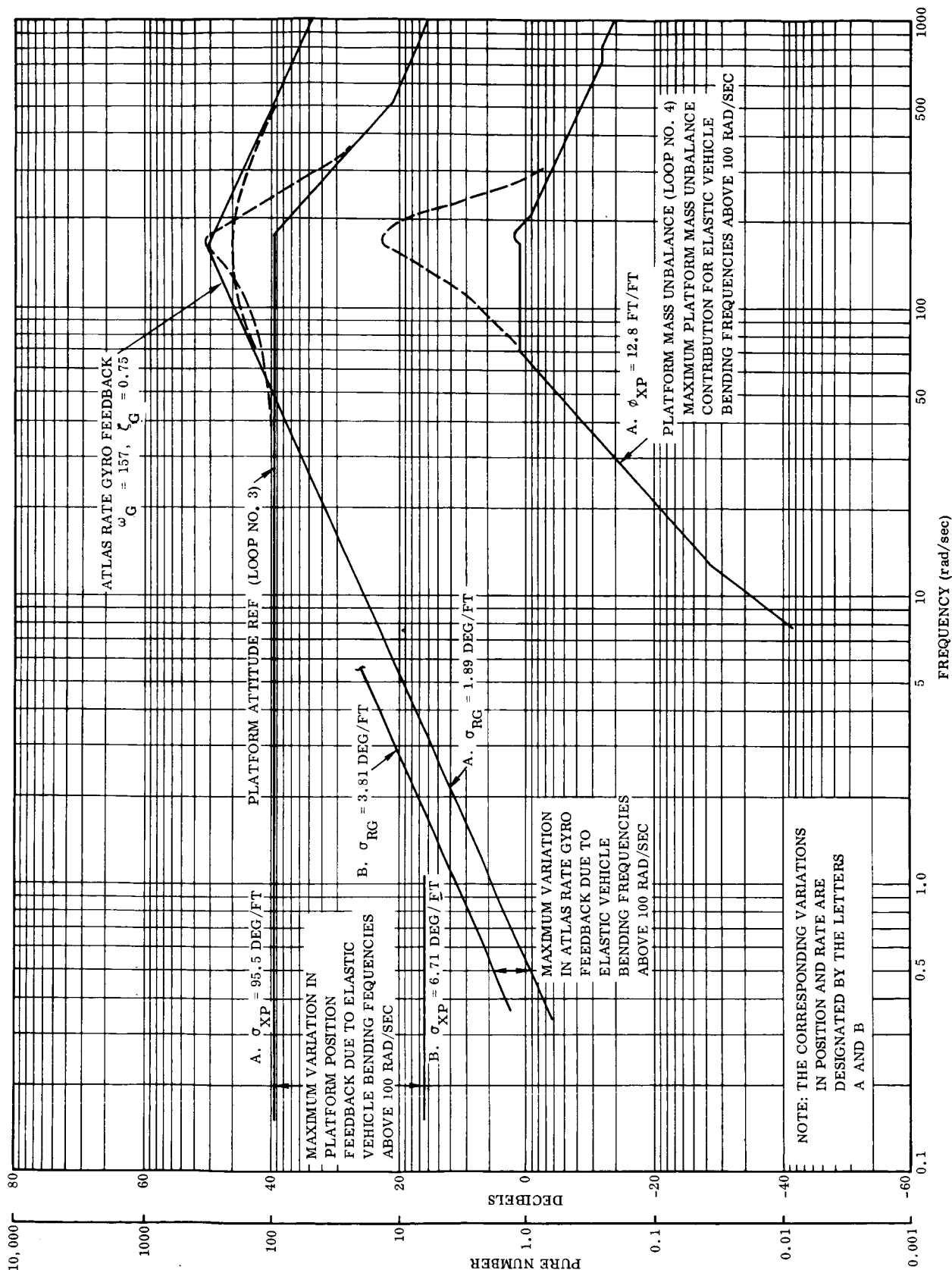


Figure B3-10. Bode Plot Comparison of the Rate, Position, and Mass Unbalance Feedback Signals for the Atlas Sustainer Phase of Flight

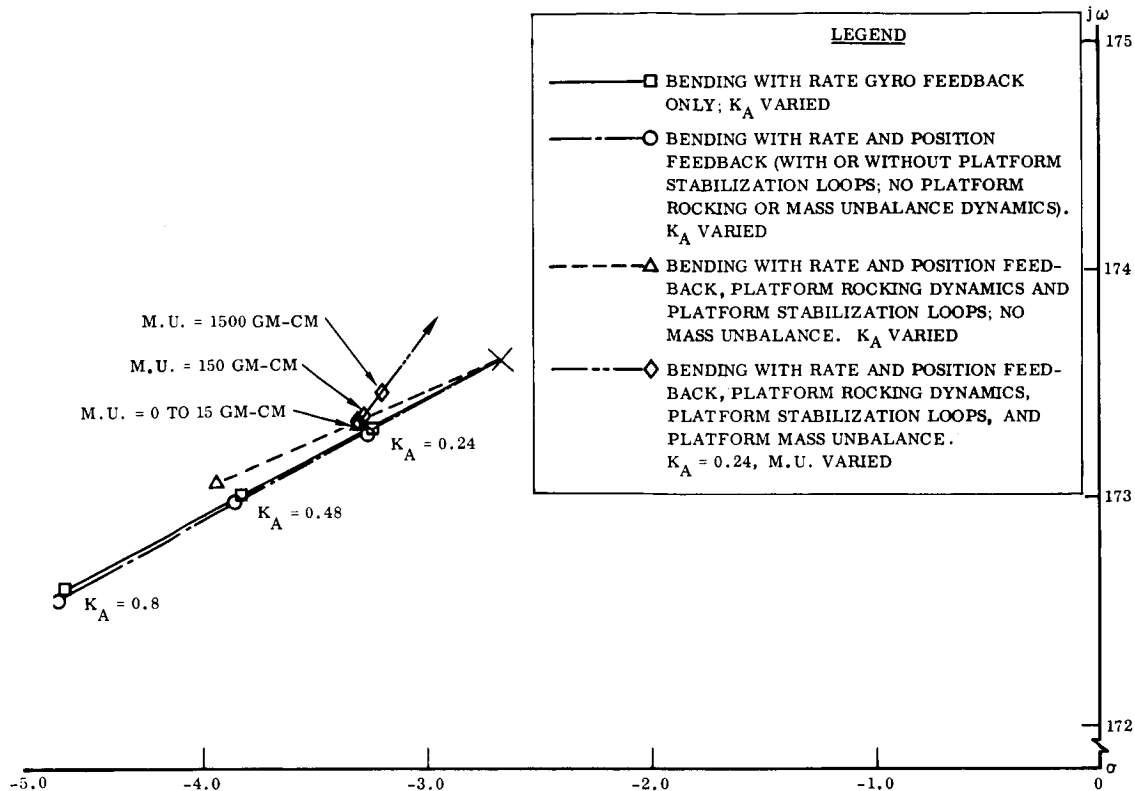


Figure B3-11. Centaur Elastic Bending Mode Stability ($\zeta = 0.001$)

B3.4.2.2.2 Atlas Sustainer Phase. There are two basic differences regarding flexible vehicle stability during the Atlas sustainer and Centaur phases of flight.

- The slenderness ratio of the Atlas/Centaur vehicle is much larger than that of the Centaur vehicle alone; hence, it is much more flexible. This results in increased elastic vehicle deflections at the inertial platform (Station 173).
- The Atlas rate gyro is located at Station 600 where elastic vehicle motion is far smaller than that sensed by the Centaur rate gyro located at Station 173.

Figure B3-10 shows that the larger bending slopes at the inertial platform, relative to the smaller slopes at the Atlas rate gyro, can result in rate and position signals of approximately the same order of magnitude for excitation frequencies between 70 and 250 rad/sec. However, the increase in the response of the platform and its effect on the position feedback signal will not cause any elastic vehicle stability problems. The high frequency characteristics of the autopilot stabilization filter and the engine servactuators greatly attenuate autopilot signals at bending frequencies in such a way that the gain variation shown in Figure B3-10 will not affect the vehicle stability characteristics (gain margins) appreciably (Reference B-15). The closed-loop elastic vehicle bending stability is illustrated by the following example.

Assume an elastic structural mode at 173 rad/sec with the minimum modal slope at the rate gyro location ($\sigma_{RG} = 1.89$ deg/ft) and the maximum slope ($\sigma_{PG} = 95.5$ deg/ft) at the inertial platform, as illustrated in Figure B3-10. This figure shows that the ratio of the rate to the position signals at the assumed modal frequency is approximately 2:1 under ideal conditions, i.e., no platform dynamics, and approximately 2:3 including the platform dynamics. The resultant closed-loop bending root for these two conditions is illustrated in Figure B3-11. Vector $\bar{A}$ represents the bending root locus assuming rate feedback only. Circle B represents the possible locations of the closed-loop bending root, including ideal position feedback,

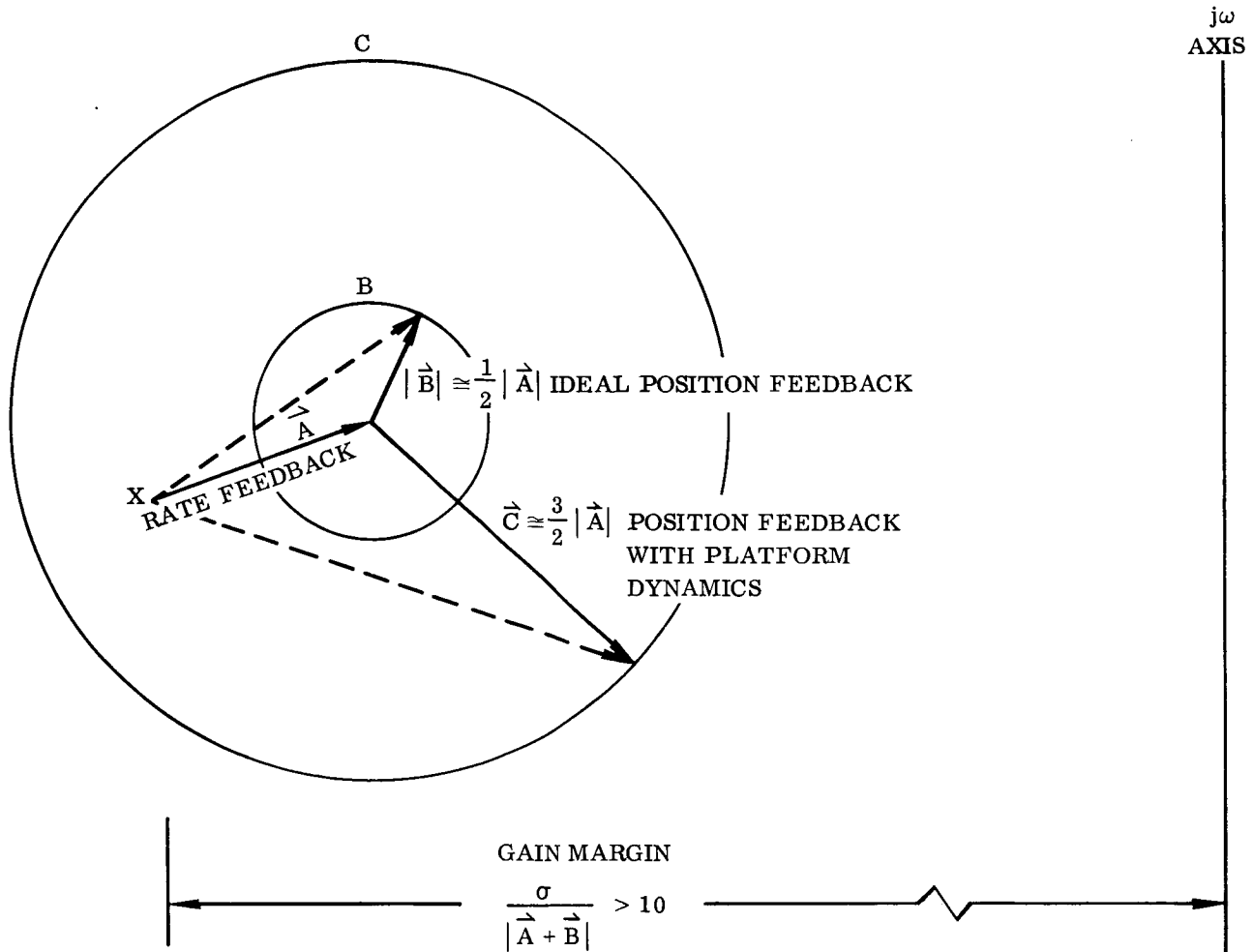


Figure B3-12. Comparison of the Effect of Position Feedback with and without Platform Dynamics on Atlas/Sustainer Bending Stability

and circle C the possible locations of the closed-loop root, including the platform dynamics. The exact location of these roots will depend on the phase of each signal relative to the rate gyro signal. However, phase is of little interest since a gain margin of greater than 10:1 exists at these frequencies (Reference B-15).

This simple example shows that the assumption of a position reference independent of frequency, while somewhat erroneous, does not materially affect the predicted elastic stability properties of the Atlas sustainer phase of flight.

B3.4.3 Platform Analysis Summary. The preceding analysis shows that no coupling of any significance exists between the Centaur autopilot/vehicle dynamics and inertial guidance platform. However, some platform/vehicle coupling appears to be possible during the Atlas sustainer phase flight. Fortunately, these high frequency characteristics are sharply attenuated by the autopilot filter and the engine servo-actuator response characteristics. It is therefore concluded that the high frequency vehicle bending stability is unaffected by the platform dynamics.

In view of the strong frequency attenuation of the flight control system, a transfer function independent of frequency between the actual vehicle attitude and the attitude reference, as sensed by the platform resolver, is adequate for the analysis of both the low and high frequency vehicle dynamics.

SECTION B4

GUIDANCE/AUTOPILOT
STABILITY EQUATIONS

LIST OF SYMBOLS FOR SECTION B4

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
$\bar{a}_m$	Total vehicle acceleration	ft/sec ²
$\bar{a}_T$	Vehicle thrust acceleration	ft/sec ²
$\bar{F}$	Resultant external force acting on vehicle, exclusive of gravity	lb
$\bar{f}$	Modified velocity to be gained	ft/sec
$\bar{f}^*$	Compensated $\bar{f}$ vector (see equations B4-6 and B4-7)	N. D.
G_R	Gravitational compensation used in $\bar{f}$ computation	ft/sec
$\bar{g}$	Acceleration due to gravity	ft/sec ²
K_G	Equivalent guidance feedback gain	deg-sec ⁻¹ /deg
k'_I	Guidance integral gain	sec ⁻¹
$\bar{l}_p$	Vector defining location of guidance accelerometers relative to the c.g. of the vehicle	ft
m	Instantaneous mass of vehicle	slugs
$\bar{r}_m$	Total vehicle displacement or position vector	ft
s	Laplace transform variable	sec ⁻¹
$\bar{V}_g$	Velocity to be gained	ft/sec
$\bar{V}_m$	Total vehicle velocity	ft/sec
$\bar{V}_r$	Required vehicle velocity for thrust cutoff	ft/sec
z	z -transform variable	N. D.
Δt_n	n^{th} compute cycle length	sec
$\delta\phi, \delta\theta, \delta\psi$	Roll, pitch and yaw angular perturbations of vehicle body axes	rad
$\bar{\omega}$	Vehicle turning rate; also used for angular velocity of reference axes with respect to inertial axes	rad/sec
$\bar{\omega}_c$	Equivalent commanded vehicle turning rate	deg/sec
$\bar{i}_x, \bar{i}_y, \bar{i}_z$	Body fixed unit vectors along the roll, pitch, and yaw axes respectively	N. D.
$\bar{i}_r, \bar{i}_n, \bar{i}_t$	Unit vectors in the radial, normal, and tangential directions respectively	N. D.

LIST OF SYMBOLS FOR SECTION B4, Contd

Symbol	Definition	Units
<u>Subscripts</u>		
x, y, z	Used to denote components along roll, pitch, and yaw axes	
r, n, t	Used to denote components in radial, normal, and tangential directions	
i, n	Used to denote sampling times	
<u>Mathematical Operators</u>		
d/dt	Used to denote differentiation with respect to time in inertial coordinates	
• (dot)	Used to denote differentiation with respect to time in rotating reference frame	
δ	Used to denote perturbations as "seen" by an inertial observer	
δ_B	Used to denote perturbations as "seen" by the body axis observer	

SECTION B4

GUIDANCE/AUTOPILOT STABILITY EQUATIONS

B4.1 INTRODUCTION. Attitude reference vector steering is based on the "velocity to be gained," $(\bar{V}_g)$ concept. That is, one calculates the difference between the current "required velocity," $(\bar{V}_r)$ and the current "actual velocity," $(\bar{V}_m)$ and then attempts to null this error by adding acceleration along the direction of this velocity error. The error vector or "velocity to be gained" is

$$\bar{V}_g = \bar{V}_r - \bar{V}_m \quad (B4-1)$$

Steering to the vector $\bar{V}_g$ itself is obviously undesirable since it would result in unnecessary maneuvering even while flying on a nominal trajectory (open loop). Thus one defines a "modified velocity to be gained" as

$$\bar{f} = \bar{V}_r - \bar{V}_m + \bar{I}_r G_r, \quad (B4-2)$$

where the function G_r is chosen so that the unnecessary maneuvering on a nominal trajectory is eliminated. A unit vector in the direction of $\bar{f}$ is defined by

$$\bar{I}_f = \frac{\bar{f}}{f}. \quad (B4-3)$$

A unit vector in the direction of the thrust acceleration is

$$\bar{I}_{a_T} = \frac{\bar{a}_T}{a_T}. \quad (B4-4)$$

The guidance system could now issue a commanded turning rate as $\bar{\omega}_c = K_G \bar{I}_{a_T} \times \bar{I}_f$ to the vehicle control system. However, this procedure is undesirable when one considers the "noise" involved. The calculation of $\bar{I}_{a_T}$ requires a numerical differentiation at an already noisy velocity. An alternate procedure, and the one followed for Centaur guidance, is to use the vehicle roll axis as a close approximation to the direction of $\bar{I}_{a_T}$. The error which would result (due to c.g. offset and other misalignments) is compensated for by adding a corrective term, $\Delta \bar{f}$, to the unit vector $\bar{I}_f$. This corrective compensation, $\Delta \bar{f}$, is so constructed that when the vehicle roll axis is coincident with $\bar{I}_f + \Delta \bar{f}$, the thrust acceleration vector will be coincident with the "unitized modified velocity to be gained vector," $\bar{I}_f$. It can be shown that the vector

$$\Delta \bar{f} = k'_I \int_{t_0}^t (\bar{I}_f - \bar{I}_{a_T}) dt \quad (B4-5)$$

will fulfill the above requirement.

B4.2 COMMANDED TURNING RATE. The commanded turning rate is given by:

$$\bar{\omega}_c = K_G \bar{I}_x \times \bar{f}^* \quad (B4-6)$$

or

$$\bar{\omega}_c = K_G \bar{I}_x \times \left[\bar{I}_f + k'_I \int_{t_0}^t (\bar{I}_f - \bar{I}_{a_T}) dt \right] \quad (B4-7)$$

The inertial platform's resolver chain is used to affect this vector or cross, ($\times$), product; $\bar{f}^*$ is computed in the inertial (platform) coordinates and the three voltages representing its components are applied to the inputs of the resolver chain. The roll axis output voltage is grounded, the sign of the yaw axis output voltage is reversed, and the pitch and yaw axis output voltages are used as commanded rates to the yaw and pitch channels of the autopilot respectively.

Since the navigational computer is a digital machine, the $\bar{f}^*$ vector is computed only once every compute cycle. Thus, $\bar{f}^*(t)$ is a discrete function of time and will be represented by the equation

$$\bar{f}^*(t) = \bar{f}_f(t_i) + k_I' \sum_{n=k}^i \left[\bar{f}_f(t_n) - \bar{f}_{a_T}(t_n) \right] \Delta t_n, \quad (B4-8)$$

where it is understood that t is in the interval $(t_i + T_D, t_{i+1} + T_D)$ where T_D is a finite time delay or transport lag. That is, $\bar{f}^*(t)$ is constant throughout the $i+1^{\text{st}}$ compute cycle and its value was determined by the instantaneous value of parameters which occurred at the instant, t_i , during the i^{th} compute cycle. The commanded vehicle turning rate as sensed by the control system becomes

$$\bar{\omega}_c(t) = K_G \bar{f}_x(t) \times \left\{ \bar{f}_f(t_i) + k_I' \sum_{n=k}^i \left[\bar{f}_f(t_n) - \bar{f}_{a_T}(t_n) \right] \Delta t_n \right\} \quad (B4-9)$$

The discrete variables such as $\bar{f}_f(t_i)$ will now be denoted by $\bar{f}_f$. Also, continuous variables such as $\bar{f}_x(t)$ will be denoted simply by $\bar{f}_x$, the time dependence being understood. Equation B4-9 is now written as

$$\bar{\omega}_c = K_G \bar{f}_x \times \left[\bar{f}_f + k_I' \sum_{n=k}^i \left(\bar{f}_f - \bar{f}_{a_T} \right) \Delta t_n \right] \quad (B4-10)$$

where $t_i + T_D \leq t < t_{i+1} + T_D$.

The analysis which follows is concerned only with the perturbations in $\bar{\omega}_c$ as measured relative to a nominal or reference trajectory. It is assumed that the unit vectors $\bar{f}_x$, $\bar{f}_f$, and $\bar{f}_{a_T}$ are all approximately coincident for the reference trajectory. The perturbation in $\bar{\omega}_c$ is denoted by $\delta \bar{\omega}_c$. Perturbations in the remaining variables are denoted similarly. From Equation B4-10, one obtains the perturbation in $\bar{\omega}_c$ as,*

$$\begin{aligned} \delta \bar{\omega}_c = K_G & \left\{ \delta \bar{f}_x \times \left[\bar{f}_f + k_I' \sum_{n=k}^i \left(\bar{f}_f - \bar{f}_{a_T} \right) \Delta t_n \right] \right. \\ & \left. + \bar{f}_x \times \left[\delta \bar{f}_f + k_I' \sum_{n=k}^i \left(\delta \bar{f}_f - \delta \bar{f}_{a_T} \right) \Delta t_n \right] \right\} \quad (B4-11) \end{aligned}$$

* The technique used throughout this appendix to derive perturbational equations (linear terms of a Taylor's series expansion) is as follows. Consider the equation $y = f(x_1, x_2, \dots, x_n)$. The perturbation in y is defined by:

$$\delta y = \sum_{i=1}^n \left[\frac{\partial f}{\partial x_i} \right]_{\text{Nom}} \delta x_i$$

where $\left[\partial f / \partial x_i \right]_{\text{Nom}}$ is, in general, a variable coefficient which is determined by evaluating $\partial f(x_1, x_2, \dots, x_k, \dots, x_n) / \partial x_i$ using nominal or reference values for the x_k . The subscript, "Nom," is not shown explicitly, its existence always being implied.

Before Equation B4-11 can be used in stability analyses, the perturbations $\delta \bar{l}_x$, $\delta \bar{l}_f$ and $\delta \bar{l}_{aT}$ must be expressed in terms of the vehicle attitude, acceleration, velocity and displacement. Since the only concern here is with small rotations relative to the nominal orientation of the body axes, the perturbation in $\bar{l}_x$ can be represented by

$$\delta \bar{l}_x = \delta \bar{\Lambda} \times \bar{l}_x \quad (B4-12)$$

where $\delta \bar{\Lambda}$ is a vector defining the angular perturbations in roll, pitch, and yaw. Written in full,

$$\delta \bar{\Lambda} = \bar{l}_x \delta \phi + \bar{l}_y \delta \theta + \bar{l}_z \delta \psi. \quad (B4-13)$$

The perturbation in $\bar{l}_{f_i}$ is determined by operating on Equation B4-3.

$$\begin{aligned} \delta \bar{l}_{f_i} &= \frac{\delta \bar{f}_i}{f_i} - \frac{f_i}{f_i^2} \delta f_i \\ &= \frac{1}{f_i} (\delta \bar{f}_i - \bar{l}_{f_i} \delta f_i). \end{aligned} \quad (B4-14)$$

The perturbation in the magnitude of f_i can be expressed in terms of δf_i as follows,

$$\begin{aligned} f_i^2 &= \bar{f}_i \cdot \bar{f}_i \\ 2 f_i \delta f_i &= 2 \bar{f}_i \cdot \delta \bar{f}_i \\ \delta f_i &= \frac{\bar{f}_i}{f_i} \cdot \delta \bar{f}_i \\ \delta f_i &= \bar{l}_{f_i} \cdot \delta \bar{f}_i \end{aligned} \quad (B4-15)$$

Substituting this result back into Equation B4-14, one obtains

$$\delta \bar{l}_{f_i} = \frac{1}{f_i} \left[\delta \bar{f}_i - (\bar{l}_{f_i} \cdot \delta \bar{f}_i) \bar{l}_{f_i} \right]. \quad (B4-16)$$

Similarly, the perturbation in $\bar{l}_{aT_i}$ is given by

$$\delta \bar{l}_{aT_i} = \frac{1}{a_{T_i}} \left[\delta \bar{a}_{T_i} - (\bar{l}_{aT_i} \cdot \delta \bar{a}_{T_i}) \bar{l}_{aT_i} \right]. \quad (B4-17)$$

Now, substituting the expressions for $\delta \bar{l}_x$, $\delta \bar{l}_f$, and $\delta \bar{l}_{a_{T_1}}$ from Equations B4-12, B4-16, and B4-17 into Equation B4-11, one obtains the following equation for the perturbation in commanded turning rate.

$$\frac{\delta \bar{\omega}_c}{K_G} = (\delta \bar{\Lambda} \times \bar{l}_x) \times \bar{l}_{f_i} + \bar{l}_x \times \left\{ \frac{1}{\bar{f}_i} [\delta \bar{f}_i - (\delta \bar{f}_i \cdot \bar{l}_{f_i}) \bar{l}_{f_i}] \right. \\ \left. + k_I' \sum_{n=k}^i \left(\frac{1}{\bar{f}_n} [\delta \bar{f}_n - (\delta \bar{f}_n \cdot \bar{l}_{f_n}) \bar{l}_{f_n}] - \frac{1}{\bar{a}_{T_n}} [\delta \bar{a}_{T_n} - (\delta \bar{a}_{T_n} \cdot \bar{l}_{a_{T_n}}) \bar{l}_{a_{T_n}}] \right) \Delta t_n \right\} \quad (B4-18)$$

where the term

$$\delta \bar{l}_x \times \sum_{n=k}^i \left(\bar{l}_{f_n} - \bar{l}_{a_{T_n}} \right) \Delta t_n$$

has been neglected since its y and z components are essentially zero and the x component is not needed since it is ultimately dropped.

As was mentioned previously, the unit vectors $\bar{l}_x$, $\bar{l}_f$, and $\bar{l}_{a_T}$ are all approximately coincident in a "nominal" trajectory. Thus, Equation B4-18 may alternately be written as

$$\frac{\delta \bar{\omega}_c}{K_G} = (\delta \bar{\Lambda} \times \bar{l}_x) \times \bar{l}_{x_i} + \bar{l}_x \times \left\{ \frac{1}{\bar{f}_i} [\delta \bar{f}_i - (\delta \bar{f}_i \cdot \bar{l}_{x_i}) \bar{l}_{x_i}] \right. \\ \left. + k_I' \sum_{n=k}^i \left(\frac{1}{\bar{f}_n} [\delta \bar{f}_n - (\delta \bar{f}_n \cdot \bar{l}_{x_n}) \bar{l}_{x_n}] - \frac{1}{\bar{a}_{T_n}} [\delta \bar{a}_{T_n} - (\delta \bar{a}_{T_n} \cdot \bar{l}_{x_n}) \bar{l}_{x_n}] \right) \Delta t_n \right\} \quad (B4-19)$$

B4.3 EQUATIONS OF MOTION (PERTURBATIONAL FORM). Referring to Figure B4-1, the perturbation in vehicle velocity as seen by the inertial observer is

$$\delta \bar{V}_m = \frac{d}{dt} (\delta \bar{r}_m) \quad (B4-20)$$

It should be noted that the perturbations in $\bar{f}$ and $\bar{a}_T$ represent the changes in these variables as seen by an inertial observer. Note also that the digital integration is actually performed in inertial coordinates. Due to the relative ease of expressing $\delta \bar{a}_T$ in terms of its components in rotating reference axes, it is also relatively easy to calculate the perturbations in vehicle velocity and displacement as seen by the reference axis observer. Thus, the velocity perturbation as seen by the reference axis observer must be corrected in order to obtain the velocity perturbation as seen by the inertial observer. In general, the only perturbation that is independent of the observer's reference frame is the perturbation in displacement, $\delta \bar{r}_m$.

where the operator $\frac{d}{dt}$ is reserved for differentiation in the u, v, w or inertial reference frame. Expressed in terms of the rate of change with respect to time as seen by the rotating reference axis observer,

$$\delta \bar{V}_m = \delta \dot{\bar{r}}_m + \bar{\omega} \times \delta \bar{r}_m \quad (B4-21)$$

where the dot (·) notation is reserved for differentiation with respect to time as seen by the rotating reference axis observer. Similarly, the perturbation in vehicle acceleration as seen by the inertial observer can be expressed as

$$\delta \bar{a}_m = \delta \ddot{\bar{r}}_m + 2\bar{\omega} \times \delta \dot{\bar{r}}_m + \left[\bar{\omega} \times \delta \bar{r}_m + \bar{\omega} \times (\bar{\omega} \times \delta \bar{r}_m) \right] \quad (B4-22)$$

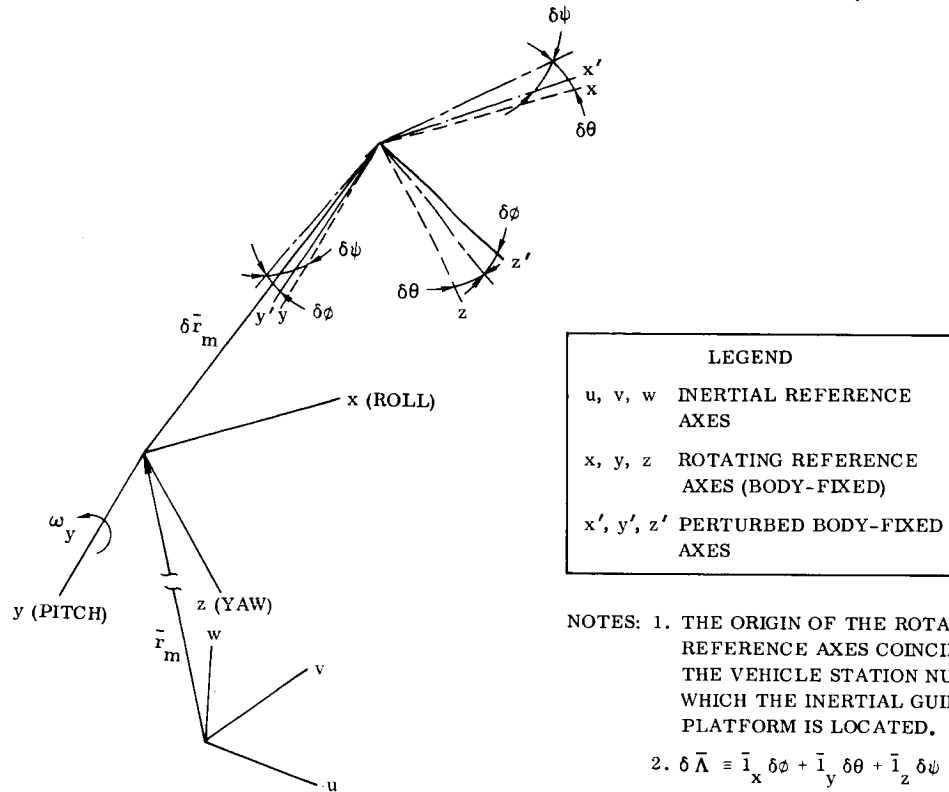


Figure B4-1. Various Frames of Reference Showing Geometry of Perturbations in Vehicle Displacements

The terms within the dashed box of Equation B4-22 will be extremely small since the nominal angular acceleration of the vehicle, $\dot{\bar{\omega}}$, is practically zero and the nominal turning rate is only on the order of 0.1 deg/sec. Retaining only the Coriolis term as the dominant effect of the rotating reference frame,

$$\delta \bar{a}_m = \delta \ddot{r}_m + 2\bar{\omega} \times \delta \dot{r}_m. \quad (\text{B4-23})$$

Expanding Equation B4-23 in terms of reference frame components,

$$\left. \begin{aligned} \delta \ddot{r}_{mx} + 2\omega_y \delta \dot{r}_{mz} - 2\omega_z \delta \dot{r}_{my} &= \delta a_{mx} \\ \delta \ddot{r}_{my} - 2\omega_x \delta \dot{r}_{mz} + 2\omega_z \delta \dot{r}_{mx} &= \delta a_{my} \\ \delta \ddot{r}_{mz} + 2\omega_x \delta \dot{r}_{my} - 2\omega_y \delta \dot{r}_{mx} &= \delta a_{mz} \end{aligned} \right\}. \quad (\text{B4-24})$$

Now, the nominal yaw and roll rates, ω_z and ω_x , are zero. Also, it will be seen later that the perturbation in vehicle acceleration in the direction of the nominal flight path, δa_{mx} , is zero. Therefore Equation B4-24 now takes the simpler form

$$\left. \begin{aligned} \delta \ddot{r}_{mx} + 2\omega_y \delta \dot{r}_{mz} &= 0 \\ \delta \ddot{r}_{my} &= \delta a_{my} \\ \delta \ddot{r}_{mz} - 2\omega_y \delta \dot{r}_{mx} &= \delta a_{mz} \end{aligned} \right\}. \quad (\text{B4-25})$$

Assuming the nominal pitch rate ω_y to be constant, the Laplace transform of Equation B4-25 is

$$\left. \begin{aligned} \delta \ddot{r}_{mx}(s) + \frac{2\omega_y}{s} \delta \ddot{r}_{mz}(s) &= 0 \\ \delta \ddot{r}_{my}(s) &= \delta a_{my}(s) \\ \delta \ddot{r}_{mz}(s) - \frac{2\omega_y}{s} \delta \ddot{r}_{mx}(s) &= \delta a_{mz}(s) \end{aligned} \right\} \cdot \quad (B4-26)$$

Solving Equation B4-26 simultaneously, the solution in the complex frequency domain is given by

$$\left. \begin{aligned} \delta \ddot{r}_{mx}(s) &= -\frac{2\omega_y}{s} \frac{\delta a_{mz}(s)}{1 + \left(\frac{2\omega_y}{s}\right)^2} \\ \delta \ddot{r}_{my}(s) &= \delta a_{my}(s) \\ \delta \ddot{r}_{mz}(s) &= \frac{\delta a_{mz}(s)}{1 + \left(\frac{2\omega_y}{s}\right)^2} \end{aligned} \right\} \cdot \quad (B4-27)$$

A typical value for the nominal pitch rate is about -0.2 deg/sec. The minimum short period frequency with which we will be concerned is the "guidance mode" frequency at approximately 0.2 rad/sec. Thus, considering only the short perturbations, the term $\left(\frac{2\omega_y}{s}\right)^2$ is negligible in comparison with unity. The following approximate solution will therefore be adequate.

$$\left. \begin{aligned} \delta \ddot{r}_{mx}(s) &= -2\omega_y \frac{\delta a_{mz}(s)}{s} \\ \delta \ddot{r}_{my}(s) &= \delta a_{my}(s) \\ \delta \ddot{r}_{mz}(s) &= \delta a_{mz}(s) \end{aligned} \right\} \cdot \quad (B4-28)$$

The Laplace transforms of the perturbations in vehicle displacement are

$$\left. \begin{aligned} \delta r_{mx}(s) &= -2\omega_y \frac{\delta a_{mz}(s)}{s^3} \\ \delta r_{my}(s) &= \frac{\delta a_{my}(s)}{s^2} \\ \delta r_{mz}(s) &= \frac{\delta a_{mz}(s)}{s^2} \end{aligned} \right\} \cdot \quad (B4-29)$$

The Laplace transforms of the perturbations in vehicle velocity are obtained by substituting the results given in Equation B4-29 back into Equation B4-21. The result, in terms of reference axis components, is

$$\left. \begin{aligned} \delta V_{mx}(s) &= -\omega_y \frac{\delta a_{mz}(s)}{s^2} \\ \delta V_{my}(s) &= \frac{\delta a_{my}(s)}{s} \\ \delta V_{mz}(s) &= \left(1 + \frac{2\omega_y^2}{s^2}\right) \frac{\delta a_{mz}(s)}{s} \end{aligned} \right\} \quad (B4-30)$$

For the same reasons presented above, the term $\frac{2\omega_y^2}{s^2}$ is negligible and will henceforth be dropped.

The perturbation in vehicle acceleration, $\delta \bar{a}_m$, must be expressed in terms of reference axis components in order to utilize Equations B4-29 and B4-30. For small perturbations, one may write

$$\delta \bar{a}_m = \delta_B \bar{a}_m + \delta \bar{\Lambda} \times \bar{a}_m \quad (B4-31)$$

where $\delta_B \bar{a}_m = \delta_B (\bar{a}_T + \bar{g})$ is the perturbation in vehicle acceleration as seen by the observer, whose frame of reference is defined by the instantaneous position (including angular perturbations) of the vehicle body axes, i.e., pitch, yaw, and roll. $\delta \bar{\Lambda}$ is a small rotation vector whose components are the perturbations in roll, pitch, and yaw respectively. That is,

$$\delta \bar{\Lambda} = \delta \phi \bar{1}_x + \delta \theta \bar{1}_y + \delta \psi \bar{1}_z \quad (B4-32)$$

As seen from the body axis reference frame,

$$\begin{aligned} \delta_B \bar{a}_m &= \delta_B (\bar{a}_T + \bar{g}) \\ &= \frac{\delta_B \bar{F}}{m} + \delta \dot{\omega} \times \bar{l}_p - \delta \bar{\Lambda} \times \bar{g} \end{aligned} \quad (B4-33)$$

where $\delta_B \bar{F}$ is the perturbation in all externally applied forces (explicitly excluding gravitation) as seen by the body axis observer. Substituting Equation B4-33 into the right side of Equation B4-31, one obtains

$$\delta \bar{a}_m = \frac{\delta_B \bar{F}}{m} + \delta \dot{\omega} \times \bar{l}_p + \delta \bar{\Lambda} \times \bar{a}_T \quad (B4-34)$$

Finally, Equation B4-34 written in component form becomes

$$\left. \begin{aligned} \delta a_{mx} &= (\ell_p)_z \delta \ddot{\theta} - (\ell_p)_y \delta \ddot{\psi} \\ \delta a_{my} &= \frac{\delta_B F_y}{m} + (\ell_p)_x \delta \ddot{\psi} + a_T \delta \psi - (\ell_p)_z \delta \ddot{\phi} \\ \delta a_{mz} &= \frac{\delta_B F_z}{m} - (\ell_p)_x \delta \ddot{\theta} - a_T \delta \theta + (\ell_p)_y \delta \ddot{\phi} \end{aligned} \right\} \quad (B4-35)$$

The perturbations in vehicle attitude, attitude rate, and attitude acceleration are determined by applying the principle of angular momentum. The mathematical statement of this principle is

$$\frac{d}{dt}(\bar{h}) = \bar{L} \quad (B4-36)$$

where $\bar{h}$ is the angular momentum of the body with respect to its center of mass and $\bar{L}$ is the externally applied torque (with respect to the center of mass). The angular momentum vector can be expressed as

$$\bar{h} = \mathcal{J} \cdot \bar{\omega} \quad (B4-37)$$

where $\mathcal{J}$ is the momental (inertial) dyad defined by

$$\mathcal{J} \triangleq \begin{bmatrix} \bar{1}_x \bar{1}_x I_{xx} & -\bar{1}_x \bar{1}_y I_{xy} & -\bar{1}_x \bar{1}_z I_{xz} \\ -\bar{1}_y \bar{1}_x I_{xy} & \bar{1}_y \bar{1}_y I_{yy} & -\bar{1}_y \bar{1}_z I_{yz} \\ -\bar{1}_z \bar{1}_x I_{xz} & -\bar{1}_z \bar{1}_y I_{yz} & \bar{1}_z \bar{1}_z I_{zz} \end{bmatrix} \quad (B4-38)$$

Expressing $\frac{d}{dt}(\bar{h})$ in terms of body axis derivatives, one has

$$\begin{aligned} \frac{d}{dt}(\bar{h}) &= \dot{\bar{h}} + \bar{\omega} \times \bar{h} \\ &= \mathcal{J} \cdot \dot{\bar{\omega}} + \mathcal{J} \cdot \bar{\omega} + \bar{\omega} \times (\mathcal{J} \cdot \bar{\omega}) \end{aligned} \quad (B4-39)$$

Substituting the right side of Equation B4-39 for the left side of Equation B4-36 and taking perturbations on both sides, one obtains the following result:

$$\mathcal{J} \cdot \delta \dot{\bar{\omega}} + \mathcal{J} \cdot \delta \bar{\omega} + \delta \bar{\omega} \times (\mathcal{J} \cdot \bar{\omega}) + \bar{\omega} \times (\mathcal{J} \cdot \delta \bar{\omega}) = \delta_B \bar{L} \quad (B4-40)$$

Here, $\delta_B \bar{L}$ is the perturbation in the externally applied torque (with respect to the mass center) as seen by the body axis observer. For most normal applications, the first term on the left side of Equation B4-40 is sufficiently dominant that the remaining three may be neglected.

B4.4 MODIFIED VELOCITY TO BE GAINED (PERTURBATIONAL FORM). The general form of the "modified velocity to be gained" vector was given in Equation B4-2. The particular form of this equation chosen for the AC-4 mission is (Reference B-16)

$$\bar{f} = V_{rt} \bar{1}_t + (V_{rr} + G_r) \bar{1}_r + V_{rn} \bar{1}_n - \bar{V}_m \quad (B4-41)$$

A triad of mutually orthogonal unit vectors $\bar{1}_r, \bar{1}_n, \bar{1}_t$ is defined as follows:

$$\left. \begin{aligned} \bar{1}_r &= \frac{\bar{r}_m}{|\bar{r}_m|} \\ \bar{1}_n &= \frac{\bar{1}_r \times \bar{V}_m}{|\bar{1}_r \times \bar{V}_m|} \\ \bar{1}_t &= \bar{1}_n \times \bar{1}_r \end{aligned} \right\} \quad (B4-42)$$

This coordinate system is based on the flight plane which is defined by the vehicle position and velocity vectors. This differs from that used for the AC-4 mission where the coordinates were defined using the position and target vectors defining the target plane.

V_{rr} , V_{rt} , G_R , and V_{rn} are trajectory-dependent functions calculated by the navigational computer. As presented in Reference B-16, these functions are defined as follows:

$$\left. \begin{aligned} V_{rt}^2 &= h_d^2 + \frac{2K_1}{r_m} - V_{rr}^2 \\ V_{rr} &= C_3 + C_4 r^* + C_5 m^* + C_6 m^{*2} \\ G_R &= (C_7 + \delta R) \epsilon^4 \\ V_{rn} &= C_8 \epsilon m \dot{\epsilon}_y \\ r^* &= r_m - C_2, \quad m^* = m - C_1 \quad \text{and} \quad m = \bar{l}_t \cdot \bar{l}_a \\ \epsilon &= h_d - \left(V_m^2 - \frac{2K_1}{r_m} \right) \\ \dot{\epsilon}_y &= \bar{N} \cdot \bar{l}_a \quad \text{and} \quad \bar{N} = \bar{l}_r \times \bar{V}_m \end{aligned} \right\} \quad (B4-43)$$

The perturbation in the "modified velocity to be gained" is obtained by operating on both sides of Equation B4-41. The result is,

$$\begin{aligned} \delta \bar{f} &= \delta V_{rt} \bar{l}_t + (\delta V_{rr} + \delta G_R) \bar{l}_r + \delta V_{rn} \bar{l}_n - \delta \bar{V}_m + V_{rt} \delta \bar{l}_t + (V_{rr} + G_R) \delta \bar{l}_r \\ &\quad + V_{rn} \delta \bar{l}_n \end{aligned} \quad (B4-44)$$

Using Equations B4-42 and B4-43, the perturbation components of Equation B4-44 can be written as follows:

$$\left. \begin{aligned} \delta V_{rt} &= \frac{-K_1}{V_{rt} r_m^2} \delta \bar{r}_m \cdot \bar{l}_r - \frac{V_{rr}}{V_{rt}} \delta V_{rr} \\ \delta V_{rr} &= C_4 \delta \bar{r}_m \cdot \bar{l}_r + [C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] \delta \bar{l}_t \cdot \bar{l}_a \\ \delta G_R &= \frac{-8K_1}{r_m^2} \epsilon^3 (C_7 + \delta R) \delta \bar{r}_m \cdot \bar{l}_r \\ \delta V_{rn} &= -\frac{2C_8 K_1}{r_m^2} (\bar{l}_t \cdot \bar{l}_a) (\bar{l}_n \cdot \bar{l}_a) \delta \bar{r}_m \cdot \bar{l}_r + C_8 \epsilon (\bar{V}_m \cdot \bar{l}_t) (\bar{l}_n \cdot \bar{l}_a) \delta \bar{l}_t \cdot \bar{l}_a \\ &\quad + C_8 \epsilon (\bar{l}_t \cdot \bar{l}_a) (\bar{l}_n \cdot \bar{l}_a) \delta \bar{V}_m \cdot \bar{l}_t + C_8 \epsilon (\bar{l}_t \cdot \bar{l}_a) (\bar{l}_n \cdot \bar{l}_a) \delta \bar{l}_t \cdot \bar{V}_m \end{aligned} \right\} \quad (B4-45)$$

$$\begin{aligned}
& + C_8 \epsilon (\bar{\mathbf{l}}_t \cdot \bar{\mathbf{l}}_a) (\bar{\mathbf{V}}_m \cdot \bar{\mathbf{l}}_t) \delta \bar{\mathbf{l}}_n \cdot \bar{\mathbf{l}}_a \\
\delta \bar{\mathbf{l}}_t = & - \left[\frac{(\bar{\mathbf{V}}_m \cdot \bar{\mathbf{l}}_r)}{r_m |\bar{\mathbf{l}}_r \times \bar{\mathbf{V}}_m|} \delta \bar{\mathbf{r}}_m \cdot \bar{\mathbf{l}}_n \right] \bar{\mathbf{l}}_n + \left[\frac{1}{|\bar{\mathbf{l}}_r \times \bar{\mathbf{V}}_m|} \delta \bar{\mathbf{V}}_m \cdot \bar{\mathbf{l}}_n \right] \bar{\mathbf{l}}_n \\
& - \left[\frac{1}{r_m} \delta \bar{\mathbf{r}}_m \cdot \bar{\mathbf{l}}_t \right] \bar{\mathbf{l}}_r \\
\delta \bar{\mathbf{l}}_r = & \frac{1}{r_m} [\delta \bar{\mathbf{r}}_m - (\delta \bar{\mathbf{r}}_m \cdot \bar{\mathbf{l}}_r) \bar{\mathbf{l}}_r] \\
\delta \bar{\mathbf{l}}_n = & - \left[\frac{(\bar{\mathbf{V}}_m \cdot \bar{\mathbf{l}}_t)}{r_m |\bar{\mathbf{l}}_r \times \bar{\mathbf{V}}_m|} \delta \bar{\mathbf{r}}_m \cdot \bar{\mathbf{l}}_n \right] \bar{\mathbf{l}}_r + \left[\frac{(\bar{\mathbf{V}}_m \cdot \bar{\mathbf{l}}_r)}{r_m |\bar{\mathbf{l}}_r \times \bar{\mathbf{V}}_m|} \delta \bar{\mathbf{r}}_m \cdot \bar{\mathbf{l}}_n \right] \bar{\mathbf{l}}_t \\
& - \left[\frac{1}{|\bar{\mathbf{l}}_r \times \bar{\mathbf{V}}_m|} \delta \bar{\mathbf{V}}_m \cdot \bar{\mathbf{l}}_n \right] \bar{\mathbf{l}}_t
\end{aligned}
\tag{B4-45}$$

Contd

The unit vectors $\bar{\mathbf{l}}_r$, $\bar{\mathbf{l}}_t$, and $\bar{\mathbf{l}}_n$ may now be expressed in terms of $\bar{\mathbf{l}}_x$, $\bar{\mathbf{l}}_y$, and $\bar{\mathbf{l}}_z$ as

$$\begin{aligned}
\bar{\mathbf{l}}_r &= (\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_x) \bar{\mathbf{l}}_x + (\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_z) \bar{\mathbf{l}}_z \\
\bar{\mathbf{l}}_t &= (\bar{\mathbf{l}}_t \cdot \bar{\mathbf{l}}_x) \bar{\mathbf{l}}_x + (\bar{\mathbf{l}}_t \cdot \bar{\mathbf{l}}_z) \bar{\mathbf{l}}_z \\
\bar{\mathbf{l}}_n &= -\bar{\mathbf{l}}_y
\end{aligned}
\tag{B4-46}$$

The perturbation in the $\bar{\mathbf{l}}_f$ vector, using Equations B4-16, B4-45, and B4-46, becomes

$$\begin{aligned}
\delta \bar{\mathbf{l}}_f = & \frac{1}{f} [(N_1 \delta V_{mx} + N_2 \delta r_{mx} + N_3 \delta V_{my} + N_4 \delta r_{my} + N_5 \delta V_{mz} + N_6 \delta r_{mz}) \bar{\mathbf{l}}_z \\
& + (N_7 \delta V_{mx} + N_8 \delta r_{mx} + N_9 \delta V_{my} + N_{10} \delta r_{my} + N_{11} \delta V_{mz} + N_{12} \delta r_{mz}) \bar{\mathbf{l}}_y]
\end{aligned}
\tag{B4-47}$$

where

$$\begin{aligned}
N_1 &= -(\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_z)(\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_x) - (\bar{\mathbf{l}}_t \cdot \bar{\mathbf{l}}_z)(\bar{\mathbf{l}}_t \cdot \bar{\mathbf{l}}_x) \\
N_2 &= \left\{ \left[C_4 - \frac{8K_1}{r_m^2} \epsilon^3 (C_7 + \delta R) \right] (\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_z) - \left[\frac{K_1}{V_{rt} r_m^2} + \frac{V_{rr}}{V_{rt}} C_4 \right] (\bar{\mathbf{l}}_t \cdot \bar{\mathbf{l}}_z) \right\} (\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_x) \\
&+ \left\{ -\frac{1}{r_m} \left[V_{rt} + [C_5 - 2C_1 C_6 + 2C_6 (\bar{\mathbf{l}}_t \cdot \bar{\mathbf{l}}_a)] (\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_a) \right] (\bar{\mathbf{l}}_r \cdot \bar{\mathbf{l}}_z) \right\}
\end{aligned}
\tag{B4-48}$$

$$\begin{aligned}
& + \frac{1}{r_m} \left[\frac{V_{rr}}{V_{rt}} [C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_r \cdot \bar{l}_a) + (V_{rr} + G_R) \right] (\bar{l}_t \cdot \bar{l}_z) \Big\} (\bar{l}_t \cdot \bar{l}_x) \\
N_3 &= \frac{-1}{|\bar{l}_r \times \bar{V}_m|} \left[[C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_n \cdot \bar{l}_a) \right] (\bar{l}_r \cdot \bar{l}_z) \\
& + \frac{1}{|\bar{l}_r \times \bar{V}_m|} \left[\frac{V_{rr}}{V_{rt}} [C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_n \cdot \bar{l}_a) - V_{rn} \right] (\bar{l}_t \cdot \bar{l}_z) \\
N_4 &= \frac{1}{r_m |\bar{l}_r \times \bar{V}_m|} \left[V_{rn} (\bar{V}_m \cdot \bar{l}_t) + [C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_n \cdot \bar{l}_a) (\bar{V}_m \cdot \bar{l}_r) \right] (\bar{l}_r \cdot \bar{l}_z) \\
& - \frac{1}{|\bar{l}_r \times \bar{V}_m|} \left[\frac{V_{rr}}{V_{rt}} [C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_n \cdot \bar{l}_a) - V_{rn} \right] (\bar{l}_t \cdot \bar{l}_z) \\
N_5 &= -1 \\
N_6 &= \left\{ \left[C_4 - \frac{8K_1}{2r_m} \epsilon^3 (C_7 + \delta R) \right] (\bar{l}_r \cdot \bar{l}_z) - \left[\frac{K_1}{V_{rt} r_m} + \frac{V_{rr}}{V_{rt}} C_4 \right] (\bar{l}_t \cdot \bar{l}_z) \right\} (\bar{l}_r \cdot \bar{l}_z) \\
& + \left\{ -\frac{1}{r_m} \left[V_{rt} + [C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_r \cdot \bar{l}_a) \right] (\bar{l}_r \cdot \bar{l}_z) \right. \\
& \left. + \frac{1}{r_m} \left[\frac{V_{rr}}{V_{rt}} [C_5 - 2C_1 C_6 + 2C_6 (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_r \cdot \bar{l}_a) + (V_{rr} + G_R) \right] (\bar{l}_t \cdot \bar{l}_z) \right\} (\bar{l}_t \cdot \bar{l}_z) \\
N_7 &= -C_8 \epsilon (\bar{l}_t \cdot \bar{l}_a) (\bar{l}_n \cdot \bar{l}_a) (\bar{l}_t \cdot \bar{l}_x) \\
N_8 &= \frac{C_8 \epsilon}{r_m} [(\bar{V}_m \cdot \bar{l}_t) (\bar{l}_r \cdot \bar{l}_a) + (\bar{V}_m \cdot \bar{l}_r) (\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_n \cdot \bar{l}_a) (\bar{l}_t \cdot \bar{l}_x) \\
N_9 &= C_8 \epsilon [(\bar{l}_n \cdot \bar{l}_a)^2 - (\bar{l}_t \cdot \bar{l}_a)^2] + \frac{V_{rt}}{|\bar{l}_r \times \bar{V}_m|} - 1 \\
N_{10} &= \frac{V_{rr} + G_R}{r_m} - \frac{V_{rt}}{r_m |\bar{l}_r \times \bar{V}_m|} (\bar{V}_m \cdot \bar{l}_r) - \frac{2C_8 K_1}{r_m^2} (\bar{l}_n \cdot \bar{l}_a) (\bar{l}_t \cdot \bar{l}_a) \\
& - \frac{C_8 \epsilon}{r_m} [(\bar{l}_t \cdot \bar{l}_a) (\bar{l}_r \cdot \bar{l}_a) (\bar{V}_m \cdot \bar{l}_t) + (\bar{l}_n \cdot \bar{l}_a)^2 (\bar{V}_m \cdot \bar{l}_r) - (\bar{l}_t \cdot \bar{l}_a)^2 (\bar{V}_m \cdot \bar{l}_r)]
\end{aligned}$$

(B4-48)
Contd

$$N_{11} = -C_8 \epsilon (\bar{l}_t \cdot \bar{l}_a)(\bar{l}_n \cdot \bar{l}_a)(\bar{l}_t \cdot \bar{l}_z)$$

$$N_{12} = \frac{C_8 \epsilon}{r_m} [(\bar{V}_m \cdot \bar{l}_t)(\bar{l}_r \cdot \bar{l}_a) + (\bar{V}_m \cdot \bar{l}_r)(\bar{l}_t \cdot \bar{l}_a)] (\bar{l}_n \cdot \bar{l}_a)(\bar{l}_t \cdot \bar{l}_z)$$

(B4-48)
Contd

The relative importance of the various terms in Equation B4-47 can be ascertained by performing a "time-slice" analysis at several time instants. Intuitively, one might expect the time interval immediately preceding Centaur MECO to be critical since the magnitude of the $\bar{l}$ vector (f) is rapidly approaching zero. Hence, the effective guidance feedback gain is at a maximum. The following parameters, taken from Reference B- , are typical for MECO - 3.2 seconds.

$C_1 = 0.234 \times 10^{-1}$	$V_{rr} = -3,130$
$C_4 = -0.795 \times 10^{-5}$	$V_{rt} = 35,800$
$C_5 = 0.176 \times 10^5$	$\delta R = -0.963 \times 10^{-34}$
$C_6 = 0.129 \times 10^4$	$\epsilon = 0.348 \times 10^8$
$C_7 = 0.133 \times 10^{-31}$	$\bar{l}_n \cdot \bar{l}_a = 1.62 \times 10^{-5}$
$C_8 = 0.794 \times 10^{-7}$	$\bar{l}_r \cdot \bar{l}_a = -1.0$
$G_R = 0.194 \times 10^{-1}$	$\bar{l}_r \cdot \bar{l}_x = -0.314$
$K_1 = 0.141 \times 10^{17}$	$\bar{l}_r \cdot \bar{l}_z = -0.949$
$r_m = 0.216 \times 10^8$	$\bar{l}_t \cdot \bar{l}_a = 0.0183$
$\bar{V}_m \cdot \bar{l}_r = -0.297 \times 10^4$	$\bar{l}_t \cdot \bar{l}_x = 0.949$
$\bar{V}_m \cdot \bar{l}_t = 0.353 \times 10^5$	$\bar{l}_t \cdot \bar{l}_z = -0.314$
$V_{rn} = 0.235 \times 10^{-1}$	

Substituting these data into Equation B4-48, one obtains:

$N_1 = 0$	$N_7 = -7.80 \times 10^{-7}$
$N_2 = 6.94 \times 10^{-4}$	$N_8 = 6.94 \times 10^{-8}$
$N_3 = 8.10 \times 10^{-6}$	$N_9 = 1.21 \times 10^{-2}$
$N_4 = -2.19 \times 10^{-11}$	$N_{10} = 7.70 \times 10^{-5}$
$N_5 = -1.0$	$N_{11} = 2.58 \times 10^{-7}$
$N_6 = -5.16 \times 10^{-4}$	$N_{12} = -2.30 \times 10^{-8}$

From a control standpoint, the only interest is in frequency components on the order of 0.2 rad/sec, or greater. Therefore, the perturbation in the $\bar{l}_f$ vector can be adequately approximated by

$$\delta \bar{l}_f = N_5 \frac{\delta V_{mz}}{f} \bar{l}_z + N_9 \frac{\delta V_{my}}{f} \bar{l}_y \quad (B4-49)$$

Since $\delta \bar{a}_m$ contains no terms involving gravity, the perturbation in thrust acceleration is equal to the perturbation in vehicle acceleration, that is, $\delta \bar{a}_T = \delta \bar{a}_m$. This obviously also implies that $\delta \bar{V}_m = \delta \bar{V}_{m_T}$ and $\delta \bar{r}_m = \delta \bar{r}_{m_T}$. The perturbation in the $\bar{l}_{a_T}$ vector may therefore be written equivalently as

$$\delta \bar{l}_{a_T} = \frac{\delta \bar{a}_m}{a_T} \quad (B4-50)$$

where a_T is still thrust acceleration only.

Utilizing Equations B4-49 and B4-50, Equation B4-19 may be rewritten in the following equivalent form.

$$\begin{aligned} \frac{\delta \bar{\omega}_c}{K_G} = & -\bar{l}_{x_i} \times (\delta \bar{A} \times \bar{l}_x) + \bar{l}_x \times \left[N_5 \frac{\delta V_{mz_i}}{f_i} \bar{l}_z + N_9 \frac{\delta V_{my_i}}{f_i} \bar{l}_y \right. \\ & \left. + k'_I \sum_{n=k}^i \left(N_5 \frac{\delta V_{mz_n}}{f_n} \bar{l}_z + N_9 \frac{\delta V_{my_n}}{f_n} \bar{l}_y - \frac{\delta \bar{a}_{m_n}}{a_{T_n}} \right) \Delta t_n \right] \end{aligned} \quad (B4-51)$$

where $t_i + T_D \leq t < t_{i+1} + T_D$

The vectors $\bar{l}_x$ and $\bar{l}_{x_i}$ are equal for all practical purposes. The subscript i will therefore be dropped in the remaining analysis. Since $\bar{a}_T$ is calculated internally in the navigational computer by numerically differentiating the thrust velocity, the perturbation in $\bar{a}_{m_n}$ must be written as

$$\delta \bar{a}_{m_n} = \frac{\delta \bar{V}_{m_n} - \delta \bar{V}_{m_{n-1}}}{\Delta t_n} \quad (B4-52)$$

Substituting the expression for $\delta \bar{a}_{m_n}$ from Equation B4-52 into Equation B4-51, one has

$$\begin{aligned} \frac{\delta \bar{\omega}_c}{K_G} = & -\bar{l}_x \times (\delta \bar{A} \times \bar{l}_x) + \bar{l}_x \times \left[N_5 \frac{\delta V_{mz_i}}{f_i} \bar{l}_z + N_9 \frac{\delta V_{my_i}}{f_i} \bar{l}_y \right. \\ & \left. + k'_I \sum_{n=k}^i \left(N_5 \frac{\delta V_{mz_n}}{f_n} \bar{l}_z + N_9 \frac{\delta V_{my_n}}{f_n} \bar{l}_y \right) \Delta t_n - k'_I \sum_{n=k}^i \frac{\delta \bar{V}_{m_n} - \delta \bar{V}_{m_{n-1}}}{a_{T_n}} \right] \end{aligned} \quad (B4-53)$$

where, again $t_i + T_D \leq t < t_{i+1} + T_D$.

Equation B4-53, written in terms of its scalar components, becomes

$$\frac{\delta \omega_{cz}}{K_G} = -\delta \psi + N_9 \frac{\delta V_{my_i}}{f_i} + k'_I N_9 \sum_{n=k}^i \frac{\delta V_{my_n}}{f_n} \Delta t_n - k'_I \sum_{n=k}^i \frac{\delta V_{my_n} - \delta V_{my_{n-1}}}{a_{T_n}} \quad (B4-54)$$

and

$$\frac{\delta \bar{\omega}_{cy}}{K_G} = -\delta \theta - N_5 \frac{\delta V_{mz_i}}{f_i} + k'_I N_5 \sum_{n=k}^i \frac{\delta V_{mz_n}}{f_n} \Delta t_n - k'_I \sum_{n=k}^i \frac{\delta V_{mz_n} - \delta V_{mz_{n-1}}}{a_{T_n}} \quad (B4-55)$$

where $t_i + T_D \leq t < t_{i+1} + T_D$. Equations B4-54 and B4-55 represent the "exact" forms for the perturbations in vehicle pitch and yaw commanded steering rates applicable to "time slice" analysis. The constants N_5 and N_9 have been previously defined. Note that the first term on the right side of these two equations is equivalent to continuous vehicle attitude feedback. The three remaining terms represent digital or sampled data feedback which is characterized by finite discontinuities due to the discrete nature of the computations involved. In addition, there is a pure delay or "transport lag" between the time (t_i) that the outputs of the signator are sampled and the time ($t_i + T_D$) that this information is reflected in the $\bar{f}^*$ vector and hence sensed by the flight control system.

The perturbations in vehicle velocity are obtained by integrating Equation B4-35. Thus, one obtains

$$\delta V_{my_n} = \left[\int_0^t \frac{\delta B_{F_y}}{m} dt + \int_0^t a_T \delta \psi dt + (\ell_p)_x \delta \dot{\psi} - (\ell_p)_z \delta \dot{\phi} \right]_{t=t_n} \quad (B4-56)$$

and

$$\delta V_{mz_n} = \left[\int_0^t \frac{\delta B_{F_z}}{m} dt - \int_0^t a_T \delta \theta dt - (\ell_p)_x \delta \dot{\theta} + (\ell_p)_y \delta \dot{\phi} \right]_{t=t_n} \quad (B4-57)$$

where $\delta \dot{\psi}$, $\delta \dot{\psi}$, $\delta \dot{\theta}$, and $\delta \dot{\theta}$ are obtained by integrating Equation B4-40. For most practical applications, the following approximate solutions suffice:

$$\left. \begin{aligned} \delta \dot{\theta} &= \int_0^t \frac{\delta B_{L_y}(\tau)}{I_{yy}(\tau)} d\tau \\ \delta \theta &= \int_0^t \delta \dot{\theta}(\tau) d\tau \end{aligned} \right\} \quad (B4-58)$$

$$\left. \begin{aligned} \delta \dot{\psi} &= \int_0^t \frac{\delta B_{L_z}(\tau)}{I_{zz}(\tau)} d\tau \\ \delta \psi &= \int_0^t \delta \dot{\psi}(\tau) d\tau \end{aligned} \right\} \quad (B4-59)$$

B4.5 AUTOPILOT/GUIDANCE LOOP CONTROL EQUATION. The "time slice" technique is employed for the stability analysis. All the time-dependent coefficients of the steering equations (Equations B4-54 and B4-55) become constant. In particular, $\bar{f}$ is constant and the sampled data portion of the guidance feedback signal (pitch plane), actually equal to $\delta f_{z_i}^*$, is given by

$$\delta f_{z_i}^* = \frac{1}{f} \left(\delta V_{mz_i} + k_I' \sum_{n=0}^i \delta V_{mz_n} \Delta t_n \right) + \frac{k_I'}{a_T} \sum_{n=0}^i (\delta V_{mz_n} - \delta V_{mz_{n-1}}) \quad (B4-60)$$

Taking the z -transform of both sides, one has

$$\delta f_z^*(z) = \left(\frac{k_I'}{a_T} + \frac{1}{f} + \frac{k_I'}{f} \frac{\Delta t}{1-z^{-1}} \right) \delta V_{mz}(z)$$

or, rearranging:

$$\delta f_z^*(z) = k_I' \left(1 + \frac{a_T}{k_I' f} \right) (1 + \lambda) \left[\frac{z - \frac{1}{1+\lambda}}{z - 1} \right] \frac{\delta V_{mz}(z)}{a_T} \quad (B4-61)$$

where the parameter λ is defined as

$$\lambda = \frac{a_T \Delta t}{f} \frac{1}{1 + \frac{a_T}{k_I' f}} \quad (B4-62)$$

In Section B2.4 of this appendix, the variables used are perturbational, although the "δ's" are omitted. In the same manner we have (from Equations B4-57 and B4-58) the following:

$$\delta V_{mz_n} = \left\{ \int_0^t \left[\ddot{z} - \ell_p \ddot{\theta} - a_T \theta \right] d\tau \right\}_{t=t_n} \quad (B4-63)$$

where $\ell_p \cong (\ell_p)_x$, with $(\ell_p)_y$ and $(\ell_p)_z$ being small relative to $(\ell_p)_x$.

The sampled data portion of the steering equation becomes in modified z -transform notation:

$$\delta f_z^*(z, m) = k_I' \left(1 + \frac{\ddot{x}}{k_I' f} \right) (1 + \lambda) \left[\frac{z - \frac{1}{1+\lambda}}{z - 1} \right] G_{H_0}(z) \mathcal{Z}_m \left[\Delta f_z(t) \right] \quad (B4-64)$$

where $G_{H_0}(z)$ is the transfer function of a zero-order hold (Figure B4-2), $\ddot{x}$ replaces a_T , $m = (1 - \frac{T_D}{\Delta t})$, and

$$\Delta f_z(t) = \int_0^t \left(\frac{\ddot{z}_p}{\ddot{x}} - \theta \right) dt \quad (B4-65)$$

The variable $\ddot{z}_p$ represents the lateral acceleration as sensed at the platform location, i.e., $\ddot{z}_p = \ddot{z} - \ell_p \ddot{\theta}$.

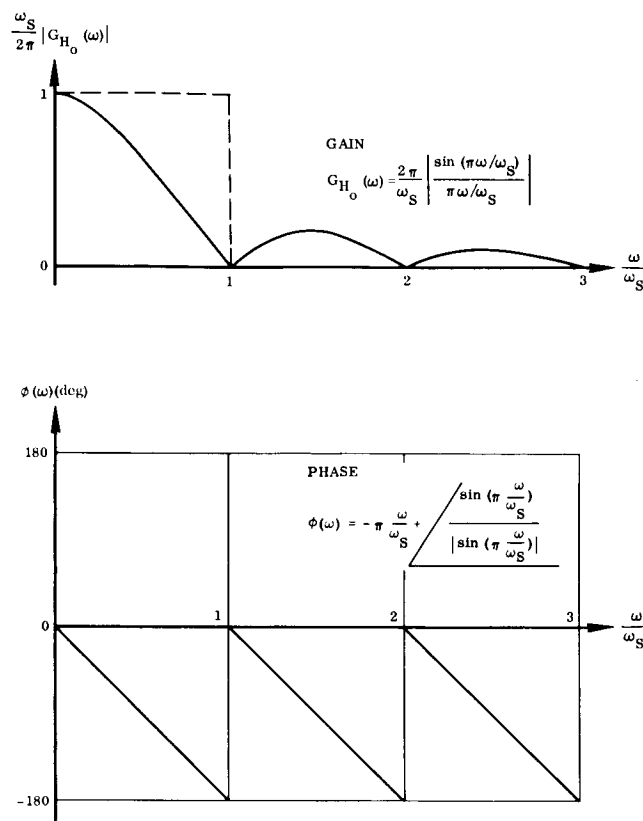


Figure B4-2. Gain and Phase Characteristics of an Ideal Zero-Order Hold

B4.5.1 Simplified Representation. Throughout most of powered flight, the magnitude of $\bar{f}$ is so large as to make the following approximation of Equation B4-64 valid (angles now in degrees):

$$k_I' \theta_{CI}^*(z) \equiv -57.3 \delta f_z^*(z, m) = k_I' G_{H_0}(z) \mathcal{Z}_m \left[\frac{57.3}{\ddot{x}_s} \left(\frac{\ddot{x}}{57.3} \theta - \ddot{z}_p \right) \right] \quad (\text{B4-66})$$

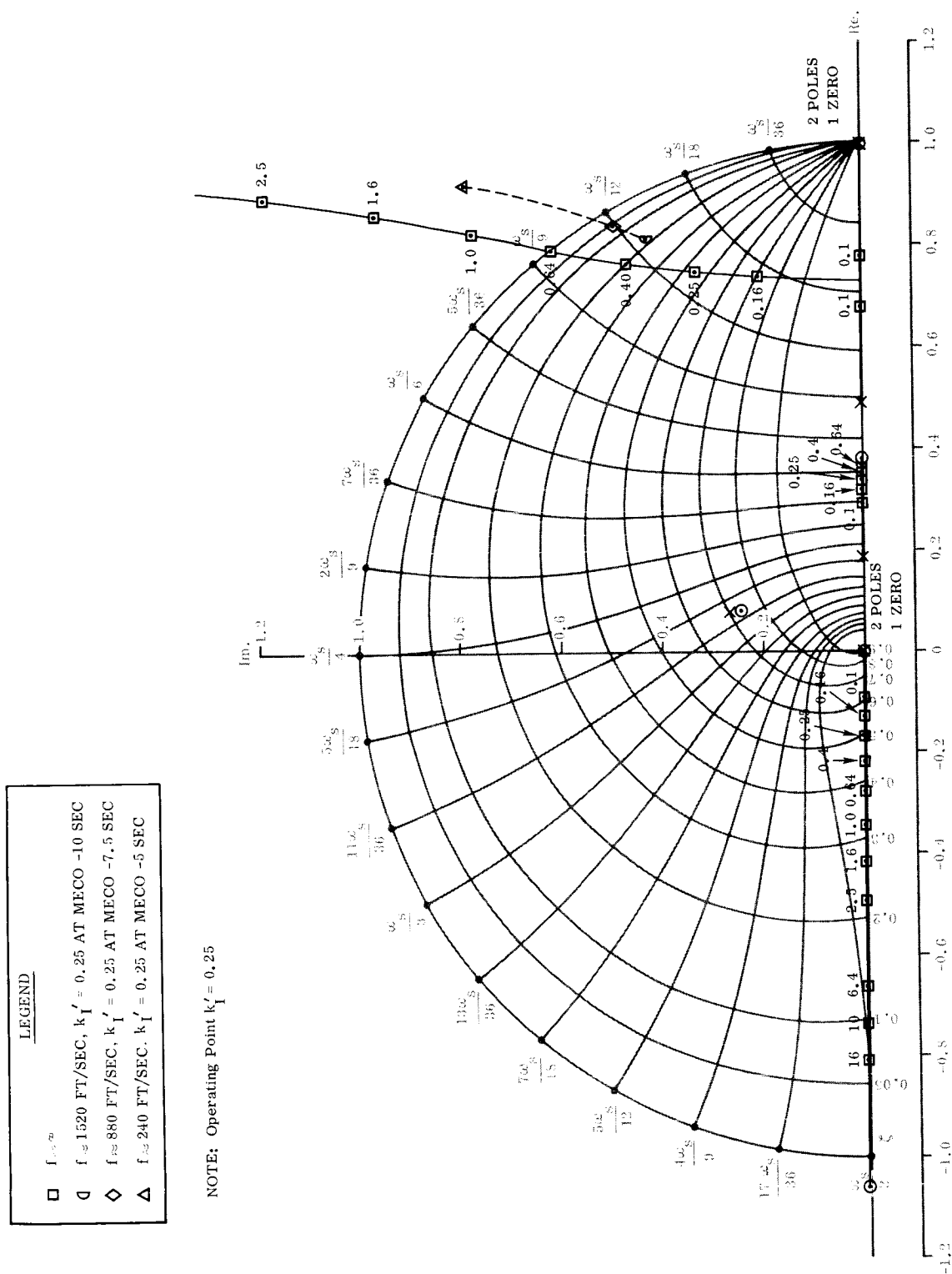
where $\ddot{x}$ is constant for the analysis. This equation is shown schematically as the sampled data feedback loop in Figure B2-4. K_G in the steering equation is a sizing constant used to scale the magnitude of $\bar{f}^*$. It is so chosen that the end-to-end position gain (with reference to the continuous portion of the feedback signal) is $k_p K_A$. This approximation is equivalent to neglecting the short period perturbations in $\bar{f}$.

B4.5.2 Time Slice Analysis. The above approximation is strictly valid for time slice studies only. The procedure is to reduce the flight control system, including the rate and position feedback loops and the vehicle dynamics, to a single transfer function (between points A and B in Figure B2-4) in the Laplace transform variable, s . Representing this transform as $GH(s)$ the end-to-end z -transform is given by:

$$G(z) = k_I' \frac{z-1}{z} \mathcal{Z}_m \left[\frac{GH(s)}{s} \right] \quad (\text{B4-67})$$

Guidance/autopilot stability is studied by plotting the roots of $1 + G(z) = 0$ in the z -plane as a function of the various gains in the system, in particular, the guidance integral gain, k_I' .

B4.5.3 Time-Varying Analysis. Although the above simplifications are strictly valid for the time-slice representation only, they are assumed valid for the time varying representation as well (Reference B-17). This is justified because the time-varying parameters change relatively slowly. The sample-and-hold



character of the guidance signal (Figure B2-4) is simulated on the analog computer, together with the remainder of the mathematical model discussed in Section B2.

B4.5.4 Conditions Prior to MECO. During the last few seconds prior to main engine cutoff the vehicle velocity is approaching the "velocity to be gained" vector rapidly, causing $\bar{f}$ to decrease toward zero. The approximation of Equation B4-66 becomes invalid with the present k_I' and Δt approximately 14 seconds prior to MECO. In effect, the lag of the system increases due to the zero introduced at $s = \frac{1}{1+\lambda}$ and the pole at $z = 1$. In addition, the loop gain increases by the factor $(1 + \frac{\ddot{x}}{k_I' \bar{f}})(1 + \lambda)$. This results in the damping ratio of the system root representing the "guidance mode" becoming less than 0.3 approximately 14 seconds prior to MECO, and zero at approximately six seconds prior to MECO as shown in Figure B4-3. Beyond this point, i.e., for the last six seconds of flight, the guidance equations may be termed unstable. In practice, this means only a slight deterioration in the guidance accuracy, as the system cannot diverge appreciably in the remaining four compute cycles before $\bar{f}^*$ is "frozen" prior to MECO. The approximation made in Equation B4-66 is therefore valid insofar as the effects on the predicted system behavior are concerned.

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APPENDIX C
COAST PHASE EQUATIONS

LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A_1, A_2, A_3, A_4	Control engine forces acting in the y_B body axes direction	lb
F_x	Forces acting along x_B in the body axes system	lb
F_y	Forces acting along y_B in the body axes system	lb
F_z	Forces acting along z_B in the body axes system	lb
I_{xx}	Moment of inertia about the roll axis	slug-ft ²
I_{xy}	Product of inertia about the roll-pitch axes	slug-ft ²
I_{xz}	Product of inertia about the roll-yaw axes	slug-ft ²
I_{yy}	Moment of inertia about the pitch axis	slug-ft ²
I_{yz}	Product of inertia about the pitch-yaw axes	slug-ft ²
I_{zz}	Moment of inertia about the yaw axis	slug-ft ²
$\bar{i}$	Unit vector along x_B in the body axes system	N. D.
$\bar{J}$	Angular momentum about the vehicle center of mass	slug-ft ² rad/sec
J_x	x_B component of angular momentum	slug-ft ² rad/sec
J_y	y_B component of angular momentum	slug-ft ² rad/sec
J_z	z_B component of angular momentum	slug-ft ² rad/sec
$\bar{j}$	Unit vector along y_B in the body axes system	N. D.
$\bar{k}$	Unit vector along z_B in the body axes system	N. D.
L	External moment about the roll axis	ft-lb
$\bar{L}$	Moment about the center of gravity due to externally applied forces	ft-lb
L_x	x_B component of externally applied moment	ft-lb
L_y	y_B component of externally applied moment	ft-lb
L_z	z_B component of externally applied moment	ft-lb
ℓ_{ra}	Radial distance from the vehicle centerline to the A Engine thrust line	ft

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
l_{x_a}	Distance along the centerline from the center of gravity to the plane of the A Engine thrust line	ft
l_{x_p}	Distance along the centerline from the center of gravity to the plane of the P Engine thrust line	ft
M	External moment about the pitch axis	ft-lb
N	External moment about the yaw axis	ft-lb
P	Instantaneous angular velocity along x_B in the body axes system	rad/sec
P_1, P_2	Control-engine forces acting in the z_B body axes direction	lb
Q	Instantaneous angular velocity along y_B in the body axes system	rad/sec
R	Instantaneous angular velocity along z_B in the body axes system	rad/sec
x, y, z	Vehicle coordinate system (roll, pitch, yaw)	N.D.
$\bar{y}$	Center-of-gravity offset along y_B in the body axis system	ft
$\bar{z}$	Center-of-gravity offset along z_B in the body axis system	ft
θ	Euler angle between the pitch axes of the inertial axes and body axes systems	deg
ϕ	Euler angle between the roll axes of the inertial axes and body axes systems	deg
ψ	Euler angle between the yaw axes of the inertial axes and body axes systems	deg
$\bar{\omega}$	Angular velocity in the inertial axes system	rad/sec

Subscripts

B	Body axes system
I	Fixed inertial axes system

C.1 INTRODUCTION. This appendix presents the derivation of the equations describing the Centaur vehicle behavior during the coast phase. Initial analysis of the AC-5 type attitude control system is documented in Reference C-1. A more complete treatment of the subject appears in Reference C-2.

C.1.1 System Description. The on-off coast phase attitude control system consists of six hydrogen peroxide (monopropellant) jets. The Centaur autopilot controls the firing of these jets by responding to error signals generated within the autopilot from rate and position reference signals. The engine locations, thrust values, and vehicle coordinate system are given in Figure C-1.

Figure C-2 is a functional block diagram of the attitude control system. The position reference is obtained from the Centaur inertial platform resolver chain with guidance supplying the reference attitude vector. The system will control rate only when guidance is inhibited. Note that a roll position reference is not required since the resolver chain automatically resolves the desired attitude vector into vehicle coordinates. The error signal, used to activate the threshold circuitry, is composed of the channel selector and rate gyro signals. The engines fire in response to the engine control logic signals in a manner that reduces the error signals to less than the threshold values. The engines cease firing when this is accomplished. Section C.3 contains a detailed description of the autopilot's functional behavior.

C.2 ANALYSIS OF ATTITUDE CONTROL DYNAMICS

C.2.1 Reference Axes. The equations used to analyze the coast phase dynamics are presented below. Since translational motions do not affect the attitude of the vehicle, this study is limited to rotational motions and thus involves only three degrees of freedom. Two sets of reference axes are used to describe the dynamics of the vehicle (see Figure C-3). The first of these involves a fixed inertial frame (x_I, y_I, z_I) to which the vehicle's attitude is referenced through the guidance computer and the inertial platform. The second is a body axes system (x_B, y_B, z_B) fixed to the vehicle and originating at its center of gravity. The body axes provide the frame of reference for locating the control engines and rate sensing gyroscopes. These two axes systems are related by Euler angles ($\psi, \theta, \text{ and } \phi$) as illustrated in Figure C-3.

C.2.2 Equations of Angular Motion. The derivation of the equations of angular motion and the simplifying approximations used in the analysis are presented below. The following equations are derived from a consideration of the angular momentum. Two assumptions are necessary for this derivation.

- The vehicle is assumed to be a rigid body since there is no completely satisfactory model to describe propellant-behavior in near zero-g conditions.
- The vehicle mass is assumed to remain constant for the duration of any particular dynamic analysis.

The angular momentum $\bar{J}$, of a rigid body about its center of mass (or center of gravity) can be expressed in terms of body axes components as

$$\bar{J} = J_x \bar{i} + J_y \bar{j} + J_z \bar{k} \quad (C-1)$$

This equation for angular momentum can be expressed in terms of angular rates and inertias such that

$$\bar{J} = (I_{xx} P - I_{xy} Q - I_{xz} R) \bar{i} + (I_{yy} Q - I_{xy} P - I_{yz} R) \bar{j} + (I_{zz} R - I_{xz} P - I_{yz} Q) \bar{k} \quad (C-2)$$

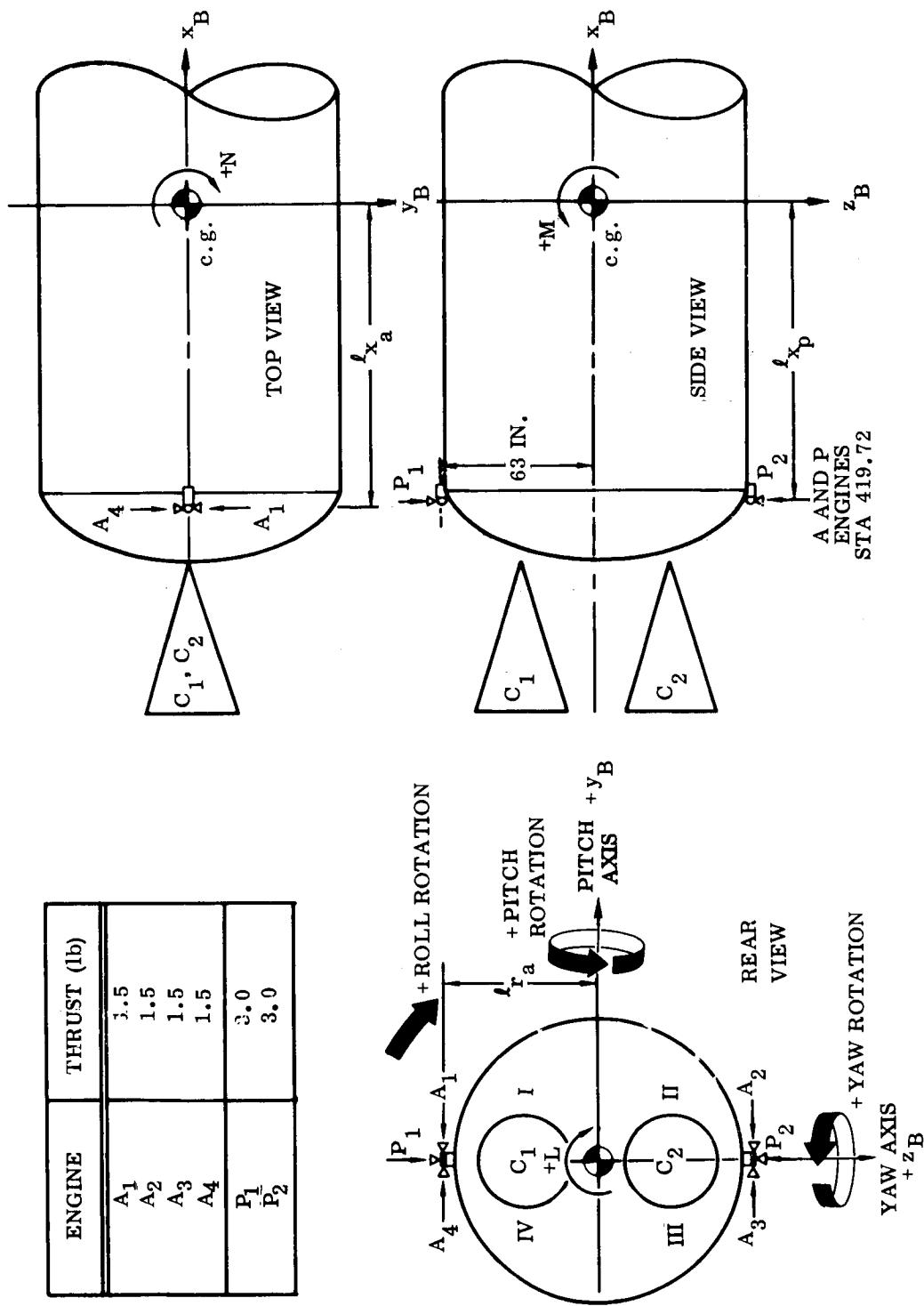
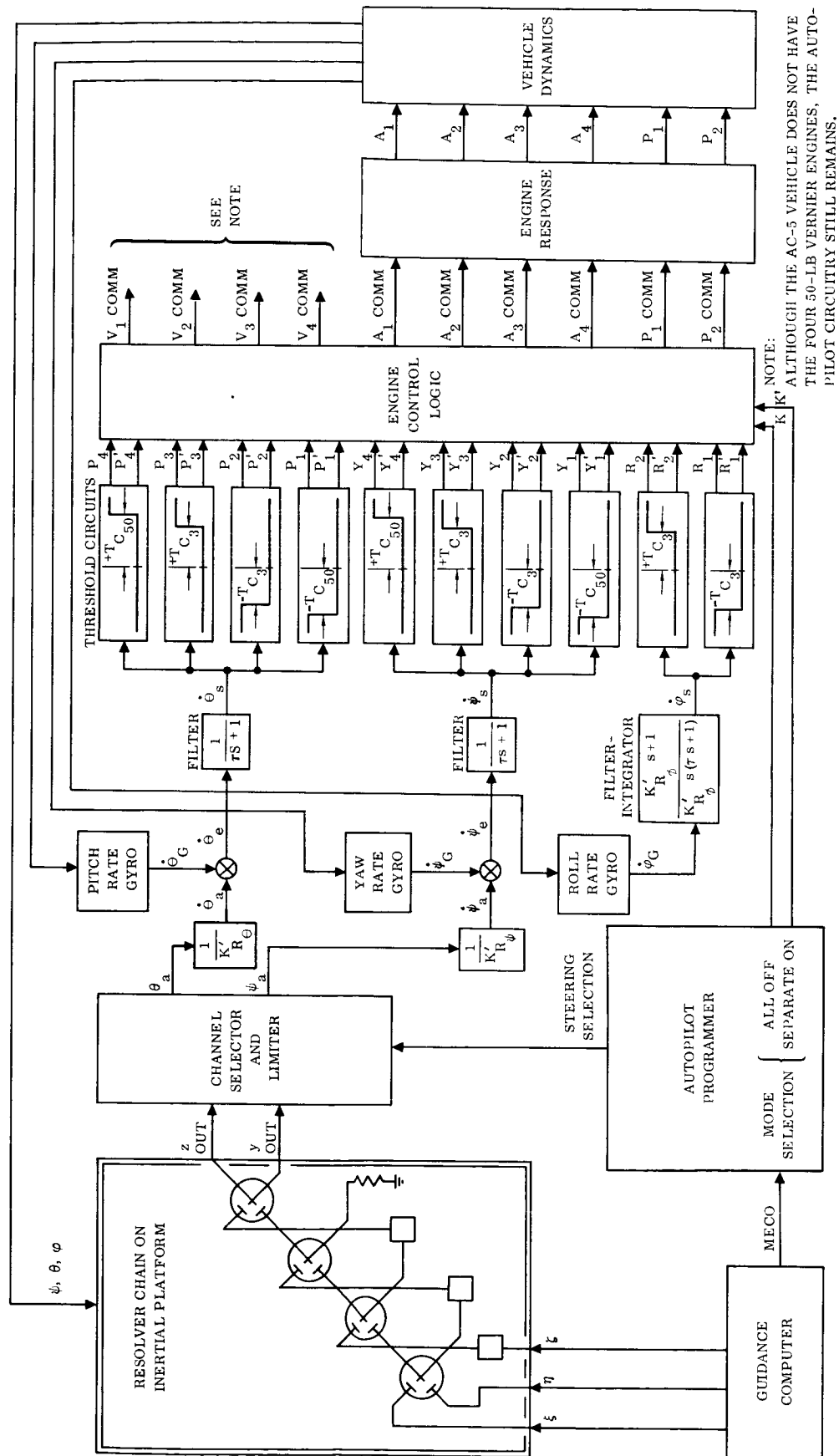


Figure C-1. Location of Attitude Control and Propellant Settling Jets



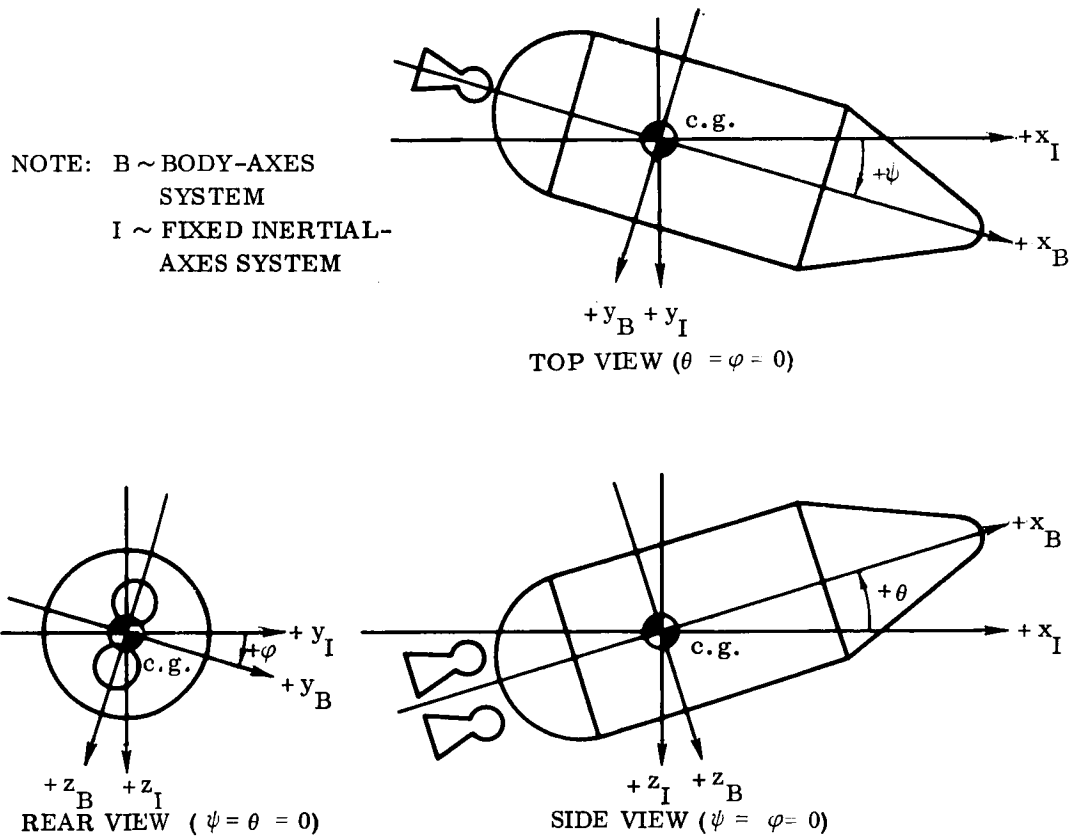


Figure C-3. Reference Axes Systems Used in the Dynamics Analysis

If the derivative of $\bar{J}$ with respect to time is calculated in relation to an inertial reference frame, then

$$\frac{d}{dt} (\bar{J}) = \bar{L} = \dot{\bar{J}} + \bar{\omega} \times \bar{J} \quad (C-3)$$

where $\bar{L}$ is the moment of the externally applied forces with respect to the body's center of gravity, and the derivative, $\dot{\bar{J}}$, on the right side of the equation is calculated with respect to a body-reference frame with angular velocity $\bar{\omega}$ in the inertial frame.

In the rotating body-axes system the components of $\bar{L}$ are given by

$$\bar{L} = L_x \bar{i} + L_y \bar{j} + L_z \bar{k} = j_x \bar{i} + j_y \bar{j} + j_z \bar{k} + \bar{\omega} \times \bar{J} \quad (C-4)$$

where

$$\bar{\omega} = P\bar{i} - Q\bar{j} + R\bar{k} \quad (C-5)$$

Substituting (C-5) in (C-4), obtaining the cross product, and combining terms, the following equation is obtained:

$$\bar{L} = (\dot{j}_x + j_z Q - j_y R) \bar{i} + (\dot{j}_y + j_x R - j_z P) \bar{j} + (\dot{j}_z + j_y P - j_x Q) \bar{k} \quad (C-6)$$

Substituting the components of the derivative of (C-2) in (C-6), and separating the resulting expression into its x, y, and z components, the following complete equations of angular motion are obtained:

Roll

$$\begin{aligned}\Sigma L &= I_{xx} \dot{P} - I_{xy} \dot{Q} - I_{xz} \dot{R} + (I_{zz} R - I_{xz} P - I_{yz} Q)Q - (I_{yy} Q - I_{xy} P - I_{yz} R)R \\ &= I_{xx} \dot{P} - I_{xy} \dot{Q} - I_{xz} \dot{R} + (I_{zz} - I_{yy})QR - I_{xz} PQ - I_{yz} Q^2 + I_{xy} PR + I_{yz} R^2\end{aligned}\quad (C-7)$$

Pitch

$$\begin{aligned}\Sigma M &= I_{yy} \dot{Q} - I_{xy} \dot{P} - I_{yz} \dot{R} + (I_{xx} P - I_{xy} Q - I_{xz} R)R - (I_{zz} R - I_{xz} P - I_{yz} Q)P \\ &= I_{yy} \dot{Q} - I_{xy} \dot{P} - I_{yz} \dot{R} + (I_{xx} - I_{zz})PR - I_{xy} QR - I_{xz} R^2 + I_{xz} P^2 + I_{yz} PQ\end{aligned}\quad (C-8)$$

Yaw

$$\begin{aligned}\Sigma N &= I_{zz} \dot{R} - I_{xz} \dot{P} - I_{yz} \dot{Q} + (I_{yy} Q - I_{xy} P - I_{yz} R)P - (I_{xx} P - I_{xy} Q - I_{xz} R)Q \\ &= I_{zz} \dot{R} - I_{xz} \dot{P} - I_{yz} \dot{Q} + (I_{yy} - I_{xx})PQ - I_{xy} P^2 - I_{yz} PR + I_{xy} Q^2 + I_{xz} QR\end{aligned}\quad (C-9)$$

By employing various approximations, however, the equations of motion can be simplified. A valid assumption for the virtually symmetrical Centaur vehicle is that xz and xy planes are planes of symmetry. This simplification is justified by an examination of the inertia terms in the above equations. This is accomplished by comparing each term with the principal moments of inertia, I_{xx} , I_{yy} , and I_{zz} by computing their ratios. The results in Table C-1 are thus obtained.

Table C-1. Ratio of Inertia Terms to Principal Moments of Inertia

PRINCIPAL MOMENT OF INERTIA		INERTIA TERM		COAST	COAST
				BEFORE PAYLOAD SEPARATION	AFTER PAYLOAD SEPARATION
$\frac{1}{I_{xx}}$	x	$(I_{zz} - I_{yy})$	=	0.036	0.041
		I_{xy}	=	0.011	0.0168
		I_{xz}	=	0.005	0.0003
		I_{yz}	=	0.003	0.00005

Table C-1. Ratio of Inertia Terms to Principal Moments of Inertia, Contd

PRINCIPAL MOMENT OF INERTIA		INERTIA TERM	COAST BEFORE PAYLOAD SEPARATION	COAST AFTER PAYLOAD SEPARATION
		$(I_{xx} - I_{zz}) =$	-0.93	-0.89
$\frac{1}{I_{yy}}$	x	$I_{xy} =$	0.00074	0.002
		$I_{xz} =$	0.00037	0.00003
		$I_{yz} =$	0.00002	0.000005
		$(I_{yy} - I_{xx}) =$	0.927	0.88
$\frac{1}{I_{zz}}$	x	$I_{xy} =$	0.00073	0.002
		$I_{xz} =$	0.00037	0.00003
		$I_{yz} =$	0.00002	0.000005

Therefore, in view of the above, the equations of angular motion reduce to,

Roll

$$\Sigma L = I_{xx} \dot{P} \quad (C-10)$$

Pitch

$$\Sigma M = I_{yy} \dot{Q} + (I_{xx} - I_{zz})PR \quad (C-11)$$

Yaw

$$\Sigma N = I_{zz} \dot{R} + (I_{yy} - I_{xx})PQ \quad (C-12)$$

The equations of angular motion presented here have been written with the assumption that the vehicle center of gravity lies on the geometric axis.

If the center of gravity is offset from the geometric axis, these equations must be modified to include this effect. There are two ways to handle this. One method is to transform the equations from the geometric axis to the center of gravity axis, which would include changing all of the moments due to external forces. The other method, and the one used here, is to retain the assumption that the forces are acting with reference to the geometric axis, and then add the kinetic reaction terms due to center-of-gravity offsets.

Let $\Sigma L'$, $\Sigma M'$, and $\Sigma N'$ be the modified equations of angular motion.

Then

$$\Sigma L' = \Sigma L + \Sigma F_y \bar{z} - \Sigma F_z \bar{y} \quad (C-13)$$

$$\Sigma M' = \Sigma M - \Sigma F_x \bar{z} \quad (C-14)$$

$$\Sigma N' = \Sigma N + \Sigma F_x \bar{y} \quad (C-15)$$

Where $\bar{y}$ and $\bar{z}$ are positive center-of-gravity displacements from the geometric axis.

The center-of-gravity offsets, for the three coast phase portions of flight, are listed in Table E-9; the resultant torques are treated as "external" (Section C.2.5).

C.2.3 Engine Moments and Control Forces. The primary forces and moments acting on the Centaur are generated by the control engines. Since the engines are fixed to the vehicle, these forces and moments are handled in the body-axes reference system. The control engine thrusts are rated at 1.5 and 3.0 pounds and are designated by A and P respectively. The control engine forces acting in the x_B , y_B and z_B directions are shown in Figure C-1. They are

$$\Sigma F_x = 0 \quad (C-16)$$

$$\Sigma F_y = -A_1 - A_2 + A_3 + A_4 \quad (C-17)$$

$$\Sigma F_z = P_1 - P_2 \quad (C-18)$$

The thrust versus time curves of the control engines have been simulated by step functions with appropriate time delays for autopilot and thrust buildup and decay delays. The step function has the same total impulse as the actual thrust versus time curve. The analysis is simplified by the assumption of a step function thrust curve. Previous analog simulation indicates this is a valid assumption that results in negligible error. Figures C-4 and C-5 show the nominal curves and assumed step functions for the 3.0 lb and the 1.5 lb thrust engines respectively. These curves are for the operating conditions subsequent to the first firing (temperature > 200° F) and indicate the delays between the time the on or off command is issued from the threshold circuit and the solenoid valve opening (Reference C-4).

The moments produced by these control engines are

Roll

$$\Sigma L = (-A_1 + A_2 - A_3 + A_4) \ell_{r_a} \quad (C-19)$$

Pitch

$$\Sigma M = \Sigma F_z (\ell_{x_p}) \quad (C-20)$$

Yaw

$$\Sigma N = \Sigma F_y (\ell_{x_a}) \quad (C-21)$$

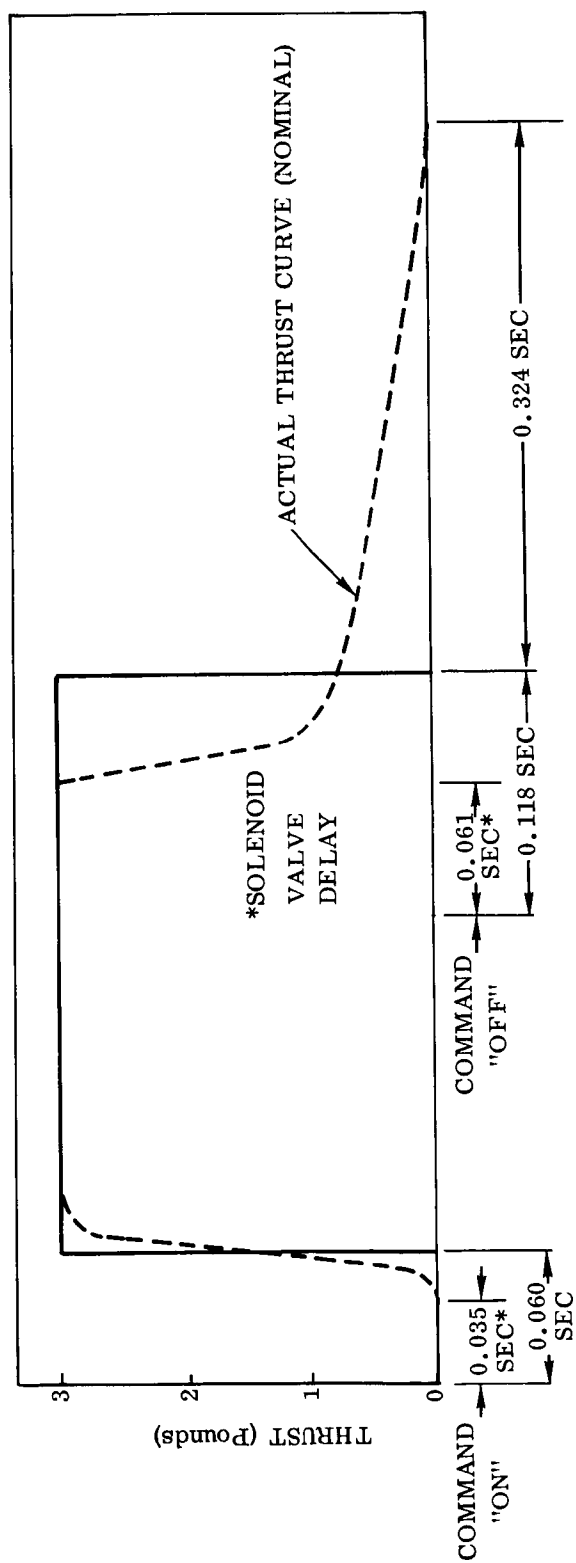


Figure C-4. Simplified Thrust and Delay Characteristics, 3-Pound Thrust Hydrogen Peroxide Engines (Nominal, Temperature > 200° F)

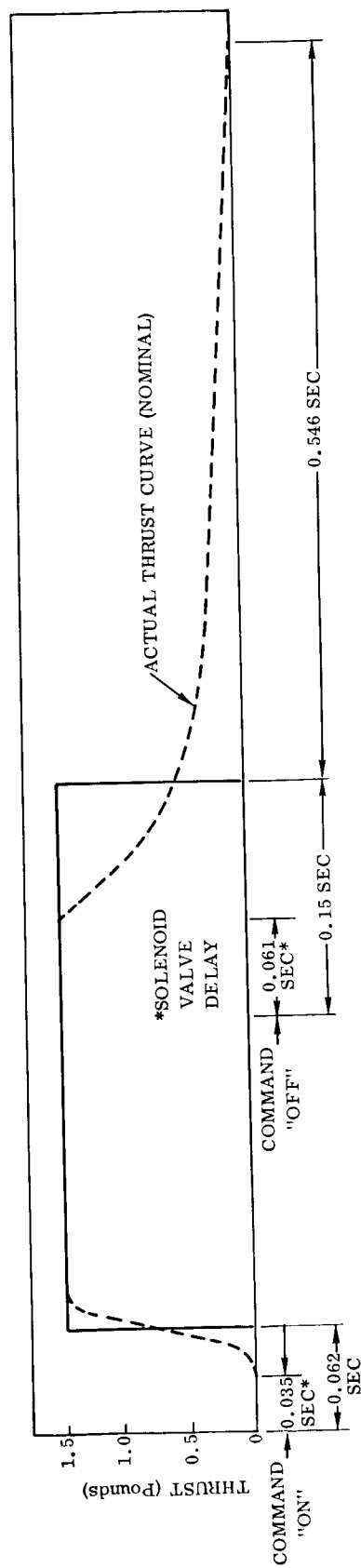


Figure C-5. Simplified Thrust and Delay Characteristics, 1.5-Pound Thrust Hydrogen Peroxide Engines (Nominal, Temperature > 200° F)

C.2.4 Transformation of Reference Axes. Since a body-axes reference frame is used as well as a fixed inertial-axes frame, the rates and positions of the vehicle in both axes systems must be related. In the fixed inertial-axes system, the body axes system and the position of the vehicle are uniquely determined by an Euler angle transformation. From the fixed inertial axes, this transformation involves, in sequence (as shown in Figure C-6),

- A yaw rotation of angle ψ about the z_I axis.
- A pitch rotation of angle θ about the new y_1 axis created by the yaw rotation.
- A roll rotation of angle ϕ about the new x_B axis created by the pitch and yaw rotations.

This sequence determines the following equations:

$$\xi_B = \xi \cos \psi \cos \theta + \eta \sin \psi \cos \theta - \zeta \sin \theta \quad (C-22)$$

$$\eta_B = \xi (\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) + \eta (\cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi) + \zeta (\cos \theta \sin \phi) \quad (C-23)$$

$$\zeta_B = \xi (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) + \eta (\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) + \zeta (\cos \theta \cos \phi) \quad (C-24)$$

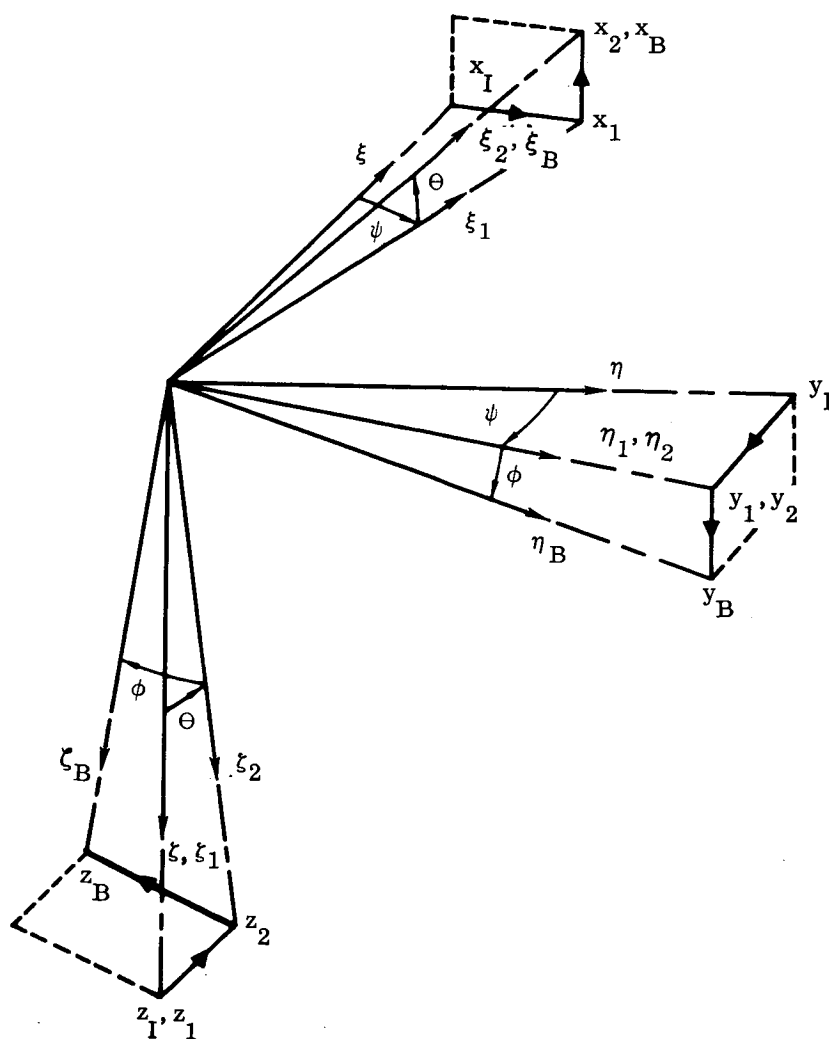


Figure C-6. Transformation of Reference Vector

where ξ , η , and ζ are components of a direction vector in the fixed inertial-axes system and ξ_B , η_B , and ζ_B are the components in the body axes system. These equations can be rewritten in matrix form:

$$\begin{bmatrix} \cos\psi \cos\theta & \sin\psi \cos\theta & -\sin\theta \\ \cos\psi \sin\theta \sin\varphi & \cos\psi \cos\theta \sin\varphi & \cos\theta \sin\varphi \\ -\sin\psi \cos\varphi & +\sin\psi \sin\theta \sin\varphi & \\ \cos\psi \sin\theta \cos\varphi & \sin\psi \sin\theta \cos\varphi & \cos\theta \cos\varphi \\ +\sin\psi \sin\varphi & -\cos\psi \sin\varphi & \end{bmatrix} = \begin{matrix} \text{Inertial} \\ \text{System} \end{matrix} \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = \begin{matrix} \text{Body} \\ \text{System} \end{matrix} \begin{bmatrix} \xi_B \\ \eta_B \\ \zeta_B \end{bmatrix} \quad (C-25)$$

To formulate the rate of change of the Euler angles, consider the vehicle at an instantaneous position displaced from the fixed reference frame by the angles ψ , θ , and φ . In the body axes system the instantaneous angular velocities, P , Q , and R , can be represented by vectors along the x_B , y_B , and z_B axes respectively. The rates of change of the Euler angles, $\dot{\psi}$, $\dot{\theta}$, and $\dot{\varphi}$, can also be represented by vectors (note that these vectors are not orthogonal).

From the positions shown in Figure C-7, the following relationships can be obtained by direct resolution:

$$P = \dot{\varphi} - \psi \sin\theta$$

$$Q = \dot{\theta} \cos\varphi + \dot{\psi} \sin\varphi \cos\theta$$

$$R = \dot{\psi} \cos\varphi \cos\theta - \dot{\theta} \sin\varphi$$

(C-26)

By rearranging these equations, the rates of change of the Euler angles may be expressed as functions of the instantaneous angular velocities. Thus,

Roll

$$\dot{\varphi} = P + Q \tan\theta \sin\varphi + R \tan\theta \cos\varphi$$

Pitch

$$\dot{\theta} = Q \cos\varphi - R \sin\varphi$$

(C-27)

Yaw

$$\dot{\psi} = R \left(\frac{\cos\varphi}{\cos\theta} \right) + Q \left(\frac{\sin\varphi}{\cos\theta} \right)$$

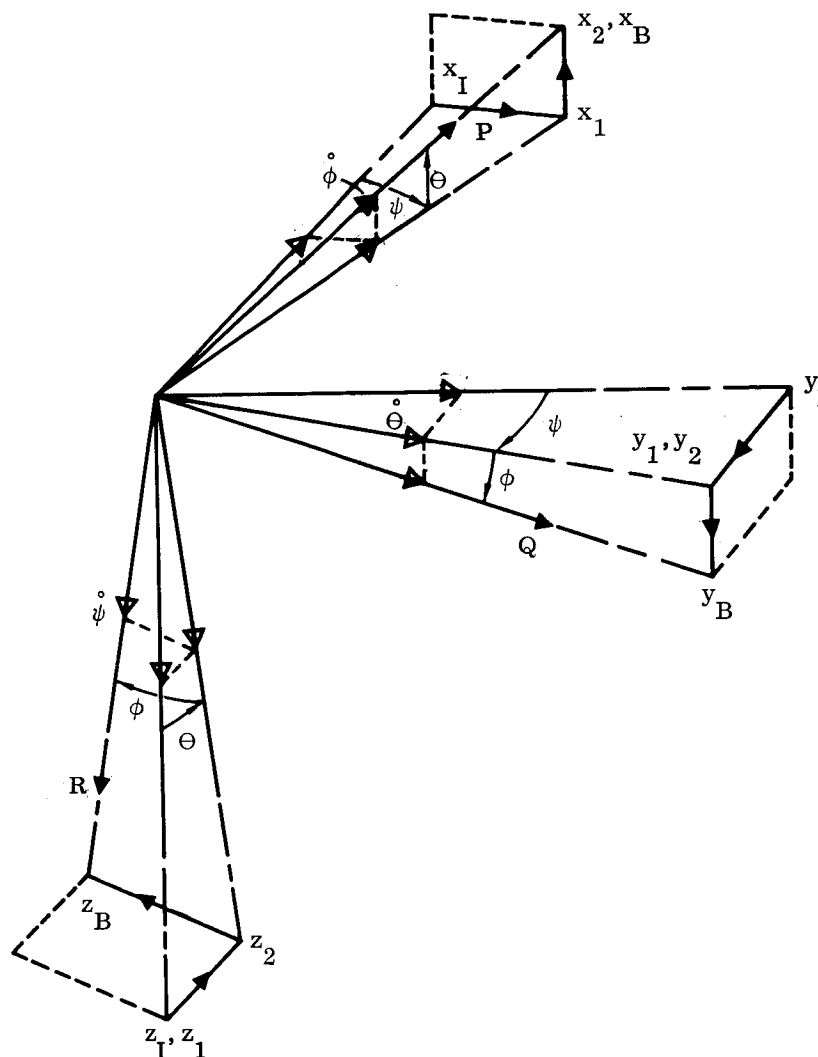


Figure C-7. Vector Representation of Instantaneous Angular Velocities and Rates of Change of the Eulerian Axes

C.2.5 Analysis Simplifications. The foregoing analysis has been considerably simplified in the results presented in Section 3.4 by considering motion in only one plane at a time. The graphical presentation of vehicle motion in the phase plane, which plots vehicle rate versus vehicle position in one plane, is easily interpreted, and the analog simulation used to obtain these results is vastly simplified, there being no need for Euler angle resolution. However, it should be noted that the actual vehicle behavior does take place in more than one plane at a time. Consequently the flight data, if presented in the form of, say, rate gyro signal versus resolver chain signal, would include the cross-coupling effects which have been neglected in the results of Section 3.4.

A second simplification has been to neglect the external torques (other than control torques) which act on the vehicle in the presentation of the phase plane behavior. These torques influence the phase plane trajectories, but if they are less than, say, 20 percent of the control torques, the influence is relatively slight. Fortunately, this assumption is valid throughout most of the coast phase period. The exceptions to this are discussed in greater detail. The approach which has been used is to summarize these torques, and draw conclusions from the relative magnitudes of the external and control torques.

C.3 AUTOPILOT EQUATIONS. This section contains the derivations of the equations describing the coast phase autopilot behavior.

C.3.1 Modes of Operation. The attitude control system has two possible modes of operation during the AC-5 mission. Either of these can be selected by the autopilot programmer at a preset time or in response to a guidance command.

C.3.1.1 "All-Off" Mode. This mode of operation is self-explanatory. The engine control logic prevents any of the attitude control jets from firing. This mode of operation applies during all powered flight phases from launch on.

C.3.1.2 "Separate On" Mode. During this mode of operation all attitude control engines are off until commanded on by the engine control logic. On earlier Centaur vehicles having the four hydrogen peroxide vernier engines, an "All-On" mode had the vernier engines on until commanded off by the engine control logic. This mode was intended for purposes of propellant settling or velocity trim. For the "Separate-On" mode, the reverse was true – the vernier engines were off until commanded on by the engine control logic for purposes of controlling large error signals. In either instance, the smaller engines operated in the same manner. For the AC-5 vehicle, the logic and circuitry to accomplish these functions is present although the vernier engines themselves are not.

C.3.2 Autopilot Switching Lines. During the "Separate-On" mode of operation, the engine control logic commands one or more attitude control engines to fire when the autopilot error signal exceeds a preset threshold. The error signal, in general, is composed of both rate and position error information; the former obtained from the rate gyros and the later from the channel selector. There is no position reference in the roll channel, although an integrator network in the error signal filter-amplifier provides the equivalent of a position reference.

The error signal is filtered for noise suppression. In addition, the threshold circuitry incorporates hysteresis. In the equations that follow, the filtering is ignored although it is included in simulations of the coast phase autopilot. The time constants of these filters are listed in Table 2.2-2.

The channel selector obtains its input from the platform resolver chain whose output is equivalent to a position error. The channel selector output is equivalent to a rate command since it is summed with the rate gyro signal. The condition for engines on in the pitch and yaw channels is given by:

$$|\dot{\eta} + \dot{\eta}_a| \geq T_c + \frac{h}{2}, \quad (C-28)$$

where

$$\dot{\eta}_a = \frac{\eta}{K_R \tau_\eta}$$

The condition for engines off is

$$|\dot{\eta} + \dot{\eta}_a| \leq T_c - \frac{h}{2} \quad (C-29)$$

For large excursions in vehicle attitude such that the magnitude of the position error is limited by the channel selector, the condition for engines on becomes:

$$|\dot{\eta} + \dot{\eta}_c| \geq T_c + \frac{h}{2} \quad (C-30)$$

and the condition for engines off is given by:

$$|\dot{\eta} + \dot{\eta}_c| \leq T_c - \frac{h}{2} \quad (C-31)$$

where:

- $\eta \sim$ Vehicle angular displacement error
- $\dot{\eta} \sim$ Vehicle angular rate
- $\dot{\eta}_c \sim$ Maximum possible commanded vehicle angular rate
- $K'_{R\eta} \sim$ Autopilot gain ratio of position to rate
- $T_c \sim$ Threshold value
- $h \sim$ Hysteresis designed in threshold circuitry

These conditions are depicted graphically by the switching lines in Figure C-8.

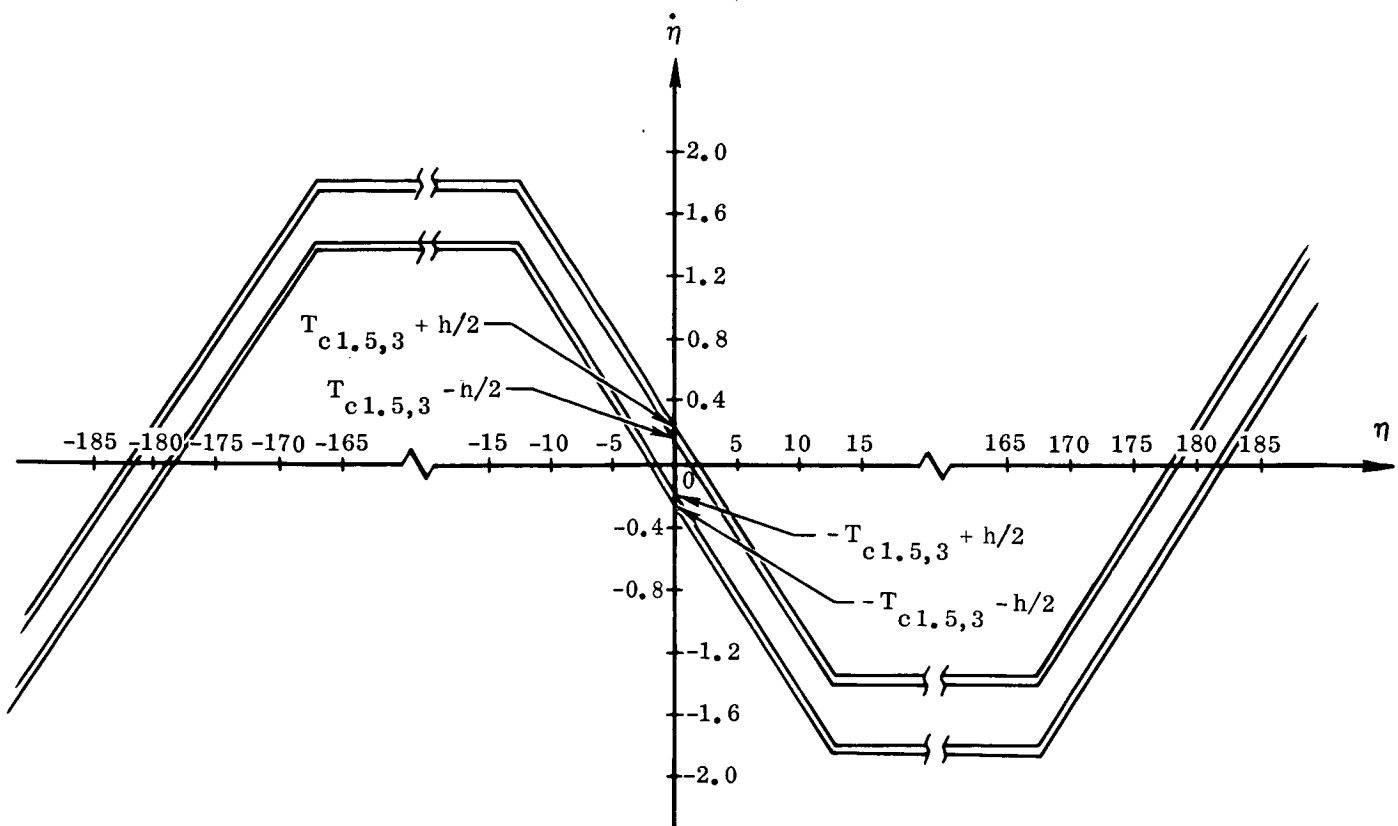


Figure C-8. Pitch and Yaw Attitude Control System Switching Lines with Attitude Reference

The threshold switching lines for the pitch, yaw, or roll planes with no attitude reference are illustrated in Figure C-9. The condition for engines on is given by:

$$|\dot{\eta}| \geq T_c + \frac{h}{2} \quad (C-32)$$

The condition for engines off is:

$$|\dot{\eta}| \leq T_c - \frac{h}{2} \quad (C-33)$$

The switching lines for the roll plane when the filter-amplifier integrator network is active are illustrated in Figure C-10. This figure does not include the effects of integrator saturation. The integrator saturates for position excursions (integrated rates) greater than ± 16.1 degrees. Upon saturation, the position reference is lost. When the rate signal passes back through zero, the position reference is restored with the vehicle roll attitude excursion during integrator saturation being lost. The amplifier will also saturate for vehicle roll rates exceeding approximately ± 1.95 deg/sec. In either case, upon saturation, the roll attitude control jets remain on until the vehicle roll attitude and rate are below the saturation level, at which time normal operation continues. These conditions are not expected in flight.

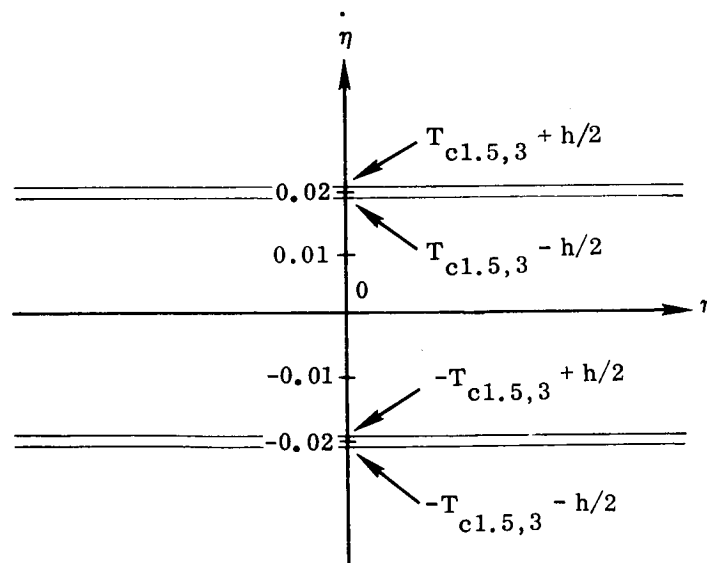


Figure C-9. Pitch, Yaw or Roll Attitude Control System Switching Lines, No Attitude Reference

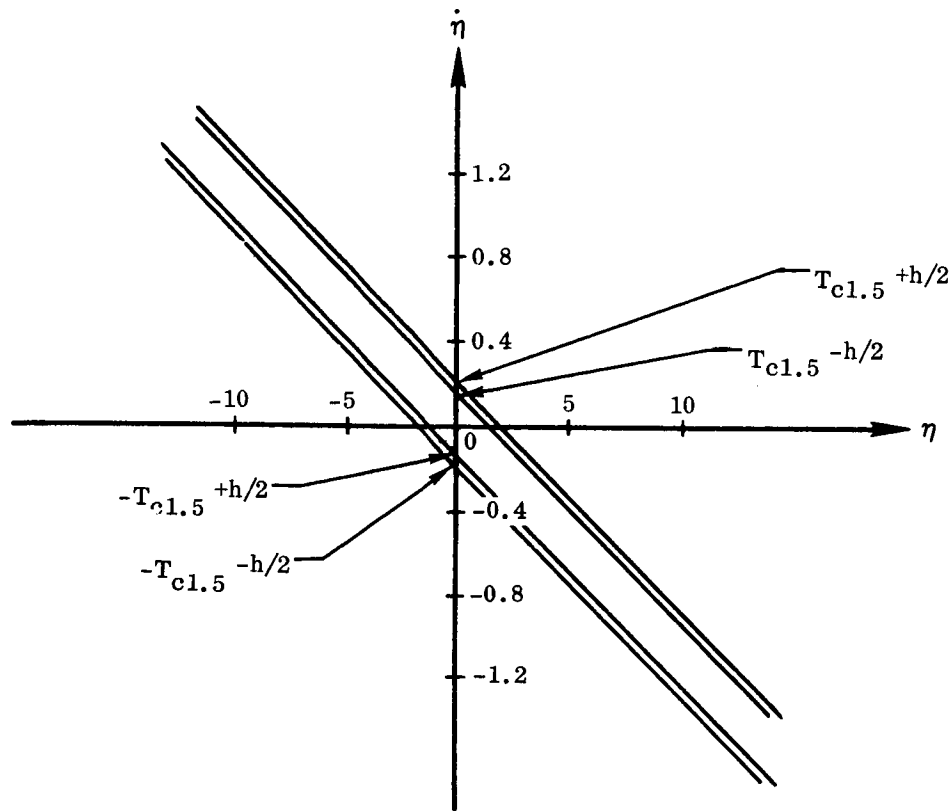


Figure C-10. Roll Attitude Control System Switching Lines with Roll Channel Integrator

The condition for engines on is expressed by

$$|\dot{\eta} + \dot{\eta}_a| \geq T_c + \frac{h}{2} \quad (C-34)$$

The condition for engines off is

$$|\dot{\eta} + \dot{\eta}_a| \leq T_c - \frac{h}{2} \quad (C-35)$$

where

$$\dot{\eta}_a = \frac{\phi}{K'_{R_\phi}}$$

and where

$K'_{R_\phi} \sim$ Autopilot roll gain ratio of position to integrated rate

$\phi \sim$ Vehicle roll angular displacement.

C.3.3 Engine Control Logic. The engine logic circuit is represented by the following Boolean equations (Reference C-3):

1.5-Pound Yaw and Roll Control Engines

$$A_1 = K' Y_3' R_1' (Y_2 + R_2) \quad (C-36)$$

$$A_2 = K' Y_3' R_2' (Y_2 + R_1) \quad (C-37)$$

$$A_3 = K' Y_2' R_1' Y_3 + R_2 \quad (C-38)$$

$$A_4 = K' Y_2' R_2' (Y_3 + R_1) \quad (C-39)$$

3-Pound Pitch Control Engines

$$P_1 = K' P_2 \quad (C-40)$$

$$P_2 = K' P_3 \quad (C-41)$$

where

$K' = 1$, SEPARATE ON command

$P_2 = 1$, If pitch rate command or error equal 0.2 deg/sec

$P_3 = 1$, If pitch rate command or error equal -0.2 deg/sec

$Y_2 = 1$, If yaw rate command or error equal 0.2 deg/sec

$Y_3 = 1$, If yaw rate command or error equal -0.2 deg/sec

$R_1 = 1$, If roll rate command or error equal 0.2 deg/sec

$R_2 = 1$, If roll rate command or error equal -0.2 deg/sec

For the above logic equations, the following definitions apply:

- Non-primed outputs (P_2 , P_3 , Y_2 , Y_3 , R_1 , and R_2) - When the non-primed outputs are true (equal to "1") the appropriate engines are commanded ON.
- Primed outputs (P_2' , P_3' , Y_2' , Y_3' , R_1' , and R_2') - When the primed outputs are true (equal to "1") the appropriate engines are commanded OFF.

Therefore, if a non-primed output is equal to "1" the associated primed output is equal to "0" and vice versa.

C.4 STATISTICAL TECHNIQUES. The disturbing torques acting on the Centaur vehicle are random and are best handled statistically. The following discussion is general in nature and provides the theoretical justification for much of the coast phase analysis as well as for the other analyses throughout the report.

C.4.1 Introduction. The output of most physical systems or processes is random to some extent. That is, either the input forcing function or the parameters defining the system, or in many cases both, are predictable only in a statistical sense. Therefore, the system output is, strictly speaking, a random variable which can only be predicted in a statistical sense.

The function which relates the random output to the random input and/or random system parameters is sometimes known explicitly. More often, however, especially in connection with processes involving complex nonlinear systems, this functional relationship is not known in explicit form. In these cases, the output can frequently be evaluated for any specified input and system parameters. In either case, a complete statistical description of the output random variable requires knowledge of its probability density function (frequency function).

Fortunately, the probability density function corresponding to a large number of physical processes is quasi-normal. Thus, the mean and standard deviation completely define the probability density function in these cases. Unfortunately, however, there also arise many practical problems for which the assumption of normality is not justified. The mean and standard deviation are still very important parameters as they usually provide the best measures of central tendency and dispersion. In fact, Tchebycheff's theorem, valid for any probability density function, can always be used when only the mean and standard deviation are known.

It is the purpose of this discussion to present a rigorous method for determining the mean and standard deviation of any dependent random variable, given the statistical parameters of the independent random variables involved.

C.4.2 Analysis and Discussion. Consider the random variable w defined as follows:

$$w = \varphi(x_1, x_2, \dots, x_n) \quad (C-42)$$

Here, the x_i are independent random variables having mean, $\bar{x}_i$, and standard deviation, σ_{x_i} , respectively. In addition, let $f_i(x_i)$ denote the probability density function corresponding to the random variable x_i . Note that no restrictions are being placed on the form of the $f_i(x_i)$. The following definitions are fundamental and will be used repeatedly in the analysis which follows. They are,

$$\int_{-\infty}^{\infty} f_i(x_i) dx_i = 1 \quad (C-43)$$

$$\int_{-\infty}^{\infty} x_i f_i(x_i) dx_i = \bar{x}_i \quad (C-44)$$

$$\int_{-\infty}^{\infty} (x_i - \bar{x}_i)^2 f_i(x_i) dx_i = \sigma_{x_i}^2 \quad (C-45)$$

The explicit functional form of φ need not be known. However, it will be assumed that the numerical value of w corresponding to any given set of the x_i ($i = 1, 2, \dots, n$) can be determined by some computational technique (e.g., manual, digital, or analog computer).

The mean and variance of the random variable w are defined as

$$\bar{w} = \int_{-\infty}^{\infty} z f(z) dz \quad (C-46)$$

and

$$\begin{aligned} \sigma_w^2 &= \int_{-\infty}^{\infty} (z - \bar{w})^2 f(z) dz \\ &= \int_{-\infty}^{\infty} z^2 f(z) dz - \bar{w}^2. \end{aligned} \quad (C-47)$$

Here, $f(z)$ is the probability density function for w and is defined as the derivative of the cumulative distribution function. That is,

$$f(z) = \frac{d F(z)}{dz} \quad (C-48)$$

where

$$F(z) = P(w \leq z). \quad (C-49)$$

Now, the probability density function for w , $F(z)$, can be expressed as

$$F(z) = P(w \leq z) = \int \dots \int f_1(x_1) \dots f_n(x_n) dx_1 \dots dx_n \quad (C-50)$$

$$\varphi(x_1, \dots, x_n) \leq z$$

where the region of integration is as indicated by the inequality, $\varphi(x_1, \dots, x_n) \leq z$. Using Equations C-48 and C-50, Equation C-46 can be written as

$$\bar{w} = \int_{-\infty}^{\infty} z \left(\frac{d}{dz} \int \dots \int f_1(x_1) \dots f_n(x_n) dx_1 \dots dx_n \right) \quad (C-51)$$

$$dz \varphi \leq z$$

which is equivalent to

$$\bar{w} = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \varphi(x_1, \dots, x_n) f_1(x_1) \dots f_n(x_n) dx_1 \dots dx_n. \quad (C-52)$$

Similarly, Equation C-47 can be written,

$$\sigma_w^2 = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} [\varphi(x_1, \dots, x_n)]^2 f_1(x_1) \dots f_n(x_n) dx_1 \dots dx_n - \bar{w}^2. \quad (C-53)$$

Now, expanding the integrand of Equation C-52 in a Taylor's Series about the point $(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$, one obtains,

$$\bar{w} = \sum_{k_1, \dots, k_n=0}^{\infty} \frac{1}{k_1! \dots k_n!} \frac{\partial^k \phi(\bar{x}_1, \dots, \bar{x}_n)}{\partial x_1^{k_1} \dots \partial x_n^{k_n}} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} y_1^{k_1} \dots y_n^{k_n} f_1(y_1) \dots f_n(y_n) dy_1 \dots dy_n \quad (C-54)$$

where

$$y_i = x_i - \bar{x}_i; i = 1, 2, \dots, n \quad (C-55)$$

and

$$k = k_1 + k_2 + \dots + k_n. \quad (C-56)$$

The following properties of the $f_i(y_i)$ follow immediately from Equation C-55:

$$\int_{-\infty}^{\infty} f_i(y_i) dy_i = 1 \quad (C-57)$$

$$\int_{-\infty}^{\infty} y_i f_i(y_i) dy_i = \bar{y}_i = 0 \quad (C-58)$$

$$\int_{-\infty}^{\infty} y_i^2 f_i(y_i) dy_i = \sigma_{y_i}^2 = \sigma_{x_i}^2 \quad (C-59)$$

Thus, one finally obtains the result

$$\bar{w} = \bar{\phi} + \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 \phi}{\partial x_i^2} \sigma_{x_i}^2 + \dots \quad (C-60)$$

where the bar (e.g., $\bar{\phi}$) signifies that the function or derivative has been evaluated at the point $(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$. Similarly, Equation C-53 becomes

$$\sigma_w^2 = \bar{\phi}^2 + \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 \phi^2}{\partial x_i^2} \sigma_{x_i}^2 + \frac{1}{8} \sum_{\substack{i,j=1 \\ i \neq j}}^n \frac{\partial^4 \phi^2}{\partial x_i^2 \partial x_j^2} \sigma_{x_i}^2 \sigma_{x_j}^2 - \bar{w}^2 + \dots \quad (C-61)$$

Now, since

$$\frac{\partial^2 \phi^2}{\partial x_i^2} = 2 \left(\frac{\partial \phi}{\partial x_i} \right)^2 + 2\phi \frac{\partial^2 \phi}{\partial x_i^2}$$

and

$$\frac{\partial^4 \varphi^2}{\partial x_i^2 \partial x_j^2} = 4 \left(\frac{\partial^2 \varphi}{\partial x_i \partial x_j} \right)^2 + 4 \frac{\partial \varphi}{\partial x_i} \frac{\partial^3 \varphi}{\partial x_i \partial x_j^2} + 4 \frac{\partial \varphi}{\partial x_j} \frac{\partial^3 \varphi}{\partial x_i^2 \partial x_j} + 2 \frac{\partial^2 \varphi}{\partial x_i^2} \frac{\partial^2 \varphi}{\partial x_j^2} + 2 \varphi \frac{\partial^4 \varphi}{\partial x_i^2 \partial x_j^2},$$

Equation C-61 can be written in the following more convenient form:

$$\begin{aligned} \sigma_w^2 = & \sum_{i=1}^n \left[\left(\overline{\frac{\partial \varphi}{\partial x_i}} \right)^2 - \frac{1}{4} \left(\overline{\frac{\partial^2 \varphi}{\partial x_i^2}} \right)^2 \sigma_{x_i}^2 \right] \sigma_{x_i}^2 + \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^n \left[\left(\overline{\frac{\partial^2 \varphi}{\partial x_i \partial x_j}} \right)^2 \right. \\ & \left. + \frac{\overline{\partial \varphi}}{\partial x_i} \frac{\overline{\partial^3 \varphi}}{\partial x_i \partial x_j^2} + \frac{\overline{\partial \varphi}}{\partial x_j} \frac{\overline{\partial^3 \varphi}}{\partial x_i^2 \partial x_j} + \frac{1}{2} \overline{\varphi} \frac{\overline{\partial^4 \varphi}}{\partial x_i^2 \partial x_j^2} \right] \sigma_{x_i}^2 \sigma_{x_j}^2 + \dots \end{aligned} \quad (C-62)$$

Most practical engineering-type processes, in particular the ones considered in this report, require the use of only the linear terms in the above series expansions. Hence, the approximate formulas

$$\begin{aligned} \bar{w} & \approx \varphi(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \\ \sigma_w^2 & \approx \sum_{i=1}^n \left[\frac{\partial \varphi(\bar{x}_1, \dots, \bar{x}_n)}{\partial x_i} \right]^2 \sigma_{x_i}^2 \end{aligned} \quad (C-63)$$

are adequate. If there is any question regarding the legitimacy of using these approximate formulas, one may calculate the next higher order derivatives in order to check their contribution to the total summation.

In those cases where $w = \varphi(x_1, x_2, \dots, x_n)$ is not known in explicit form, or where the known functional form is extremely complicated, the required derivatives must be calculated numerically. Using the first forward difference, one has

$$\frac{\overline{\partial \varphi}}{\partial x_i} \approx \frac{\varphi(\bar{x}_1, \dots, \bar{x}_i + \Delta x_i, \dots, \bar{x}_n) - \varphi(\bar{x}_1, \dots, \bar{x}_i, \dots, \bar{x}_n)}{\Delta x_i} \quad (C-64)$$

where Δx_i is an arbitrary increment in the random variable x_i from its mean value, $\bar{x}_i$. The higher-order derivatives such as

$$\frac{\partial^2 \varphi}{\partial x_i^2} \text{ and } \frac{\partial^2 \varphi}{\partial x_i \partial x_j}$$

can be evaluated in a similar manner using standard numerical techniques.

C.5 REFERENCES.

- C-1 Yoshioka, V. T., Centaur Attitude Control System Study-Removal of The 50 lb H₂O₂ Engines, General Dynamics/Astronautics Memo CD-64-039-CEN, 4 March 1964.
- C-2 Frech, E., Parameter Study and Analysis of Centaur On-Off Attitude Control System, General Dynamics/Astronautics Report AE60-0374, 22 April 1960.
- C-3 Trotter, B. E., and Thacker, I. M., The Autopilot for the Centaur Space Vehicle, Model 55, General Dynamics/Astronautics Report AE60-0569, 1 August 1960.
- C-4 Rosenbaum, E. S., Attitude Control Engine Transients, General Dynamics/Astronautics Memo 962-2-ESR-4, 30 November 1964.

APPENDIX D
CENTAUR/SPACECRAFT
SEPARATION DYNAMICS

LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
F_i	Force exerted by the i^{th} spring	lb
I_{ij} ($i, j = x, y, z$)	Mass moment or product of inertia referenced to a set of centroidal axes parallel to the geometric axes of the vehicle. Vehicle indicated by superscript	slug-ft ²
K_i	Spring constant of the i^{th} spring	lb/ft
ℓ_i	Undeformed length of the i^{th} spring	ft
M_1, M_2	Total mass of the spacecraft and of Centaur respectively	slug
r	Radial distance from the X axis to the centerline of the springs in the separation plane (cf. Δ_i)	ft
s	Maximum distance the springs are allowed to recoil	ft
X, Y, Z	Rectangular coordinate axes of the inertial frame of reference defined by the position of the combined Centaur/spacecraft body axis system at the instant of separation	
X_1, Y_1, Z_1 X_2, Y_2, Z_2	Rectangular coordinate axes of the spacecraft (subscript 1) and Centaur (subscript 2); rigidly attached to the respective vehicles	
X_{1X}, Y_{1Y}, Z_{1Z} X_{2X}, Y_{2Y}, Z_{2Z}	Linear displacements - in X, Y, and Z directions - of the spacecraft (subscript 1) and Centaur (subscript 2) with respect to the inertial reference frame	ft
X_{12}	Relative longitudinal distance between the spacecraft and Centaur	ft
$\bar{y}_1, \bar{y}_2$	Y component of center-of-gravity offset for the spacecraft and Centaur respectively	ft
$\bar{z}_1, \bar{z}_2$	Z component of center-of-gravity offset for the spacecraft and Centaur respectively	ft
Δ_i	Angle between r and the -Z axis in the separation plane; considered positive when rotated counterclockwise from the -Z axis for the spacecraft and clockwise for Centaur (cf. r)	deg
δ_i	Distance the i^{th} spring moves in time (t), including angular effects	ft
$\delta_{\psi_{1(t)}}, \delta_{\theta_{1(t)}}$	Variation of $\dot{\psi}_1(t)$ and of $\dot{\theta}_1(t)$	rad/sec
σ	Standard deviation of a normal distribution	
σ^2	Statistical variance	
ψ, θ, ϕ	Eulerian angles in yaw, pitch, and roll for the spacecraft when subscripted with 1 and for Centaur with 2	rad
ω	Vector sum angular velocity of the spacecraft	deg/sec

D.1 INTRODUCTION. This appendix presents the derivation of the equations describing the behaviour of the Centaur vehicle and the spacecraft during their separation. A more complete presentation is given in Reference D-1.

D.1.1 System Description. The Centaur and spacecraft are designed to separate through the action of three sets of helical compression springs which impart longitudinal and small angular motions to the spacecraft. The springs and latches on which the spacecraft is mounted are located on the forward interface of the payload adapter. As shown in Figure D-1, the three latches are equally spaced around the adapter perimeter, and the associated springs are offset 3.50 inches from the latches, having their centerline on a 46.24-inch diameter circle. Each separation-spring assembly consists of a spring support and plunger holding two helical springs concentrically mounted to act together in a direction parallel to the vehicle longitudinal axis. The plunger, running aft through the spring support, terminates in a head which limits its motion to one inch. The inner springs (Numbers 1, 3, and 5) exert a smaller force than the outer ones (Numbers 2, 4, and 6).

D.1.2 Latch Operation. The latch assembly, consisting of a coupling stud on the spacecraft, is gripped by a latch which in turn held by a piston. The latches are released when the pistons are driven aft by three pairs of pyrotechnic charges which fire in unison upon receiving a discrete signal through the separation switch. In the ensuing one-seventh second, the springs recoil against the spacecraft, imparting a forward velocity and angular rates to the spacecraft.

D.2 SUMMARY OF ANALYSIS AND ASSUMPTIONS. A set of differential equations is first formulated to express the accelerations due to the sum of all forces exerted by the six springs. The following forces impart negligible motions to the spacecraft and are not considered:

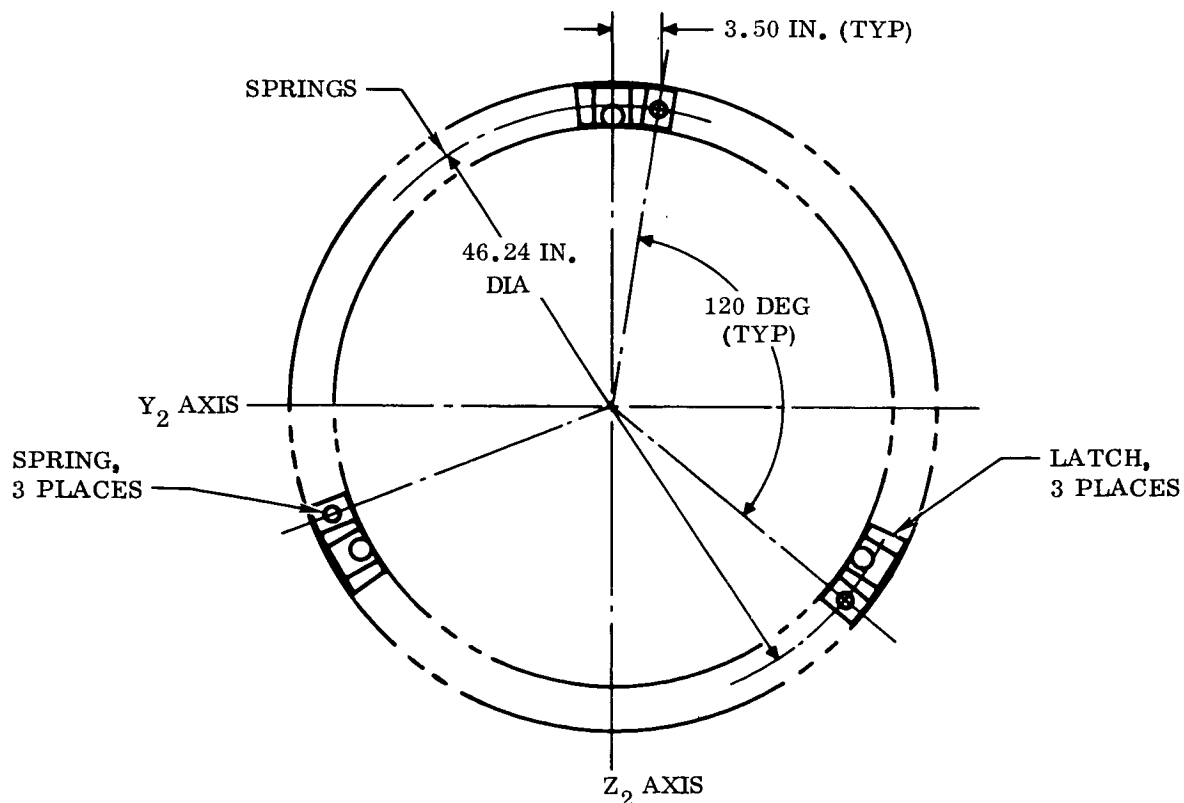


Figure D-1. Payload Adapter with Springs and Latches.
Looking Aft Toward Centaur.

- a. Separation Switches. The separation switches are mounted on the spacecraft at its interface with the payload adapter, one at each of three attachment points. When separation is initiated, each switch imparts a nominal thrust of 1.8 pounds to the spacecraft. This thrust falls off linearly over the first half-inch of travel, at which point the thrust becomes zero. Each switch parallels the other two in thrust within ± 0.1 pound over the entire range of travel.
- b. Extensometers. The extensometers are mounted at three points around the periphery of the inter-stage adapter. Each has a wiper arm connected to the aft end of one of the spring plungers (which pass through the spring supports). Their purpose is to measure the position of the spring (as a function of time) and thus the velocity with which the spring moves. Each extensometer has a nominal force of 1.5 pounds which parallels the other two forces within $\pm 1/4$ pound.

The resultant changes due to these additional forces are less than one percent and are therefore ignored.

The motions imparted to the two vehicles depend, in part, on the spring constants, the lengths of the springs, and the distance they are allowed to recoil. They also depend on the masses of the Centaur and the spacecraft and on the offsets of their respective centers of gravity from the geometric axes.

In computing the longitudinal acceleration of the spacecraft with respect to the inertial reference frame (that of the combined Centaur/spacecraft vehicle), it is necessary to consider the relative motion of the two vehicles. An "exact" formulation in which all cross-coupling effects are considered yields equations which are too unwieldy for practical use. Fortunately, the equations can be considerably simplified by neglecting some of these cross-coupling effects without incurring any appreciable error in the final solution. The separation distance between the two vehicles, for example, turns out to be essentially unaltered by the presence of small angular rates; hence, these terms may be dropped from the differential equation describing separation distance. To justify such deletions, it was necessary first to compute the time at which the springs reach exactly one inch, their full recoil distance. Nominal values were employed for the spring constants, the undeformed lengths of the springs, and their geometrical locations. The desired interval was found to be 0.139 second and is the same for all springs within a thousandth of a second.

Inspection of the solution reveals that five of its terms are small compared to the sixth and can be deleted. The resulting equations, thus simplified in several ways, are finally compared with the exact equations, and the solutions are found to differ by less than two percent. These approximations are employed in the calculations of the mean longitudinal velocity and its variance.

Similarly, in computing angular rates, terms in ψ (yaw) are deleted from pitch equations and vice versa, since these terms introduce only negligible coupling effects. For the same reason, any offsets of the centers of gravity from the geometrical axis can be omitted in terms where they are coupled or squared. The solutions to these simplified equations, evaluated at the time of full recoil, fall within two percent of the corresponding exact values, thus justifying the use of the simplified equations. In their final simplified form, the solution to the equations of motion for pitch and yaw reduce, in each case, to two terms. Of these, the term involving the spacecraft center-of-gravity offset predominates over the other so that it alone need be considered. It is thus concluded that the angular rates are in simple proportion to the spacecraft center-of-gravity offset.

Finally, applying the fundamental principles of statistics with data adapted from specified design tolerances, the standard deviations from the resulting mean angular and linear rates are computed.

D.2.1 Dynamic Equations. Three sets of axes are used as illustrated in Figure D-2. The coordinates (X, Y, Z) refer to the inertial frame of reference defined by the position of the combined Centaur/spacecraft body axes at the instant of separation. The coordinates (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2) represent body axes rigidly attached to the spacecraft and Centaur respectively. In these expressions, as in those following, a subscript or superscript 1 denotes the spacecraft and a 2 the Centaur vehicle.

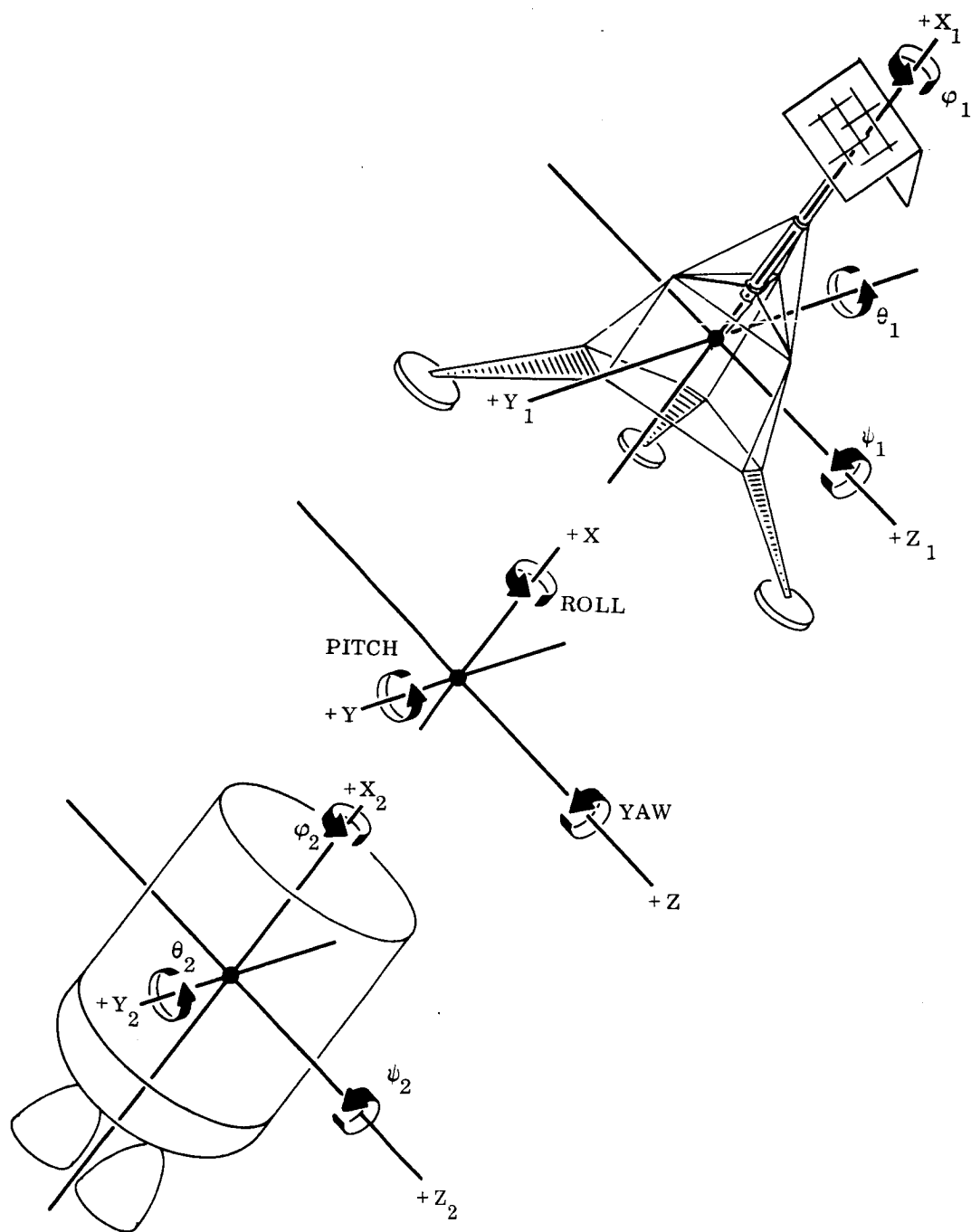


Figure D-2. Centaur/Spacecraft Coordinate System

D.2.1.1 Summation of Torques. Let θ_1 and ψ_1 represent angular displacements and $I_{yy}^{(1)}$ and $I_{zz}^{(1)}$ moments of inertia in pitch and yaw respectively for the spacecraft. Summing the torques about the centroidal axes of the two vehicles yields the following equations for the accelerations with respect to the inertial frame:

$$\ddot{\psi}_1 = \frac{1}{I_{zz}^{(1)}} \sum_{i=1}^6 (r \sin \Delta_1 + \bar{y}_1) F_i \quad (D-1)$$

$$\ddot{\theta}_1 = -\frac{1}{I_{yy}^{(1)}} \sum_{i=1}^6 (r \cos \Delta_1 + \bar{z}_1) F_i \quad (D-2)$$

$$\ddot{\psi}_2 = \frac{1}{I_{yy}^{(2)}} \sum_{i=1}^6 (r \sin \Delta_1 + \bar{y}_2) F_i \quad (D-3)$$

$$\ddot{\theta}_2 = \frac{1}{I_{yy}^{(2)}} \sum_{i=1}^6 (r \cos \Delta_1 + \bar{z}_2) F_i \quad (D-4)$$

where the summation refers to the six separation springs, each located around the interface by the polar coordinates r and Δ_1 (Figures D-3 and D-4) and exerting a force, F_i (Table E-10). The roll angle and all product-of-inertia terms are assumed to be negligible.

D.2.1.2 Separation Spring Movements and Forces. The distances (δ_i) that the springs move are given by

$$\delta_i = X_{12} + (r \sin \Delta_1 + \bar{y}_1) \psi_1 - (r \cos \Delta_1 + \bar{z}_1) \theta_1 - (r \sin \Delta_1 + \bar{y}_2) \psi_2 + (r \cos \Delta_1 + \bar{z}_2) \theta_2 \quad (D-5)$$

$$i=1, \dots, 6$$

where the distance X_{12} is the relative longitudinal separation between the two vehicles.

Now, assuming that each spring force (F_i) varies linearly with displacement, we have

$$F_i = K_i (\ell_i - \delta_i) \quad (D-6)$$

$$i=1, \dots, 6$$

where K_i is the spring constant of the i^{th} spring and ℓ_i is its undeformed length (see Table E-10).

D.2.1.3 Basic Dynamic Equations. The general equations expressing the dynamics of the separation process are then derived by substituting Equation D-5 into Equation D-6 and the result into Equations D-1 through D-4. The resulting angular accelerations are therefore

$$\ddot{\psi}_1 = \frac{1}{I_{zz}^{(1)}} \sum_{i=1}^6 K_i (r \sin \Delta_1 + \bar{y}_1) \left\{ \ell_i - X_{12} - (r \sin \Delta_1 + \bar{y}_1) \psi_1 \right. \\ \left. + (r \cos \Delta_1 + \bar{z}_1) \theta_1 + (r \sin \Delta_1 + \bar{y}_2) \psi_2 - (r \cos \Delta_1 + \bar{z}_2) \theta_2 \right\} \quad (D-7)$$

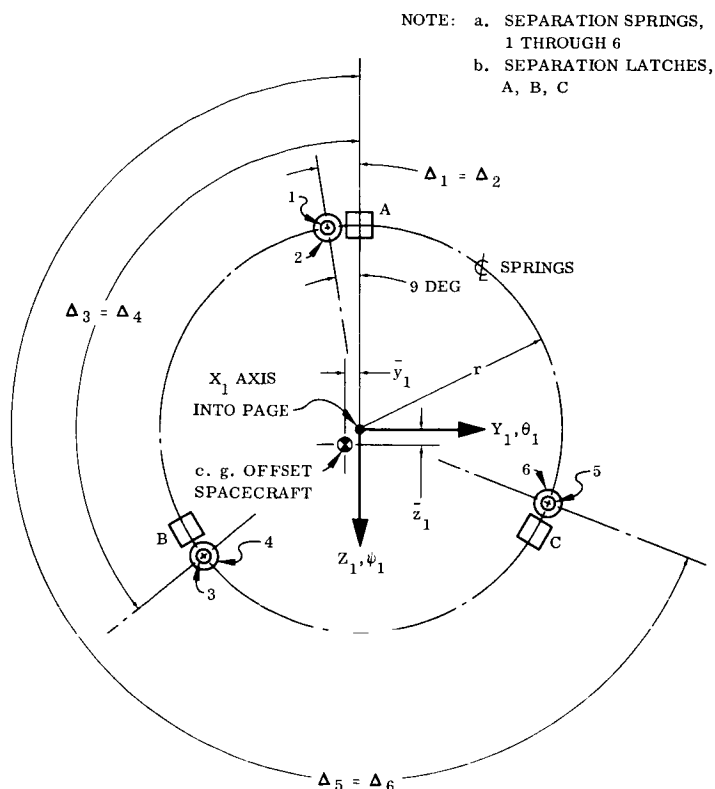


Figure D-3. Parameters of Separation System. Looking Forward Toward the Spacecraft

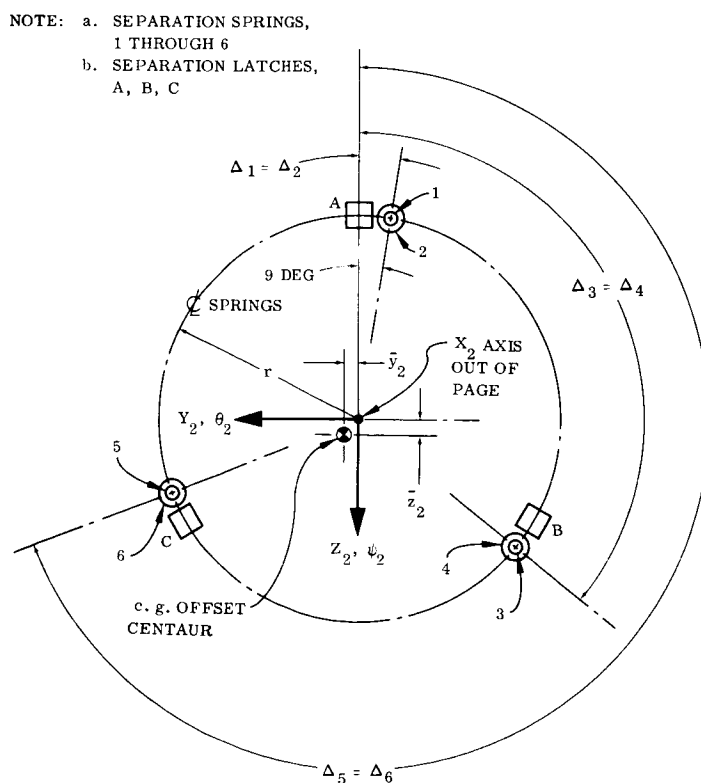


Figure D-4. Parameters of Separation System. Looking Aft Toward Centaur

$$\ddot{\theta}_1 = -\frac{1}{I_{yy}^{(1)}} \sum_{i=1}^6 K_i (r \cos \Delta_i + \bar{z}_1) \left\{ \ell_i - X_{12} - (r \sin \Delta_i + \bar{y}_1) \psi_1 \right. \\ \left. + (r \cos \Delta_i + \bar{z}_1) \theta_1 + (r \sin \Delta_i + \bar{y}_2) \psi_2 - (r \cos \Delta_i + \bar{z}_2) \theta_2 \right\} \quad (D-8)$$

$$\ddot{\psi}_2 = -\frac{1}{I_{zz}^{(2)}} \sum_{i=1}^6 K_i (r \sin \Delta_i + \bar{y}_2) \left\{ \ell_i - X_{12} - (r \sin \Delta_i + \bar{y}_1) \psi_1 \right. \\ \left. + (r \cos \Delta_i + \bar{z}_1) \theta_1 + (r \sin \Delta_i + \bar{y}_2) \psi_2 - (r \cos \Delta_i + \bar{z}_2) \theta_2 \right\} \quad (D-9)$$

$$\ddot{\theta}_2 = \frac{1}{I_{yy}^{(2)}} \sum_{i=1}^6 K_i (r \cos \Delta_i + \bar{z}_2) \left\{ \ell_i - X_{12} - (r \sin \Delta_i + \bar{y}_1) \psi_1 \right. \\ \left. + (r \cos \Delta_i + \bar{z}_1) \theta_1 + (r \sin \Delta_i + \bar{y}_2) \psi_2 - (r \cos \Delta_i + \bar{z}_2) \theta_2 \right\} \quad (D-10)$$

D.2.1.4 Linear Acceleration: Longitudinal. Neglecting initial turning rates, the linear accelerations of the spacecraft and Centaur with respect to the inertial frame are given by

$$\ddot{X}_{1X} = \frac{1}{M_1} \sum_{i=1}^6 F_i \quad (D-11)$$

$$\ddot{X}_{2X} = -\frac{1}{M_2} \sum_{i=1}^6 F_i \quad (D-14)$$

$$\ddot{Y}_{1Y} = \frac{\psi_1}{M_1} \sum_{i=1}^6 F_i \quad (D-12)$$

$$\ddot{Y}_{2Y} = -\frac{\psi_2}{M_2} \sum_{i=1}^6 F_i \quad (D-15)$$

$$\ddot{Z}_{1Z} = -\frac{\theta_1}{M_1} \sum_{i=1}^6 F_i \quad (D-13)$$

$$\ddot{Z}_{2Z} = \frac{\theta_2}{M_2} \sum_{i=1}^6 F_i \quad (D-16)$$

where M_1 and M_2 are the masses of the spacecraft and Centaur respectively, and the angular displacements ψ and θ are assumed to be small.

Major concern is with motions along the X axis. Subtracting Equation D-14 from Equation D-11, the relative longitudinal acceleration between the two vehicles is

$$\ddot{X}_{12} = \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \sum_{i=1}^6 F_i \quad (D-17)$$

To express the acceleration in terms of the spring parameters, the above equations are expanded by substituting, in place of F_i , the expression in Equation D-6 for the forces of the springs, where δ_i is given by Equation D-5. Then, the following general equation for the longitudinal acceleration of the spacecraft relative to the Centaur vehicle results.

$$\ddot{X}_{12} = \frac{M_1 + M_2}{M_1 M_2} \left\{ \sum_{i=1}^6 K_i [\ell_i - X_{12} - (r \sin \Delta_i + \bar{y}_1) \psi_1 + (r \cos \Delta_i + \bar{z}_1) \theta_1 + (r \sin \Delta_i + \bar{y}_2) \psi_2 - (r \cos \Delta_i + \bar{z}_2) \theta_2] \right\} \quad (D-18)$$

D.2.2 Simplifying Assumptions. Equations D-7 through D-10 and Equation D-18 may be combined to form a set of simultaneous differential equations governing the rotation and longitudinal translation of the spacecraft and the Centaur vehicle. These can be solved, using nominal values for all parameters, to yield the mean angular and longitudinal displacements as functions of time. It was determined, through a solution of these equations (Reference D-1), that the following simplifications could be made without incurring significant error in the results:

- a. Angular displacements do not appreciably affect the longitudinal motion.
- b. Certain second-order coupling terms may be ignored on the basis of relative magnitude.

D.2.2.1 Simplified Equations. Considering rotation in yaw, Equations D-7 and D-9 are simplified to:

$$\begin{aligned} \ddot{\psi}_1 = & -\frac{1}{I_{zz}^{(1)}} \sum_{i=1}^6 K_i (r \sin \Delta_i)^2 \psi_1 + \frac{1}{I_{zz}^{(1)}} \sum_{i=1}^6 K_i (r \sin \Delta_i)^2 \psi_2 \\ & + \frac{1}{I_{zz}^{(1)}} \sum_{i=1}^6 K_i \ell_i (r \sin \Delta_i + \bar{y}_1) - \frac{1}{I_{zz}^{(1)}} \sum_{i=1}^6 K_i (r \sin \Delta_i \\ & + \bar{y}_1) \left\{ \frac{\sum_{j=1}^6 K_j \ell_j}{\sum_{j=1}^6 K_j} \left[1 - \cos \left(\sqrt{\frac{\sum_{j=1}^6 K_j}{M}} t \right) \right] \right\} \end{aligned} \quad (D-19)$$

and

$$\begin{aligned} \ddot{\psi}_2 = & -\frac{1}{I_{zz}^{(2)}} \sum_{i=1}^6 K_i (r \sin \Delta_i)^2 \psi_2 + \frac{1}{I_{zz}^{(2)}} \sum_{i=1}^6 K_i (r \sin \Delta_i)^2 \psi_1 \\ & - \frac{1}{I_{zz}^{(2)}} \sum_{i=1}^6 K_i \ell_i (r \sin \Delta_i + \bar{y}_2) + \frac{1}{I_{zz}^{(2)}} \sum_{i=1}^6 K_i (r \sin \Delta_i \\ & + \bar{y}_2) \left\{ \frac{\sum_{j=1}^6 K_j \ell_j}{\sum_{j=1}^6 K_j} \left[1 - \cos \left(\sqrt{\frac{\sum_{j=1}^6 K_j}{M}} t \right) \right] \right\} \end{aligned} \quad (D-20)$$

where the longitudinal separation distance, $X_{12}(t)$, is found to be

$$X_{12}(t) = \frac{\sum_{i=1}^6 K_i \ell_i}{\sum_{i=1}^6 K_i} \left[1 - \cos \left(\sqrt{\frac{\sum_{i=1}^6 K_i}{M}} t \right) \right] \quad (D-21)$$

and

$$M = \frac{M_1 M_2}{M_1 + M_2} \quad (D-22)$$

D.2.3 Angular Velocities. The yaw component of the angular velocity is derived by solving Equations D-19 and D-20, yielding Equation D-23 (see Figure D-5).

Evaluating Equation D-23 for mean values of the parameters, and rearranging terms, yields an expression for the mean angular rate of the spacecraft measured in the yaw plane at time (t),

$$\begin{aligned} \bar{\psi}_1(t) = & \frac{\bar{F}}{I_{zz}^{(1)} \bar{K} \left[\frac{3}{M} - \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2 \right]} \left\{ (M) \left[\frac{3}{M} - \frac{1}{I_{zz}^{(2)}} \sum_{i=1}^3 (r \sin \Delta_{2i})^2 \right] \left(\sqrt{\frac{3\bar{K}}{M}} \right) \sin \left(\sqrt{\frac{3\bar{K}}{M}} t \right) \right. \\ & - \frac{3}{I_{zz}^{(1)} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right)} \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} \right] \sin \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] \left. \right\} \bar{y}_1 \\ & + \frac{\bar{F}}{I_{zz}^{(1)} \bar{K} \left[\frac{3}{M} - \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2 \right]} \left\{ (M) \left[\frac{1}{I_{zz}^{(2)}} \sum_{i=1}^3 (r \sin \Delta_{2i})^2 \right] \left(\sqrt{\frac{3\bar{K}}{M}} \right) \sin \left(\sqrt{\frac{3\bar{K}}{M}} t \right) \right. \\ & - \frac{3}{I_{zz}^{(2)} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right)} \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} \right] \sin \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] \left. \right\} \bar{y}_2 \end{aligned} \quad (D-24)$$

(D-23)

Figure D-5. Yaw Component of Angular Velocity Equation.

Where, for simplicity of presentation:

$$\bar{F} = \bar{F}_{2i-1} + \bar{F}_{2i} \quad \text{for } i = 1, \dots, 3$$

and

$$\bar{K} = \bar{K}_{2i-1} + \bar{K}_{2i} \quad \text{for } i = 1, \dots, 3$$

This equation consists of only two terms and is thus essentially of the form

$$\dot{\bar{\psi}}_1(t) = f(t) \bar{y}_1 + g(t) \bar{y}_2 \quad (\text{D-25})$$

Examining the parameters in the coefficients of $\bar{y}_1$ and $\bar{y}_2$, it is clear that

$$\frac{3}{M} \gg \frac{1}{I_{zz}^{(2)}} \sum_{i=1}^3 (r \sin \Delta_{2i})^2 \quad (\text{D-26})$$

and

$$I_{zz}^{(2)} \gg I_{zz}^{(1)} \quad (\text{D-27})$$

so that the coefficient of $\bar{y}_2$ is much less than the coefficient of $\bar{y}_1$. We can therefore write

$$\dot{\bar{\psi}}_1 \cong c_1 \bar{y}_1 \quad (\text{D-28})$$

where c_1 appears in place of $f(t)$ because, at a given time, this coefficient is approximately constant.

Similarly, it can be shown that

$$\dot{\bar{\theta}}_1 \cong d_1 \bar{z}_1 \quad (\text{D-29})$$

As future design modifications shift the spacecraft center of gravity to new locations, Equations D-28 and D-29 will afford a ready insight to resulting changes in angular velocities.

D.2.4 Longitudinal Velocity. To compute the longitudinal velocity of the spacecraft with respect to the inertial frame, consider first $X_{12}(t)$, the relative distance between the two vehicles. This was given in Equation D-21 and is repeated below.

$$X_{12}(t) = \frac{\sum_{i=1}^6 K_i \ell_i}{\sum_{i=1}^6 K_i} \left[1 - \cos \left(\sqrt{\frac{\sum_{i=1}^6 K_i}{M}} t \right) \right]$$

Differentiating $X_{12}(t)$ then yields the rate as

$$\dot{X}_{12}(t) = \frac{\sum_{i=1}^6 K_i l_i}{\sum_{i=1}^6 K_i} \left(\sqrt{\frac{\sum_{i=1}^6 K_i}{M}} \right) \sin \left(\sqrt{\frac{\sum_{i=1}^6 K_i}{M}} t \right) \quad (D-30)$$

which can also be written as

$$\dot{X}_{12}(t) = \frac{1}{M_1} \frac{\sum_{i=1}^6 K_i l_i}{\sqrt{\frac{\sum_{i=1}^6 K_i}{M}}} \sin \left(\sqrt{\frac{\sum_{i=1}^6 K_i}{M}} t \right) + \frac{1}{M_2} \frac{\sum_{i=1}^6 K_i l_i}{\sqrt{\frac{\sum_{i=1}^6 K_i}{M}}} \sin \left(\sqrt{\frac{\sum_{i=1}^6 K_i}{M}} t \right) \quad (D-31)$$

It can now be shown that, relative to the inertial frame of reference (X, Y, Z), the velocity of the spacecraft, $\dot{X}_{1X}(t)$, is given by

$$\dot{X}_{1X}(t) = \frac{1}{M_1} \frac{\sum_{i=1}^6 K_i l_i}{\sqrt{\frac{\sum_{i=1}^6 K_i}{M}}} \sin \left(\sqrt{\frac{\sum_{i=1}^6 K_i}{M}} t \right) \quad (D-32)$$

Equation D-21 is now solved for the time that it takes the springs to reach their full recoil distance (s) – one inch as presently designed. Substituting this expression into Equation D-32, the velocity of the spacecraft at spring distance, s, is

$$\dot{X}_{1X}(s) = \sqrt{\frac{\sum_{i=1}^6 (l_i s - \frac{1}{2} s^2) K_i}{\frac{1}{2} M_1 \left(1 + \frac{M_1}{M_2} \right)}} \quad (D-33)$$

and therefore the mean velocity is given by

$$\bar{X}_{1X}(s) = \sqrt{\frac{6 s (\bar{F} - \frac{1}{2} s \bar{K})}{M_1 \left(1 + \frac{M_1}{M_2} \right)}} \quad (D-34)$$

D.3 STANDARD DEVIATIONS FOR SEPARATION VELOCITIES. The range within which the motions of the spacecraft may be expected to remain can be shown by a statistical analysis of the deviations due to dispersions in the design parameters of the vehicles and springs.

D.3.1 Gaussian Form of the Joint Density Function. If a straightforward application of simple probability theory is to be employed, it must first be shown that the joint density function of the aggregate of all variables takes the form of a normal or Gaussian curve.

Prior to computing the joint function of the 15 variables involved, K_i , ℓ_i , $\bar{y}_1$, $\bar{y}_2$, and $\dot{\psi}_1(0)$, it should be noted that some of the functions taken separately are not normal. The probability function of the free length of the separation spring, ℓ_i , for example, can be assumed to be rectangular. As portrayed in Figure D-6, this distribution is therefore such that it is equally probable for the spring to have any length deviation, $\Delta \ell_i$, within $\pm a_1$; yet the probability that it will surpass these limits is effectively zero. The density or frequency function representing the tolerance in the spring constants is also assumed to be rectangular in form.

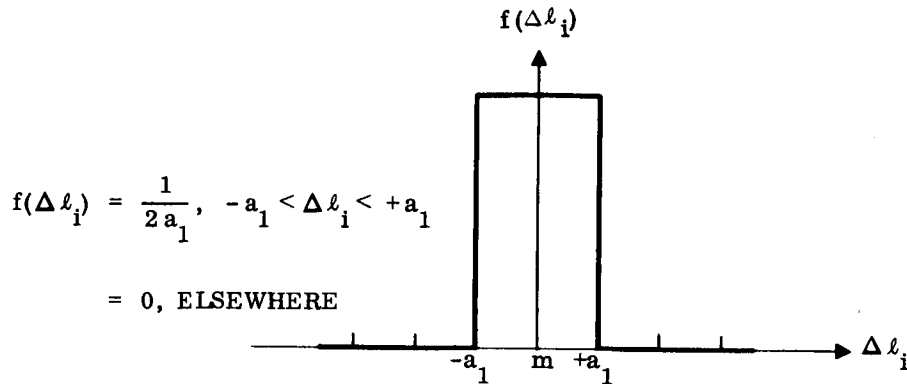


Figure D-6. Probability Density Function of the Free Spring Length Deviation, Showing Rectangular Distribution

The probability density function of the lateral center-of-gravity offset is assumed to be normal. This is a departure from previous assumptions (Reference D-1) but is felt to be more realistic. The value of the uncertainty in location of the lateral center of gravity is taken to be three times the standard deviation (3σ) of a normal distribution, and is shown as $\pm a_2$, Figure D-7.

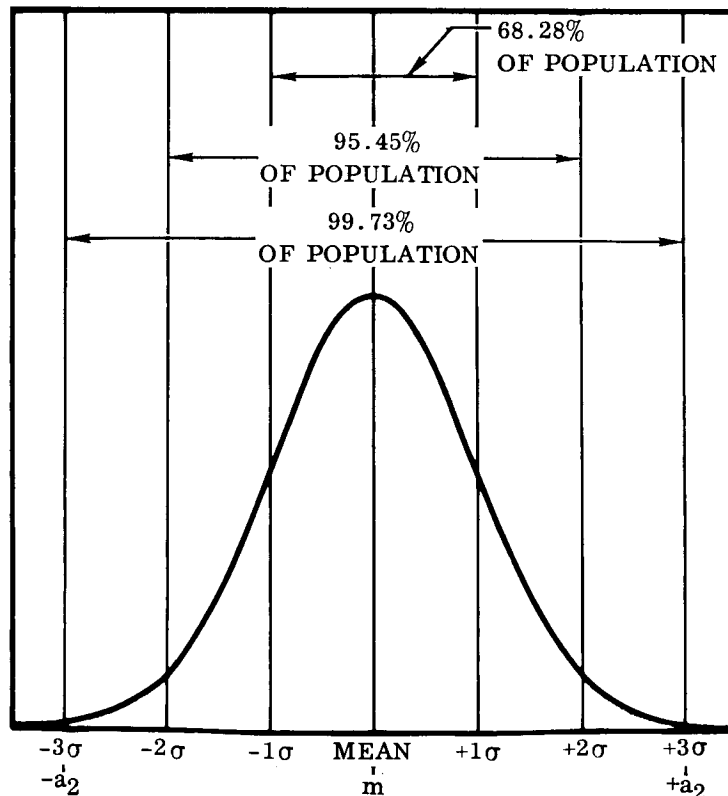


Figure D-7. Probability Density Function of the Lateral Center-of-Gravity Offset, $\bar{y}$, Showing Normal Distribution

The initial angular velocities of the spacecraft, $\dot{\psi}_1(0)$ and $\dot{\theta}_1(0)$, are within the rate thresholds of the attitude control system. The probability density function of this variable is approximately discrete at $\pm a_3$ where a_3 is close to the rate threshold of the attitude control system. The variance, $\sigma_{\dot{\psi}_1}^2(0)$, as computed in the following section has little significance by itself; however, it results in no appreciable error when used in determining the variance of the velocity dispersions in pitch $(\sigma_{\dot{\theta}_1}^2)$ and yaw $(\sigma_{\dot{\psi}_1}^2)$.

The joint probability density function of the 15 random variables, K_i and ℓ_i ($i=1, \dots, 6$), $\bar{y}_1$, $\bar{y}_2$, and $\dot{\psi}_1(0)$ - is approximately normal, since, by the Central Limit Theorem, the sum of a large number of independent random variables, not necessarily normal themselves, will be almost normal. This is approximately true in this case when only a few of the density functions are summed. Hence, where as many as 15 variables are involved, the resultant joint density function comes very close to being normal, provided that the variances are approximately of the same order of magnitude.

D.3.1.1 Variances of the Random Variables. The variance of a random variable, u , with density function $f(u)$, is given by

$$\sigma^2 = \int_{-\infty}^{\infty} u^2 f(u) du - \left[\int_{-\infty}^{\infty} u f(u) du \right]^2 \quad (D-35)$$

In the present case, u can be either ℓ_{2i-1} , or ℓ_{2i} . Therefore, solving Equation D-35 for each of these random variables yields

$$\sigma_{\ell_{2i-1}}^2 = \sigma_{\ell_{2i}}^2 = \frac{a_1^2}{3} \quad (D-36)$$

where a_1 refers to half the interval over which the parameter varies. The variances for the free lengths of all six springs were assumed to be equal (see Table E-10). Likewise, the variances of $\bar{y}_1$ and $\bar{y}_2$ are assumed to be the same.

In the case of the spring constants (K_{2i-1} and K_{2i}), the variances are calculated in the same manner from the data given in the drawings (Reference D-5).

For the normal distribution of $\bar{y}_1$ and $\bar{z}_1$, the frequency function is given by.

$$f(\bar{y}_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\bar{y}_1 - m)^2}{2\sigma^2}} \quad (D-37)$$

where m is the mean (a real constant), σ is the standard deviation (a positive constant), and the constant $1/\sigma\sqrt{2\pi}$ is a normalizing factor which makes the infinite integral of Equation D-37 equal to unity.

The variance for a normal distribution is equal to the square of the standard deviation. From Figure D-7,

$$\sigma_{\bar{y}_1}^2 = \left(\frac{a_2}{3} \right)^2 \quad (D-38)$$

For the discrete distribution of $\dot{\psi}_1(0)$ and/or $\dot{\theta}_1(0)$,

$$\sigma_{\dot{\psi}_1(0)}^2 = \sigma_{\dot{\theta}_1(0)}^2 = \int_{-\infty}^{\infty} [\dot{\psi}_1(0)]^2 f[\dot{\psi}_1(0)] d[\dot{\psi}_1(0)] \quad (D-39)$$

where m , in Figure D-8 is equal to zero and

$$f[\dot{\psi}_1(0)] = 0, \quad -\infty < \dot{\psi}_1(0) < -a_3, \quad -a_3 < \dot{\psi}_1(0) < +a_3, \quad +a_3 < \dot{\psi}_1(0) < +\infty$$

$$\int_{-a_3-\epsilon}^{-a_3+\epsilon} f[\dot{\psi}_1(0)] d[\dot{\psi}_1(0)] = \frac{1}{2}; \quad \dot{\psi}_1(0) = -a_3$$

$$\int_{a_3-\epsilon}^{a_3+\epsilon} f[\dot{\psi}_1(0)] d[\dot{\psi}_1(0)] = \frac{1}{2}; \quad \dot{\psi}_1(0) = +a_3$$

Evaluating Equation D-39 yields

$$\sigma_{\dot{\psi}_1(0)}^2 = a_3^2 \quad (D-40)$$

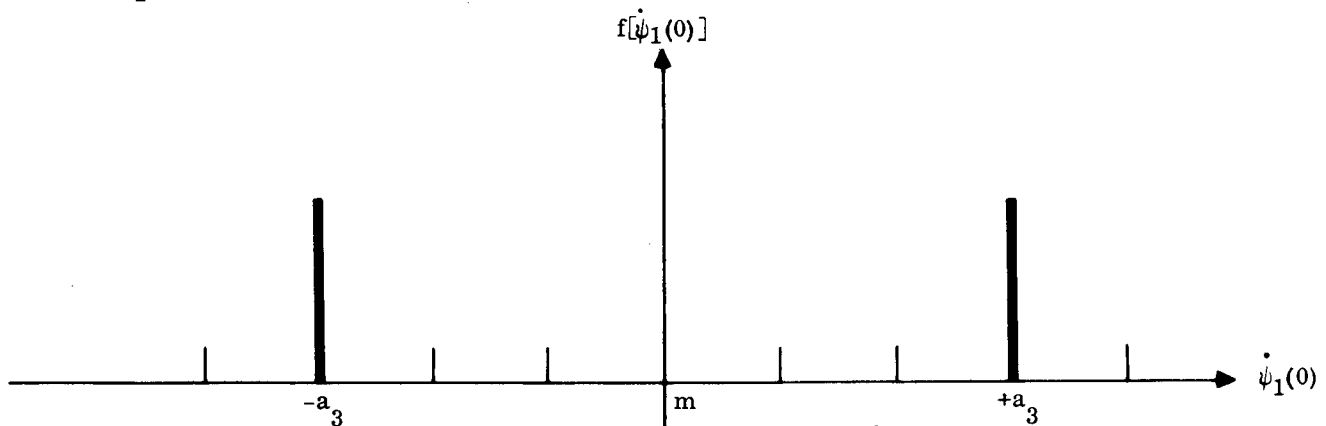


Figure D-8. Probability Density Function of Initial Angular Velocity, $\dot{\psi}_1(0)$, Showing Discrete Distribution

D.3.2 Variances for Velocity Dispersions. Having derived the variances for the 15 random variables, we can now determine the variances for the angular rates, $\dot{\psi}_1(t)$ and $\dot{\theta}_1(t)$, which are functions of these variables.

D.3.2.1 Dispersions in Angular Velocity. First consider the yaw velocity, $\dot{\psi}_1(t)$. Its variation, $\delta\dot{\psi}_1(t)$, is expressible in terms of the mean values of: the numerators, $\bar{N}_i$ ($i=1, \dots, 4$), of the four terms in Equation D-23; the denominators, $\bar{D}_i$; and the dispersions in initial turning rate, $\dot{\psi}_1(0)$, spring constants (K_i), undeformed spring lengths (ℓ_i), and center-of-gravity offsets ($\bar{y}_1$ and $\bar{y}_2$). This variation, then, is given by Equation D-41 (Figure D-9).

where all second-order terms (involving products of small variations) have been neglected.

Now, since $\delta\dot{\psi}_1(t)$ comprises a linear combination of the 15 random variables, δK_i and $\delta \ell_i$ ($i=1, \dots, 6$), $\delta \bar{y}_1$, $\delta \bar{y}_2$, and $\delta \dot{\psi}_1(0)$, its variance is given in Equation D-42 (Figure D-10).

$$\begin{aligned}
\delta \dot{\psi}_1(t) = & \sum_{i=1}^6 \left\langle \frac{1}{D_1} \left[\left(\frac{1}{I_{zz}^{(1)}} \right) (\dot{y}_1 - \dot{y}_2) \left(\frac{\rho_i - \bar{\rho}}{I_i \bar{K}} \right) t + \frac{3\bar{F}}{D_2} \left\{ \frac{1}{I_{zz}^{(1)}} \left[\sum_{j=1}^3 (r \sin \Delta_{2j})^2 + (r \sin \Delta_1)^2 \right] + \frac{1}{M} (r \sin \Delta_1 + \bar{y}_1) \right\} + \frac{3\bar{N}_2 \bar{K}}{D_2^2 M} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) (r \sin \Delta_1)^2 \right] \left(\sqrt{\frac{3\bar{K}}{M}} \right) \sin \left(\sqrt{\frac{3\bar{K}}{M}} t \right) \right. \\
& - \left. \left[\left(\frac{\bar{N}_3}{D_3^2} + \frac{\bar{N}_4}{D_4^2} \right) \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) (r \sin \Delta_1)^2 + \frac{1}{I_{zz}^{(1)}} \left[\frac{1}{I_{zz}^{(1)}} \right] (r \sin \Delta_1 + \bar{y}_1) + \frac{1}{I_{zz}^{(2)}} (r \sin \Delta_1 + \bar{y}_2) \right] \left[\frac{1}{D_4} \left(\frac{\rho_i - \bar{\rho}}{I_i \bar{K}} \right) + \frac{1}{D_3} \left(\frac{\bar{F}}{\bar{K}} \right) \right] \right. \\
& + \left. \left[\frac{\bar{N}_3}{2D_3} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) (r \sin \Delta_1)^2 \right] \sin \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} t \right] + \cos \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} t \right] \right. \\
& + \left. \left. \left[\frac{\bar{N}_3}{2D_3} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) (r \sin \Delta_1)^2 \right] \sin \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} t \right] + \cos \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} t \right] \right] \right\} \delta K_i \\
& + \sum_{i=1}^6 \left\{ \frac{\bar{K}_i}{D_1} \left[\left(\frac{1}{I_{zz}^{(1)}} \right) (\dot{y}_1 - \dot{y}_2) \right] t + \frac{\bar{K}_i}{D_4} \left[\frac{1}{I_{zz}^{(1)}} \left[\sum_{j=1}^3 (r \sin \Delta_{2j})^2 + \frac{1}{I_{zz}^{(2)}} (r \sin \Delta_1 + \bar{y}_1) + \frac{1}{I_{zz}^{(2)}} (r \sin \Delta_1 + \bar{y}_2) \right] \right. \right. \\
& + \left. \left. \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} t \right] \sin \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} t \right] + \cos \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} t \right] \right] \right\} \delta \rho_i \\
& + \left\{ \frac{3\bar{F}\bar{K}}{D_2} \left(\frac{1}{I_{zz}^{(1)}} \right) \left[\frac{1}{I_{zz}^{(2)}} \sum_{i=1}^3 (r \sin \Delta_{2i})^2 + \frac{1}{M} \left(\sqrt{\frac{3\bar{K}}{M}} \right) \sin \left(\sqrt{\frac{3\bar{K}}{M}} t \right) + \left[\frac{3\bar{F}}{D_3} \left(\frac{1}{I_{zz}^{(1)}} \right) + \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} \right] \right] \right. \right. \\
& + \left. \left. \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} \right] \right] \right\} \delta \gamma_i \\
& + \left\{ \frac{3\bar{F}\bar{K}}{D_2} \left(\frac{1}{I_{zz}^{(1)}} \right) \left[\frac{1}{I_{zz}^{(2)}} \sum_{i=1}^3 (r \sin \Delta_{2i})^2 + \frac{1}{M} \left(\sqrt{\frac{3\bar{K}}{M}} \right) \sin \left(\sqrt{\frac{3\bar{K}}{M}} t \right) + \left[\frac{3\bar{F}}{D_3} \left(\frac{1}{I_{zz}^{(1)}} \right) + \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} \right] \right] \right. \right. \\
& + \left. \left. \left[\sqrt{\frac{1}{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2}} \right] \right] \right\} \delta \gamma_2 + \delta \psi_1(0) \quad (D-41)
\end{aligned}$$

Figure D-9. Equation for Variation in Yaw Velocity

$$\begin{aligned}
\sigma_{\psi_1}^{2,2}(t) = & \sum_{i=1}^6 \left\langle \frac{1}{D_1} \left(\frac{1}{I_{zz}^{(1)}} \right) (\bar{y}_1 - \bar{y}_2) \left(\frac{\bar{F}}{1 - \bar{K}} \right) \right\rangle t + \frac{3\bar{F}}{D_2} \left\{ \frac{1}{I_{zz}^{(1)}} \right\} t + \frac{1}{D_2} \left[\sum_{j=1}^3 (r \sin \Delta_{2j})^2 + (r \sin \Delta_1)^2 \right] + \frac{1}{M} (r \sin \Delta_1 + \bar{y}_1) \left\{ \frac{3\bar{K}}{D_2 M} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) (r \sin \Delta_{2i})^2 \left(\frac{\sqrt{3\bar{K}}}{M} \right) \sin \left(\frac{\sqrt{3\bar{K}}}{M} t \right) \right\} \\
& - \left\langle \frac{\bar{N}_3}{(D_3)^2} + \frac{\bar{N}_4}{D_4^2} \left[\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right] (r \sin \Delta_{2i})^2 \right\rangle + \frac{1}{I_{zz}^{(1)}} \left[\frac{1}{I_{zz}^{(1)}} \right] (r \sin \Delta_1 + \bar{y}_1) + \frac{1}{I_{zz}^{(2)}} (r \sin \Delta_1 + \bar{y}_2) \left[\frac{1}{D_4} \left(\frac{\bar{F}}{1 - \bar{K}} \right) + \frac{1}{D_3} \left(\frac{\bar{F}}{I_{zz}^{(1)}} \right) \right] \left\{ \sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} \sin \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] \right\} \\
& + \frac{\bar{N}_3}{2D_3} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) (r \sin \Delta_1)^2 \left\{ \sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} \sin \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] + \cos \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] \right\} \sigma_{K_1}^2 \\
& + \sum_{i=1}^6 \left\{ \frac{\bar{K}_i}{D_1} \left[\left(\frac{1}{I_{zz}^{(1)}} \right) \left(\frac{1}{I_{zz}^{(2)}} \right) (\bar{y}_1 - \bar{y}_2) \right] + \frac{\bar{K}_i}{D_4} \left\{ \frac{1}{I_{zz}^{(1)}} \right\} t + \frac{1}{D_4} (r \sin \Delta_1 + \bar{y}_1) + \frac{1}{I_{zz}^{(2)}} (r \sin \Delta_1 + \bar{y}_2) \right\} \left\{ \sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} \sin \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] \right\} \sigma_{\theta_i}^2 \\
& + \frac{3\bar{F}\bar{K}}{D_2} \left(\frac{1}{I_{zz}^{(1)}} \right) \left[-\frac{1}{I_{zz}^{(2)}} \sum_{i=1}^3 (r \sin \Delta_{2i})^2 + \frac{1}{M} \left(\frac{\sqrt{3\bar{K}}}{M} \right) \sin \left(\frac{\sqrt{3\bar{K}}}{M} t \right) + \frac{3\bar{F}}{D_3} \left(\frac{1}{I_{zz}^{(1)}} \right) \right] \left\{ \sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} \sin \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] \right\} \sigma_{\bar{y}_1}^2 \\
& + \left\{ \frac{3\bar{F}\bar{K}}{D_2} \left(\frac{1}{I_{zz}^{(1)}} \right) \left[\frac{1}{I_{zz}^{(2)}} \sum_{i=1}^3 (r \sin \Delta_{2i})^2 + \frac{3\bar{F}}{D_3} \left(\frac{1}{I_{zz}^{(1)}} \right) \left(\frac{1}{I_{zz}^{(2)}} \right) \right] + \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} \sin \left[\sqrt{\bar{K} \left(\frac{1}{I_{zz}^{(1)}} + \frac{1}{I_{zz}^{(2)}} \right) \sum_{i=1}^3 (r \sin \Delta_{2i})^2} t \right] \right\} \sigma_{\bar{y}_2}^2 + \sigma_{\psi_1}^2(0) \quad (D-42)
\end{aligned}$$

Figure D-10. Equation for Statistical Variance in Yaw Velocity

where, in Equations (D-41) and D-42), instead of the summation, $\sum_{i=1}^3 (r \sin \Delta_{2i})^2$, we could also have used $\sum_{i=1}^3 (r \sin \Delta_{2i-1})^2$.

In the derivation of $\sigma_{\dot{\psi}_1}^2$, the longitudinal distance X_{12} is assumed to be held constant; or equivalently, the terms being summed over j in Equation D-23 are kept constant. This is permissible because the standard deviation X_{12} is very small.

The variance for the pitch velocity, $\dot{\theta}_1(t)$, is computed exactly as in Equation D-42 except that $I_{zz}^{(1)}$, $I_{zz}^{(2)}$, $\sin \Delta_1$, $\bar{y}_1$, $\bar{y}_2$, and $\dot{\psi}_1(0)$ are replaced by $I_{yy}^{(1)}$, $I_{yy}^{(2)}$, $\cos \Delta_1$, $\bar{z}_1$, $\bar{z}_2$, and $\dot{\theta}_1(0)$ respectively.

D.3.2.2 Variance for Longitudinal Velocity. To compute the variance for the longitudinal velocity of the spacecraft with respect to the inertial reference frame, consider $\dot{X}_{IX}(s)$, as given in Equation D-33, which is repeated below for convenience.

$$\dot{X}_{IX}(s) = \sqrt{\frac{\sum_{i=1}^6 (\ell_i s - \frac{1}{2} s^2) K_i}{\frac{1}{2} M_1 \left(1 + \frac{M_1}{M_2}\right)}}$$

Since $\dot{X}_{IX}(s)$ is a linear combination of 12 random variables, i.e., ℓ_i and K_i ($i=1, \dots, 6$), its variance is given by

$$\sigma_{\dot{X}_{IX}}^2(s) = \frac{s \left[(\ell_1 - \frac{1}{2}s)^2 \sigma_{K_1}^2 + (\ell_2 - \frac{1}{2}s)^2 \sigma_{K_2}^2 + (\bar{K}_1^2 + \bar{K}_2^2) \sigma_{\ell_1}^2 \right]}{2 M_1 \left(1 + \frac{M_1}{M_2}\right) (\bar{F} - \frac{1}{2}s \bar{K})} \quad (D-43)$$

where, instead of ℓ_1 and ℓ_2 , we could also have used ℓ_3 and ℓ_5 or ℓ_4 and ℓ_6 respectively; and the same applies to the spring constants.

The linear velocities in the Y and Z directions are negligible and are not considered.

D.4 SEPARATION CLEARANCES. Since transverse velocities are negligible, the separation clearance is a function of the relative longitudinal velocity, $\dot{X}_{12}$, the vector sum of the angular velocity, ω , and the dimensions of the spacecraft and the Centaur payload adapter. Figure D-11 shows the principle parameters.

In the case of $\dot{\psi}_1(t)$ and $\dot{\theta}_1(t)$, where the frequency functions about the origin are

$$f[\dot{\psi}_1(t)] = \frac{1}{\sigma \sqrt{2\pi}} e^{-[\dot{\psi}(t)]^2 / 2 \sigma^2} \quad (D-44)$$

$$f[\dot{\theta}_1(t)] = \frac{1}{\sigma \sqrt{2\pi}} e^{-[\dot{\theta}_1(t)]^2 / 2 \sigma^2} \quad (D-45)$$

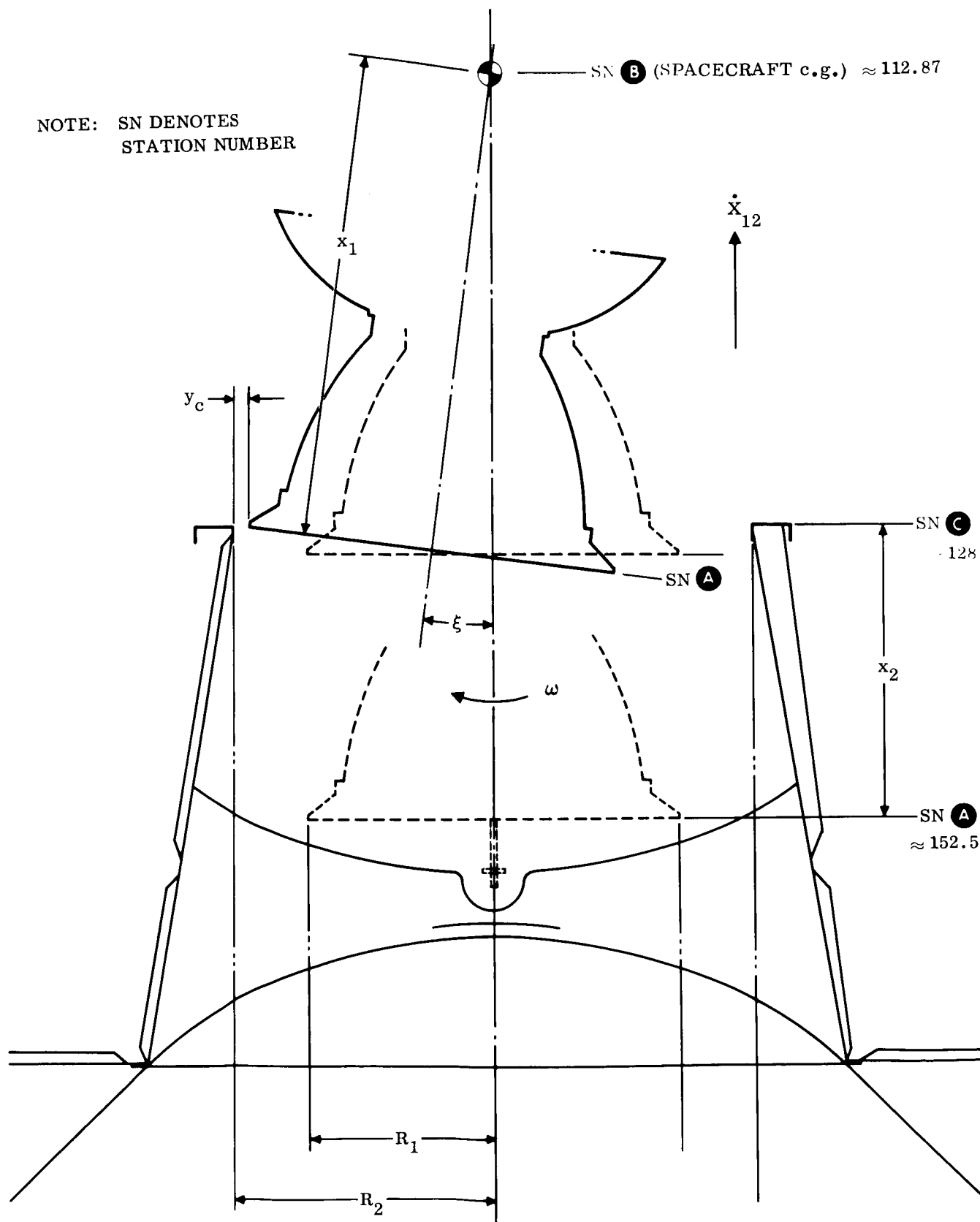


Figure D-11. Surveyor Spacecraft Separation Parameters

it can be shown that the probability of the vector sum angular rate, ω , lying within a circle of radius R centered at the origin is

$$1 - e^{-R^2/2\sigma^2}$$

Solving for R , where 99.73 percent of the population of ω lies within the circle,

$$R = 3.44\sigma \quad (D-46)$$

Now the vector sum angular rate (3σ) of the spacecraft may be expressed.

$$\omega = \bar{\omega} \pm 3\sigma_{\omega} \quad (D-47)$$

where

$$3\sigma_{\omega} \equiv 3.44\sigma_{\dot{\psi}_1(t)} \equiv 3.44\sigma_{\dot{\theta}(t)}$$

For purposes of separation clearance, the maximum (3σ) angular velocity is used. Consequently, ω as subsequently used is defined:

$$\omega = \sqrt{[\dot{\psi}_1(t)]^2 + [\dot{\theta}_1(t)]^2} + 3.44\sigma_{\dot{\psi}_1(t)} \quad (D-48)$$

From Figure D-11, the clearance, y_c , is

$$y_c = R_2 - R_1 \cos \xi_1 + x_1 \sin \xi_1 \quad (D-49)$$

where

$$\xi = \omega t$$

$$t = \frac{x_2}{\dot{X}_{12}}$$

D.4.1 Allowable Lateral Center-of-Gravity Envelope. To determine the spacecraft lateral center-of-gravity offset envelope, the following assumptions and equations were used.

- A minimum acceptable clearance of $y_{c \min} = 1$ inch was assumed.
- Small angles were assumed.
- The allowable center-of-gravity offset (radial) is assumed to lie within a circle about the geometric centerline.
- The coefficients c_1 and d_1 (Equations D-28 and D-29) are assumed to be equal, and of the value 14.4 deg/sec/ft. (The value used was obtained from Reference D-1 and is assumed to be a representative number for the current Surveyor spacecraft.)

In addition to the collision limitation, there is a constraint on angular velocity

$$\omega \leq 3 \text{ deg/sec} \quad (\text{D-50})$$

The vector sum angular velocity may be expressed as

$$\omega = c_1 \bar{R} + \Delta\omega = c_1 \bar{R} + c_1 \Delta\bar{R} \quad (\text{D-51})$$

where

$$\Delta\omega = 3.44 \sqrt{0.316 \sigma_{\psi_1(t)}^2}$$

and

$$\Delta\bar{R} = \frac{\Delta\omega}{c_1}$$

$\bar{R}$ is the radius representing the maximum boundary if the radial center of gravity is exactly known. The value 0.316 represents the fraction of the total variance, $\sigma_{\psi_1(t)}^2$, contributed by all factors other than center-of-gravity offset uncertainty. This fraction was determined from the evaluation of Equation D-42 for the Surveyor A-21 configuration.

Now substituting into Equation D-49 and rearranging terms, we have an expression for the collision constraint in terms of the longitudinal center-of-gravity position (represented by x_1) and the lateral center-of-gravity offset (radial) $\bar{R}$,

$$x_1 \bar{R} = \left(-y_{c_{\min}} + R_2 - R_1 \right) \frac{57.3 \dot{X}_{12}}{c_1 x_2} - x_1 \Delta\bar{R} \quad (\text{D-52})$$

Similarly, an expression for the collision constraint in terms of the longitudinal center of gravity and the nominal radial center of gravity ($\bar{R}'$) may be derived:

$$x_1 \bar{R}' = \left(-y_{c_{\min}} + R_2 - R_1 \right) \frac{57.3 \dot{X}_{12}}{c_1 x_2} - x_1 \Delta\bar{R}' \quad (\text{D-53})$$

where

$$\omega = c_1 \bar{R}' + 3\sigma\omega = c_1 \bar{R}' + c_1 \Delta\bar{R}' \quad (\text{D-54})$$

and

$$\Delta\bar{R}' = \frac{3.44 \sigma_{\psi_1(t)}}{c_1} \quad (\text{D-55})$$

D.5 REFERENCES.

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- D-5. Spring - Jettison, Surveyor, General Dynamics/Astronautics Drawing No. 55-46065.
- D-6. Monthly Configuration, Performance, and Weight Status Report, General Dynamics/Astronautics Report No. GD/A63-0495, 21 November 1963.
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APPENDIX E
VEHICLE BASIC DATA

LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A	Actuating piston area	ft ²
A _A	Accumulator piston area	ft ²
A _{BS}	"Bootstrap" piston area	ft ²
A _f	Feedback piston area	ft ²
A _{RA}	Reservoir piston area	ft ²
A _{RH}	A _{RA} + A _{BS}	ft ²
B	Bulk modulus of hydraulic fluid	lb/ft ²
C _B	Coulomb friction coefficient	ft-lb
C _f	Discharge coefficient for washout orifice	$\frac{\text{ft}^3/\text{sec}}{\sqrt{\text{lb}/\text{ft}^2}}$
C _V	Viscous friction coefficient	$\frac{\text{ft-lb}}{\text{deg/sec}}$
I _R	Engine "wet" moment of inertia of gimballed mass about gimbal point (pitch)	slug-ft ²
I _{xxR}	Reduced moment of inertia about the roll axis	slug-ft ²
I _{yyR}	Reduced moment of inertia about the pitch axis	slug-ft ²
I _{zzR}	Reduced moment of inertia about the yaw axis	slug-ft ²
I _{yy}	Moment of inertia about pitch axis	slug-ft ²
I _{zz}	Moment of inertia about yaw axis	slug-ft ²
K	Power of adiabatic compression	N. D.
K _A	End-to-end position gain	$\frac{\text{deg } \delta}{\text{deg } \theta}$
K _A K _R	End-to-end rate gain	$\frac{\text{deg } \delta}{\text{deg/sec } \theta}$
K _a	Servoamplifier gain	ma/volt
K _a K _t	Product of servoamplifier gain and feedback transducer gain	volt/deg
K _c	Actuator no-load open-loop velocity gain	(sec) ⁻¹
K _f	Feedback spring constant	lb/ft
K _I	Autopilot integrator gain	sec ⁻¹
K _m	Spring constant of actuator support structure	lb/ft

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
K_R	Rate gyro gain	$\frac{\text{deg}}{\text{deg/sec}}$
K_S	DPF valve pressure feedback gain	$\frac{\text{ma}}{\text{lb/ft}^2}$
K_V	Servovalve discharge coefficient	$\frac{\text{ft}^3/\text{sec}}{\text{ma} \sqrt{\text{lb/ft}^2}}$
k_I'	Steering equation integral gain	sec^{-1}
L_{PCL}, L_{PCH}	Liquid oxygen and liquid hydrogen pendulum lengths	ft
l_c	Rocket engine pitch control moment arm	ft
l_{CL}, l_{CH}	Liquid oxygen and liquid hydrogen pendulum moment arms	ft
l_p	Distance between vehicle center of gravity and inertial guidance platform	ft
l_R	Distance between rocket engine center of gravity and gimbal point	ft
l_{ra}	Radial distance from the vehicle centerline to the A Engine thrustline	ft
l_{xa}	Distance along the centerline from the center-of-gravity to the plane of the A Engine thrustline	ft
l_{xp}	Distance along the centerline from the center-of-gravity to the plane of the P Engine thrustline	ft
l_z	Rocket engine roll control moment arm	ft
M_R	Reduced mass of vehicle (without sloshing mass)	slugs
M_{PCL}, M_{PCH}	Liquid oxygen and liquid hydrogen pendulum masses	slugs
M_R	Rocket engine gimballed mass, one engine "wet"	slugs
M_T	Total mass of vehicle	slugs
$m^{(i)}$	Generalized mass, i^{th} mode	slugs
P_{ACI}	Initial pressure in Region d of accumulator (See Figure B-6)	lb/ft^2
P_C	Relief valve cracking pressure	lb/ft^2
P_{FF}	Relief valve pressure drop at full flow	lb/ft^2

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
P_R	Hydraulic system reservoir pressure (return pressure from servovalve)	lb/ft ²
P_{Reseat}	Relief valve reseat pressure	lb/ft ²
P_S	Hydraulic system supply pressure to servovalve	lb/in ²
Q_{FF}	Relief valve full flow	ft ³ /sec
R	Engine actuator moment arm	ft
T_c	Control engine thrust	lb
T_D	Guidance computer computation delay	sec
T_i	Temperature of hydraulic system at launch	° F
T_o	Temperature of hydraulic system at MES	° F
t	Time	sec
Δt	Guidance computer compute cycle length	sec
V_A	Total volume of hydraulic fluid in one actuator cylinder	ft ³
V_{ACi}	Initial volume, region d of accumulator (See Figure B-6)	ft ³
V_{T_o}	Volume of manifold, power pack, and tubing on high pressure side at temperature T_o	ft ³
$\ddot{x}$	Vehicle axial acceleration	ft/sec ²
x_{CG}	Roll center of gravity	Station Number (in.)
$\bar{y}$	Yaw center-of-gravity offset	ft
$\bar{z}$	Pitch center-of-gravity offset	ft
$\beta_{\text{XT}}^{(i)}$	Normalized slope-coupling term, i^{th} mode ($\beta_{\text{XT}}^{(i)} = \sigma_{\text{XT}}^{(i)} - \sigma_{\text{XE}}^{(i)}$)	deg/ft
Δ	Angular offset of the rocket engine center-of-gravity from the engine centerline	deg
∇	Thrust misalignment	deg
ϵ	Thrust offset	ft
$\zeta^{(i)}$	Damping ratio, i^{th} mode	N. D.

LIST OF SYMBOLS, Contd

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
ζ_{CL}, ζ_{CH}	Liquid oxygen and liquid hydrogen slosh damping ratios	N. D.
ζ_G	Rate gyro damping ratio	N. D.
$\sigma_{PG}^{(i)}$	Normalized slope of i^{th} mode at position reference location	deg/ft
$\sigma_{RG}^{(i)}$	Normalized slope of i^{th} mode at rate gyro location	deg/ft
$\sigma_{XT}^{(i)}$	Normalized slope of i^{th} mode at engine gimbal point	deg/ft
τ_R	Equivalent rate gyro time constant	sec
$\phi_{XT}^{(i)}$	Normalized displacement of i^{th} mode at engine gimbal point	ft/ft
$\omega^{(i)}$	Natural frequency, i^{th} mode	rad/sec
ω_{CL}, ω_{CH}	Liquid oxygen and liquid hydrogen slosh frequencies	rad/sec
ω_G	Rate gyro natural frequency	rad/sec

E.1 ATLAS/CENTAUR SEPARATION SYSTEM PARAMETERS. The parameters used are given in Figure E-1 and Table E-1. The idealized thrust forces (time measured from "Fire Retrorockets" command) are

$$\bar{F}_R(t) = \begin{cases} 644 \text{ lb}; & 0 < t < 0.71 \text{ sec} \\ 0; & t > 0.71 \text{ sec} \end{cases}$$

$$T_R(t) = \begin{cases} 720 - 145.4 t; & 0 < t < 4.95 \text{ sec} \\ 0; & t > 4.95 \text{ sec} \end{cases}$$

$$\varphi_1 = 58$$

$$\varphi_5 = 215$$

$$\varphi_2 = 23$$

$$\varphi_6 = 199$$

$$\varphi_3 = 341$$

$$\varphi_7 = 157$$

$$\varphi_4 = 325$$

$$\varphi_8 = 122$$

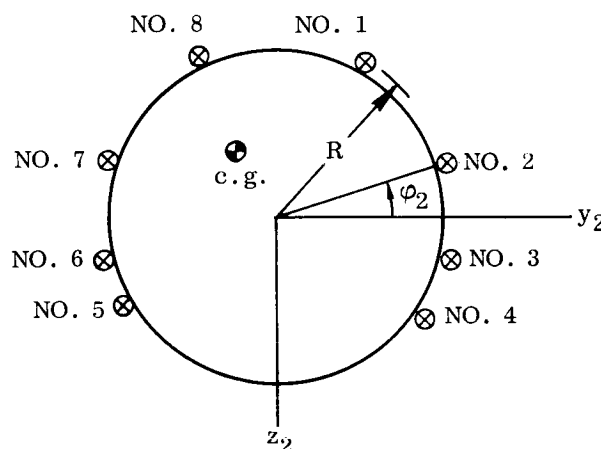


Figure E-1. Circumferential Retrorocket Locations (degrees)

Table E-1. Separated Sustainer Stage Inertial Properties

PARAMETER	VALUE
m_2 (SLUGS)	258
I_2 (SLUG-FT ²)	117×10^3
ℓ_2 (FT)	45.7
R (FT)	5.25
$\bar{y}_2$ (IN.)	-6.69
$\bar{z}_2$ (IN.)	-11.35

E.2 SERVOVALVE, ACTUATOR, ENGINE, AND HYDRAULIC SYSTEM PARAMETERS. The parameters used to describe the electrohydraulic engine/actuator dynamics and the hydraulic power supply are given in Tables E-2 and E-3. These parameter values have been obtained from References E-1, E-2, and E-3. Table E-2 presents those parameters that were used for the rigid body/propellant sloshing, autopilot/guidance, and elastic vehicle stability analyses. Ignition phase analysis uses parameters in both Tables E-2 and E-3.

E.3 RIGID-BODY/PROPELLANT-SLOSHING AND AUTOPILOT/GUIDANCE PARAMETERS. The parameter values used for the rigid-body/propellant-sloshing and autopilot/guidance stability analyses in this report are listed in Tables E-4, E-5, and E-6. These values, used for the pitch and roll plane analyses, were obtained from Reference E-8, which is based on data contained in References E-4, E-5, E-6, and E-7. The parameters used in the main engine cutoff transient are presented in Table E-7.

E-4. ELASTIC VEHICLE BENDING PARAMETERS. Table E-8 gives the normalized (at the engine gimbal point) deflections and slopes, generalized masses, frequencies, and structural damping. These are the parameters used to analyze the stability of the first three elastic modes corresponding to four flight conditions: propellant tanks full, half full, quarter full, and empty (MECO). This data was obtained from Reference E-9.

The rate gyro parameters used in the bending stability analysis are:

- a. Rate gyro natural frequency (ω_G) = 105 rad/sec
- b. Rate gyro damping ratio (ζ_G) = 1.70

E.5 COAST PHASE VEHICLE PARAMETERS. Table E-9 gives the vehicle mass, inertia, and moment arms for the three coast phase portions of flight.

E.6 CENTAUR/SPACECRAFT SEPARATION PARAMETER. Tables E-10 and E-11 gives the parameters used in the separation analysis. This data was obtained from References E-10, E-11, and E-12.

Table E-2. Servovalve, Actuator, and Engine Parameters for RL-10A-3 Propulsion System

PARAMETER	VALUE	REMARKS
$A \text{ (ft}^2\text{)}$	1.06×10^{-2}	
$B \text{ (lb/ft}^2\text{)}$	3.89×10^7	At 100° F
$C_B \text{ (ft-lb)}$	200	Maximum Specified Value
$C_V \left(\frac{\text{ft-lb}}{\text{deg/sec}} \right)$	3.62	
$I_R \text{ (slug-ft}^2\text{)}$	66.7	Pitch
	65.3	Yaw
$M_R \text{ (slugs)}$	11.0	Nominal
$\ell_R \text{ (ft)}$	2.04	
$R \text{ (ft)}$	1.16	Nominal
$K_a \text{ (ma/volt)}$	3.6	
$K_a K_t \text{ (volt/deg)}$	7.416	$K_t = 2.06 \text{ volt/deg}$
$K_m \text{ (lb/ft)}$	5.1×10^5	Pitch Plane (30 psig Tank Pressure)
	9.84×10^5	Yaw Plane (30 psig Tank Pressure)

Table E-2. Servovalve, Actuator, and Engine Parameters for RL-10A-3 Propulsion System, Contd

PARAMETERS	VALUE	REMARKS
$K_v \left(\frac{\text{ft}^3/\text{sec}}{\text{ma} \sqrt{\text{lb}/\text{ft}^2}} \right)$	1.02×10^{-6}	
$P_R \text{ (lb}/\text{ft}^2\text{)}$	7.2×10^3	
$P_S \text{ (lb}/\text{ft}^2\text{)}$	1.512×10^5	$P_S - P_R = 1000 \text{ psi} = 1.44 \times 10^5 \text{ lb}/\text{ft}^2$
$T_c \text{ (lb)}$	1.5×10^4	
$V_A \text{ (ft}^3\text{)}$	1.33×10^{-3}	
$K_c \text{ (sec}^{-1}\text{)}$	13.1	No-load Open-loop Actuator Velocity Gain
$A_f \text{ (ft}^2\text{)}$	3.41×10^{-4}	Area of Washout Piston
$C_f \left(\frac{\text{ft}^3/\text{sec}}{\sqrt{\text{lb}/\text{ft}^2}} \right)$	1.34×10^{-7}	Discharge Coefficient for DPF Orifice
$K_f \text{ (lb}/\text{ft}\text{)}$	9.36×10^3	Spring Constant Washout Piston
$K_s \left(\frac{\text{ma}}{\text{lb}/\text{ft}^2} \right)$	1.22×10^{-4}	Pressure Feedback Gain

Table E-3. Hydraulic Power Supply and Miscellaneous Engine Parameters

PARAMETER	VALUE	REMARKS
$\nabla \text{ (deg)}$	0.536 0.356	Pitch Yaw } Assumed Values
$\Delta \text{ (deg)}$	8.47	Pitch; Positive for C-1, Negative for C-2 Engine.
	3.22	Yaw; Negative for C-1, Positive for C-2 Engine.
$\epsilon \text{ (ft)}$	0.01	Maximum
$A_{BS} \text{ (ft}^2\text{)}$	0.0033	
$A_{RA} \text{ (ft}^2\text{)}$	0.0819	
$A_{RH} \text{ (ft}^2\text{)}$	0.0854	
$K \text{ (N. D.)}$	1.4	Nitrogen
$P_{ACi} \text{ (lb}/\text{ft}^2\text{)}$	86,400	At 70° F

Table E-3. Hydraulic Power Supply and Miscellaneous Engine Parameters, Contd

PARAMETER	VALUE	REMARKS
$V_{ACi} (ft^3)$	1.805×10^{-3}	
$T_i (^\circ F)$	70	
$T_o (^\circ F)$	80	
$A_A (ft^2)$	0.0778	
$Q_{FF} (ft^3/sec)$	3.345×10^{-3}	
$P_C (lb/ft^2)$	146,900	
$P_{FF} (lb/ft^2)$	158,400	
$P_{Reseat} (lb/ft^2)$	141,110	
$V_{T_o} (ft^3)$	2.778×10^{-3}	Volume of Manifold, Power Pack, and Tubing on High Pressure Side Only.

Table E-4. AC-5 Rigid Body (Pitch Plane), Autopilot, and Guidance Parameters

PARAMETER	TIME (SECONDS)				
	MES	MES +210	MES +310	MES +370	MECO
$T_c (lb)$	30,000	30,000	30,000	30,000	30,000
$l_c (ft)$	7.83	9.56	10.48	10.8	14.04
$l_p (ft)$	15.6	13.9	-	12.1	9.42
$\ddot{x} (ft/sec^2)$	27.4	47.1	71.8	105	154
$I_{yy_r} \times 10^{-3} (slug-ft^2)$	56.1	39.8	37.5	35.6	37.5
$M_r (slugs)$	834	371	286	246	193
$K_A \left(\frac{deg}{deg} \right)$	Varied*	Varied*	Varied*	Varied*	Varied*
$K_R \left(\frac{deg}{deg/sec} \right)$	1.33	1.33	1.33	1.33	1.33
$K_I (sec^{-1})$	0.785	0.785	0.785	0.785	0.785
$\tau_R (sec)$	0.04	0.04	0.04	0.04	0.04
$\Delta t (sec)$	1.25	1.25	-	1.25	1.25
$T_D (sec)$	1.25	1.25	-	1.25	1.25

* The operating point is: $K_A = 0.24$

Table E-4. AC-5 Rigid Body (Pitch Plane), Autopilot, and Guidance Parameters, Contd

PARAMETER	TIME (SECONDS)				
	MES	MES +210	MES +310	MES +370	MECO
$k'_I (\text{sec}^{-1})$	Varied*	Varied*	-	Varied*	Varied*

* The operating point is: $k'_I = 0.25$

Table E-5. AC-5 Propellant Sloshing Parameters

PARAMETER	TIME (SECONDS)				
	MES	MES +210	MES +310	MES +370	MECO
$\zeta_{CL} \times 10^3 (\text{N.D.})$	2	7	27	71	0
$\zeta_{CH} \times 10^4 (\text{N.D.})$	0.80	0.80	0.80	0.80	0
$\omega_{CL} (\text{rad/sec})$	3.74	3.32	3.19	2.69	0
$\omega_{CH} (\text{rad/sec})$	3.68	4.17	4.80	4.37	0
$L_{PCL} (\text{ft})$	1.96	4.28	7.03	14.4	0
$L_{PCH} (\text{ft})$	2.02	2.71	3.11	5.47	0
$\ell_{CL} (\text{ft})$	-3.71	-3.98	-2.47	3.73	0
$\ell_{CH} (\text{ft})$	7.87	-0.527	-3.07	-2.75	0
$M_{PCH} (\text{slugs})$	240	240	109	27.4	0
$M_{PCL} (\text{slugs})$	17.9	23.8	20.9	11.8	0

Table E-6. AC-5 Rigid Body Roll Parameters*

T_c	ℓ_z	I_{xx_r}	τ_R	K_I	$\frac{K_A K_R}{\left(\frac{\text{deg}}{\text{deg/sec}}\right)}$
(lb)	(ft)	(slug-ft ²)	(sec)	(sec ⁻¹)	
30,000	2.08	2,106	0.04	2.4	0.11

*These parameters remain fixed throughout the powered flight phase.

Table E-7. Parameters Used in Calculation of Main Engine Cutoff Transients

PARAMETER		VALUE	
l_c (ft)		14.05	
l_z (ft)		2.08	
I_{yyr}, I_{zzr} (slug-ft ²)		29,000	
I_{xxr} (slug-ft ²)		2,100	
Actuator Correction Angle for Thrust Structure Loading			
ENGINE	ACTUATOR	BIAS ANGLE (deg)	DIRECTION OF BIAS
C-1	Pitch	-0.681	Toward LO ₂ boost pump sump
C-1	Yaw	+0.426	Toward LH ₂ boost pump sump
C-2	Pitch	+0.605	Toward LO ₂ boost pump sump
C-2	Yaw	-0.752	Toward H ₂ O ₂ bottle

Table E-8. AC-5 Elastic Bending Parameters, Engines in the Mode

TANKS FULL			
	FIRST MODE	SECOND MODE	THIRD MODE
$m^{(i)}$ (slugs)	1795	445	2028
$\omega^{(i)}$ (rad/sec)	138	258	338
$\zeta^{(i)}$ (N.D.)	0.005	0.005	0.005
$\sigma_{RG}^{(i)}$ (deg/ft)	19.8	2.93	-2.54
$\sigma_{PG}^{(i)}$ (deg/ft)	19.8	2.93	-2.54
$\sigma_{XT}^{(i)}$ (deg/ft)	-9.61	-6.14	-2.0
$\phi_{XT}^{(i)}$ (ft/ft)	1.0	1.0	1.0
$\beta_{XT}^{(i)}$ (deg/ft)	-65.0	-29.6	-22.0

Table E-8. AC-5 Elastic Bending Parameters, Engines in the Mode, Contd

TANKS 1/2 FULL			
	FIRST MODE	SECOND MODE	THIRD MODE
$m^{(i)}$ (slugs)	1048	712	1461
$\omega^{(i)}$ (rad/sec)	167	327	710
$\zeta^{(i)}$ (N.D.)	0.005	0.005	0.005
$\sigma_{RG}^{(i)}$ (deg/ft)	13.4	14.8	-65.7
$\sigma_{PG}^{(i)}$ (deg/ft)	13.4	14.8	-65.7
$\sigma_{XT}^{(i)}$ (deg/ft)	-11.4	-11.0	-10.0
$\phi_{XT}^{(i)}$ (ft/ft)	1.0	1.0	1.0
$\beta_{XT}^{(i)}$ (deg/ft)	-49.3	-32.5	-22.1
TANKS 1/4 FULL			
	FIRST MODE	SECOND MODE	THIRD MODE
$m^{(i)}$ (slugs)	1150	333	1772
$\omega^{(i)}$ (rad/sec)	177	441	791
$\zeta^{(i)}$ (N.D.)	0.005	0.005	0.005
$\sigma_{RG}^{(i)}$ (deg/ft)	13.0	18.9	-160
$\sigma_{PG}^{(i)}$ (deg/ft)	13.0	18.9	-160
$\sigma_{XT}^{(i)}$ (deg/ft)	-12.2	-14.7	-9.40
$\phi_{XT}^{(i)}$ (ft/ft)	1.0	1.0	1.0
$\beta_{XT}^{(i)}$ (deg/ft)	-47.5	-33.5	-19.0
TANKS EMPTY			
	FIRST MODE	SECOND MODE	THIRD MODE
$m^{(i)}$ (slugs)	303	85.5	82.7
$\omega^{(i)}$ (rad/sec)	185	581	862
$\zeta^{(i)}$ (N.D.)	0.005	0.005	0.005

Table E-8. AC-5 Elastic Bending Parameters, Engines in the Mode, Contd

	TANKS EMPTY (Contd)		
	FIRST MODE	SECOND MODE	THIRD MODE
$\sigma_{RG}^{(i)}$ (deg/ft)	5.11	19.5	-34.2
$\sigma_{PG}^{(i)}$ (deg/ft)	5.11	19.5	-34.2
$\sigma_{XT}^{(i)}$ (deg/ft)	-7.2	-14.6	-18.1
$\phi_{XT}^{(i)}$ (ft/ft)	1.0	1.0	1.0
$\beta_{XT}^{(i)}$ (deg/ft)	-38.2	-30.1	-25.7

Table E-9. AC-5 Coast Phase Vehicle Parameters

PARAMETER	TIME		
	PRIOR TO SPACECRAFT SEPARATION	AFTER SPACECRAFT SEPARATION	DYNAMIC MODEL SD-1
M_T (slugs)	195.9	151.8	44.1
I_{xx} (slug-ft ²)	2,100	1,932	125
I_{yy} and I_{zz} (slug-ft ²)	29,000	17,800	129
x_{CG} (Station Number - in.)	285	332.6	109.8
$\bar{y}$ (in.)	1.76	2.3	0.0
$\bar{z}$ (in.)	1.07	1.33	0.52
ℓ_{ra} (in.)	63	63	
ℓ_{xa} (in.)	134.72	87.12	
ℓ_{xp} (in.)	134.72	87.12	

Table E-10. Centaur/Payload Separation Mechanism Parameters
(Symbols are defined in Appendix D)

PARAMETER	VALUE
$\bar{K}_{2i-1} \quad i = 1, \dots, 3 \quad (\text{lb/ft})$	260.870
$\bar{K}_{2i} \quad i = 1, \dots, 3 \quad (\text{lb/ft})$	757.282
$\bar{K} \quad (\text{lb/ft})$	1018.152
$\Delta_1 = \Delta_2 \quad (\text{degrees})$	9.0
$\Delta_3 = \Delta_4 \quad (\text{degrees})$	129.0
$\Delta_5 = \Delta_6 \quad (\text{degrees})$	249.0
$\sigma_{K2i-1}^2 \quad i = 1, \dots, 3 \quad (\text{lb/ft})^2$	26.40
$\sigma_{K2i}^2 \quad i = 1, \dots, 3 \quad (\text{lb/ft})^2$	341.93
$\sigma_{\ell_i}^2 \quad i = 1, \dots, 6 \quad (\text{ft})^2$	208.33×10^{-8}
$\bar{F}_{2i-1} \quad i = 1, \dots, 3 \quad (\text{lb})$	45.869
$\bar{F}_{2i} \quad i = 1, \dots, 3 \quad (\text{lb})$	96.554
$\bar{F} \quad (\text{lb})$	142.423
$\bar{\ell}_{2i-1} \quad i = 1, \dots, 3 \quad (\text{ft})$	0.176
$\bar{\ell}_{2i} \quad i = 1, \dots, 3 \quad (\text{ft})$	0.127
$r \quad (\text{ft})$	1.934

Table E-11. Centaur/Payload Separation Parameters
(Symbols are defined in Appendix D)

PARAMETER	VALUE		
	6C/SD-1 (AC-5)	3D/A21 AC-7/SURVEYOR	TYPICAL VALUES FROM REFERENCE E-10
$\bar{Y}_1$ (ft)	0.0	0.011	-0.014
$\bar{Z}_1$ (ft)	0.043	0.01	0.005
SN B (in.)	109.8	112.87	*
SN A (in.)	130.5	152.5	*
$\bar{Y}_2$ (ft)	2.3	0.056	0.198
$\bar{Z}_2$ (ft)	1.33	0.05	0.138
SN C (in.)	128.32	128.32	*
R_1 (ft)	0.667	1.33	*
R_2 (ft)	1.79	1.79	*
M_1 (slugs)	44.1	67	66.7
M_2 (slugs)	151.8	128.14	136.6
$M = \frac{M_1 M_2}{M_1 + M_2}$ (slugs)	34.1	44	44.6
$\dot{\psi}_1$ (0) (rad/sec)	681×10^{-8}	681×10^{-8}	** 25.39×10^{-8}
$\sigma_{\bar{Y}_1}^2 = \sigma_{\bar{Y}_2}^2$ (ft)	23.1×10^{-4}	23.1×10^{-4}	*** 5.79×10^{-4}
$\sigma_{\bar{Z}_1}^2 = \sigma_{\bar{Z}_2}^2$ (ft)	23.1×10^{-4}	23.1×10^{-4}	*** 5.79×10^{-4}
$I_{yy_1} = I_{zz_1}$ (slug-ft ²)	129	200	170
$I_{yy_2} = I_{zz_2}$ (slug-ft ²)	17,800	14,200	16,800

* Not given, assumed to be same as for operational - Surveyor configuration

** With 50-lb thrust vernier attitude control engines. Without these engines and with a 1.6 deg/sec rate, this would become 780×10^{-8} *** Based on an uncertainty of ± 0.5 inches. More recent data indicates that this uncertainty (3σ) is, more realistically, ± 1.0 inch

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